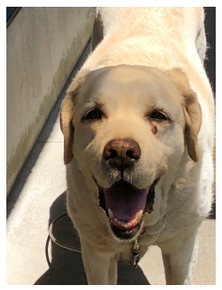
Weak-binding relation in the zero range limit



Tomona Kinugawa　**Tetsuo Hyodo**

Department of Physics, Tokyo Metropolitan University

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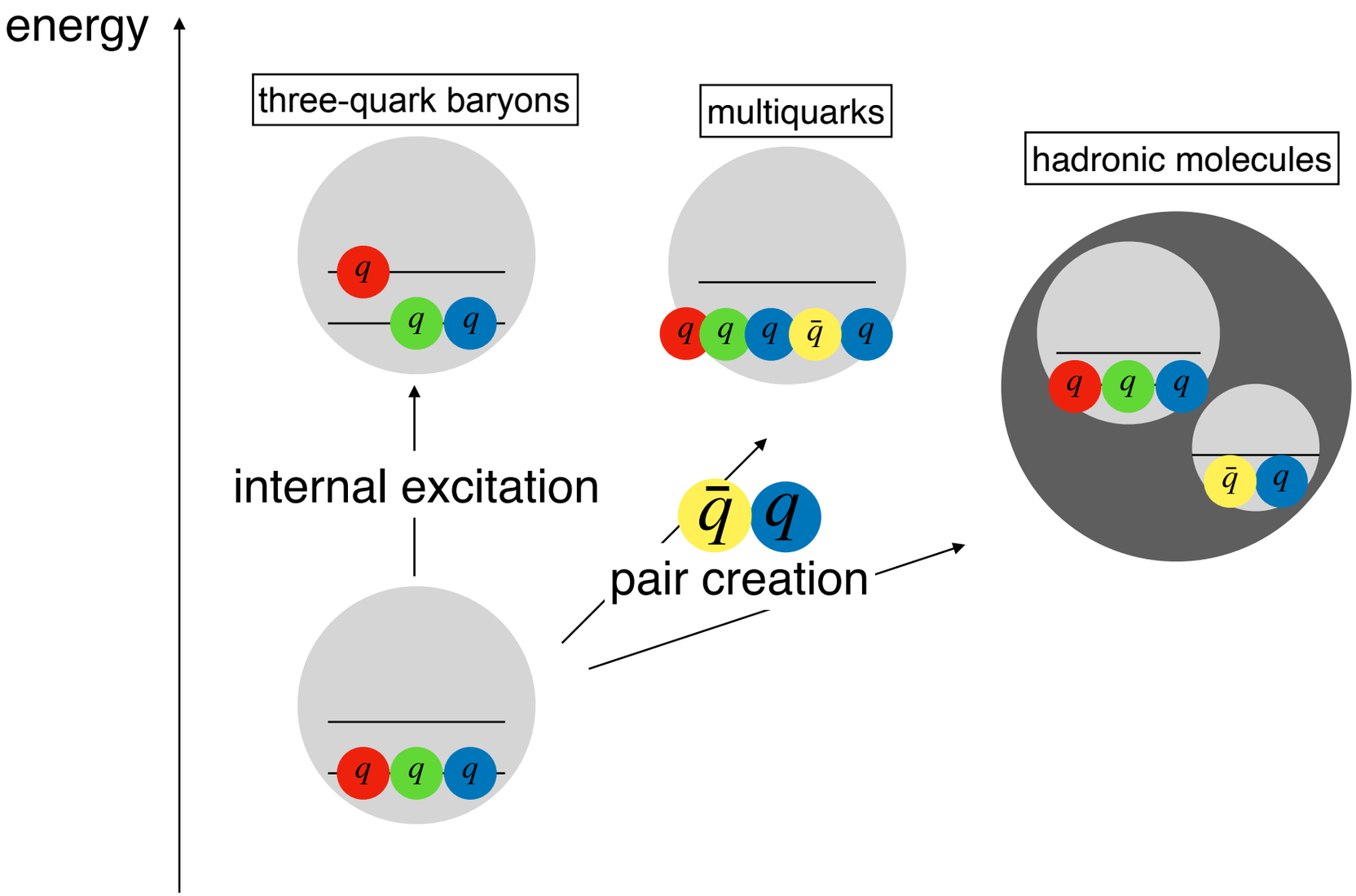
Background

