Evolution of the hot giant dipole resonance as a manifestation of the order-and-chaos coexistence

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The onset of chaos in the characteristics of the hot giant dipole resonance (GDR) is studied through the evolution of the hot GDR as a function of temperature and time. The strength distribution, the time correlation function, and the logarithm of the total number of transitions of magnitude smaller than a given threshold value have been calculated using the phonon-damping model for the hot GDR in ^{120}Sn. The results show that the existence of the hot GDR, even at high temperatures, is a manifestation of the coexistence of order and chaos in highly excited nuclei. [S0556-2813(99)01009-2]

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I. INTRODUCTION

Collective motion (order) [1] and nuclear chaotic dynamics [2] are two fundamental features of the atomic nucleus that seem to be diametrically opposed to each other. An interesting example of the coexistence of order and chaos in finite nuclear systems is the existence of various giant resonances. They carry a large fraction of the total strength and are located far above the ground state (g.s.) in an energy region that is expected to be dominated by chaotic dynamics. The best-known one among them is the giant dipole resonance (GDR), which is built on the ground-state or in highly excited (hot) compound nuclei [3]. A GDR is a coherent motion of all protons against neutrons, which is described by a collective particle-hole (ph) vibrational mode (the GDR phonon) in the nucleus. Such a special state is characterized in experiments by a large dipole moment, which corresponds to a strong collective γ decay. The mixing of the GDR with the dense background of incoherent complicated states leads to the fragmentation of the GDR over all available configurations, which form the fine structure of the GDR strength function. As a result the GDR acquires a width. In a fully microscopic description, the width of the g.s. GDR is mainly the spreading width $\Gamma^{\downarrow}$ due to mixing of $1p1h$ configurations with more complicated ones such as $2p2h$ configurations, etc. As such, the spreading with $\Gamma^{\downarrow}$ is caused by pure quantal effects (the quantal width). The increase of the excitation energy E^* enhances the interparticle collisions. This makes the time necessary for the compound nucleus to reach thermal equilibrium become much shorter than the time of the nuclear deexcitation. The initial intrinsic energy E^* is then redistributed equally over all available degrees of freedom, and the compound nucleus is characterized thermodynamically by a temperature *T*. This is the limit where many-body chaos are expected to be fully developed. At $T\neq 0$ the channel for pp and hh configurations is open due to the deformation of the Fermi surface. The coupling of the GDR to pp and hh configurations at $T\neq 0$ induces the thermal damping width Γ_T [4]. It has been shown in Refs. [5,6], for the first time in realistic nuclei, that this makes the total full width at the half maximum (FWHM) of the GDR increase with excitation energy E^* (temperature *T*) up to some moderate value of E^* ($\sim$130 MeV in tin isotopes) [7–10] and saturates thereafter [11,12]. The domination of the coupling to noncollective pp and hh configurations allows us to discuss about many-body chaos as a stochastic limit of dynamics where the incoherent collisionlike processes play the major role [13]. The observation of the GDR in highly exited nuclei (the hot GDR) and its behavior as a function of temperature are, therefore, one of the clear evidences of the coexistence of order in chaos at nonzero temperature. The evolution of the hot GDR as a function of excitation energy E^* (temperature *T*) is ultimately related to the evolution of nuclear dynamics from regular to chaotic motion.

In the present paper we will attempt to shed some light on the issue of the order-to-chaos transition or the coexistence of order and chaos in the phenomenon of hot GDR. This study is based on two well-recognized conjectures, according to which, (i) the collectivity (regularity) is manifest as an enhancement of the strength function, while (ii) a classically chaotic system is characterized by the fluctuations of the Gaussian orthogonal ensemble (GOE) (see, e.g., Ref. [14]). The latter corresponds to a Gaussian distribution for the shape of the GDR in the strong coupling limit instead of a Lorentzian one at low excitation energies [15]. The basic question, to which we will try to give an answer, is whether the saturation of the GDR width can serve as a signature of the onset of chaos or whether the hot GDR can be still considered as regular motion, but embedded in chaos. This can also help to clarify if the GDR still persists at that high temperature as $T\sim 5-6$ MeV. Throughout the paper we will follow the phonon damping model (PDM), which has been proposed and applied successfully by us in Refs. [5,6] to describe the behavior of the hot GDR as a function of temperature. We will use the strength function and the correlation function calculated within the PDM to search for the onset of chaos in the hot GDR.

