Extended renormalized random phase approximation

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The random phase approximation (RPA) is fully renormalized to take into account the ground-state correlations (g.s.c.) beyond the RPA, which arise due to the fermion structure of the collective RPA operators. As compared to the conventional renormalized RPA (RRPA), the novelty of the present extended renormalized RPA (ERRPA) is the inclusion of the particle-particle (pp) and hole-hole (hh) g.s.c. beyond the RPA along with the ph ones in the complete set of ERRPA equations. The formalism is illustrated by numerical calculations within a schematic four-level model. The results show that the effects of g.s.c. beyond the RPA due to pp (and hh) transitions can cause a partial or complete compensation of the ones due to ph transitions. This reduces the deviation from the RPA energy-weighted sum of strength within the ERRPA as compared to the one within the conventional RRPA.

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I. INTRODUCTION

The set of linear equations of the random phase approximation (RPA) is usually obtained within the quasiboson approximation, which treats the particle-hole (ph) pair creation $B^{\dagger}_{ph}$ and annihilation B_{ph} operators as if they were bosons. This leads to a violation of the Pauli principle. As a consequence, the RPA breaks down at a certain critical value of the interaction strength, where the RPA solution becomes imaginary. The renormalized RPA (RRPA) is an approach to correct for this inconsistency, taking into account the effect of ground-state correlations (g.s.c.) beyond the RPA (see, e.g., [1–4]). This approach has been reformulated in the recent Ref. [5], where an improvement in the estimation of the g.s.c. has been obtained compared to the approach originally proposed by Hara [1].

Recently, there has been a renewed interest in the RRPA for the application in the microscopic description of single and double β decays. Further studies of the RRPA and renormalized quasiparticle RPA (RQRPA) have been reported [6–8]. The studies of RRPA have been so far limited within the ph channel, neglecting the pp and hh interactions. In the RQRPA, this corresponds partly to the omission of the so-called scattering terms in the model Hamiltonian, whose effect is strongest for particle-particle (pp) and/or hole-hole (hh) configurations. It has been suggested that the omission of such terms may be the reason for the strong violation within the RRPA of certain model-independent sum rules, which are well fulfilled within the RPA, such as the Ikeda sum rule for the charge-exchange transitions [6].

In the present paper, we will extend the RRPA to include also the effects due to pp and hh interactions along with the ph one, and the pp and hh g.s.c. beyond the RPA. It will be shown that, with increasing the interaction, the effect of pp and hh g.s.c. also increases to be comparable with the one of ph ones, and, therefore, cannot be neglected. With increasing the pp and hh interaction, the contribution from pp and hh g.s.c. can compensate partially until completely the one coming from the ph ones. This may shed some light on the issue of the sum-rule violation within the RRPA.

The paper is organized as follows. The ERRPA is presented in Sec. II, where the complete set of the ERRPA is derived for the first time. Although the formalism is developed using the representation of real particles and holes, the extension to the quasiparticle picture is straightforward as demonstrated in the Appendix. Section III represents the application of the formalism in a schematic four-level model. The paper is summarized in the last section, where some conclusions are drawn.

II. FORMALISM

Let us consider the creation and annihilation operators of the nucleon pairs, $B^{\dagger}_{kl}=a^{\dagger}_{k}a_{l}$ and $B_{kl}=(B^{\dagger}_{lk})^{\dagger}$, respectively, with kl running over all ph, pp, and hh indices. The operators $B^{\dagger}_{kl}$ and B_{kl} satisfy the following exact commutation relation:

$$[B_{kl},B^{\dagger}_{k'l'}]=\delta_{kk'}B^{\dagger}_{ll'}-\delta_{ll'}B_{kk'}\,. \tag{1}$$

Following the standard procedure [1,5], we introduce the renormalized phonon operator

$$\begin{aligned}Q^{\dagger}_{\nu}&=\sum_{kl}\frac{1}{\sqrt{D_{kl}}}[\mathcal{X}^{(\nu)}_{kl}B^{\dagger}_{kl}-\mathcal{Y}^{(\nu)}_{kl}B_{kl}]\\&=\sum_{ph(n_h>n_p)}\frac{1}{\sqrt{D_{ph}}}[X^{(\nu)}_{ph}B^{\dagger}_{ph}-Y^{(\nu)}_{ph}B_{ph}]\\&\quad+\sum_{ss'(n_{s'}>n_s)}\frac{1}{\sqrt{D_{ss'}}}[x^{(\nu)}_{ss'}B^{\dagger}_{ss'}-y^{(\nu)}_{ss'}B_{ss'}]\end{aligned} \tag{2}$$

to generate the ''excited state'' of the system

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$$|\nu\rangle = Q_\nu^\dagger|0\rangle, \tag{3}$$

where $|0\rangle$ is the (correlated) ground-state (phonon vacuum)

$$Q_\nu|0\rangle = 0. \tag{4}$$

The phonon operator (2) is different from the conventional RRPA one by the presence of the pp and hh configurations ($ss'=pp'$ or hh') in the second sum beside the ph ones ($kl=ph$) in the first sum at the right-hand side (rhs) of Eq. (2). Hereafter, we use the indices ph to denote the ph configurations with the amplitudes $\mathcal{X}_{ph}^{(\nu)}=X_{ph}^{(\nu)}$ and $\mathcal{Y}_{ph}^{\nu}=Y_{ph}^{\nu}$, while ss' denote the pp and hh configurations with the amplitudes $\mathcal{X}_{ss'}^{(\nu)}=x_{ss'}^{\nu}$ and $\mathcal{Y}_{ss'}^{(\nu)}=y_{ss'}^{\nu}$.

