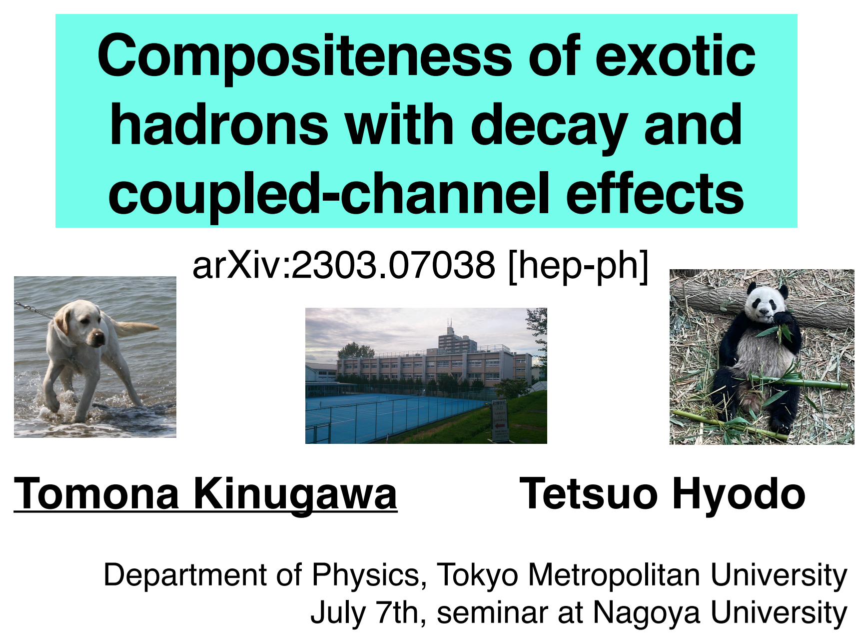
Internal structure of near-threshold states with Coulomb plus short range interaction



Tomona Kinugawa

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Jun 6th, Seminar @Quark-Hadron Theory Group, Nagoya Univ.

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Apr. 2010 - Mar. 2016
Student at Nagoya Univ. affiliated highschool

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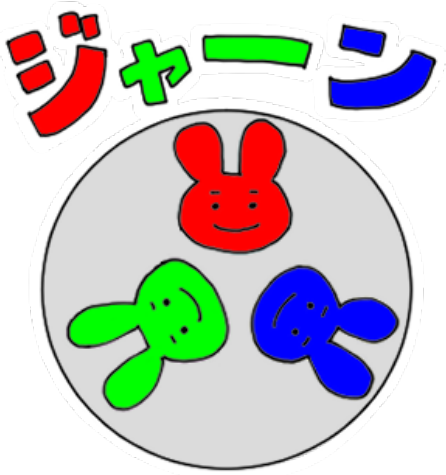
Apr. 2022 - Mar. 2025
Doctor student at Tokyo Metropolitan Univ.

- Apr. 2023 - Mar. 2024 JST Fellow
- Apr. 2023 - Mar. 2025 JSPS Fellow (DC2)
- Mar. 2025 Doctor of Science

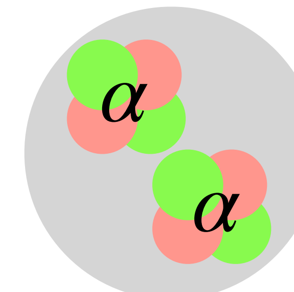
Apr. 2025 - Postdoc. at RIKEN



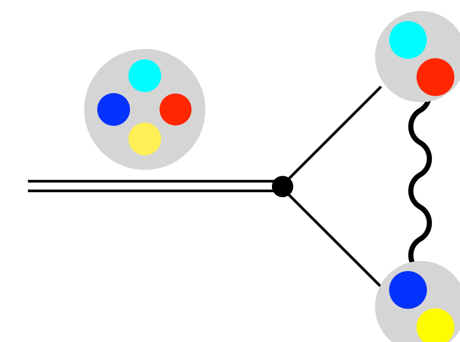
Outline

near-threshold states with **short-range** interaction

near-threshold bound states & resonances
with **Coulomb + short-range** interaction



framework



- bare state which couples to Coulomb scattering
- Coulomb scattering length, Coulomb effective range, a_B

numerical calculations & discussion

- investigate pole trajectory
- analyze internal structure with compositeness
- study universal nature of near-threshold states

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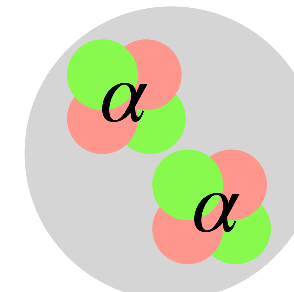

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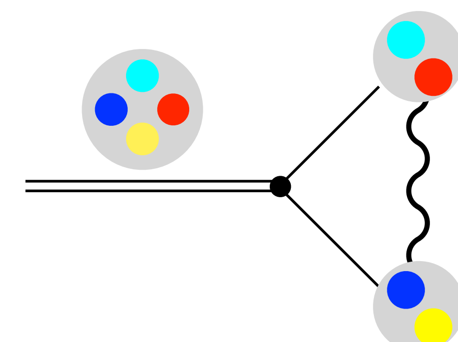

- bare state which couples to Coulomb scattering
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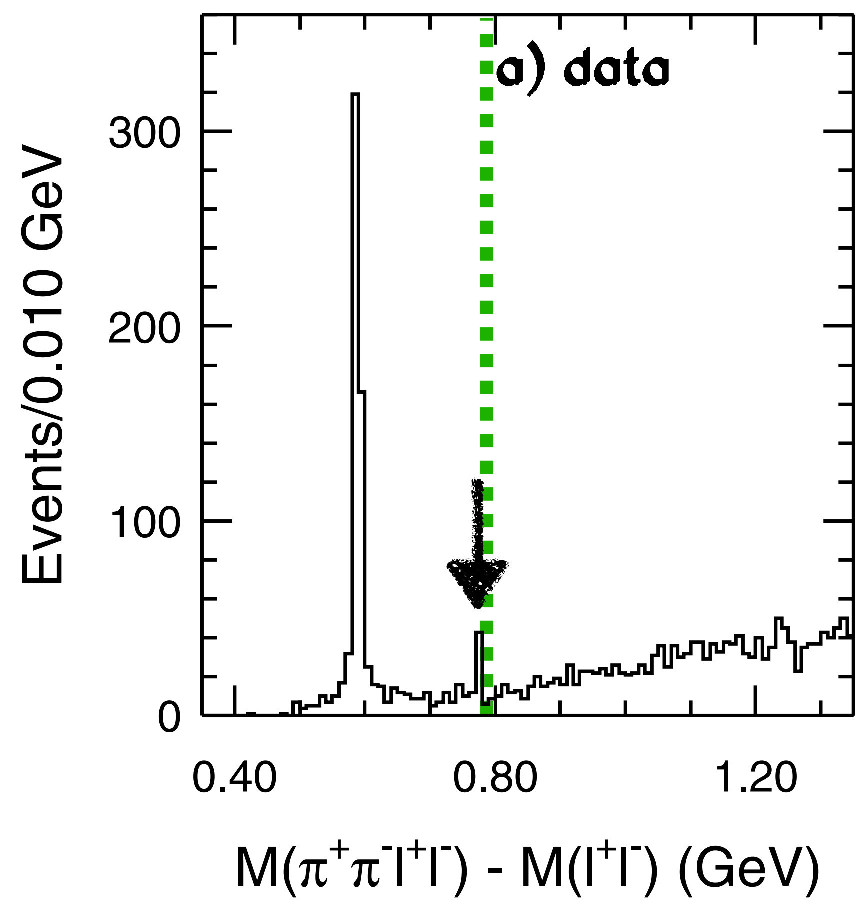
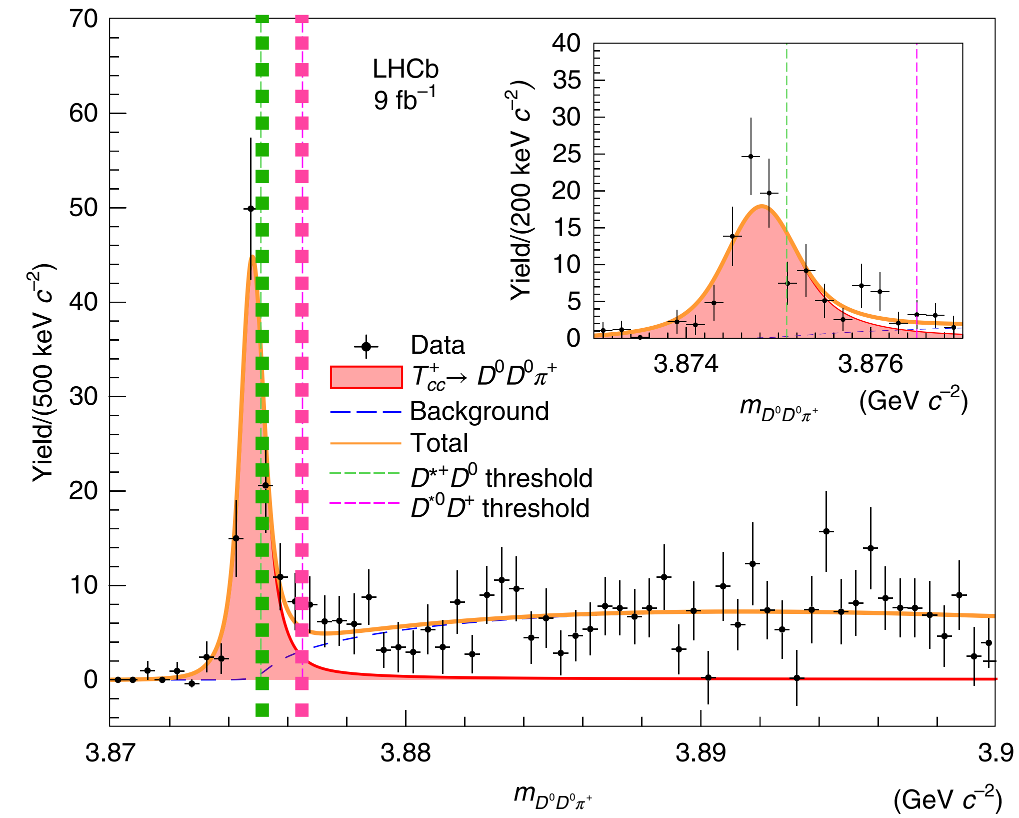
Near-threshold exotic hadrons

- exotic hadrons are tend to be observed near threshold

$X(3872) \to \pi^+\pi^- J/\psi$ $\quad$ $T_{cc}(3875)^+ \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$



S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).



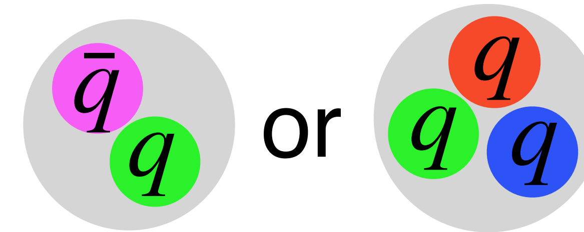
LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;
LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

→ internal structure of near-threshold states?

Exotic hadrons

A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, PTEP 2016, 062C01 (2016)

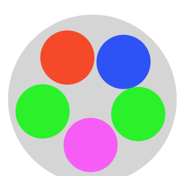
- ordinaly hadron: $q\bar{q}$ (meson) or qqq (baryon)



- other than $q\bar{q}$ or qqq → **exotic hadron** → structure?

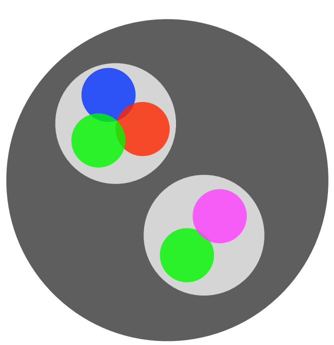
◉ <u>possible structure</u> F-K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, and B-S. Zou, RevModPhys.90.015004 (2018).

- **multiquarks**: quarks → hadron



size $\lesssim$ 1 fm (**compact**)

- **hadronic molecule**: quarks → hadrons (subunit) → hadron



size $\gtrsim$ 1 fm (spatially large radious)

e.g. deuteron (pn molecule), $R = 4.32$ fm

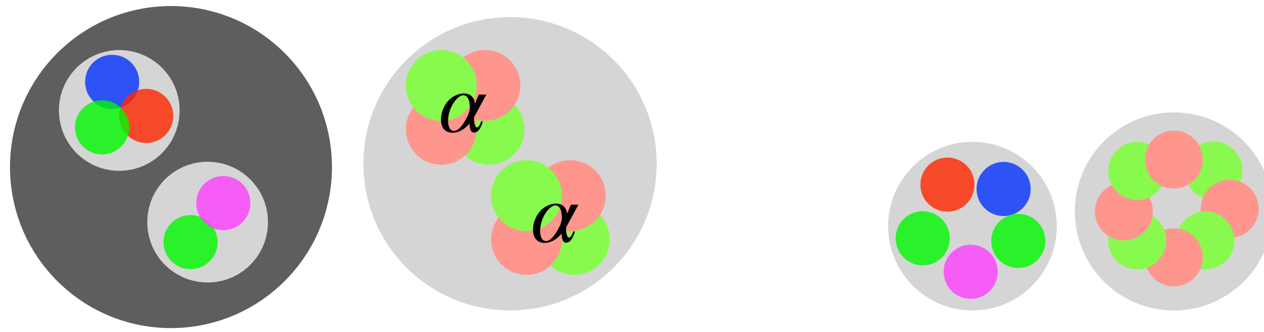
→ **superposition** of possible components

internal structure ← investigate weight of each component

Compositeness

Weinberg, S. Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Kinugawa, T. Hyodo, arXiv:2411.12285 [hep-ph] (accepted in EPJ A).

◉ Definition



$$|\text{state}\rangle = \sqrt{X}\,|\text{composite}\rangle + \sqrt{Z}\,|\text{others}\rangle$$

compositeness (X) **elementarity** (Z)

- **quantitative measure**

$$0 \le X \le 1$$
$$X + Z = 1$$

⟶

$X > 0.5 \Leftrightarrow$ **composite dominant**

$X < 0.5 \Leftrightarrow$ **non-composite dominant**

- applicable to various states

$f_0(980),\ a_0(980)$ V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004); Y. Kamiya and T. Hyodo, PTEP 2017; Phys. Rev. C 93, 035203 (2016) etc.

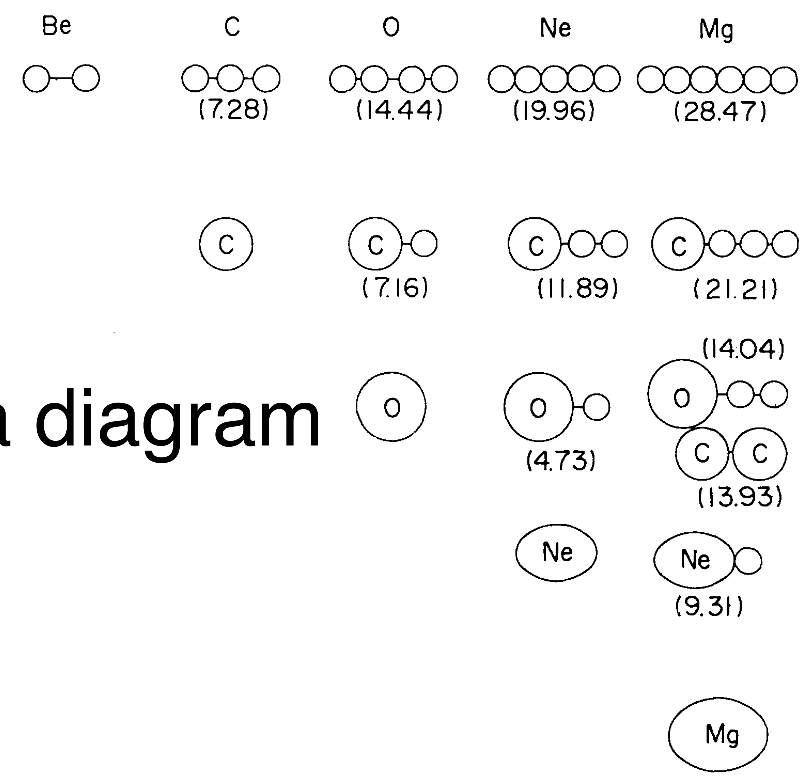
nuclei, atomic systems E. Braaten, H. W. Hammer, and M. Kusunoki, (2003), cond-mat/0301489; T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022).

Clustering phenomena

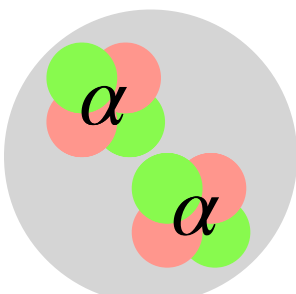
- near-threshold states ⟶ clustering phenomena

- α clustering phenomena

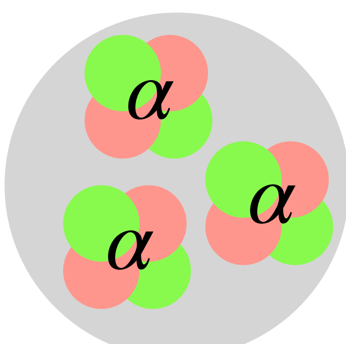
K. Ikeda, and N. Takizawa, and H. Horiuchi, Prog. Theor. Phys. Suppl. E68, 464-475 (1968).



Ikeda diagram

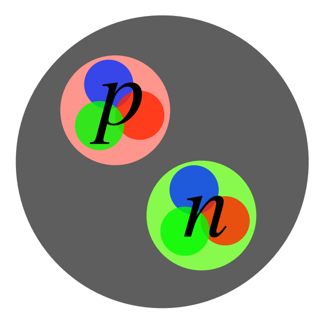


^{8}Be nuclei



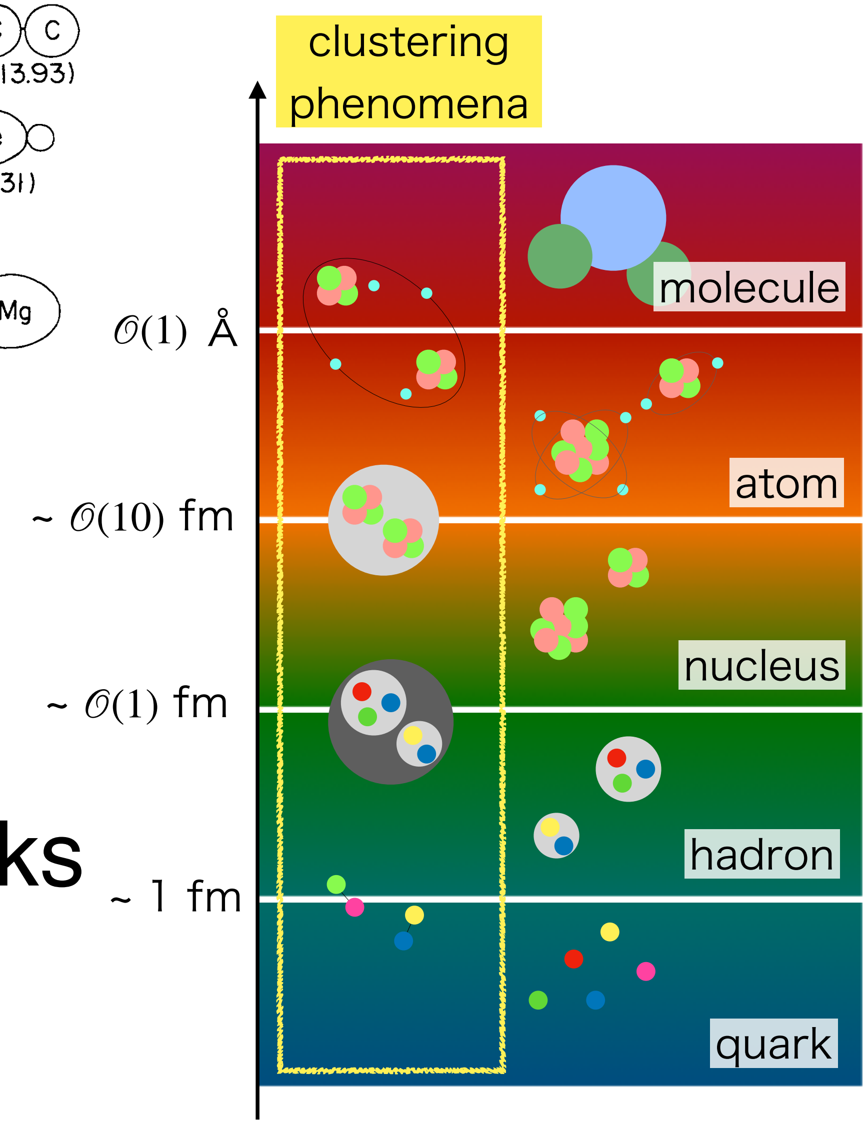
^{12}C Hoyle state

- deuteron $B = 2.22$ MeV



hadronic molecule ⟶ clustering of quarks

- clustering phenomena: **universal**!



Low-energy universality

- near-threshold energy region **in s-wave & short-range systems**

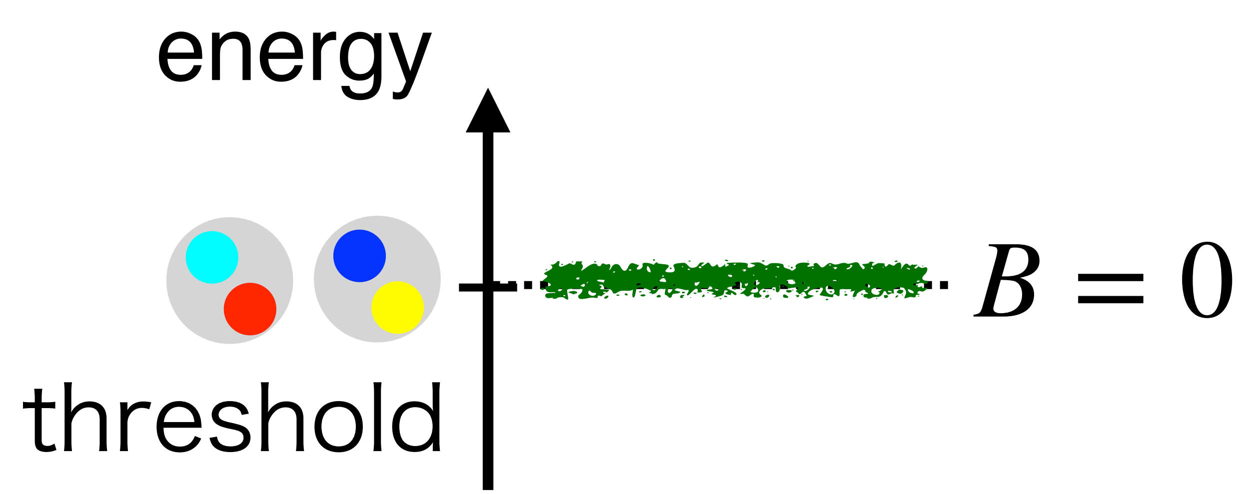
  ← universal phenomena

  E. Braaten and H.-W. Hammer, Phys. Rept. **428**, 259 (2006);
  P. Naidon and S. Endo, Rept. Prog. Phys. **80**, 056001 (2017).

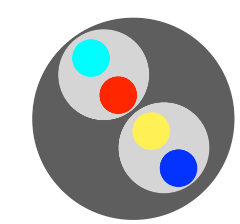
- scattering amplitude ← scattering legth a_s

$$f(k)^{-1} = -\frac{1}{a_s} - ik$$

eigenmomentum: $k = \frac{i}{a_s}$

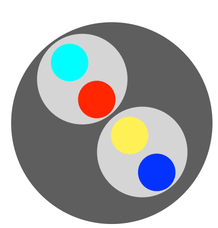


- binding energy $B = 0$ (exactly at threshold)

  → that state is **always** completely composite ($X = 1$) 

  compositeness is model-**independently** determined

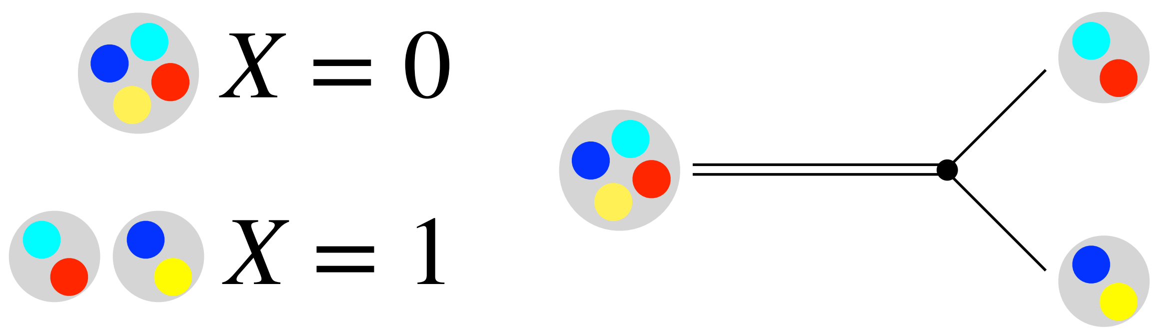
  T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

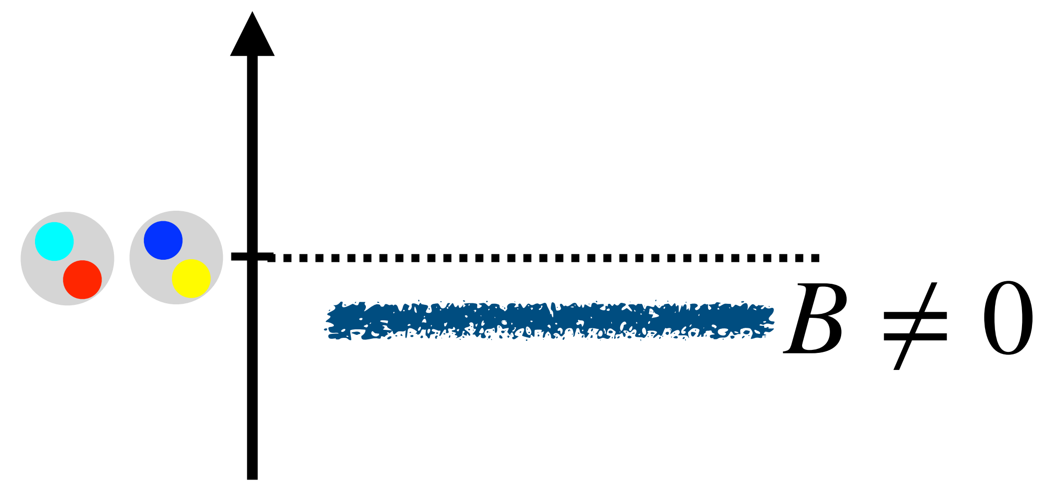
→ Is state still  even beyond this ideal case?

Shallow bound states

T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024).

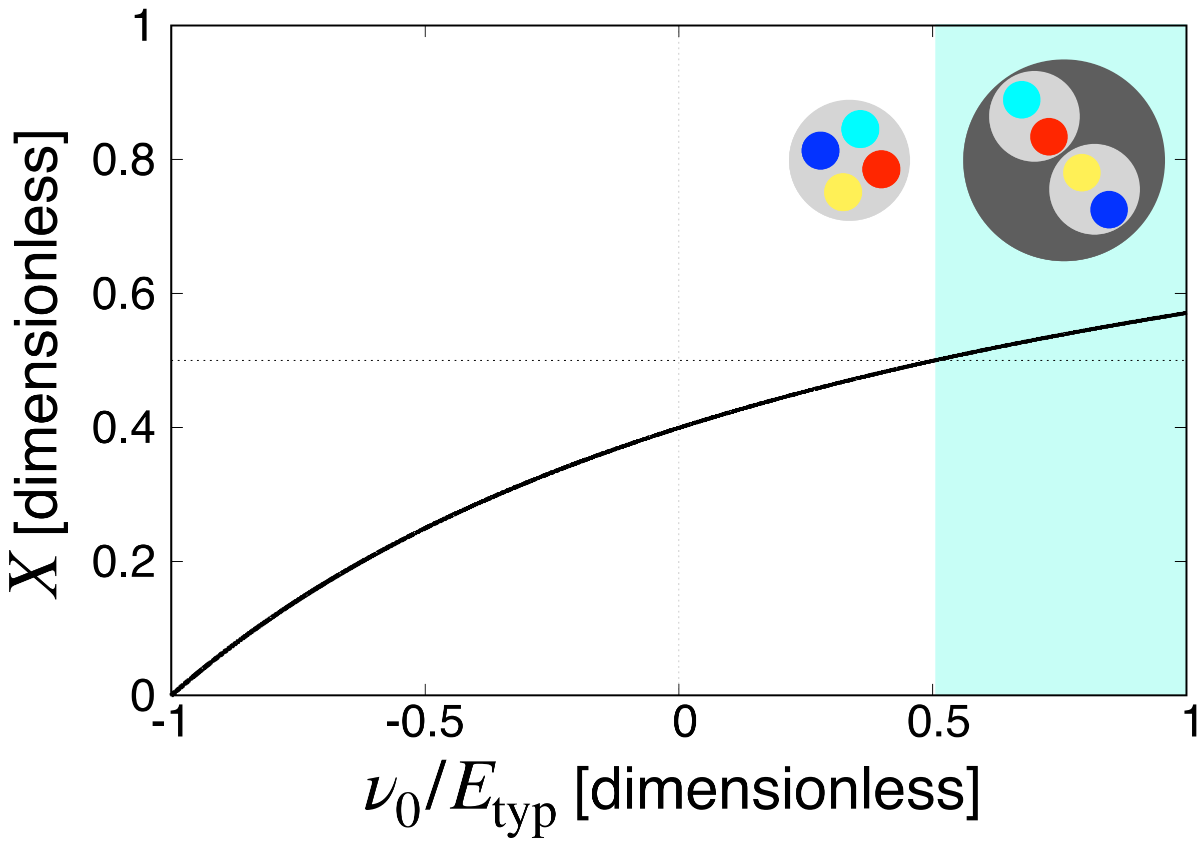
- EFT model E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).



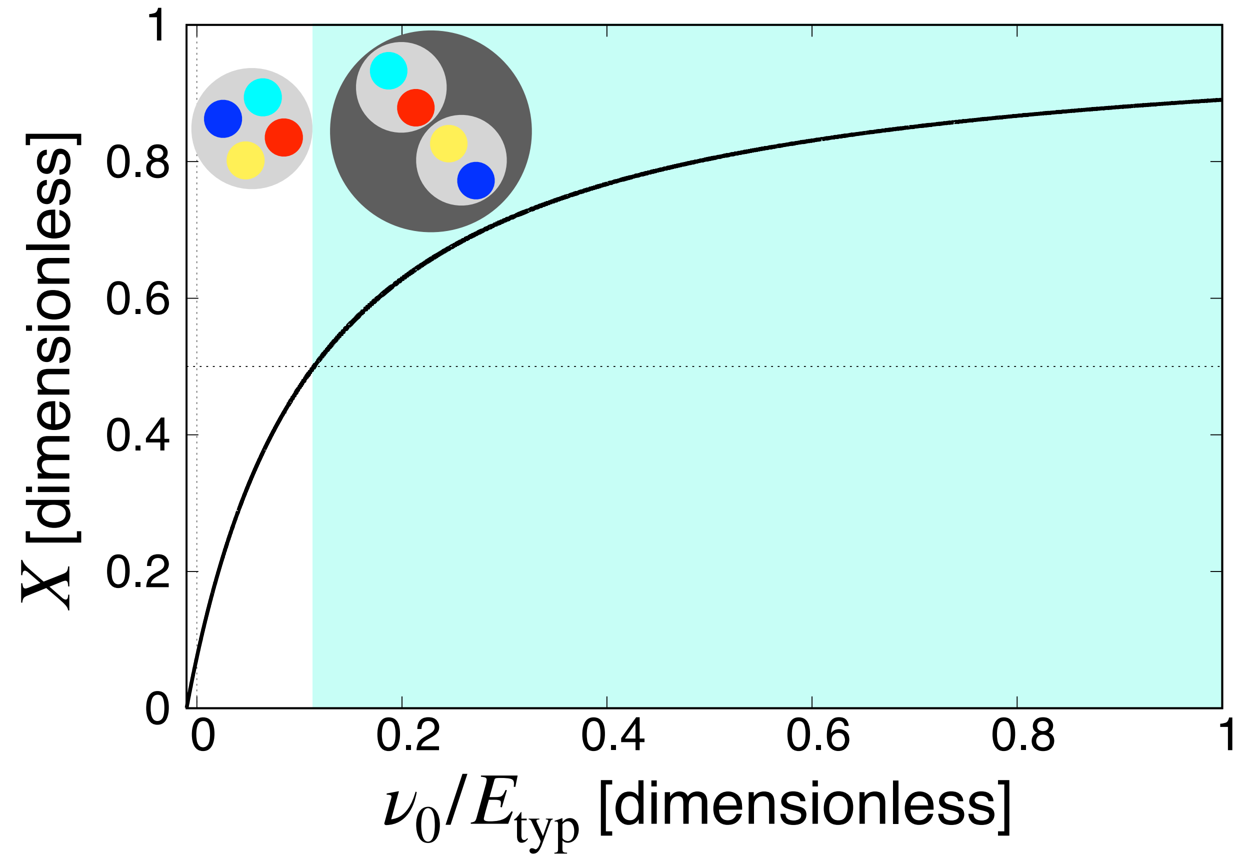


construct models that reproduce same bound state with B

typical bound state



shallow bound state



→ **shallow bound states tend to be composite dominant!**

Near-th. resonances

T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

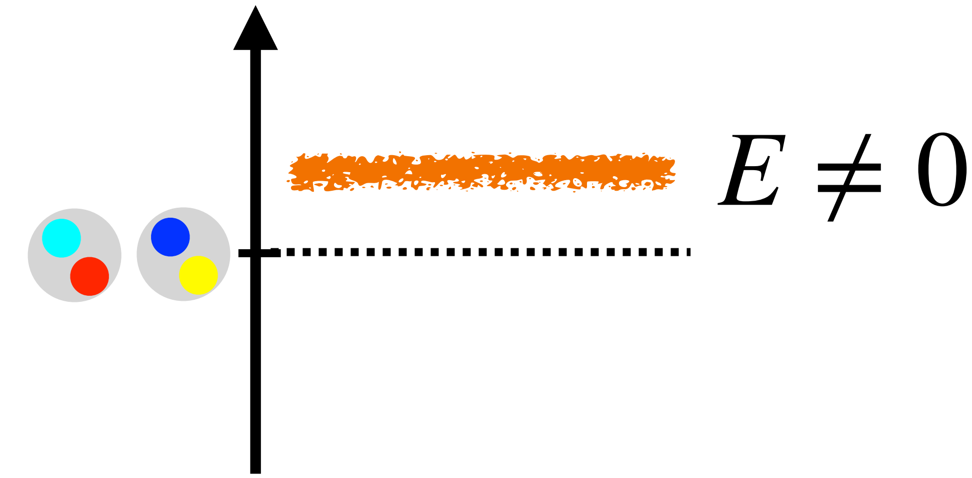
- Effective range expansion

$$k = \frac{i}{r_e} - \frac{1}{r_e}\sqrt{\frac{2r_e}{a_s} - 1}$$

k : momentum

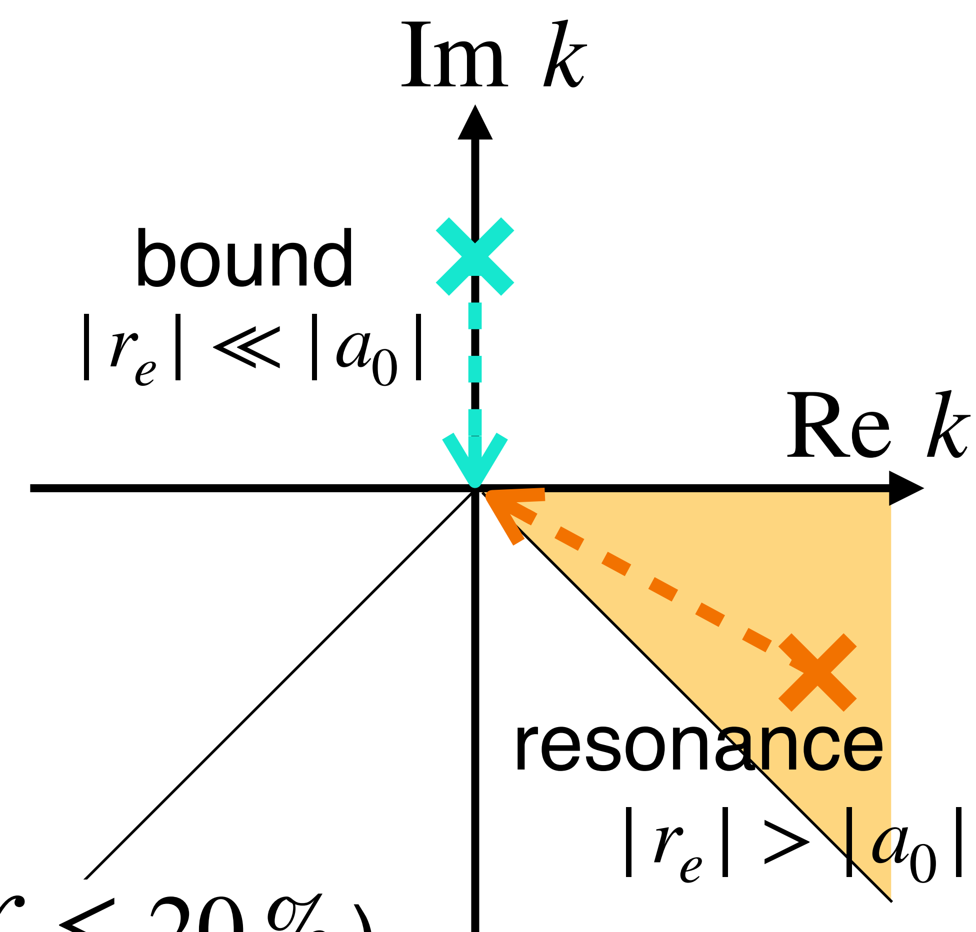
a_s : scattering legth

r_e : effective range



resonance ⇔ $r_e < 0$ & $|r_e| > |a_0|$

⟶ $a_0 \to \infty$ & $|r_e| \to \infty$ at threshold

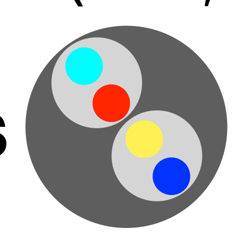


- compositeness T. Hyodo, Phys. Rev. C **90**, 055208 (2014).

$$X = \sqrt{\frac{1}{1 - 2r_e/a_0}}$$

- resonances are non-composite dominant ($\mathscr{X} \lesssim 20\,\%$)

⟶ **near-threshold states is not always**



s-wave short-range systems

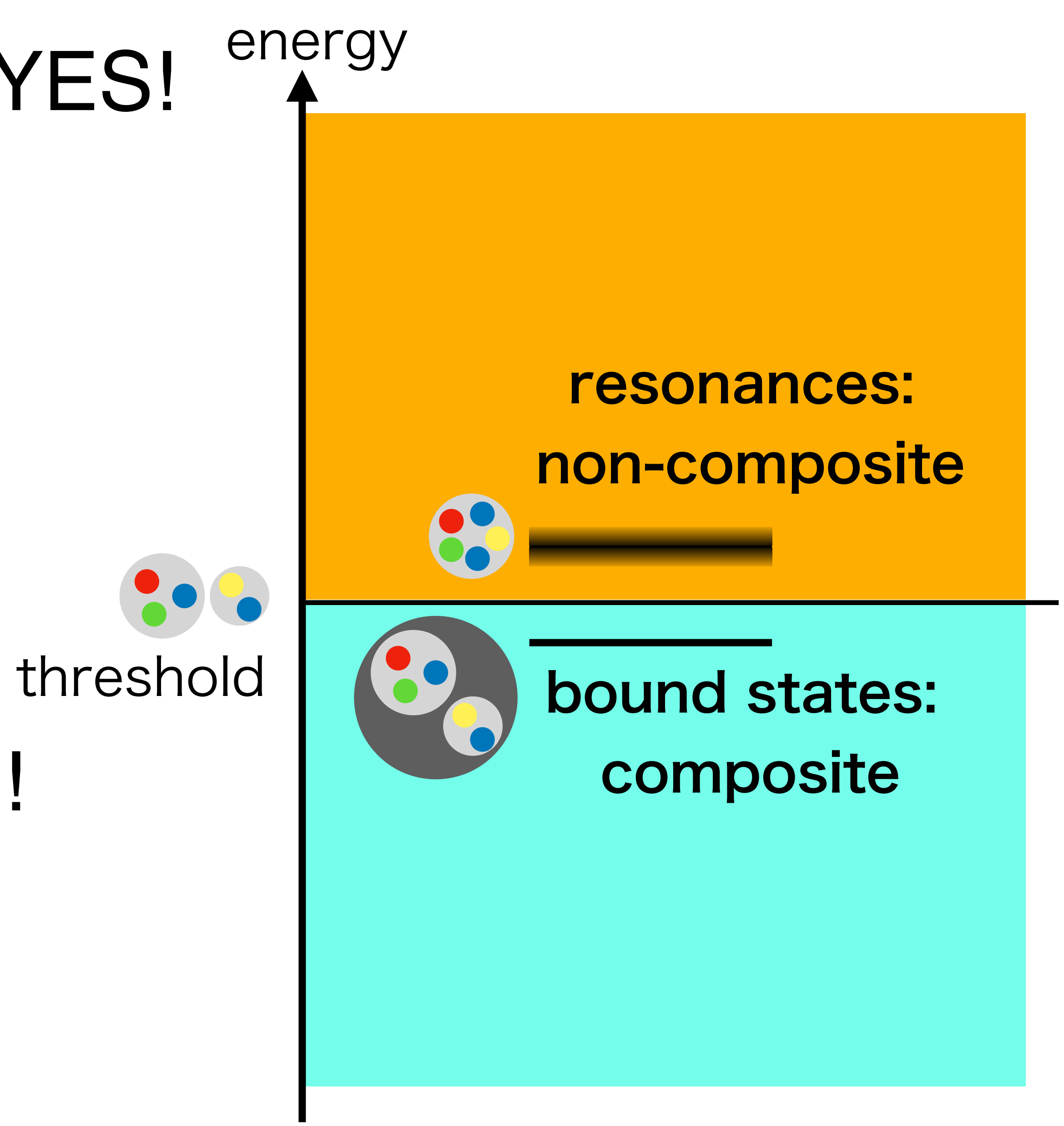
◉ Are near-threshold states composite dominant ?