The paper is organized as follows. In Sec. II, the outline of the PDM is presented and applied to the calculation of the

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correlation function. In Sec. III, the numerical results for the strength distribution and correlation function of the hot GDR in ^{120}Sn are discussed. Conclusions are drawn in the last section.

II. FORMALISM

This section represents only the outline of the PDM, which is necessary for the application in the present paper. Details of the model are given in Refs. [5,6].

The PDM uses a model Hamiltonian that is composed of three terms

$$H=\sum_s E_s a_s^\dagger a_s+\sum_q \omega_q Q_q^\dagger Q_q+\sum_{ss'q} F_{ss'}^{(q)} a_s^\dagger a_{s'}(Q_q^\dagger+Q_q). \tag{2.1}$$

The first term is the single-particle field, where $a_s^\dagger$ and a_s are creation and destruction operators of a particle or hole state with energy $E_s=\epsilon_s-\epsilon_F$ with ϵ_s being the single-particle energy and ϵ_F the Fermi energy. The second term is the phonon field, where $Q_q^\dagger$ and Q_q are the creation and destruction operators of a phonon with energy ω_q. The last term describes the coupling between the first two terms with $F_{ss'}^{(q)}$ denoting the coupling matrix elements. The indices s and s' denote particle (p, $E_p>0$) or hole (h, $E_h<0$), while the index q is reserved for the phonon state $q=\{\lambda,i\}$ with multipolarity λ (the projection μ of λ in the phonon index is omitted for simplicity). The sums over q run over $\lambda\geqslant 1$. It has been shown in Ref. [4] that the form (2.1) can be rigorously derived from a microscopic Hamiltonian that includes a two-body residual interaction beyond the single-particle mean field. The structure of phonon is defined, in this case, by solving the random-phase approximation (RPA) equation. The matrix element $F_{ss'}^{(q)}$ can be expressed in terms of the single-particle matrix elements and the RPA X and Y amplitudes.

The PDM uses the double-time Green function method [16] to derive the propagation of the GDR phonon through the field of incoherent nucleon pairs $ss'=ph$, pp', and hh'. The coupling of phonon to this single-particle field causes the damping of both phonon and single-particle motion. The latter has been shown to be relatively small so that we will not mention it hereafter. The formal expression of the Green function associated with such phonon propagation can be written as

$$G_q(E)=\frac{1}{2\pi}\frac{1}{E-\omega_q-P_q(E)}, \tag{2.2}$$

where the polarization operator $P_q(E)$ occurs because of the coupling between the phonon field and the field of nucleon pairs, which is given by the last term of the Hamiltonian in Eq. (2.1). The phonon damping $\gamma_q(\omega)$ is derived as the imaginary part of the analytic continuation of $P_q(E)$ into the complex energy plane, namely,