The procedure of incorporating the g.s.c. beyond RPA suggests replacing the exact commutator (1) by its expectation value (in the diagonal approximation) in the correlated ground state $|0\rangle$ to obtain

$$[B_{kl},B_{k'l'}^\dagger] \simeq \delta_{kk'}\delta_{ll'}D_{kl} = \delta_{kk'}\delta_{ll'}(n_l-n_k), \tag{5}$$

with $n_j=\langle 0|a_j^\dagger a_j|0\rangle$ being the particle ($j=p$) or hole ($j=h$) occupation number. Within the RPA, the ground state $|0\rangle$ is replaced by the Hartree-Fock (HF) ground state $|\mathrm{HF}\rangle$, for which $n_h=1$ and $n_p=0$ so that $D_{ph}=1$ and $D_{ss'}=0$ ($ss'=pp'$ or hh'). In this case Eq. (1) becomes a boson commutation relation. This approximation is called the quasiboson approximation (QBA). The fact that D_{kl} are not 1 or 0 means that there are correlations in the ground state $|0\rangle$ beyond the QBA. The RRPA partly takes these g.s.c. into account, considering $D_{ph}\neq 1$. This part belongs to the ph g.s.c. beyond the RPA. In fact, D_{ph} is defined within the RRPA by solving a set of nonlinear self-consistent RPA-like equations for the amplitudes $X_{ph}^{(\nu)}$ and $Y_{ph}^{(\nu)}$, the phonon energy ω_ν and the factor D_{ph} [5]. These RRPA equations were obtained using the equation-of-motion method with replacing the exact commutator (1) for $kl=ph$ with the ground-state expectation value (5). The RRPA, however, neglects the factor

$$D_{ss'} = n_{s'} - n_s \quad (ss'=pp' \;\; or \;\; hh'), \tag{6}$$

which arises due to the pp and hh g.s.c. beyond the RPA, even though the latter may be comparable with D_{ph}.

In the present section we will derive a self-consistent set of the extended RRPA (ERRPA) equations, where the contributions from all ph as well as pp and hh g.s.c. are taken into account. For this purpose we consider the following general model Hamiltonian with separable residual interactions:

$$H=\sum_s \epsilon_s B_{ss}^\dagger - \frac{1}{4}kF^2, \quad F=\sum_{kl} f_{kl}(B_{kl}^\dagger + B_{kl}). \tag{7}$$

The first term at the rhs of the definition of H in Eq. (7) describes the motion of independent nucleons with single-particle energies $\epsilon_s \equiv E_s - E_F$ (E_F is the Fermi energy) within the mean field. A level with $\epsilon_s > 0$ corresponds to a particle (p) energy, while $\epsilon_s<0$ is a hole (h) energy. The second term of Eq. (7) includes the interactions in both ph (kl and $k'l'=ph$) and pp (or hh) (kl and $k'l'=pp$ or hh) channels as well as the interference terms between them. Within the RPA or the RRPA, the factor $D_{ss'}$ [Eq. (6)] is neglected, so the terms corresponding to pp and hh interactions in the Hamiltonian (7) do not contribute. It is also worth noticing that, so long as the g.s.c. beyond the RPA is taken into account ($n_h\neq 1$ and $n_p\neq 0$), $D_{ss'}$ ($s\neq s'$) in Eq. (6) is not zero even when the pp and hh interactions in the Hamiltonian (7) are neglected. However, as we will see later, $D_{ss'}$ will not contribute to the ERRPA equations when the pp and hh interactions are zero, or omitted. As a matter of fact, both $D_{ss'}$ and the pp or/and hh interactions have been omitted in a number of papers, where the RRPA was studied. The ERRPA does not neglect any term in Eq. (7). The introduction of the separable interaction F in Eq. (7) allows us to derive the set of ERRPA equations in analytic form. The generality of the conclusions is, however, not affected by this choice. For simplicity, we do not distinguish proton and neutron systems, as well as the isoscalar and isovector parts of the interaction parameter k either. The extension to such details is, however, straightforward.

Following the same procedure as in Ref. [5] and making use of Eq. (5), we come to the following approximated commutation relations:

$$[H,B_{ph}]=(\epsilon_h-\epsilon_p)B_{ph}+kf_{ph}D_{ph}\hat{M}, \tag{8}$$

$$[H,B_{ss'}]=(\epsilon_{s'}-\epsilon_s)B_{ss'}+kf_{ss'}D_{ss'}\hat{M}, \tag{9}$$

where

$$\hat{M}=\sum_{ph} f_{ph}(B_{ph}^\dagger + B_{ph}) + \sum_{ss'} f_{ss'}(B_{ss'}^\dagger + B_{ss'}). \tag{10}$$