- shallow bound states: (usually) YES!

  ← low-energy universality with large a_s

- near-threshold resonances: NO!

  ← kind of universality with large a_s and $|r_e|$

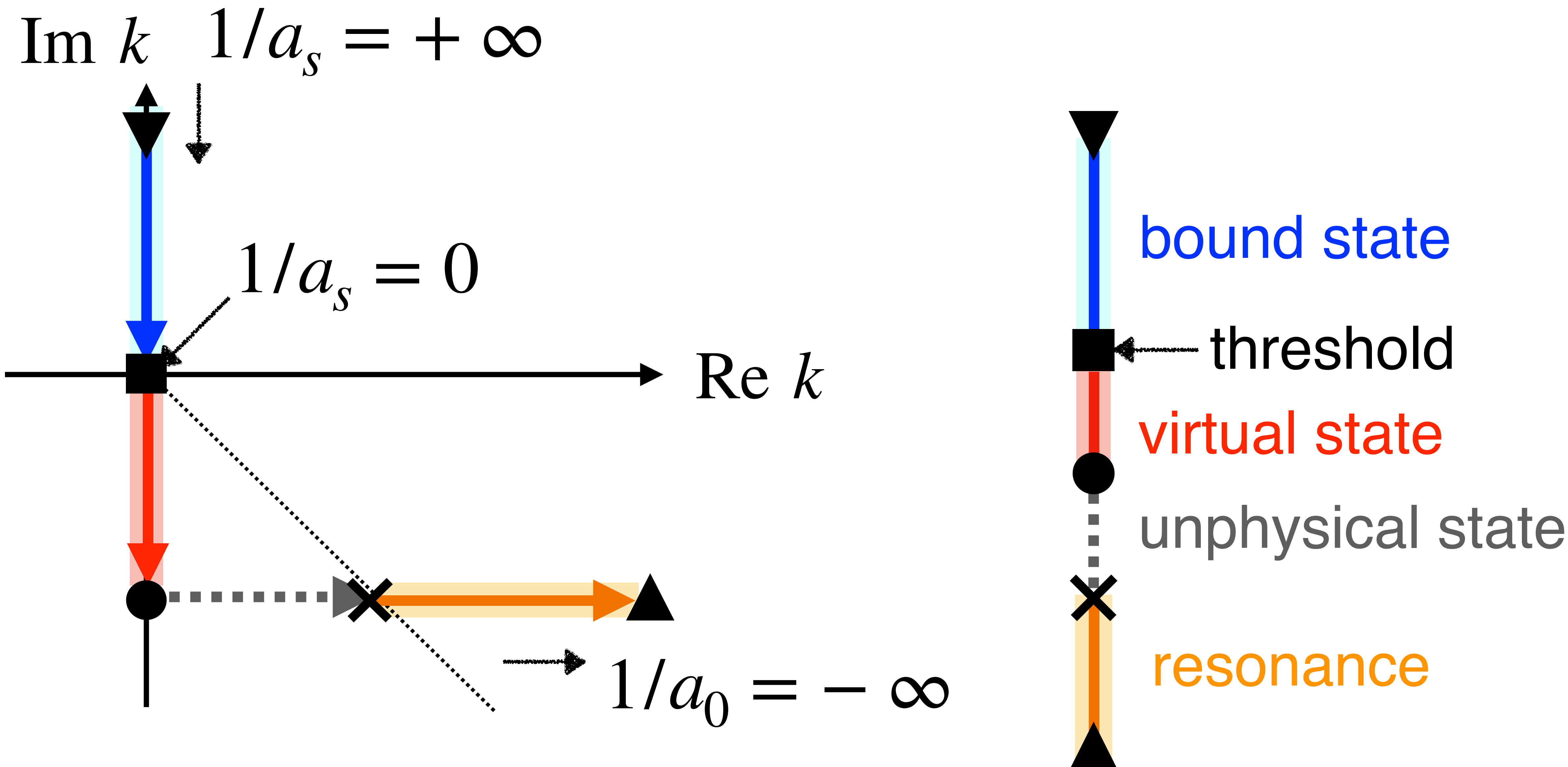


Pole trajectory in k plane

- low-energy behavior of scattering amplitude (ERE)

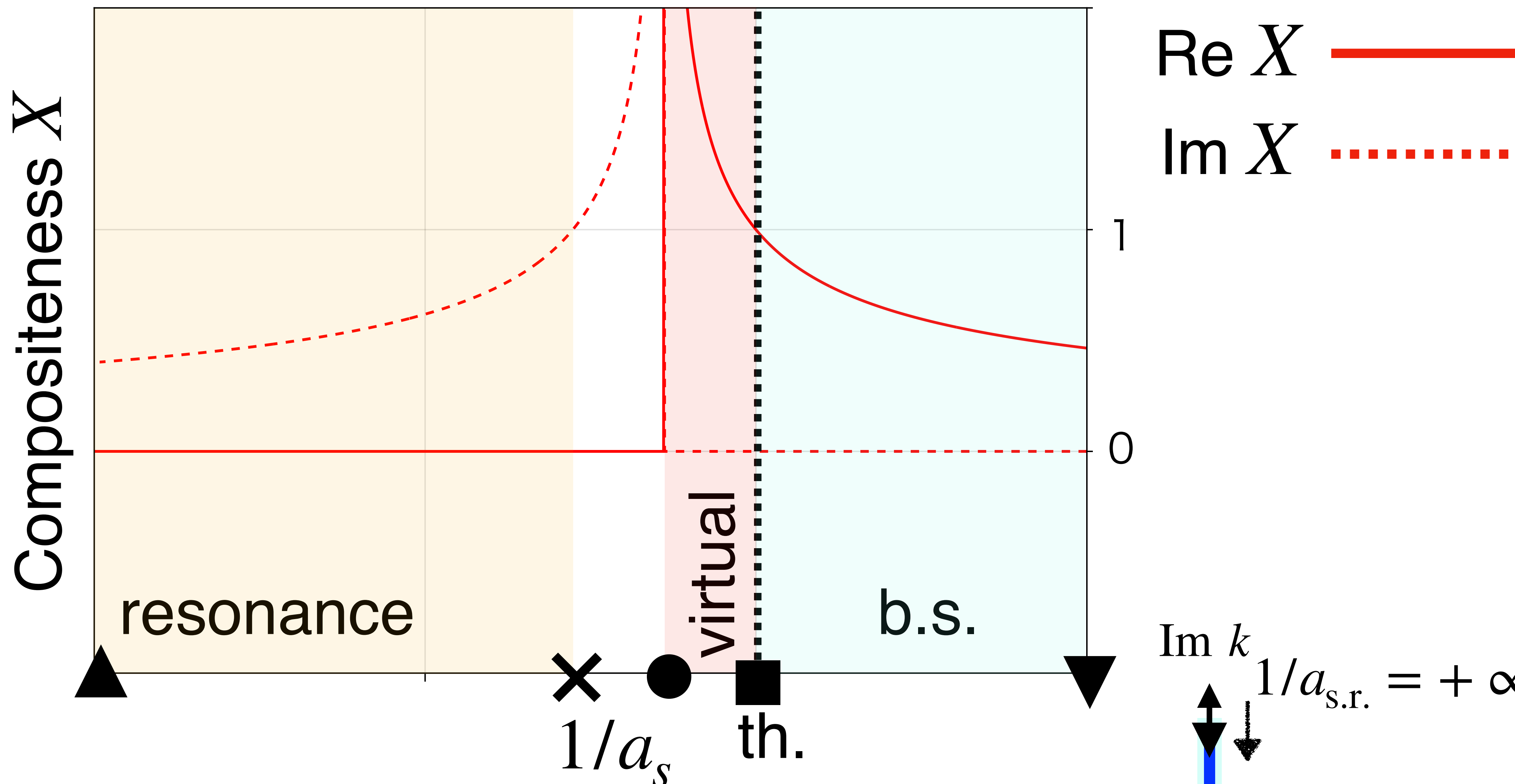
$$f(k)^{-1} = -\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik$$

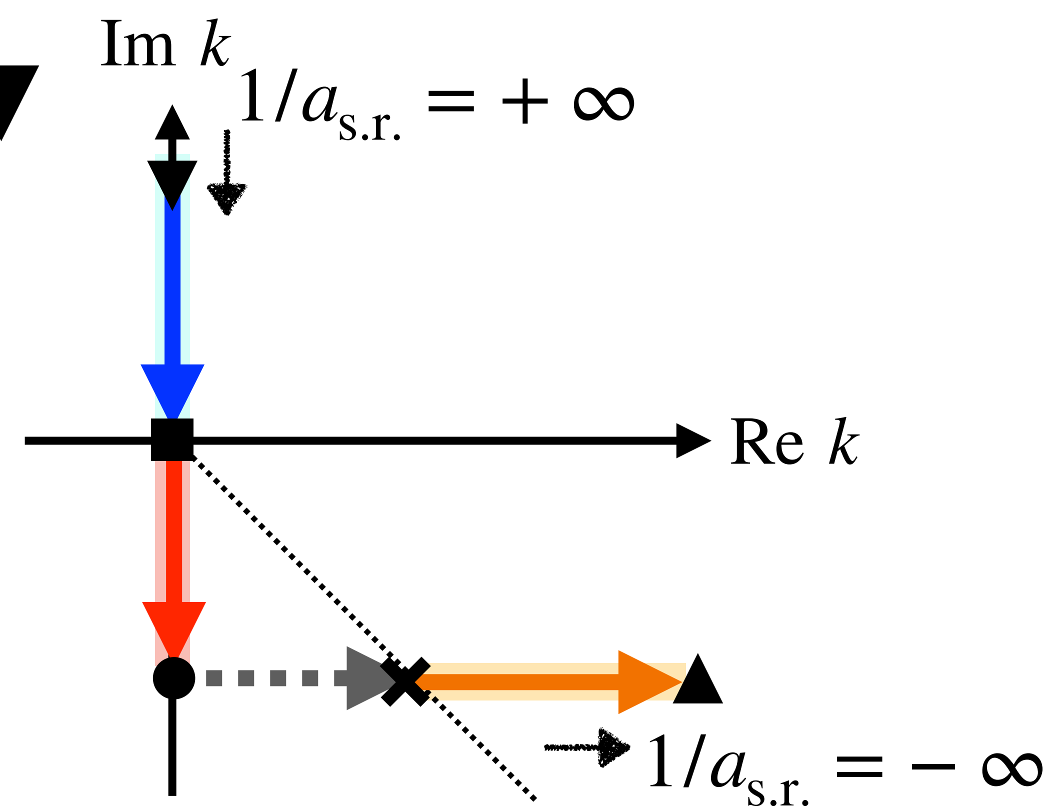
- eigenmomentum = pole of $f(k)$ ← scattering observables



T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) .

Compositeness (only w/ s.r.)



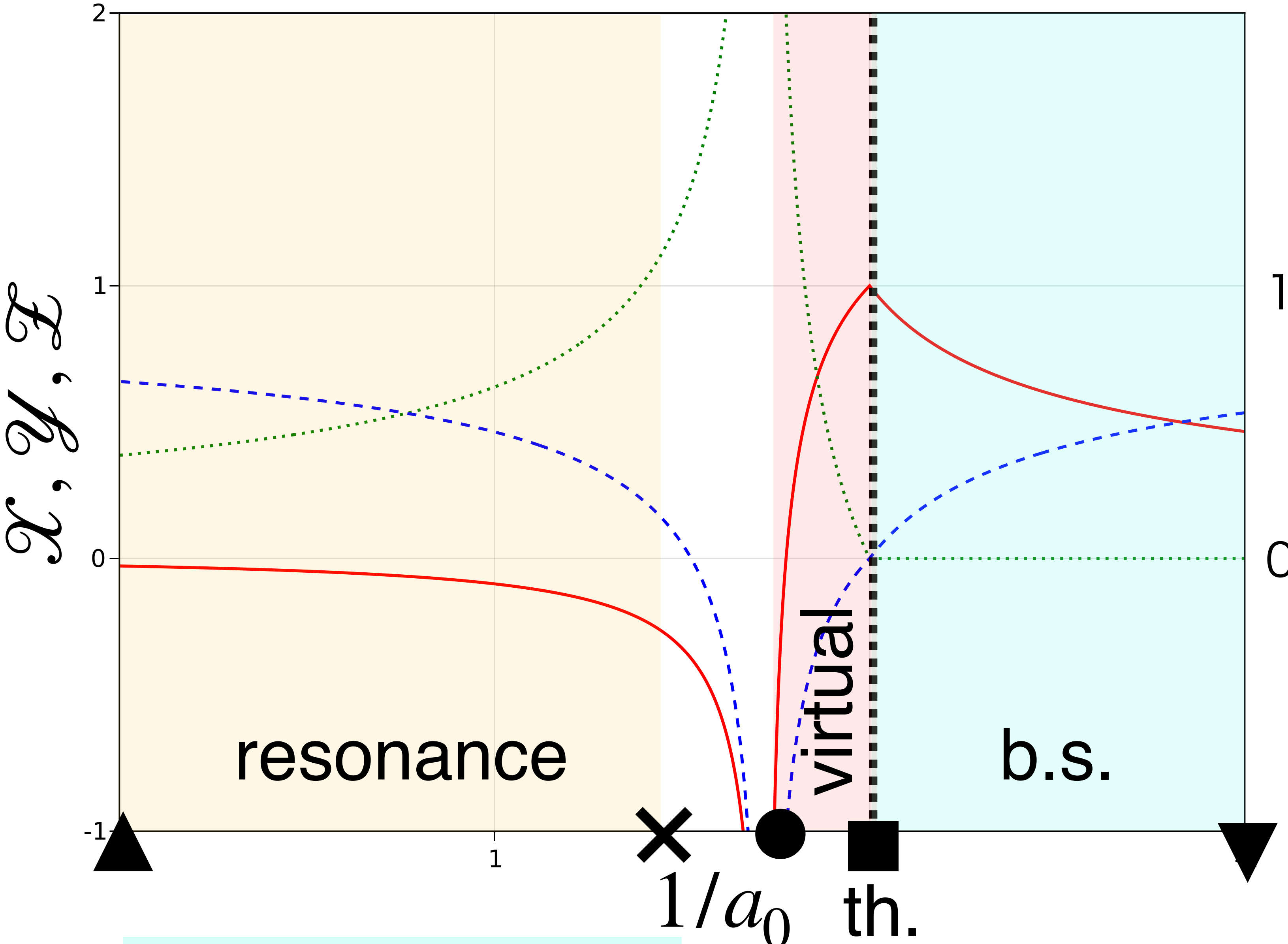


- b.s. : $0 \leq X \leq 1$
- virtual : $1 < X$

● divergence $X \to \infty$

- resonance : $\text{Im } X \leq 1$

$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ from b.s. to resonance



- interpretation of $X \in \mathbb{C}$

$\mathscr{X}$ —— compositeness

$\mathscr{Y}$ ····· uncertain

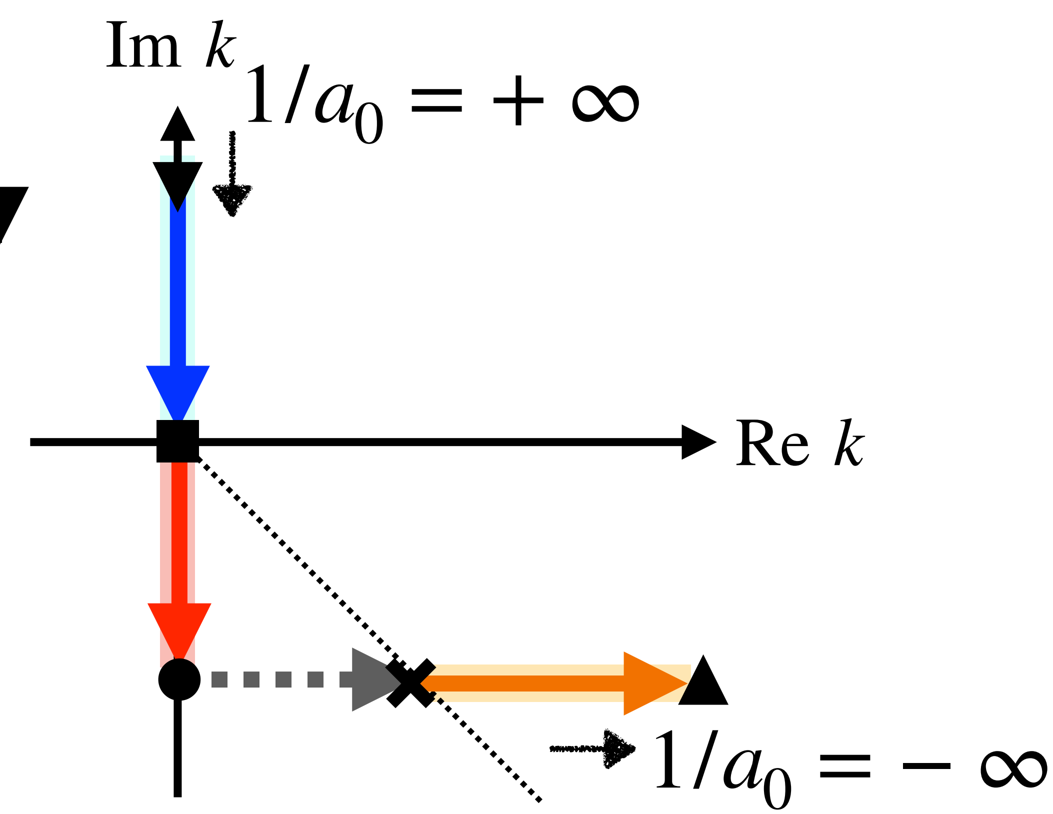
$\mathscr{Z}$ ······ elementarity

T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

- b.s. : $0 \leq X \leq 1$
- virtual : $\mathscr{Z} < 0$ (non-interpretable)

● divergence $\mathscr{X}, \mathscr{Y}, \mathscr{Z} \to \infty$

- resonance : $\mathscr{Z}$ dominant



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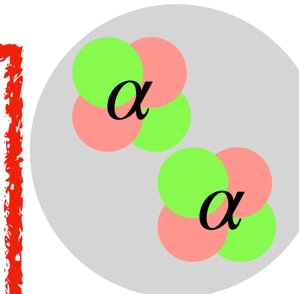
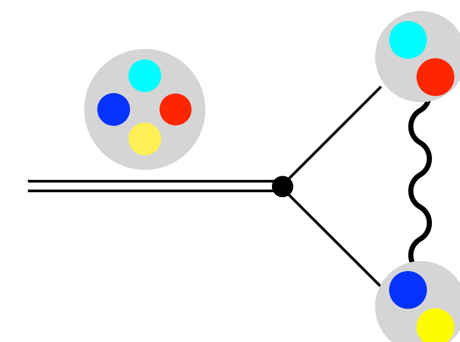
near-threshold states with **short-range** interaction

near-threshold bound states & resonances
with **Coulomb + short-range** interaction



framework



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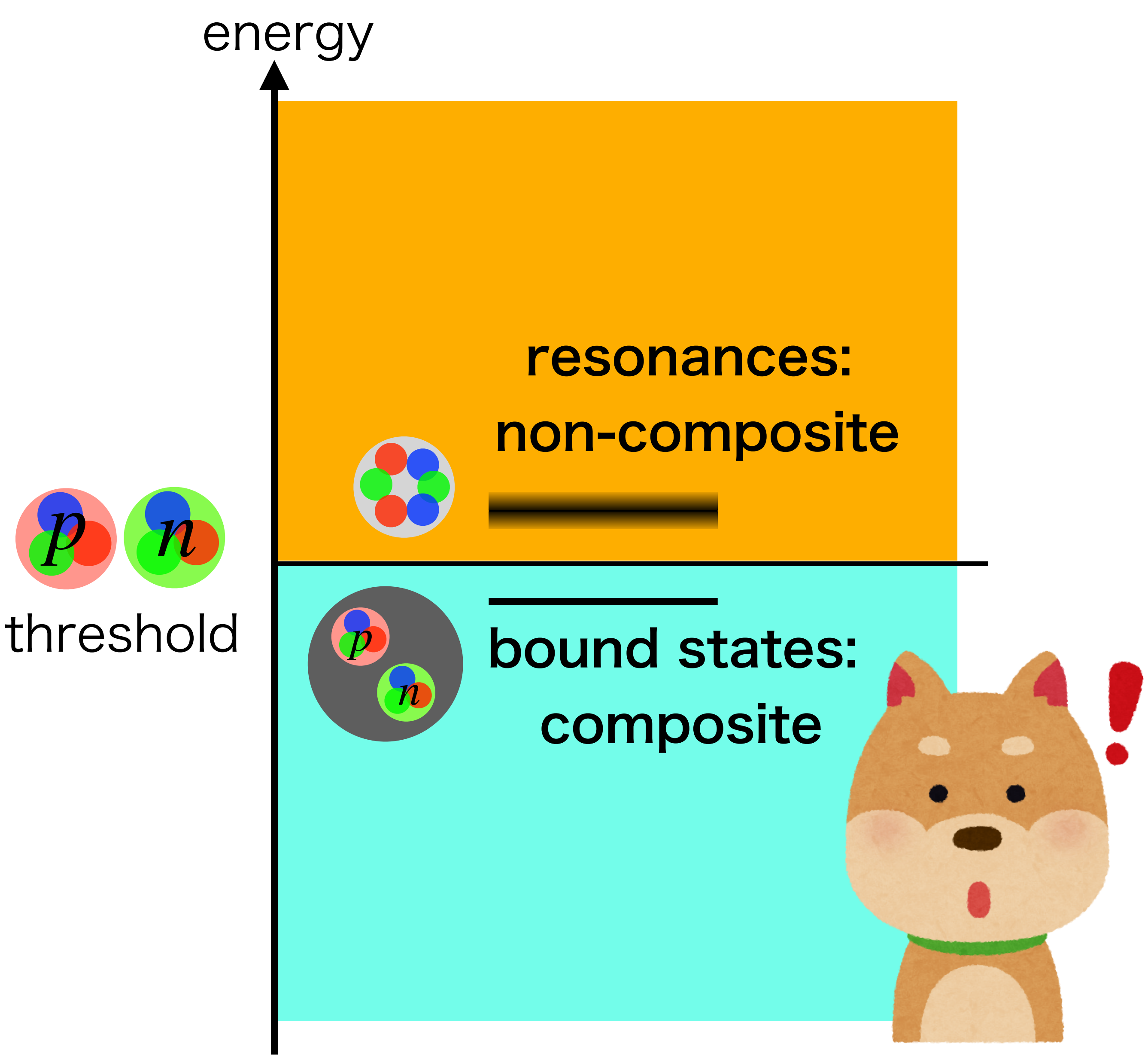
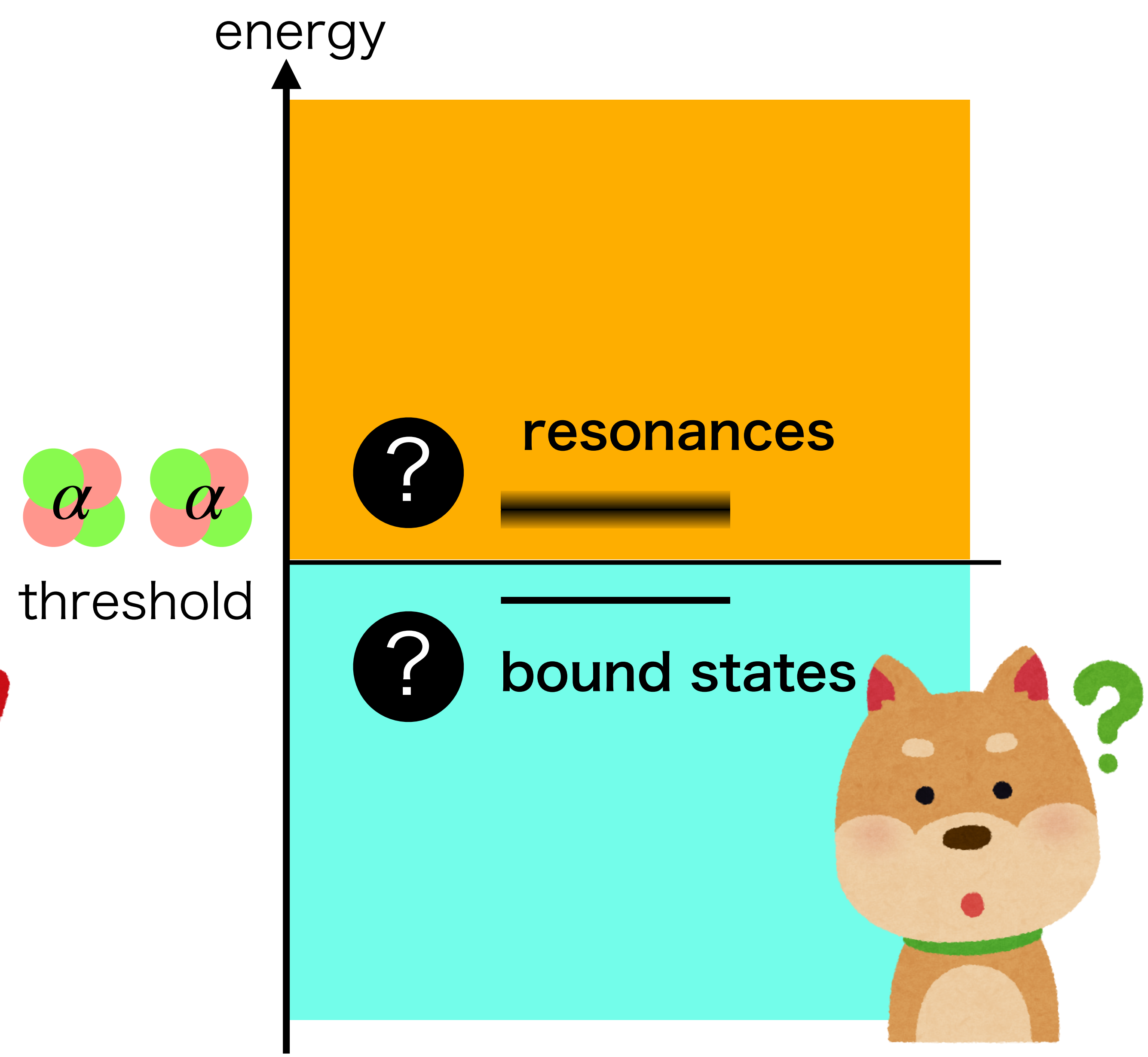
Near-th. states with Coulomb + s.r.



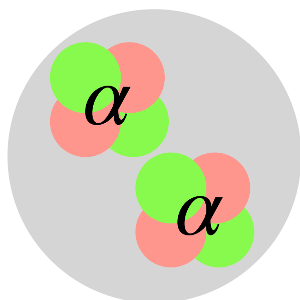


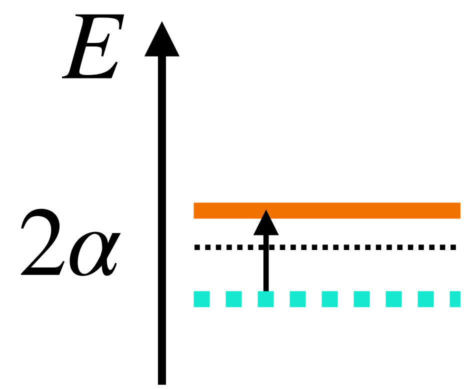
scatterings with **short range interaction**

scatterings with **Coulomb + short range** interaction

Coulomb + short range systems

- ^{8}Be nuclei ($2\alpha^{++}$) J. Hiura, and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 52, 25 (1972); R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809, 171 (2008).



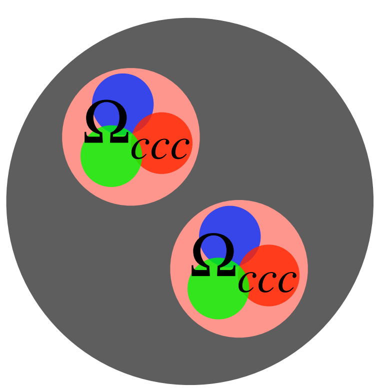


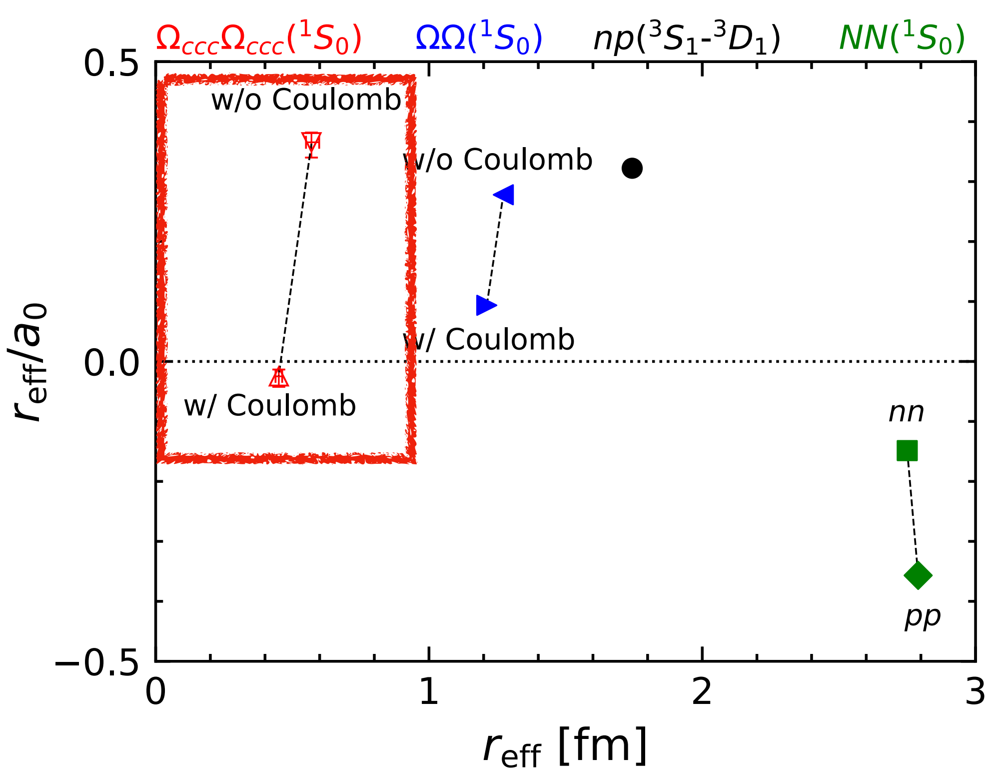
resonance (w/ Coulomb)

↑

bound (w/o Coulomb)

- $\Omega_{ccc}^{++}\,\Omega_{ccc}^{++}$ (HAL QCD) Y. Lyu, H.Tong, *et al*. [HAL QCD Coll.], Phys. Rev. Lett. 127 (2021) 072003.





resonance (w/ Coulomb)

↑

bound (w/o Coulomb)

- $\Xi^{-}\alpha$: Coulomb assisted bound state E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, Phys. Rev. C 106, 064318 (2022).

→ Coulomb is important for near-threshold states!

Coulomb + short range systems

◉ Coulomb + short range interaction

H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

R. Oppenheim Berger and Larry Spruch, Phys. Rev. 138, B1106-B1115 (1965).

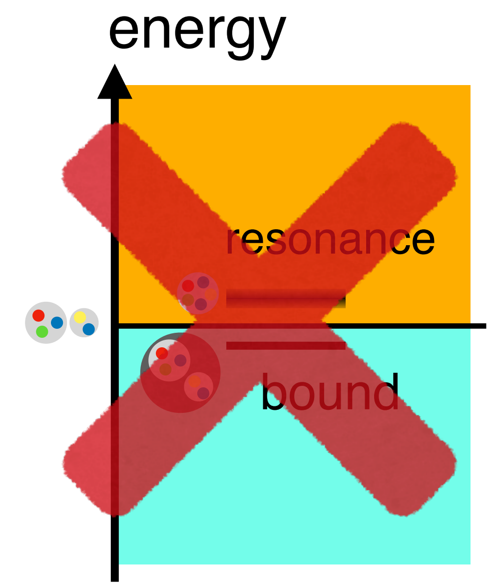
W. Domcke, Atom. Mol. Phys. 16, 359 (1983).

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809, 171 (2008).

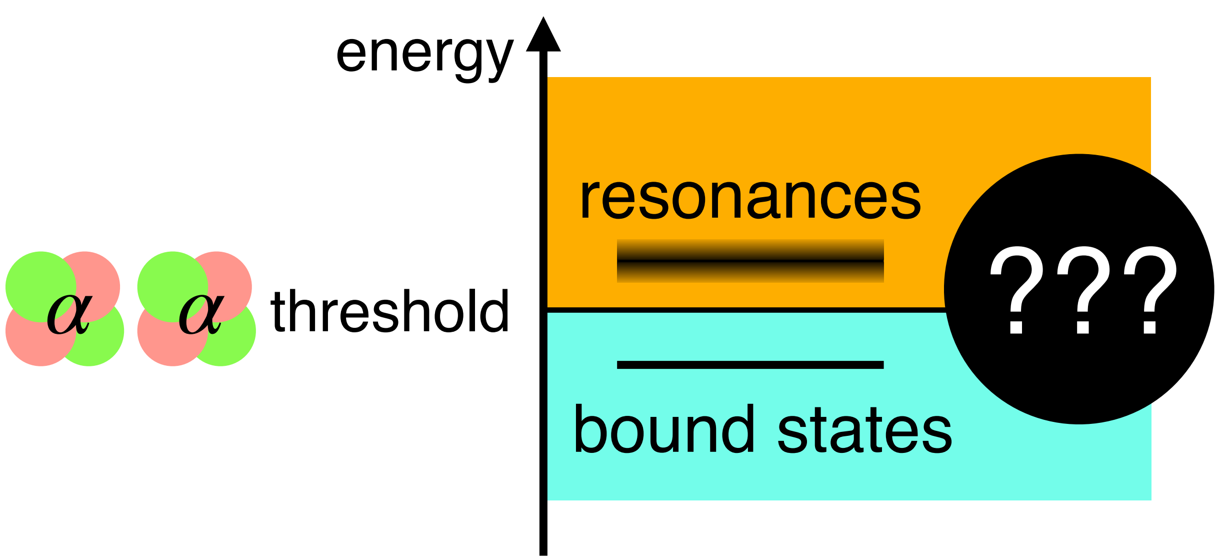
C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798, 135016 (2019).

S. Mochizuki, and Y. Nishida, Phys. Rev. C 110 , 064001 (2024).

- low-energy behavior of scattering amplitude is different from that of short range interaction



◉ nature of near-threshold state with Coulomb + short range interaction?



- two body
- small k region ⟶ s-wave
- Coulomb repulsive and attractive

⟶ pole trajectories?

⟶ compositeness?

