

Internal structure of near-threshold states using compositeness



T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024).

T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

T. Kinugawa and T. Hyodo, arXiv:2507.22399 [hep-ph].

Tomona Kinugawa, RIKEN Nishina Center

Aug. 19th, SEA-NHP 2025 @ Universitas Gadjah Mada

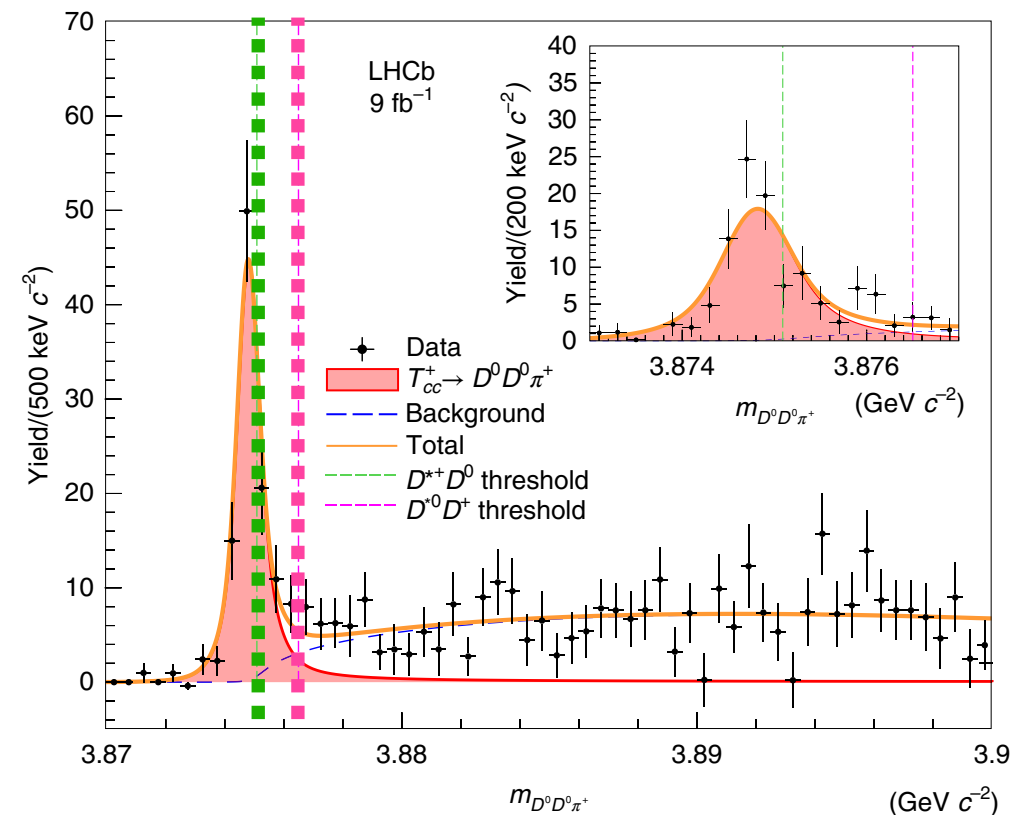
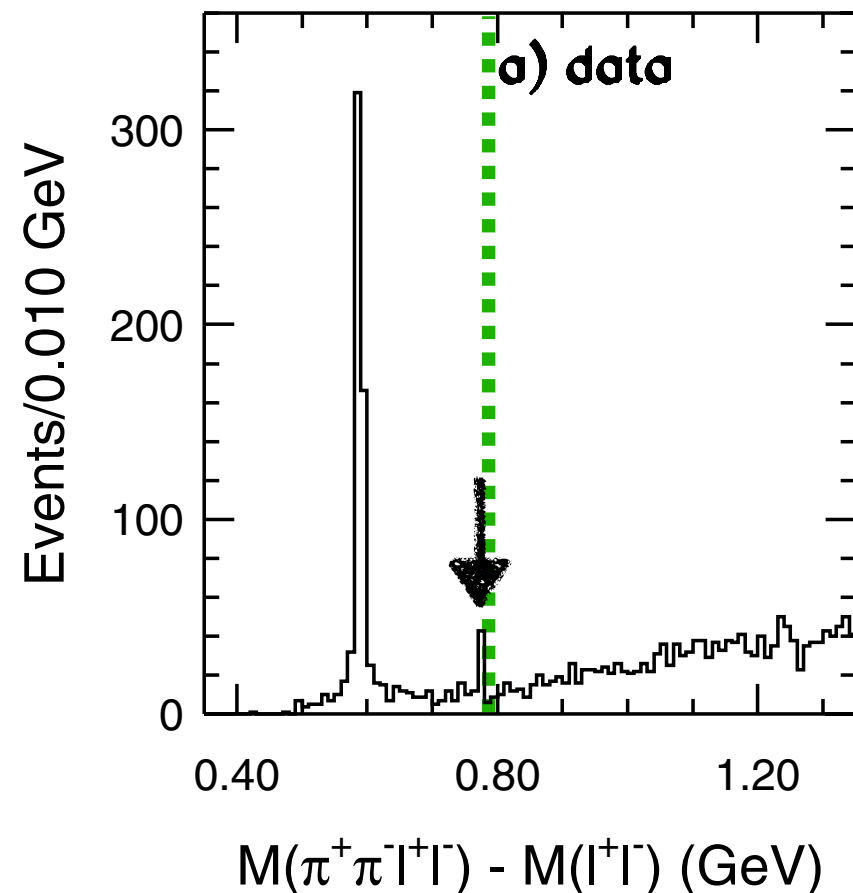
Near-threshold exotic hadrons

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- exotic hadrons tend to be observed near threshold

$$X(3872) \rightarrow \pi^+ \pi^- J/\psi$$

$$T_{cc}(3875)^+ \rightarrow D^0 D^0 \pi^+ \quad (cc\bar{u}\bar{d})$$



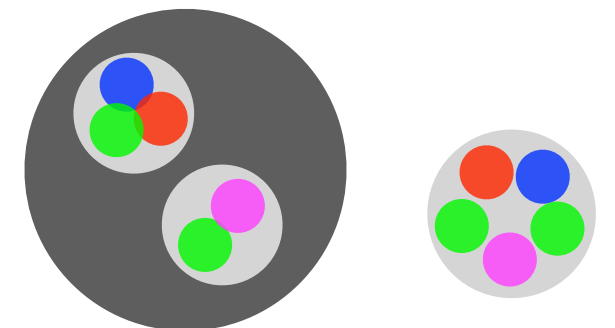
LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

→ internal structure of near-threshold states?

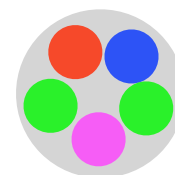
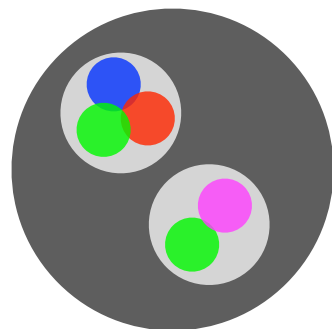
hadronic molecule, multiquark, etc...?



Compositeness

Weinberg, S. Phys. Rev. 137, 672–678 (1965);
 T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
 T. Kinugawa, T. Hyodo, Eur. Phys. J. A 61, 154 (2025).

● Definition



$$|\text{state}\rangle = \sqrt{X} |\text{composite}\rangle + \sqrt{Z} |\text{others}\rangle$$

compositeness

elementarity

- quantitative measure

$$0 \leq X \leq 1$$

$$X + Z = 1$$



$$X > 0.5 \Leftrightarrow \text{composite dominant}$$

$$X < 0.5 \Leftrightarrow \text{non-composite dominant}$$

- applicable to various states

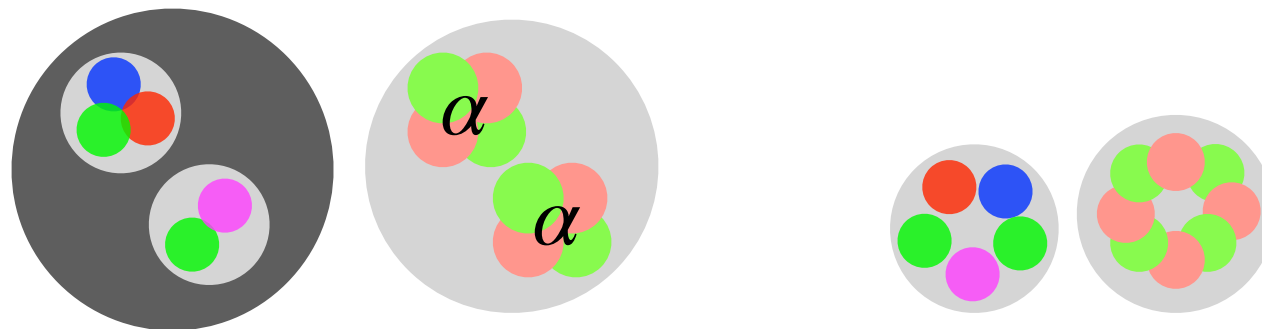
$$f_0(980), a_0(980)$$

V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev,
 Phys. Lett. B 586, 53–61 (2004);
 Y. Kamiya and T. Hyodo, PTEP 2017; Phys. Rev. C 93, 035203 (2016) etc.

Compositeness

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● Definition



$$|\text{state}\rangle = \sqrt{X} |\text{composite}\rangle + \sqrt{Z} |\text{others}\rangle$$

compositeness

elementarity

- quantitative measure

$$\begin{array}{l} 0 \leq X \leq 1 \\ X + Z = 1 \end{array} \longrightarrow \begin{array}{l} X > 0.5 \Leftrightarrow \text{composite dominant} \\ X < 0.5 \Leftrightarrow \text{non-composite dominant} \end{array}$$

- applicable to various states

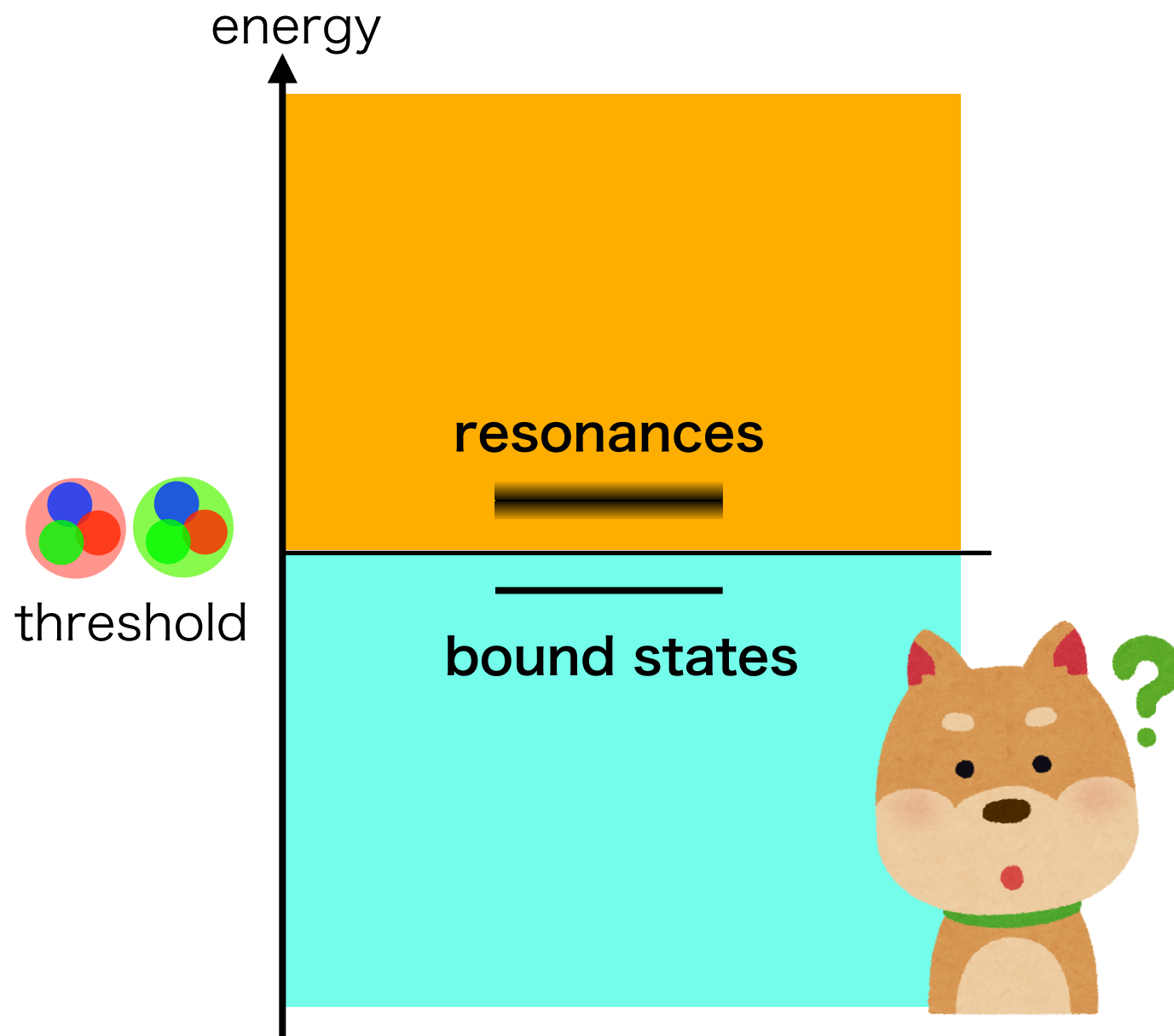
$f_0(980)$, $a_0(980)$

V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev,
Phys. Lett. B 586, 53–61 (2004);
Y. Kamiya and T. Hyodo, PTEP 2017; Phys. Rev. C 93, 035203 (2016) etc.

nuclei, atomic systems

E. Braaten, H. W. Hammer, and M. Kusunoki, (2003), cond-mat/0301489;
T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022).

Near-th. s-wave states



near-threshold states
= composite?



scatterings with **short range interaction**

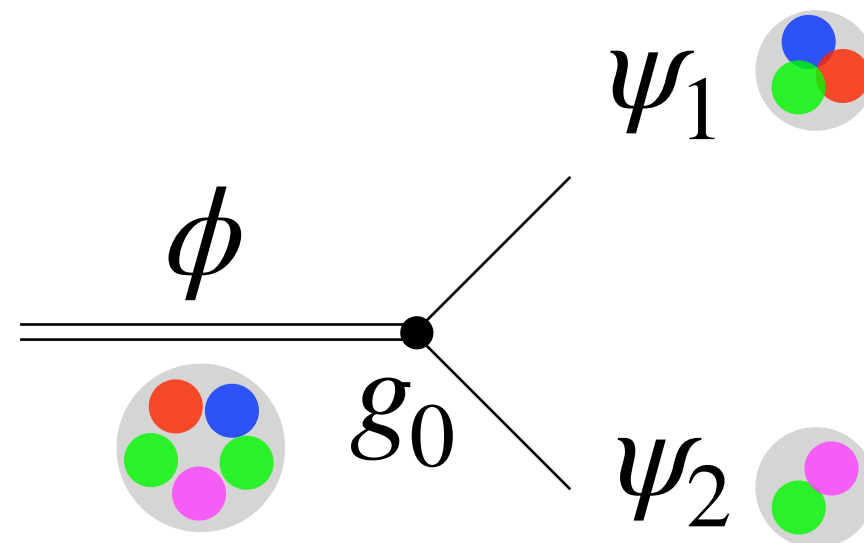
T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024).

T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

E. Braaten, M. Kusunoki, and D. Zhang,
Annals Phys. 323, 1770 (2008).

$$\mathcal{H}_{\text{int}} = g_0(\phi^\dagger \psi_1 \psi_2 + \psi_1^\dagger \psi_2^\dagger \phi).$$

1. single-channel scatterings of $\psi_{1,2}$
2. couplings of $\psi_{1,2}$ to ϕ



● scattering amplitude

$$V = \frac{g_0^2}{E - \nu_0}, \quad G = -\frac{\mu}{\pi^2} \left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right) \right]. \quad \Lambda : \text{cutoff}$$

$$\xrightarrow{T = \frac{1}{V^{-1} - G}} f(k) = -\frac{\mu}{2\pi} \left[\frac{\frac{k^2}{2\mu} - \nu_0}{g_0^2} + \frac{\mu}{\pi^2} \left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right) \right] \right]^{-1}.$$

Compositeness in model

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● compositeness X

scattering amplitude : $T = \frac{1}{V^{-1} - G}$ Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

$$X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2 \kappa}{g_0^2 \mu^2} \left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1 + (\Lambda/\kappa)^2} \right)^{-1} \right]^{-1}.$$

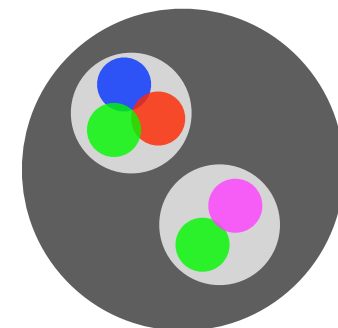
$$\kappa = \sqrt{2\mu B}$$

- compositeness X of bound state with given B ?

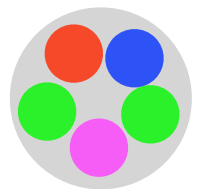
fix binding energy B

→ parameter dependence of X ?

$X > 0.5$



$X < 0.5$



or

Model scale and parameters

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● scale in model

- determined by cutoff Λ : $E_{\text{typ}} = \Lambda^2/(2\mu)$

● model parameters g_0, ν_0, Λ

1. fix binding energy B

coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \frac{\pi^2}{\mu}(B + \nu_0) \left[\Lambda - \kappa \arctan(\Lambda/\kappa) \right]^{-1}$

$\therefore$ pole condition $f^{-1}(i\kappa) = 0$ $\kappa = \sqrt{2\mu B}$.

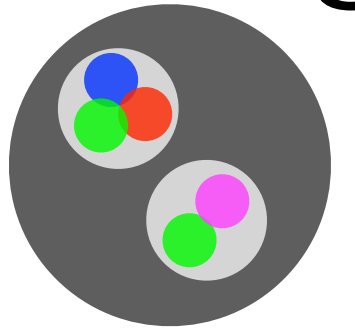
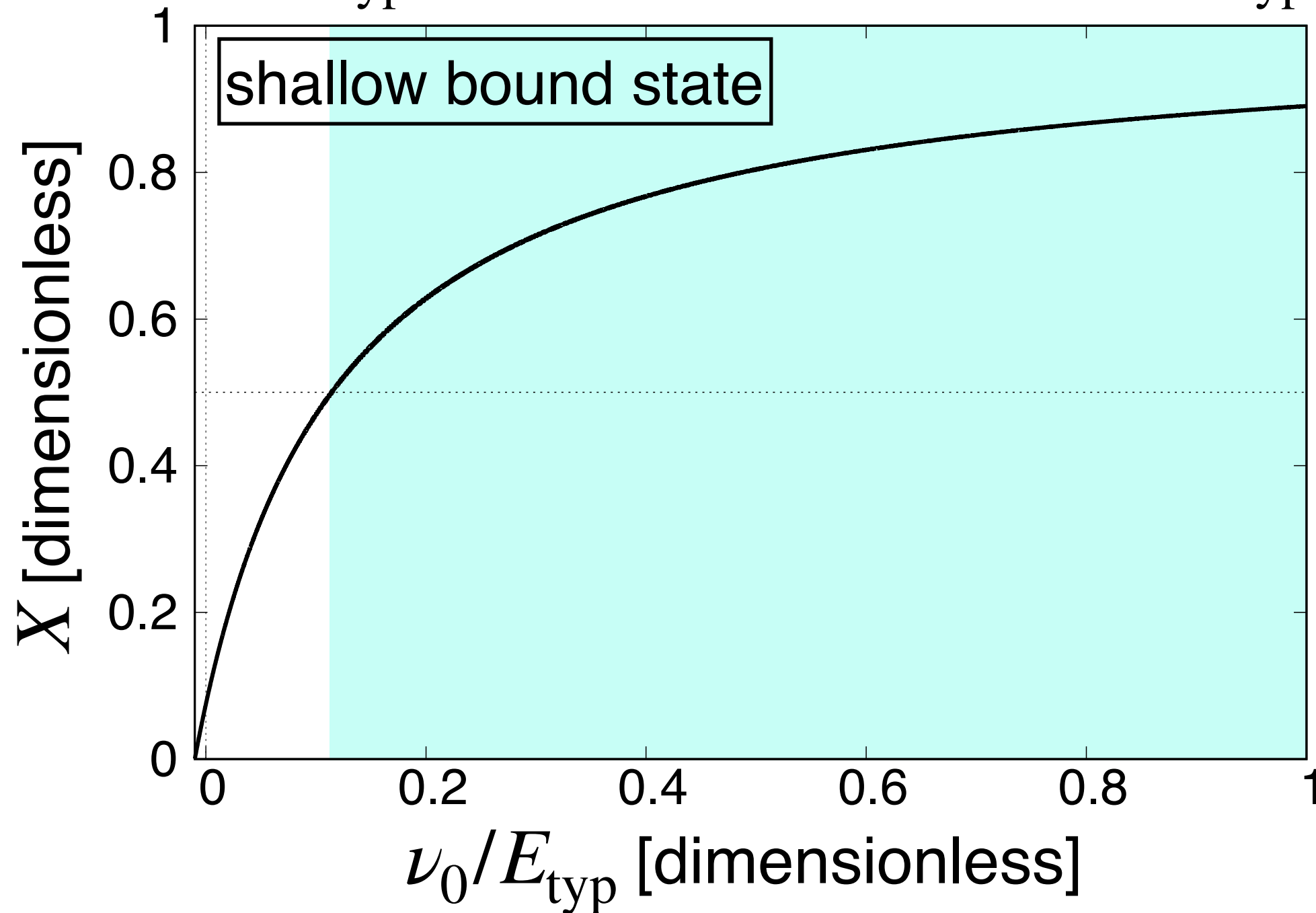
2. use dimensionless quantities by Λ e.g. B/E_{typ}

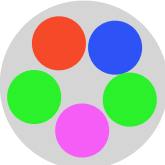
3. free parameter $\longrightarrow$ bare state energy ν_0

vary in $-B/E_{\text{typ}} \leq \nu_0/E_{\text{typ}} \leq 1$ region

$\therefore g_0^2 \geq 0$ (lower) & applicable limit of model (upper)

● parameter ν_0/E_{typ} dependence of X ($B = 0.01E_{\text{typ}}$)



- $X > 0.5$ in 88 % of $\nu_0 \longrightarrow$ emergence of **universality**
- fine tune of ν_0 is nessecary to obtain  shallow bound state
- $\longrightarrow$ shallow bound states tend to be composite dominant

Near-th. resonances in ERE

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T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

● eigenenergy of resonance

- renormalized EFT model $\longrightarrow$ effective range expansion (ERE)

$$f(k)^{-1} = -\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik \longrightarrow k^\pm = \frac{i}{r_e} \pm \frac{1}{r_e} \sqrt{\frac{2r_e}{a_0} - 1}$$

a_0 : scattering length

r_e : effective range

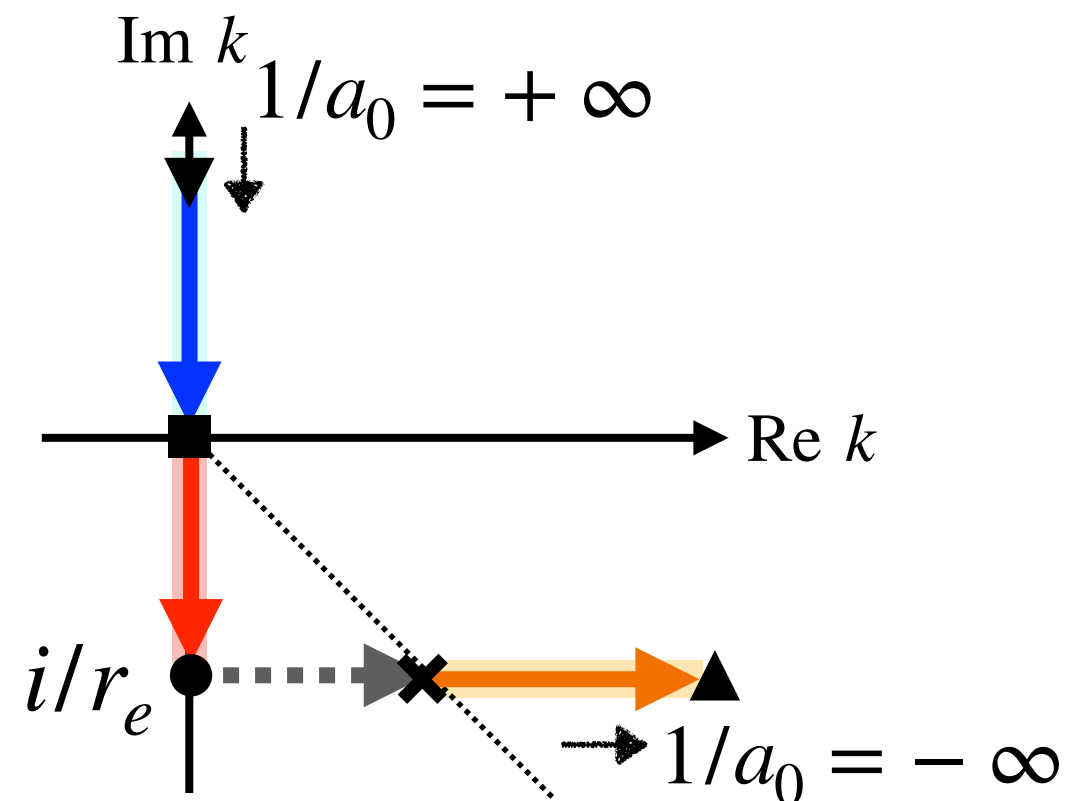
T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

- pole position ($k^\pm$) $\longleftrightarrow a_0$ and r_e

- bound state: positive pure imaginary

- virtual state: negative pure imaginary

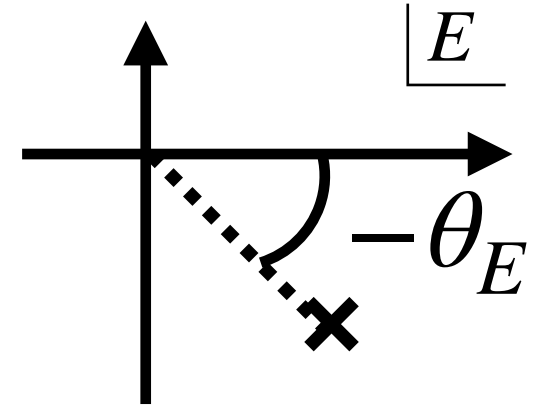
- resonances: $\text{Re } k > 0$ and $\text{Im } k < 0$



Interpretation of complex X

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$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan \theta_k = -i \tan(\theta_E/2)$$



- i) $\mathcal{X}$: probability of certainly finding $|\text{composite}\rangle$
- ii) $\mathcal{Z}$: probability of certainly finding $|\text{elementary}\rangle$

iii) $\mathcal{Y}$: probability of uncertain identification T. Berggren, Phys. Lett. B 33, 547 (1970).

$$\mathcal{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$1. \mathcal{X} \rightarrow X, \mathcal{Z} \rightarrow Z$$

$$\text{and } \mathcal{Y} \rightarrow 0 \text{ (Im } X \rightarrow 0)$$

$$\mathcal{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$2. \mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$$

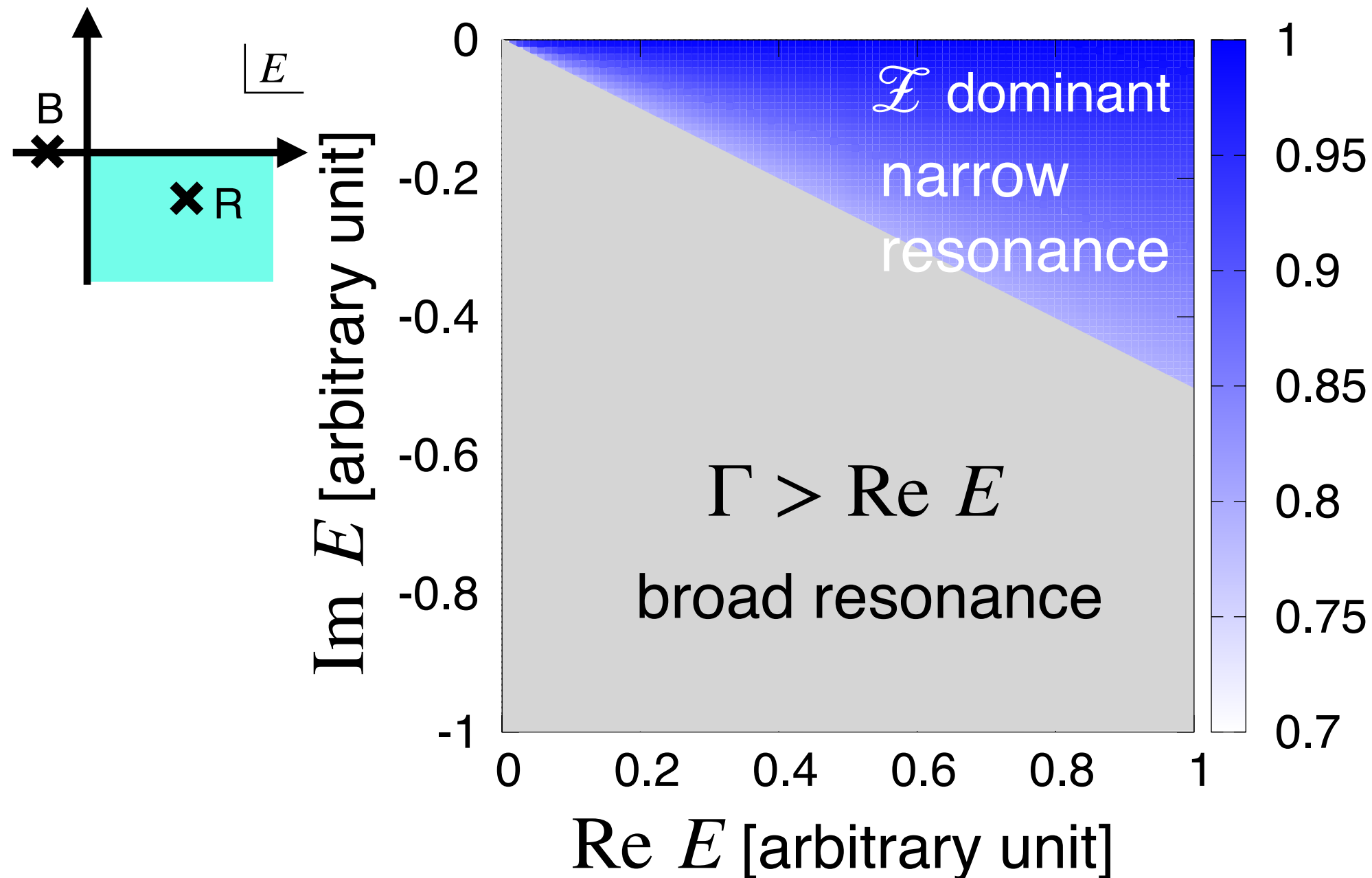
α : uncertain nature of resonances

$$\mathcal{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

$$\text{T. Kinugawa and T. Hyodo, } \alpha = \frac{\sqrt{5} - 1 + \sqrt{10 - 4\sqrt{5}}}{2} \approx 1.1318. \\ \text{arXiv:2403.12635 [hep-ph].}$$