Applying Eqs. (8)–(10) in the equation-of-motion method [2], we find that the amplitudes X, Y, x, and y satisfy the following ERRPA equations:

$$[(\epsilon_h-\epsilon_p)+\omega_\nu]X_{ph}^{(\nu)}+kf_{ph}\sqrt{D_{ph}}M_\nu=0, \tag{11}$$

$$[(\epsilon_h-\epsilon_p)-\omega_\nu]Y_{ph}^{(\nu)}+kf_{ph}\sqrt{D_{ph}}M_\nu=0, \tag{12}$$

$$[(\epsilon_{s'}-\epsilon_s)+\omega_\nu]x_{ss'}^{(\nu)}+kf_{ss'}\sqrt{D_{ss'}}M_\nu=0, \tag{13}$$

$$[(\epsilon_{s'}-\epsilon_s)-\omega_\nu]y_{ss'}^{(\nu)}+kf_{ss'}\sqrt{D_{ss'}}M_\nu=0, \tag{14}$$

where

$$M_\nu=\sum_{ph} f_{ph}\sqrt{D_{ph}}(X_{ph}^{(\nu)}+Y_{ph}^{(\nu)})+\sum_{ss'} f_{ss'}\sqrt{D_{ss'}}(x_{ss'}^{(\nu)}+y_{ss'}^{(\nu)}). \tag{15}$$

Using Eqs. (2) and (5), we also find that the ERRPA phonon operators Q_ν and $Q_\nu^\dagger$ satisfy the boson commutation relation

$$[Q_\nu,Q_{\nu'}^\dagger]=\delta_{\nu\nu'}, \tag{16}$$

if the amplitudes X, Y, x, and y fulfill the orthonormalization condition

$$\sum_{kl} (\mathcal{X}_{kl}^{(\nu)}\mathcal{X}_{kl}^{(\nu')}-\mathcal{Y}_{kl}^{(\nu)}\mathcal{Y}_{kl}^{(\nu')})$$

$$=\sum_{ph} (X_{ph}^{(\nu)}X_{ph}^{(\nu')}-Y_{ph}^{(\nu)}Y_{ph}^{(\nu')})+\sum_{ss'} (x_{ss'}^{(\nu)}x_{ss'}^{(\nu')}-y_{ss'}^{(\nu)}y_{ss'}^{(\nu')})$$

$$=\delta_{\nu\nu'} . \tag{17}$$

From Eqs. (11)–(14) we obtain the explicit form of the phonon amplitudes as

$$X_{ph}^{(\nu)}=\frac{kf_{ph}\sqrt{D_{ph}}}{\epsilon_p-\epsilon_h-\omega_\nu}M_\nu, \quad Y_{ph}^{(\nu)}=\frac{kf_{ph}\sqrt{D_{ph}}}{\epsilon_p-\epsilon_h+\omega_\nu}M_\nu, \tag{18}$$

$$x_{ss'}^{(\nu)}=\frac{kf_{ss'}\sqrt{D_{ss'}}}{\epsilon_s-\epsilon_{s'}-\omega_\nu}M_\nu, \quad y_{ss'}^{(\nu)}=\frac{kf_{ss'}\sqrt{D_{ss'}}}{\epsilon_s-\epsilon_{s'}+\omega_\nu}M_\nu. \tag{19}$$

Replacing X, Y, x, and y at the lhs of Eq. (17) with their expressions from Eqs. (18) and (19), we derive the expression for M_ν as

$$M_\nu=\frac{1}{k}\Bigg\{\sum_{ph} f_{ph}^2 D_{ph}\Bigg[\frac{1}{(\epsilon_p-\epsilon_h-\omega_\nu)^2}-\frac{1}{(\epsilon_p-\epsilon_h+\omega_\nu)^2}\Bigg] +\sum_{ss'} f_{ss'}^2 D_{ss'}\Bigg[\frac{1}{(\epsilon_s-\epsilon_{s'}-\omega_\nu)^2} -\frac{1}{(\epsilon_s-\epsilon_{s'}+\omega_\nu)^2}\Bigg]\Bigg\}^{-1/2}. \tag{20}$$

Doing the same for the rhs of Eq. (15) and eliminating M_ν from both sides, we obtain the following secular equation for the phonon energy ω_ν:

$$1+2k\Bigg[\sum_{ph} \frac{f_{ph}^2 D_{ph}(\epsilon_h-\epsilon_p)}{(\epsilon_h-\epsilon_p)^2-\omega_\nu^2}+\sum_{ss'} \frac{f_{ss'}^2 D_{ss'}(\epsilon_{s'}-\epsilon_s)}{(\epsilon_{s'}-\epsilon_s)^2-\omega_\nu^2}\Bigg]=0. \tag{21}$$

The ERRPA equations (11)–(14) and the secular equation (21) must be solved self-consistently with the equations for the factors D_{ph} and $D_{ss'}$. The latter are derived as follows.