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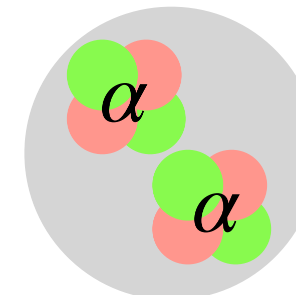
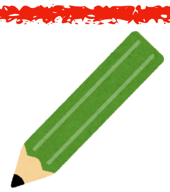
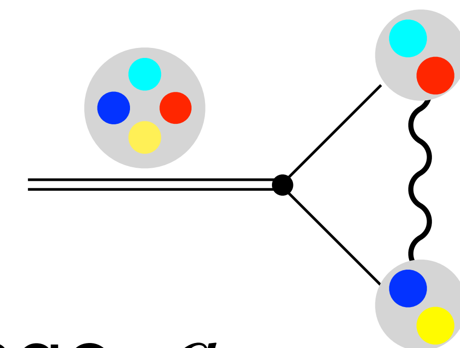
near-threshold states with **short-range** interaction

near-threshold bound states & resonances
with **Coulomb + short-range** interaction





<u>framework : model with Feshbach method</u>



- bare state which couples to Coulomb scattering
- Coulomb scattering length, Coulomb effective range, a_B

<u>numerical calculations & discussion</u>

- investigate pole trajectory
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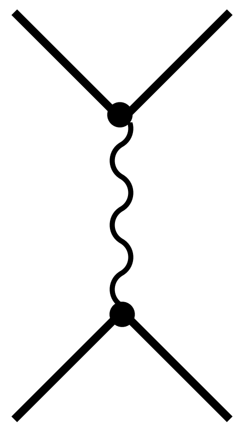
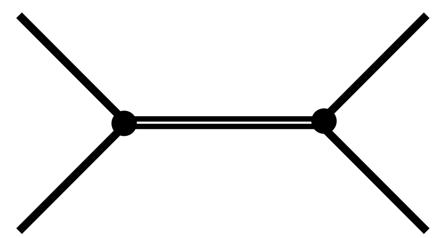
Coulomb+short range model

◉ model

Coulomb



short range (s.r.)

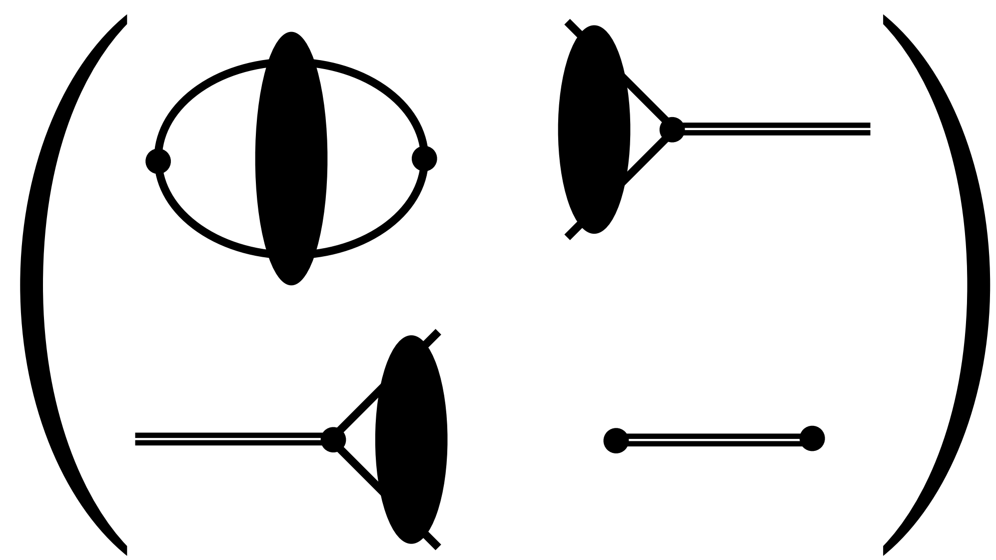


S. Weinberg, Phys. Rev. 137, 672–678 (1965);
V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004);
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

◉ Hamiltonian

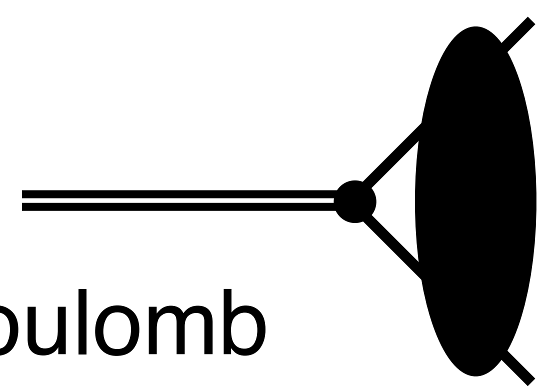
W. Domcke, Atom. Mol. Phys. 16 359 (1983);
H. Feshbach, Annals Phys. 19 287-313 (1962).
C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798, 135016

$$\hat{H} = \begin{pmatrix} \hat{H}_{PP} & \hat{H}_{PQ} \\ \hat{H}_{QP} & \hat{H}_{QQ} \end{pmatrix} =$$



$X = 0$ channel

⇔ b.s. w/o Coulomb



$X = 1$ channel

⇔ scattering w/ Coulomb

EFT framework

◉ Hamiltonian

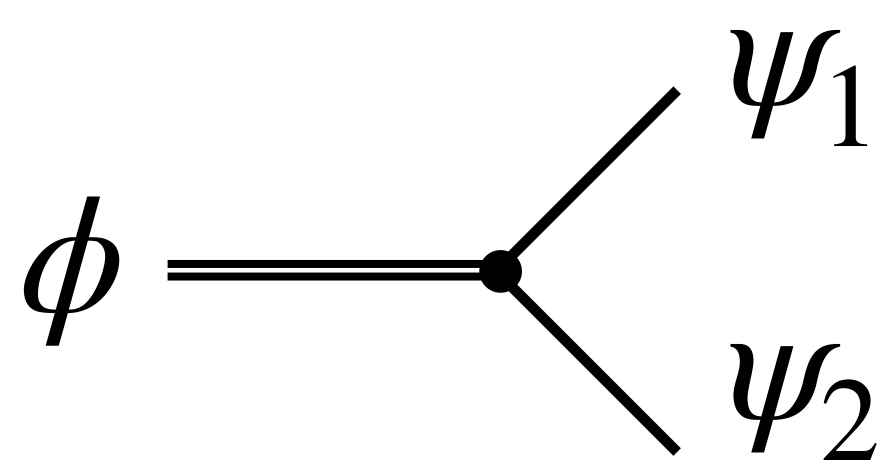
X. Kong and F. Ravndal, Phys. Lett. B 450, 320 (1999); Nucl. Phys. A 665, 137 (2000);
S.-i. Ando, J. W. Shin, C. H. Hyun, and S. W. Hong, Phys. Rev. C 76, 064001 (2007);
R. Higa, H. W. Hammer, and U. van Kolck, Nucl. Phys. A 809, 171 (2008).

free Hamiltonian

$$H_0 = \int d\boldsymbol{r} \ \left[\frac{1}{2m_1} \nabla \psi_1^\dagger \cdot \nabla \psi_1 + \frac{1}{2m_2} \nabla \psi_2^\dagger \cdot \nabla \psi_2 \right]$$

short-range interaction

$$H_S = \int d\boldsymbol{r} \ \left[\frac{1}{2M} \nabla \phi^\dagger \cdot \nabla \phi + \nu_0 \phi^\dagger \phi + g_0 (\phi^\dagger \psi_1 \psi_2 + \psi_1^\dagger \psi_2^\dagger \phi) \right]$$



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◉ Hamiltonian

X. Kong and F. Ravndal, Phys. Lett. B 450, 320 (1999); Nucl. Phys. A 665, 137 (2000);
S.-i. Ando, J. W. Shin, C. H. Hyun, and S. W. Hong, Phys. Rev. C 76, 064001 (2007);
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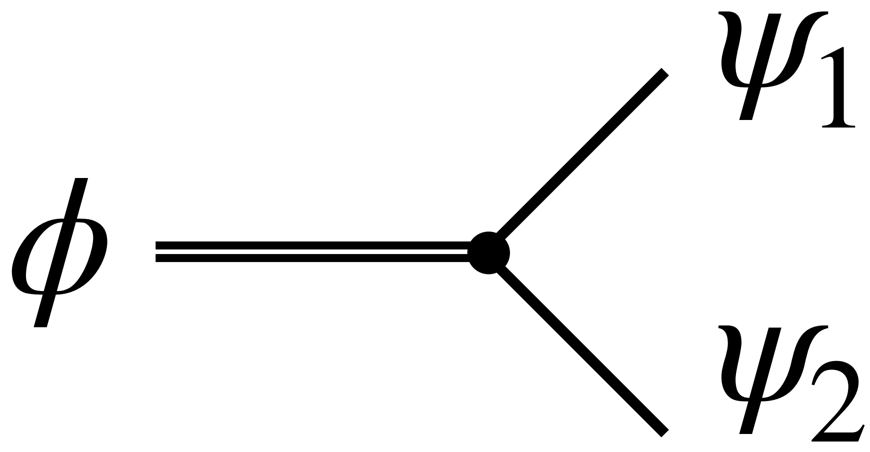
$$H_S = \int d\boldsymbol{r} \left[\frac{1}{2M} \nabla \phi^\dagger \cdot \nabla \phi + \nu_0 \phi^\dagger \phi + g_0 (\phi^\dagger \psi_1 \psi_2 + \psi_1^\dagger \psi_2^\dagger \phi) \right]$$

Coulomb interaction

$$V_C = \frac{Z_1 Z_2 \alpha}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}$$

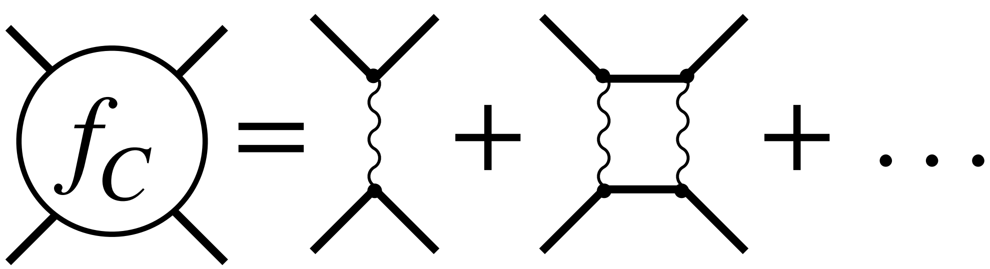


Scattering amplitude

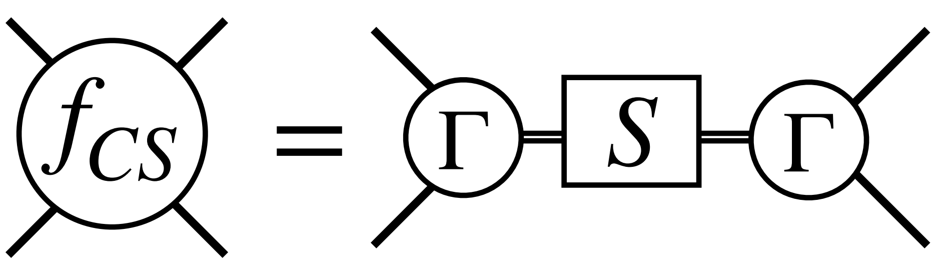
H. Feshbach, Annals Phys. 19 287-313 (1962);
W. Domcke, Atom. Mol. Phys. 16 359 (1983);
R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

decomposition of amplitude $f = f_C + f_{CS}$

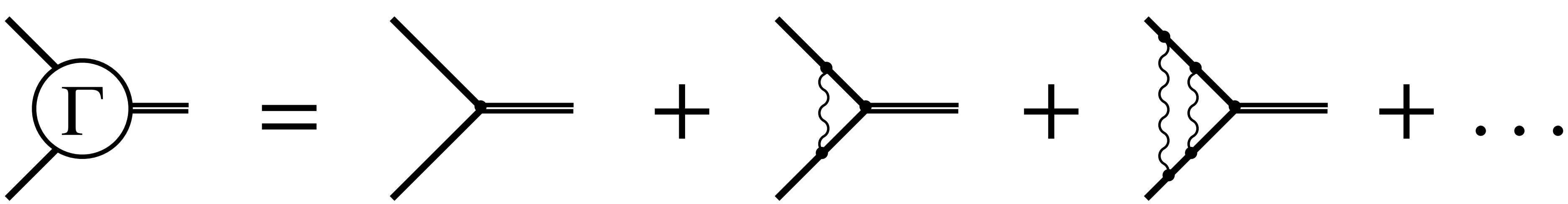
c.f. Lippmann-Schwinger eq.: $f \propto [V^{-1} - G^0]^{-1}$

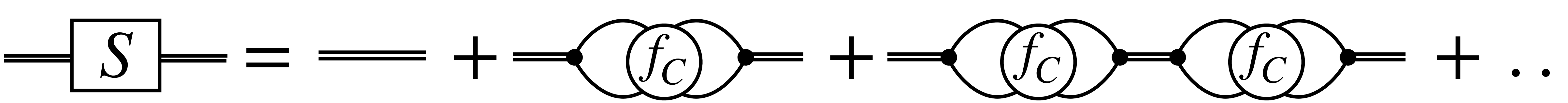


pure Coulomb amplitude



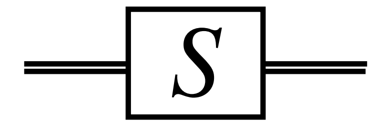
Coulomb distorted short-range amplitude



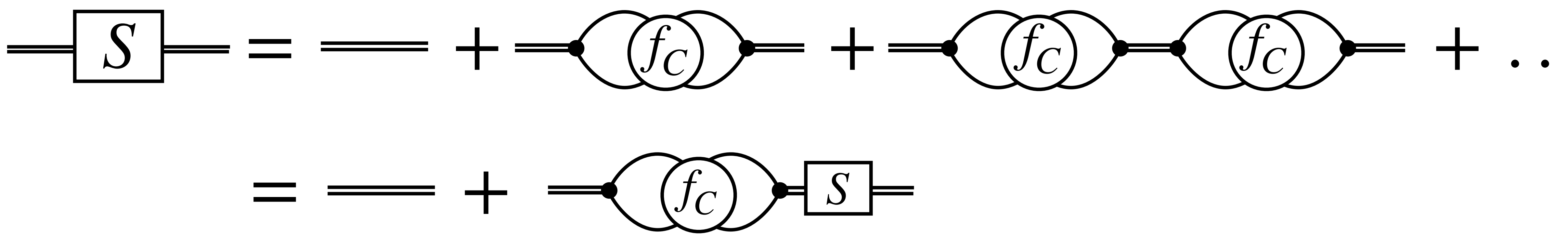


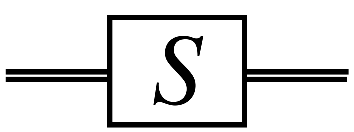
Coulomb distorted amplitude

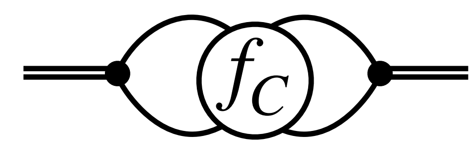
◉ <u>pole condition</u> $f(k) = f_C(k) + f_{CS}(k)$

pole of $f(k)$ $\Leftrightarrow$ pole of $f_{CS}(k)$ $\Leftrightarrow$ 

$= \infty$





$= S(E)$,  $= (E - \nu_0)^{-1}$, 

$= \Sigma(E)$

$$\longrightarrow S(E) = (E - \nu_0)^{-1} + (E - \nu_0)^{-1}\Sigma(E)S(E)$$
$$= [E - \nu_0 - \Sigma(E)]^{-1}$$

$\longrightarrow$ pole condition : $E - \boxed{\nu_0} - \boxed{\Sigma(E)} = 0$

bare state energy — self energy

Coulomb distorted amplitude

◉ scattering amplitude R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

$$f_{CS}(k) = C_\eta^2 e^{2i\sigma_0} \left[\frac{2\pi\Delta^{(R)}}{\mu g_0^2} - \frac{\pi}{\mu^2 g_0^2}k^2 - 2k_C H(\eta) \right]^{-1}.$$

$$C_\eta^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1}$$: Sommerfeld factor

σ_0 : s-wave pure Coulomb phase shift

$\Delta^{(R)}$: renormalized mass parameter

$$\eta = \frac{k_C}{k},\ \ k_C = \frac{1}{a_B}\mathrm{sgn}(Z_1 Z_2),\ a_B = \frac{\hbar c}{\mu c^2 |Z_1 Z_2|}$$: Borh radius

$$H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \log[i\eta\ \mathrm{sgn}(k_C)],\ \psi(x) = \frac{d}{dx}\log(\Gamma(x))$$

Coulomb distorted amplitude

◉ scattering amplitude by scattering observables

- ERE with Coulomb

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

$$f(k) \propto \left[-\frac{1}{a_s^C} + \frac{r_e^C}{2} k^2 + \mathcal{O}(k^4) - 2k_C H(\eta) \right]^{-1} .$$

$$\longrightarrow f_{CS}(k) = C_\eta^2 e^{2i\sigma_0} \left[-\frac{1}{a_s} + \frac{r_e}{2} k^2 - 2k_C H(\eta) \right]^{-1} .$$

a_s : Coulomb scattering length

r_e : Coulomb effective range

◉ compositeness X

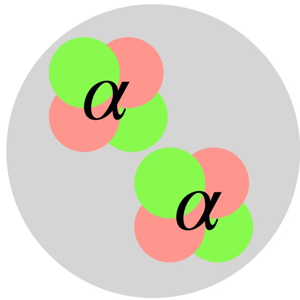
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

$$X = 1 - \frac{1}{1 - \frac{d}{dE}\Sigma(E)}, \quad \frac{d}{dE}\Sigma(E) = \frac{d}{dE} H(\eta) .$$

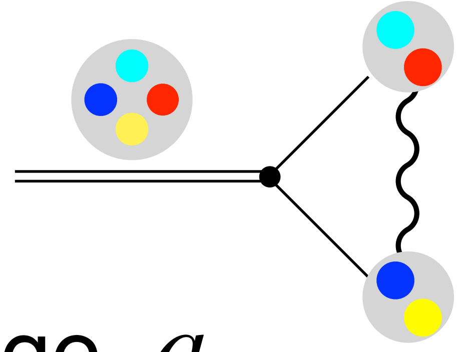
Outline

near-threshold states with **short-range** interaction

near-threshold bound states & resonances
with **Coulomb + short-range** interaction



<u>framework : model with Feshbach method</u>



- bare state which couples to Coulomb scattering
- Coulomb scattering length, Coulomb effective range, a_B

<u>numerical calculations & discussion</u>

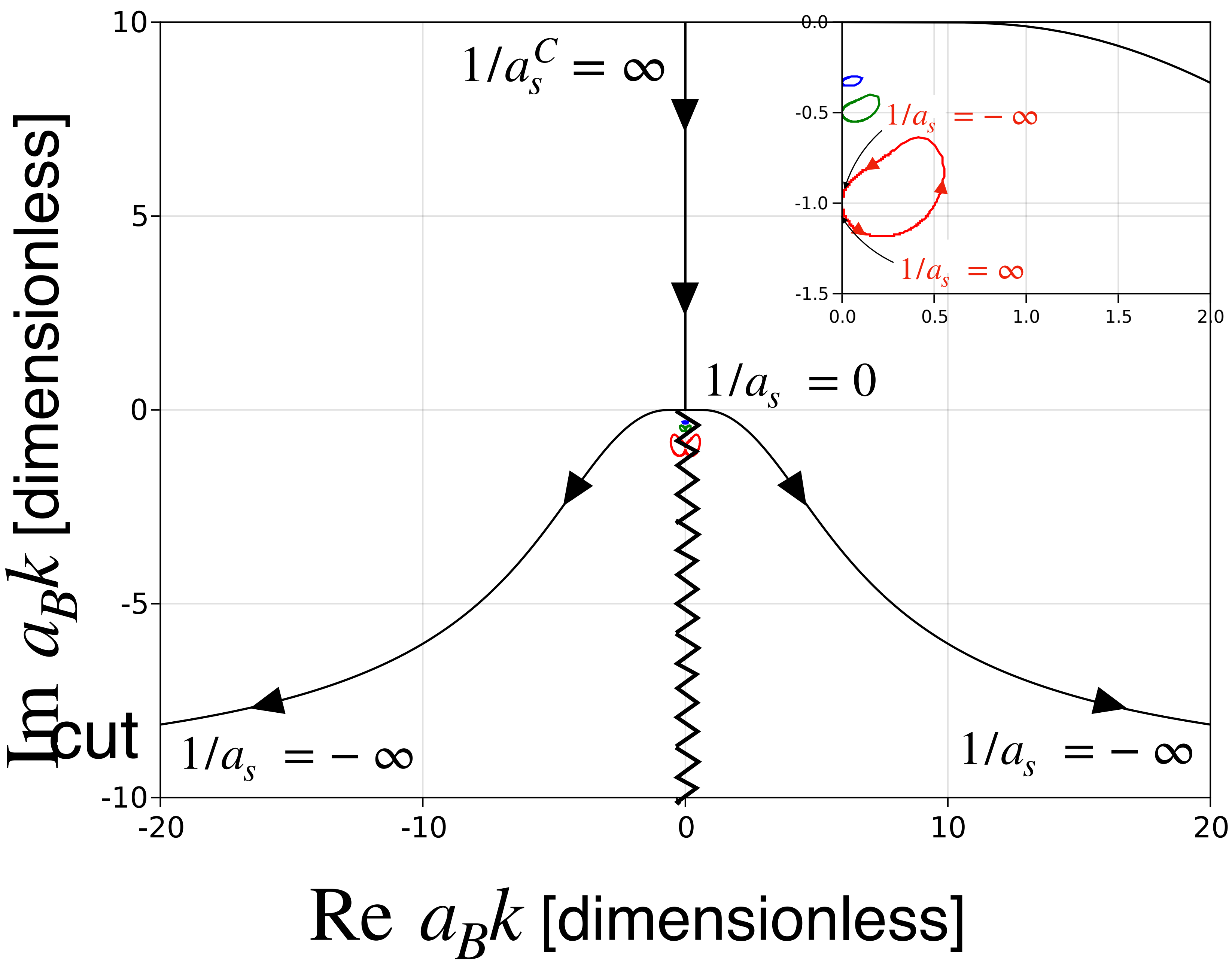
- investigate pole trajectory
- analyze internal structure with compositeness
- study universal nature of near-threshold states

Pole trajectory (repulsive Coulomb)

◉ pole trajectory in complex momentum k plane

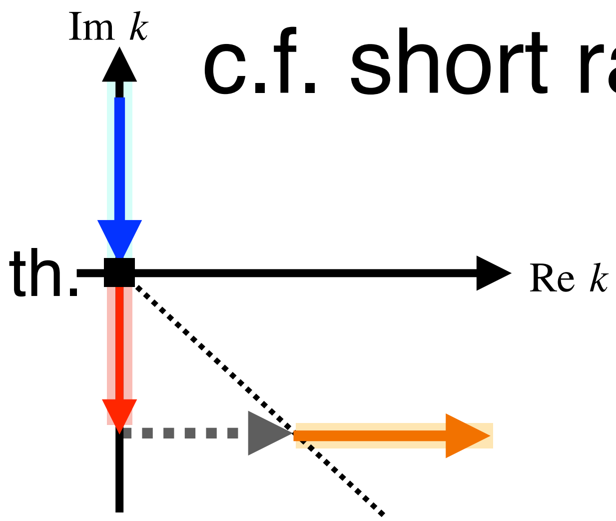
- varying Coulomb scattering length a_s with fixed r_e and a_B

→ pole position (eigenmomentum) moves



- b.s directory goes to resonance

c.f. short range



- $a_s \to \infty$ at threshold
- but no universality

∵ radius of w.f. $< \infty$

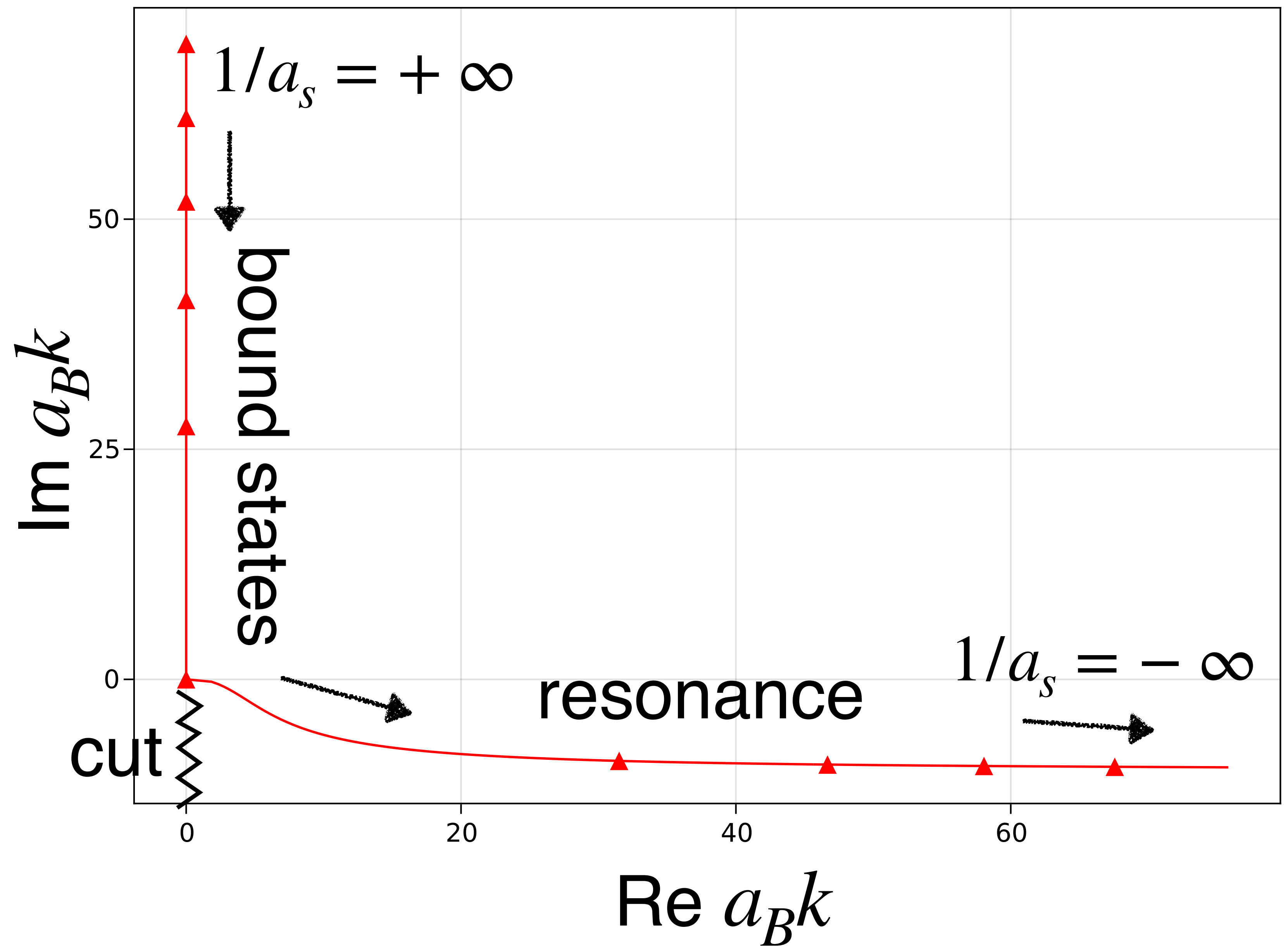
S. Mochizuki, and Y. Nishida, Phys. Rev. C 110 , 064001 (2024).

Pole trajectory (repulsive Coulomb)

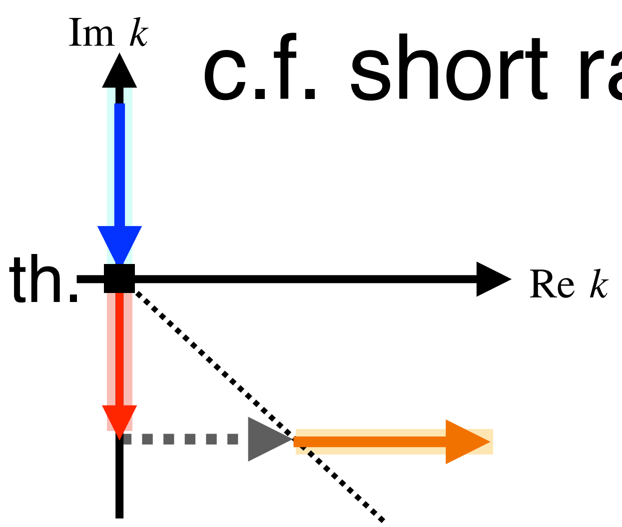
◉ pole trajectory in complex momentum k plane

- varying Coulomb scattering length a_s with fixed r_e and a_B

  → pole position (eigenmomentum) moves



- b.s directory goes to resonance



c.f. short range

- $a_s \to \infty$ at threshold
- but no universality

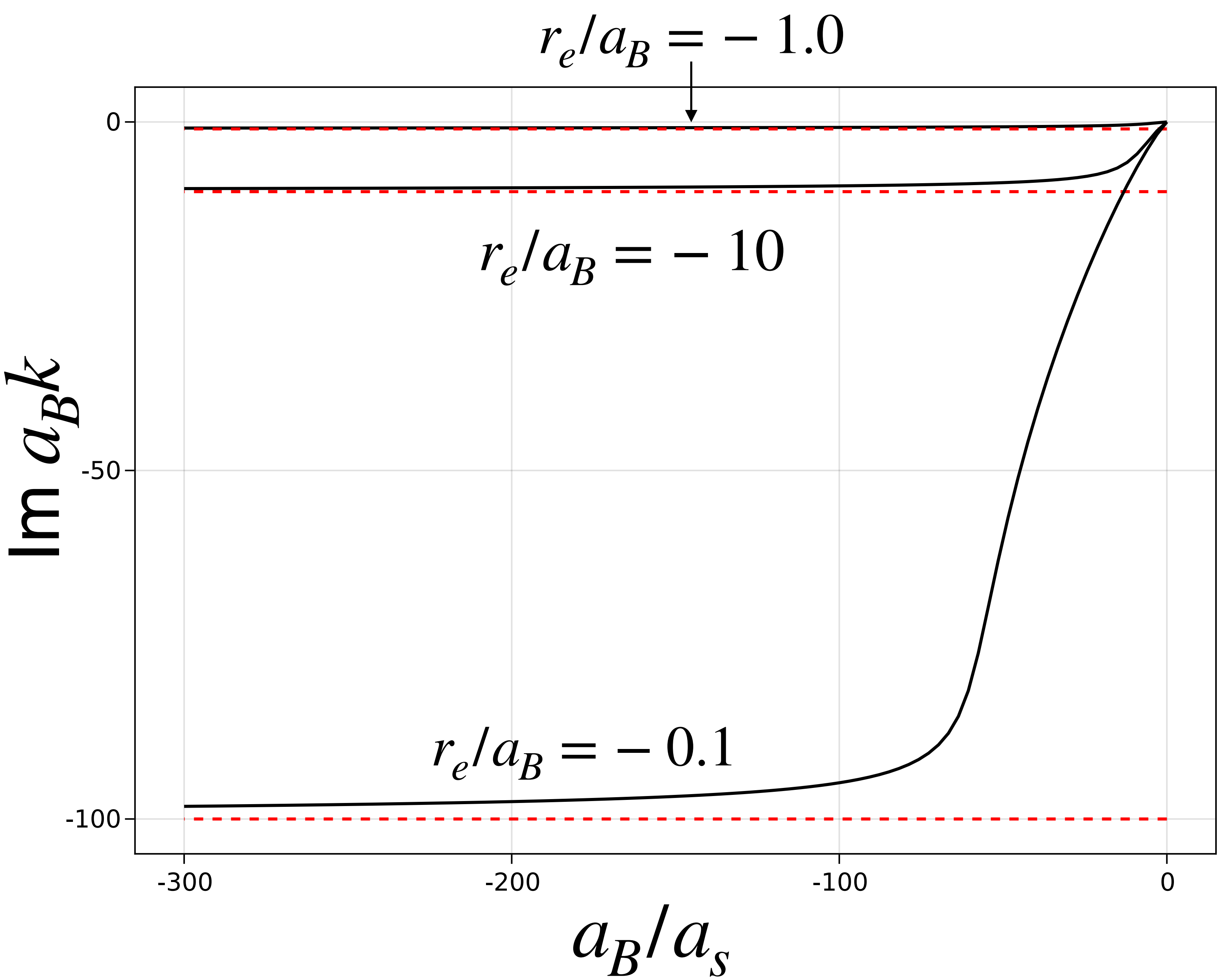
  ∵ radius of w.f. $< \infty$

S. Mochizuki, and Y. Nishida, Phys. Rev. C 110 , 064001 (2024).

far from threshold (repulsive Coulomb)

◉ imaginary part of eigenenergy in complex momentum k plane

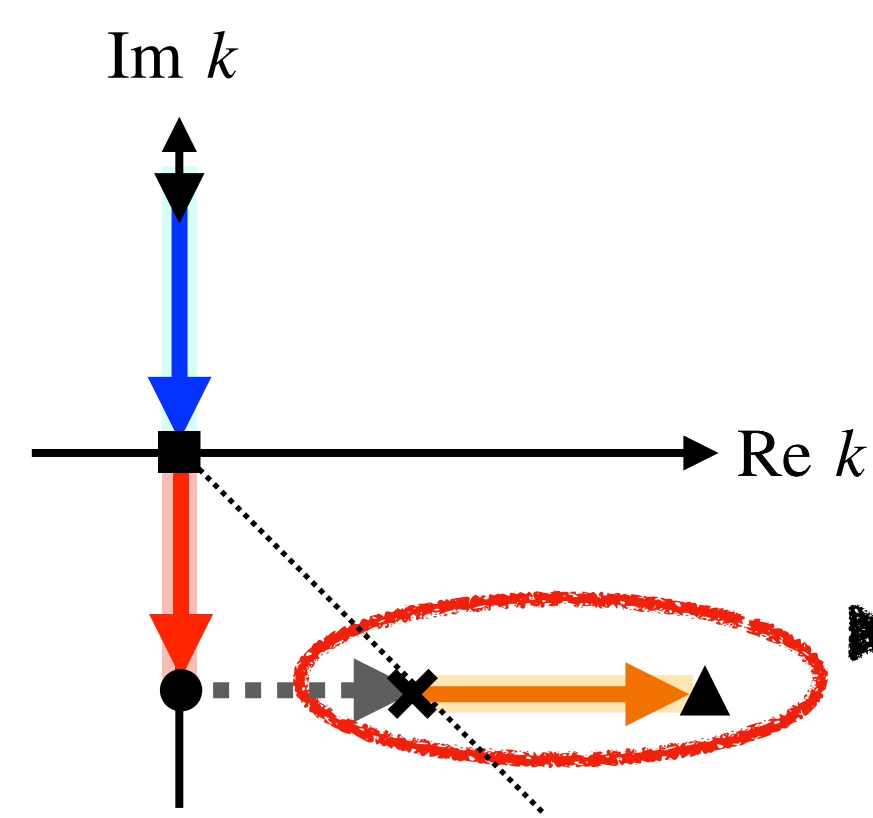
- far from threshold in $1/a_s \to -\infty$ limit



dotted lines :a_B/r_e

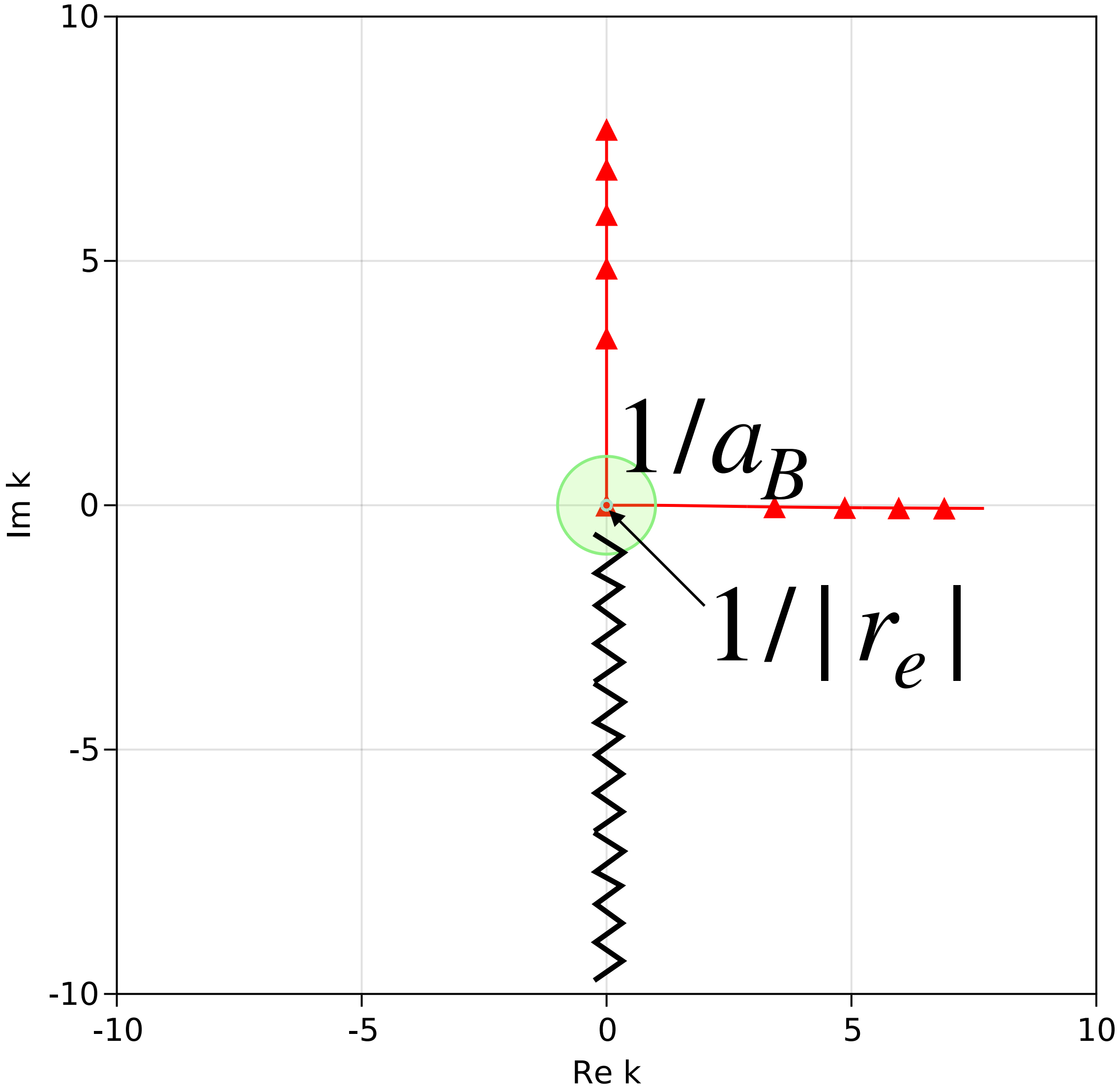
- Im $k \to 1/r_e$

→ trajectory close to that of resonance in ERE

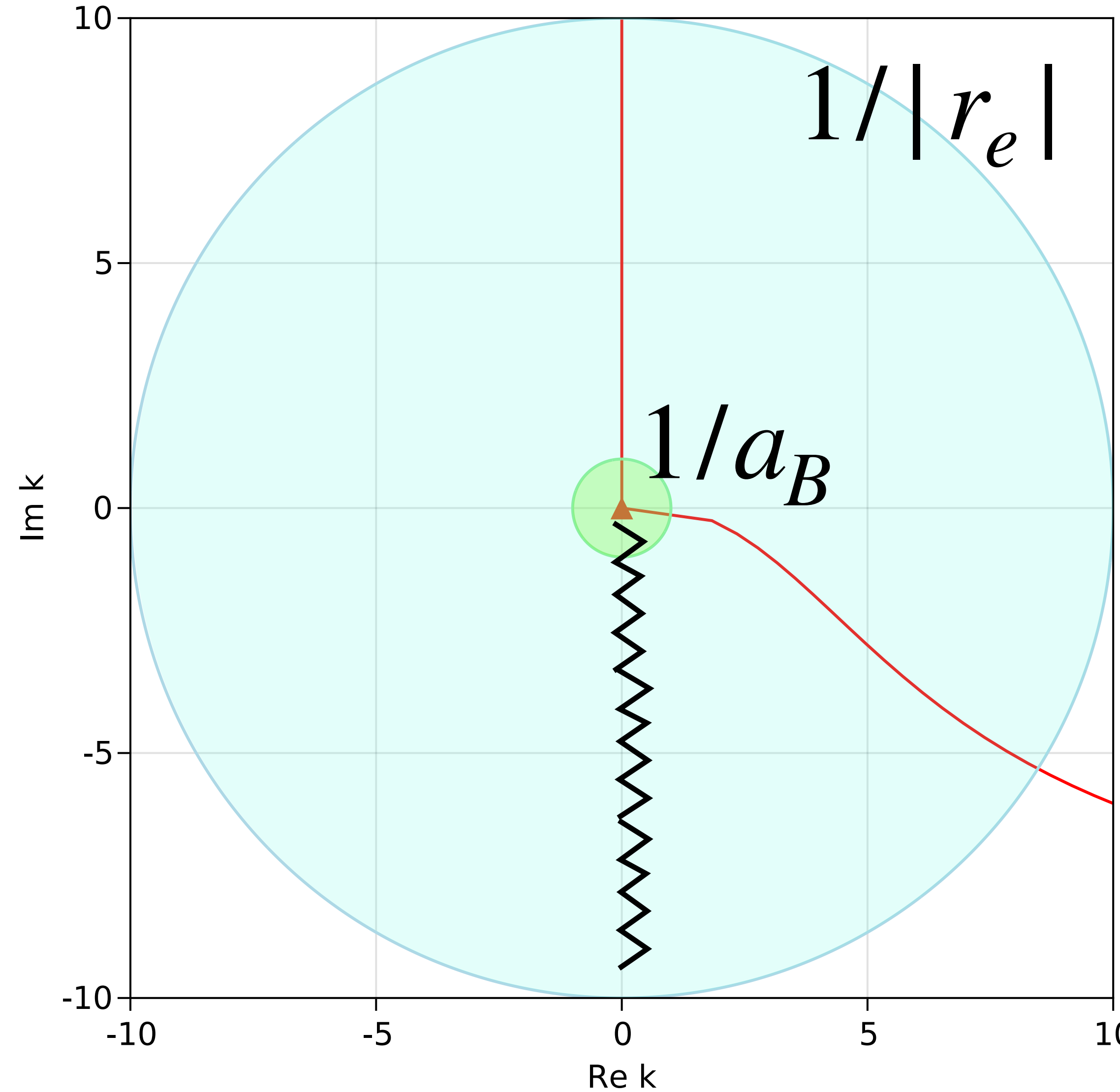


Pole trajectory (repulsive Coulomb)

