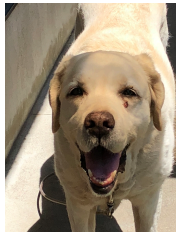
Range correction in the weak-binding relation



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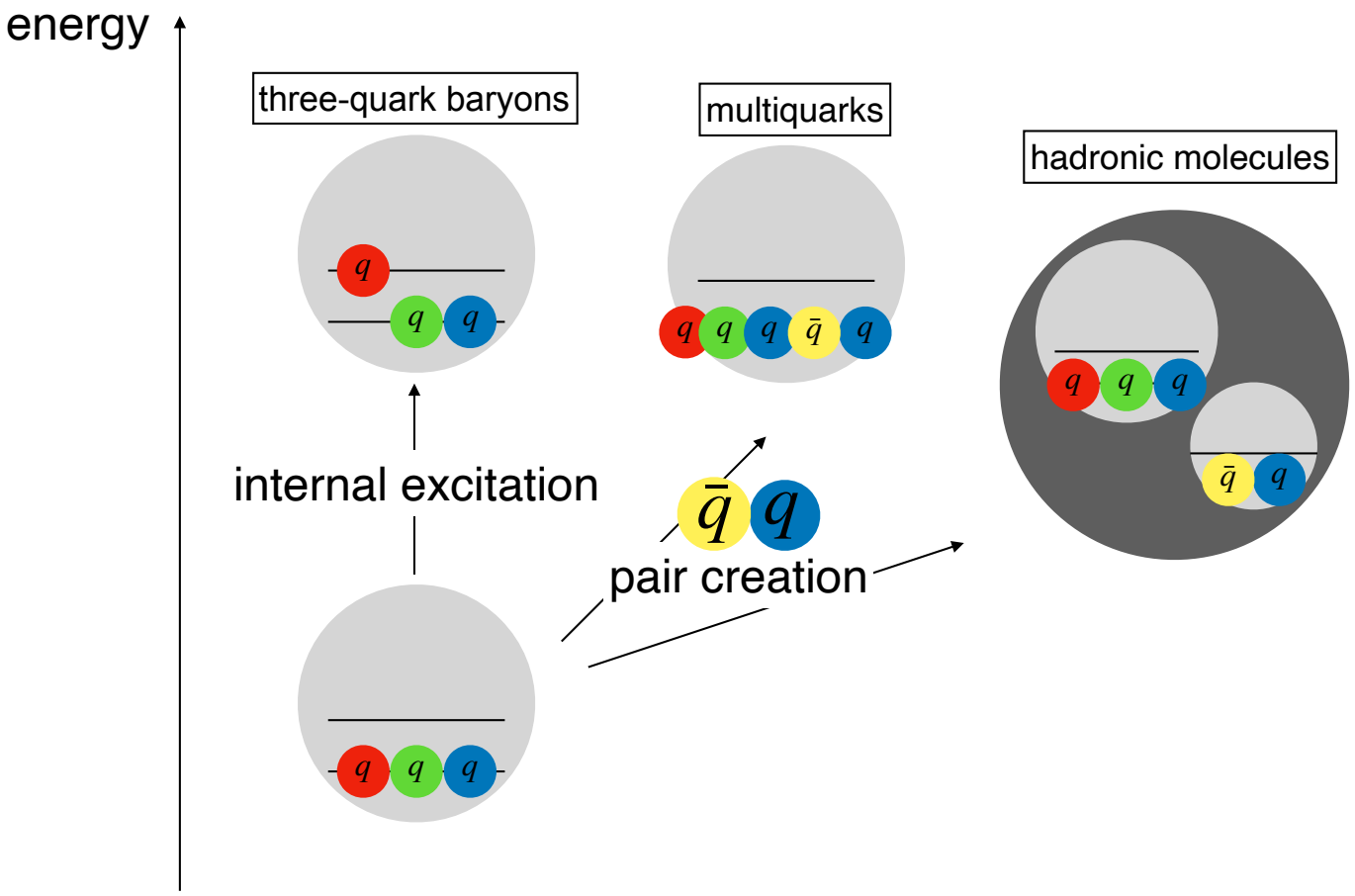
Background