Structure of near-th. resonances

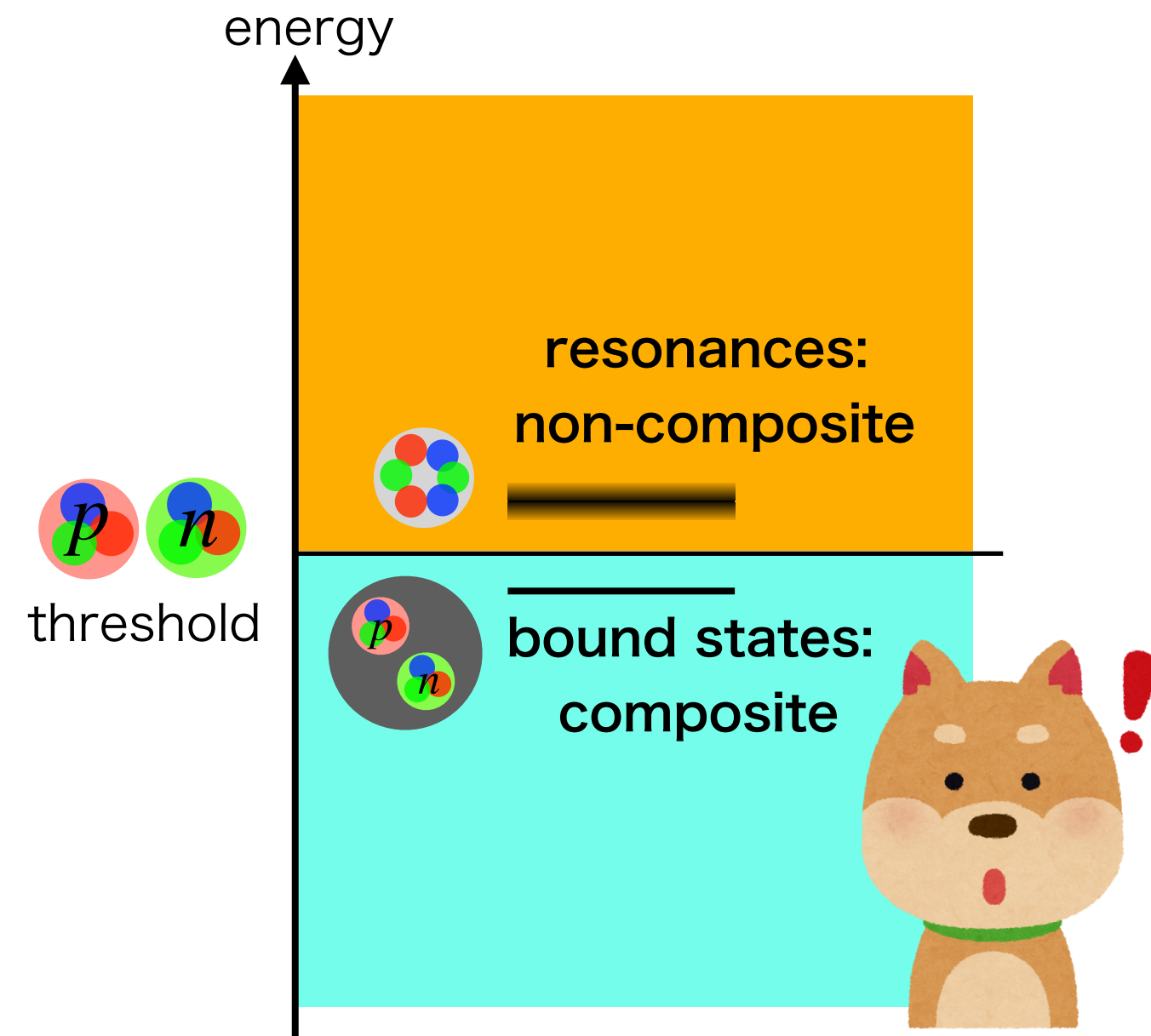
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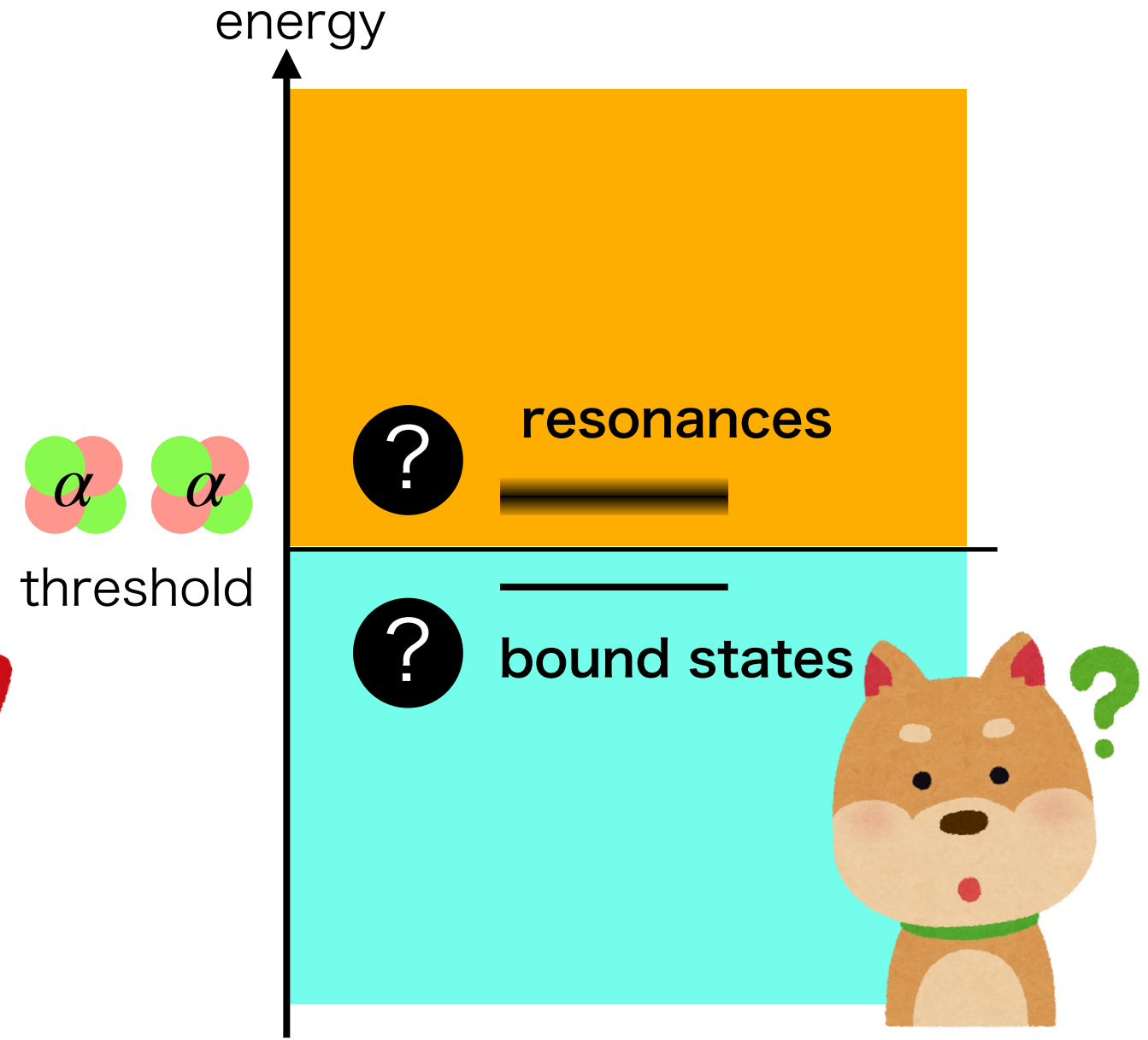
- resonances are **not composite dominant state** ($\mathcal{I} \gtrsim 0.8$)
- different from near-threshold bound states (composite dominant $X \sim 1$ and $Z \sim 0$)

Near-th. states with Coulomb + s.r.

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scatterings with **short range**
interaction



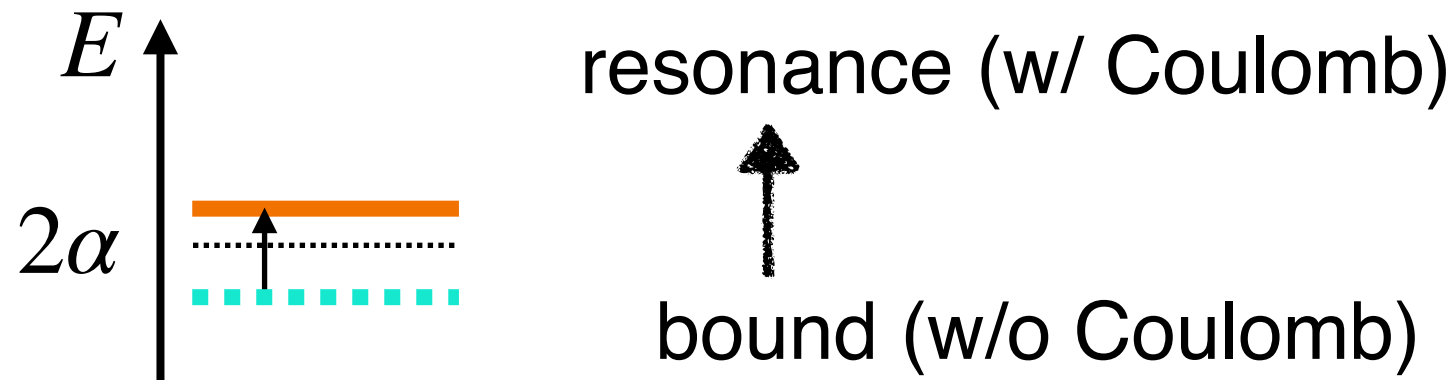
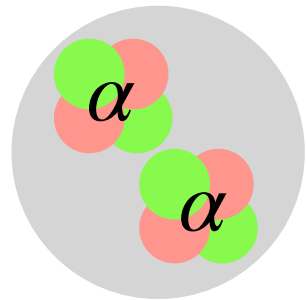
scatterings with **Coulomb +**
short range interaction

T. Kinugawa and T. Hyodo, arXiv:2507.22399 [hep-ph].

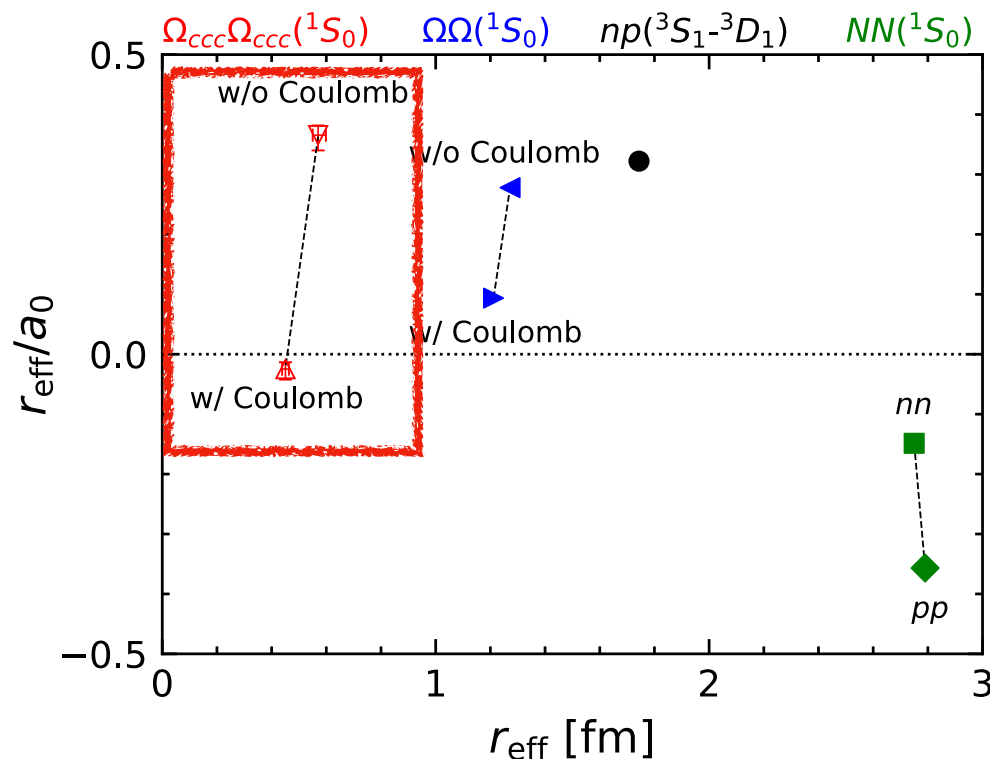
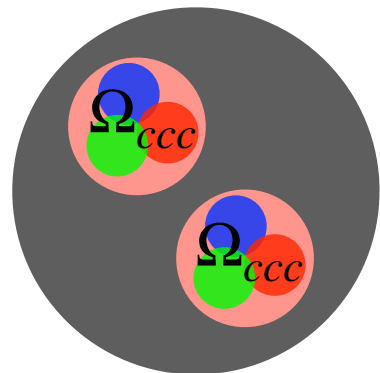
Coulomb + short range systems

13

- ${}^8\text{Be}$ nuclei ($2\alpha^{++}$) J. Hiura, and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 52, 25 (1972);
R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809, 171 (2008).



- $\Omega_{ccc}^{++}\Omega_{ccc}^{++}$ (HAL QCD) Y. Lyu, H.Tong, *et al.* [HAL QCD Coll.], Phys. Rev. Lett. 127 (2021) 072003.



resonance (w/ Coulomb)

bound (w/o Coulomb)

- $\Xi^{-}\alpha$: Coulomb assisted bound state E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, Phys. Rev. C 106, 064318 (2022).

→ Coulomb is important for near-threshold states!

Coulomb+short range model

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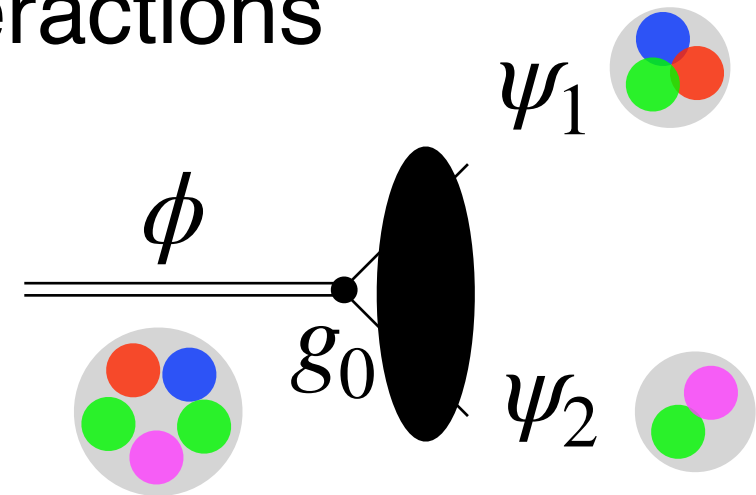
● EFT model

W. Domcke, Atom. Mol. Phys. 16 359 (1983). R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

- EFT model for Coulomb plus short-range interactions

- pole condition (ERE)

$$-\frac{1}{\boxed{a_s}} + \frac{\boxed{r_e}}{2}k^2 - ik \pm \frac{2}{\boxed{a_B}} \left[\underbrace{\log(-ia_Bk)}_{\text{log cut}} + \psi \left(1 \mp \frac{i}{a_Bk} \right) \right] = 0$$



$$a_B = \frac{\hbar c}{\mu c^2 Z_1 Z_2} : \text{Bohr radius} \quad \psi(z) = \frac{d}{dz} \log \Gamma(z) : \text{digamma function}$$

- compositeness X

$$X = 1 - \frac{1}{1 - \frac{d}{dE} \boxed{F(E)}} = \left[1 - \frac{r_e}{R^C} \right]^{-1} \quad R^C : \text{radius with Coulomb contribution}$$

T. Hyodo, Phys. Rev. C **90**, 055208 (2014).

self energy

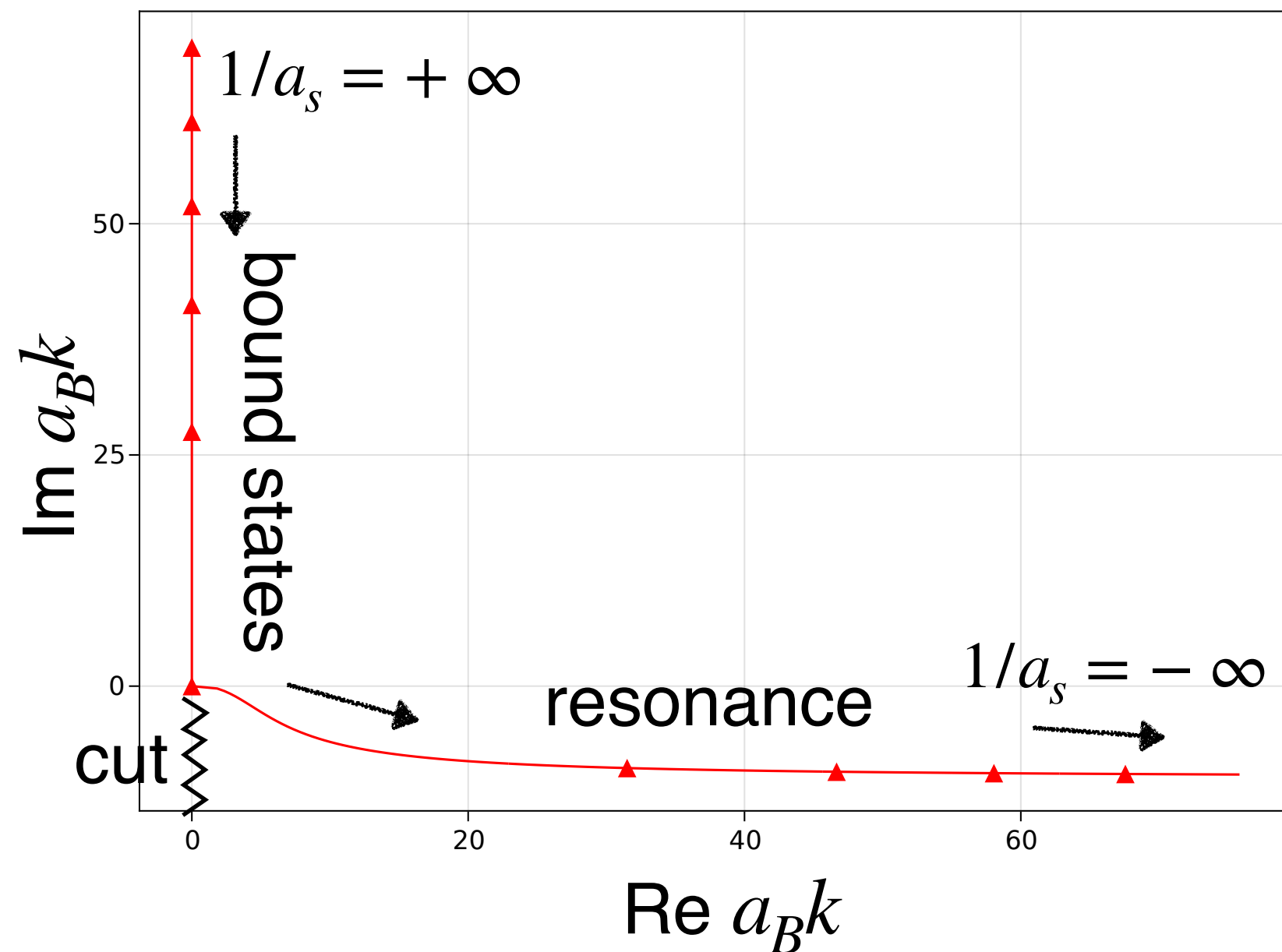
$$R^C = -\frac{1}{k} \left[2i \left(\frac{1}{a_Bk} \right)^2 \psi_1 \left(\mp i \frac{1}{a_Bk} \right) + i - 2 \left(\mp \frac{1}{a_Bk} \right) \right]$$

Pole trajectory (repulsive Coulomb) ¹⁵

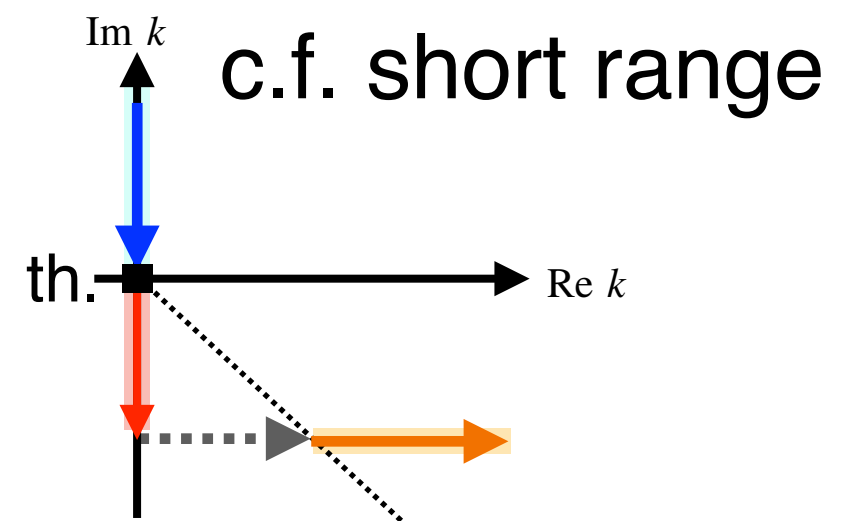
● pole trajectory in complex momentum k plane

- varying Coulomb scattering length a_s with fixed r_e and a_B

→ pole position (eigenmomentum) moves



- b.s. directory goes to resonance



- $a_s \rightarrow \infty$ at threshold

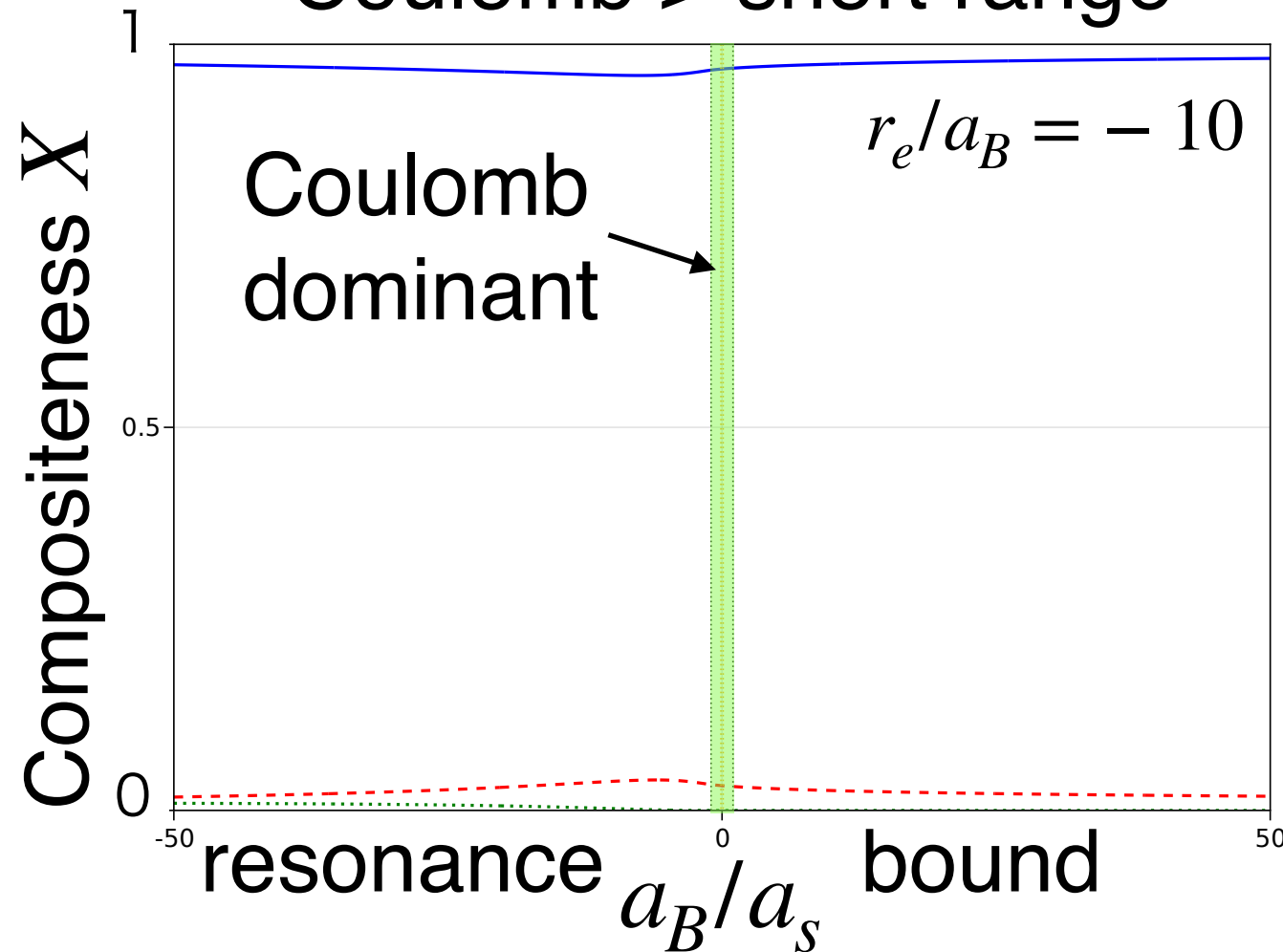
- but no universality

$\therefore$ radius of w.f. $< \infty$

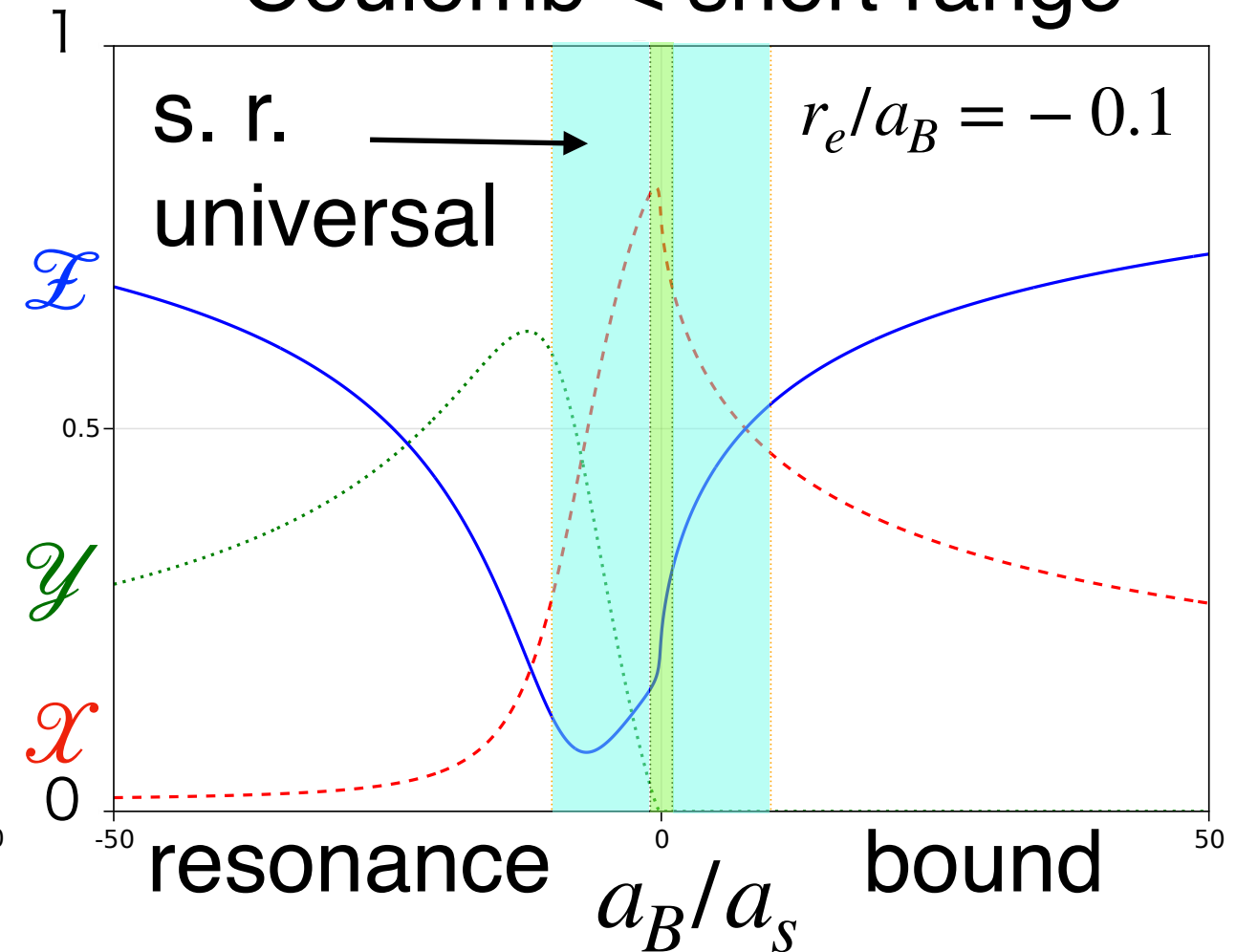
Compositeness (repulsive Coulomb)

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Coulomb > short range



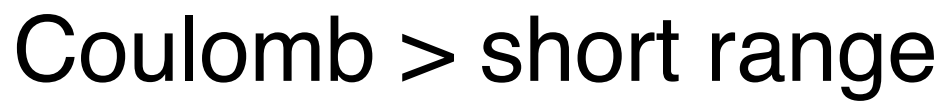
Coulomb < short range



- complex compositeness $\leftarrow X, Y, Z$
- states with large $|1/a_s|$ are elementary Z dominant
- structure of bound states $\approx$ resonances $\because$ continuous X

T. Kinugawa and T. Hyodo,
arXiv:2403.12635 [hep-ph].

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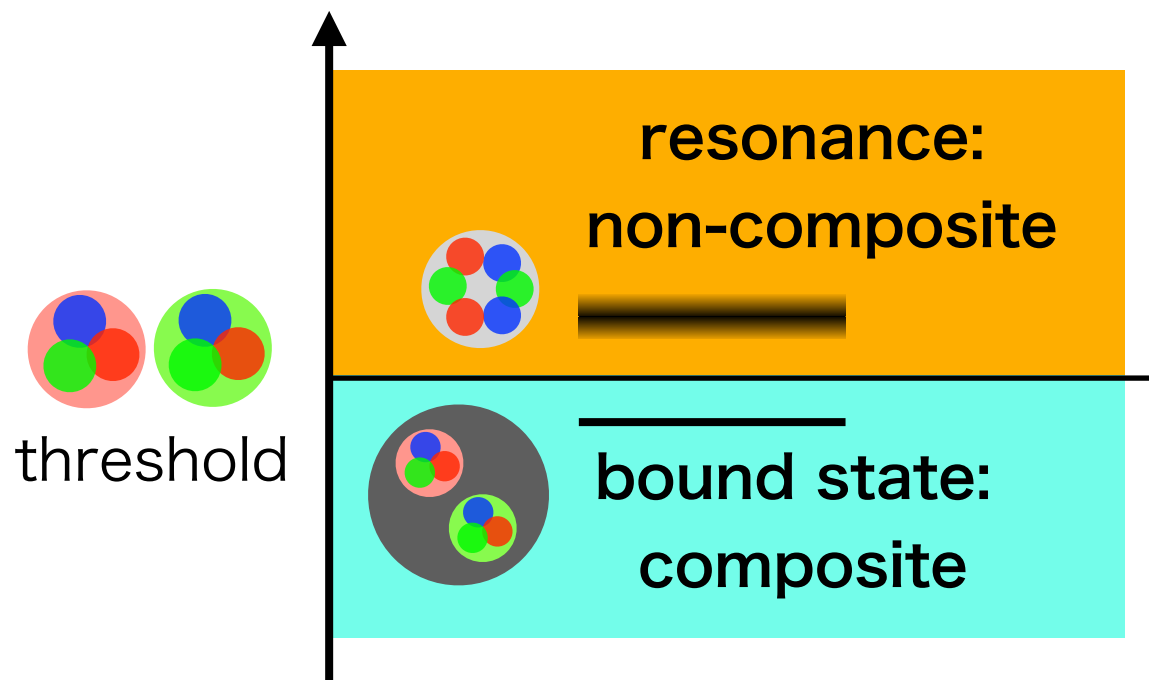