$a_B = 1, r_e = -10$



$a_B = 1, r_e = -0.1$

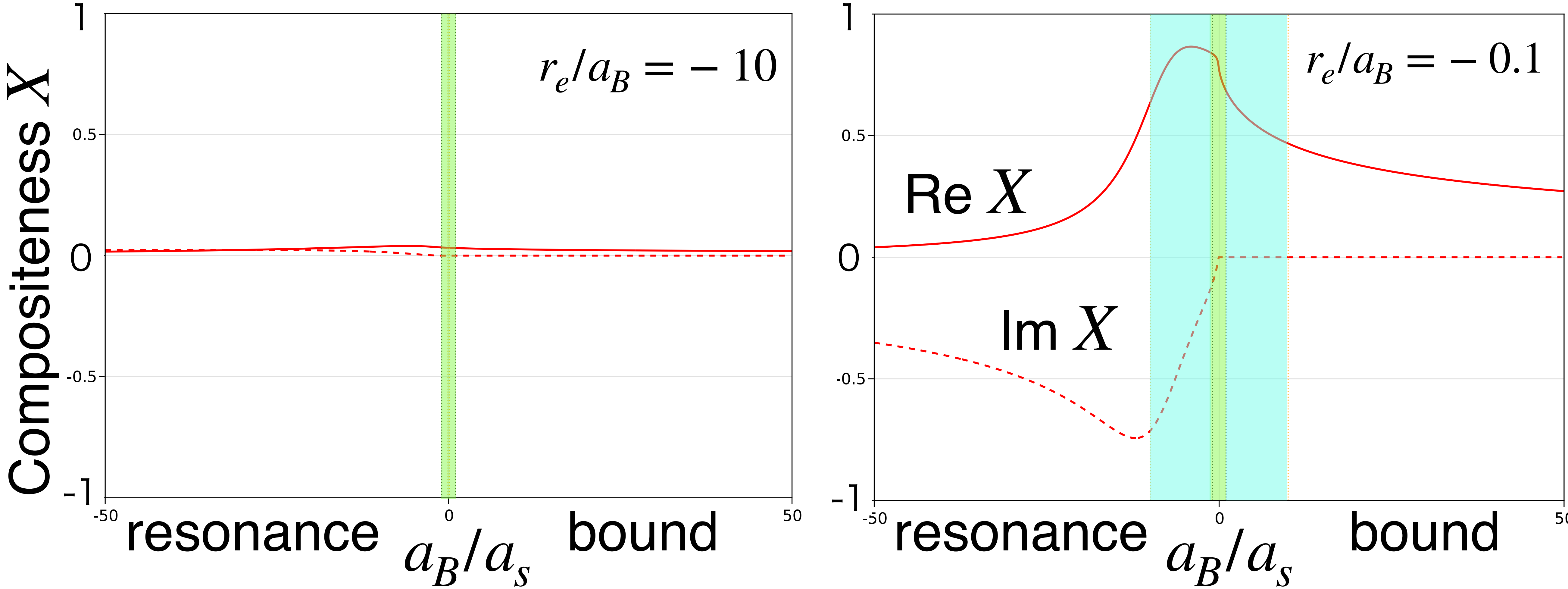


short range universality $\wedge$ Coulomb dominant region

short range universality $\vee$ Coulomb dominant region

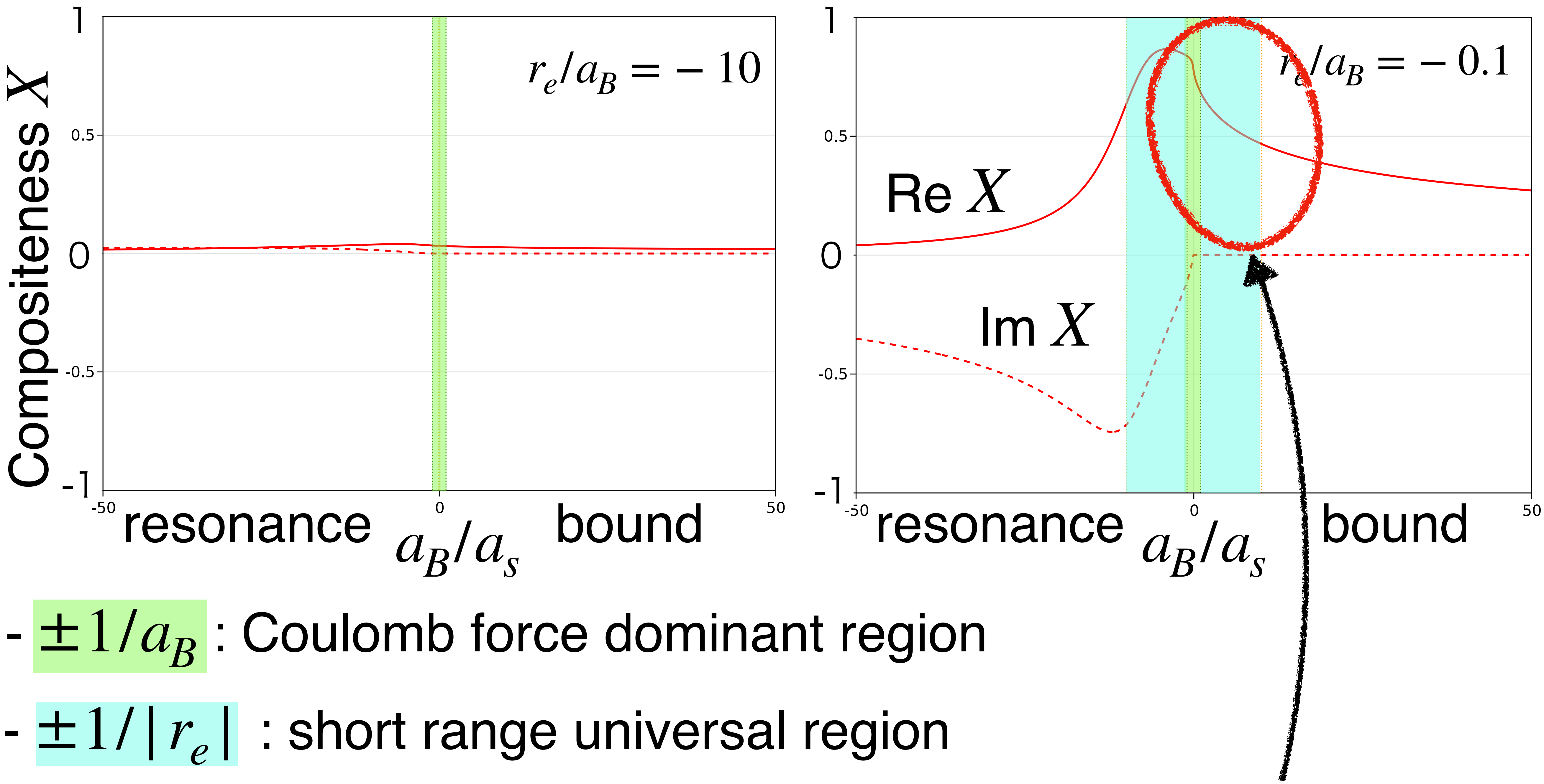
Compositeness (repulsive Coulomb)

Coulomb > short range　　　　Coulomb < short range



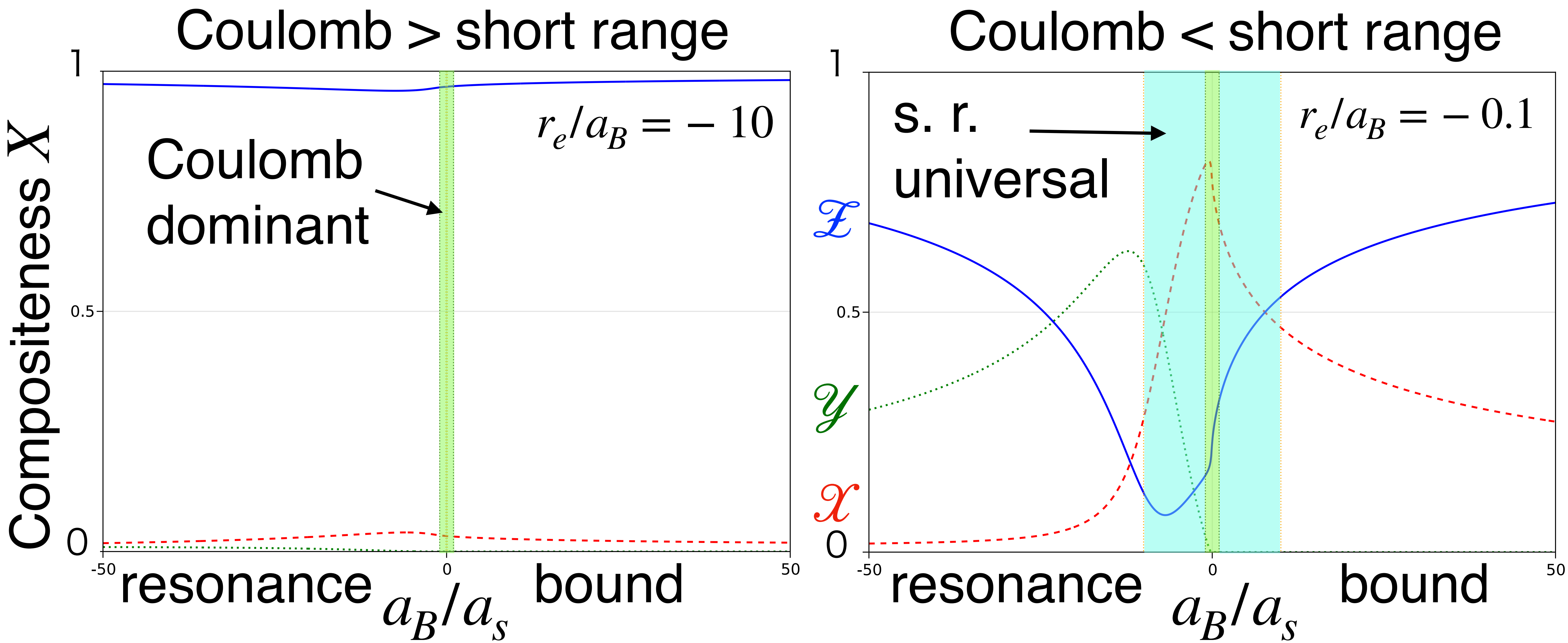
- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region

Compositeness (repulsive Coulomb)



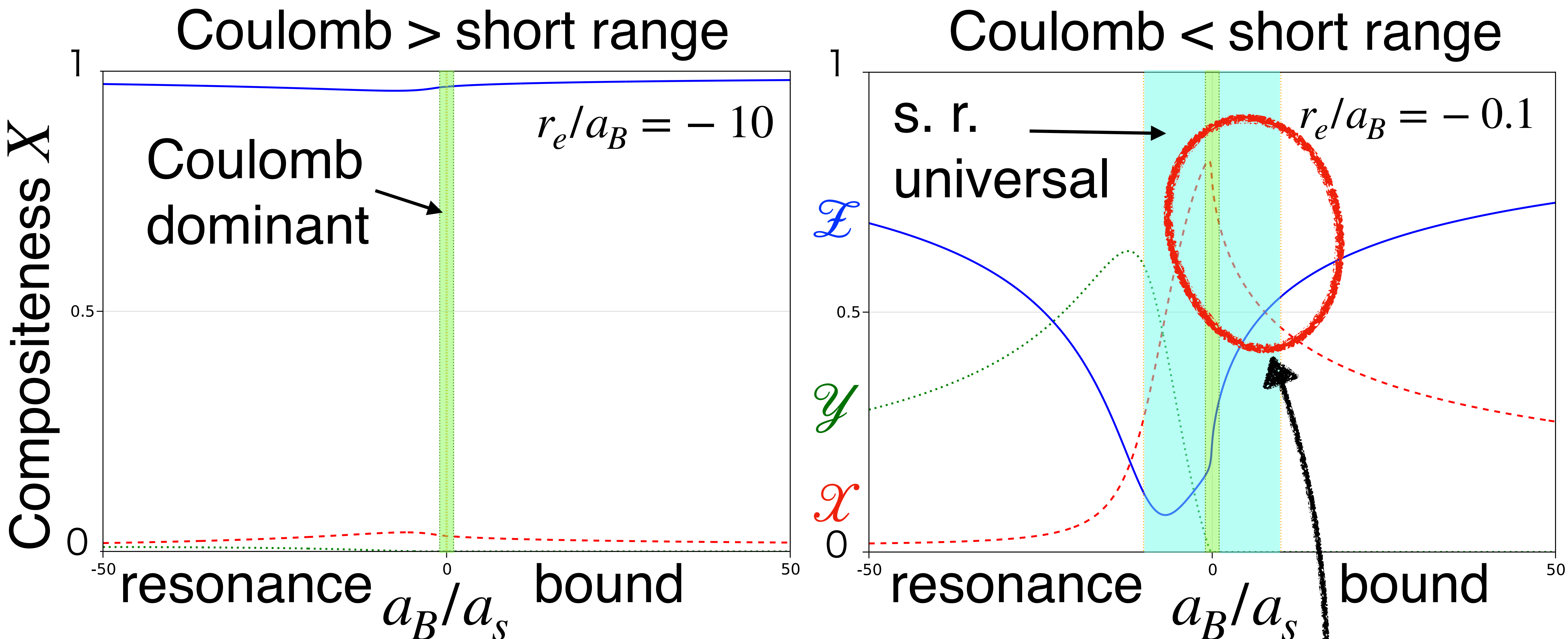
- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- remnant of short range universality in $|r_e| \ll |a_B|$ case
  $X \to 1$ in $B \to 0$ limit in short range

Compositeness (repulsive Coulomb)



- complex compositeness ⟵ $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].
- states with large $|1/a_s|$ are elementary $\mathscr{Z}$ dominant
- structure of bound states ≈ resonances ∵ continuous X

Compositeness (repulsive Coulomb)



- complex compositeness ← $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].
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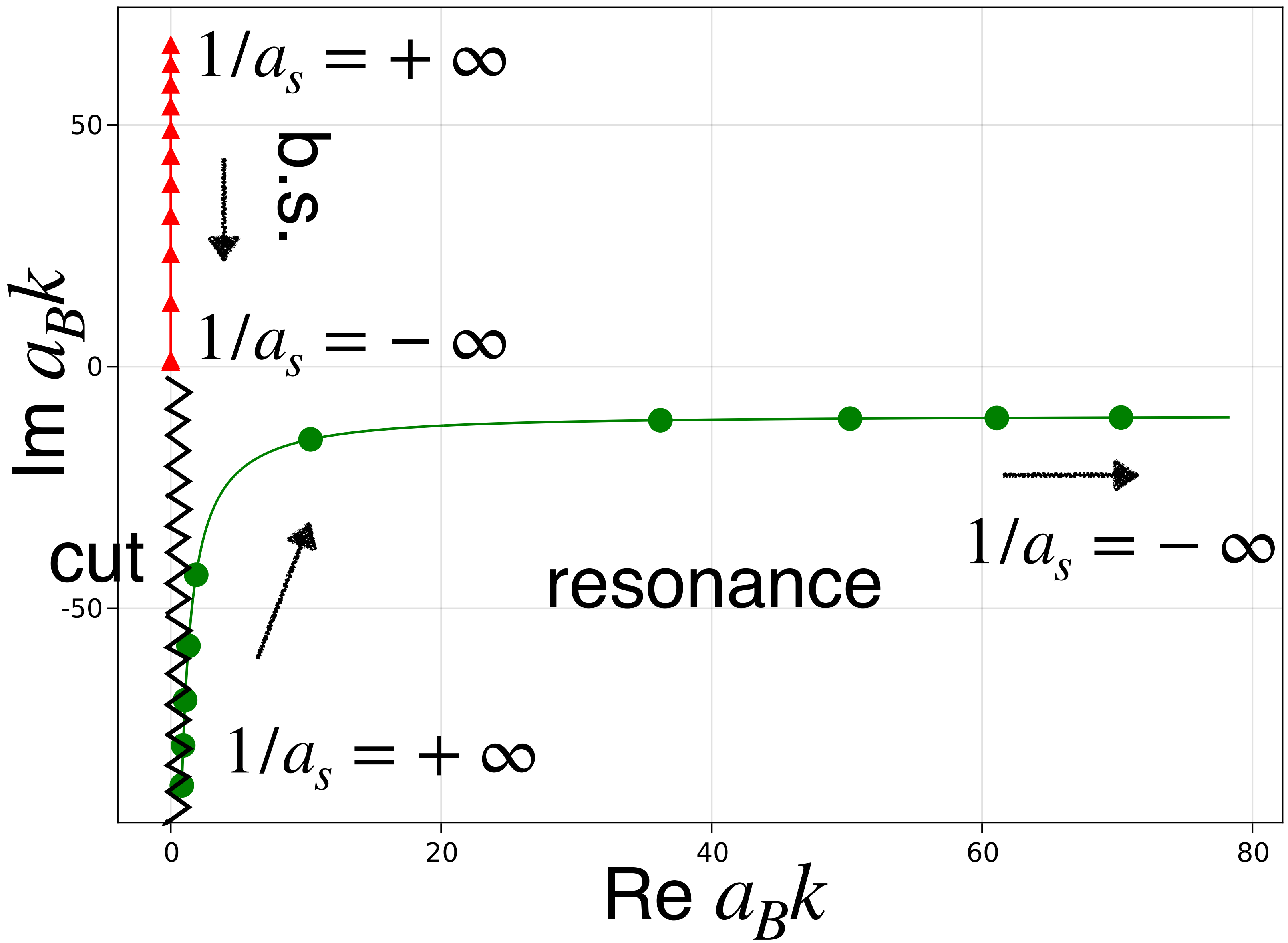
- remnant of short range universality in $|r_e| \ll |a_B|$ case
  $X \to 1$ in $B \to 0$ limit in short range

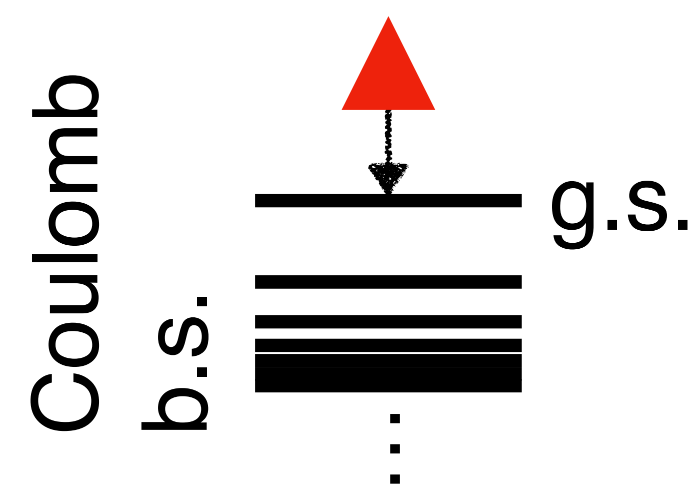
Pole trajectory (attractive Coulomb)

◉ pole trajectory in complex momentum k plane



- bound ≠ resonance

- bound pole
 → Coulomb g.s.

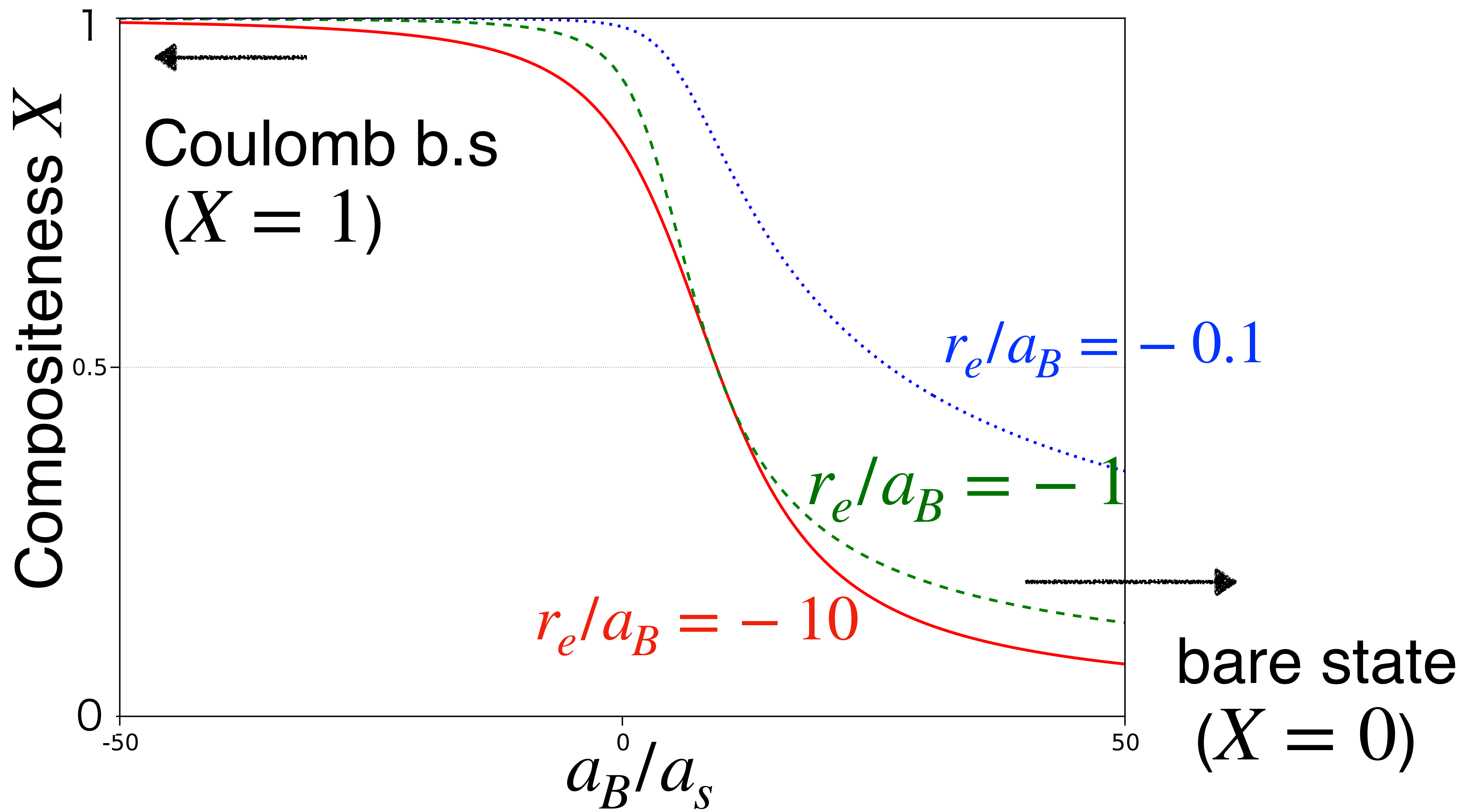


- pole cannot go to $k = 0$

W. Domcke, Atom. Mol. Phys. 16 359 (1983);

S. Mochizuki, and Y. Nishida,
Phys. Rev. C 110 , 064001 (2024).

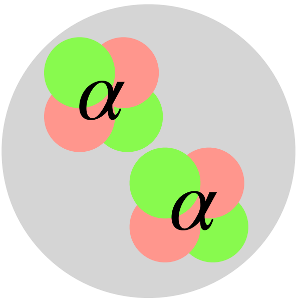
Compositeness (att. Coulomb b.s.)

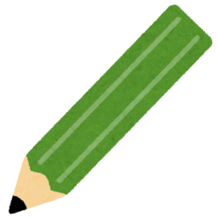


- $1/a_s \to +\infty$: states becomes elementary dominant ($X \to 0$)
- no short range universality but $X \to 1$ in $B \to B_{\text{Coulomb g.s.}}$ limit
  ∵ Coulomb g.s. has no bare state contribution (i.e. $X = 1$)
- Coulomb < short range ($r_e = -0.1$) : remnant of universality

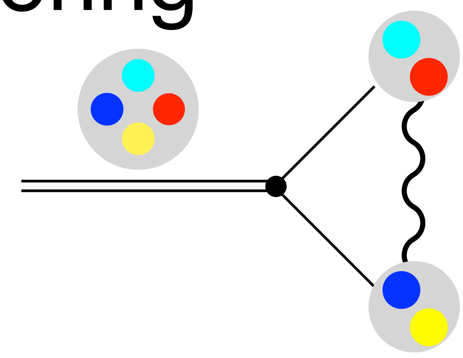
Summary

near-threshold bound states & resonances
with **Coulomb + short range** interaction





- bare state which couples to Coulomb scattering
- pole condition ⟵ $a_s,\ r_e,\ a_B$



- **repulsive** Coulomb

  bound → resonance (does not become virtual states)

  X is not necessary to be unity at threshold

  if Coulomb < s.r., remnant of s.r. universality can be seen

  structure of b.s. ≈ nature of resonance

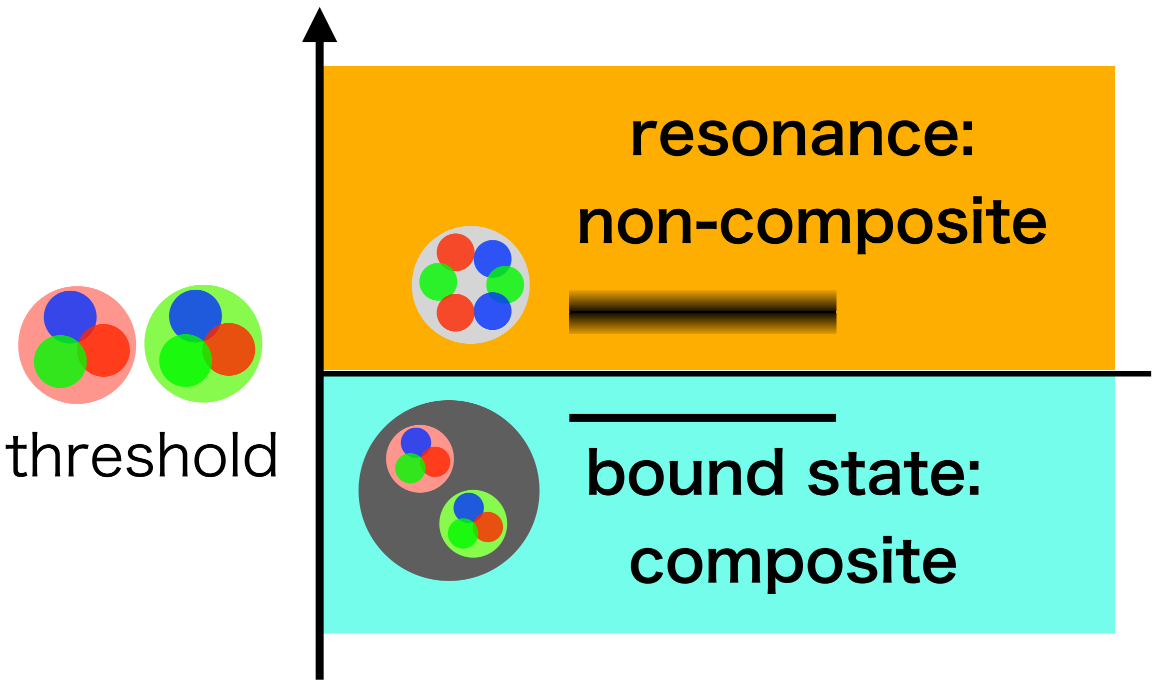
- **attractive** Coulomb

  bound → Coulomb g.s. & virtual → resonance

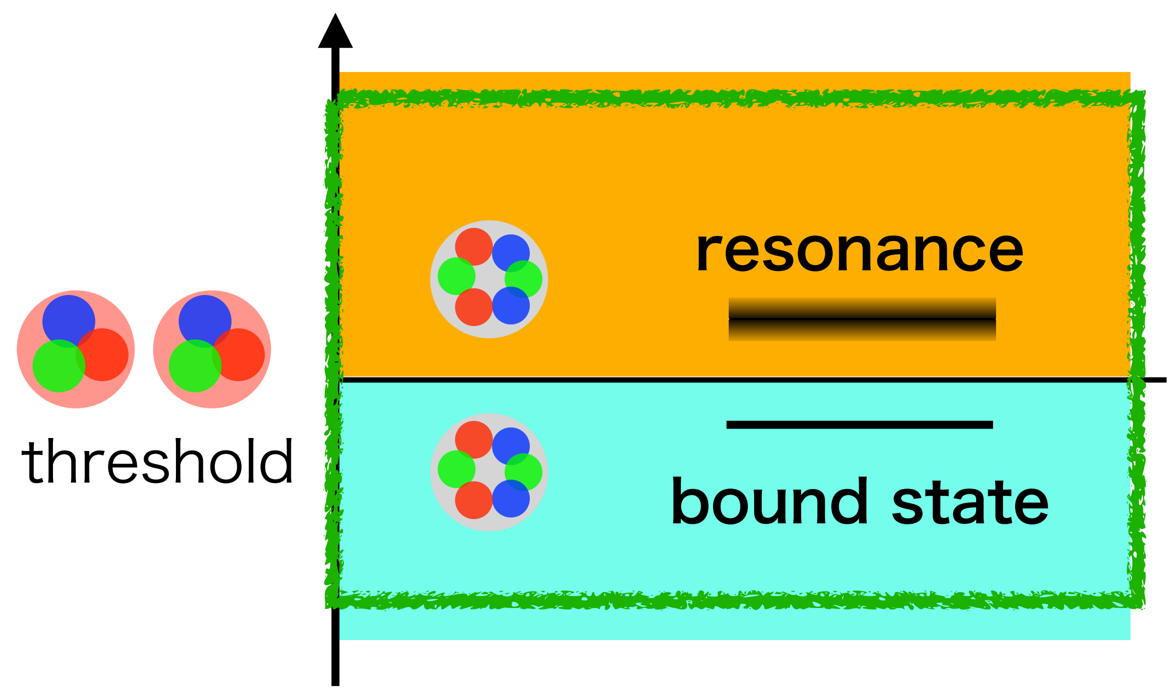
  $X \to 1$ for bound state

Summary: repulsive Coulomb

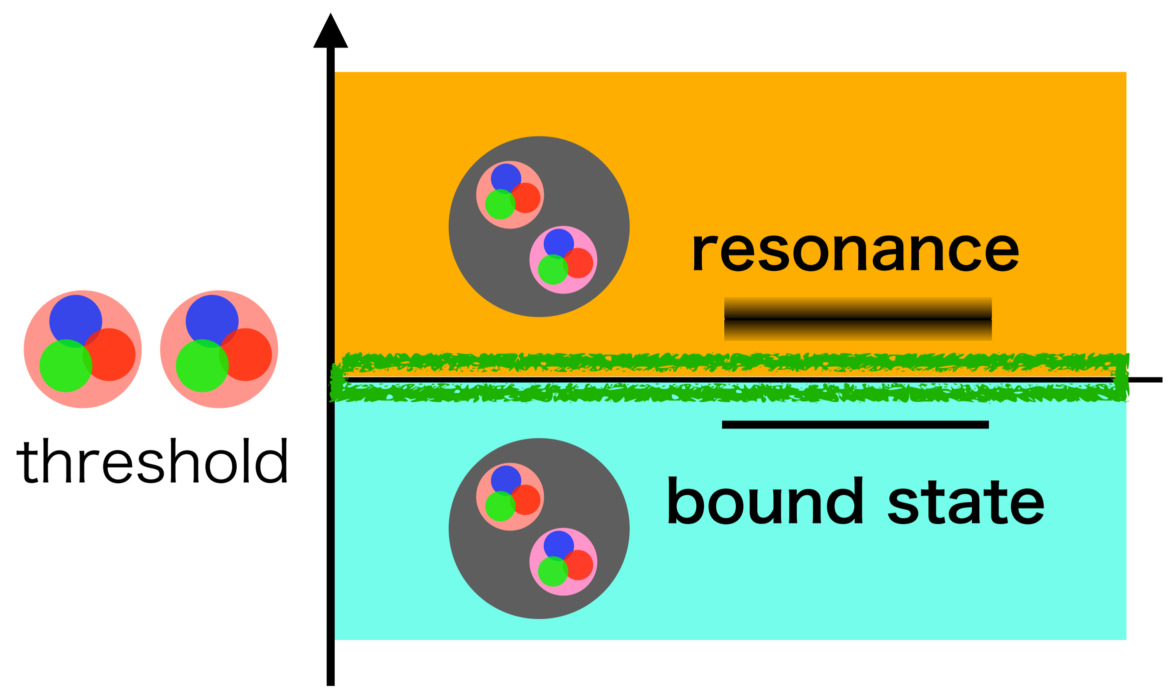
short range interaction



Coulomb > short range

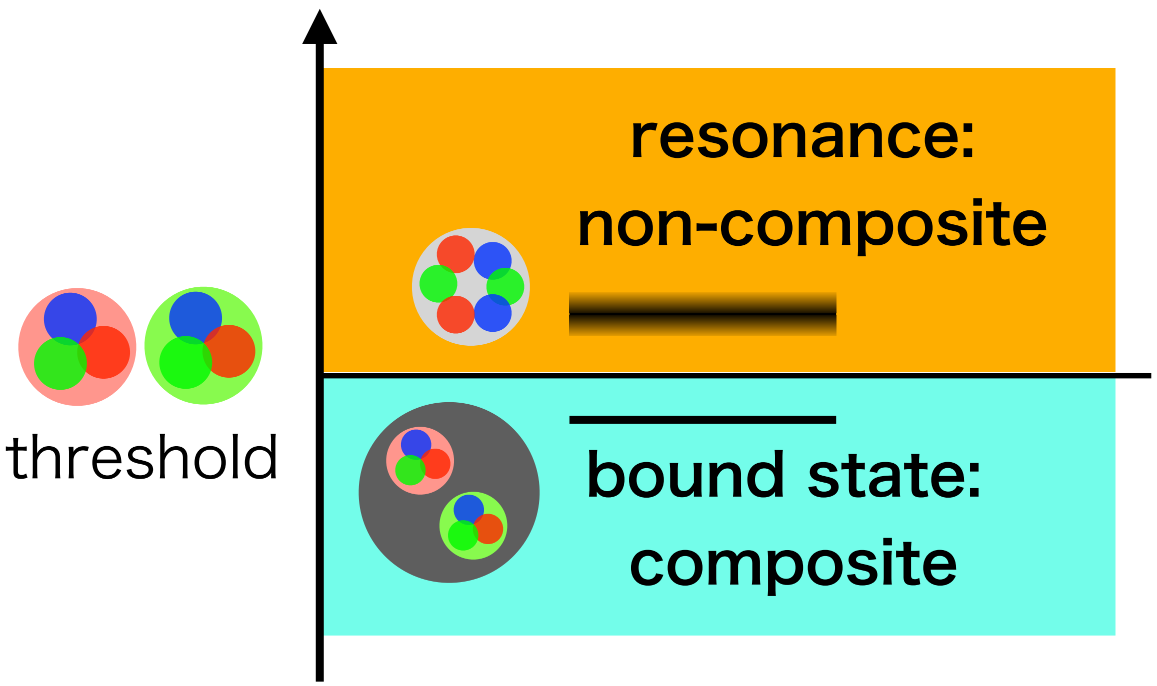


Coulomb < short range

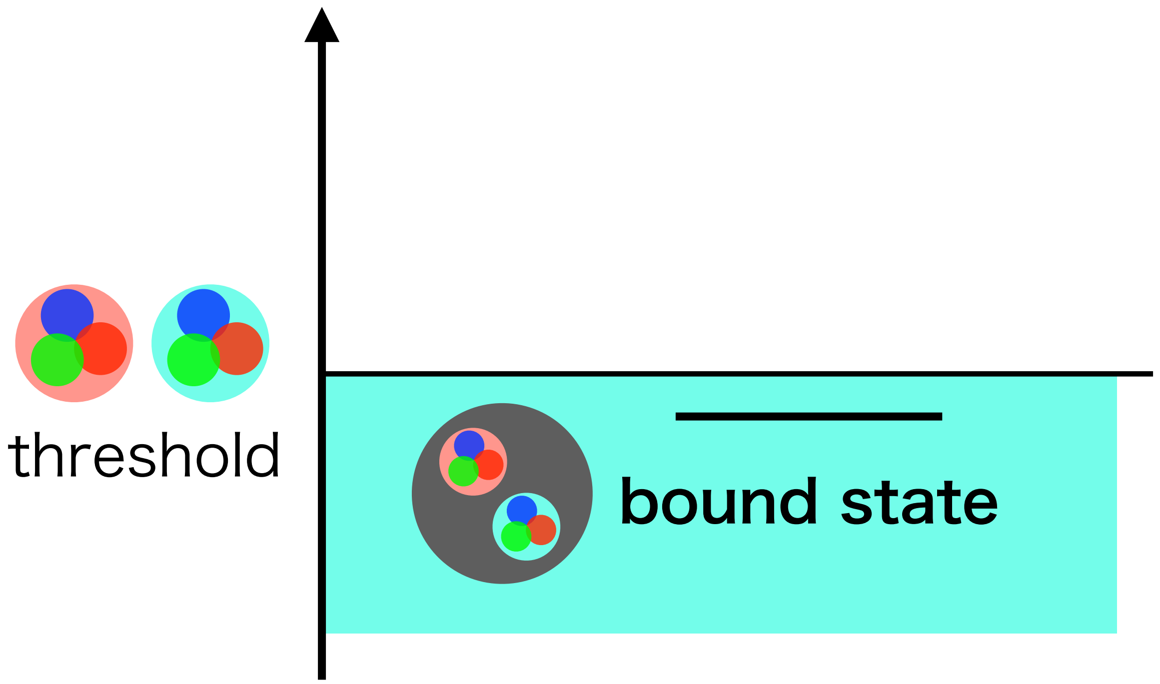


Summary: attractive Coulomb

short range interaction

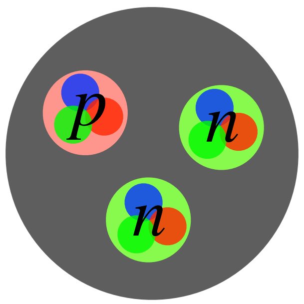


Coulomb + short range

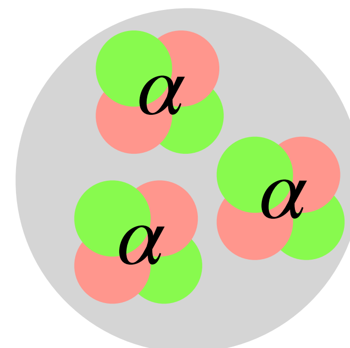


Future perspective

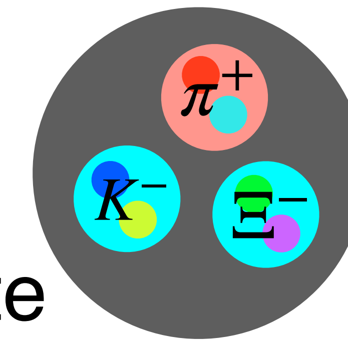
- three body near-threshold states



triton



^{12}C Hoyle state



$\Omega(2012)$

short range interaction

Coulomb + short range

weak-binding relation for three body system?