candidates for exotic hadrons

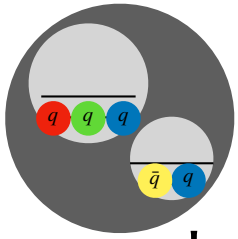
$\Lambda(1405)$, T_{cc} etc…

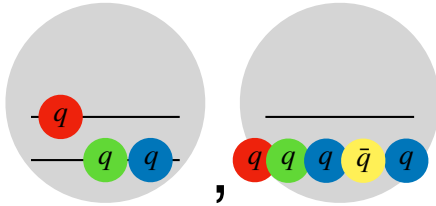
multiquarks
hadronic molecules



structure of hadrons ← model independent → observable

Previous work



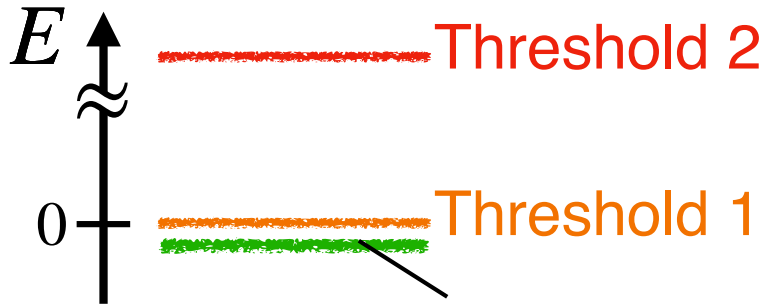


Hadron wave function $|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$

Compositeness (weight of hadronic molecule)

Weak-binding relation for bound state

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}$$



a_0 (scattering length) R_{typ}(interaction range)

$R \equiv (2\mu B)^{-1/2}$, B (binding energy)

When $R \gg R_{\text{typ}}$: observables(a_0, B) → compositeness(X)

S. Weinberg, Phys. Rev. 137, B672 (1965); Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Aim of this talk

We study the range correction in the weak-binding relation by introducing the effective range r_e.

Improved weak-binding relation

T. Kinugawa, T. Hyodo, (2021), arXiv:2112.00249

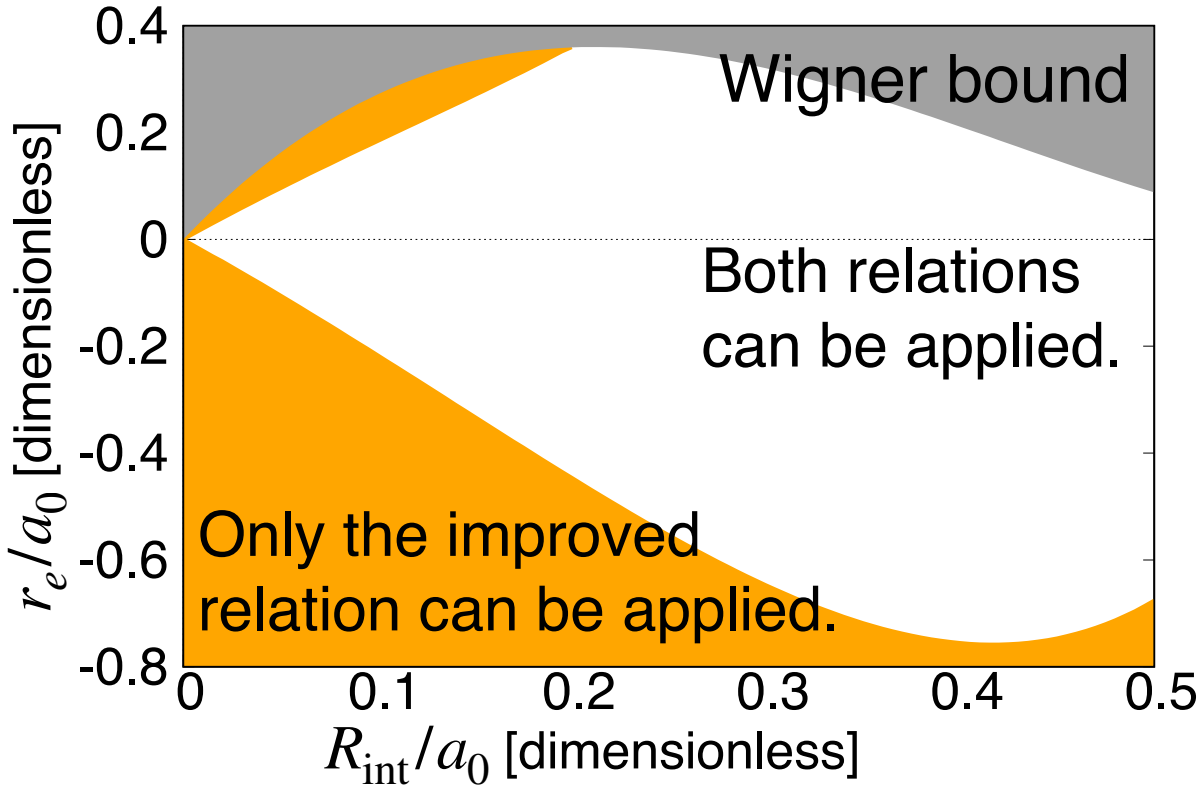
Redefinition of $R_{\rm typ}$ $\quad$ $R_{\rm int}$: interaction range

$$R_{\rm typ} = \max\left\{ R_{\rm int}, R_{\rm eff} \right\},\ R_{\rm eff} = \max\left\{ |r_e|, \frac{|P_s|}{R^2}, \cdots \right\}.$$