$$\gamma_q(\omega)=|\mathrm{Im}\,P_q(\omega\pm i\varepsilon)|, \tag{2.3}$$

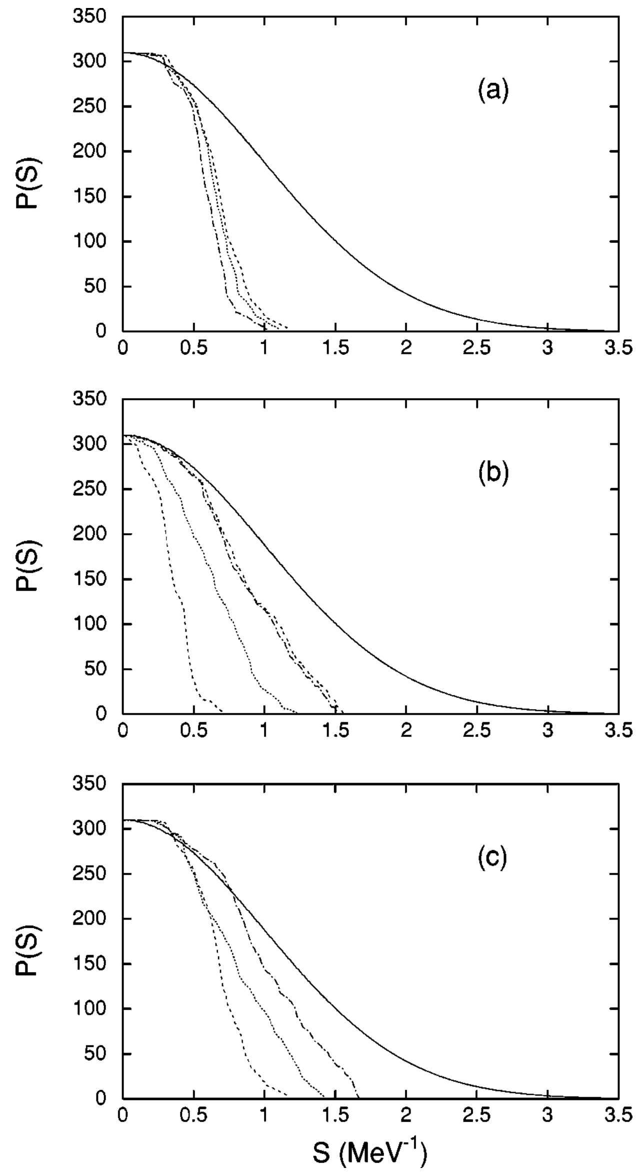

FIG. 1. GDR quantal (a), thermal (b), and total (c) strength distributions at various temperatures. The dashed, dotted, dash-dotted, and double-dashed lines correspond to $T=0.1$, 2, 5, and 6 MeV, respectively. The solid line denotes the value obtained by replacing $S_q(\omega)$ with the corresponding Gaussian distribution in the calculation of the function P.

where ω is real and $\varepsilon\rightarrow 0$. The FWHM $\Gamma_{\rm GDR}$ of the GDR is twice as much as the phonon damping $\gamma_q(\omega)$ at $\omega=\omega_{\rm GDR}$, i.e.,

$$\Gamma_{\rm GDR}=2\gamma_q(\omega_{\rm GDR}), \tag{2.4}$$

where $\omega_{\rm GDR}$ is the GDR energy, which is defined as the solution of the equation for the pole of the Green function $G_q(\omega)$:

$$\omega_{\rm GDR}-\omega_q-P_q(\omega_{\rm GDR})=0. \tag{2.5}$$

In practical applications ε in Eq. (2.3) plays the role of a smearing parameter, which smooths out narrow peaks and