Using the inverse transformation of Eq. (2)

$$B_{kl}^\dagger=\sqrt{D_{kl}}\sum_\nu (\mathcal{X}_{kl}^{(\nu)}Q_\nu^\dagger+\mathcal{Y}_{kl}^{(\nu)}Q_\nu), \tag{22}$$

we can express $B_{ph}^\dagger$ $(kl=ph)$ in terms of phonon operators as $Q_\nu^\dagger$ and Q_ν:

$$B_{ph}^\dagger=\sqrt{D_{ph}}\sum_\nu (X_{ph}^{(\nu)}Q_\nu^\dagger+Y_{ph}^{(\nu)}Q_\nu). \tag{23}$$

In order to compare the effects of g.s.c. within the ERRPA with the ones within the RRPA we need to evaluate all the factors D_{ph}, $D_{pp'}$, and $D_{hh'}$ in terms of the backward-going ph amplitude $Y_{ph}^{(\nu)}$. For this purpose we apply Eq. (23) and the mappings used in Ref. [5]

$$B_{pp'}^\dagger\rightarrow\sum_h B_{ph}^\dagger B_{p'h}, \quad B_{hh'}^\dagger\rightarrow\delta_{hh'}-\sum_p B_{ph'}^\dagger B_{ph} \tag{24}$$

to calculate $D_{ph}\equiv\langle 0|B_{hh}^\dagger|0\rangle-\langle 0|B_{pp}^\dagger|0\rangle$ and $D_{ss'}\equiv\langle 0|B_{s's'}^\dagger|0\rangle-\langle 0|B_{ss}^\dagger|0\rangle$. As a result, we obtain

$$D_{ph}=1-\sum_\nu \Bigg\{\sum_{p'} D_{p'h}[Y_{p'h}^{(\nu)}]^2+\sum_{h'} D_{ph'}[Y_{ph'}^{(\nu)}]^2\Bigg\}, \tag{25}$$

$$D_{pp'}=\sum_\nu \sum_h \Bigg\{D_{p'h}\sum_\nu [Y_{p'h}^{(\nu)}]^2-D_{ph}[Y_{ph}^{(\nu)}]^2\Bigg\}, \tag{26}$$

$$D_{hh'}=\sum_\nu \sum_p \Bigg\{D_{ph'}\sum_\nu [Y_{ph'}^{(\nu)}]^2-D_{ph}[Y_{ph}^{(\nu)}]^2\Bigg\}. \tag{27}$$

Expression (25) for D_{ph} is the same as has been derived in Ref. [5]. The set of Eqs. (11)–(14) and Eqs. (25)–(27) is the complete set of self-consistent and nonlinear ERRPA equations. If the pp and hh g.s.c. or/and the pp and hh interactions are neglected, Eqs. (13), (14), (19), (26), and (27) vanish. In this case, one recovers the set of the conventional RRPA equations (cf. [5]). The secular equation (21) is reduced accordingly to the one for the RRPA phonon energy, in which the last sum at the lhs of Eq. (21) is absent.

With the solution of the ERRPA equations (11)–(14) and (25)–(27), we can calculate the matrix element of any one-body operator that generates an electromagnetic transition from the ground state $|0\rangle$ to the one-phonon state $|\nu\rangle$, and, consequently all the moments of the transition. Let us consider the same one-body operator F, which forms the separable interaction in Hamiltonian (7). Using the inverse transformation (22) and the definition (4) of the ground state, we obtain easily the matrix element $\langle\nu|F|0\rangle$ in the form

$$\langle\nu|F|0\rangle=\sum_{ph} f_{ph}\sqrt{D_{ph}}[X_{ph}^{(\nu)}+Y_{ph}^{(\nu)}] +\sum_{pp'} f_{pp'}\sqrt{D_{pp'}}[x_{pp'}^{(\nu)}+y_{pp'}^{(\nu)}] +\sum_{hh'} f_{hh'}\sqrt{D_{hh'}}[x_{hh'}^{(\nu)}+y_{hh'}^{(\nu)}]. \tag{28}$$

The k moment of the electromagnetic transition is then proportional to

$$m_k=\sum_\nu |\langle\nu|F|0\rangle|^2\omega_\nu^k. \tag{29}$$

For the dipole transitions, the energy-weighted sum (EWS) m_1 is proportional to the model-independent Thomas-Reich-Kuhn (TRK) sum rule if the phonon energy ω_ν, the amplitudes $X_{ph}^{(\nu)}$ and $Y_{ph}^{(\nu)}$ are calculated within the RPA, where

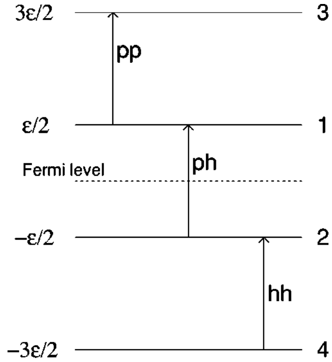

FIG. 1. The schematic model used in the present numerical calculations.

$D_{ph}=1$ and $D_{ss'}=0$. The zero moment is proportional to the non-energy-weighted sum (NEWS) m_0.

Finally, in view of a renewed interest in the RQRPA using the ph interactions [6–8], we derive the set of ERQPA equations in the Appendix.

III. NUMERICAL ILLUSTRATION WITHIN A SCHEMATIC MODEL

In this section we test the ERRPA within a schematic model and compare the results with the ones of the RPA and the RRPA. The simplest model for this purpose is a four-level model, in which the ERRPA equations can be solved analytically.