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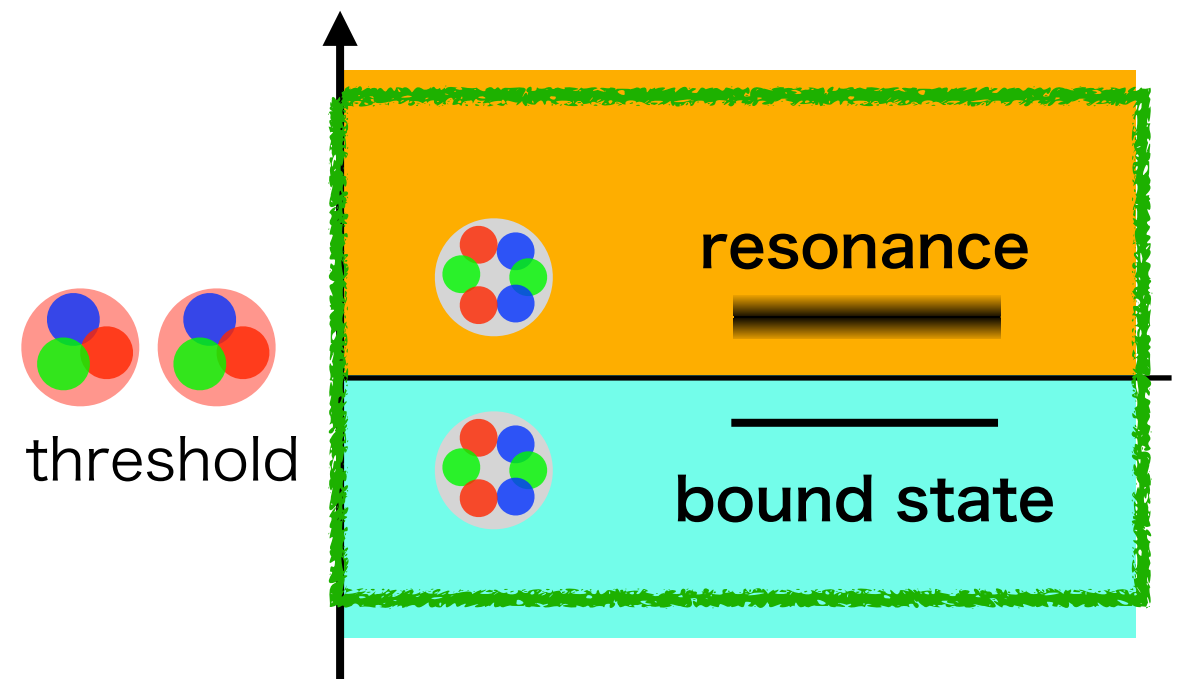
Summary

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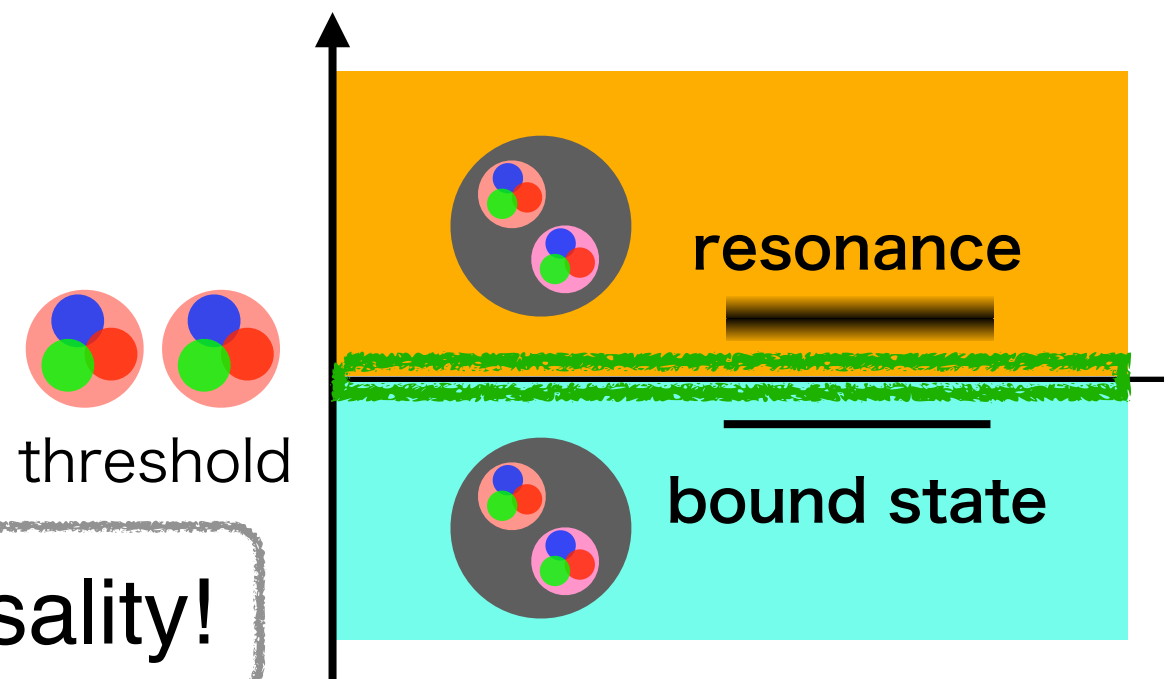
short range interaction



Coulomb > short range



Coulomb < short range



remnant of short-range universality!



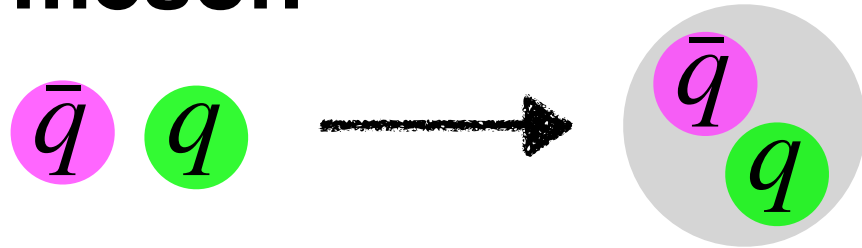
Back up

Exotic hadron

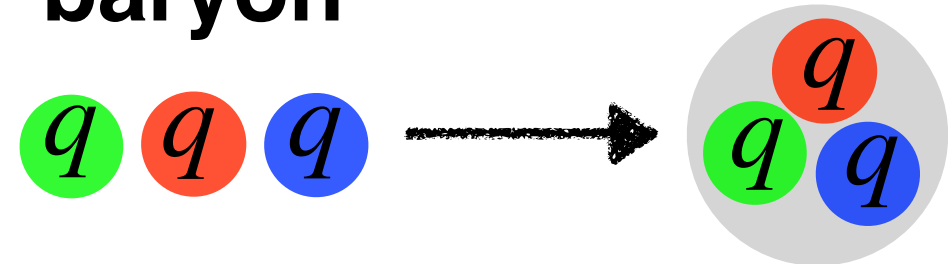
quark q , gluon $\xrightarrow{\text{strong interaction}}$ hadron

● ordinary hadron

- meson



- baryon



- ~ 400 kinds have been observed so far

Particle Data Group, S. Navas et al., Phys. Rev. D 110, 030001 (2024).

● exotic hadron

- composed of four or more quarks



- **rarely** observed $\leftarrow$ Why?

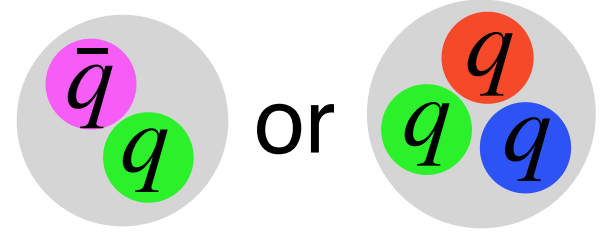
- hint for understanding low-energy phenomena in QCD

Exotic hadrons

A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui,
PTEP 2016, 062C01 (2016)

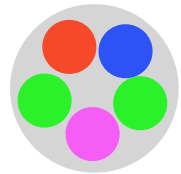
20

- ordinary hadron: $q\bar{q}$ (meson) or qqq (baryon)
- other than $q\bar{q}$ or $qqq \rightarrow$ **exotic hadron** $\rightarrow$ structure?



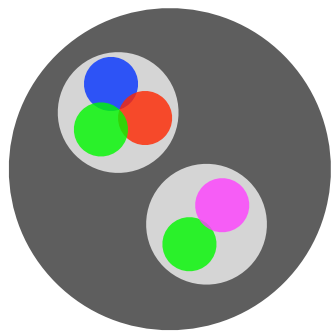
● possible structure F-K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, and B-S. Zou,
RevModPhys.90.015004 (2018).

- **multiquarks**: quarks $\rightarrow$ hadron



size $\lesssim 1$ fm (**compact**)

- **hadronic molecule**: quarks $\rightarrow$ hadrons (subunit) $\rightarrow$ hadron



size $\gtrsim 1$ fm (spatially large radius)

e.g. deuteron (pn molecule), $R = 4.32$ fm

$\rightarrow$ **superposition** of possible components

internal structure $\leftarrow$ investigate weight of each component

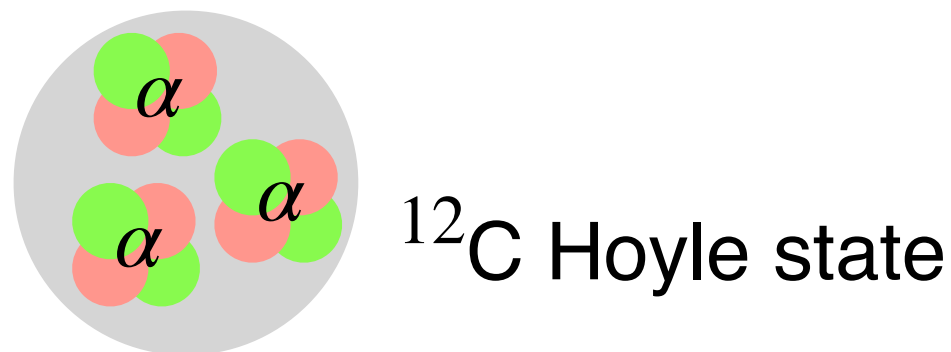
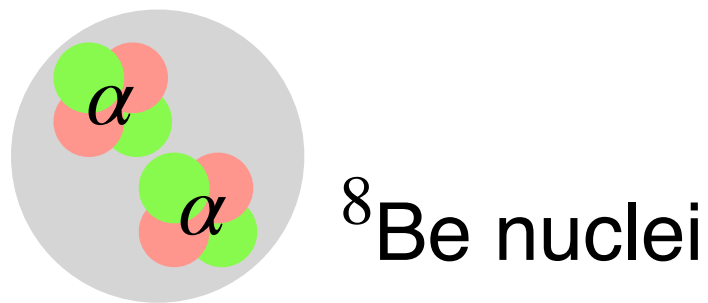
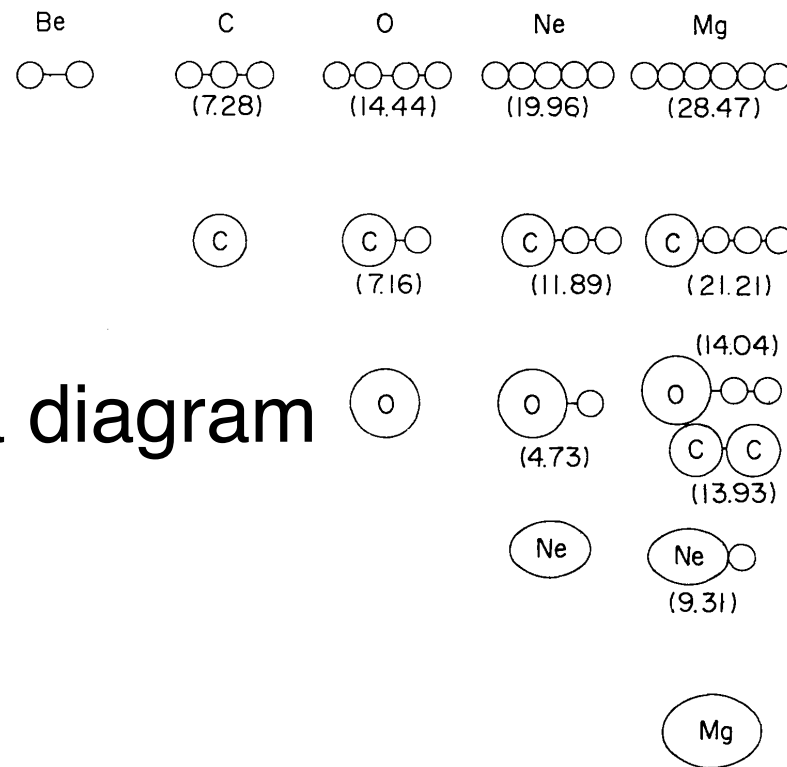
Clustering phenomena

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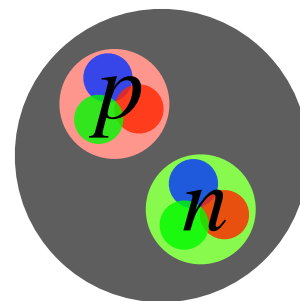
- near-threshold states $\longrightarrow$ clustering phenomena

- α clustering phenomena

K. Ikeda, and N. Takizawa, and H. Horiuchi, Prog. Theor. Phys. Suppl. E68, 464-475 (1968).

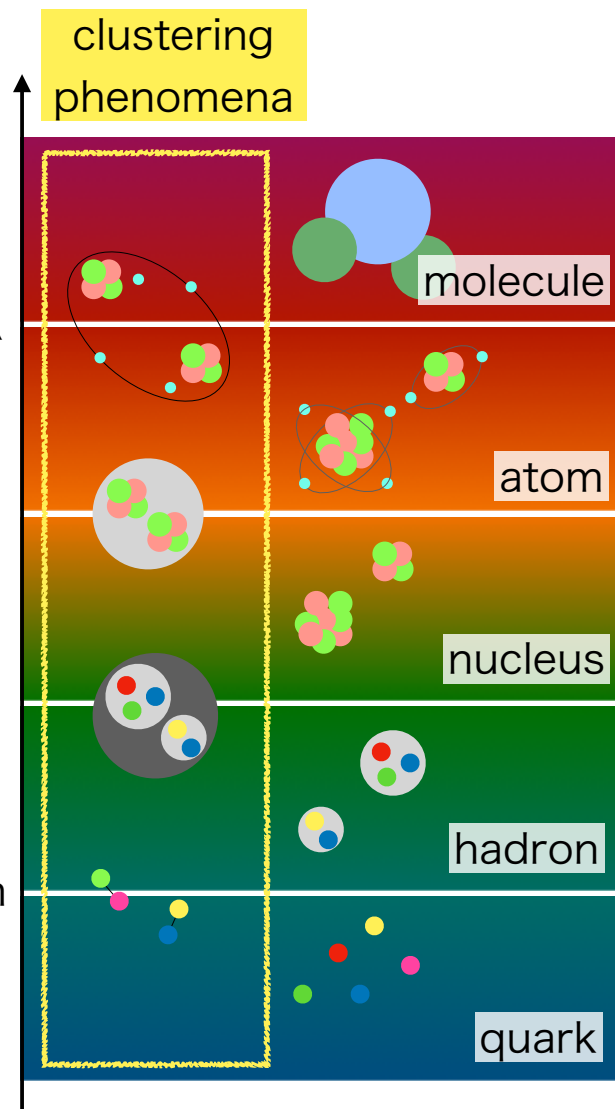


- deuteron $B = 2.22$ MeV



hadronic molecule $\longrightarrow$ clustering of quarks

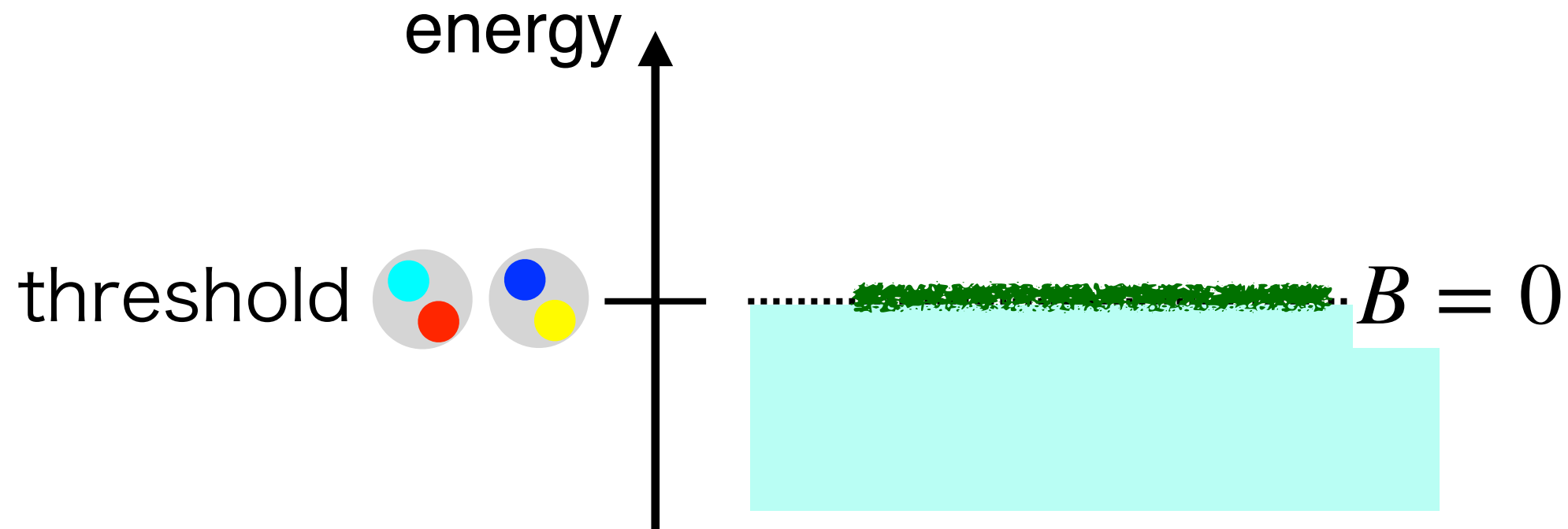
- clustering phenomena: **universal!**



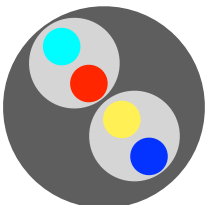
Low-energy universality

- near-threshold energy region ← universal phenomena

E. Braaten and H.-W. Hammer, Phys. Rept. **428**, 259 (2006); P. Naidon and S. Endo, Rept. Prog. Phys. **80**, 056001 (2017).



- binding energy $B = 0$ (exactly at threshold)

→ that state is **always** completely composite ($X = 1$) 
 compositeness is model-**independently** determined

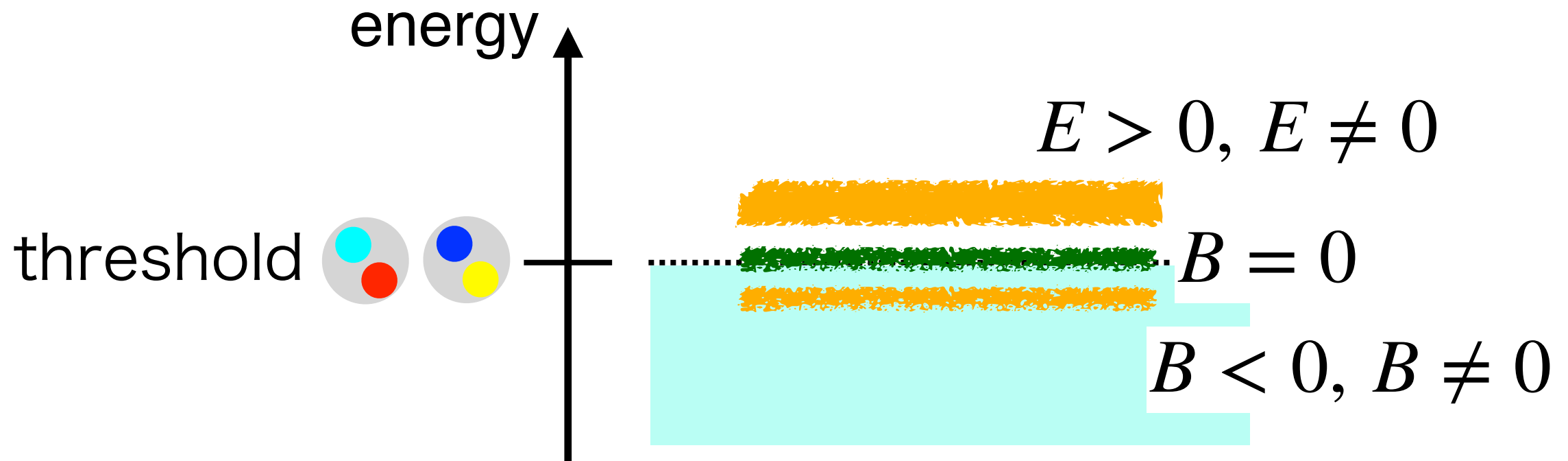
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

← this work

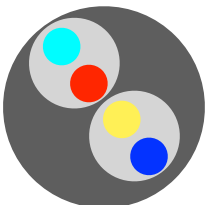
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T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

- structure of near-threshold states ($B \neq 0$)? $\leftarrow$ this work
- similar to $B = 0$ state? $X \sim 1$?

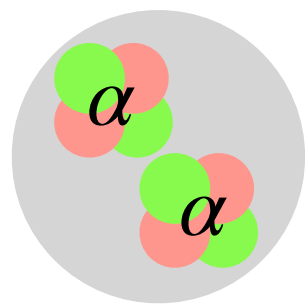
Near-threshold states

23

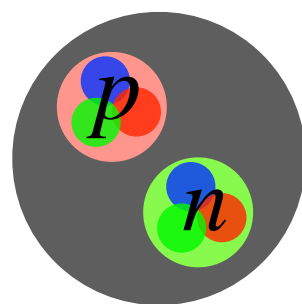
● Are near-threshold states composite dominant ($X \sim 1$)?

- **empirically** : Yes! “threshold energy rule”

K. Ikeda, and N. Takizawa, and H. Horiuchi, Prog. Theor. Phys. Suppl. E68, 464-475 (1968).



^8Be nuclei



deuteron



threshold

energy

resonance

bound state

- **theoretically**

Even non-composite dominant
shallow bound states ($X \sim 0$) can be realized!

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;

C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014).

→ Why do we rarely observe such a state?

structure of resonances slightly above threshold?

→ provide theoretical foundation of threshold energy rule

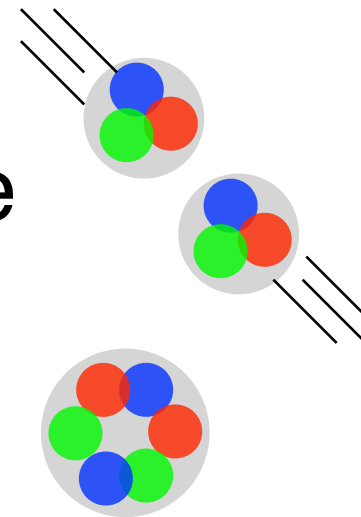
- decomposing Hamiltonian into free part and remainder

$$\hat{H} = \hat{H}_0 + \hat{V}$$

free Hamiltonian

$$\hat{H}_0 |\mathbf{p}\rangle = \frac{\mathbf{p}^2}{2\mu} |\mathbf{p}\rangle \quad \text{free scattering state}$$

$$\hat{H}_0 |\phi\rangle = \nu_0 |\phi\rangle \quad \text{bare discrete state}$$



- compositeness X , elementarity Z

$$X = \int d\mathbf{p} \, |\langle \mathbf{p} | B \rangle|^2, \quad Z = |\langle \phi | B \rangle|^2$$

overlaps between bound state $|B\rangle$ and $|\mathbf{p}\rangle$ or $|\phi\rangle$

- decomposition is not unique! $\longrightarrow$ **model dependence of X**

Compositeness

● model calculation

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **85**, 015201 (2012);
F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

$$T = \frac{1}{V^{-1} - G}$$

V : effective interaction

G : loop function

residue of scattering amplitude g

$$X = -g^2 G'(E) \Big|_{E=-B}$$

$$\alpha'(E) = d\alpha/dE$$

$$= \frac{G'(E)}{G'(E) - [V^{-1}(E)]'} \Big|_{E=-B}$$

$E=-B$ Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

g^2 : model independent $\leftarrow T_{\text{on}}(-B)$ (observable)

$G(E)$: model dependent $\leftarrow$ cutoff dependent

History of compositeness

- Weinberg's work (1960s) Weinberg, S. Phys. Rev. 137, 672–678 (1965) etc.
 deuteron is not an elementary particle $\leftarrow$ weak-binding relation
 Z : field renormalization factor
- application to exotic hadrons (2000s-)
 X : “compositeness”
 generalization to unstable states
 with spectral function V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004) etc.
 with effective range expansion T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) etc.
 with effective field theory Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017) etc.
 application to ...
 $f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016);
 T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.
 $\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ;
 Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.
 nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

Weak-binding relation

S. Weinberg, Phys. Rev. 137, 672–678 (1965).

$$X = \frac{a_0}{2R - a_0} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)$$

a_0 : scattering length
 R_{typ} : typical length scale in system
 $R = 1/\sqrt{2\mu B}$

- for weakly bound states, $R \gg R_{\text{typ}}$

compositeness $\longleftarrow$ observables (a_0, B)

Y. Li, F.-K. Guo, J.-Y. Pang, and J.-J. Wu, Phys. Rev. D 105, L071502 (2022);

J. Song, L. R. Dai, and E. Oset, Eur. Phys. J. A 58, 133 (2022);

M. Albaladejo, J. Nieves, Eur. Phys. J. C 82, 724 (2022);

T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022);

Z. Yin and D. Jido, Phys. Rev. C 110, no.5, 055202 (2024).

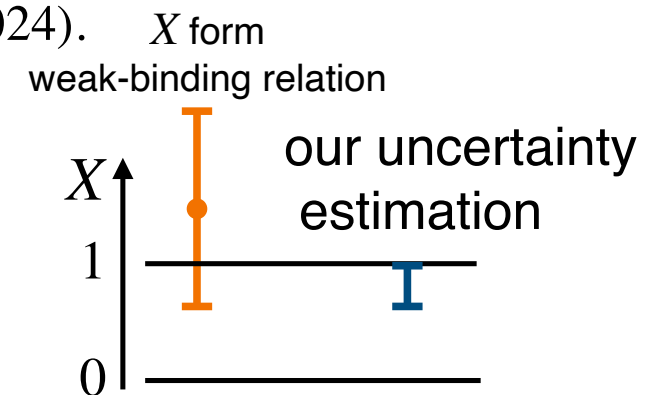
● range correction

compositeness of deuteron $X \sim 1.7 > 1$

$\longrightarrow$ important to consider effective range

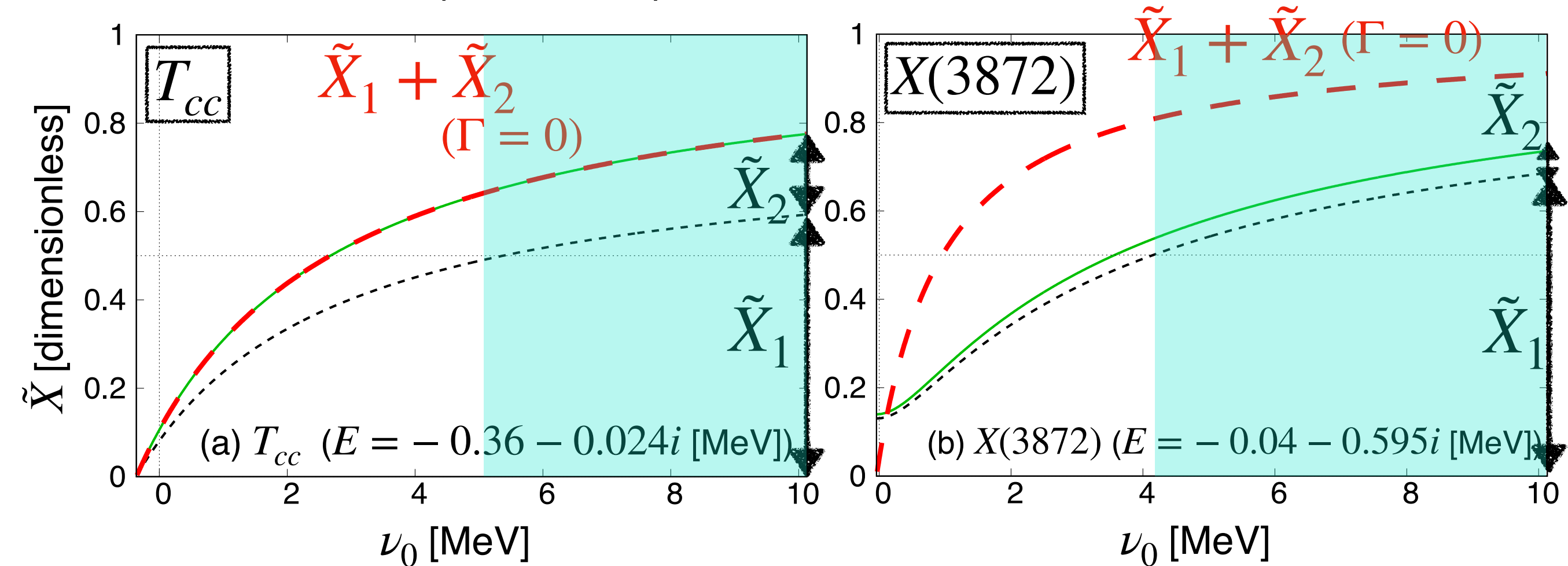
- our work : range correction $\longleftarrow$ uncertainty estimation

compositeness of deuteron : $0.74 \leq X \leq 1$



Application to T_{cc} and $X(3872)$

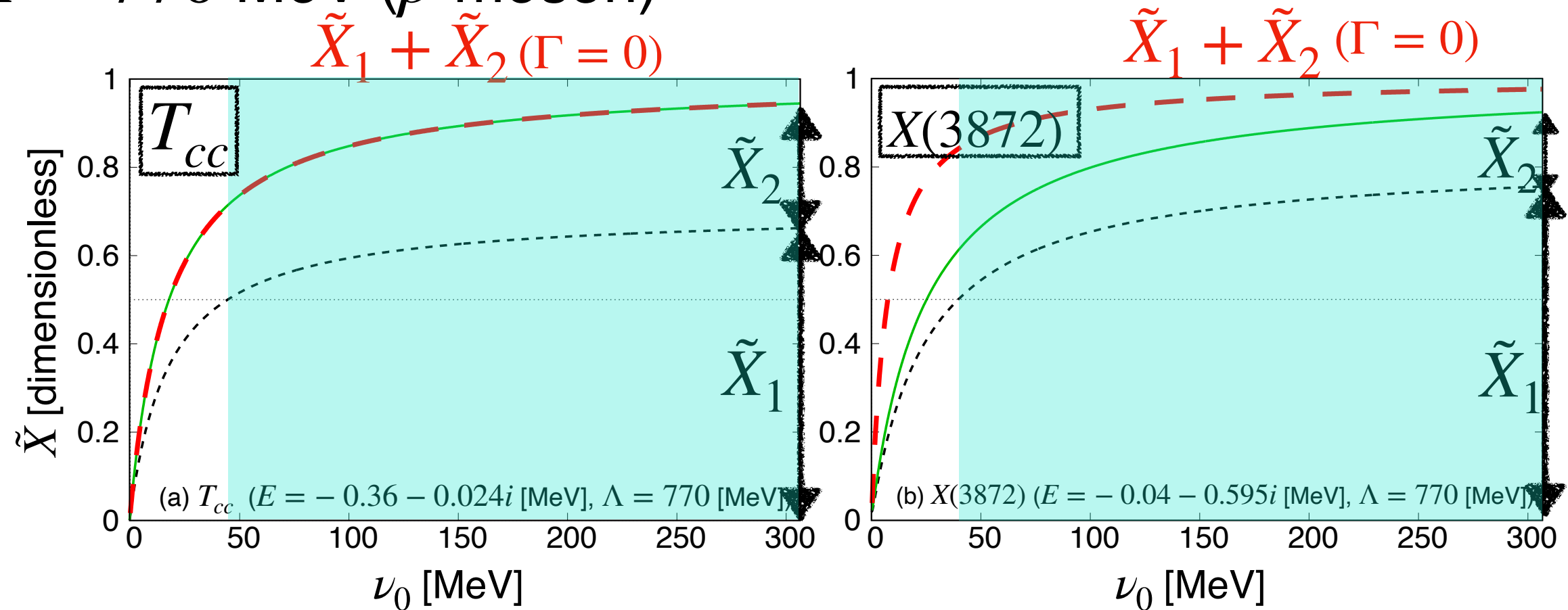
- $\Lambda = 140$ MeV (π meson)