candidates for exotic hadrons

$\Lambda(1405)$, XYZ meson etc…

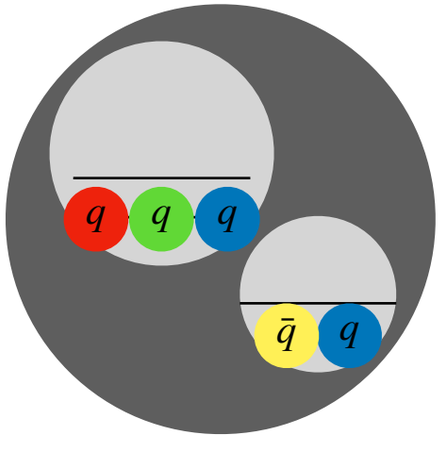
multiquarks
hadronic molecules

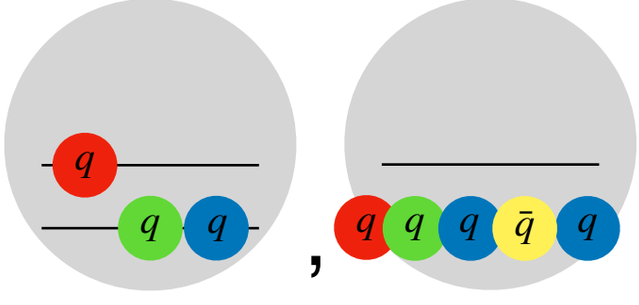


structure of hadrons ← model independent → observable

Previous work

Hadron wave function





$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

Compositeness (weight of hadronic molecule)

Weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

a_0 (scattering length)

$R \equiv (2\mu B)^{-1/2}$, B (binding energy)

$R_{\rm typ}$(interaction range)

When $R \gg R_{\rm typ}$: observable(a_0, B) → compositeness(X)

S. Weinberg, Phys. Rev. 137, B672 (1965); Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Motivation

Weak-binding relation $a_0 = R\left\{ \dfrac{2X}{1+X} + \mathcal{O}\left(\dfrac{R_{\rm typ}}{R}\right)\right\}$

Apply to the following model :

Single channel: $|$ hadronic molecule$\rangle$ only $\Rightarrow X = 1$ $\Leftrightarrow a_0 = R$?

Zero range limit: $R_{\rm typ} \to 0 \Rightarrow \mathcal{O}(R_{\rm typ}/R) \to 0$

This work

Effective range model (single channel, zero range limit)

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008), 0709.0499.

$$\mathscr{H}_{\rm int} = \frac{1}{4}\lambda_0(\psi^\dagger\psi)^2 + \frac{1}{4}\rho_0\nabla(\psi^\dagger\psi)\cdot\nabla(\psi^\dagger\psi) \quad\Rightarrow\quad f(k) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik\right]^{-1}$$

$$a_0 = R\frac{2r_e/R}{1-(r_e/R-1)^2} = R\left[1+\mathcal{O}\left(\left|\frac{r_e}{R}\right|\right)\right] \Rightarrow a_0 \neq R$$

Weak-binding relation should be improved.

Conclusion and future prospect

- Weak-binding relation : observable → compositeness (X)

$$a_0 = R\left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right) \right\}$$

- $R_{\rm typ} \to 0$ in effective range model : $a_0 = R + \mathcal{O}(|r_e/R|)$

→ Weak-binding relation is not satisfied.

- Redefinition of $R_{\rm typ}$ $R_{\rm int}$: interaction range

$$R_{\rm typ} = R_{\rm int} \rightarrow \max\left\{R_{\rm int}, R_{\rm eff}\right\}, \quad R_{\rm eff} = \max\{|r_e|, \cdots\}$$

- Apply the improved relation to the system with $R_{\rm eff} \gtrsim R_{\rm typ}$