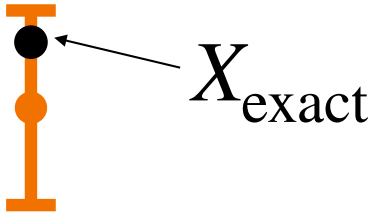
Numerical calculation

Effective range model ($R_{\rm int} \neq 0$)

Applicable regions in $R_{\rm int}/a_0$-r_e/a_0 plane



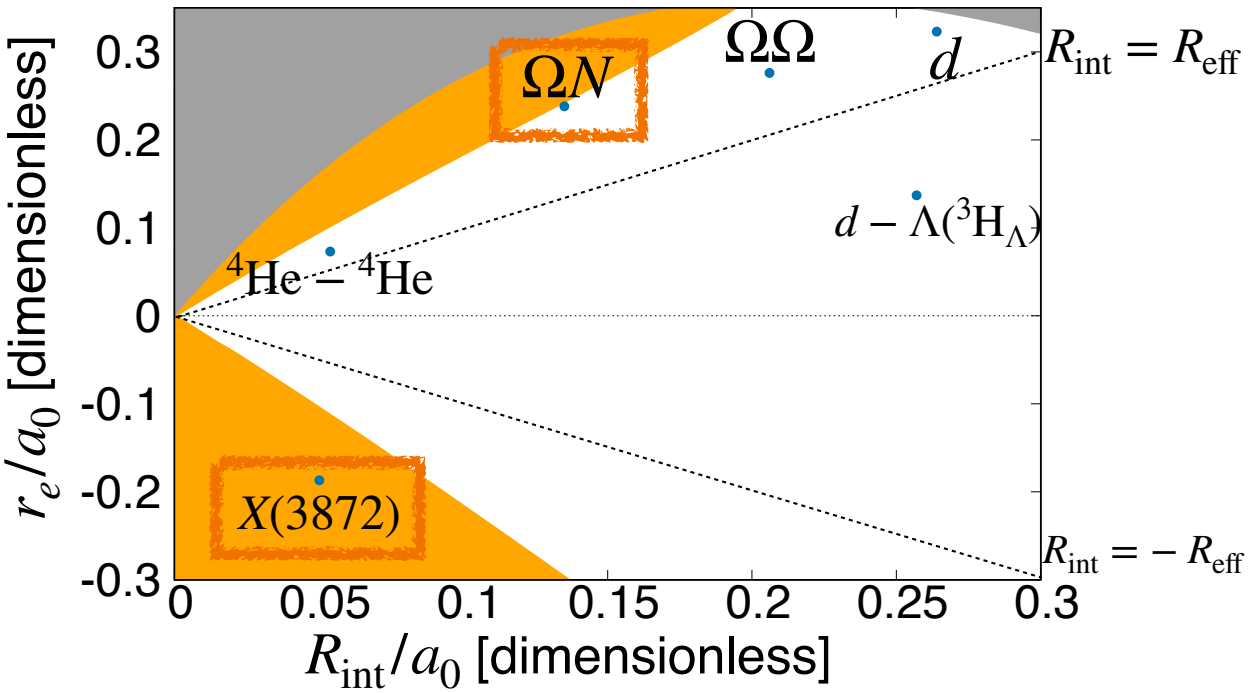
Validity condition



T. Kinugawa, T. Hyodo, (2021), arXiv:2111.06619

Applicable region of the improved weak-binding relation is larger.

Application



Lower bounds of compositeness X_l

system	This work	Previous work
deuteron	0.738	0.857
$X(3872)$	0.530	0.681
ΩN	0.787	1.27
$\Omega\Omega$	0.775	0.918
d-$\Lambda(^3_\Lambda{\rm H})$	0.745	0.745
^{4}He-^{4}He	0.929	0.967

Only improved weak-binding relation is applicable to $X(3872)$ and ΩN

Summary and future prospects

- Weak-binding relation : observable → compositeness (X)
- Improve the weak-binding relation by redefinition of $R_{\rm typ}$
- Apply to the actual hadrons, hypernuclei and atoms
- Future prospect: extend the improved relation to the unstable states