- T_{cc} : $\tilde{X}_1 > 0.5$ for 45 % of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for 59 % of ν_0 region
- coupled ch. effect is more important for T_{cc} than $X(3872)$
- decay effect is more important for $X(3872)$ than T_{cc}

Application to T_{cc} and $X(3872)$

- $\Lambda = 770$ MeV (ρ meson)

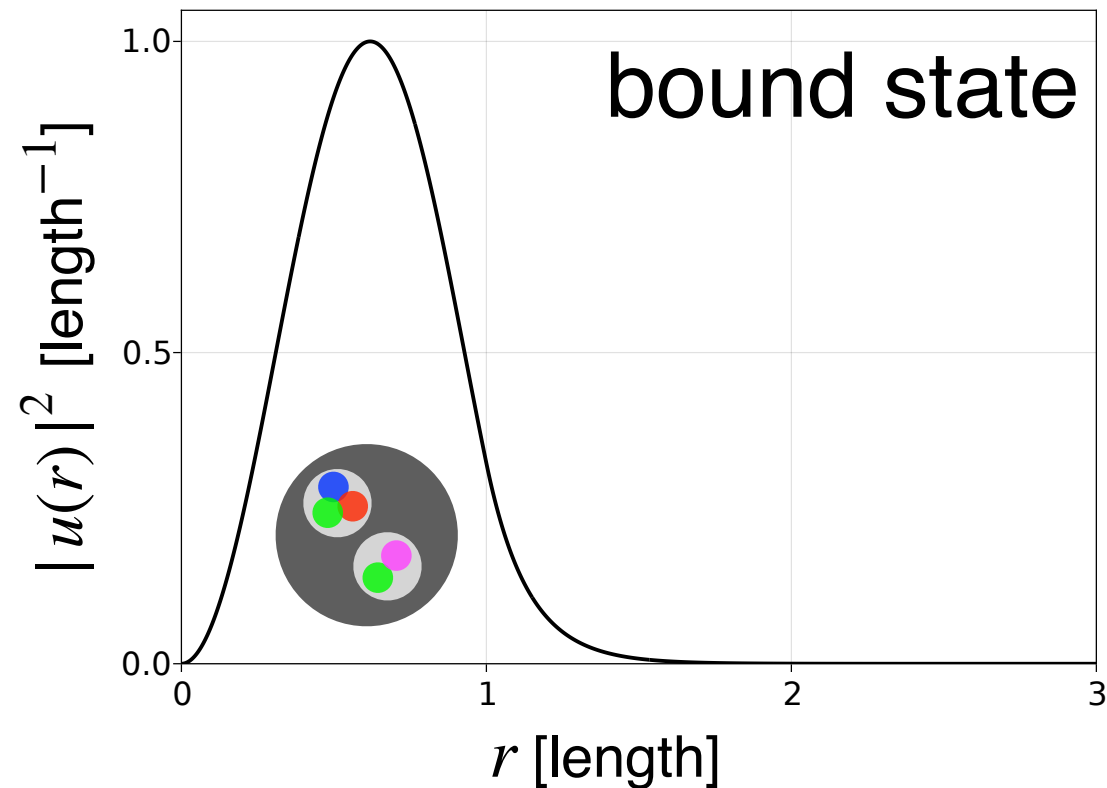


- T_{cc} : $\tilde{X}_1 > 0.5$ for 85 % of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for 87 % of ν_0 region
- typical energy scale E_{typ} is larger
 - states becomes close to universality limit $X \rightarrow 1$
 - decay effect : suppressed coupled ch. effect : enhanced

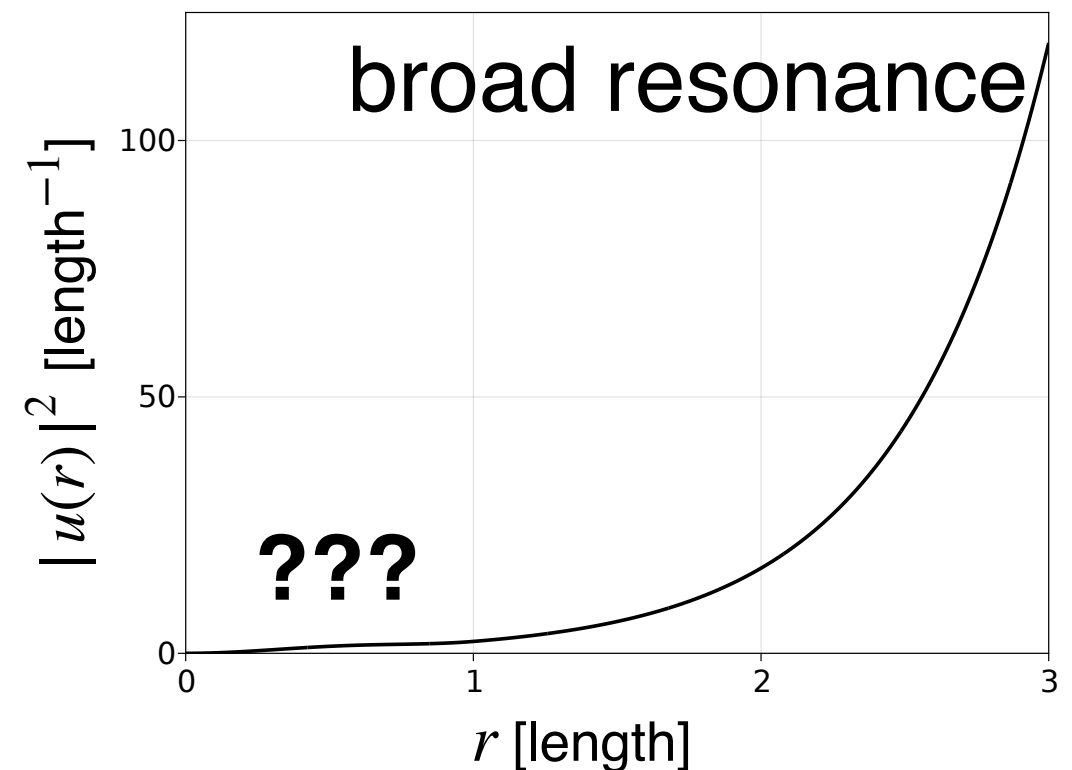
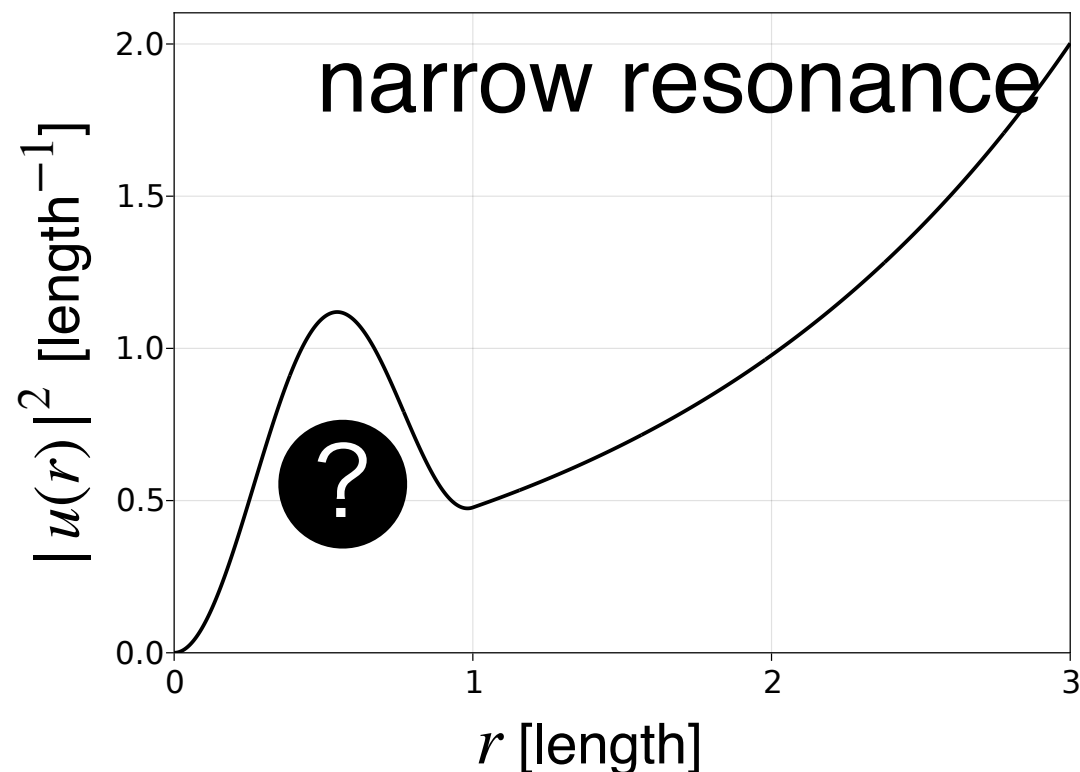
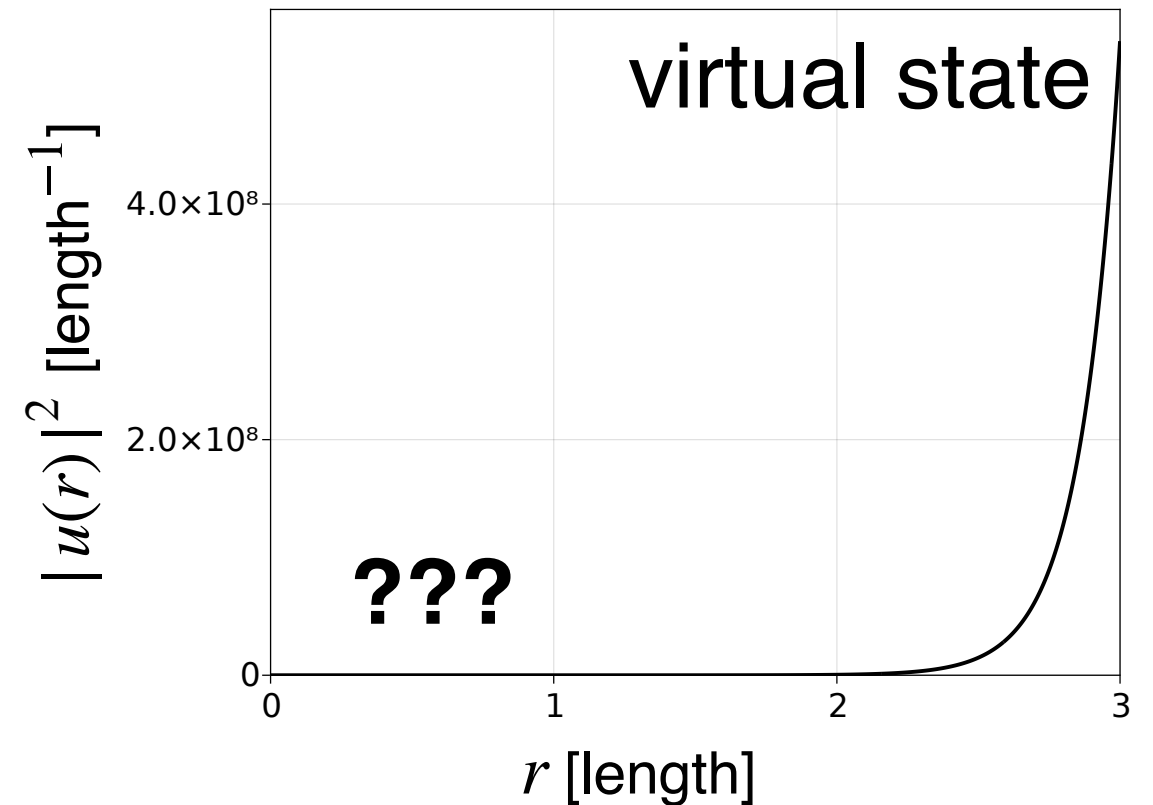
Wavefunction of eigenstates

30

✓ internal “structure”!



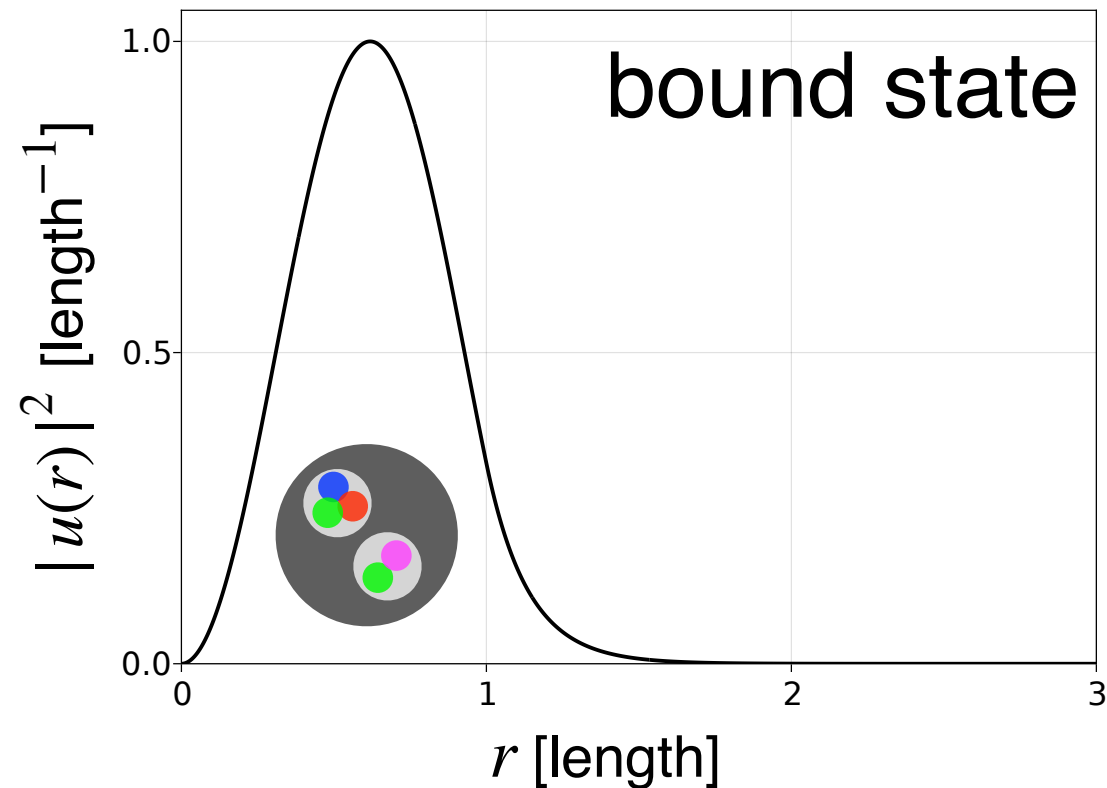
✗ internal “structure”?



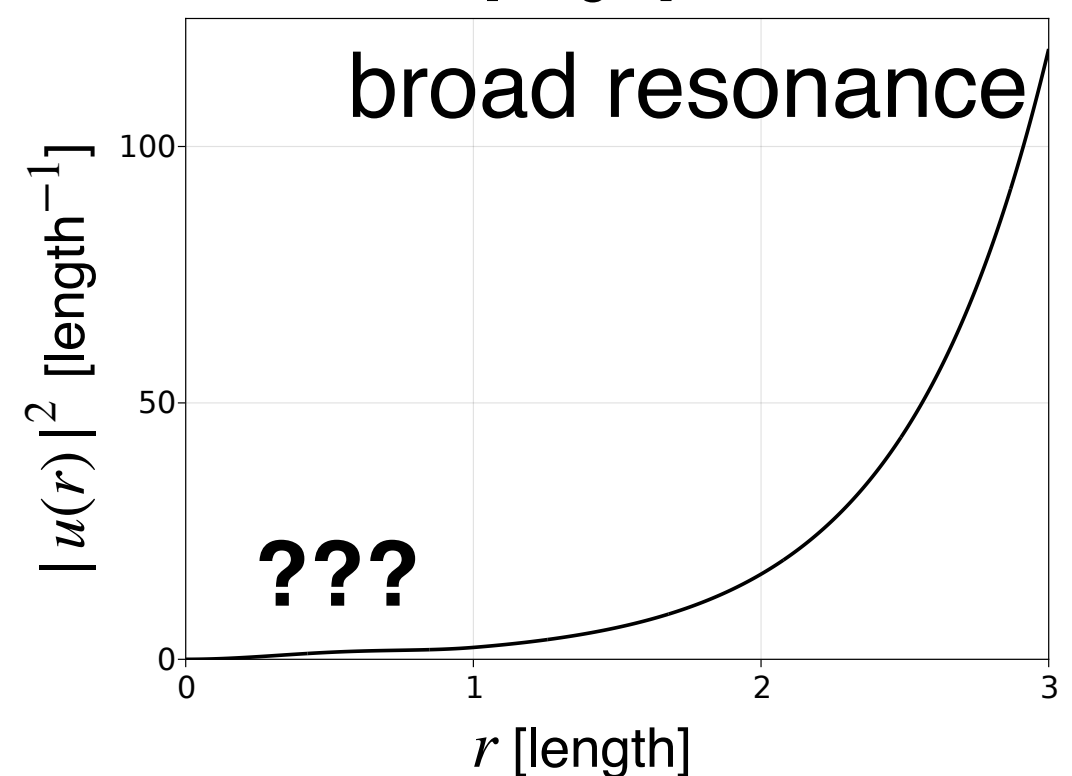
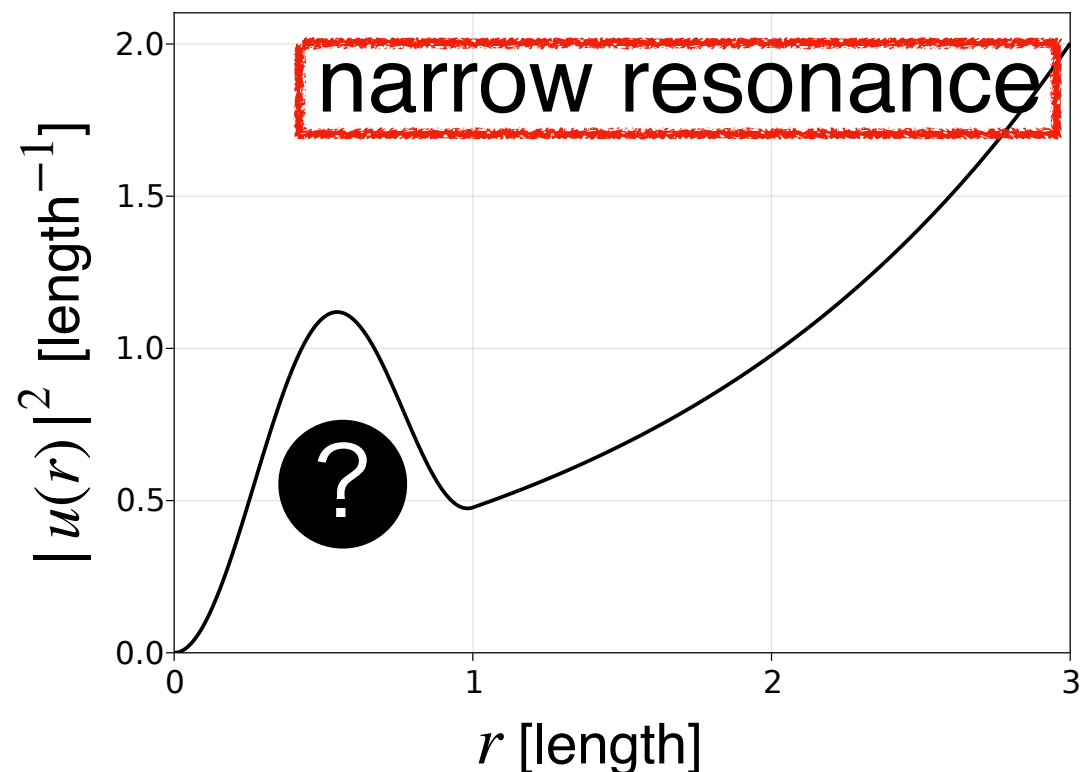
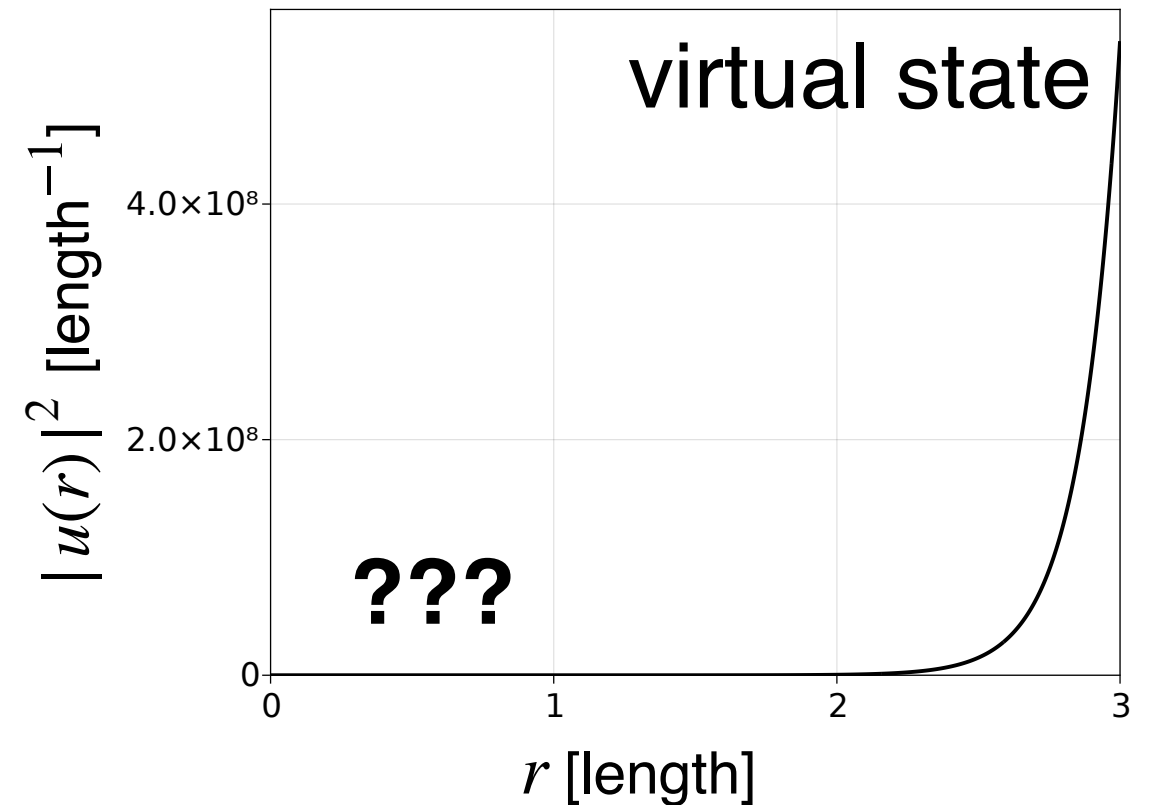
Wavefunction of eigenstates

30

✓ internal “structure”!



✗ internal “structure”?



● resonance pole written by effective range expansion (ERE)

$$f(k)^{-1} = -\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik \longrightarrow k^\pm = \frac{i}{r_e} \pm \frac{1}{r_e} \sqrt{\frac{2r_e}{a_0} - 1}$$

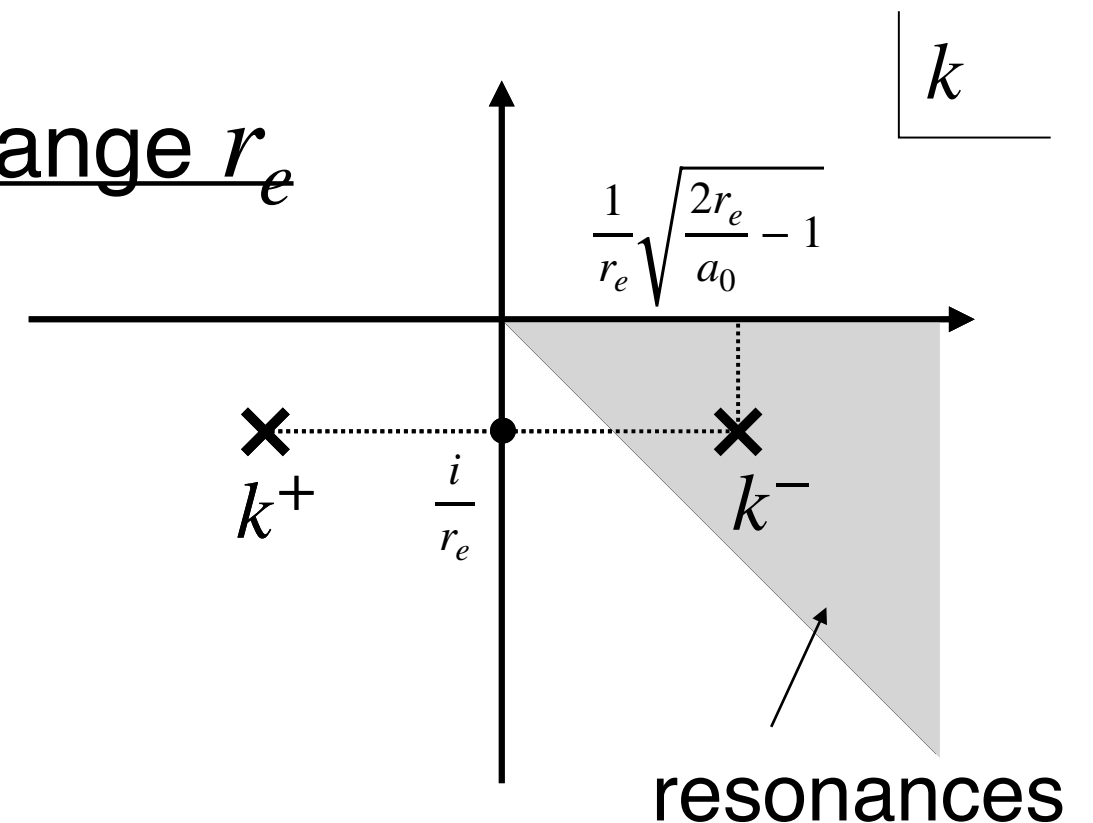
T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

- pole position ($k^\pm$) $\longleftrightarrow a_0$ and r_e

● scattering length a_0 and effective range r_e

- r_e should be negative to obtain resonances
- $\text{Im}k = |1/r_e|$ should be small to obtain narrow resonances

→ Effective range should be **large and negative** for near-threshold resonances



Universality for near-th. resonances ³²

- near-threshold **bound** (and virtual) states

$a_0 \rightarrow \infty$ and universality holds in $B \rightarrow 0$ limit 

$\rightarrow X \rightarrow 1$ (completely composite)

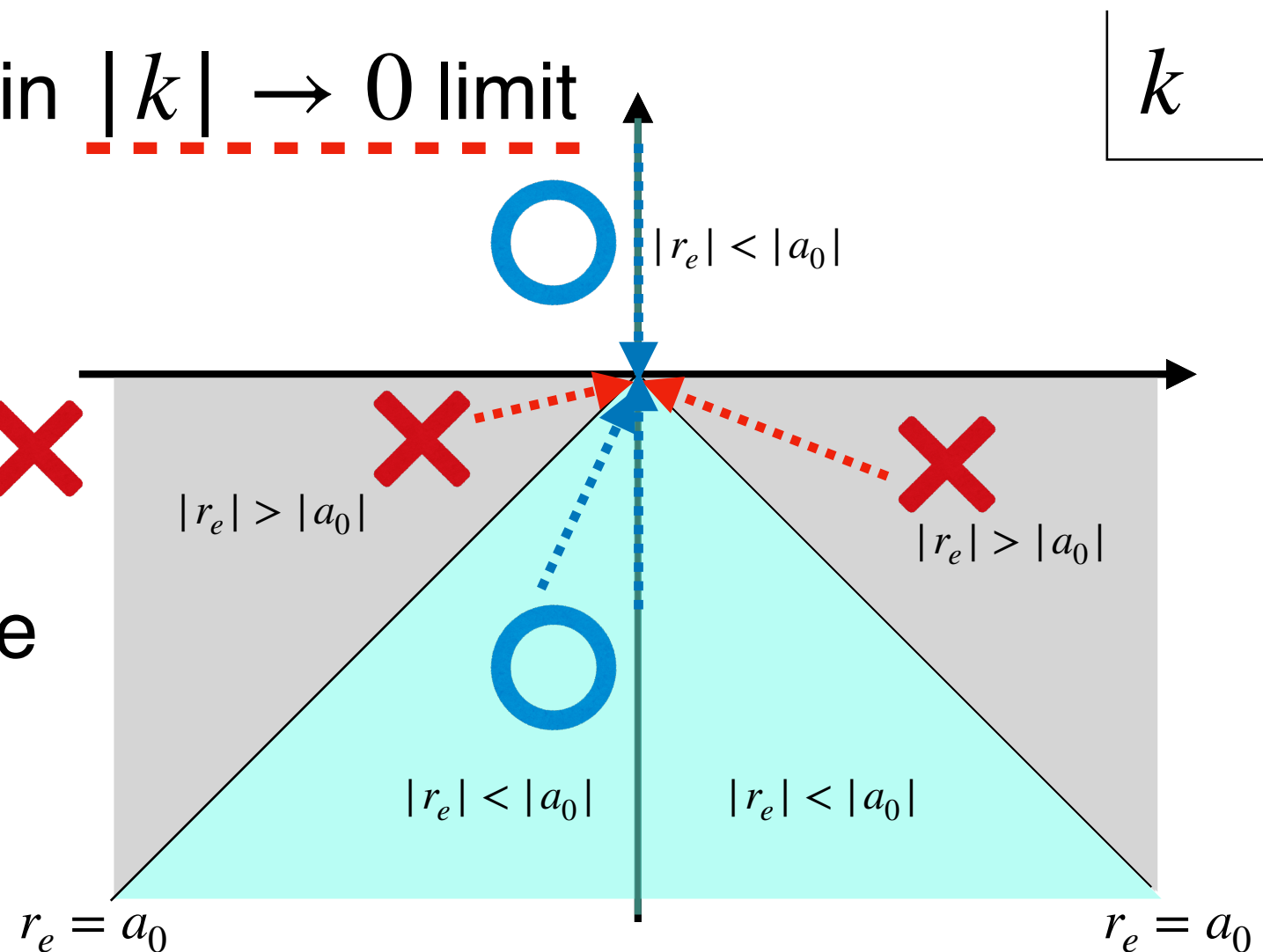
- near-threshold **resonances**

$a_0 \rightarrow \infty$ but also $|r_e| \rightarrow \infty$ in $|k| \rightarrow 0$ limit

$$\because |a_0| \leq |r_e|$$

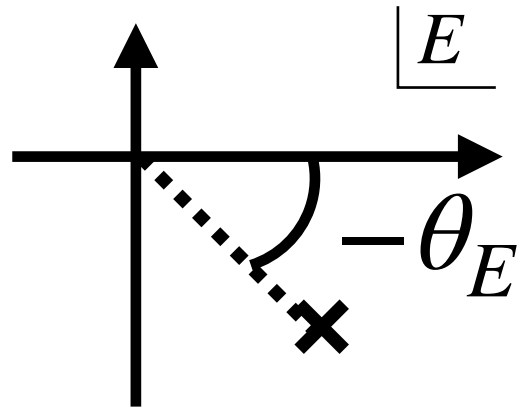
$\rightarrow$ universality does not hold 

Near-threshold resonances are **not** necessarily composite dominant



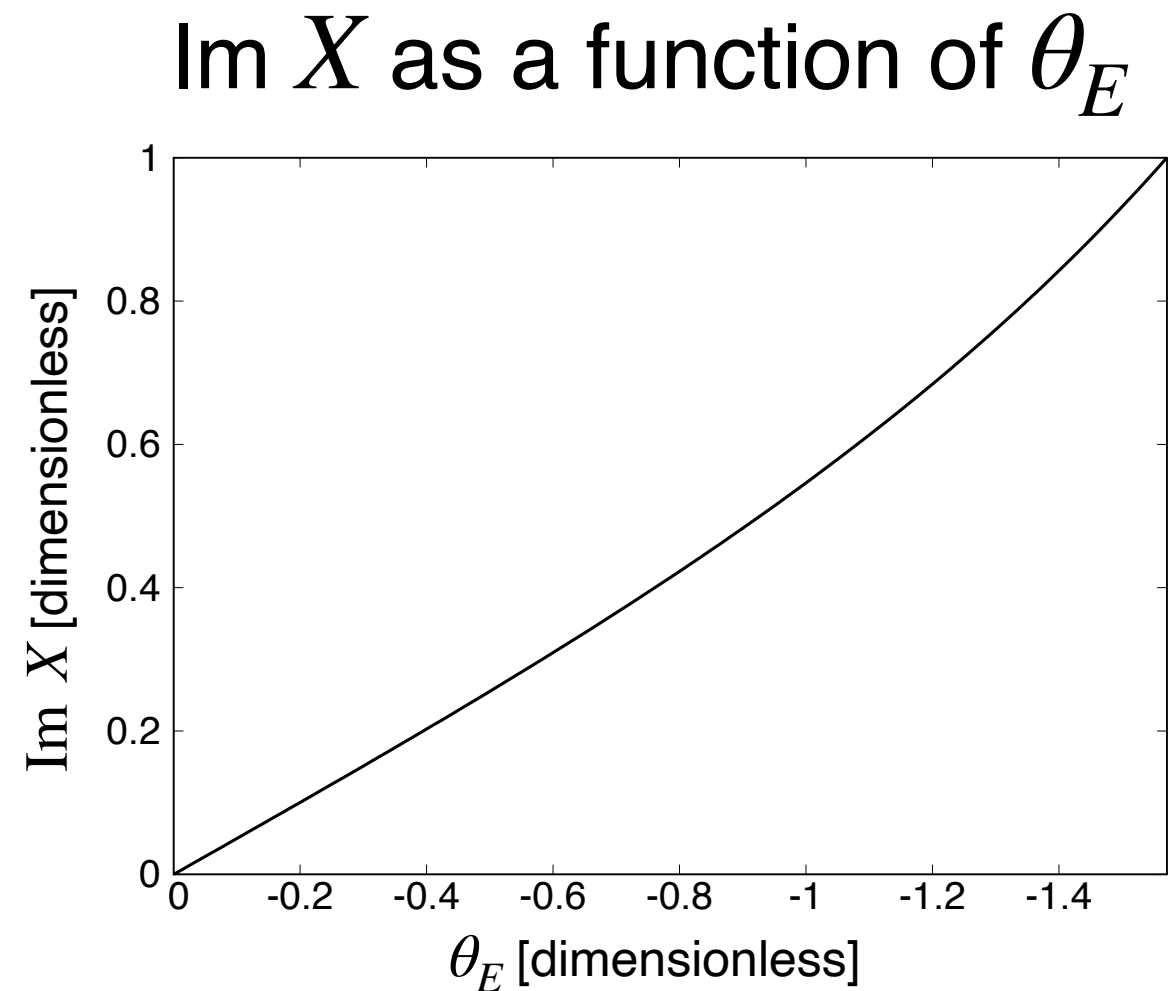
Compositeness in ERE

$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan \theta_k = -i \tan(\theta_E/2)$$



$$(k = |k| e^{i\theta_k}, E = |E| e^{i\theta_E})$$

→ X in ERE is pure imaginary



- in general, compositeness X of unstable resonances becomes **complex** by definition
- complex X **cannot** be directly interpreted as a probability



- interpretation scheme of complex X is **not** unique

- $|X|$ F. Aceti, E. Oset, Phys. Rev. D 86, 014012 (2012); T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
Z.H. Guo, J.A. Oller, Phys. Rev. D 93(9), 096001 (2016); J.A. Oller, Annals Phys. 396, 429 (2018).