The model consists of four equidistant levels, which are symmetrically situated from both sides of the Fermi level, as shown in Fig. 1. The distance between the levels is equal to ϵ. Therefore, the energies of levels 1 and 3, which are above the Fermi level, are $\epsilon/2$ and $3\epsilon/2$, respectively. The levels 2 and 4 below the Fermi level have the energies equal to $-\epsilon/2$ and $-3\epsilon/2$, respectively. We assume, furthermore, that only neighbor levels interact with each other via the following matrix elements f_{kl} in Eq. (7): $f_{21}=f_1$, $f_{13}=f_{42}=f_2$. Hence, f_1 is the ph matrix element, and f_2 is the pp (or hh) matrix element.

The g.s.c. factors D_{ph} from Eq. (25) and $D_{ss'}$ from Eqs. (26) and (27) have the simple form

$$D_{21}=\frac{1}{1+2Y_{21}^2}, \quad D_{42}=D_{13}=D_{21}Y_{21}^2. \tag{30}$$

In Eq. (30) the factor D_{21} has the same form as has been derived within the Lipkin model in [5]. Using Eq. (30), we obtain the analytic solution of the secular equation (21) as

$$\omega=\epsilon\sqrt{1-2k\frac{f_1^2}{\epsilon}\frac{\left[1+2\left(\frac{f_2}{f_1}\right)^2 Y_{21}^2\right]}{1+2Y_{21}^2}}. \tag{31}$$

The amplitudes X, Y, x, and y are found as

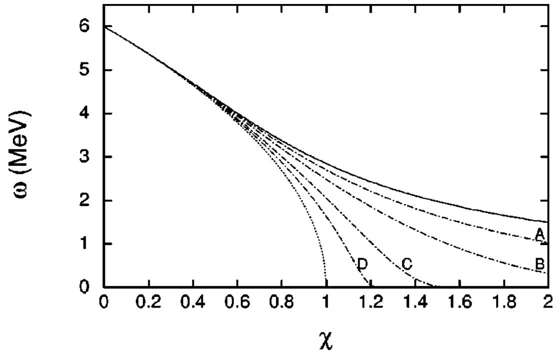

FIG. 2. Energy ω of the excited state $|\nu\rangle$ in the schematic model of Fig. 1 as a function of the strength χ. The ERRPA solution obtained with $f_2=f_1$ is plotted as a dotted line, which coincides with the RPA solution. The ERRPA solution obtained with $f_2=0$ (the conventional RRPA one) is plotted with a solid line, while the ERRPA solutions obtained with $f_2/f_1=0.2$, 0.3, 0.4, and 0.45 are plotted as dot-dashed lines marked with A, B, C, and D, respectively.

$$X_{21}=\frac{kf_1\sqrt{D_{21}}}{\epsilon-\omega}M, \quad Y_{21}=\frac{kf_1\sqrt{D_{21}}}{\epsilon+\omega}M, \tag{32}$$

$$x_{42}=\frac{kf_2\sqrt{D_{42}}}{\epsilon-\omega}M=x_{13}, \quad y_{42}=\frac{kf_2\sqrt{D_{42}}}{\epsilon+\omega}M=y_{13}, \tag{33}$$

with

$$M=\frac{1}{2kf_1}\frac{\epsilon^2-\omega^2}{\sqrt{\epsilon\omega}}\sqrt{\frac{1+2Y_{21}^2}{1+2\left(\frac{f_2}{f_1}\right)^2 Y_{21}^2}}. \tag{34}$$

Equations (31)–(34) have two limits. The first one takes place when the $pp(hh)$ interaction f_2 is zero. In this case Eqs. (31)–(34) are reduced to the RRPA solution, as has been mentioned in the previous section. The second limit is reached when the $pp(hh)$ interaction f_2 becomes equal to the ph one, f_1. In this case, from Eqs. (31)–(34), one recovers the usual RPA solution. Therefore, this schematic model shows that when the $pp(hh)$ interaction f_2 increases from 0 to the value of the ph one (f_1) the total effects of g.s.c. beyond the RPA will decrease until they vanish completely when $f_2=f_1$, where the solution ω again breaks down at the same critical value of the interaction as of the RPA.

The numerical calculations have been carried out using $\epsilon=6.0$ MeV and $f_1=0.5$ MeV at several values of f_2 $(0\leqslant f_2\leqslant f_1)$ with varying the interaction parameter k (in MeV^{-1}). The obtained results are plotted below as a function of the dimensionless interaction parameter $\chi=2kf_1^2/\epsilon$, which will be referred to as the strength hereinafter.