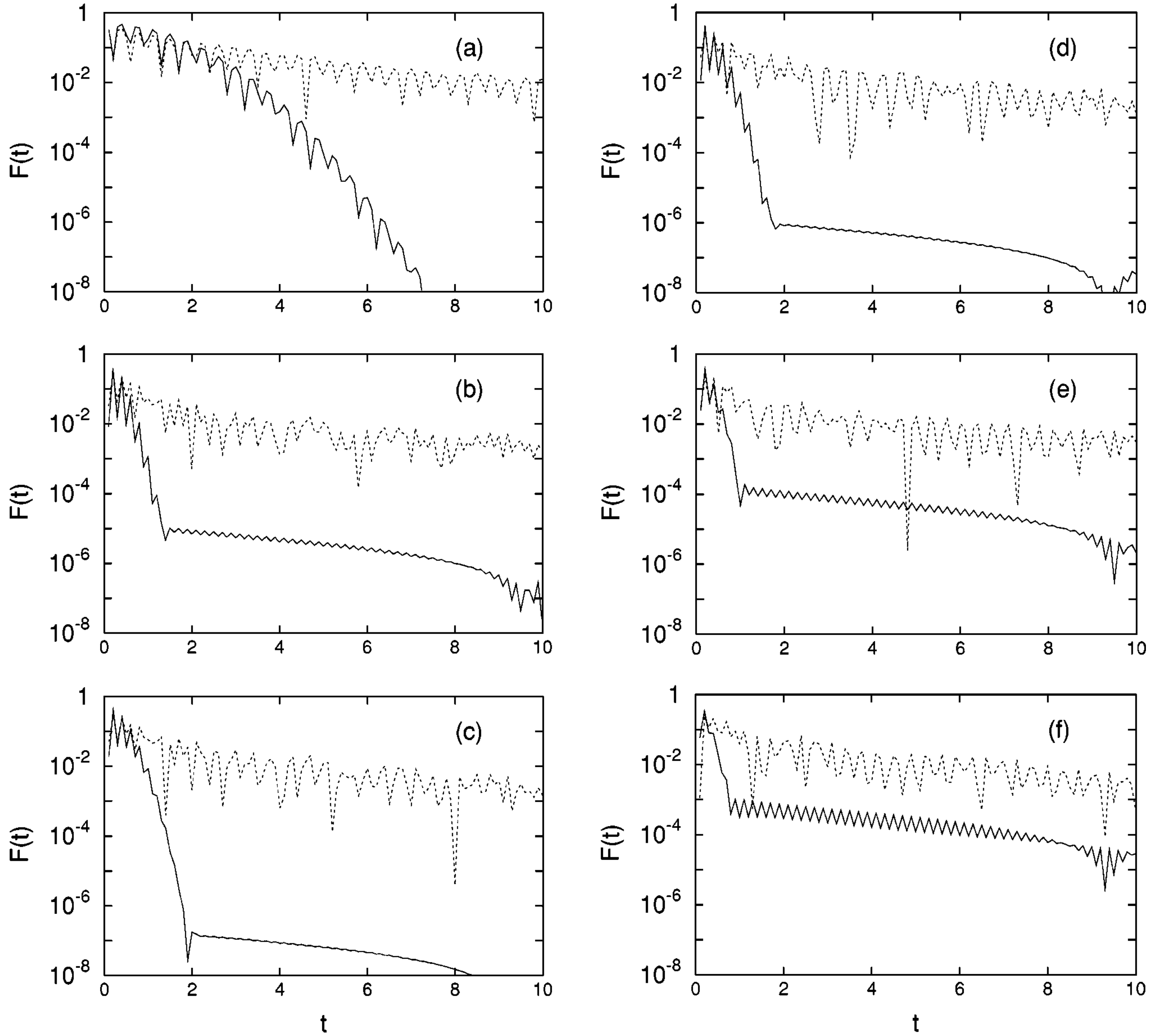

FIG. 2. GDR quantal correlation function at various temperatures (dashed curves) in comparison with those obtained using a Gaussian strength function (solid curves) instead of $S_q(\omega)$. Displayed in (a), (b), (c), (d), (e), and (f) are the values obtained at $T=0.1$, 1, 2, 3, 4, and 5 MeV, respectively.

therefore allows calculations on a coarse energy mesh. To avoid spurious results, ε has to be chosen sufficiently small. We found that the results of all the calculations within the PDM did not vary appreciably in the interval 0.2 MeV$\leqslant\varepsilon\leqslant$1.0 MeV. Therefore, we will discuss the results that are obtained by using the value of $\varepsilon=0.5$ MeV. The explicit form of the polarization operator $P_q(\omega)$ and the phonon damping $\gamma_q(\omega)$ depend on how explicitly and microscopically the phonon is coupled to the incoherent nucleon pairs. This leads to two versions of the model, namely, the PDM-1 [5] and PDM-2 [6]. The PDM-1 considers the coupling of the GDR phonon to the nucleon pairs in the lowest order in the graph expansion, including the contribution of higher-order graphs effectively via adjusting the parameters of the model at $T=0$. The PDM-2 treats explicitly the coupling to higher-order processes up to two-phonon ones. In practice, the numerical results obtained within these two versions are very similar. The PDM-2 will be applied in the calculations in the present paper.

The shape of the GDR is described by the strength function $S_q(\omega)$, which is derived from the spectral intensity $J_q(\omega)$ in the standard way using the analytic continuation of the Green function $G_q(\omega\pm i\varepsilon)$ and by expanding $P_q(\omega)$ around ω_{GDR} [16]. The final form of $S_q(\omega)$ has been given in Ref. [6] as

$$S_q(\omega)=\frac{1}{\pi}\frac{\gamma_q(\omega)}{(\omega-\omega_{\mathrm{GDR}})^2+[\gamma_q(\omega)]^2}. \tag{2.6}$$

The spectral intensity $J_q(\omega)$ is related to the strength function $S_q(\omega)$ simply as