- $\text{Re } X$ F. Aceti, L.R. Dai, L.S. Geng, E. Oset, Y. Zhang, Eur. Phys. J. A 50, 57 (2014).

- introduce new quantities defined by complex X

Y. Kamiya, T. Hyodo, PTEP 2017(2), 023D02 (2017);

T. Sekihara, T. Arai, J. Yamagata-Sekihara, S. Yasui, Phys. Rev.C 93(3), 035204 (2016).

- introduce new quantities based on ERE

T. Hyodo, Phys. Rev. Lett. 111, 132002 (2013);

I. Matuschek, V. Baru, F.K. Guo, C. Hanhart, Eur. Phys. J. A 57(3), 101 (2021).

➡ How can we propose a “reasonable” scheme?

● condition for reasonable measure

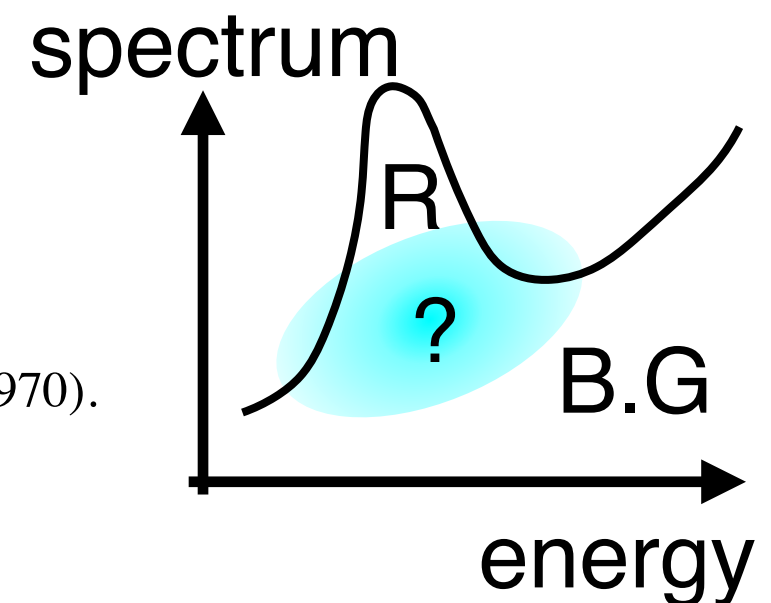
1. reduce to compositeness of bound states when $\text{Im } X \rightarrow 0$
2. probabilistic quantities (normalized and between 0 and 1)
3. take into account only **narrow** resonance automatically

Our proposal

- introduce new quantities defined by complex X
- i) $\mathcal{X}$: probability of certainly finding $|\text{composite}\rangle$
- ii) $\mathcal{Z}$: probability of certainly finding $|\text{elementary}\rangle$
- iii) $\mathcal{Y}$: probability of uncertain identification

uncertain appears from

- finite lifetime (uncertainty in energy)
- separation from B.G. T. Berggren, Phys. Lett. B 33, 547 (1970).



$$\mathcal{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathcal{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$\mathcal{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

$$1. \mathcal{X} \rightarrow X, \mathcal{Z} \rightarrow Z$$

$$\text{and } \mathcal{Y} \rightarrow 0 \text{ (Im } X \rightarrow 0)$$

$$2. \mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$$

α reflects uncertain nature of resonances

Definition

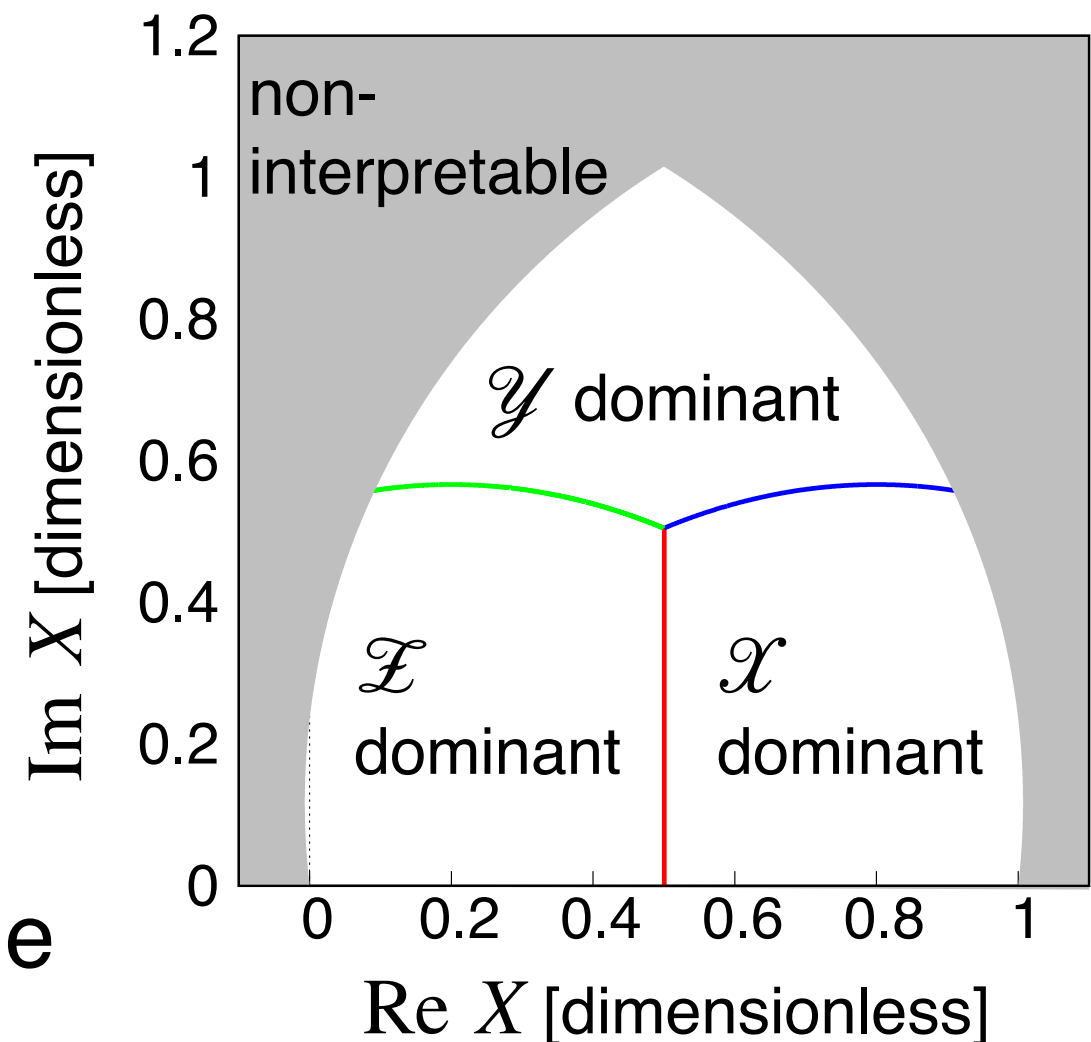
- if $\alpha > 1/2$, $\mathcal{Y}$ is always positive but $\mathcal{X}, \mathcal{Z}$ can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		non-interpretable

3. our criterion for narrow resonance

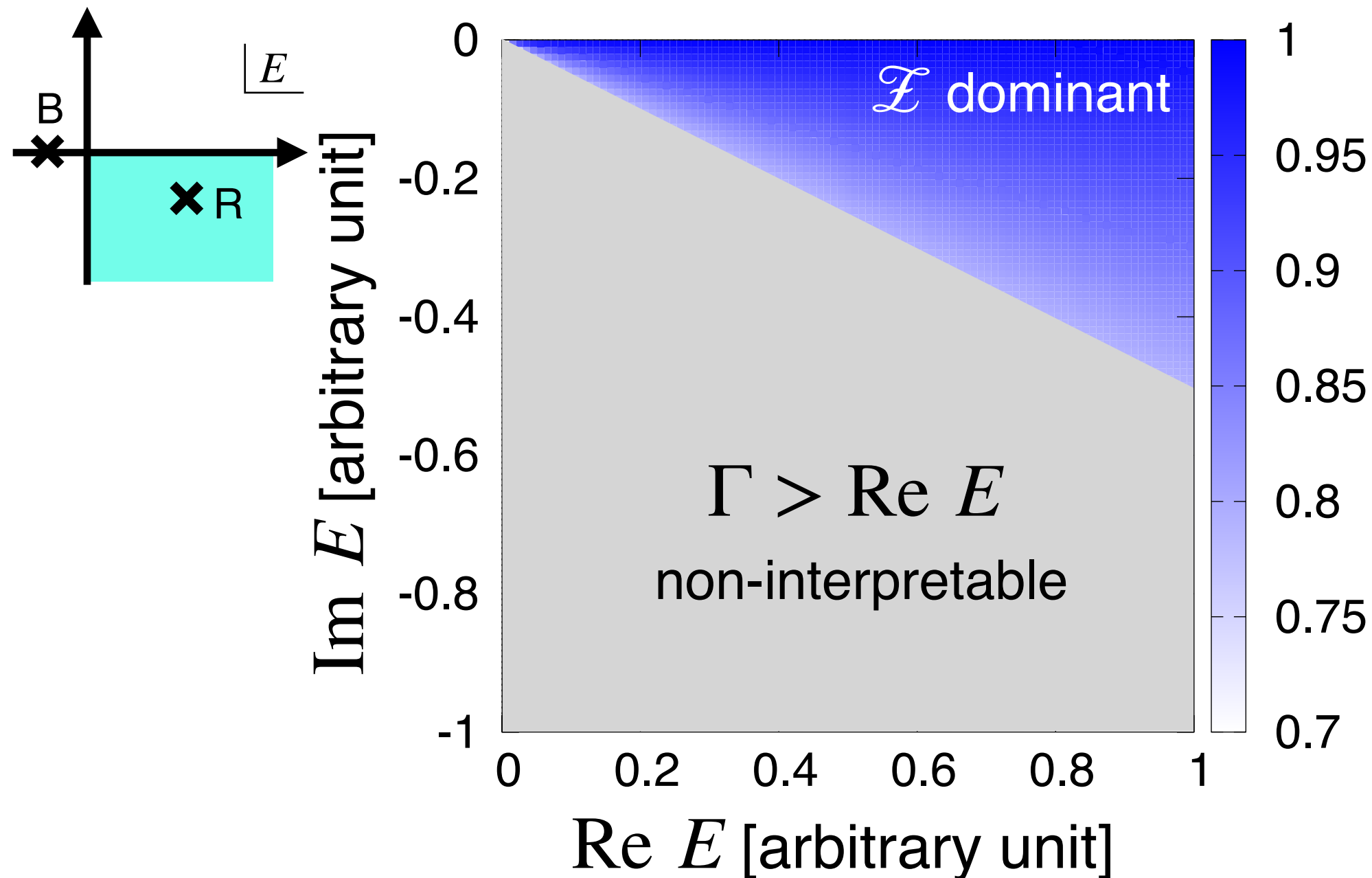
$$\longrightarrow \Gamma \leq \text{Re } E$$

- $\alpha \sim 1.1318$ using ERE
- narrow resonances: interpretable
 $\longrightarrow \mathcal{X}, \mathcal{Y}$, or $\mathcal{Z}$ dominant
- broad resonances: non-interpretable



Structure of near-th. resonances

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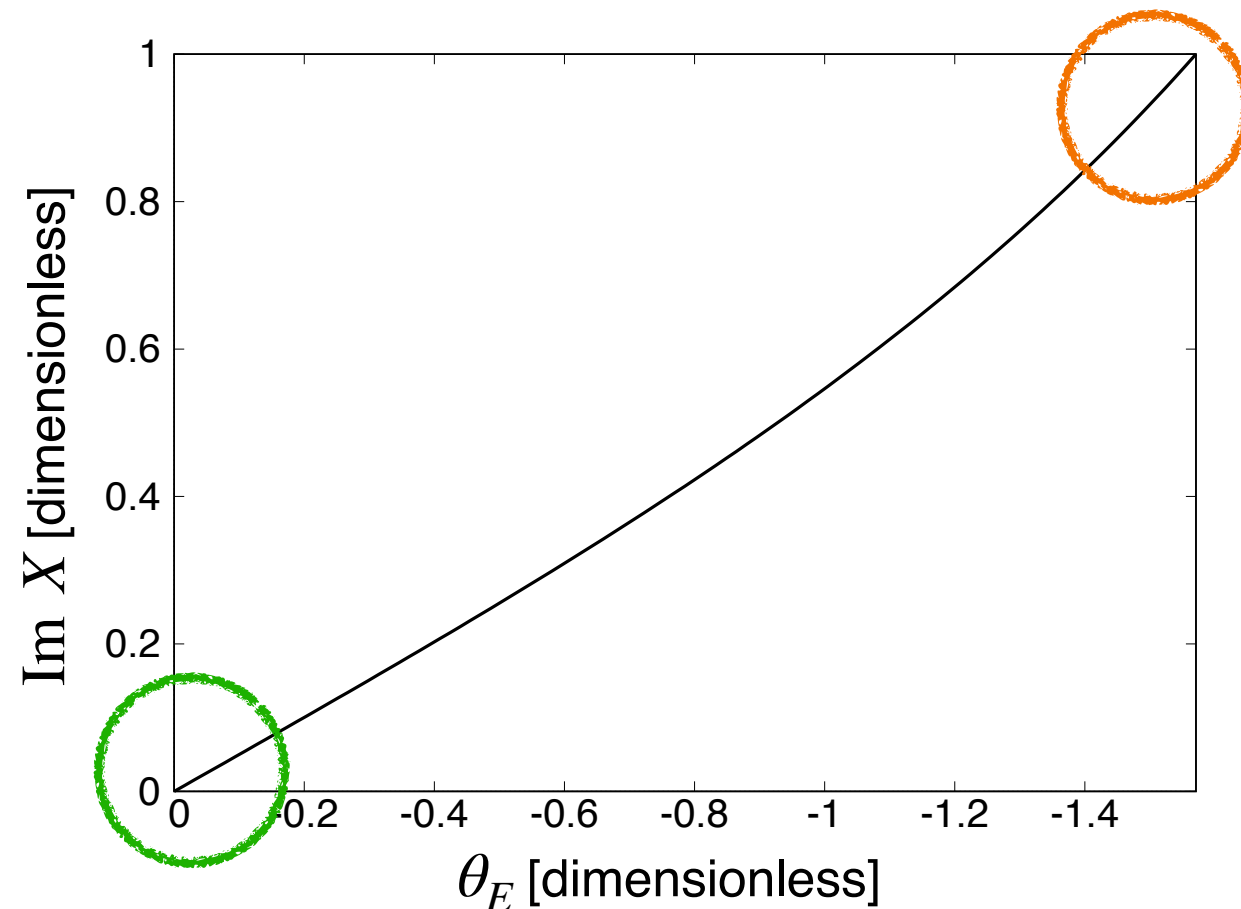
- resonances are **not composite dominant state** ($\mathcal{I} \gtrsim 0.8$)
- different from near-threshold bound states
(composite dominant $X \sim 1$ and $Z \sim 0$)

Compositeness in ERE

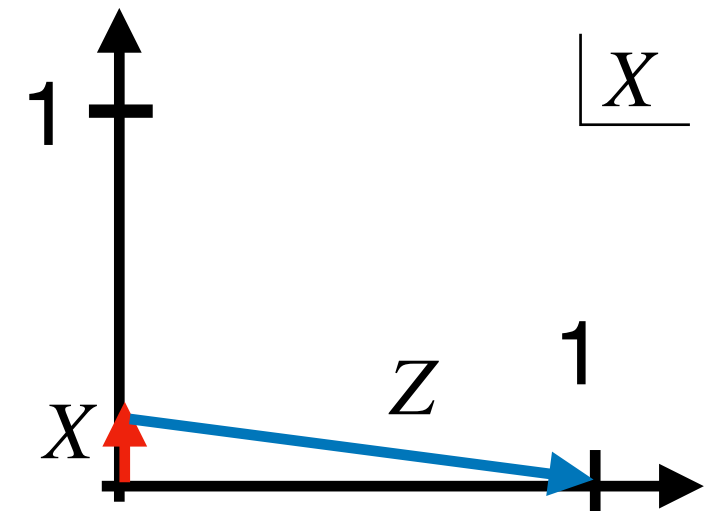
$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan \theta_k \quad (k = |k| e^{i\theta_k})$$

→ X in ERE is pure imaginary

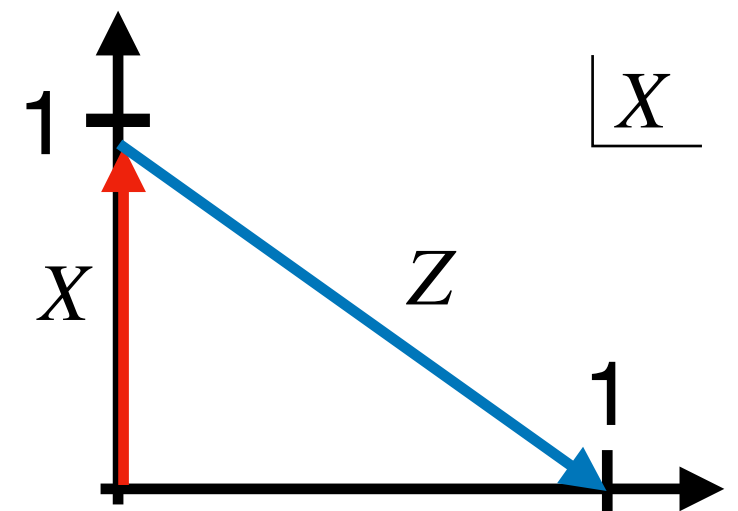
X as a function of θ_E ($E = |E| e^{i\theta_E}$)



small width ($\theta_E \sim 0$)

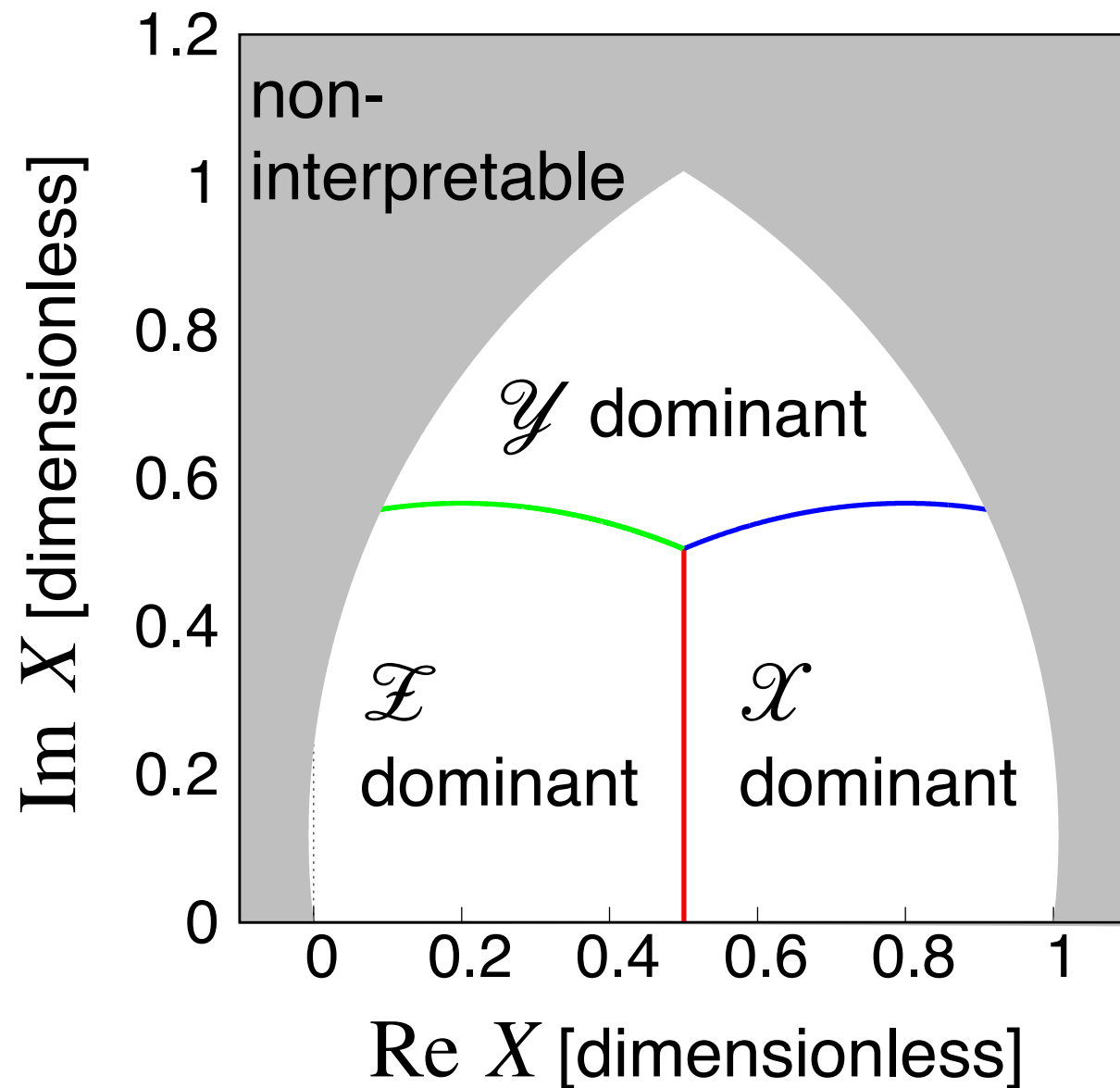


large width ($\theta_E \sim -\pi/2$)



$\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ dominant region

- $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ dominant region in complex X plane

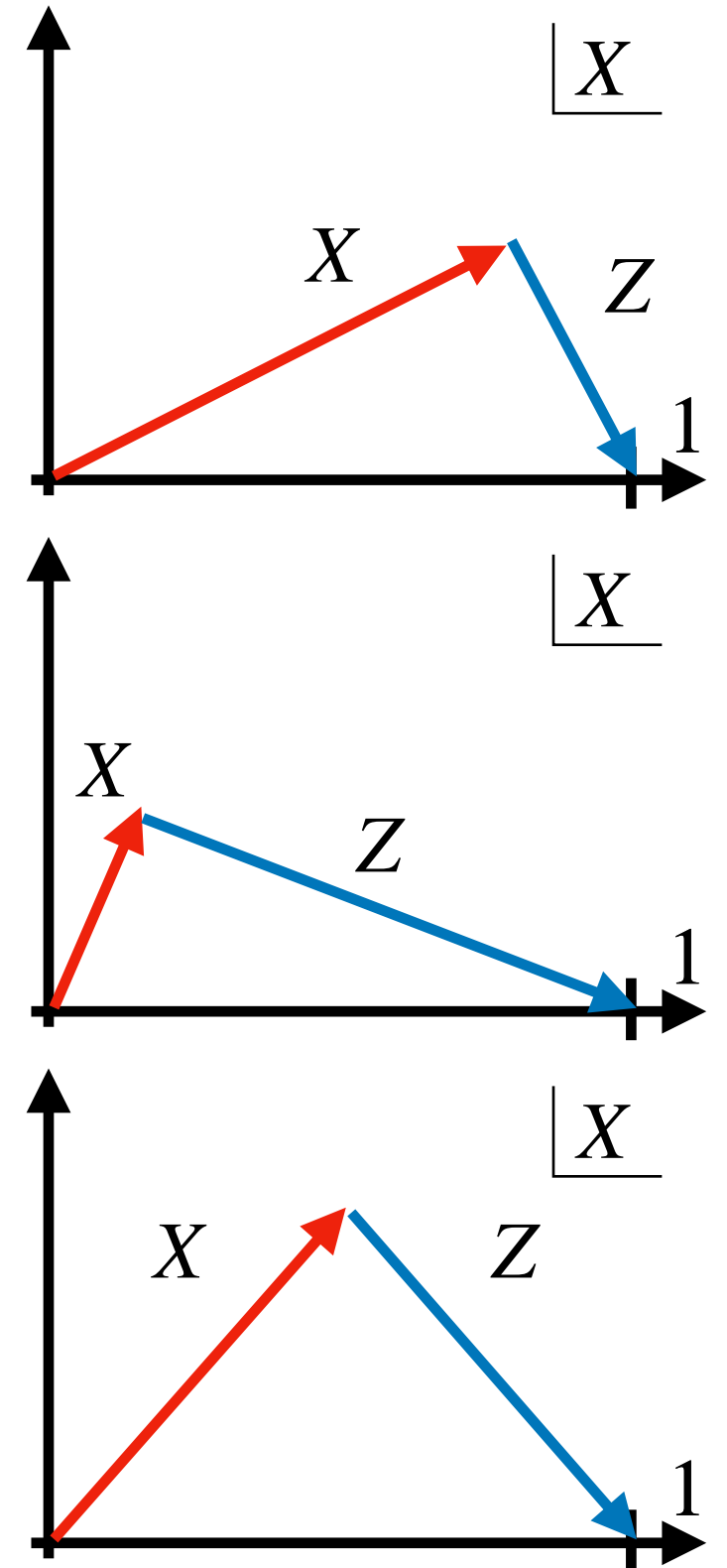


large $\text{Im } X$ is assigned to non-interpretable cases

composite dominant

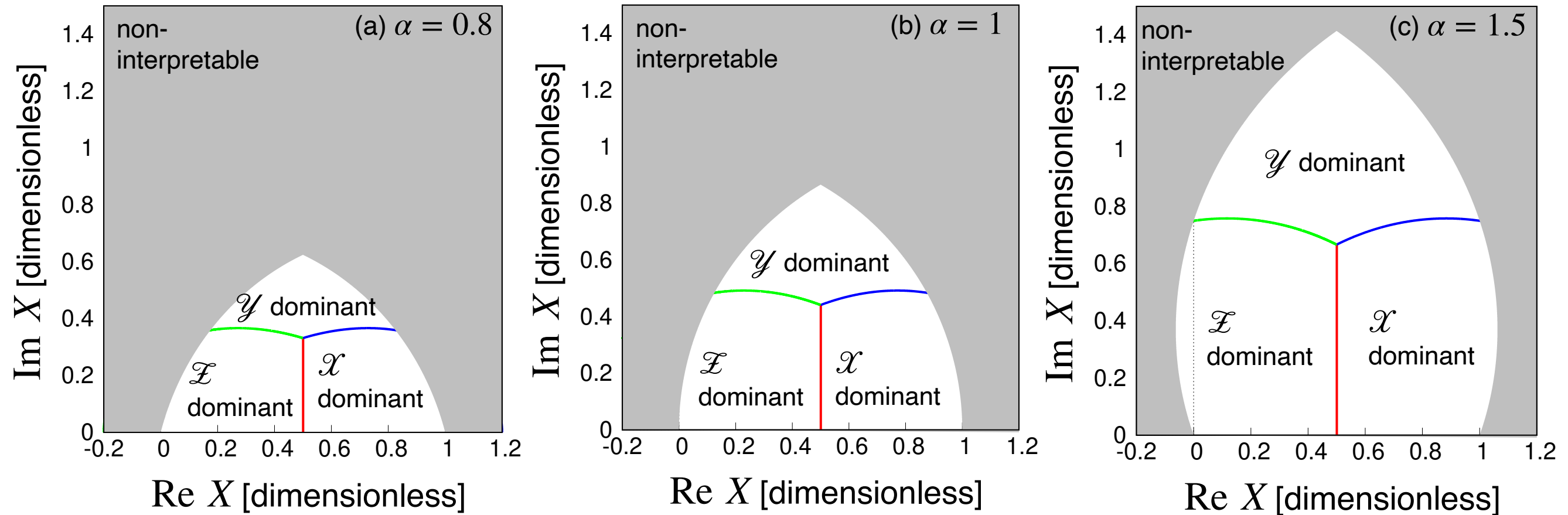
elementary dominant

uncertain



$\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ dominant region

40



- Interpretable regions become large with increase of α

$\alpha \rightarrow \infty \longrightarrow \mathcal{X}, \mathcal{Y}, \mathcal{Z}$ reduce to interpretation in previous work