- near-threshold states in higher-partial wave scatterings

p-wave amplitude

$$f(k,\theta) = \frac{k^2\cos\theta}{-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik^3}.$$

- compositeness in medium?

V. Montesinos, M. Albaladejo, J. Nieves, and L. Tolos, Phys. Rev. C 108, 035205 (2023).

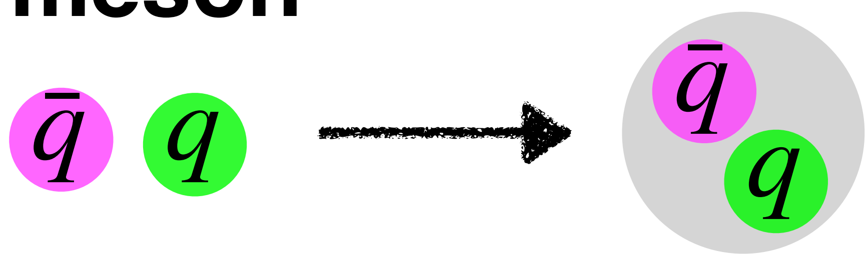
Back up

Exotic hadron

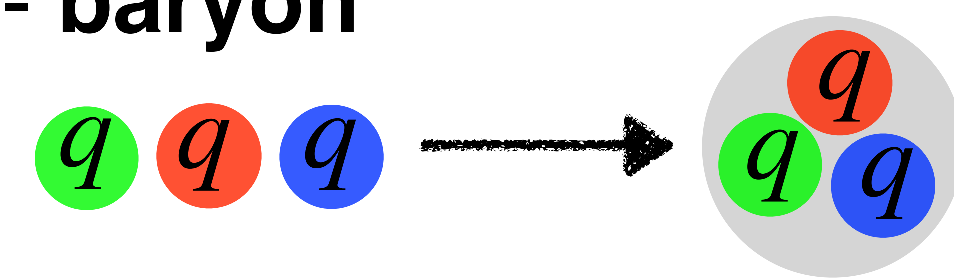
quark q, gluon → hadron
strong interaction

◉ ordinary hadron

- **meson**



- **baryon**

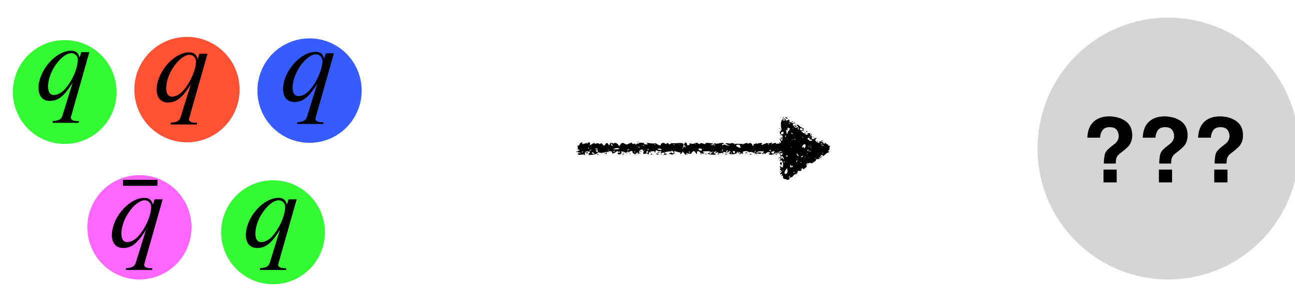


- ~400 kinds have been observed so far

Particle Data Group, S. Navas et al., Phys. Rev. D 110, 030001 (2024).

◉ exotic hadron

- composed of four or more quarks



internal structure?

- **rarely** observed ← Why?

- hint for understanding low-energy phenomena in QCD

Compositeness

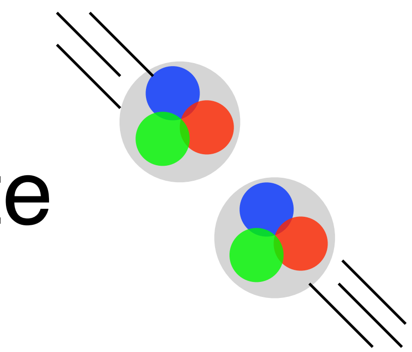
Weinberg, S. Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Kinugawa, T. Hyodo, arXiv:2411.12285 [hep-ph] (accepted in EPJ A).

- decomposing Hamiltonian into free part and remainder

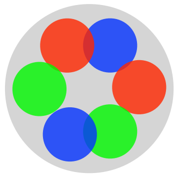
$$\hat{H} = \hat{H}_0 + \hat{V}$$

free Hamiltonian

$$\hat{H}_0 |\boldsymbol{p}\rangle = \frac{\boldsymbol{p}^2}{2\mu} |\boldsymbol{p}\rangle$$ free scattering state



$$\hat{H}_0 |\phi\rangle = \nu_0 |\phi\rangle$$ bare discrete state



- compositeness X, elementarity Z

$$X = \int d\boldsymbol{p}\ |\langle \boldsymbol{p} | B\rangle|^2, \quad Z = |\langle \phi | B\rangle|^2$$

overlaps between bound state $|B\rangle$ and $|\boldsymbol{p}\rangle$ or $|\phi\rangle$

- decomposition is not unique! ⟶ **model dependence** of X

S. Weinberg, Phys. Rev. 130, 776 (1963); D. Luri´e and A. J. Macfarlane, Phys. Rev. 136, B816 (1964).

Compositeness

◉ model calculation

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **85**, 015201 (2012);
F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

$$T = \frac{1}{V^{-1} - G}$$

V : effective interaction

G : loop function

residue of scattering amplitude g

$$X = -\, g^2\, G'(E)\Big|_{E=-B}$$

$$= \left.\frac{G'(E)}{G'(E) - [V^{-1}(E)]'}\right|_{E=-B}$$

$\alpha'(E) = d\alpha/dE$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

g^2 : model independent ← $T_{\rm on}(-B)$ (observable)

$G(E)$: model dependent ← cutoff dependent

History of compositeness

- Weinberg's work (1960s) Weinberg, S. Phys. Rev. 137, 672–678 (1965) etc.

  deuteron is not an elementary particle ⟵ weak-binding relation

  Z : field renormalization factor

- application to exotic hadrons (2000s-)

  X: "compositeness"

  generalization to unstable states

  - with spectral function V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004) etc.
  - with effective range expansion T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) etc.
  - with effective field theory Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017) etc.

  application to …

  - $f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016); T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.
  - $\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.
  - nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

Weak-binding relation

S. Weinberg, Phys. Rev. 137, 672–678 (1965).

$$X = \frac{a_0}{2R - a_0} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)$$

a_0 : scattering length

$R_{\rm typ}$: typical length scale in system

$R = 1/\sqrt{2\mu B}$

- for weakly bound states, $R \gg R_{\rm typ}$

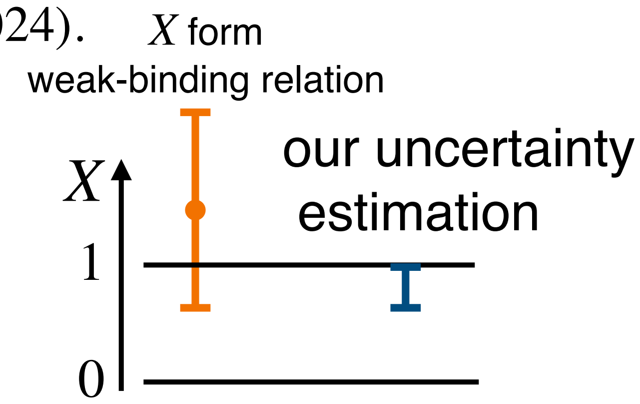
compositeness ⟵ observables (a_0, B)

Y. Li, F.-K. Guo, J.-Y. Pang, and J.-J. Wu, Phys. Rev. D 105, L071502 (2022);
J. Song, L. R. Dai, and E. Oset, Eur. Phys. J. A 58, 133 (2022);
M. Albaladejo, J. Nieves, Eur. Phys. J. C 82, 724 (2022);
T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022);
Z. Yin and D. Jido, Phys. Rev. C 110, no.5, 055202 (2024).

◉ range correction

compositeness of deuteron $X \sim 1.7 > 1$

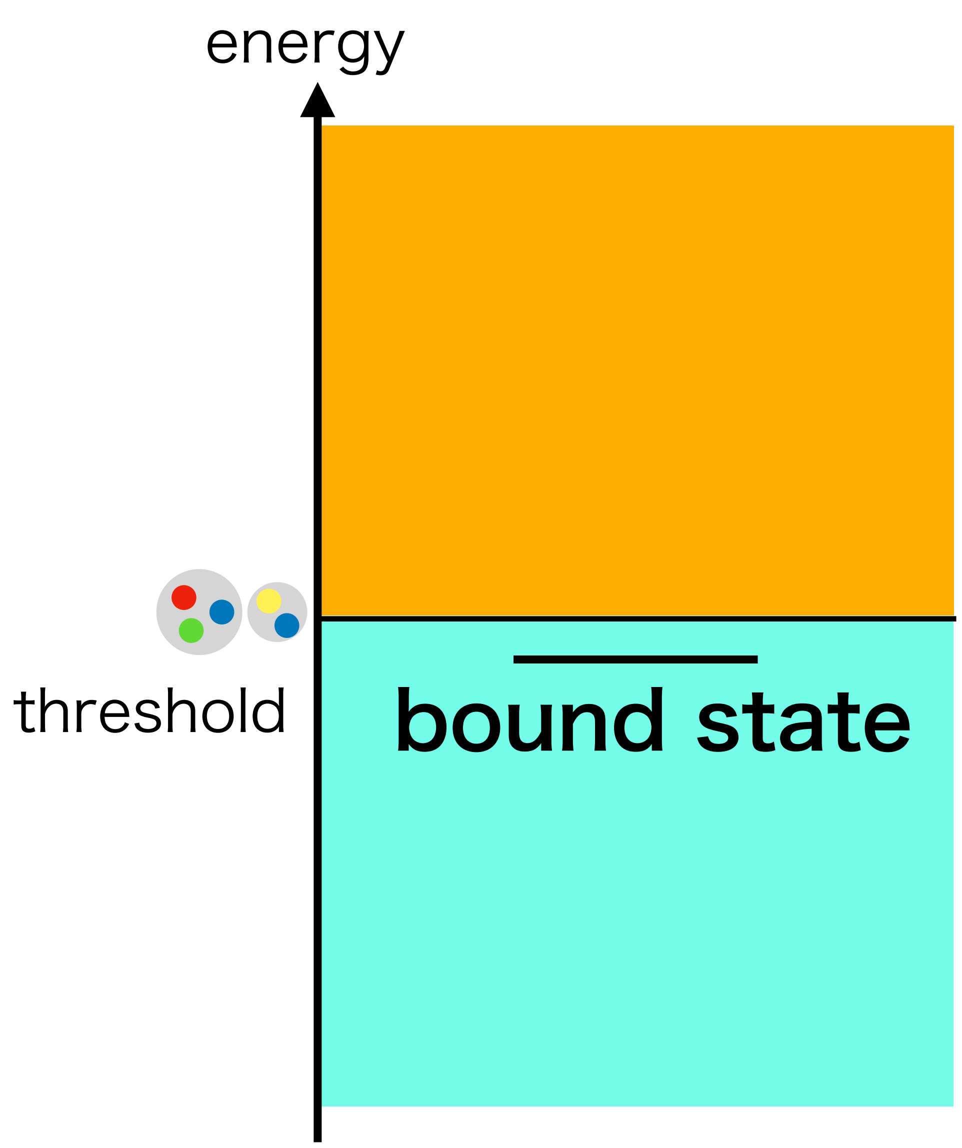
⟶ important to consider effective range



- our work : range correction ⟵ uncertainty estimation

compositeness of deuteron : $0.74 \leq X \leq 1$

Near-threshold bound states



T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024).

non-rel. EFT model

◉ single-channel resonance model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).

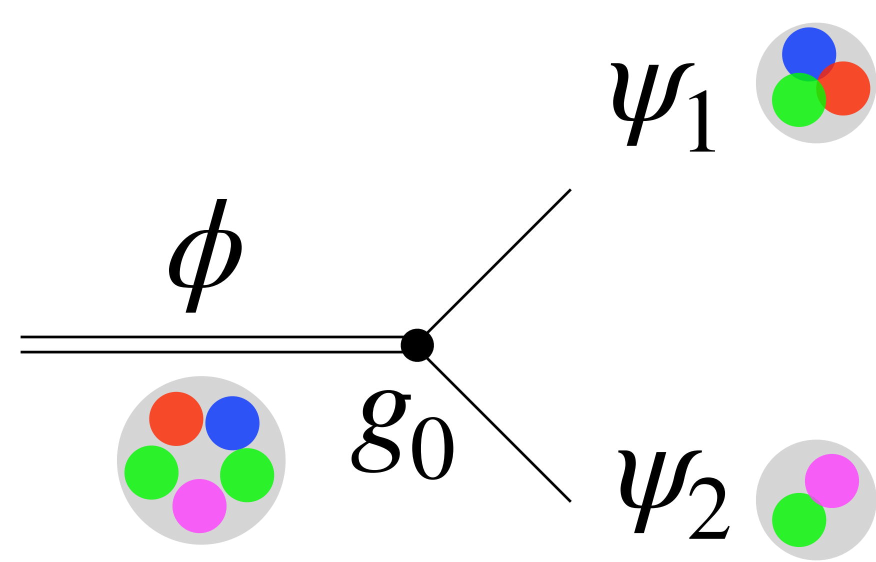
$$\mathscr{H}_{\mathrm{free}} = \frac{1}{2m_1}\nabla\psi_1{}^\dagger\cdot\nabla\psi_1 + \frac{1}{2m_2}\nabla\psi_2{}^\dagger\cdot\nabla\psi_2 + \frac{1}{2m_\phi}\nabla\phi^\dagger\cdot\nabla\phi + \nu_0\phi^\dagger\phi,$$

1.

$$\mathscr{H}_{\mathrm{int}} = g_0(\phi^\dagger\psi_1\psi_2 + \psi_1{}^\dagger\psi_2^\dagger\phi)\,.$$

2.

1. single-channel scatterings of $\psi_{1,2}$
2. couplings of $\psi_{1,2}$ to ϕ



◉ scattering amplitude

$$V = \frac{g_0^2}{E-\nu_0}, \quad G = -\frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right]. \quad \Lambda\text{ : cutoff}$$

$$\xrightarrow[T = \frac{1}{V^{-1}-G}]{} \quad f(k) = -\frac{\mu}{2\pi}\left[\frac{\frac{k^2}{2\mu}-\nu_0}{g_0^2} + \frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right]\right]^{-1}.$$

Compositeness in model

◉ compositeness X

scattering amplitude : $T = \dfrac{1}{V^{-1} - G}$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

$$X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1 + (\Lambda/\kappa)^2}\right)^{-1}\right]^{-1}.$$

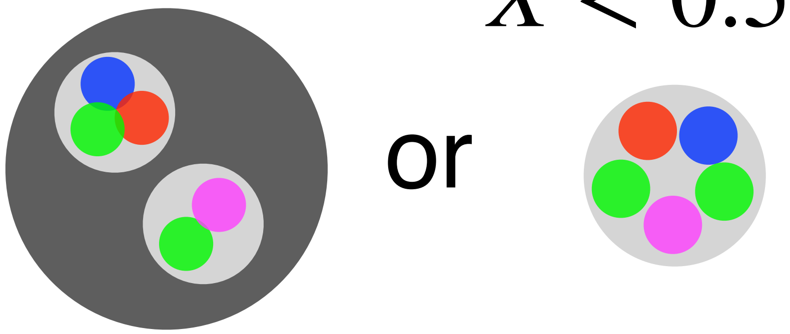
$\kappa = \sqrt{2\mu B}$

- compositeness X of bound state with given B?

fix binding energy B

→ parameter depenence of X?

$X > 0.5$ or $X < 0.5$



Model scale and parameters

◉ scale in model

- determined by cutoff Λ: $E_{\rm typ} = \Lambda^2/(2\mu)$

◉ model parameters g_0, ν_0, Λ

1. fix binding energy B

   coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \dfrac{\pi^2}{\mu}(B + \nu_0)\Big[\Lambda - \kappa \arctan(\Lambda/\kappa)\Big]^{-1}$

   ∵ pole condition $f^{-1}(i\kappa) = 0$ $\quad\quad \kappa = \sqrt{2\mu B}$.

2. use dimentionless quantities by Λ e.g. $B/E_{\rm typ}$

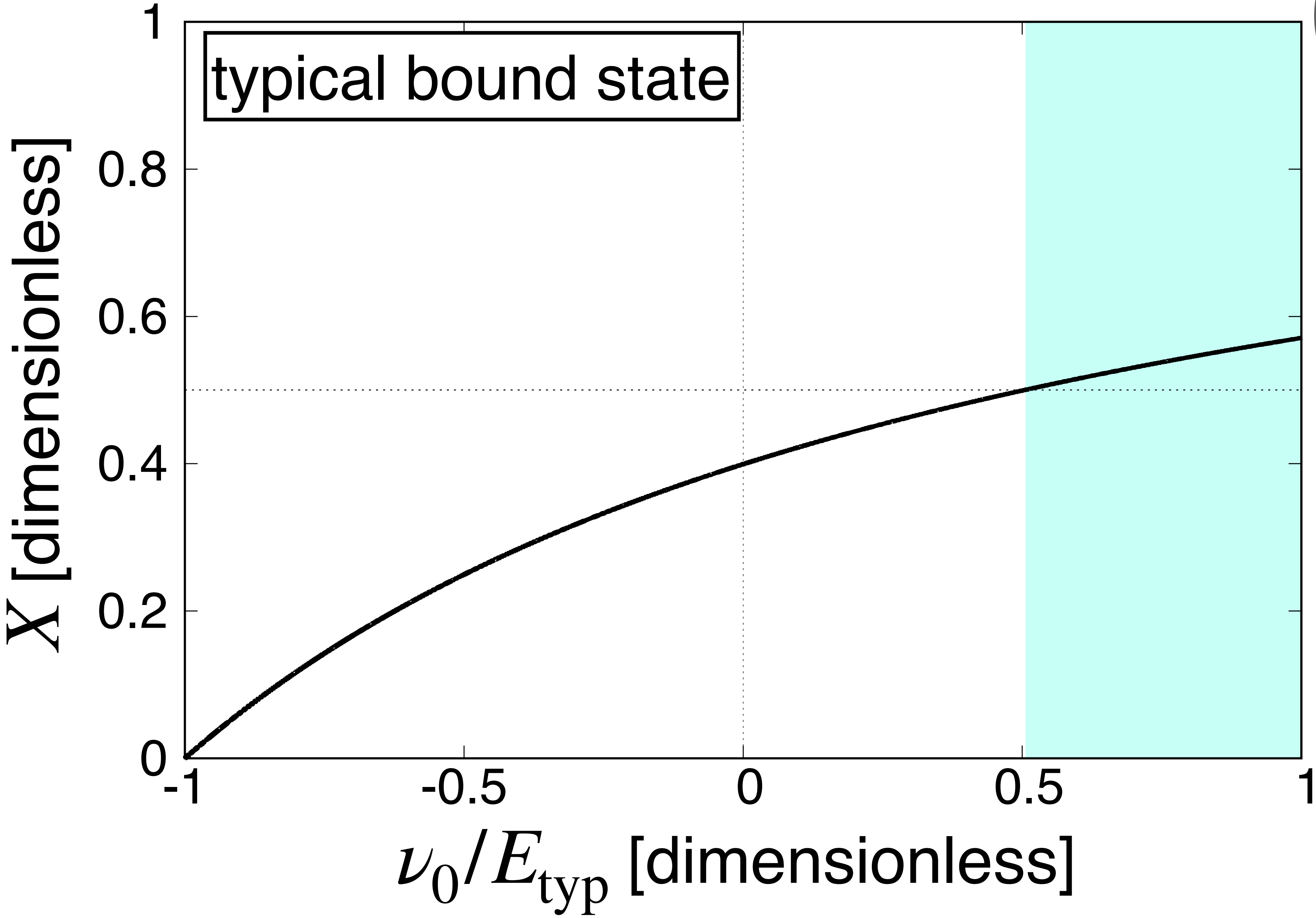
3. free parameter ⟶ bare state energy ν_0

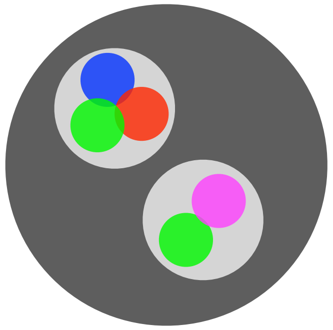
   vary in $-B/E_{\rm typ} \le \nu_0/E_{\rm typ} \le 1$ region

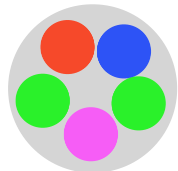
   ∵ $g_0^2 \ge 0$ (lower) & applicable limit of model (upper)

Model-dependence of X

◉ parameter ν_0/E_{typ} dependence of X ($B = E_{\mathrm{typ}}$)

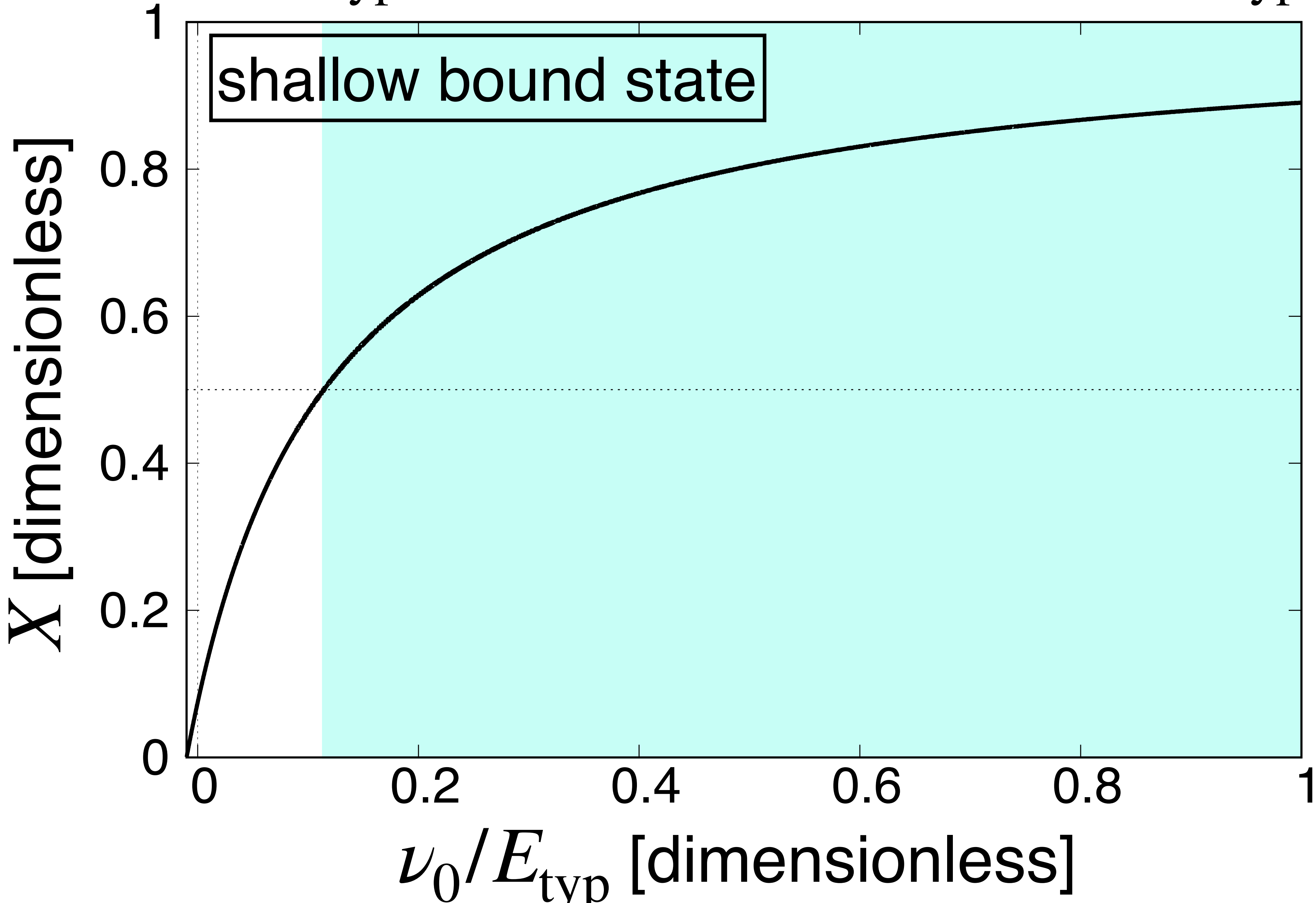


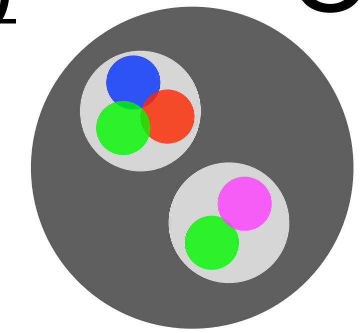


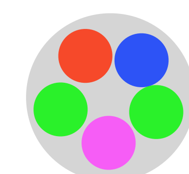


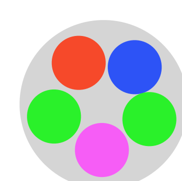
- $X < 0.5$ in 75 % of ν_0 ∵ **bare state ($X = 0$) origin**

◉ parameter $\nu_0/E_{\rm typ}$ dependence of X ($B = 0.01E_{\rm typ}$)





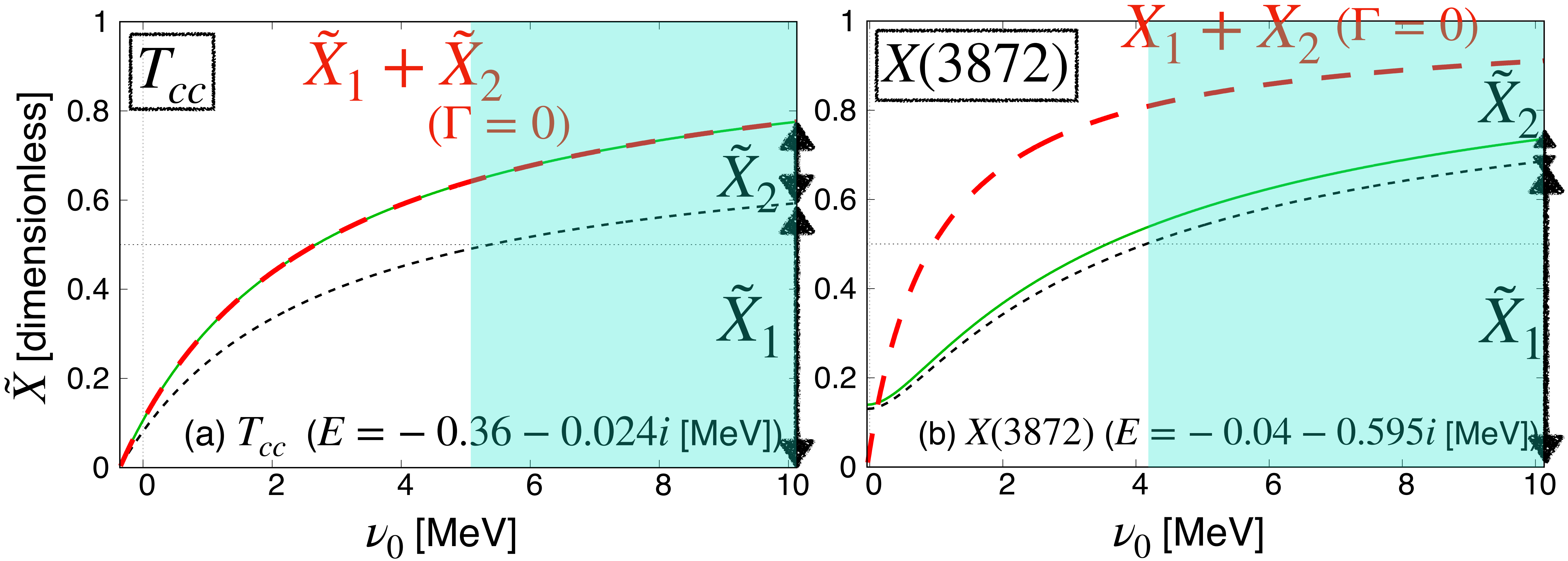


- $X > 0.5$ in $88\ \%$ of ν_0 ⟶ emergence of **universality**
- fine tune of ν_0 is nessecary to obtain  shallow bound state

⟶ shallow bound states tend to be composite dominant

Application to T_{cc} and $X(3872)$

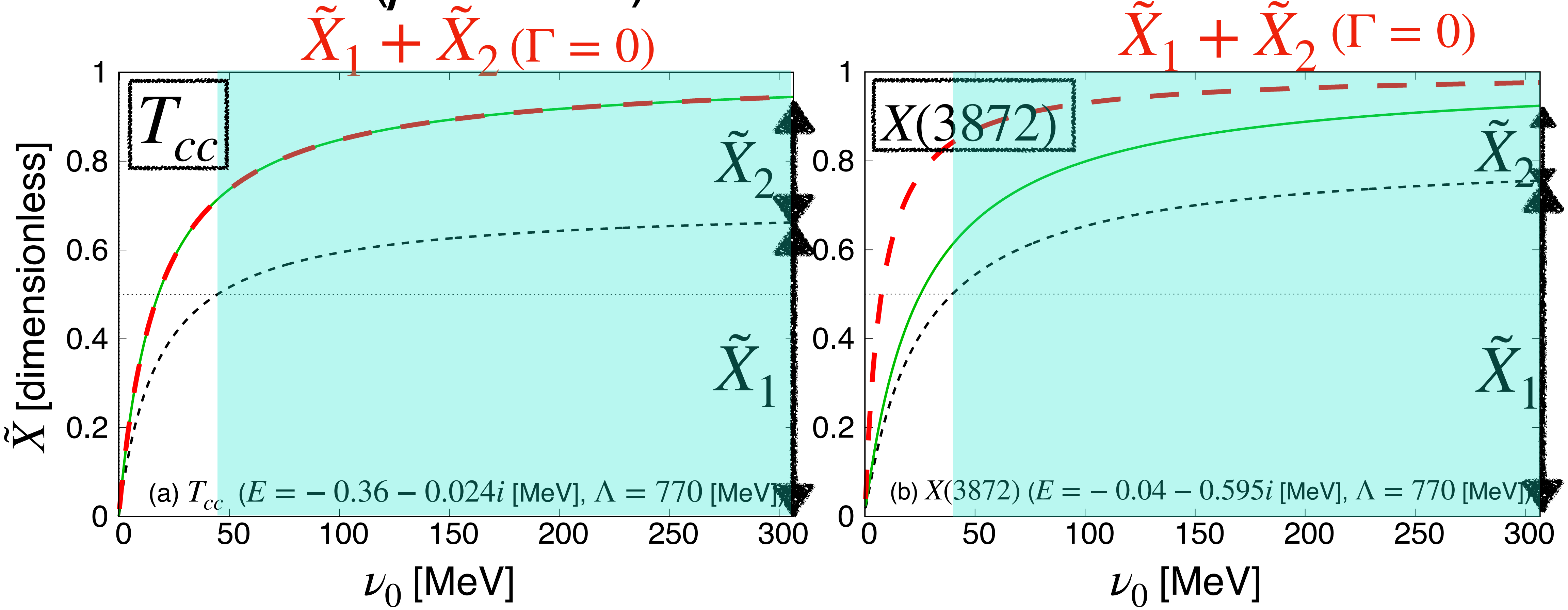
- $\Lambda = 140$ MeV (π meson)



- T_{cc} : $\tilde{X}_1 > 0.5$ for $45\ \%$ of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for $59\ \%$ of ν_0 region
- coupled ch. effect is more important for T_{cc} than $X(3872)$
- decay effect is more important for $X(3872)$ than T_{cc}

Application to T_{cc} and $X(3872)$

- $\Lambda = 770$ MeV (ρ meson)

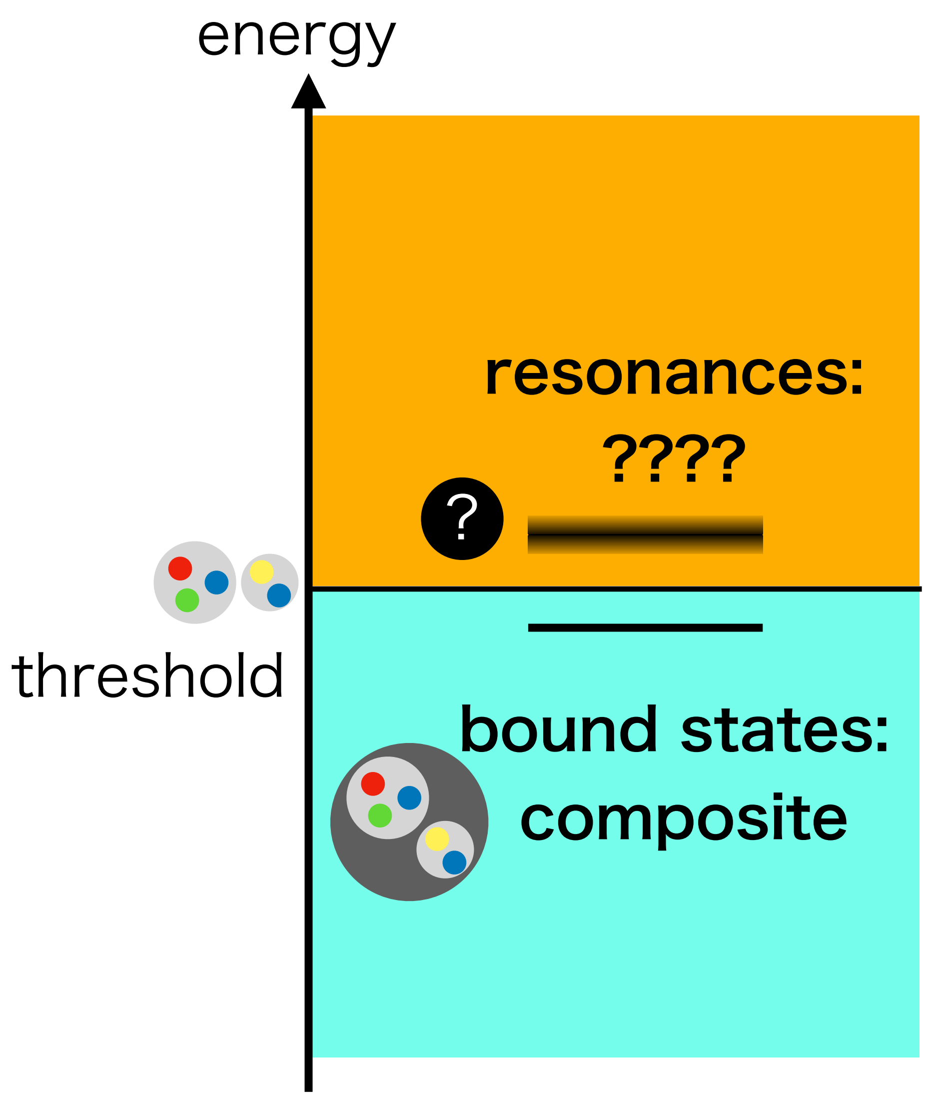


- T_{cc} : $\tilde{X}_1 > 0.5$ for 85 % of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for 87 % of ν_0 region
- typical energy scale $E_{\rm typ}$ is larger