The energy ω of the one-phonon state $|\nu\rangle$ is displayed in Fig. 2 as a function of the strength χ at several values of f_2. With increasing the $pp(hh)$ interaction f_2 from 0 to the value f_1 of the ph one, the energy ω decreases to reach the RPA value at $f_2=f_1$. The secular equation (31) for the ERRPA energy starts to break down at a critical strength χ_c, where the expression under the square root in Eq. (31) van-

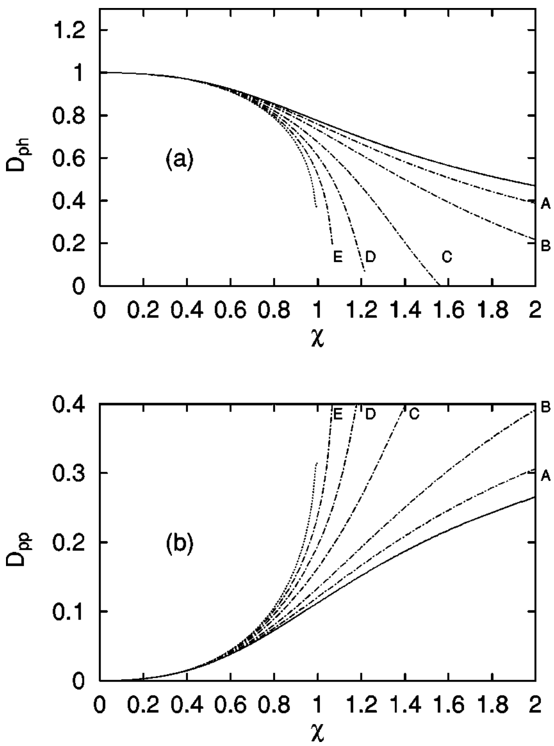

FIG. 3. The g.s.c. factors in the schematic model of Fig. 1 as a function of the strength χ. Figure 3(a) displays the ph g.s.c. factor D_{ph}; Fig. 33(b) shows the $pp(hh)$ g.s.c. factor $D_{pp}=D_{hh}$ (b). The notations of the curves are the same as in Fig. 2. The curve marked with E is obtained with $f_2/f_1=0.48$.

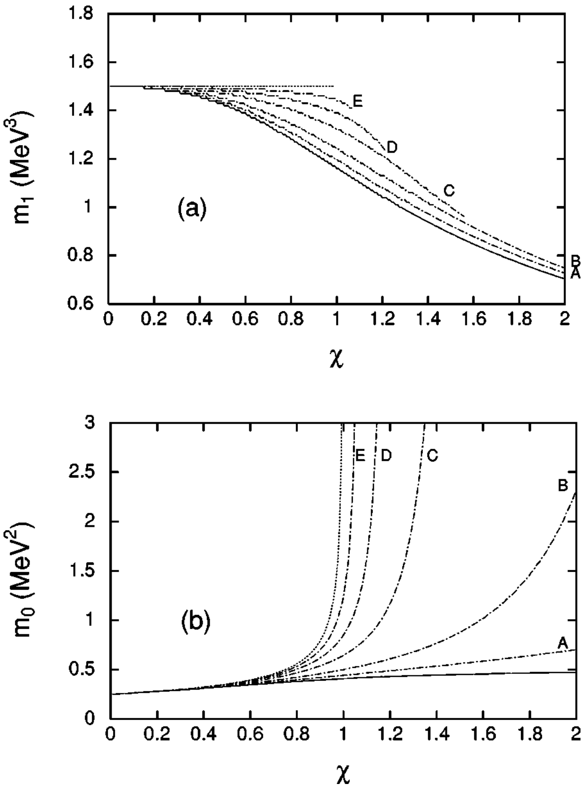

FIG. 4. The sums of transition strengths generated by the operator F from the ground state $|0\rangle$ to the excited state $|\nu\rangle$ in the schematic model of Fig. 1 as a function of the strength χ. Figure 4(a) displays the EWS m_1; Fig. 4(b) shows the NEWS m_0. The notations of the curves are the same as in Fig. 3.

ishes. The value of χ_c decreases from ∞ to 1 as increasing f_2 from 0 to f_1. The value $\chi_c=1$ is the critical value at which the RPA breaks down. The fact that the ERRPA solution is approaching the RPA one with increasing the $pp(hh)$ strength means that the total effect of the g.s.c. beyond the RPA on the energy is decreasing to a point where it completely vanishes when $f_2=f_1$. At this point the ERRPA solution is identical to the RRPA one.

Both of the ph and $pp(hh)$ g.s.c. beyond RPA themselves increase with increasing the strength χ. This is clearly shown in Fig. 3, where the deviation from 1 of the ph g.s.c. D_{ph} [Fig. 3(a)] and the deviation from 0 of the $pp(hh)$ factor $D_{pp}=D_{hh}$ [Fig. 3(b)] become stronger when χ and f_2 increase. However, as D_{ph} decreases while D_{pp} increases with increasing χ and f_2, a compensation takes place leading to a complete cancellation between the contribution due to the ph g.s.c. and the one due to the $pp(hh)$ g.s.c. at $f_2=f_1$, where the RPA solution is restored (dotted curve in Fig. 2).

The EWS m_1 and NEWS m_0 of strengths of the transition between the ground state $|0\rangle$ and the excited state $|\nu\rangle$ are plotted as a function of χ at several values of f_2 in Figs. 4(a) and 4(b), respectively. In the RPA, the EWS is equal to 1.5 MeV^3 independently of the strength χ within the whole interval of χ where the RPA is valid [dotted curve in Fig. 4(a)]. The value of m_1 is not conserved in RRPA [solid curve in Fig. 4(a)]. It strongly decreases with increasing χ. Including the contribution from pp and hh g.s.c., the deviation of m_1 from the RPA value becomes weaker with increasing f_2. This is the result of the sum of the products between ω (Fig. 2) and m_0 [Fig. 4(b)], which approach the values within the RPA when f_2 is approaching f_1. As has been discussed above, the ERRPA solution still breaks down but at a value of χ_c that is higher than the one in RPA.