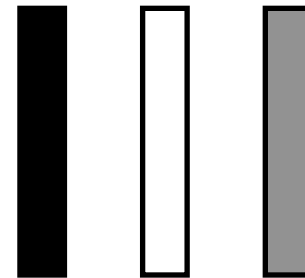
$$\mathcal{X} \rightarrow \tilde{X}, \mathcal{Z} \rightarrow \tilde{Z}, \mathcal{Y} \rightarrow 0$$

Y. Kamiya and T. Hyodo,
Phys. Rev. C **93**, 035203 (2016).

uncertainty in resonances

41

a single
measurement



sum of measurements of a bound states / resonances

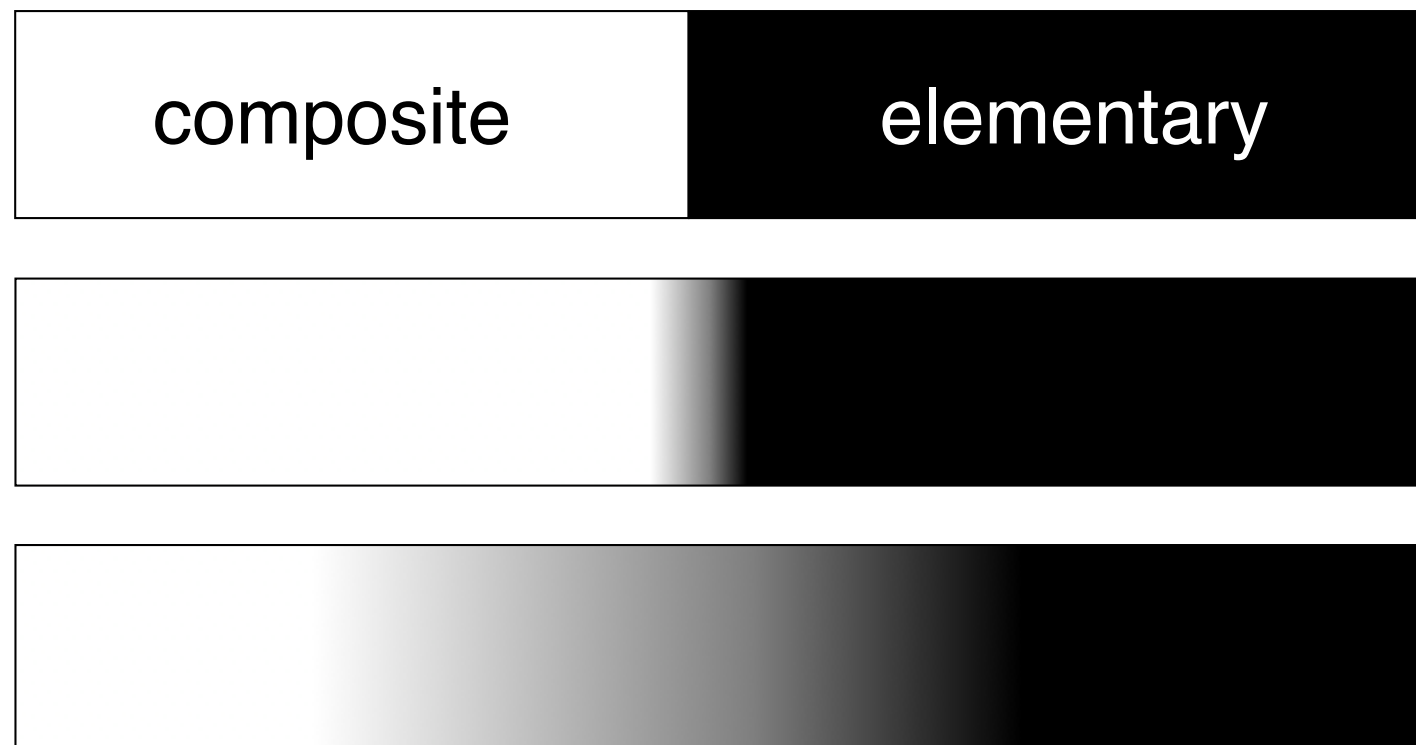
bound state

composite

elementary

narrow
resonance

broad
resonance

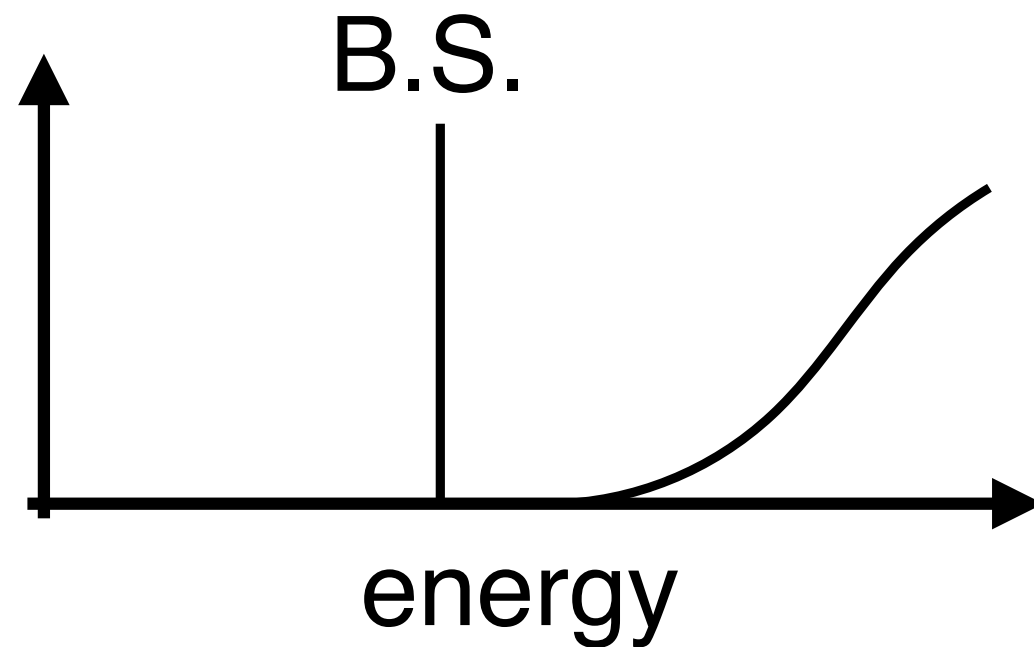


measurements

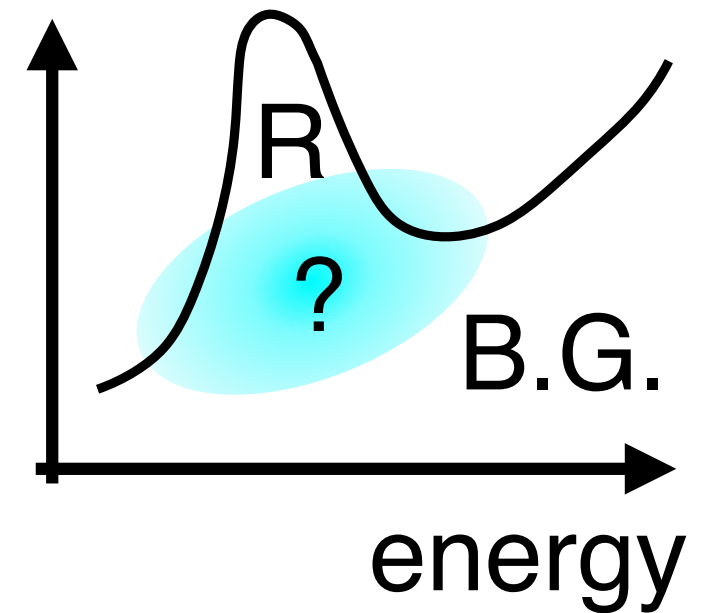
uncertainty in resonances

42

(a) spectrum



(b) spectrum

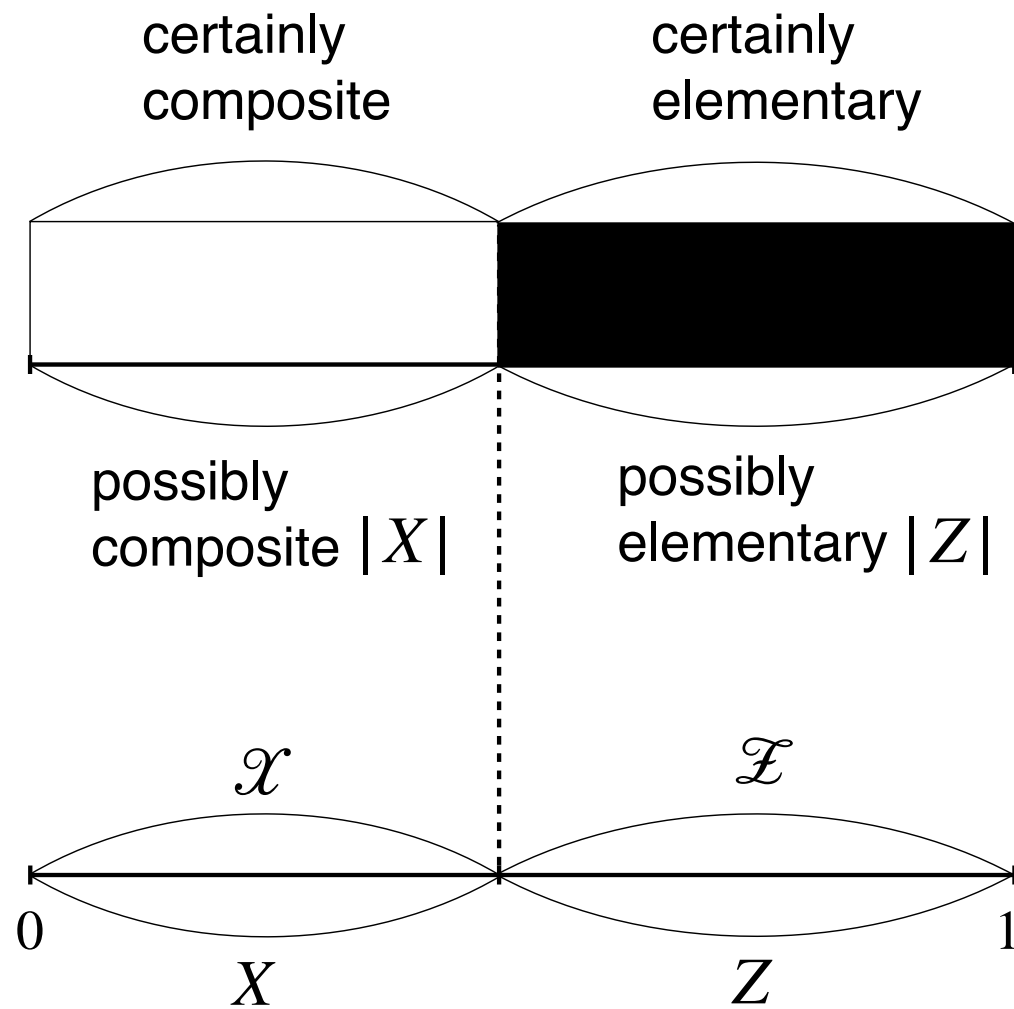


uncertain appears from

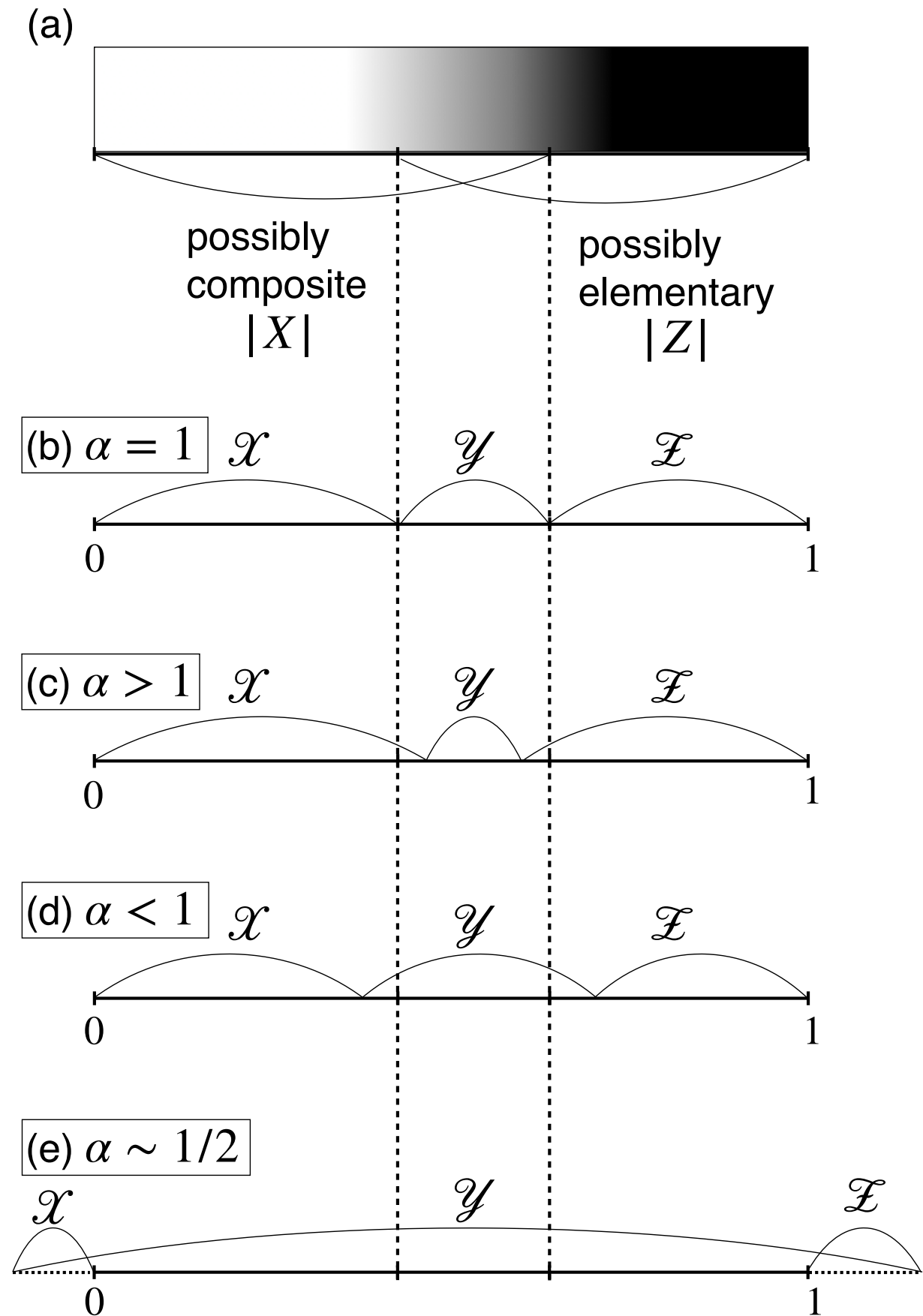
- finite lifetime (uncertainty in energy)
- separation from B.G.

Interpretation

43



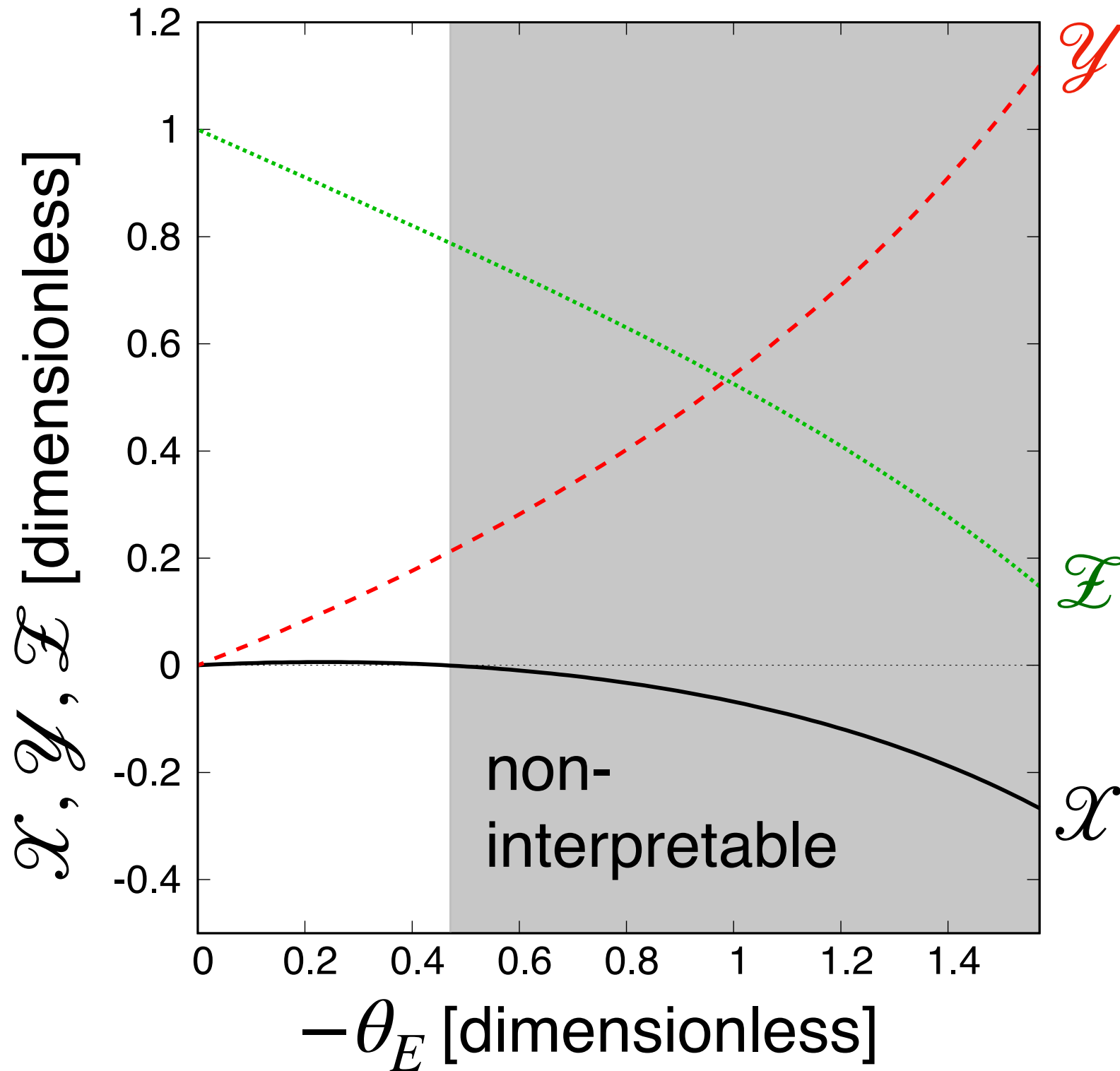
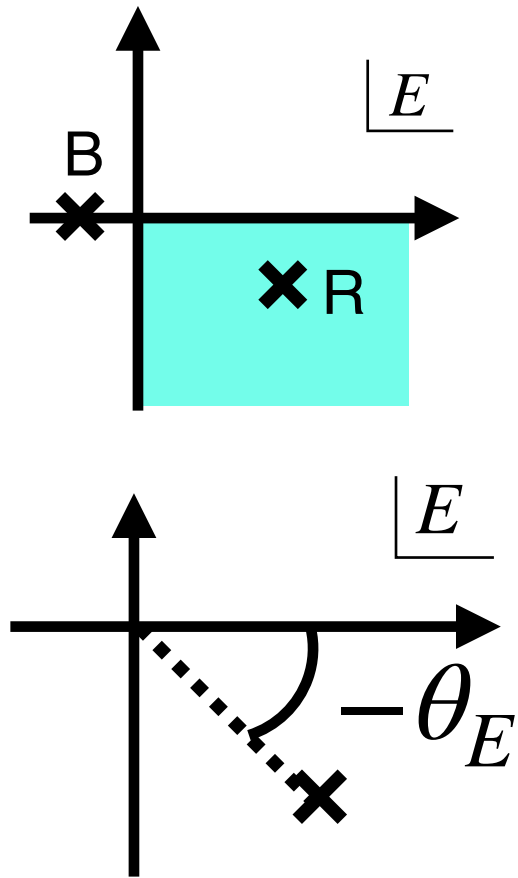
bound state



resonance

Structure of near-th. resonances

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resonances with previous works

45

$$\bar{Z} = 1 - \sqrt{\left| \frac{1}{1 - 2r_e/a_0} \right|}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

$$\tilde{Z}_{\text{KH}} = \frac{1 + |Z| - |X|}{2}$$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

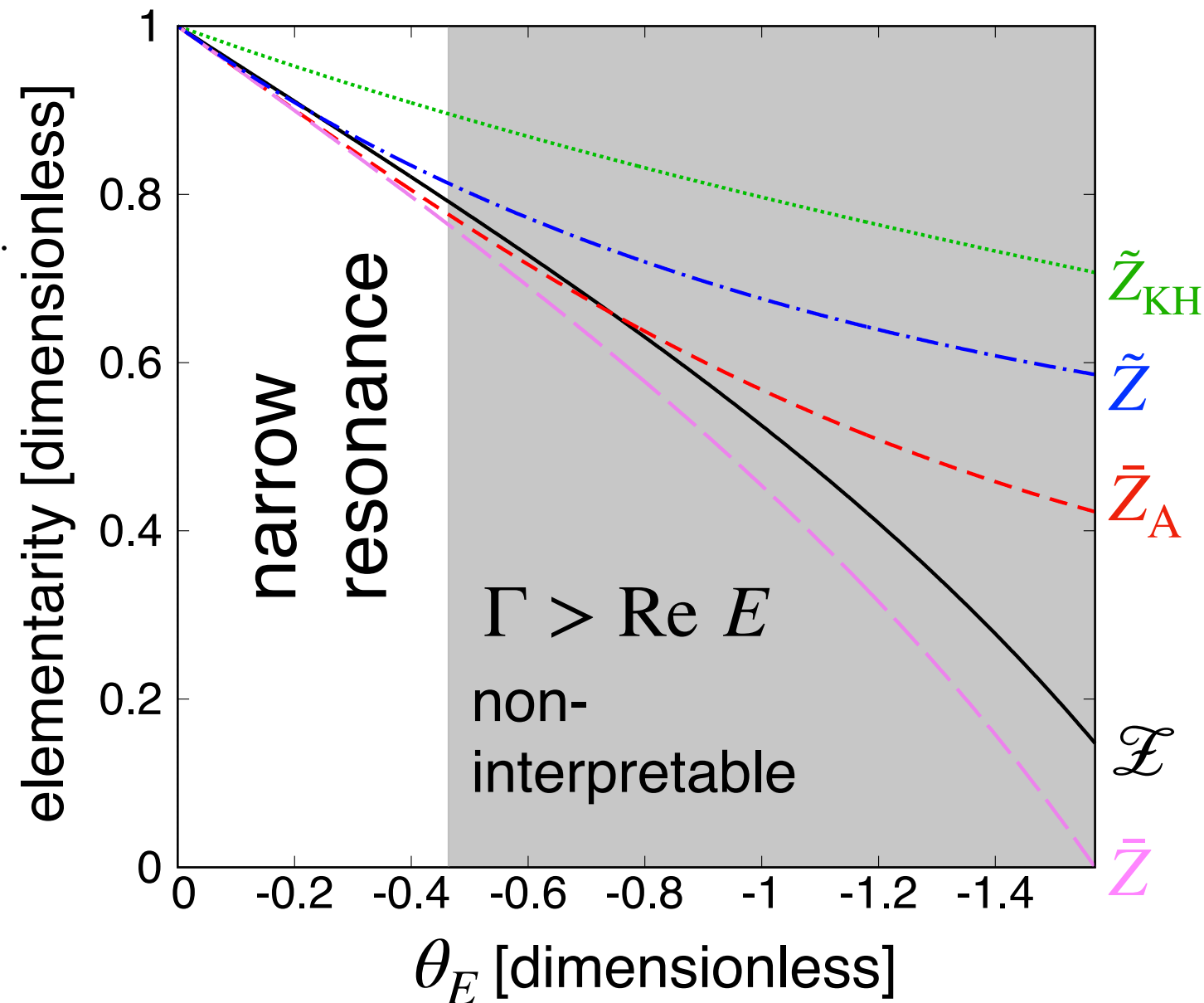
$$\tilde{Z} = \frac{|Z|}{|X| + |Z|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC **93**, 035204 (2016).

$$\bar{Z}_A = 1 - \sqrt{\frac{1}{1 + |2r_e/a_0|}}$$

I. Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A **57**, 101 (2021).

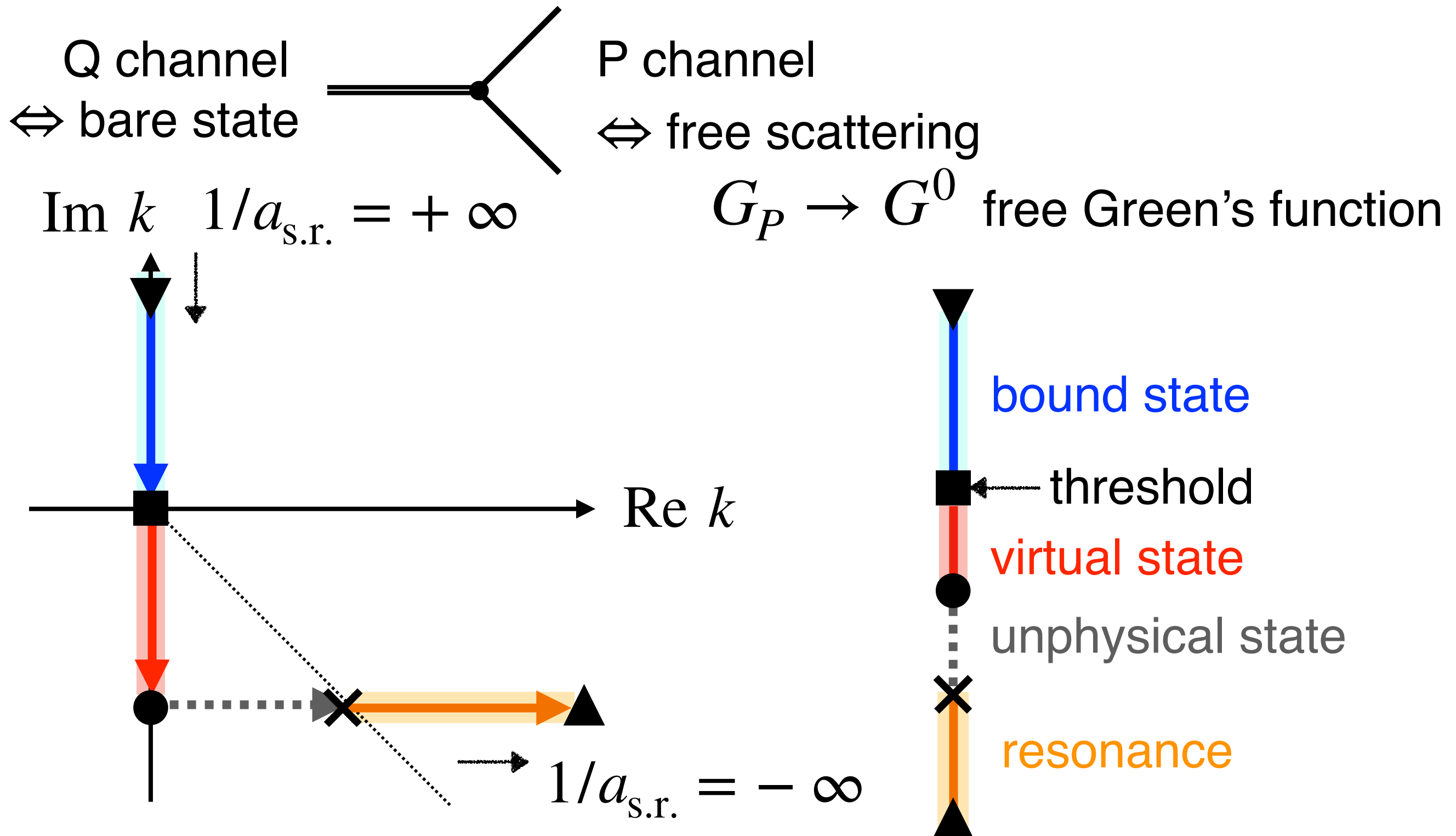
interpretations as a function of θ_E



- all schemes show resonances are elementary dominant!

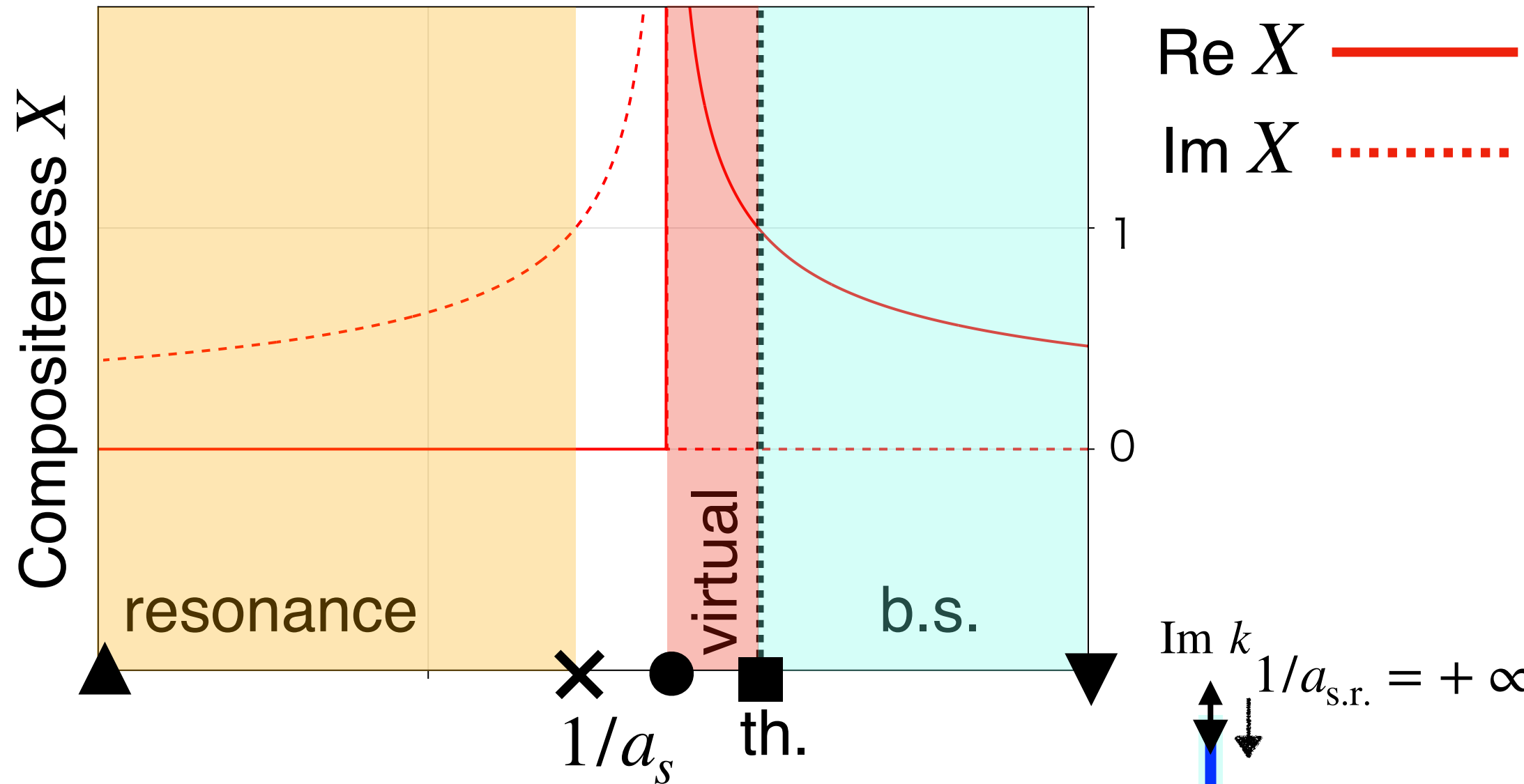
Pole trajectory (only w/ s.r.)