→ states becomes close to universality limit $X \to 1$

decay effect : suppressed coupled ch. effect : enhanced

Near-threshold resonances

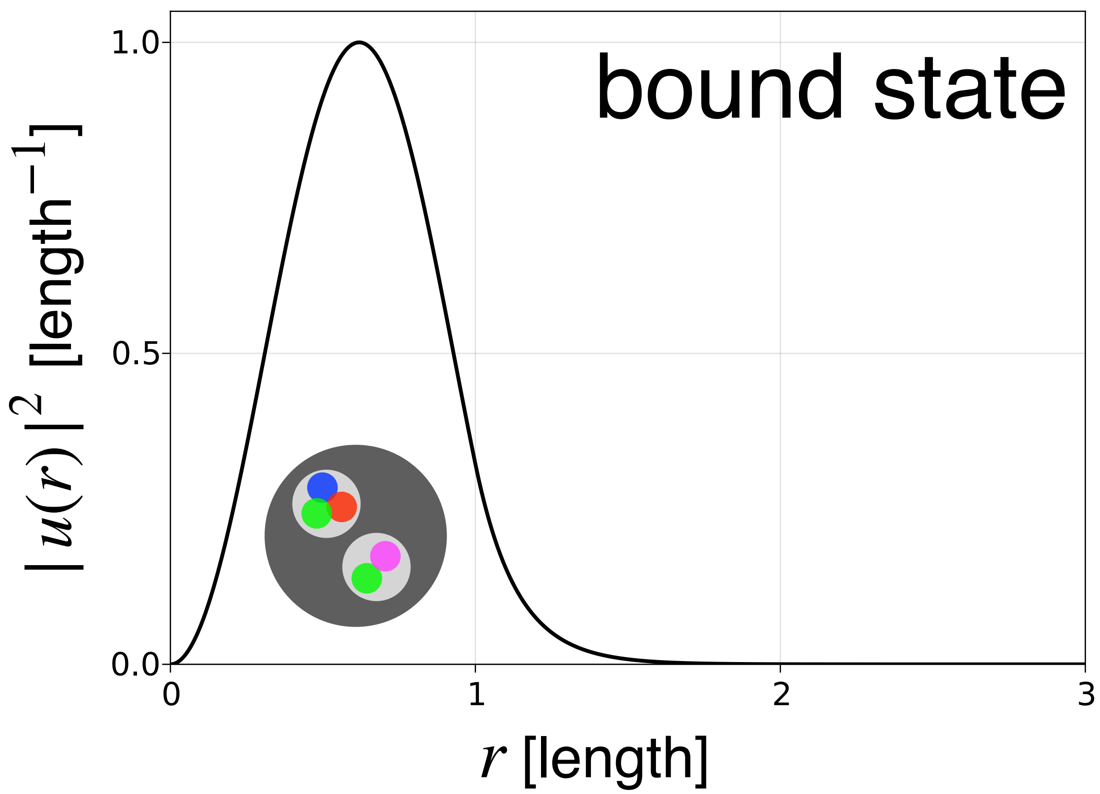


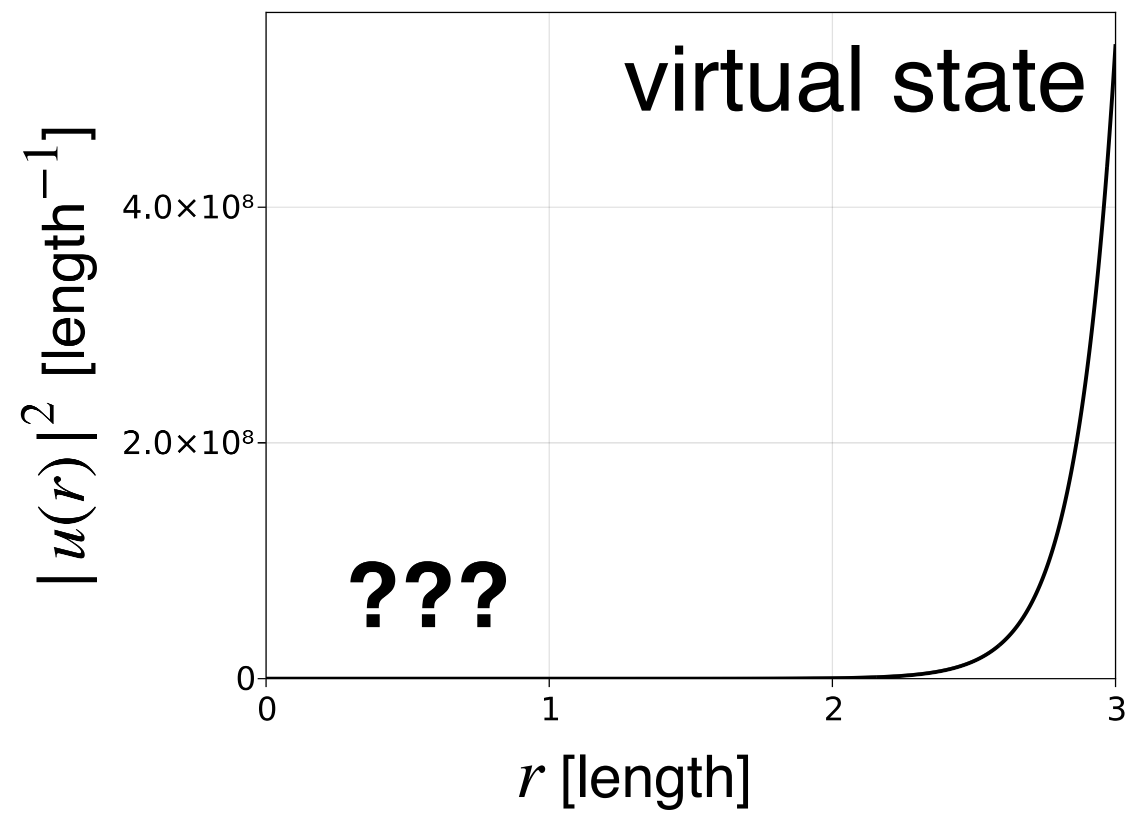
T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

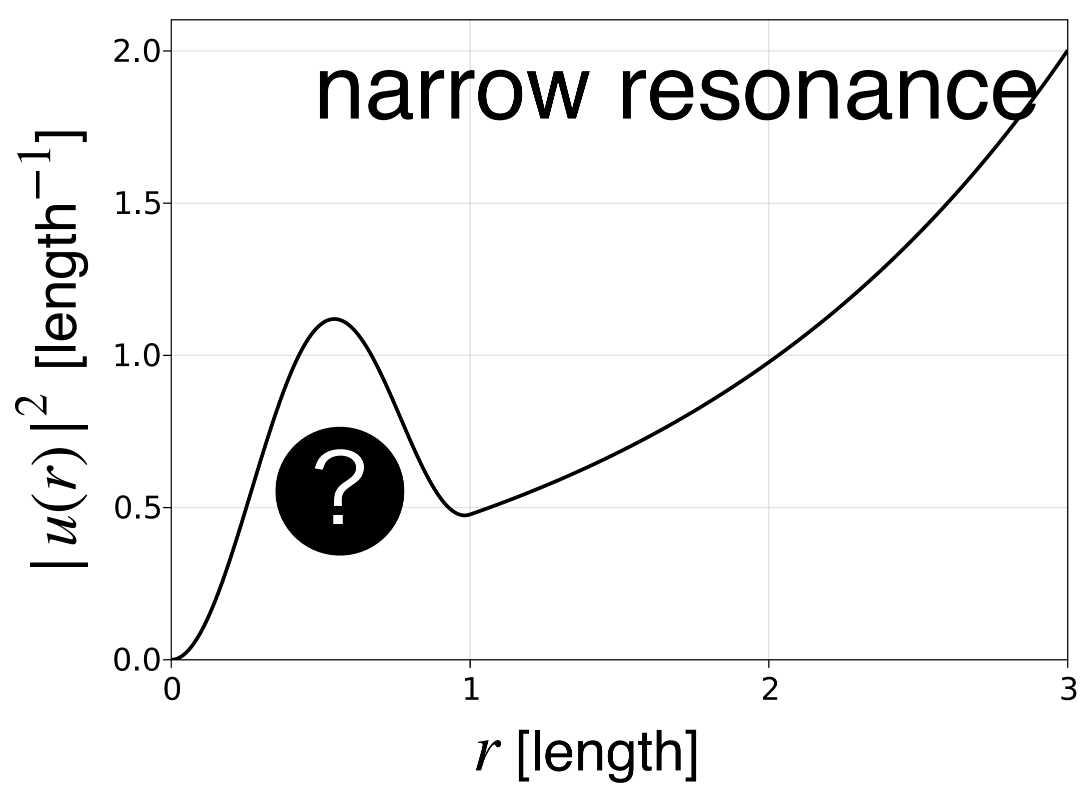
Wavefunction of eigenstates

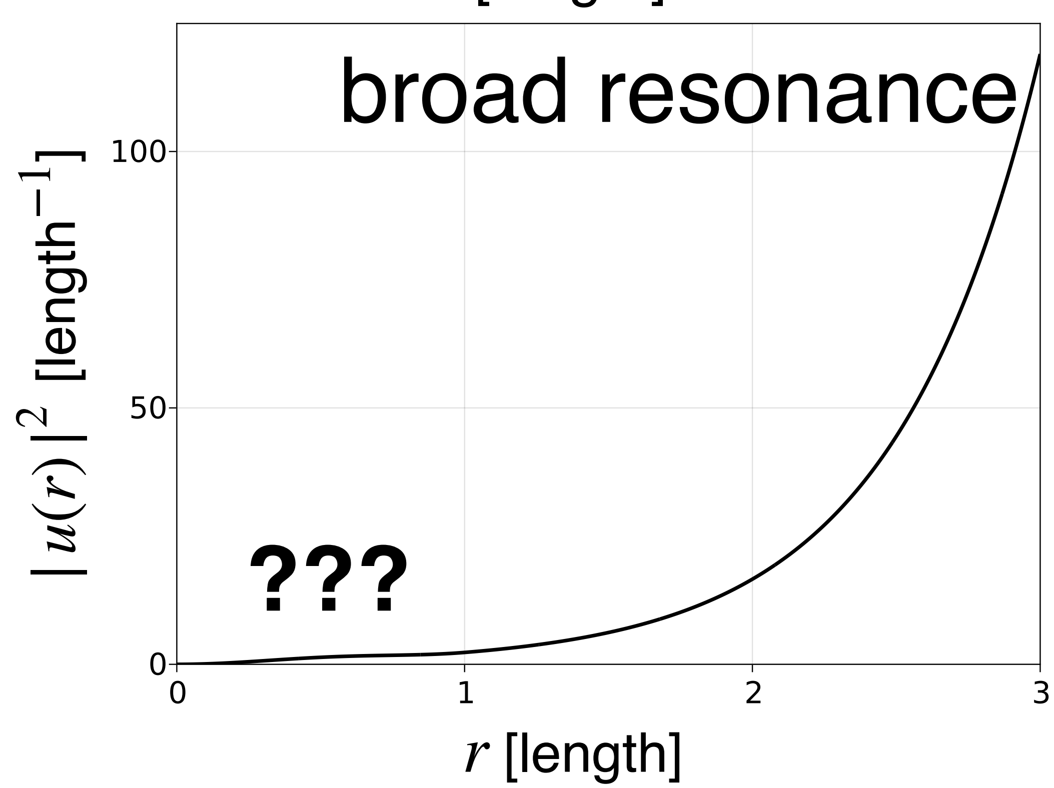
✅ internal “structure”!

❌ internal “structure”?





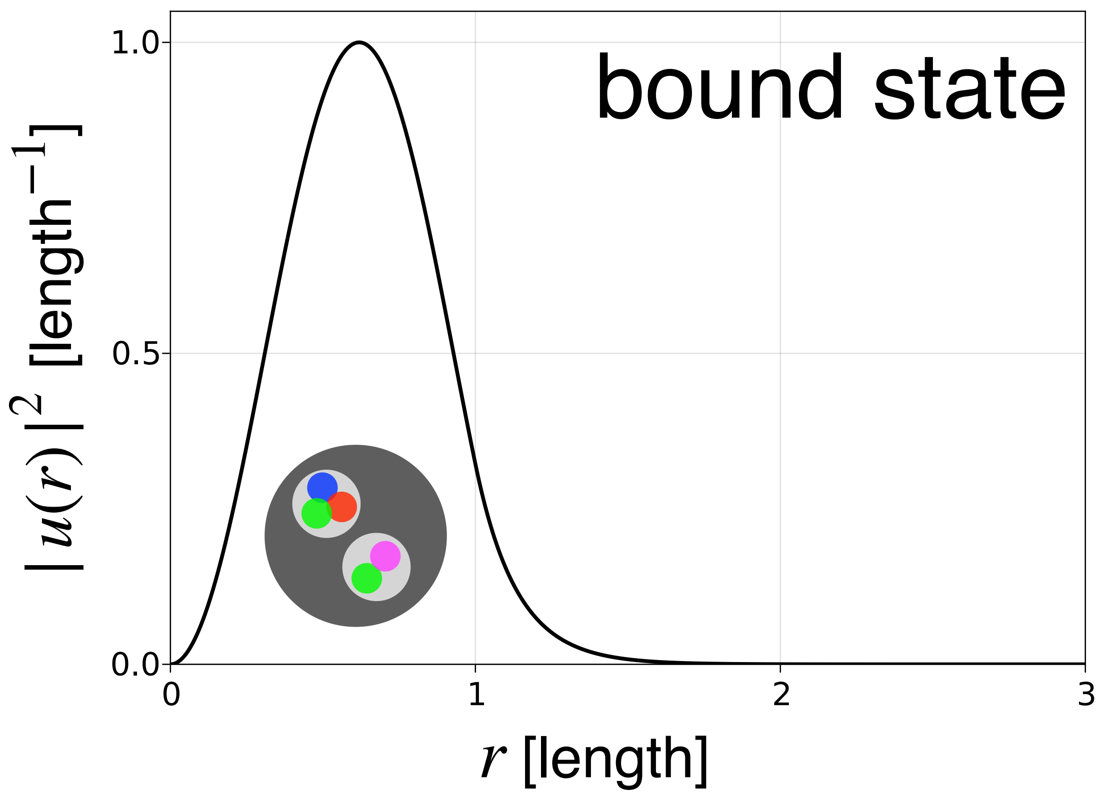


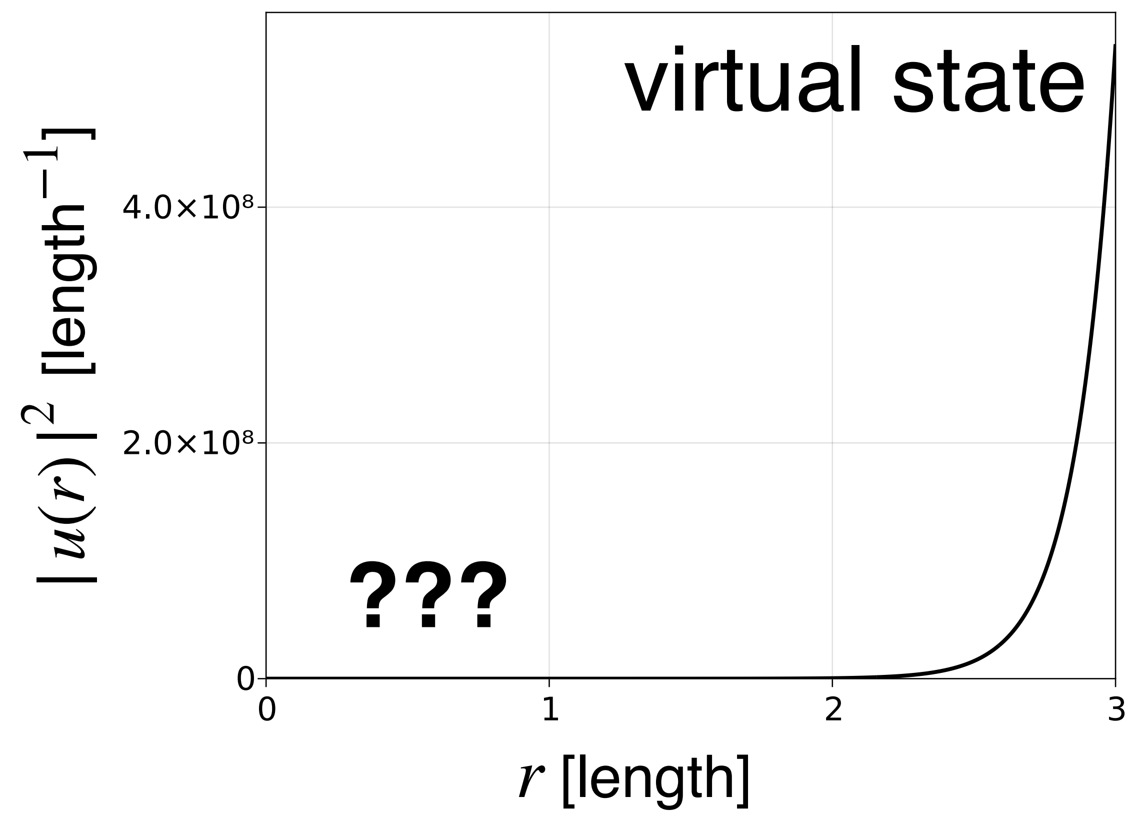


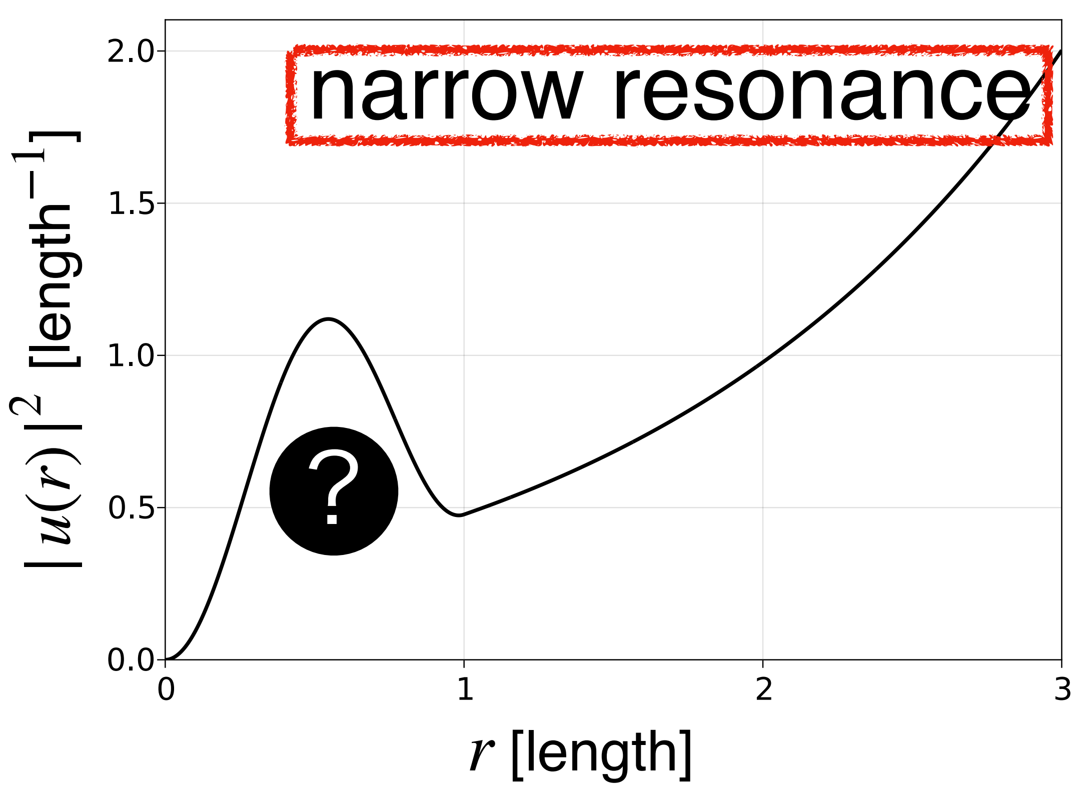
Wavefunction of eigenstates

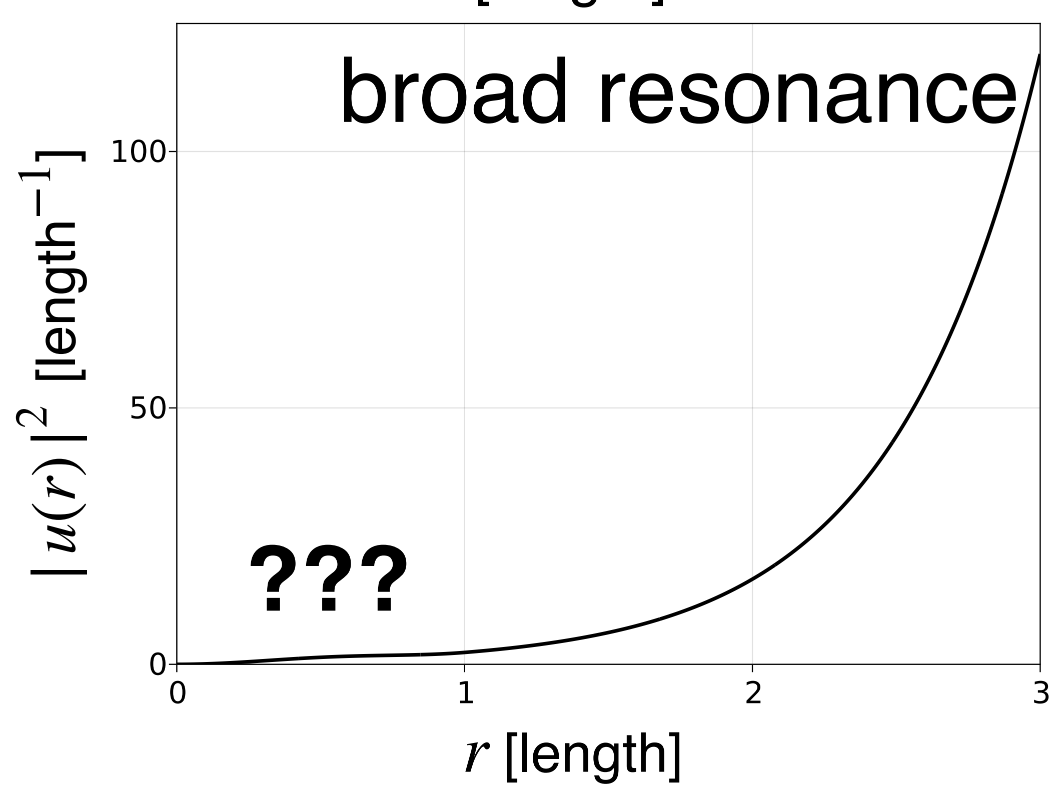
✅ internal “structure”!

❌ internal “structure”?









Near-th. resonances in ERE

T. Kinugawa and T. Hyodo arXiv:2403.12635 [hep-ph].

◉ resonance pole written by effective range expansion (ERE)

$$f(k)^{-1} = -\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik \longrightarrow k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1}$$

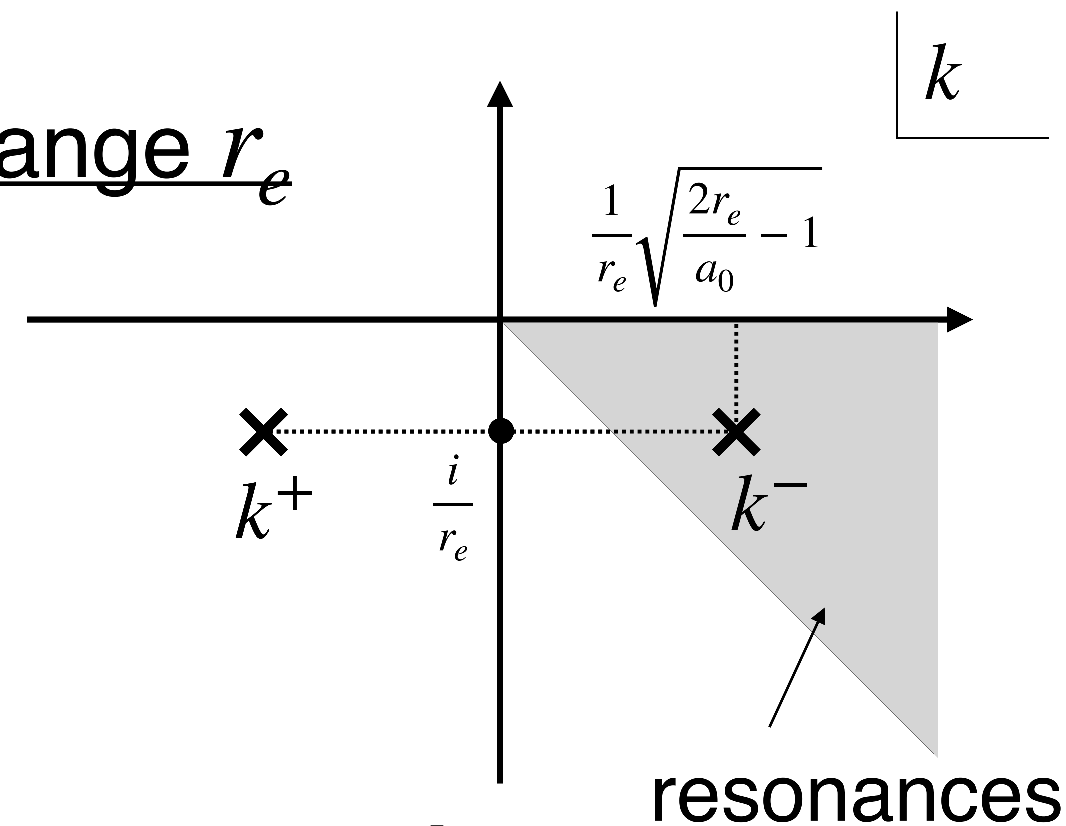
T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

- pole position ($k^{\pm}$) ⟷ a_0 and r_e

◉ scattering length a_0 and effective range r_e

- r_e should be negative to obtain resonances
- $\mathrm{Im}k = |1/r_e|$ should be small to obtain narrow resonances

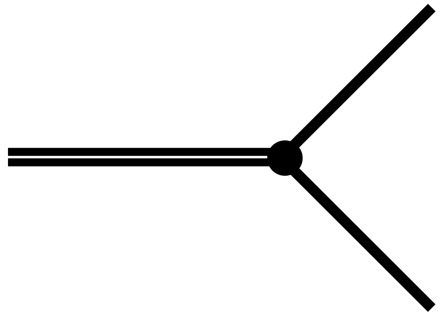
⟶ Effective range should be **large and negative** for near-threshold resonances



Pole trajectory (only w/ s.r.)

◉ pole trajectory in complex momentum k plane (No Coulomb)

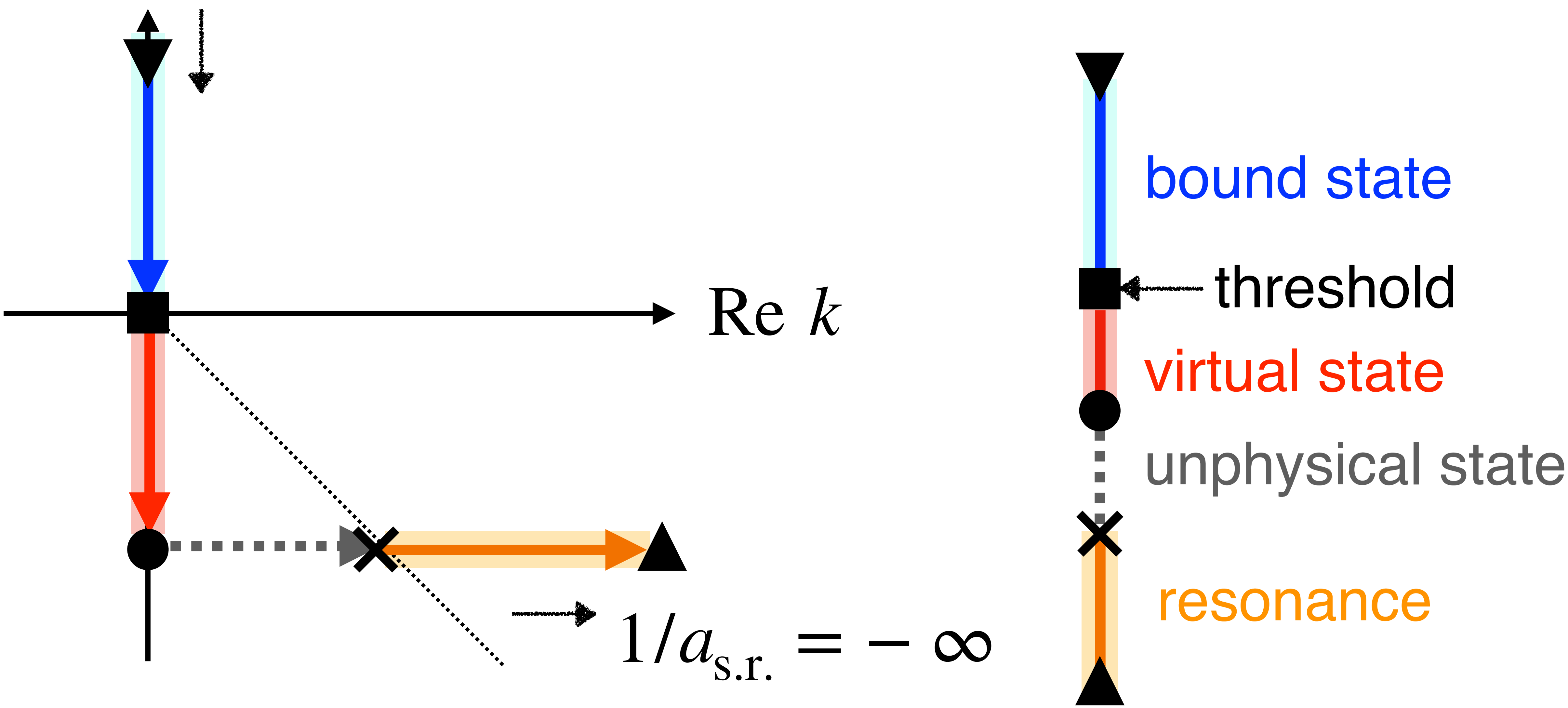
Q channel
⇔ bare state



P channel
⇔ free scattering

$G_P \to G^0$ free Green's function

$\mathrm{Im}\, k \quad 1/a_{\mathrm{s.r.}} = +\infty$



T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) .

Universality for near-th. resonances

- near-threshold **bound** (and virtual) states

$a_0 \to \infty$ and universality holds in $B \to 0$ limit ○

→ $X \to 1$ (completely composite)

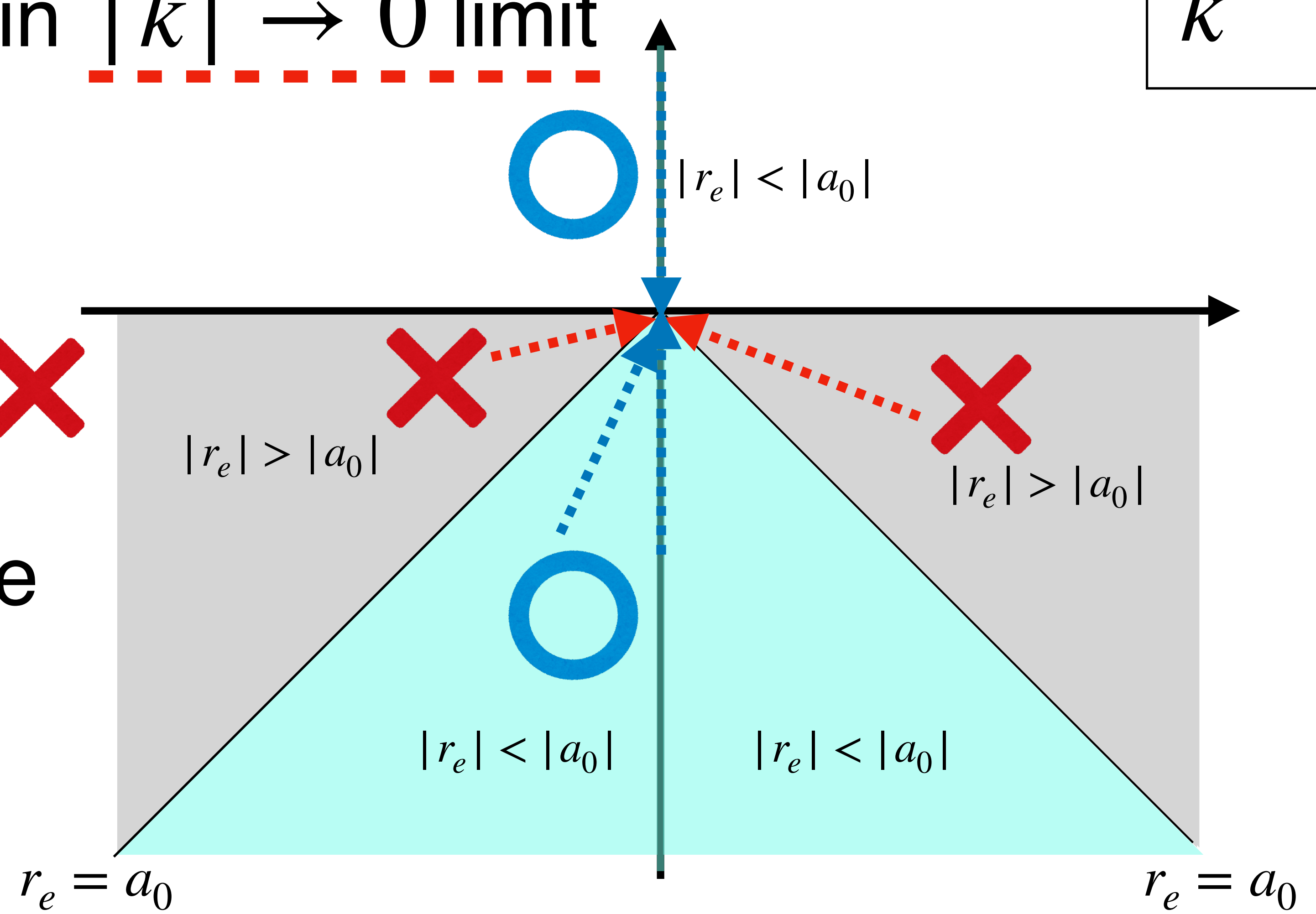
- near-threshold **resonances**

$a_0 \to \infty$ but also $|r_e| \to \infty$ in $|k| \to 0$ limit

$\because |a_0| \leq |r_e|$

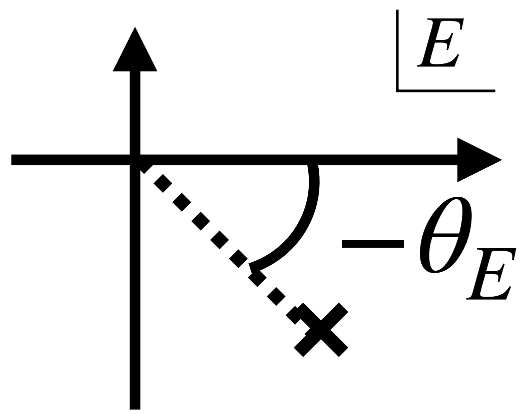
→ universality does not hold ✕

Near-threshold resonances are **not** necessarily composite dominant



Compositeness in ERE

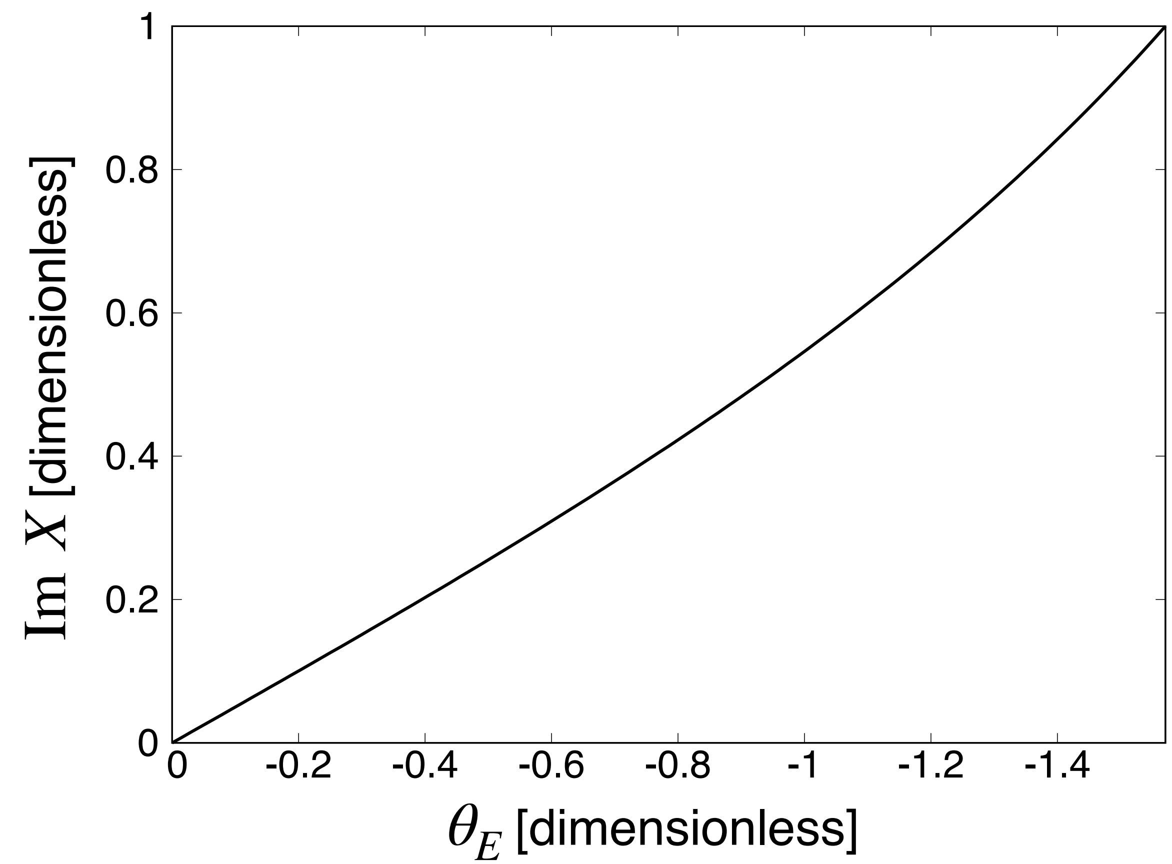
$$X = \sqrt{\frac{1}{1-\frac{2r_e}{a_0}}} = -\,i\tan\theta_k = -\,i\tan(\theta_E/2)$$



$$(k = |k|e^{i\theta_k},\ E = |E|e^{i\theta_E})$$

→ X in ERE is pure imaginary

Im X as a function of θ_E



- in general, compositeness X of unstable resonances becomes **complex** by definition