Finally, we would like to point out that the ERRPA equations derived in the present work is formally identical to the equations of the finite-temperature RPA (FT-RPA) [9,10]. The difference is in the physics which make the factors D_{ph} and $D_{ss'}$ differ from 1 and 0, respectively. Within the FT-RPA, the single-particle occupation numbers n_p and n_h, which enter in the definition of D_{ph} and $D_{ss'}$, are derived from the Fermi-Dirac statistics after the average over the grand canonical ensemble at a temperature T (see, e.g., [11]). In [9], it has been shown that the EWS of the dipole strength decreases with increasing the temperature when only the contribution from ph configurations are taken into account. However, the fulfillment of the TRK sum rule is practically restored after the inclusion of the pp and hh configurations in the FT-RPA equations along with the ph ones.

IV. CONCLUSIONS

In the present work, we have extended the RRPA to include also the the effects due to the pp and hh interactions and pp and hh g.s.c. beyond the RPA along with the ph ones. A complete set of nonlinear ERRPA equations have been derived using the equation-of-motion method. The formalism has been illustrated by numerical calculations using a schematic four-level model, in which the secular equation for the one-phonon energy has an analytic solution.

The analysis of the numerical calculations show that the contribution of the pp and hh g.s.c. beyond the RPA to the phonon energy, EWS and NEWS of the electromagnetic transition strengths can be important if the pp and hh interactions are comparable with the ph ones. The increase of the pp and hh interactions can lead to a partial or complete compensation of the total effect due to g.s.c. beyond RPA. When the pp and hh interactions are smaller than the ph ones, the ERRPA solution breaks down at a critical value of the interaction parameter which is higher than the one in the RPA. The deviation of the EWS sum of strength within the ERRPA from the interaction-independent RPA value is smaller than the one within the RRPA. This deviation also decreases with increasing the pp and hh interactions.

We conclude that, for the systems where the pp and hh interactions are not negligible, the pp and hh g.s.c. must be taken into account. In this case the ERRPA formalism developed here should be used instead of the conventional RRPA.

APPENDIX: THE EXTENDED RENORMALIZED QRPA (ERQRPA) USING PARTICLE-HOLE INTERACTIONS

As has been discussed in Sec. II, the pp and hh g.s.c. exist even when the pp and hh interactions are neglected because the factor $D_{ss'}=n_{s'}-n_s$ ($\{ss'\}=\{pp'\}$ or $\{hh'\}$) does not vanish. This is, indeed, the case when g.s.c. beyond the QRPA are considered. However, all the studies within the RQRPA have neglected either the so-called scattering terms in the Hamiltonian, which contains the sum of products of two creation-annihilation quasiparticle pairs $\alpha^\dagger\alpha$, or have just ignored the commutator between these scattering quasiparticle operators. The aim of the present appendix is to derive the set of ERQRPA equations, where the effect of g.s.c. due to these scattering terms is included.

The complete model Hamiltonian under consideration within the QRPA can be written in the form:

$$H_{\rm QRPA}=\sum_{jm} E_j\alpha^\dagger_{jm}\alpha_{jm}-\frac{1}{2}\sum_{\lambda\mu} k^{(\lambda)}M^\dagger_{\lambda\mu}M_{\lambda\mu}. \quad (A1)$$

This is the standard quasiparticle Hamiltonian for the one-component system described by the quasiparticle creation $\alpha^\dagger$ and annihilation α operators and the separable multipole interaction in terms of the multipole operator $M_{\lambda\mu}$:

$$M^\dagger_{\lambda\mu}=\frac{(-)^{\lambda-\mu}}{\sqrt{\lambda+\mu}}\sum_{jj'} f^{(\lambda)}_{jj'}\times\left\{\frac{1}{2}u^{(+)}_{jj'}[A^\dagger_{\lambda\mu}(jj')+A_{\lambda\tilde{\mu}}(jj')]+v^{(-)}_{jj'}B_{\lambda\mu}(jj')\right\}. \quad (A2)$$

In Eq. (A2) $f^{(\lambda)}_{jj'}=\langle j'||iR_\lambda(r)Y_\lambda||j\rangle$ is the single-particle matrix element corresponding to the separable multipole interaction in the ph channel; $u^{(+)}_{jj'}=u_jv_{j'}+u_{j'}v_j$ and $v^{(-)}_{jj'}=u_ju_{j'}-v_jv_{j'}$ are the combinations of the u_j and v_j coefficients of the Bogolyubov transformation, which expresses particles with energies ϵ_s in terms of the quasiparticles with energies $E_j=\sqrt{(\epsilon_j-\epsilon_F)^2+\Delta^2}$ in a system with a superfluid pairing gap Δ. The creation quasiparticle-pair $A^\dagger_{\lambda\mu}(jj')$ and the scattering quasiparticle $B_{\lambda\mu}(jj')$ operators are defined as

$$A^\dagger_{\lambda\mu}(jj')=\sum_{mm'}\langle jmjm'|\lambda\mu\rangle\alpha^\dagger_{jm}\alpha_{j'm'},$$