● pole trajectory in complex momentum k plane (No Coulomb)



Compositeness (only w/ s.r.)

47

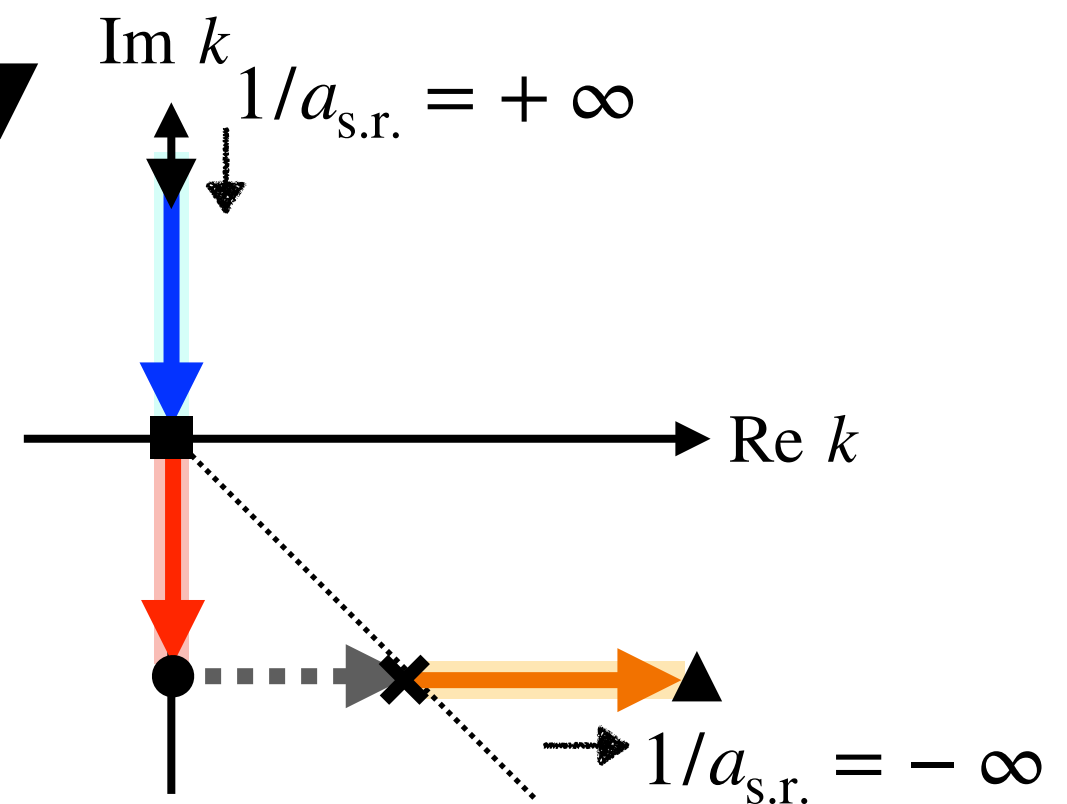


- b.s. : $0 \leq X \leq 1$

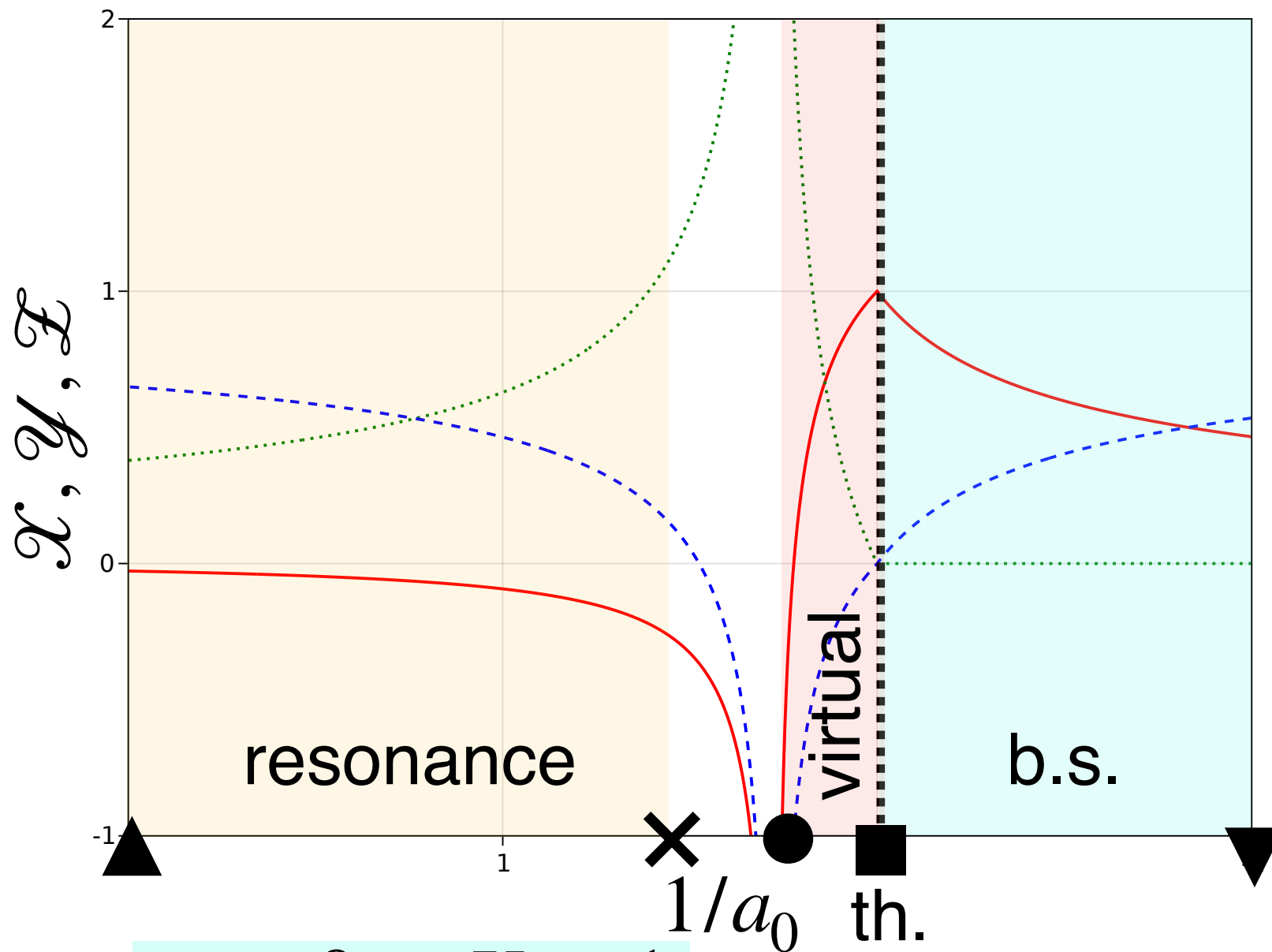
- virtual : $1 < X$

● divergence $X \rightarrow \infty$

- resonance : $\text{Im } X \leq 1$



$\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ from b.s. to resonance ⁴⁸



- b.s. : $0 \leq X \leq 1$

- virtual : $\mathcal{Z} < 0$ (non-interpretable)

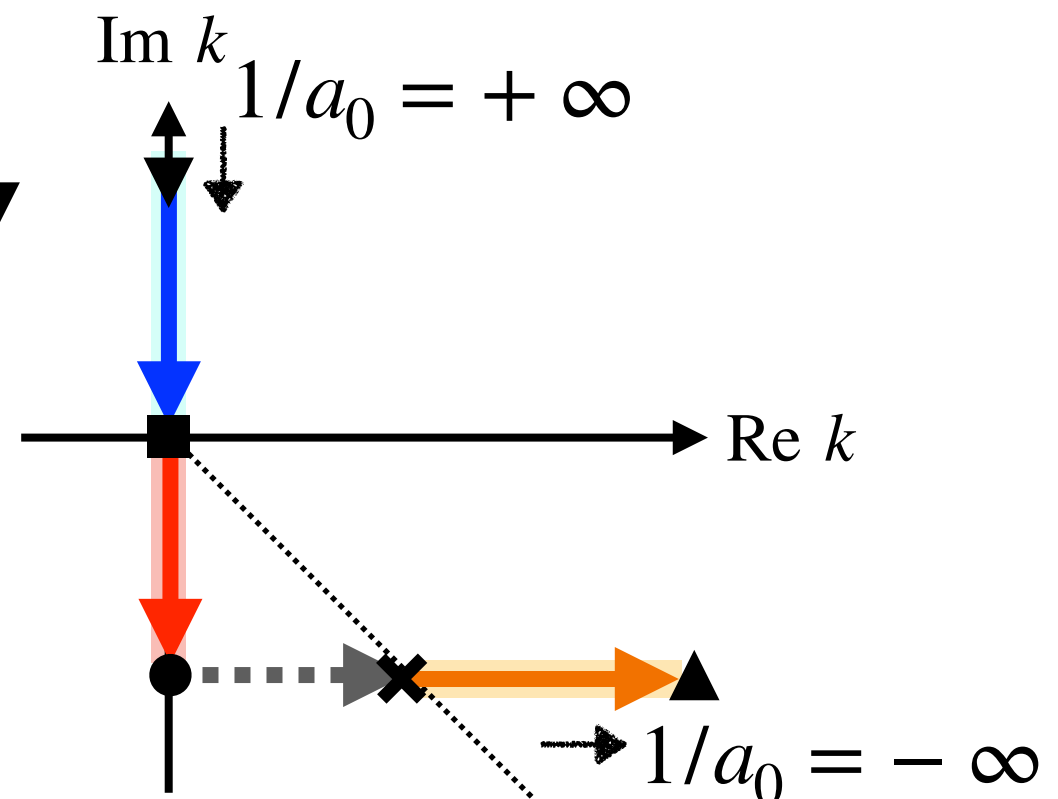
● divergence $\mathcal{X}, \mathcal{Y}, \mathcal{Z} \rightarrow \infty$

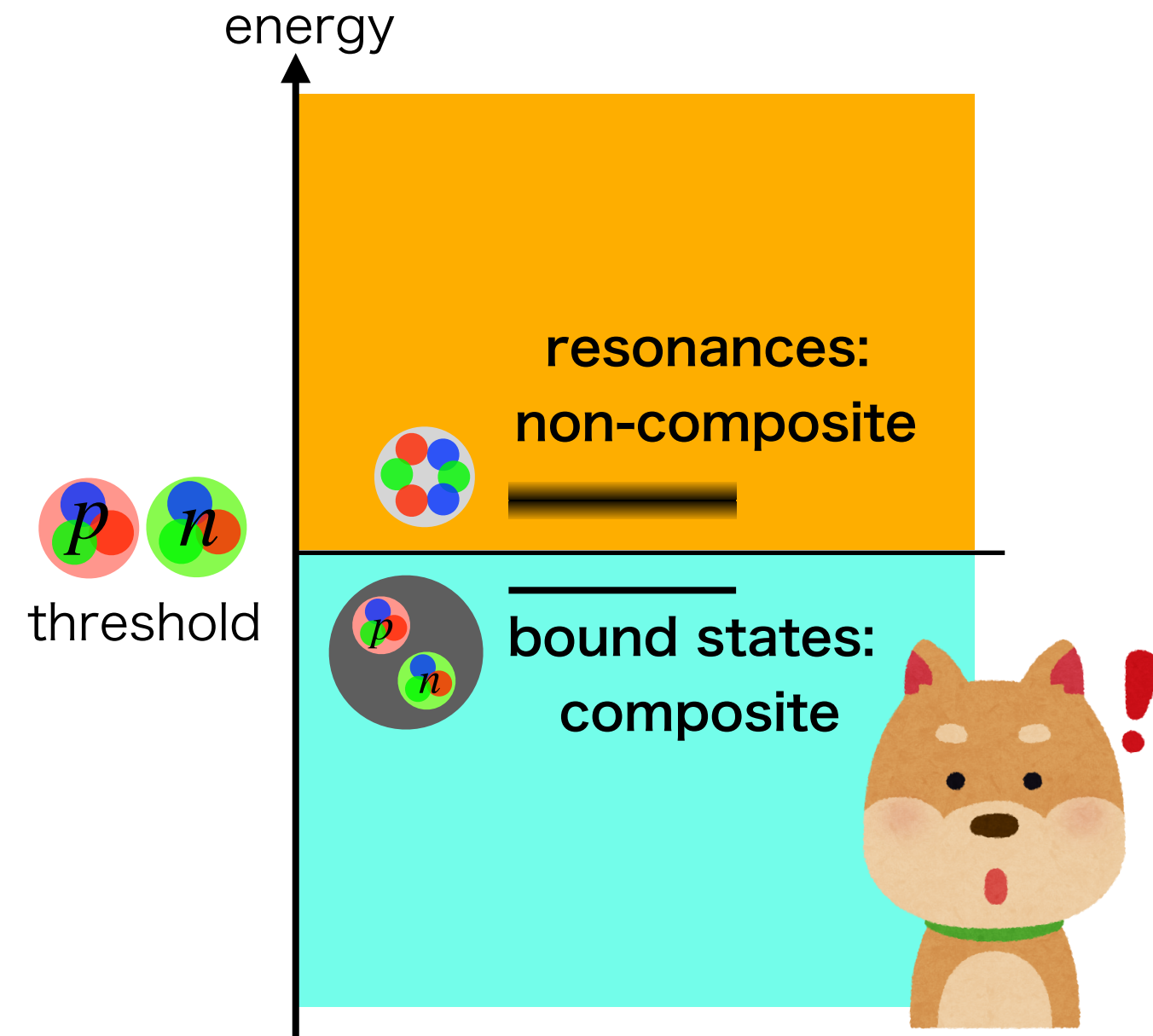
- resonance : $\mathcal{Z}$ dominant

$\mathcal{X}$ ———

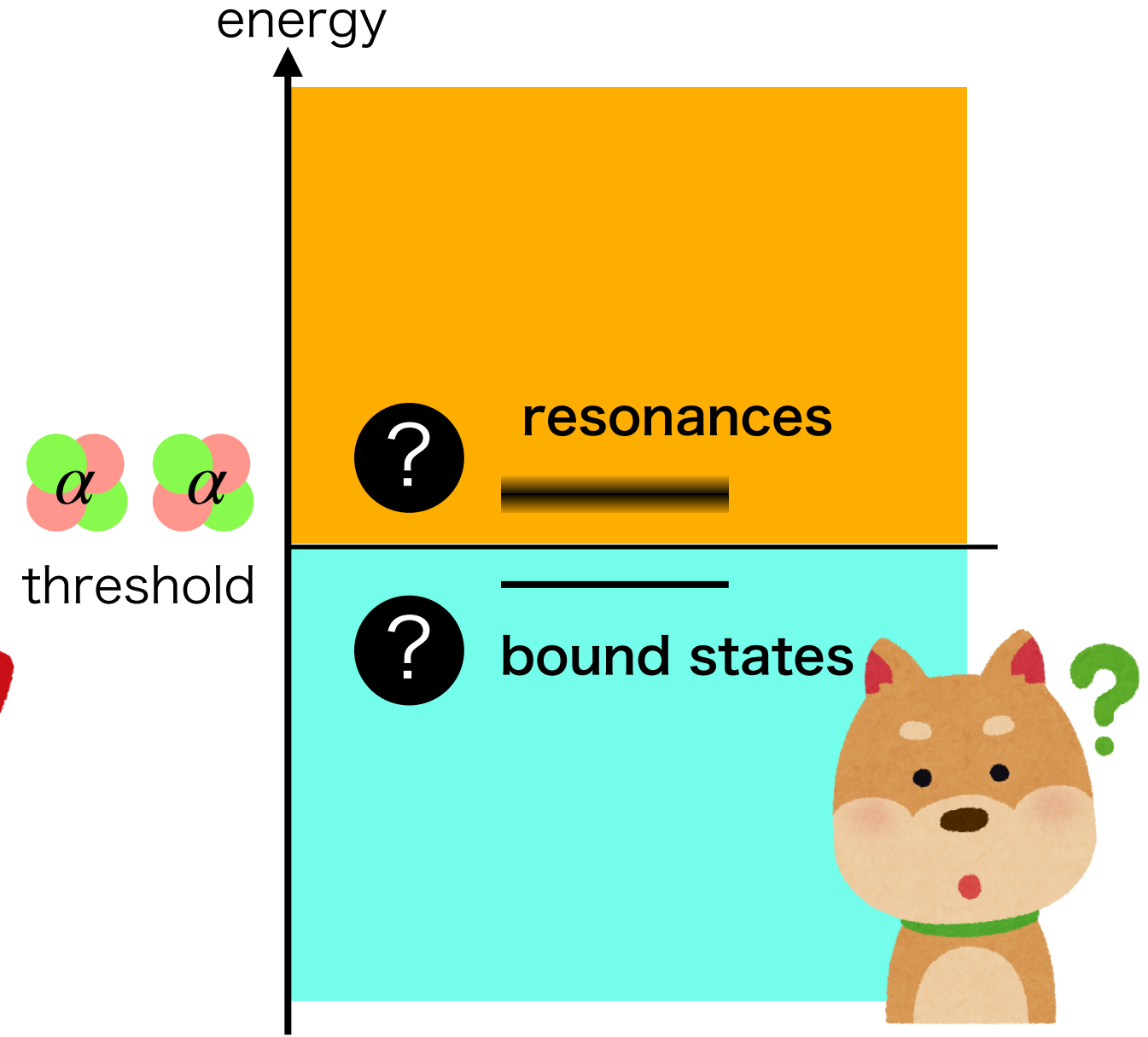
$\mathcal{Y}$
.....

$\mathcal{Z}$
.....





scatterings with **short range**
interaction



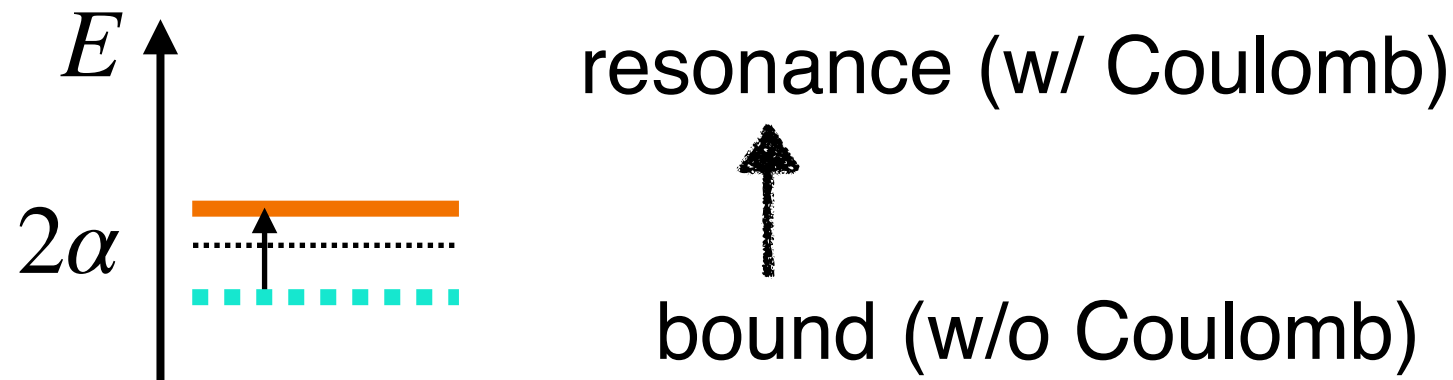
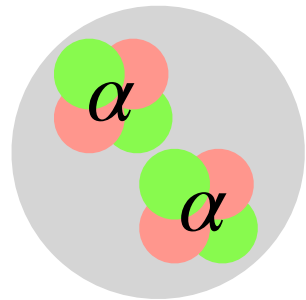
scatterings with **Coulomb +**
short range interaction

T. Kinugawa and T. Hyodo, in preparation.

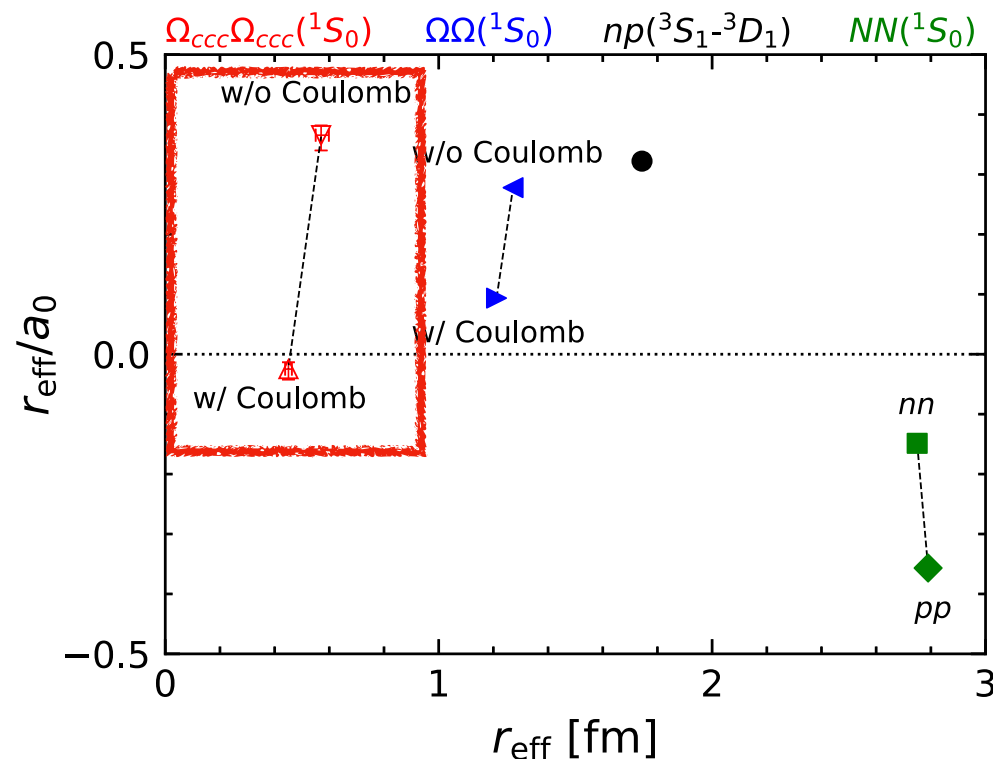
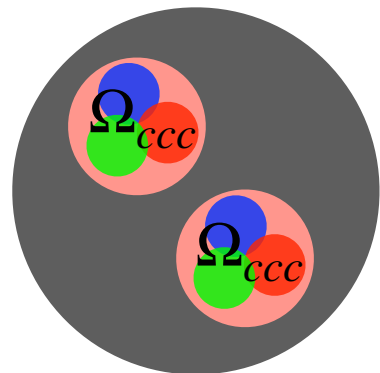
Coulomb + short range systems

50

- ${}^8\text{Be}$ nuclei ($2\alpha^{++}$) J. Hiura, and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 52, 25 (1972);
R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809, 171 (2008).



- $\Omega_{ccc}^{++}\Omega_{ccc}^{++}$ (HAL QCD) Y. Lyu, H.Tong, *et al.* [HAL QCD Coll.], Phys. Rev. Lett. 127 (2021) 072003.



resonance (w/ Coulomb)
↑
bound (w/o Coulomb)

- $\Xi^{-}\alpha$: Coulomb assisted bound state E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, Phys. Rev. C 106, 064318 (2022).

→ Coulomb is important for near-threshold states!

Coulomb + short range systems

51

● Coulomb + short range interaction

H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

R. Oppenheim Berger and Larry Spruch, Phys. Rev. 138, B1106-B1115 (1965).

W. Domcke, Atom. Mol. Phys. 16, 359 (1983).

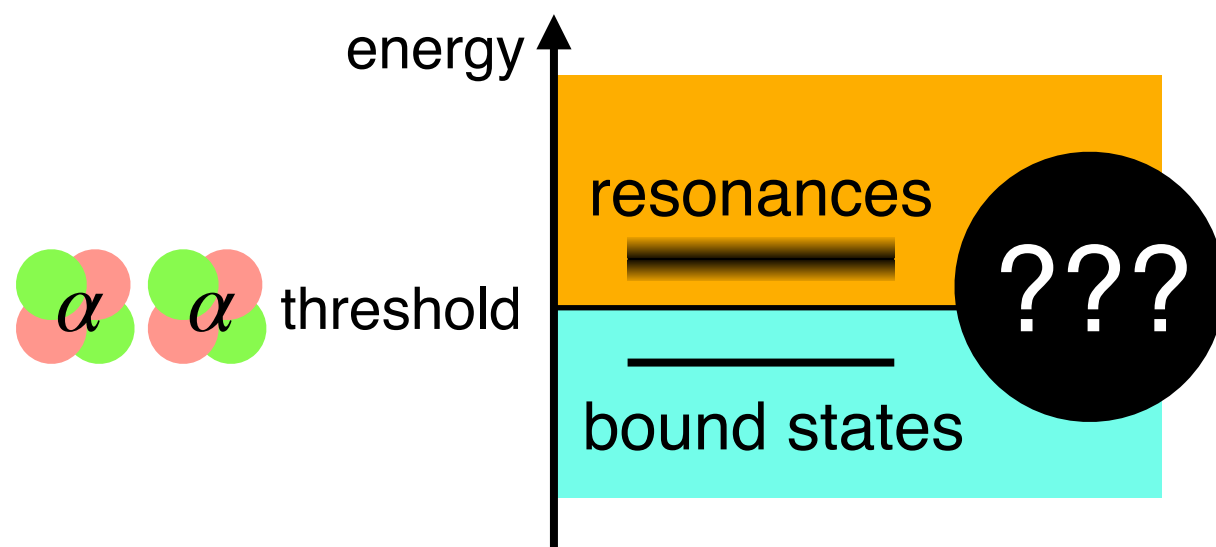
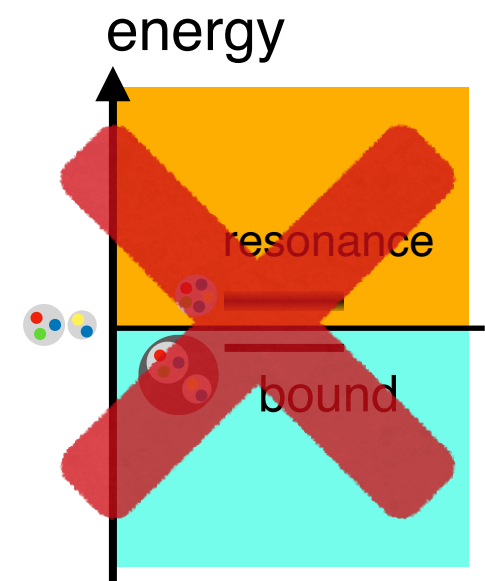
R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809, 171 (2008).

C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798, 135016 (2019).

S. Mochizuki, and Y. Nishida, Phys. Rev. C 110 , 064001 (2024).

- low-energy behavior of scattering amplitude is different from that of short range interaction

● nature of near-threshold state with Coulomb + short range interaction?

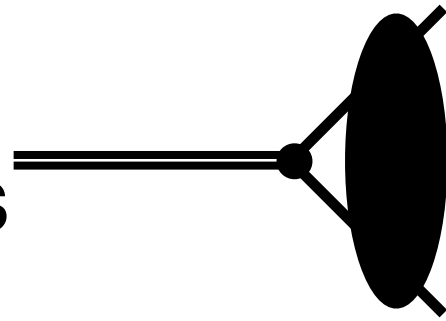


- two body
- small k region $\rightarrow$ s -wave
- Coulomb repulsive
 - $\rightarrow$ pole trajectories?
 - $\rightarrow$ compositeness?

Coulomb+short range model

● Hamiltonian W. Domcke, Atom. Mol. Phys. 16 359 (1983). R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

Q channel
 $\Leftrightarrow$ bound states
 w/ short range



P channel

$\Leftrightarrow$ scattering w/ Coulomb

$$\hat{H} = \begin{pmatrix} \hat{H}_{PP} & \hat{H}_{PQ} \\ \hat{H}_{QP} & \hat{H}_{QQ} \end{pmatrix} = \begin{pmatrix} \text{Coulomb} & \text{Short range} \\ \text{Short range} & \text{Coulomb} \end{pmatrix}$$

H. Feshbach, Annals Phys. 19 287-313 (1962).

● pole condition H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798 (2019).

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B} \left[\underbrace{\log(-ia_B k)}_{\text{log cut}} + \psi \left(1 \mp \frac{i}{a_B k} \right) \right] = 0$$

● compositeness X T. Hyodo, Phys. Rev. C 90, 055208 (2014) .

Bohr radius

$$X = 1 - \frac{1}{1 - \frac{d}{dE} F(E)} \text{ self energy}$$

$$a_B = \frac{\hbar c}{\mu c^2 Z_1 Z_2}$$

Coulomb+short range model

● short range limit $a_B \rightarrow \infty$

$$a_B = \frac{\hbar c}{\mu c^2 Z_1 Z_2}$$

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B} \left[\log(-ia_B k) + \psi \left(1 + \frac{i}{a_B k} \right) \right] = 0$$

$\rightarrow 0$

$$\rightarrow -\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik = 0 \quad \text{short range interaction}$$

● further low-energy limit $r_e \rightarrow 0$

- zero-range theory S. Mochizuki, and Y. Nishida, Phys. Rev. C 110 , 064001 (2024).