- complex X **cannot** be directly interpreted as a probability

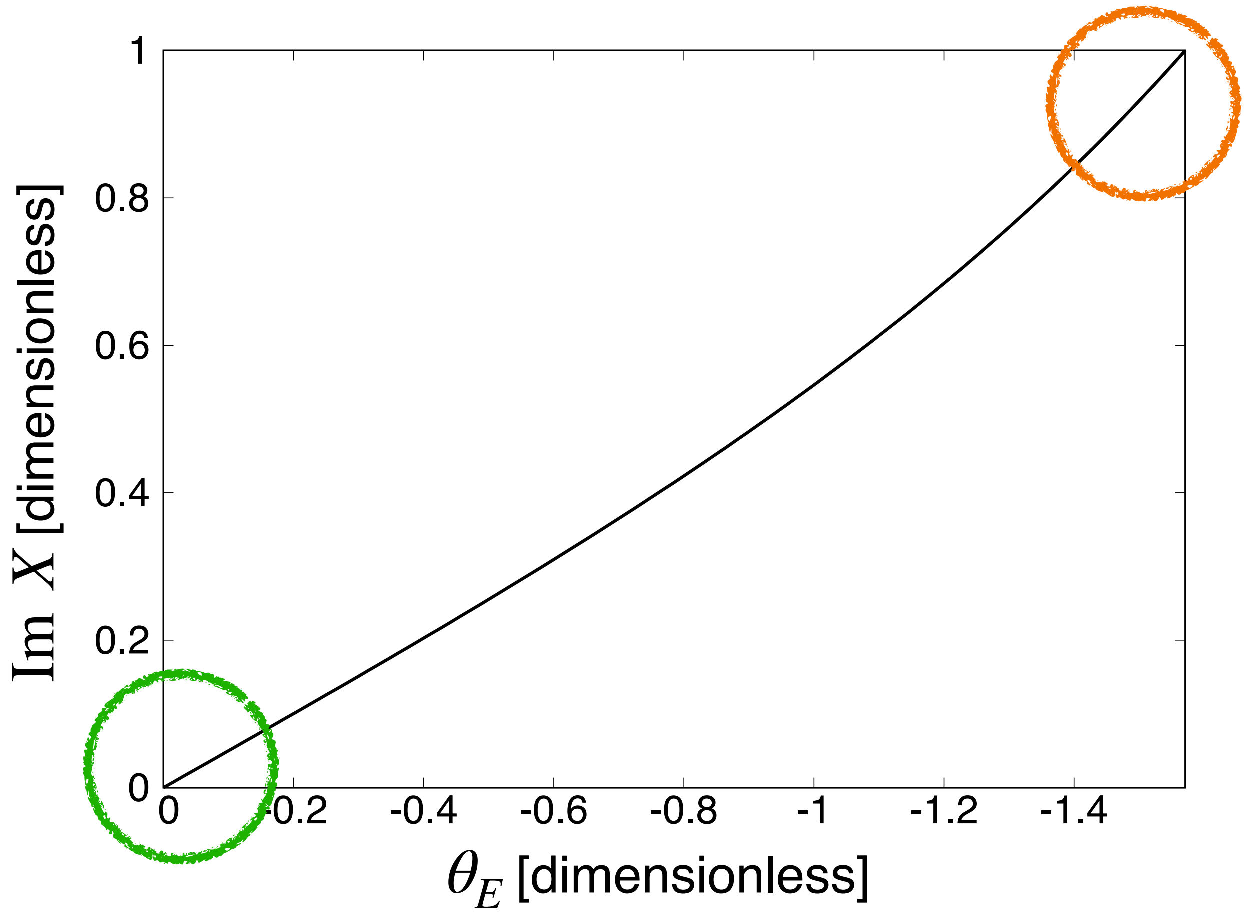
Compositeness in ERE

$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan\theta_k$$

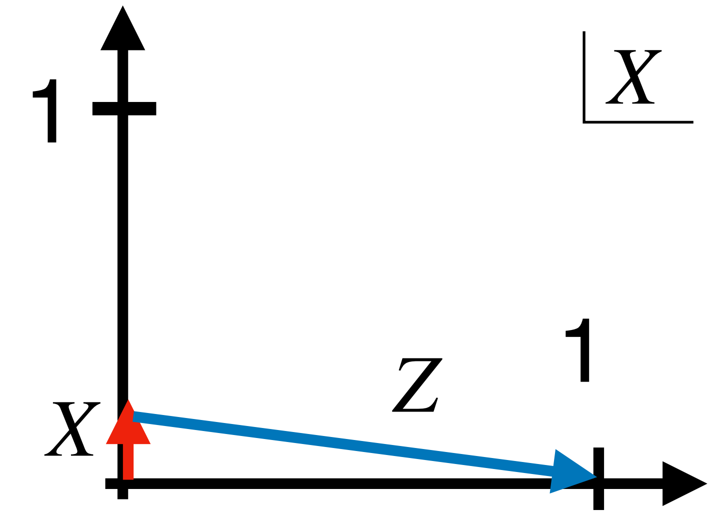
$$(k = |k| e^{i\theta_k})$$

→ X in ERE is pure imaginary

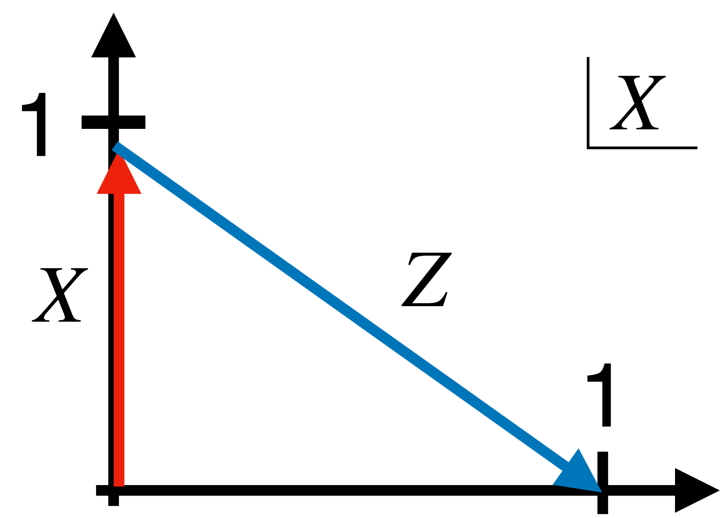
X as a function of θ_E $(E = |E| e^{i\theta_E})$



small width ($\theta_E \sim 0$)



large width ($\theta_E \sim -\pi/2$)



Complex compositeness

- interpretation sheme of complex X is **not** unique
  - $|X|$ F. Aceti, E. Oset, Phys. Rev. D 86, 014012 (2012); T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013); Z.H. Guo, J.A. Oller, Phys. Rev. D 93(9), 096001 (2016); J.A. Oller, Annals Phys. 396, 429 (2018).
  - Re X F. Aceti, L.R. Dai, L.S. Geng, E. Oset, Y. Zhang, Eur. Phys. J. A 50, 57 (2014).
  - introduce new quantities defined by complex X

    Y. Kamiya, T. Hyodo, PTEP 2017(2), 023D02 (2017);
    T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, Phys. Rev.C 93(3), 035204 (2016).
  - introduce new quantities based on ERE

    T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013);
    I. Matuschek, V. Baru, F.K. Guo, C. Hanhart, Eur. Phys. J. A 57(3), 101 (2021).

→ How can we propose a “reasonable” scheme?

● condition for reasonable measure

1. reduce to compositeness of bound states when $\mathrm{Im}\ X \to 0$
2. probabilistic quantities (normalized and between 0 and 1)
3. take into account only **narrow** resonance automatically

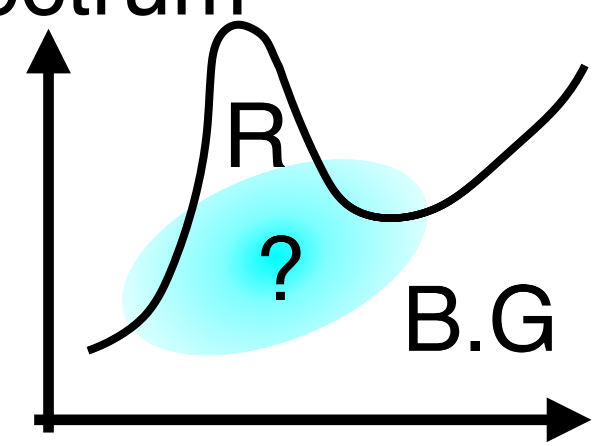
Our proposal

- introduce new quantities defined by complex X

i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$

ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$

iii) $\mathscr{Y}$: probability of uncertain identification



uncertain appears from

- finite lifetime (uncertainty in energy)
- separation from B.G. T. Berggren, Phys. Lett. B 33, 547 (1970).

$$\mathscr{X} = \frac{(\alpha-1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

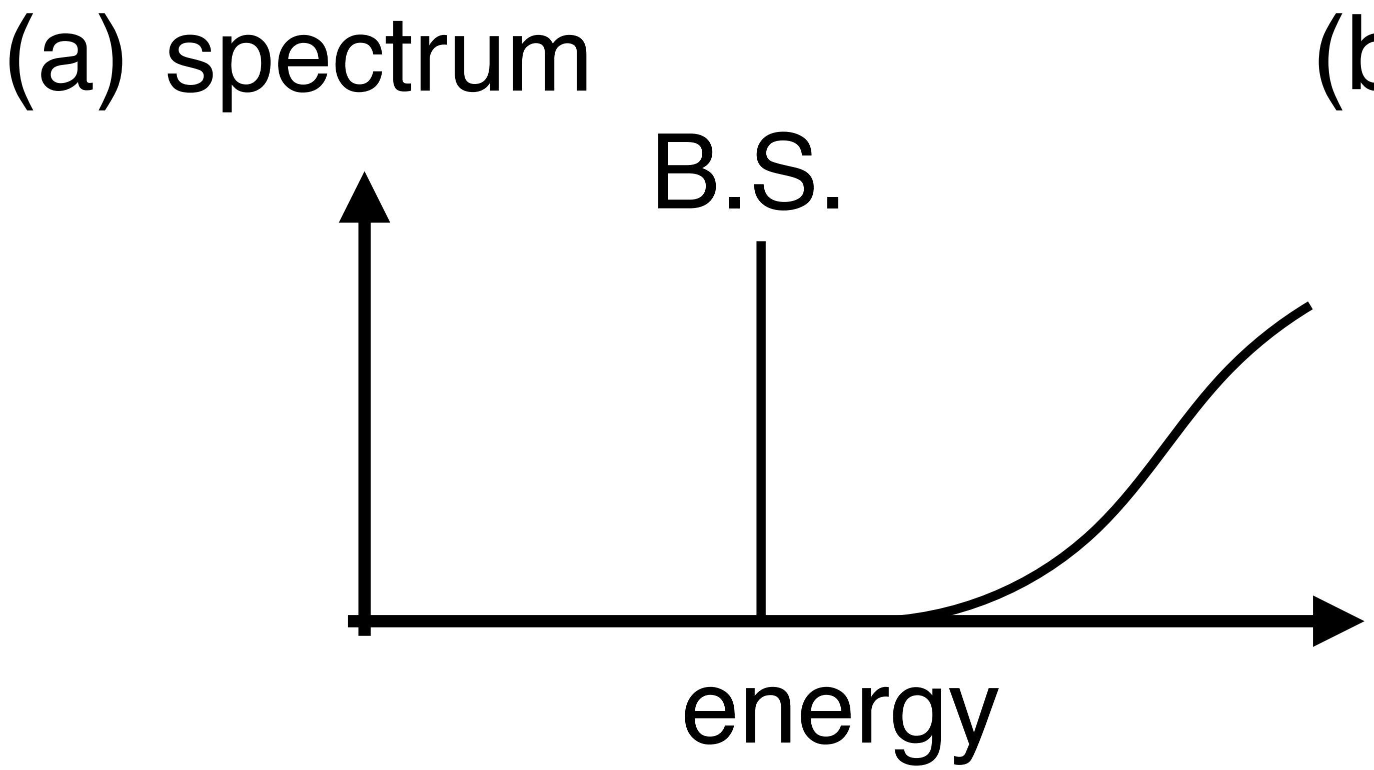
$$\mathscr{Z} = \frac{(\alpha-1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

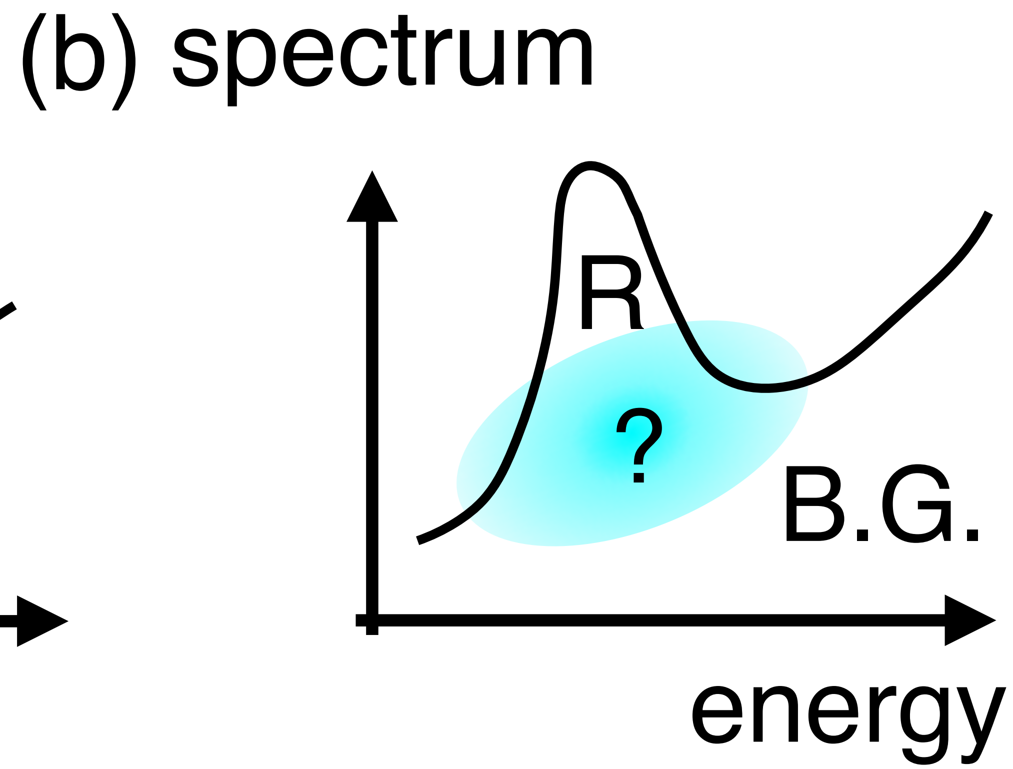
$$\mathscr{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

1. $\mathscr{X} \to X$, $\mathscr{Z} \to Z$ and $\mathscr{Y} \to 0$ $(\mathrm{Im}\ X \to 0)$
2. $\mathscr{X} + \mathscr{Y} + \mathscr{Z} = 1$

α reflects uncertain nature of resonances

uncertainty in resonances

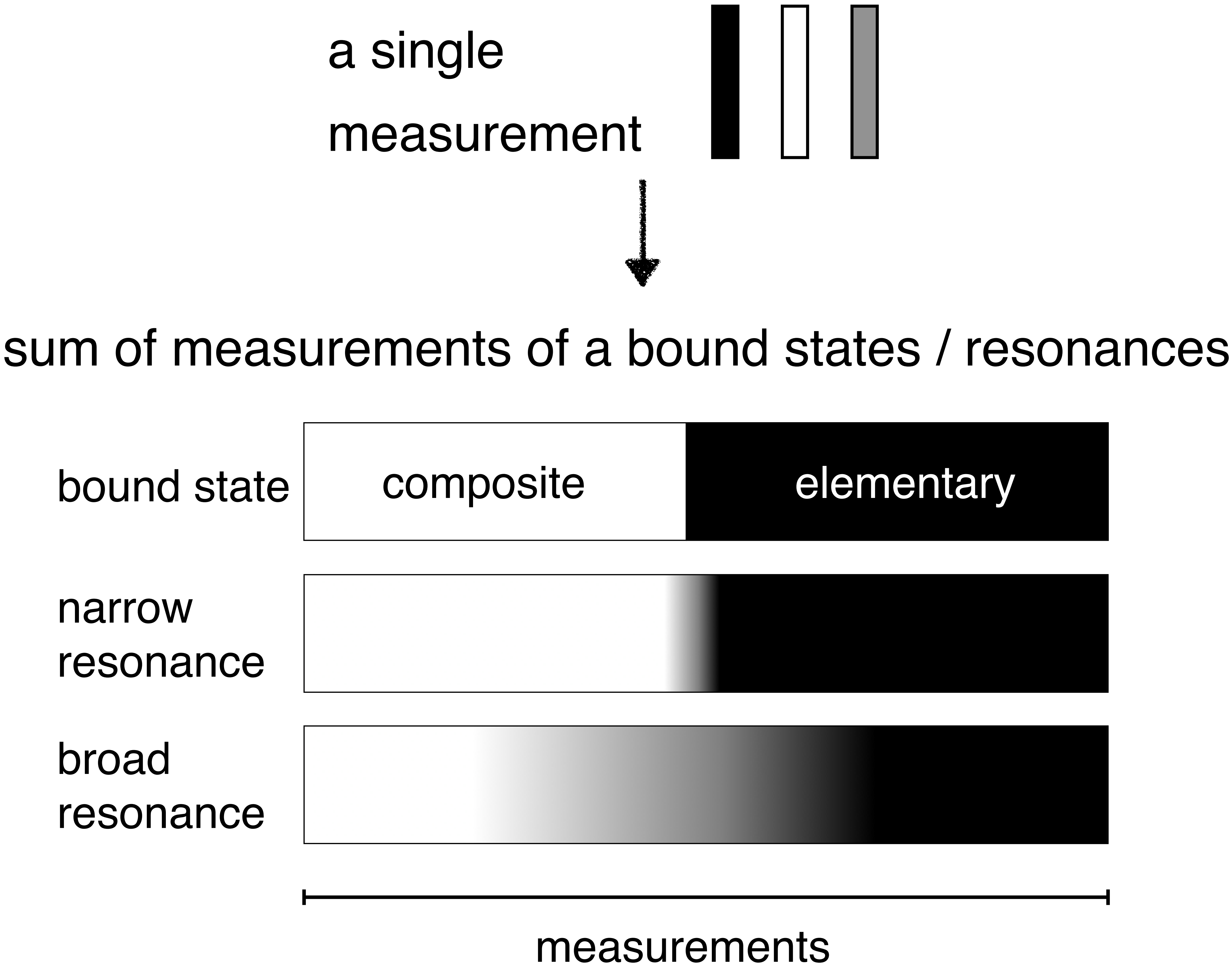




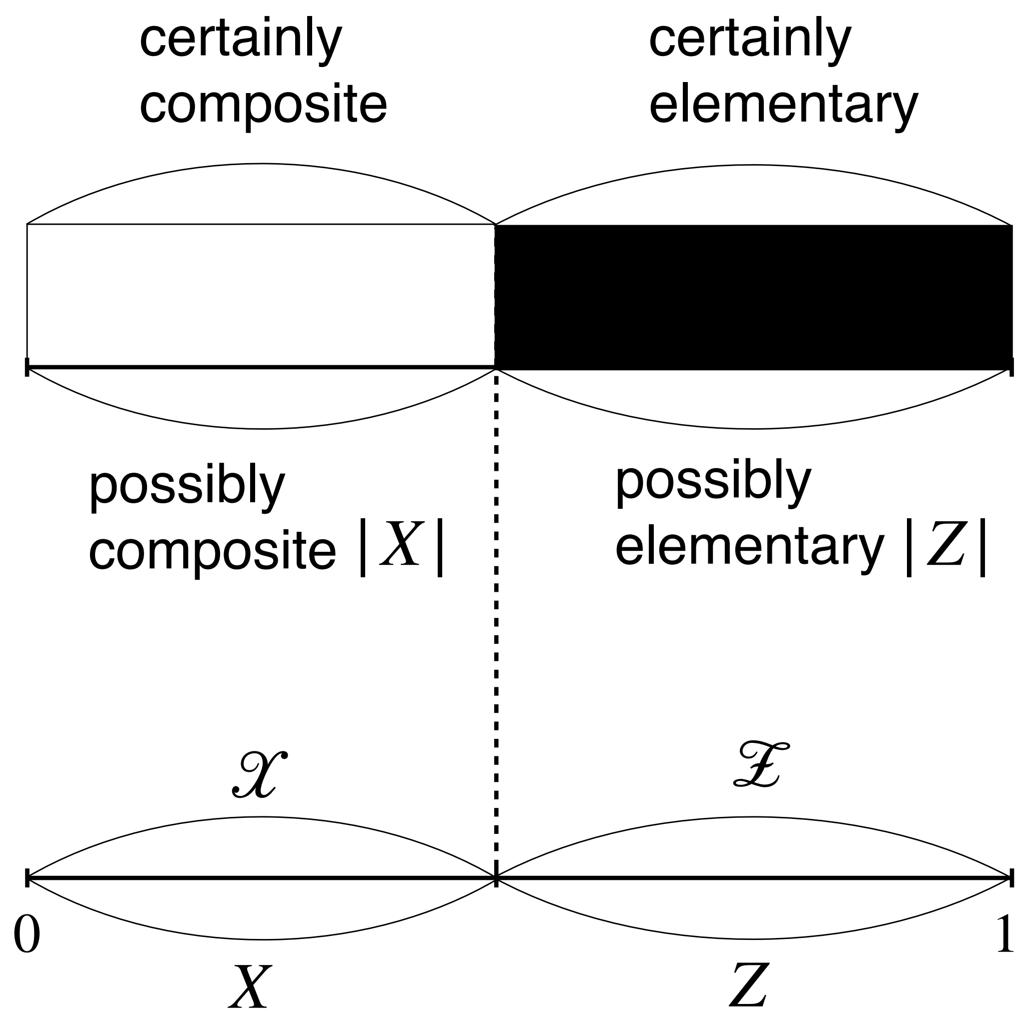
uncertain appears from

- finite lifetime (uncertainty in energy)
- separation from B.G.

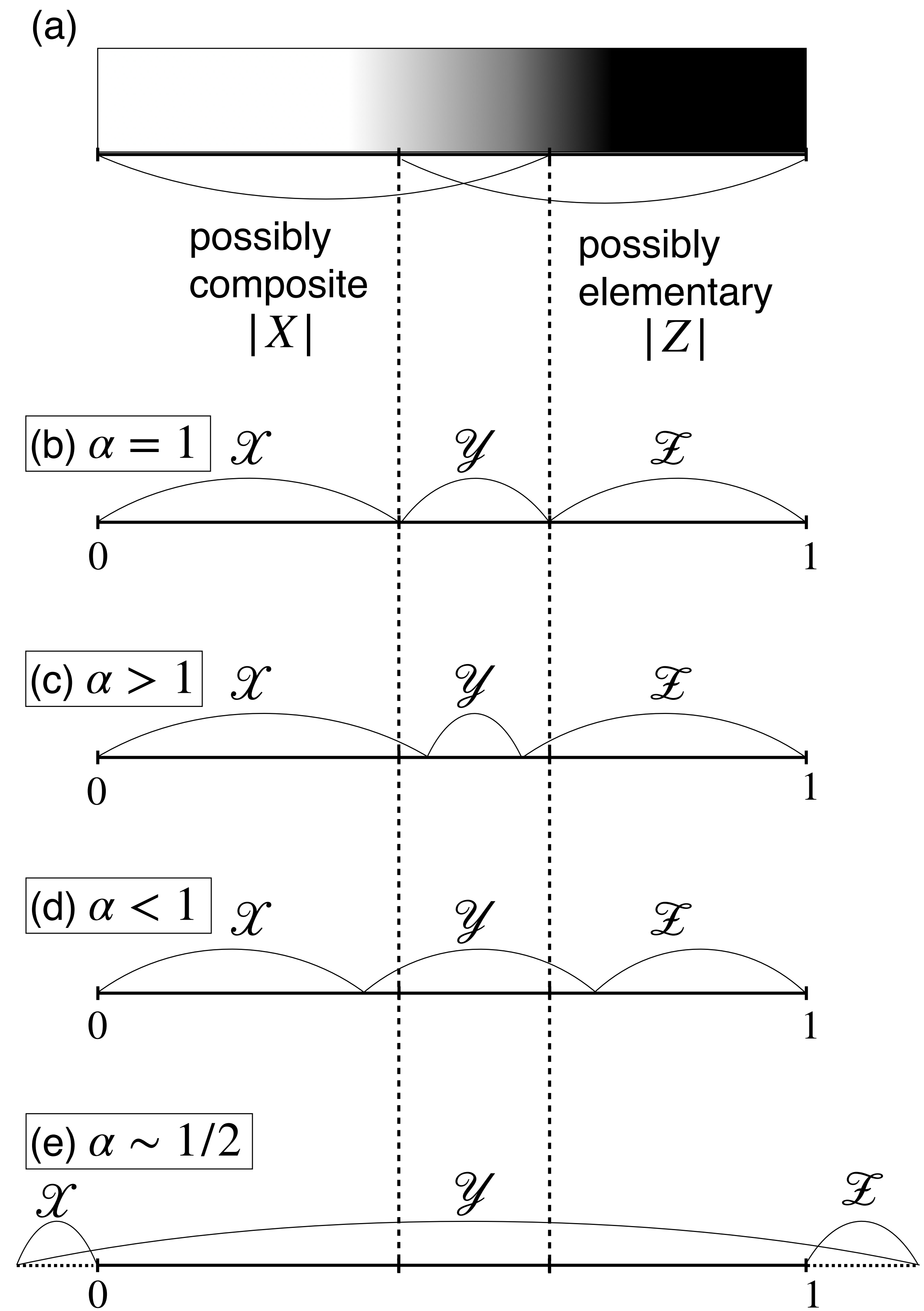
uncertainty in resonances



Interpretation



bound state



resonance

Definition

- if $\alpha > 1/2$, $\mathscr{Y}$ is always positive but $\mathscr{X}$, $\mathscr{Z}$ can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		non-interpretable

3. our criterion for narrow resonance

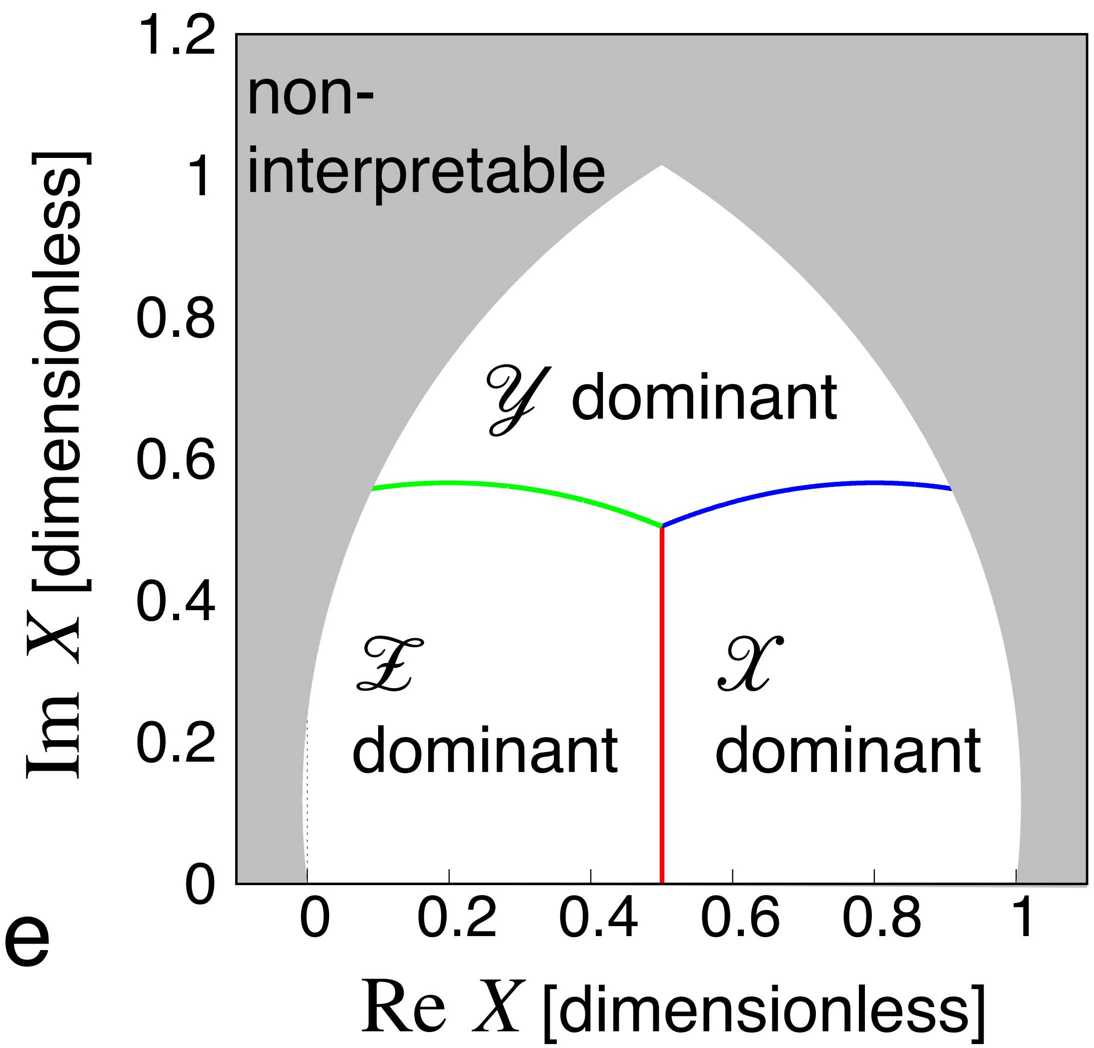
→ $\Gamma \leq \mathrm{Re}\ E$

- $\alpha \sim 1.1318$ using ERE

- narrow resonances: interpretable

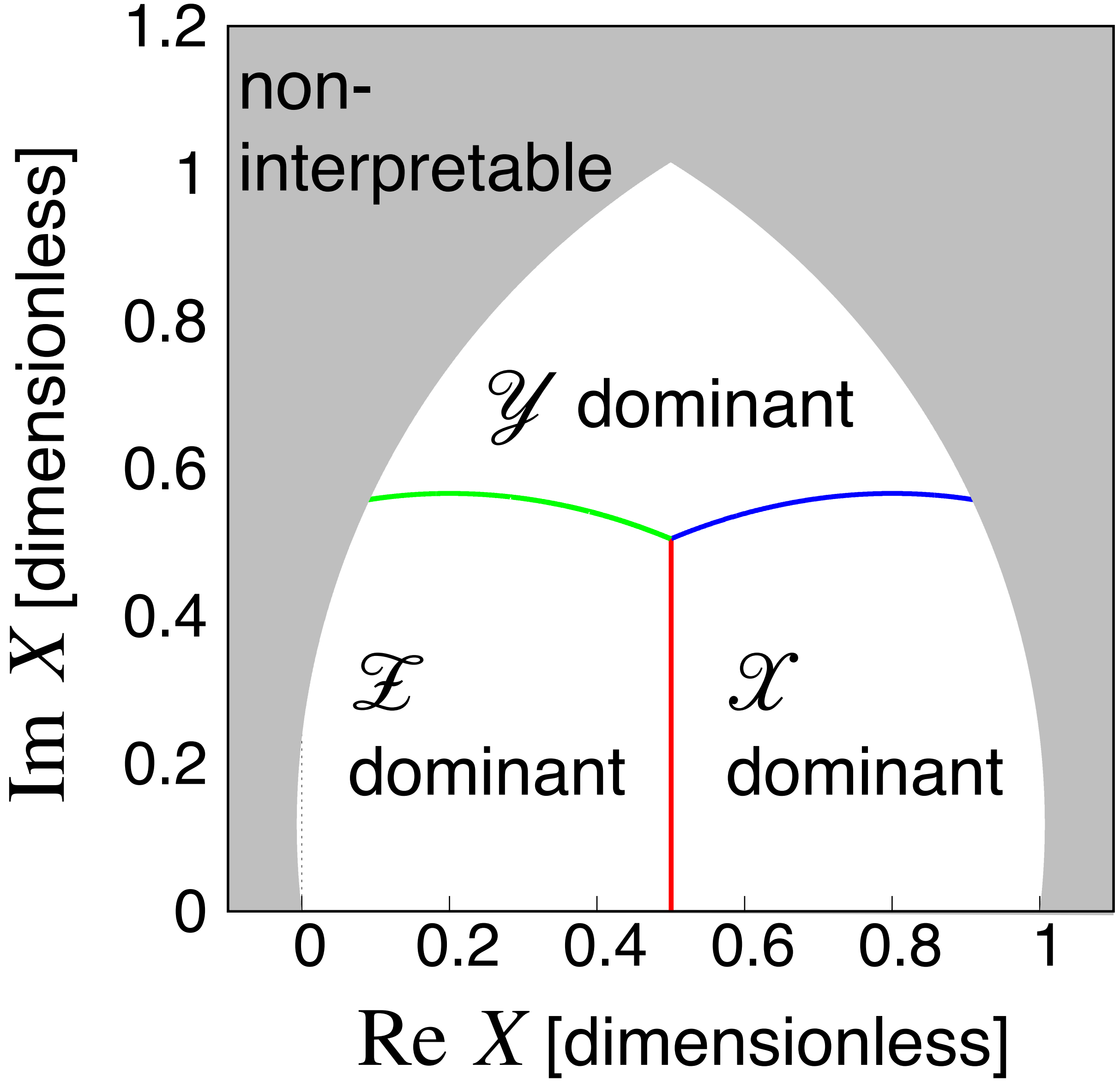
→ $\mathscr{X}$, $\mathscr{Y}$, or $\mathscr{Z}$ dominant

- broad resonances: non-interpretable



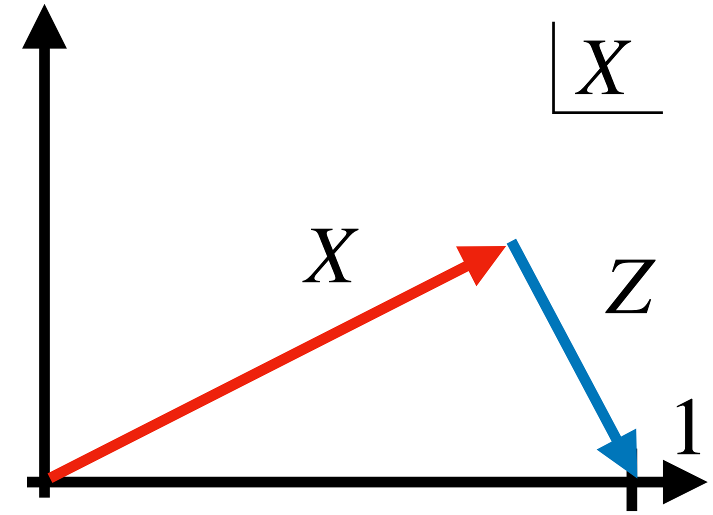
$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region

- $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region in complex X plane

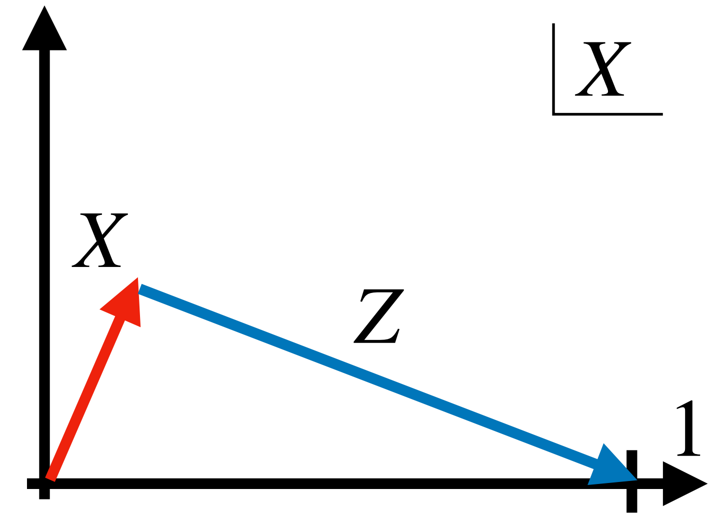


large Im X is assigned to non-interpretable cases

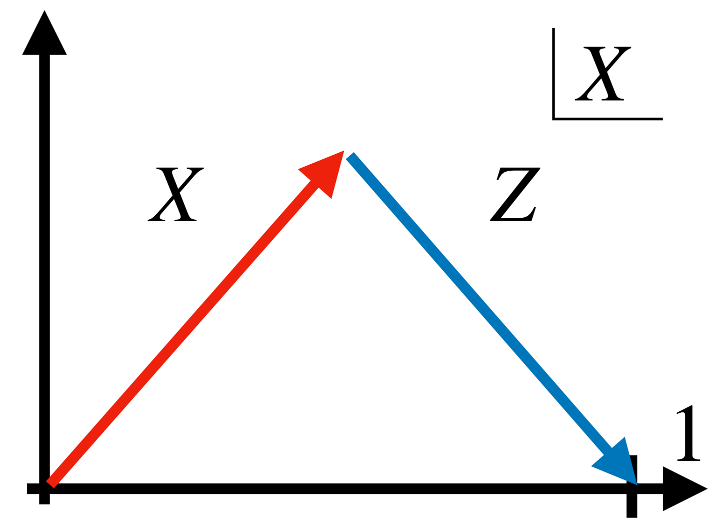
composite dominant



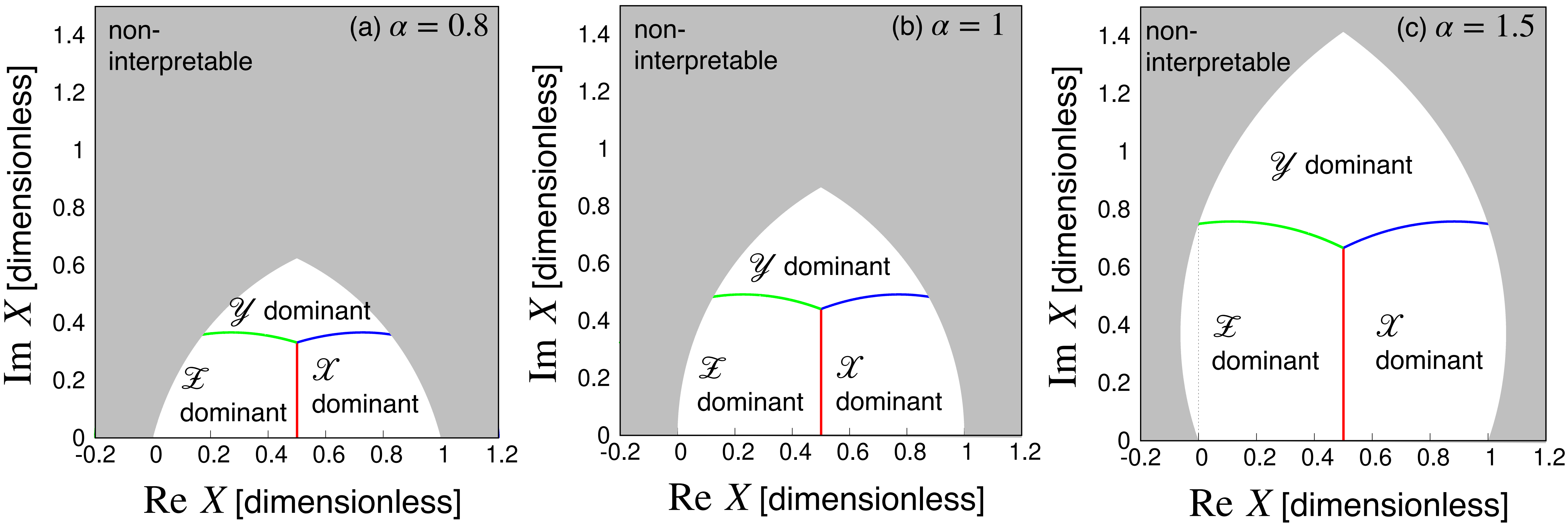
elementary dominant



uncertain



$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region



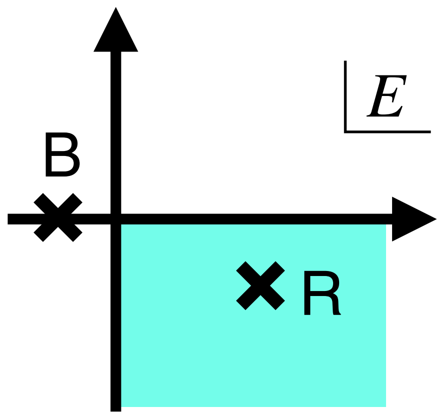
- Interpretable regions become large with increase of α