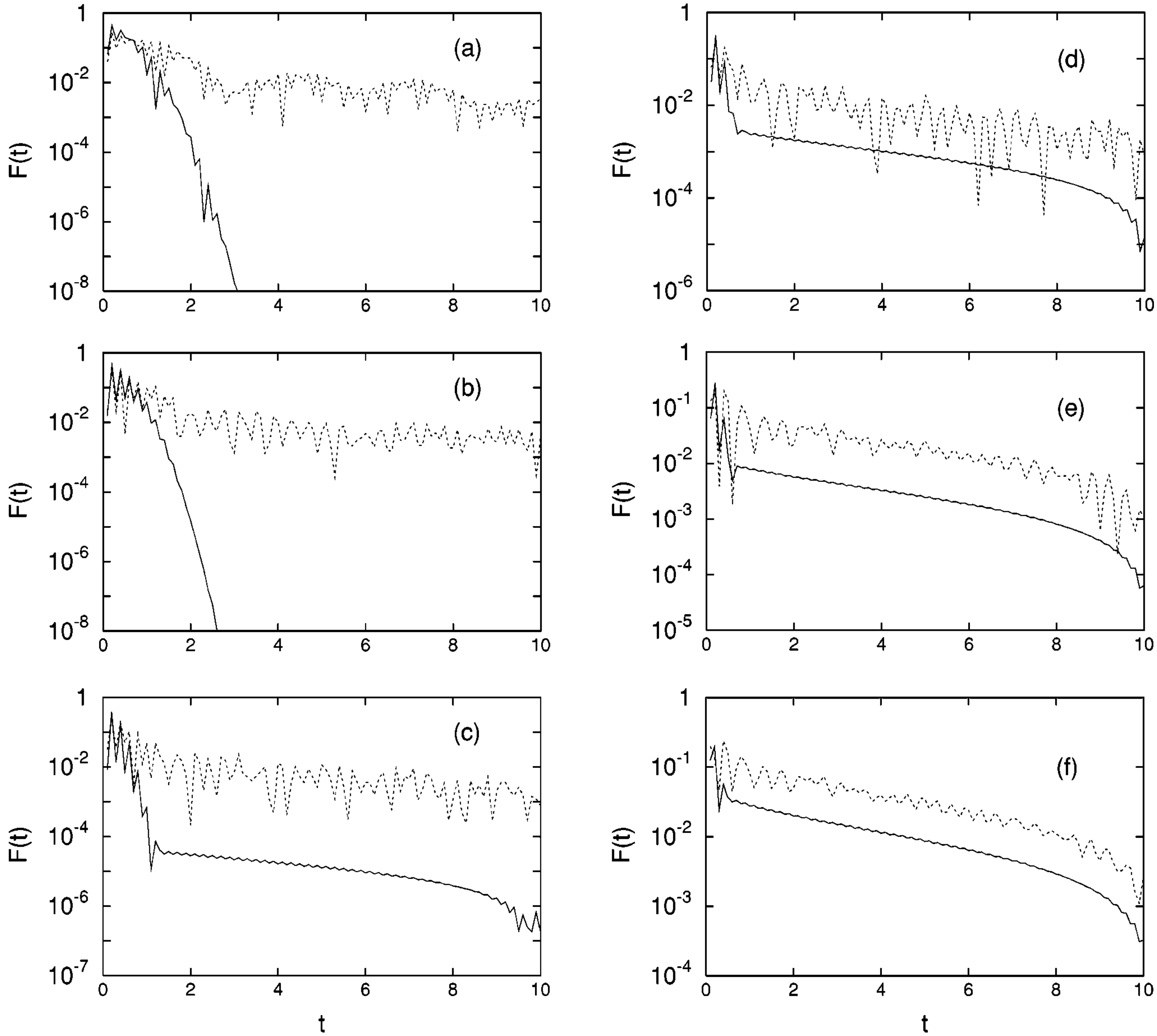

FIG. 3. GDR thermal correlation function at various temperatures. The notation is the same as in Fig. 2.

$$J_q(\omega)=\frac{S_q(\omega)}{\exp(\omega/T)-1}. \qquad (2.7)$$

The dynamics of the hot GDR can be studied through the evolution in time of the correlation function $\mathcal{F}_q(t-t')$ in the form

$$\mathcal{F}_q(t-t')=\frac{1}{2}\int_{-\infty}^{\infty}J_q(\omega)(\mathrm{e}^{\omega/T}+1)\mathrm{e}^{-i\omega(t-t')}d\omega, \quad (2.8)$$

which is the arithmetic average of two correlation functions $\mathcal{F}_{Q^\dagger Q}(t-t')=\langle Q_q^\dagger(t')Q_q(t)\rangle$ and $\mathcal{F}_{QQ^\dagger}(t-t')=\langle Q_q(t)Q_q^\dagger(t')\rangle$ [16].