$$B_{\lambda\mu}(jj')=-\sum_{mm'}\langle jmjm'|\lambda\mu\rangle\alpha^\dagger_{jm}\alpha_{j'\tilde{m}'}, \quad (A3)$$

where $\alpha_{j\tilde{m}}=(-)^{j-m}\alpha_{jm}$. If there is no pairing ($\Delta=0$), we have $u_p=1$, $u_h=0$, $v_p=0$, and $v_h=1$. Hence, the quasiparticle-pair operators $A^\dagger_{\lambda\mu}(jj')$ and $A_{\lambda\mu}(jj')$ change to the ph pair operators $B^\dagger_{ph}$ and B_{ph}, while the quasiparticle scattering operators $B_{\lambda\mu}(jj')$ and $B^\dagger_{\lambda\mu}(jj')$ change to the pp and/or hh operators $B^\dagger_{ss'}$ and $B_{ss'}$.

We introduce the ERQRPA phonon operator similarly to Eq. (2)

$$Q^\dagger_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\left\{\frac{1}{\sqrt{D^A_{jj'}}}[X^{(\lambda i)}_{jj'}A^\dagger_{\lambda\mu}(jj')-Y^{(\lambda i)}_{jj'}A_{\lambda\tilde{\mu}}(jj')]+\frac{1}{\sqrt{D^B_{jj'}}}[x^{(\lambda i)}_{jj'}B_{\lambda\mu}(jj')-y^{(\lambda i)}_{jj'}B^\dagger_{\lambda\tilde{\mu}}(jj')]\right\}, \quad (A4)$$

and define the ground state $|0\rangle$ as the vacuum of the phonon operator (A4) as $Q_{\lambda\mu i}|0\rangle=0$. Following the same approximation as in Eqs. (5) and (6), we replace the commutators $[A,A^\dagger]$ and $[B^\dagger,B]$ with their expectation values in the ground state as

$$[A_{\lambda\mu}(jj'),A^\dagger_{\lambda'\mu'}(j_1j_1')]\simeq\delta_{\lambda\lambda'}\delta_{\mu\mu'}[\delta_{jj_1}\delta_{j'j_1'}-(-)^{j+j'-\lambda}\delta_{jj_1'}\delta_{j'j_1}]D^A_{jj'}, \quad (A5)$$

$$[B^\dagger_{\lambda\mu}(jj'),B_{\lambda'\mu'}(j_1j_1')]\simeq\delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{jj_1}\delta_{j'j_1'}D^B_{jj'}. \quad (A6)$$

The g.s.c. factors $D^A_{jj'}=1-n_j-n_{j'}$ and $D^B_{jj'}=n_{j'}-n_j$, where $n_j=\langle 0|\alpha^\dagger_{jm}\alpha_{jm}|0\rangle$ is the quasiparticle occupation number, are evaluated in the same way as in Eqs. (25)–(27), namely

$$D^A_{jj'}=1-\sum_{\lambda i}\sum_{j''}\{D^A_{j''j'}[Y^{(\lambda i)}_{j''j'}]^2+D^A_{jj''}[Y^{(\lambda i)}_{jj''}]^2\}, \quad (A7)$$

$$D^B_{jj'}=\sum_{\lambda i}\sum_{j''}\{D^A_{j'j''}[Y^{(\lambda i)}_{j'j''}]^2-D^A_{jj''}[Y^{(\lambda i)}_{jj''}]^2\}. \quad (A8)$$

The ERQRPA secular equation for the phonon energy has the form

$$\frac{1}{k^{(\lambda)}}=\frac{1}{2\lambda+1}\sum_{jj'}[f^{(\lambda)}_{jj'}]^2\times\left[\frac{u^{(+)}_{jj'}D^{A}_{jj'}(E_j+E_{j'})}{(E_j+E_{j'})^2-\omega^2}-\frac{v^{(-)}_{jj'}D^{B}_{jj'}(E_j-E_{j'})}{(E_j-E_{j'})^2-\omega^2}\right]. \tag{A9}$$

The amplitudes X, Y, x, and y are evaluated following the same expressions as in Eqs. (18)–(20), replacing k with $k^{(\lambda)}$, f_{ph} with $f^{(\lambda)}_{jj'}u^{(+)}_{jj'}$, $f_{ss'}$ with $f^{(\lambda)}_{jj'}v^{(-)}_{jj'}$, D_{ph} with $D^{A}_{jj'}$, $D_{ss'}$ with $D^{B}_{jj'}$, $(\epsilon_p-\epsilon_h)$ with $(E_j+E_{j'})$, $(\epsilon_s-\epsilon_{s'})$ with $(E_j-E_{j'})$, (ν) with (λi), and (ph) and (ss') with jj'.

In Refs. [1,6–8], the scattering term $\sim v^{(-)}_{jj'}B_{\lambda\mu}(jj')$ in the Hamiltonian (A1) is neglected, therefore the factor $D^{B}_{jj'}$ has no contribution in Eqs. (A9), and x and y vanish. Reference [12] ignores the commutator (A6), therefore the last term at the rhs of Eq. (A9) is missing.

We conclude that for a complete study of the g.s.c. beyond the QRPA the ERQRPA equations derived in this Appendix should be used instead of the conventional RQRPA ones.

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