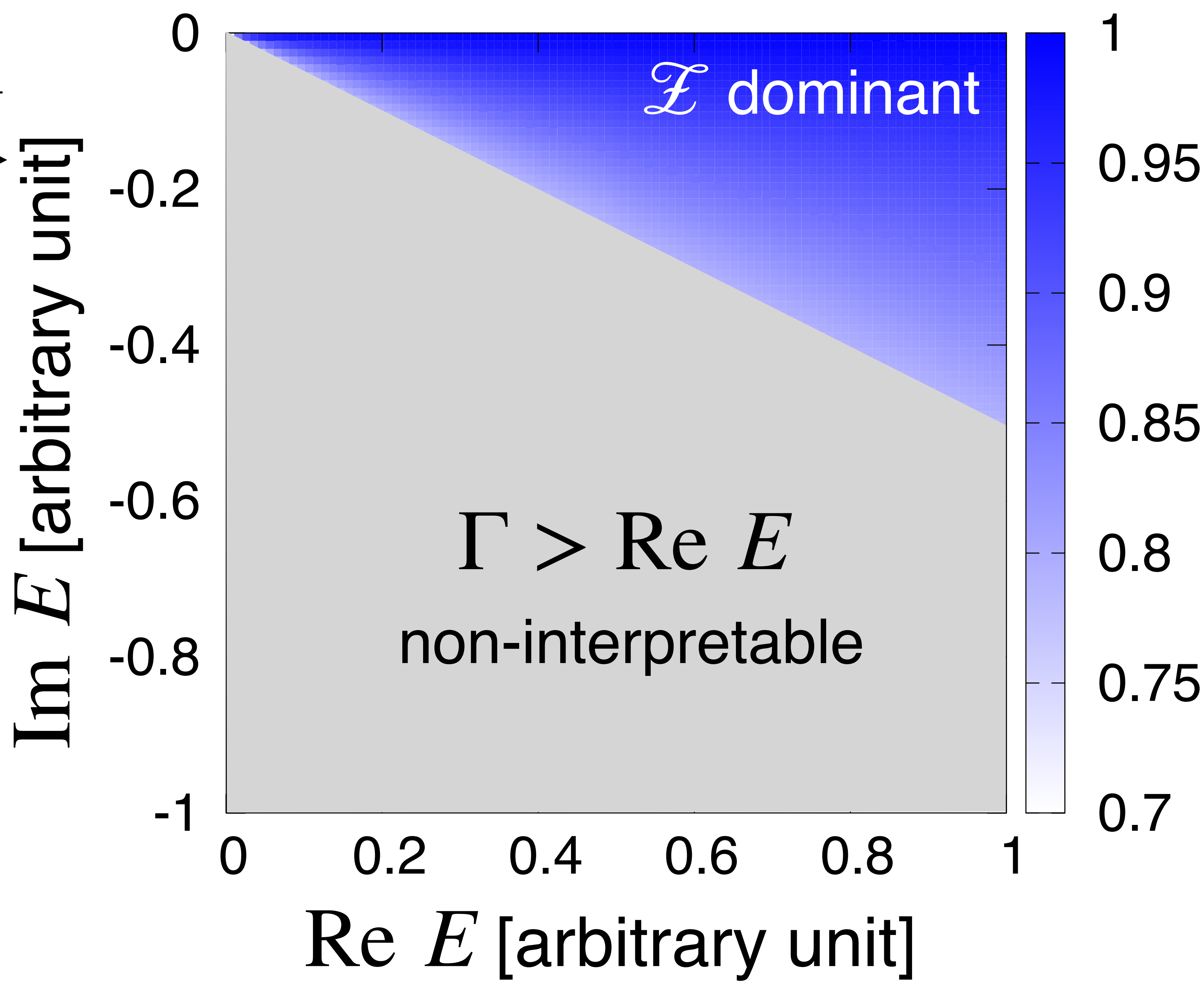
$\alpha \to \infty \longrightarrow \mathscr{X}, \mathscr{Y}, \mathscr{Z}$ reduce to interpretation in previous work

$$\mathscr{X} \to \tilde{X},\ \mathscr{Z} \to \tilde{Z},\ \mathscr{Y} \to 0$$

Y. Kamiya and T. Hyodo,
Phys. Rev. C **93**, 035203 (2016).

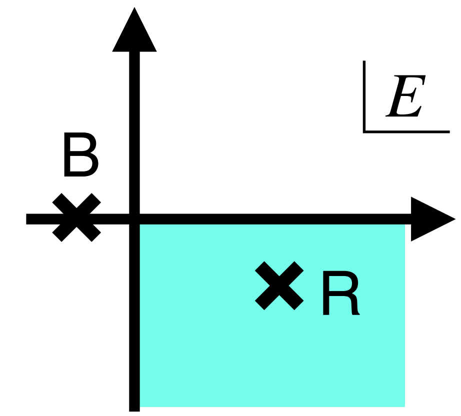
Structure of near-th. resonances

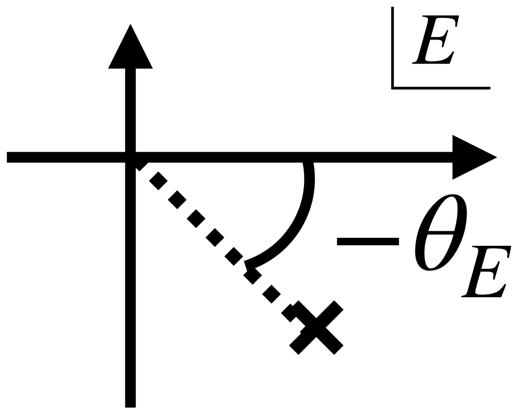


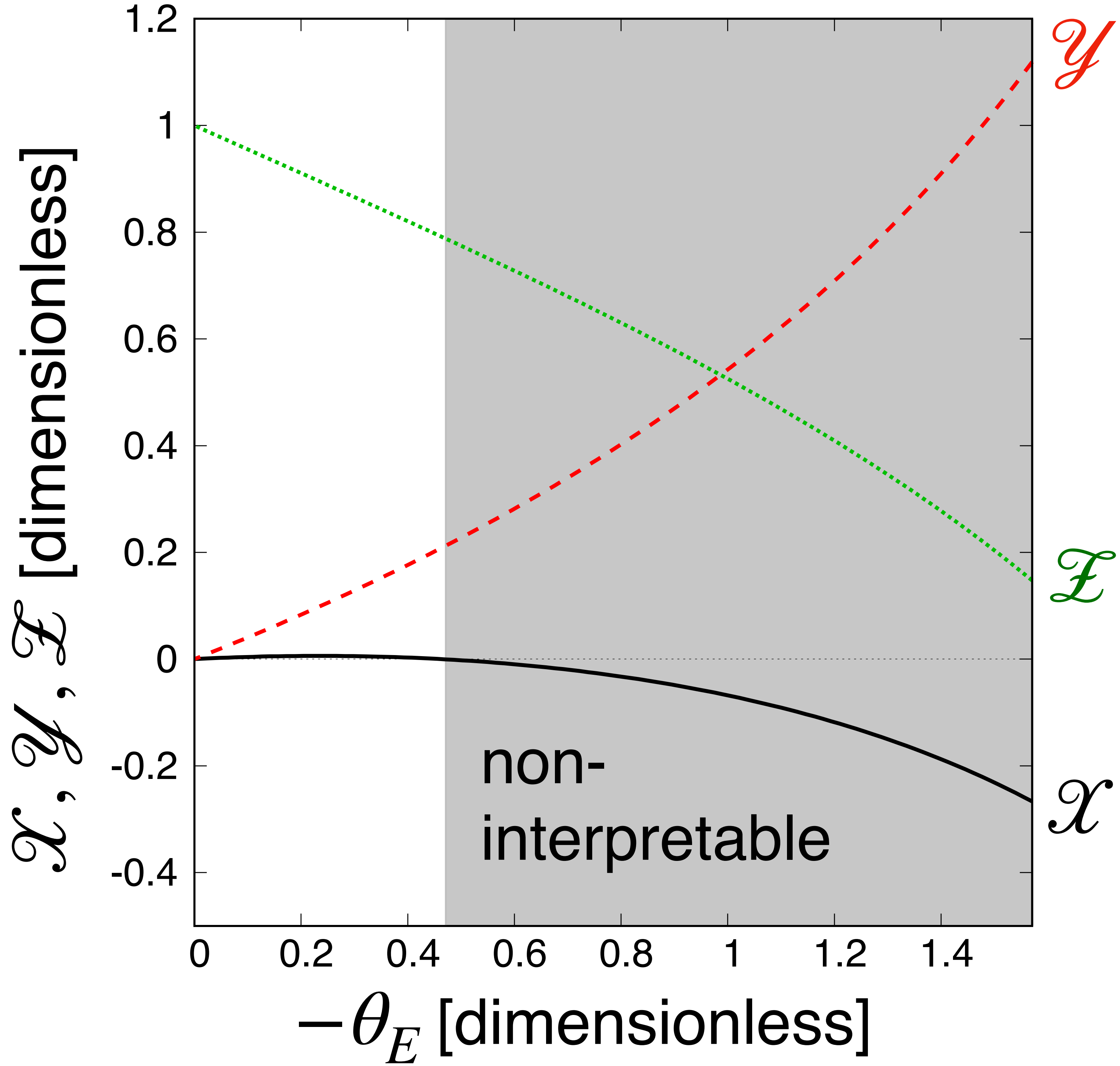


- resonances are **not composite dominant state** ($\mathcal{Z} \gtrsim 0.8$)
- different from near-threshold bound states (composite dominant $X \sim 1$ and $Z \sim 0$)

Structure of near-th. resonances







resonances with previous works

$$\bar{Z} = 1 - \sqrt{\left|\frac{1}{1-2r_e/a_0}\right|}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

$$\tilde{Z}_{\rm KH} = \frac{1+|Z|-|X|}{2}$$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

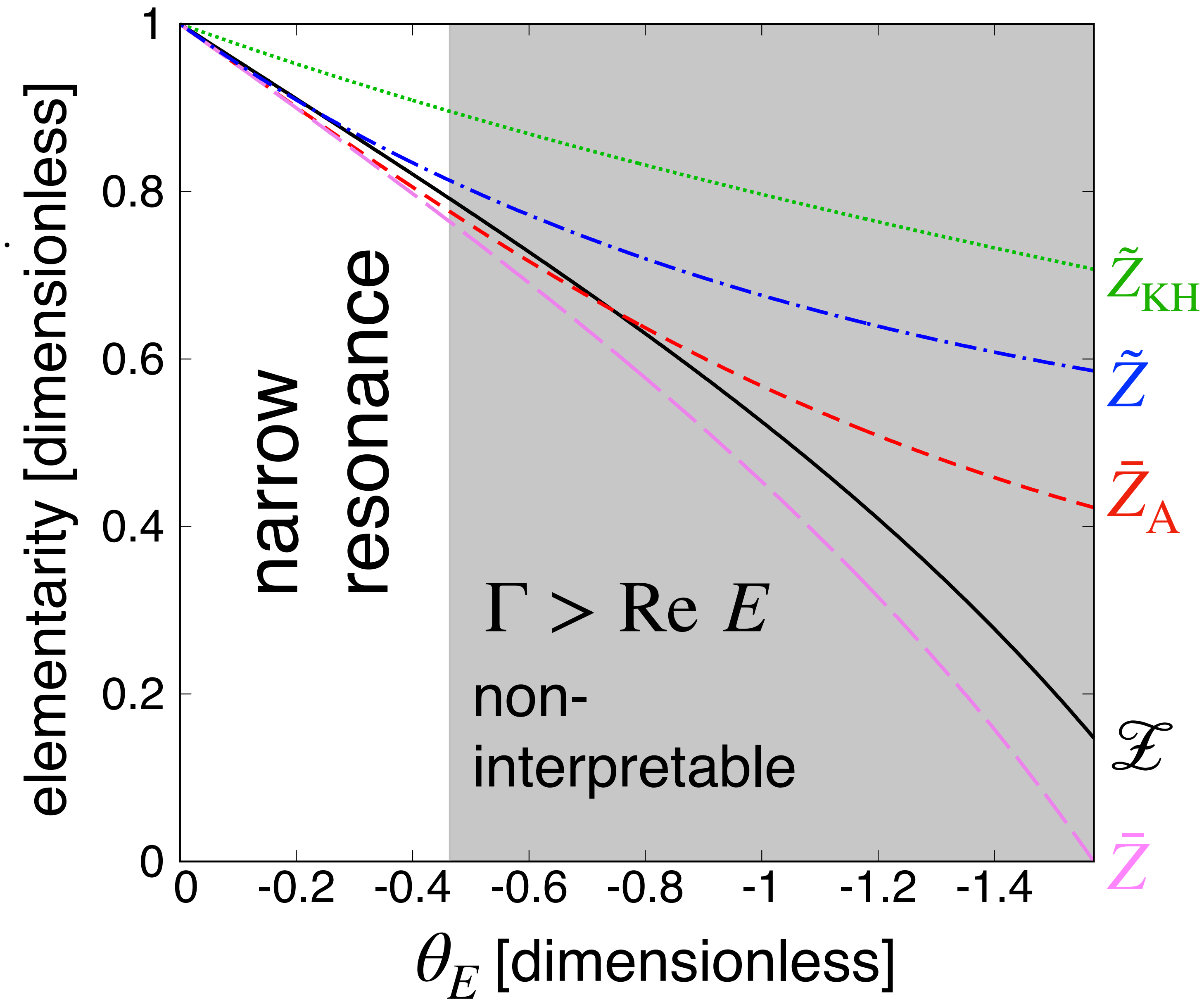
$$\tilde{Z} = \frac{|Z|}{|X|+|Z|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC **93**, 035204 (2016).

$$\bar{Z}_A = 1 - \sqrt{\frac{1}{1+|2r_e/a_0|}}$$

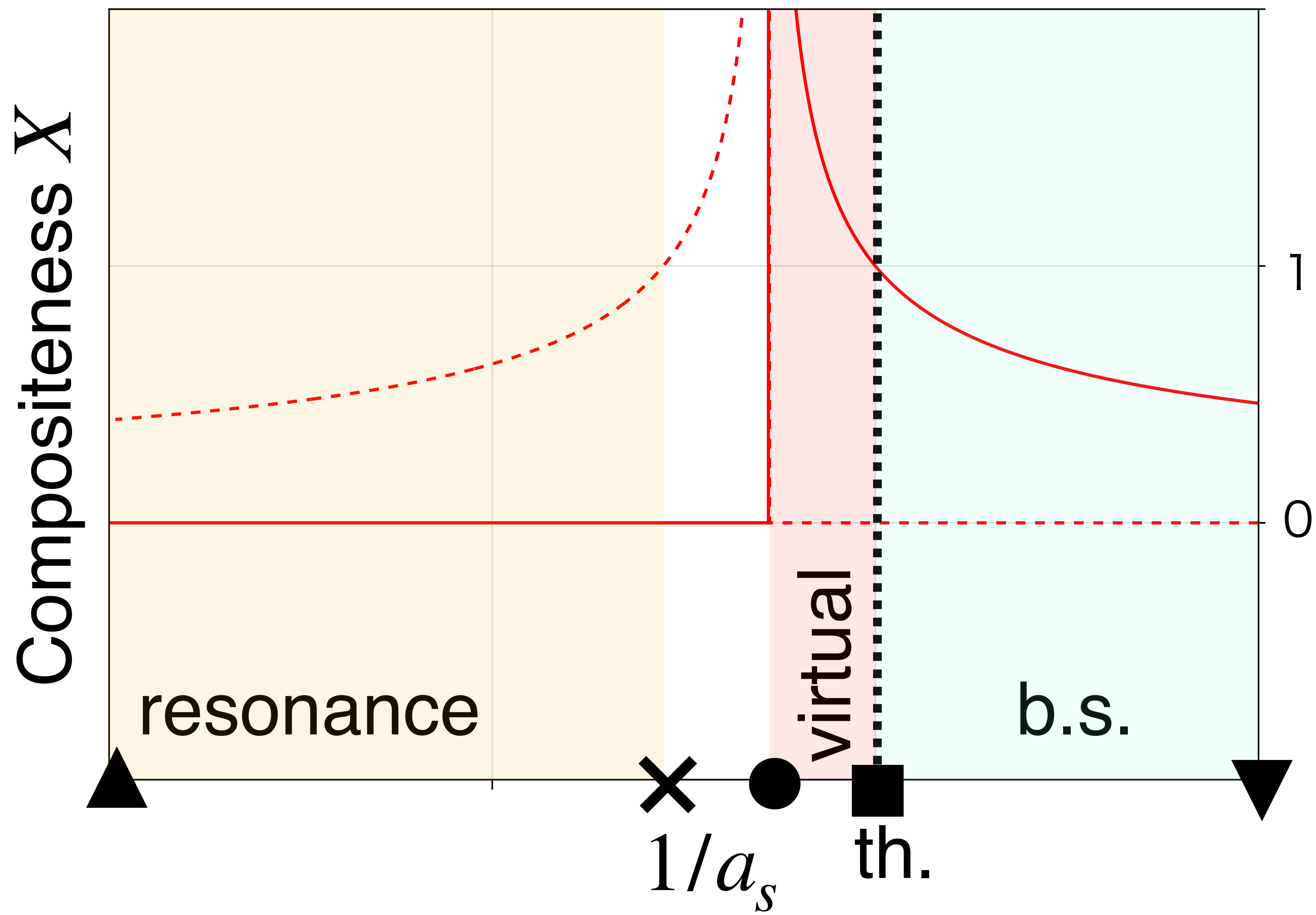
I. Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A **57**, 101 (2021).

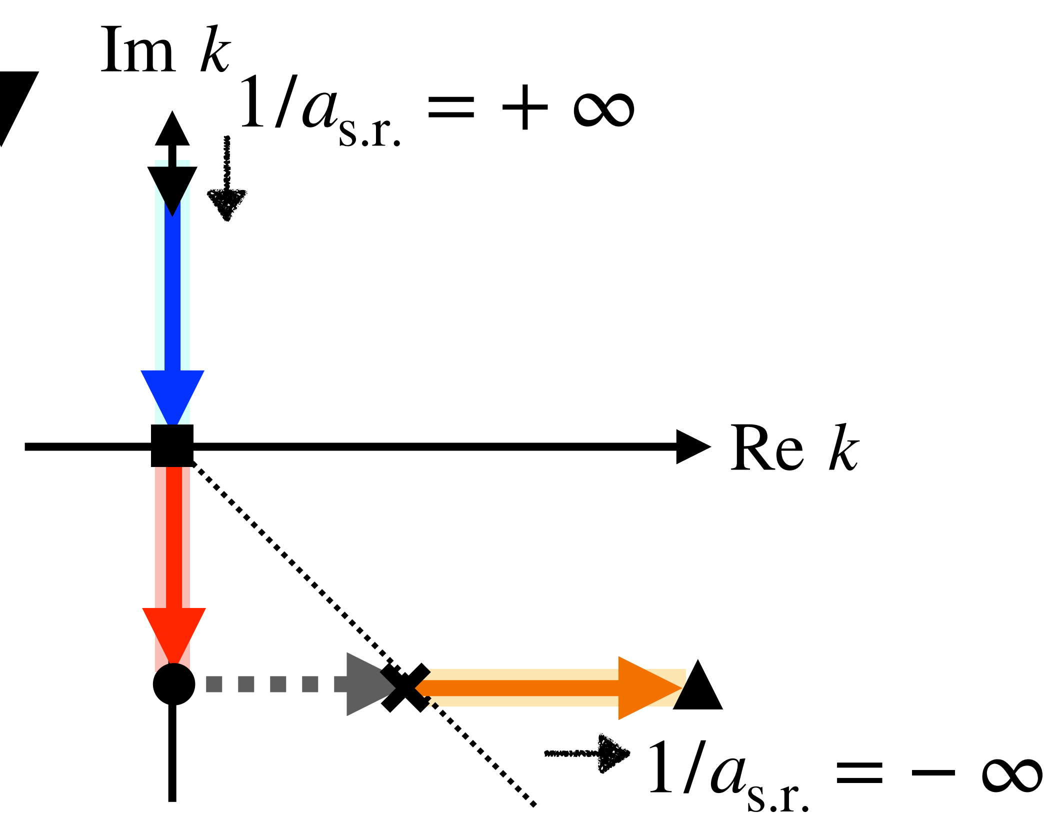
interpretations as a function of θ_E



- all shcemes show resonances are elementary dominant!

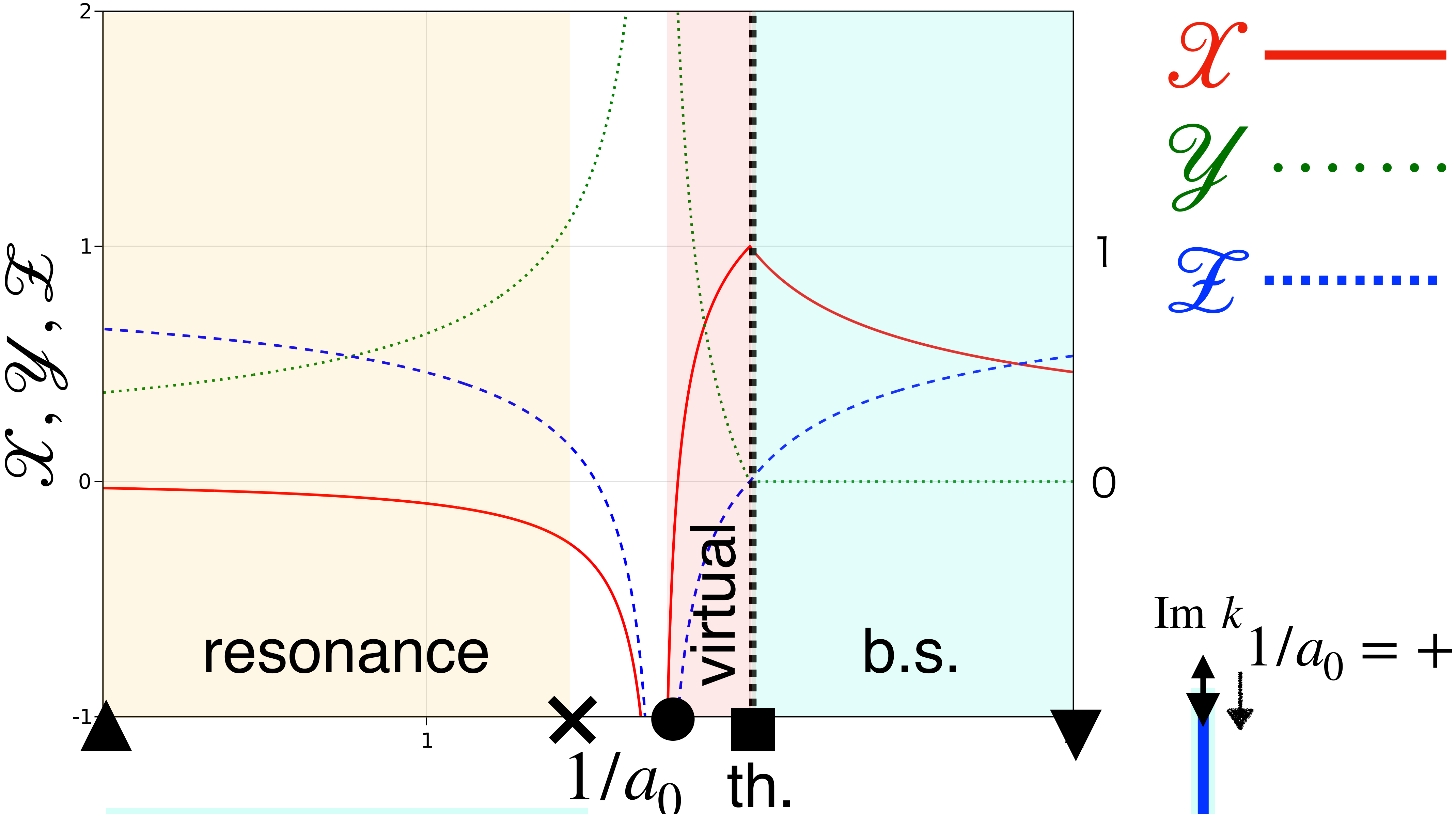
Compositeness (only w/ s.r.)

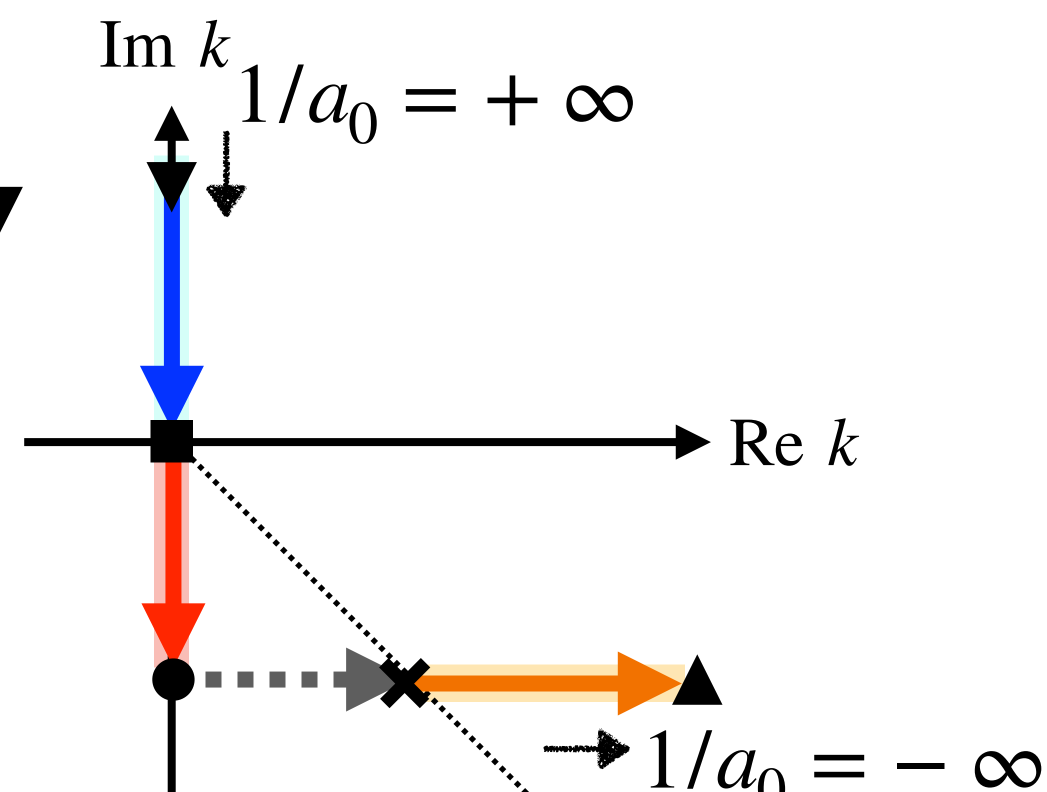




- b.s. : $0 \leq X \leq 1$
- virtual : $1 < X$
- ● divergence $X \to \infty$
- resonance : $\mathrm{Im}\ X \leq 1$

$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ from b.s. to resonance





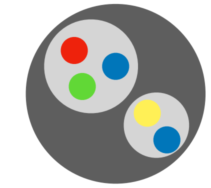
- b.s. : $0 \leq X \leq 1$
- virtual : $\mathscr{Z} < 0$ (non-interpretable)

● divergence $\mathscr{X}, \mathscr{Y}, \mathscr{Z} \to \infty$

- resonance : $\mathscr{Z}$ dominant

Summary so far

T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024); arXiv:2403.12635 [hep-ph].

- near-threshold bound states are composite dominant 

∵ low-energy universality

- new interpretation of complex compositeness and elementarity

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
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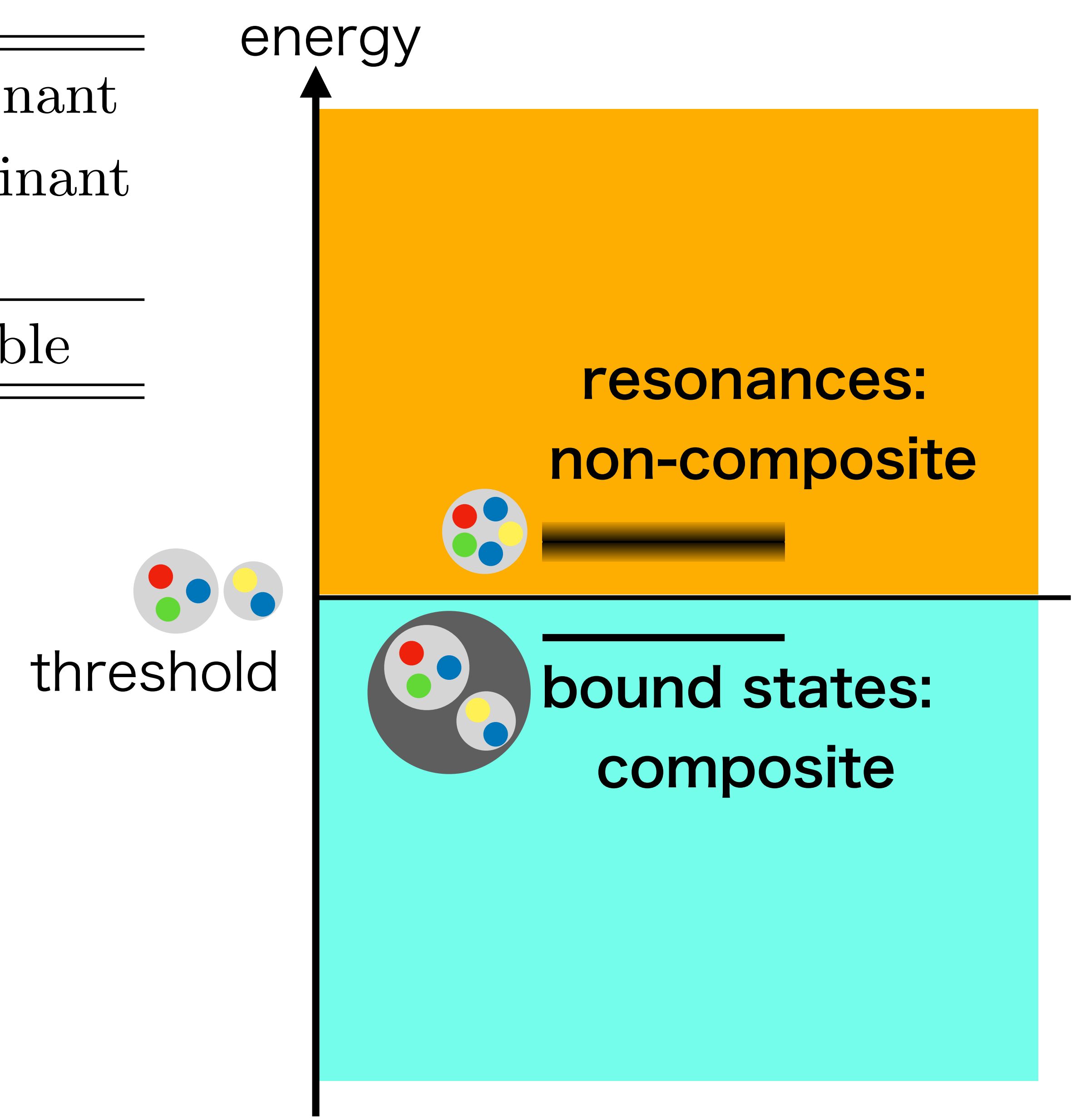
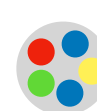


- near-threshold resonances are **not composite dominant!**

qualitatively different from near-threshold bound states 

Coulomb+short range model

◉ short range limit $a_B \to \infty$

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B}\left[\log(-ia_B k) + \psi\left(1 + \frac{i}{a_B k}\right)\right] = 0$$

$$\to 0$$

$$\longrightarrow -\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik = 0$$ ERE in s.r. interaction

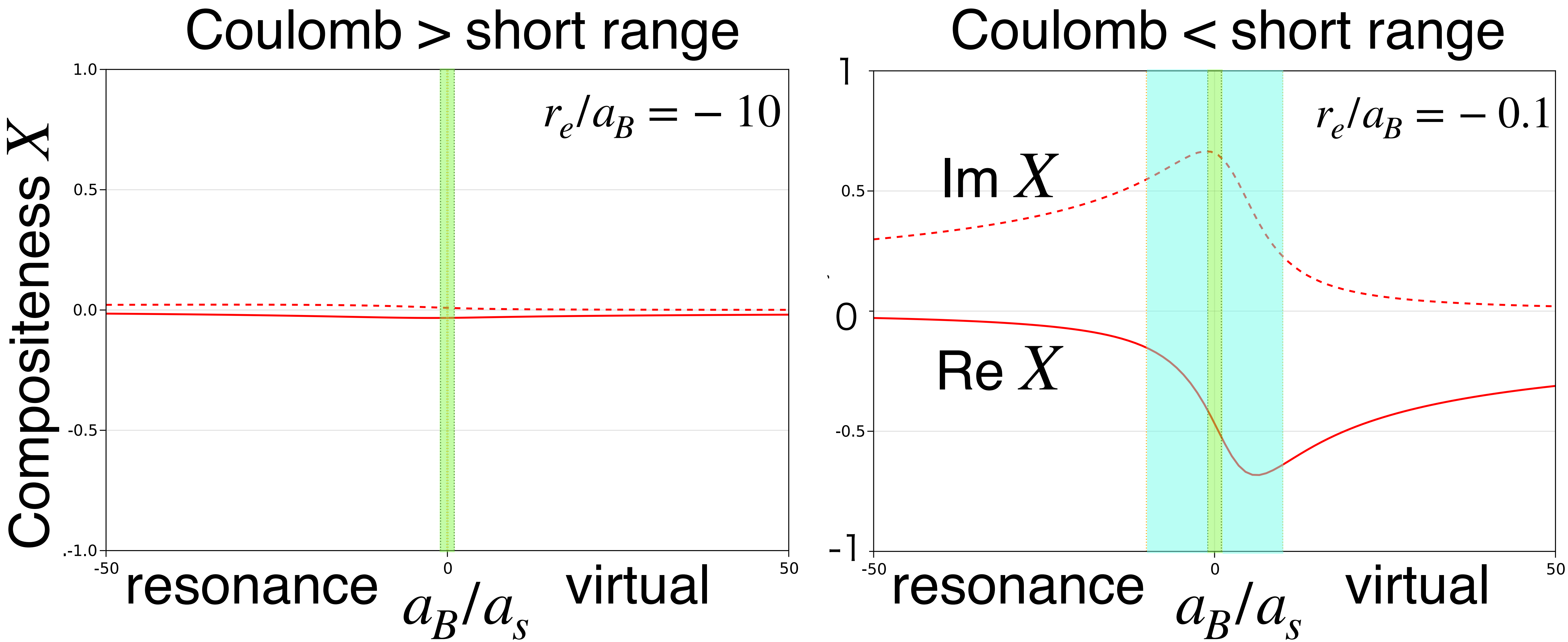
- larger a_B ⟶ weaker Coulomb $a_B = \frac{\hbar c}{\mu c^2 \alpha Z_1 Z_2}$

◉ further low-energy limit $r_e \to 0$

- zero-range theory S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

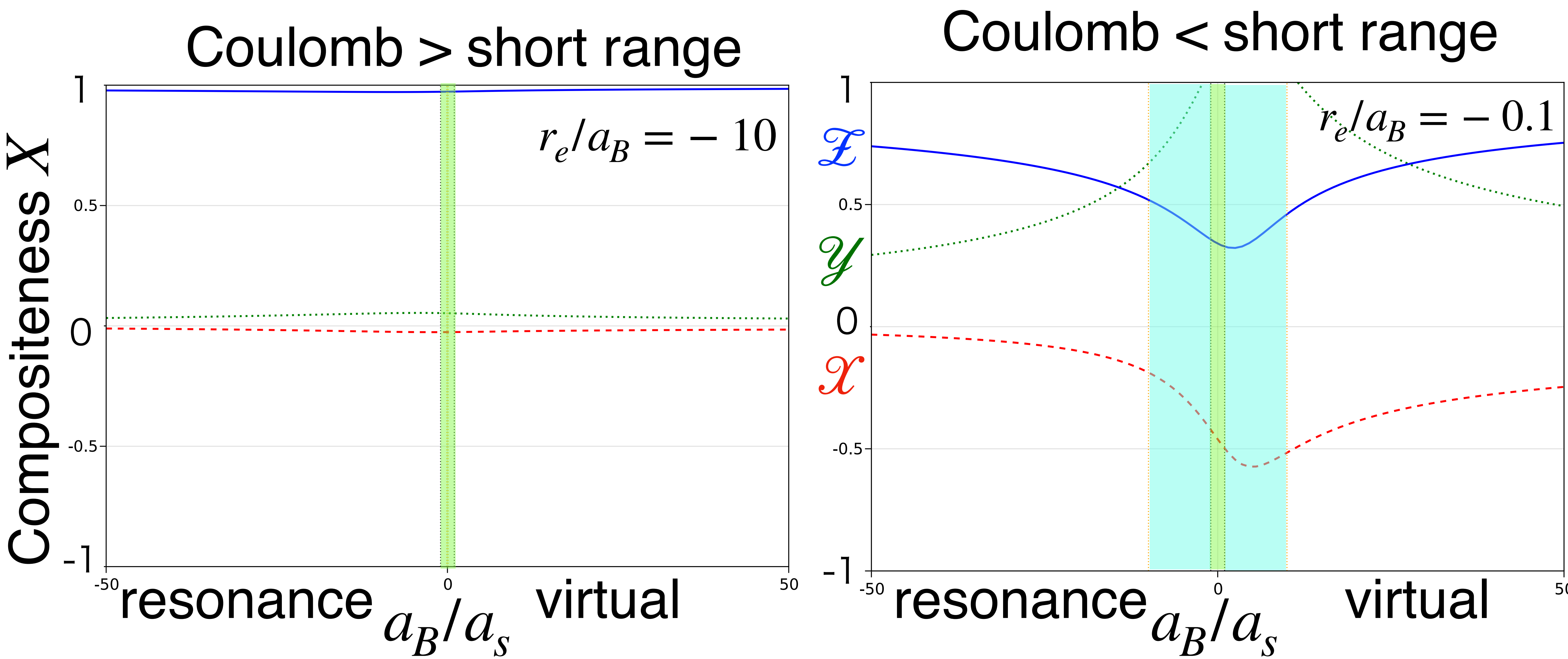
$$\frac{ia_B k}{2} \mp \log(-ia_B k) + \psi\left(1 + \frac{i}{a_B k}\right) + \frac{a_B}{2a_s} = 0$$

Compositeness (att. Coulomb resonance)



- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- compositeness of unstable resonances are complex $X \in \mathbb{C}$

Compositeness (att. Coulomb resonance)



- $\mathcal{X} < 0$ → non-interpretable in this region
- but $\mathcal{X} \geq 0$ in far-threshold region with large $|1/a_s|$

→ states are $\mathcal{Z}$ dominant with large bare state contribution