In the Appendix of Ref. [4] the evolution of the collective GDR vibration due to its coupling to incoherent pp and hh configurations at $T\neq 0$ has been considered using theory of stochastic process in the random frequency modulation [17]. It has been shown that the stochastic limit of the random perturbation caused by the incoherent pp and hh configurations on the hot GDR can be described by a power spectrum in the Gaussian form. Therefore, in order to see a possible signature of the order-to-chaos transition in the hot GDR with increasing temperature we calculate, in the present paper, the following functions: (i) the strength distribution $P(\sqrt{\tilde{S}_q})$, where $\tilde{S}_q=S_q[(\omega-\omega_{\mathrm{GDR}})^2+\gamma_q(\omega)^2]$ (this distribution $P(x)$ sums up all the strengths whose values are not smaller than x), (ii) the logarithm $\log N$ of the total number N of transitions of magnitude smaller than a given threshold value S as a function of $\log S$, (iii) the correlation function $\mathcal{F}_q(t-t')$, and compare them with their corresponding Gaussian values obtained by using a Gaussian strength function instead of S_q.

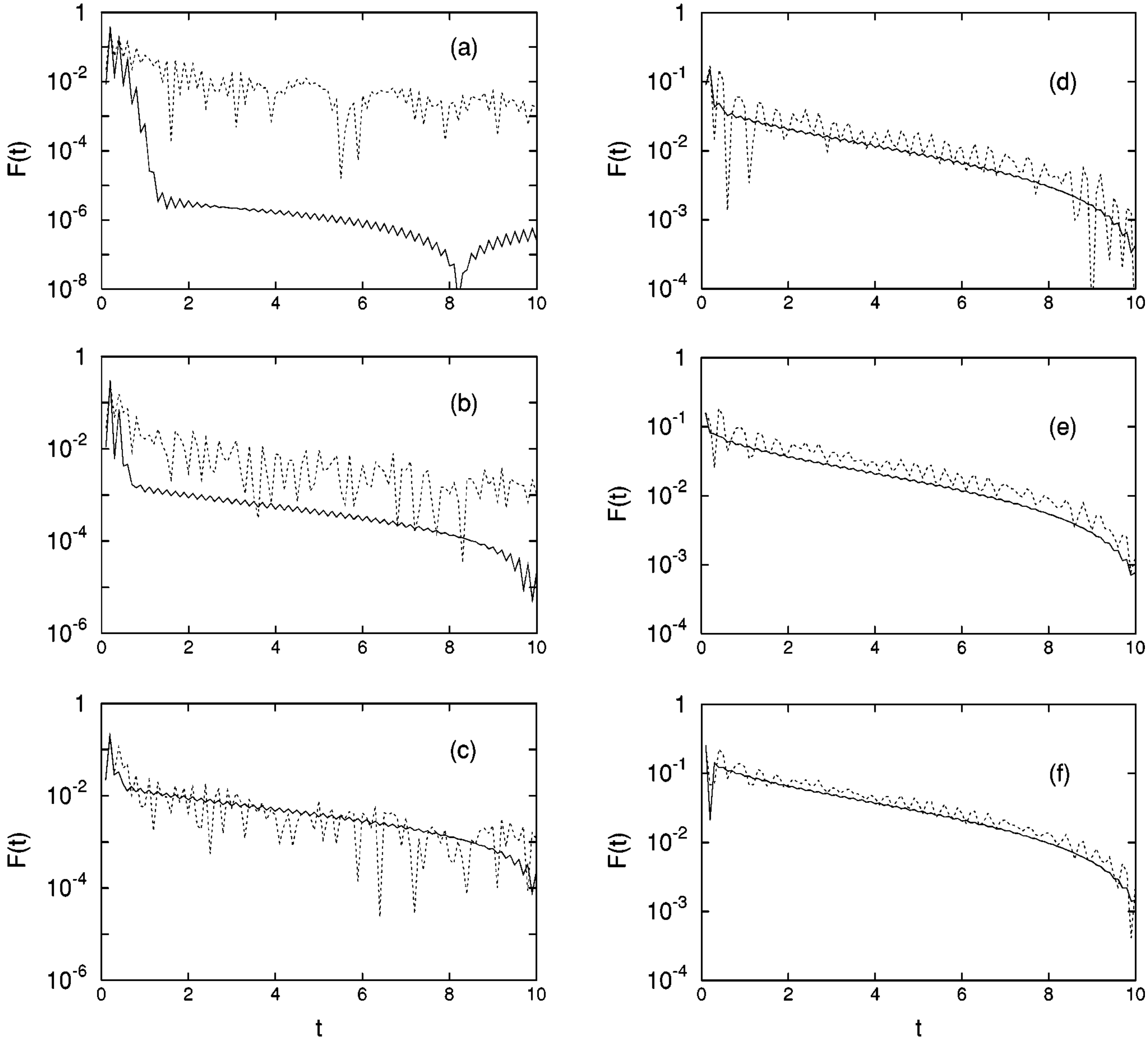

FIG. 4. GDR total correlation function. The notation is the same as in Fig. 2.

III. NUMERICAL RESULTS

The numerical calculations have been performed within the PDM-2 for the damping of the GDR in ^{120}Sn at nonzero temperature. The detail of the model, the selection of the parameters, and the values of the selected parameters themselves have been presented in Ref. [6]. The GDR phonon is damped via coupling to all nucleon pairs $ss'=ph$, pp, and hh, among which the pp and hh configurations appear only at $T\neq 0$ because of the deformation of the Fermi surface. The coupling to ph configurations leads to the quantal damping width $\Gamma^{\downarrow}=\Gamma_Q$, which has been shown to decrease slightly with increasing temperature [5,6]. The coupling to pp and hh configurations causes the thermal damping width Γ_T, which increases sharply as T increases in the low and moderate temperature region ($T\leqslant 3-4$ MeV), but reaches a saturation in the high temperature region ($T\geqslant 4$ MeV) [5,6]. The temperature dependence of the total damping width of the hot GDR is mostly driven by this thermal damping width. In the following we will refer to the results that are obtained taking into account the coupling to ph, to pp and hh configurations, and to all of them, as the quantal, thermal, and total results, respectively.

The quantal, thermal, and total strength distributions $P(S)$ are displayed in Figs. 1(a), 1(b), and 1(c), respectively. It is seen from Fig. 1(a) that the quantal distribution remain rather stable against varying temperature. This shows the persistence of the quantal coupling, that, in fact, leads to the damping of the g.s. GDR as of a zero sound at $T=0$, even at rather high temperature $T\sim 5$ MeV. At the same time, the channel of coupling to the incoherent pp and hh configurations at $T\neq 0$ leads to the thermal strength distribution in Fig. 1(b), which is approaching the Gaussian distribution with increasing T. As a result, the full strength distribution in Fig. 1(c) has a similar behavior as of the thermal one. We notice that both the thermal and total strength distributions do not reach the Gaussian distribution (the solid line), which corresponds to the fully chaotic limit.

Shown in Figs. 2–4 are the quantal, thermal, and total correlation functions $\mathcal{F}(t)$, respectively, where the panels in each figure display the values obtained at various temperatures. It is remarkable that the fine structure of the oscillation in the quantal correlation function depends weakly on temperature, and the function itself differs strongly from the Gaussian distribution. Meanwhile, the curves that describe the thermal and total correlation functions are getting smoother, approaching the Gaussian correlation function, with increasing temperature. These figures clearly show that, if we assume the fully chaotic dynamics to be described by the GOE, the evolution of the hot GDR is a process where the regular collective GDR vibration becomes more and more embedded in a chaotic background of incoherent pp and hh configurations due to the coupling to this background as the temperature increases. At $T=5$ MeV the gross structure of the total correlation function is nearly the same as of the Gaussian one. However, the fine structure of the total correlation function can still clearly be seen, showing that the regular motion still persists.

In Ref. [18] it has been observed that the fully chaotic case of the isovector resonance corresponds to a total number N of transitions of magnitude smaller than a given threshold value $S_{\rm th}$ that can be described by a scaling law $N\sim S_{\rm th}^{\alpha}$ ($\alpha \simeq 0.5$). This power law signals a self-organized system at its critical state where the equilibrium balance reduces the dimensionality. The quantal, thermal, and total $\log N$ are plotted against $\log S_{\rm th}$ in Figs. 5(a), 5(b), and 5(c), respectively. We can see that even though the thermal damping makes this quantity get closer to the straight solid line indicating the scaling law $\log[\sim\sqrt{S}]$, it is still far away from the straight line. This confirms the conclusion drawn from the analysis of the strength distributions and correlation functions, that the evolution of the hot GDR indeed seems to show a transition from the region of regular motion toward the chaotic region. However, the fully chaotic regime has not been yet reached. As a temperature as high as $T\sim5$ MeV, the collective motion, although strongly damped via coupling to noncollective background, still can be seen.

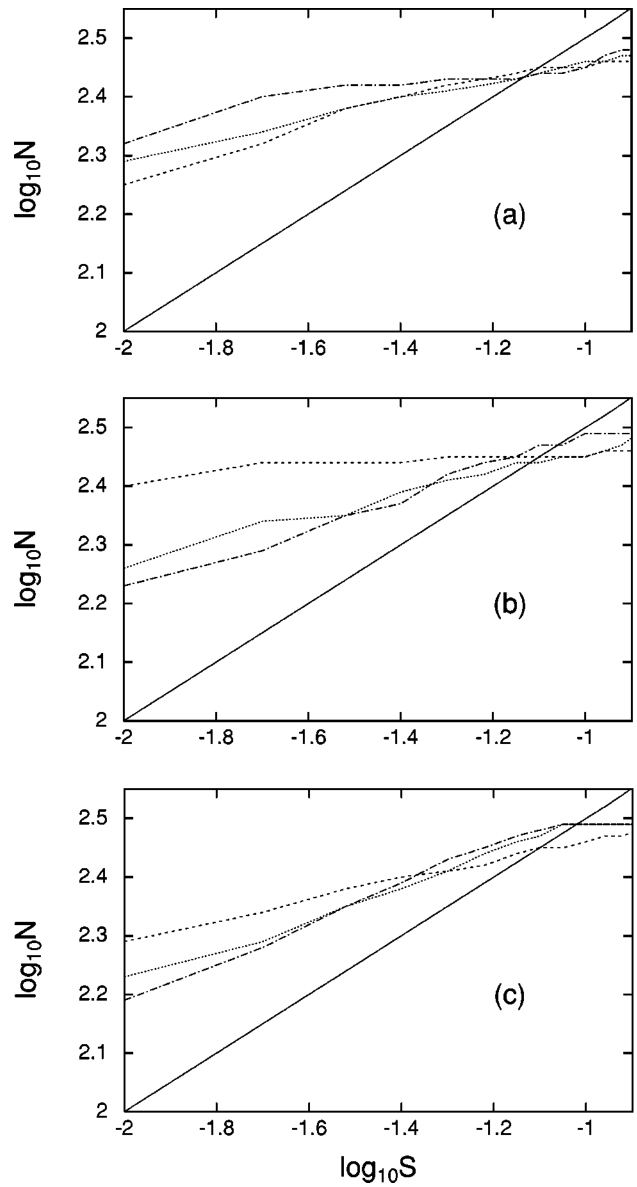

FIG. 5. Quantal (a), thermal (b), and total (c) $\log N$ (see text) at various temperatures. The solid line is $\log(\sim\sqrt{N})$. The notation for dashed, dotted, and dash-dotted lines is the same as in Fig. 1.

IV. CONCLUSIONS

In the present paper we have attempted to search for the onset of chaos in the characteristics of the hot GDR. Assuming the strength distribution and the correlation function in the fully chaotic regime to be described by the GOE, we have studied the order-to-chaos transition through the evolution of the hot GDR as a function of temperature and time. The PDM has been employed to calculate the strength distribution, the correlation function, and the logarithm $\log N$ of the total number N of transitions of magnitude smaller than a given threshold value S as a function of $\log S$. We expected that the regular motion of the collective GDR vibration becomes embedded in, and its regularity becomes distorted by the background of incoherent pp and hh configurations, which becomes more chaotic as the temperature increases. Indeed, the analysis of the results obtained shows that the fine structure of the hot GDR is getting smoother with increasing temperature. However, the hot GDR does not yet reach the fully chaotic regime even at a temperature as high as $T\sim5$ MeV. We conclude that the persistence of the GDR collective mode as regular motion at that high temperature, even though strongly damped, is a direct indication that the observation of GDR in hot nuclei is a manifestation of the coexistence of order and chaos in highly excited and finite nuclear systems.

From the experimental point of view, it is currently hard to distinguish the results obtained by fitting the γ spectrum in the heavy-ion fusion reactions with a Lorentzian and with a Gaussian because of the large error bars. The new method to excite the compound nucleus in the inelastic α scattering has shown that the hot GDR shape can be well approximated by a Lorentzian with relatively smaller error bars [10]. If these measurements can also be carried out at higher temperatures $T>3$ MeV, their results can serve as a direct experimental verification of the theoretical study performed in the present paper.

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