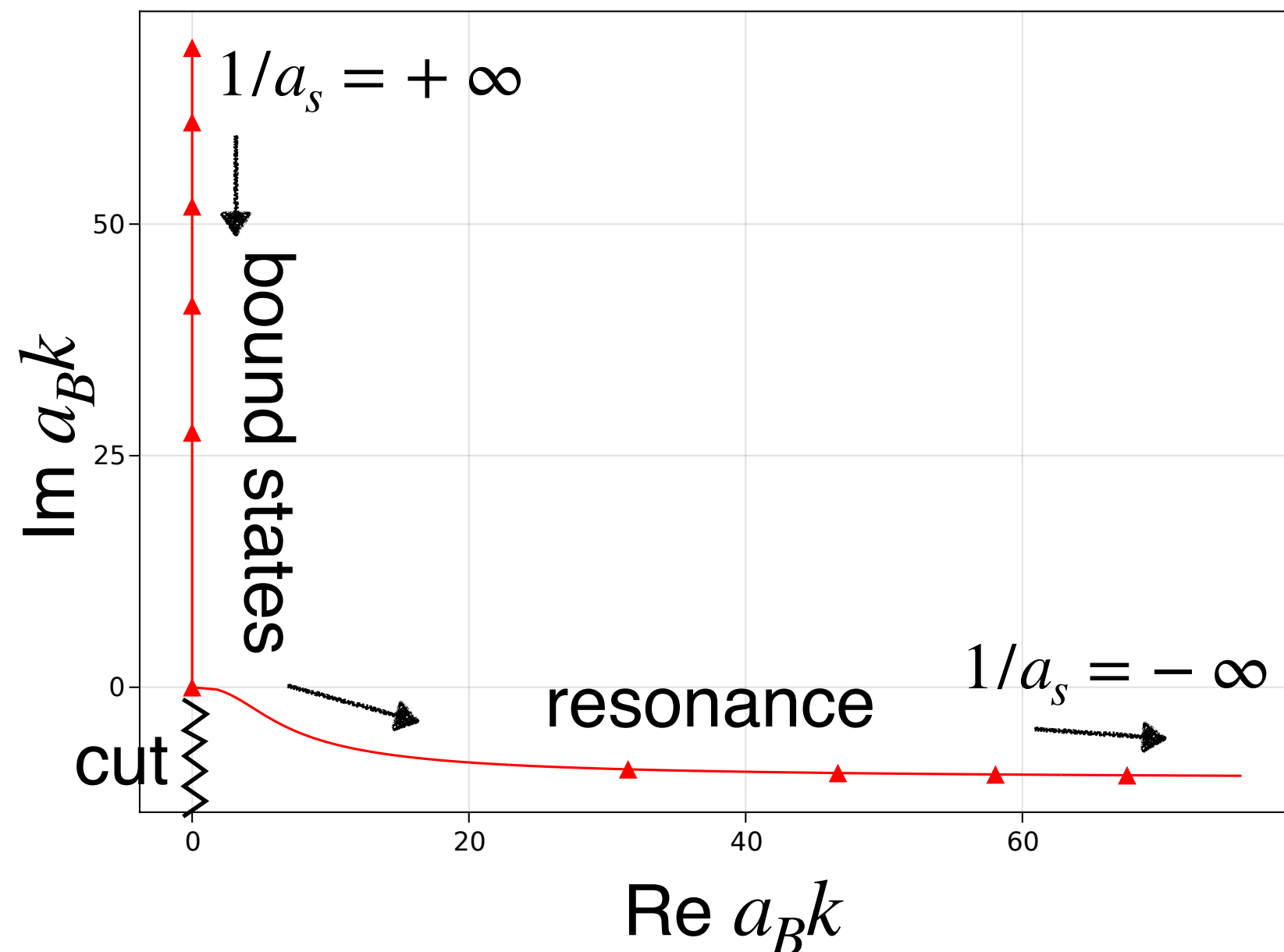
$$\frac{ia_B k}{2} \mp \log(-ia_B k) + \psi \left(1 + \frac{i}{a_B k} \right) + \frac{a_B}{2a_s} = 0$$

Pole trajectory (repulsive Coulomb) ⁵⁴

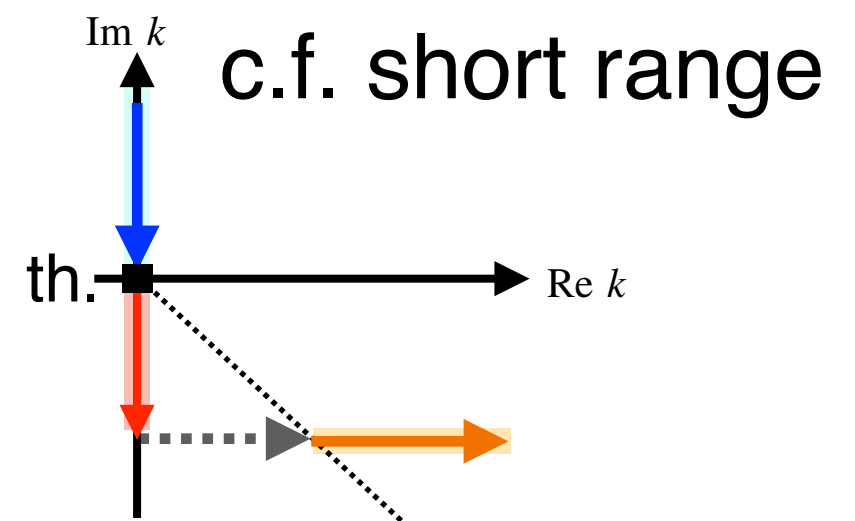
● pole trajectory in complex momentum k plane

- varying Coulomb scattering length a_s with fixed r_e and a_B

→ pole position (eigenmomentum) moves



- b.s. directory goes to resonance



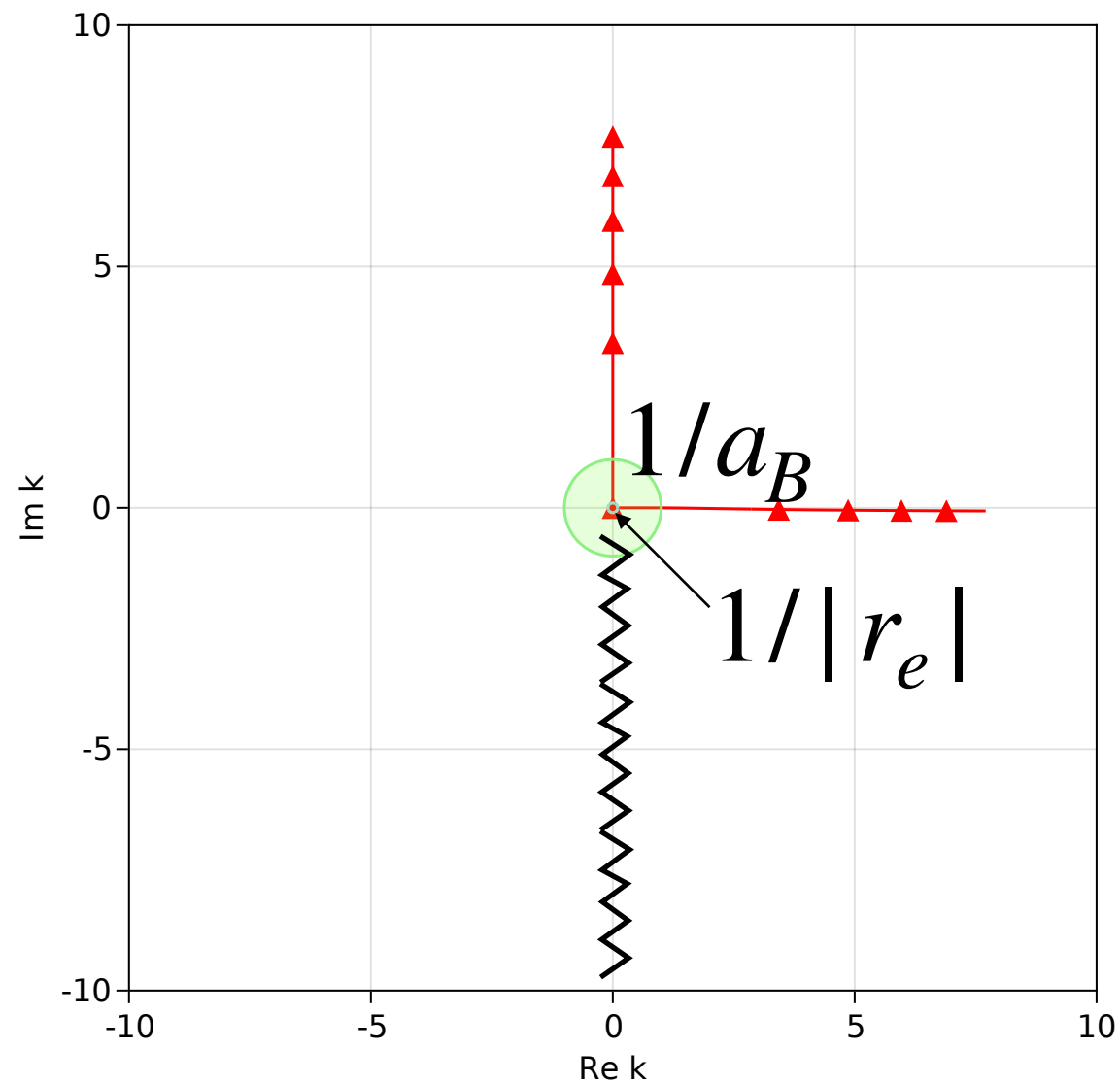
- $a_s \rightarrow \infty$ at threshold

- but no universality

$\therefore$ radius of w.f. $< \infty$

Pole trajectory (repulsive Coulomb) 55

$$a_B = 1, r_e = -10$$

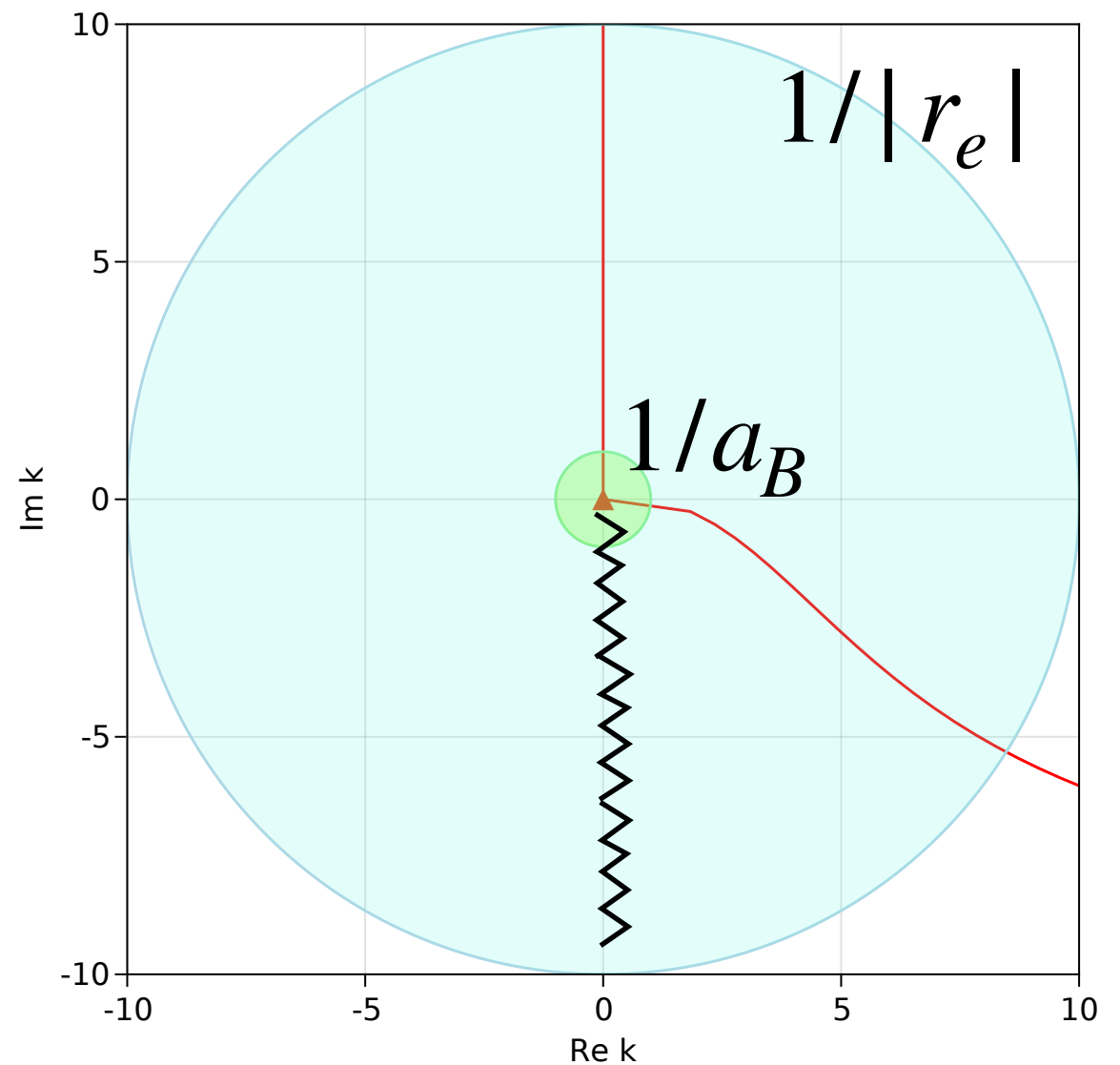


short range universality

$\wedge$

Coulomb dominant region

$$a_B = 1, r_e = -0.1$$



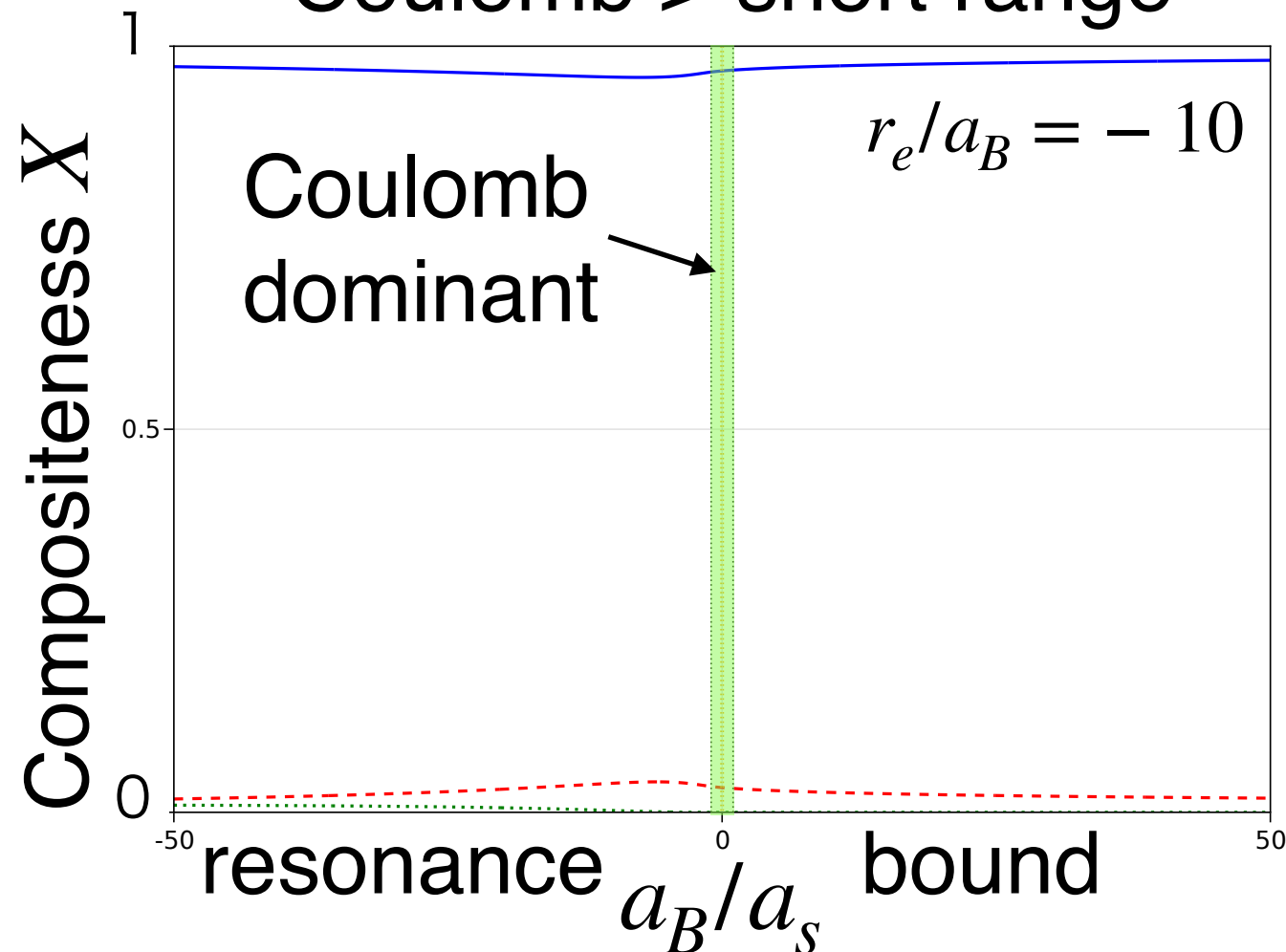
short range universality

$\vee$

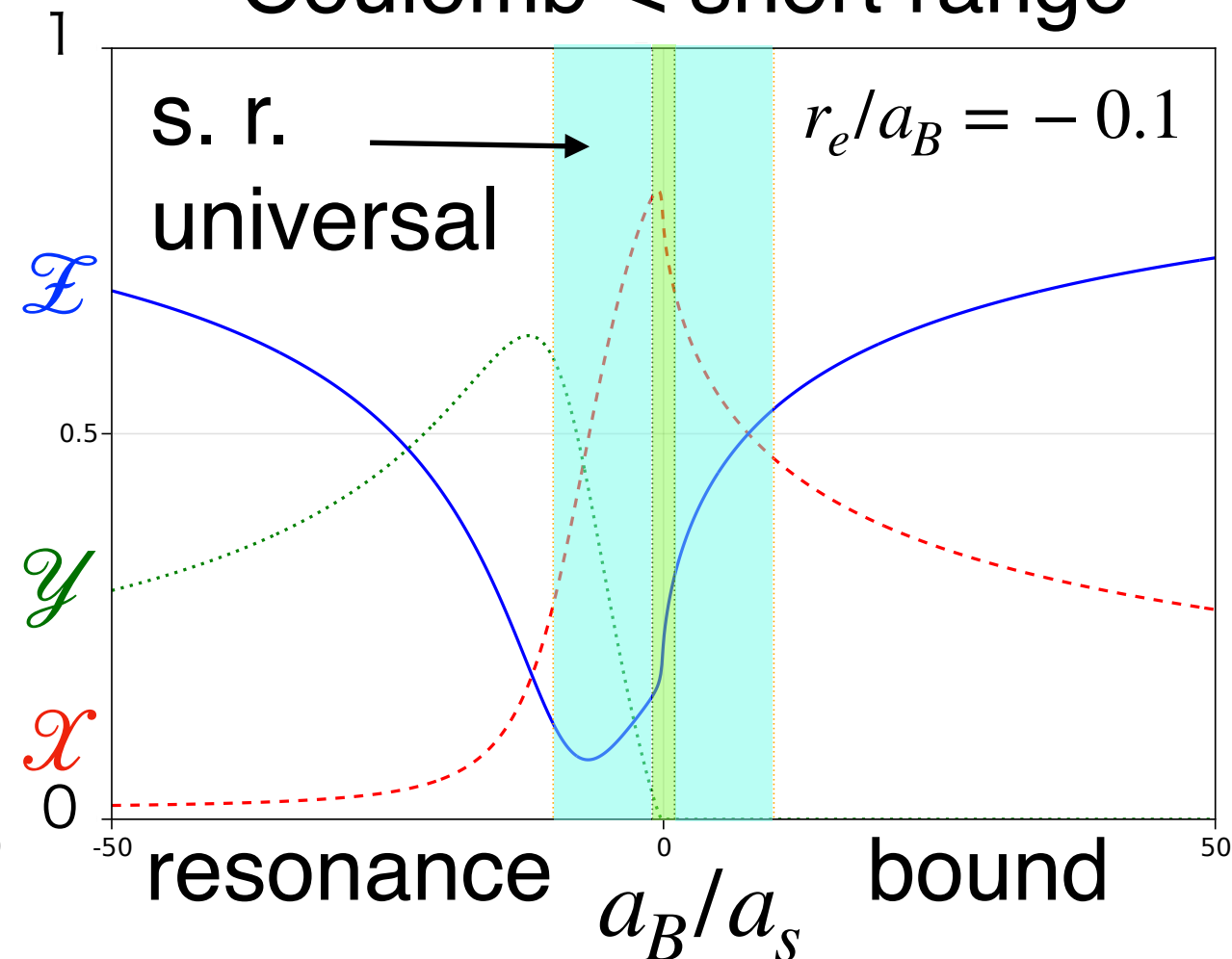
Coulomb dominant region

Compositeness (repulsive Coulomb) 56

Coulomb $>$ short range



Coulomb $<$ short range



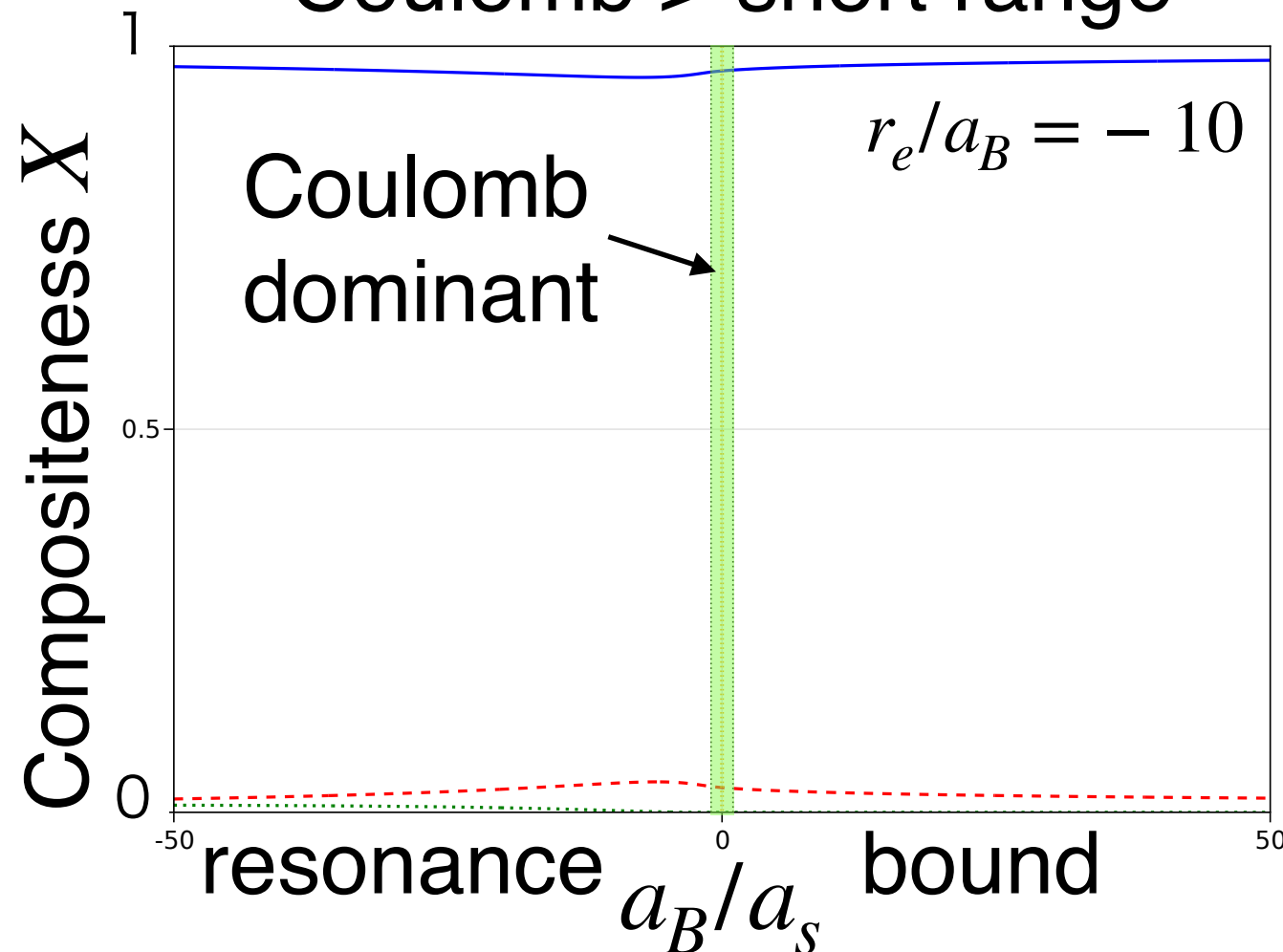
- complex compositeness $\leftarrow \mathcal{X}, \mathcal{Y}, \mathcal{Z}$
- states with large $|1/a_s|$ are elementary $\mathcal{Z}$ dominant
- structure of bound states $\approx$ resonances $\therefore$ continuous X

T. Kinugawa and T. Hyodo,
arXiv:2403.12635 [hep-ph].

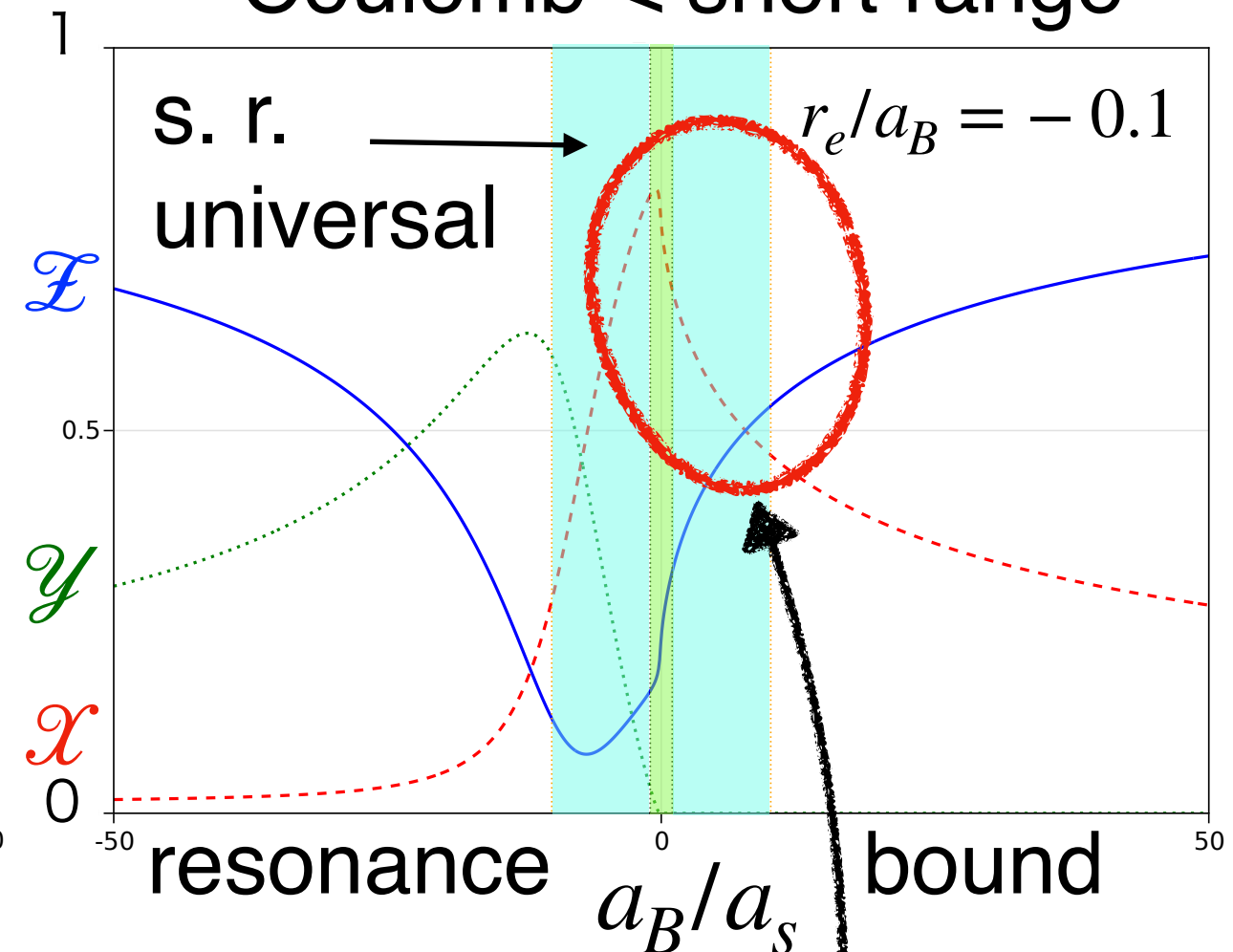
Compositeness (repulsive Coulomb)

56

Coulomb $>$ short range



Coulomb $<$ short range

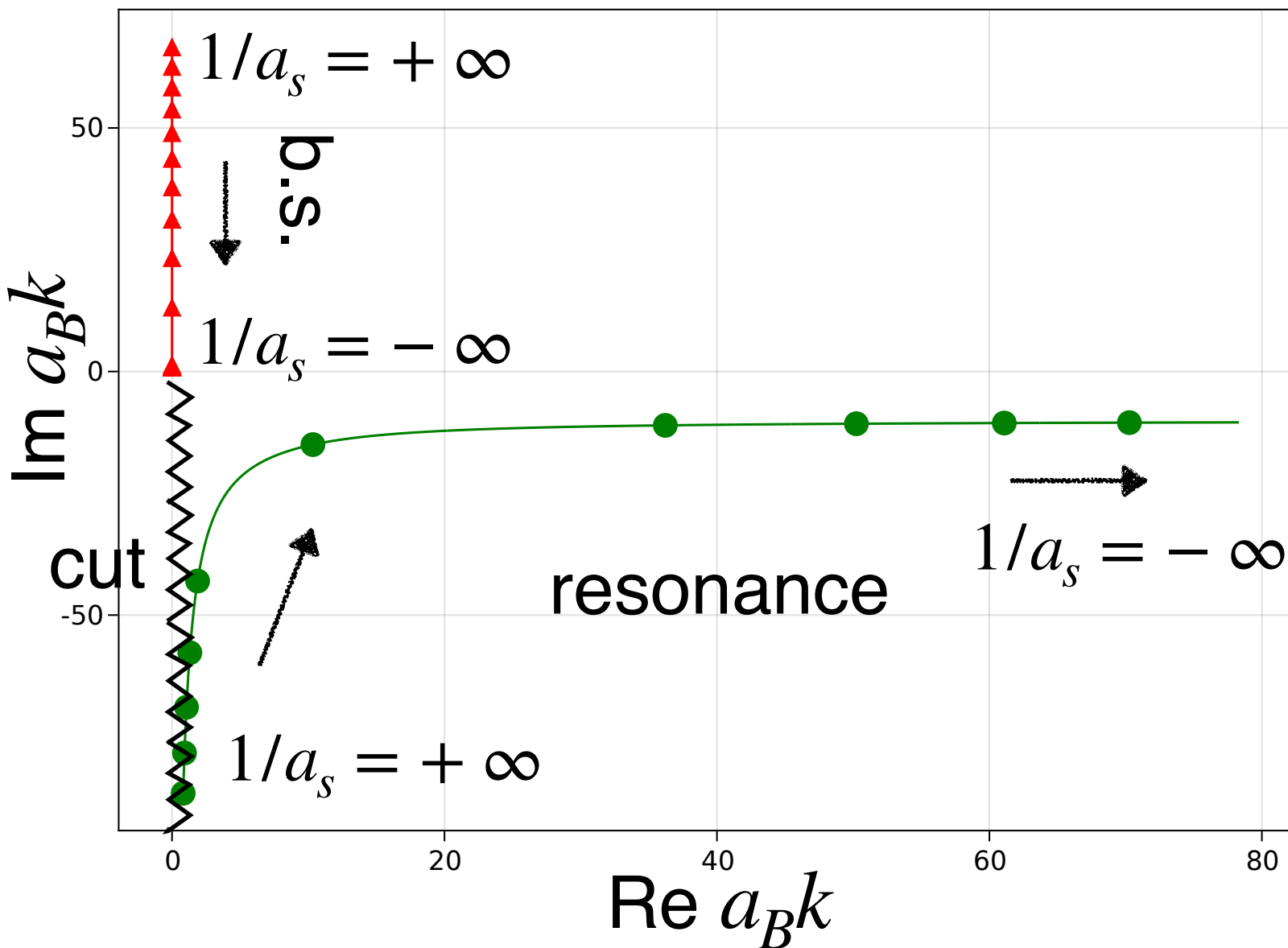


- complex compositeness $\leftarrow X, Y, Z$
- states with large $|1/a_s|$ are elementary Z dominant
- structure of bound states $\approx$ resonances $\because$ continuous X
- remnant of short range universality in $|r_e| \ll |a_B|$ case
 $X \rightarrow 1$ in $B \rightarrow 0$ limit in short range

T. Kinugawa and T. Hyodo,
arXiv:2403.12635 [hep-ph].

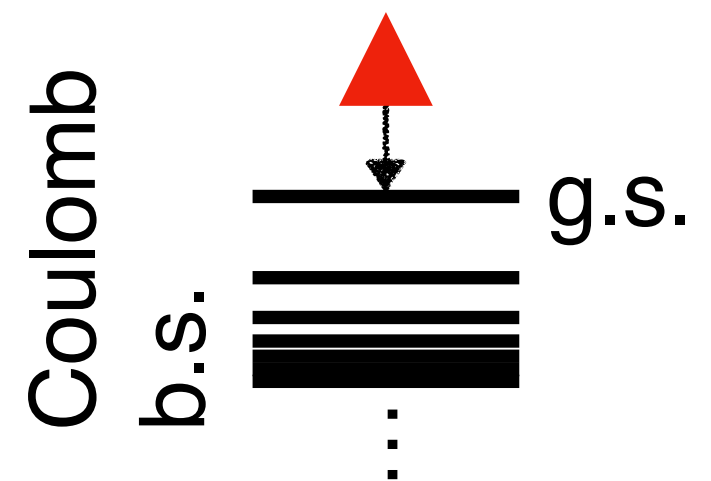
Pole trajectory (attractive Coulomb)⁵⁷

● pole trajectory in complex momentum k plane



- bound $\neq$ resonance

- bound pole
→ Coulomb g.s.

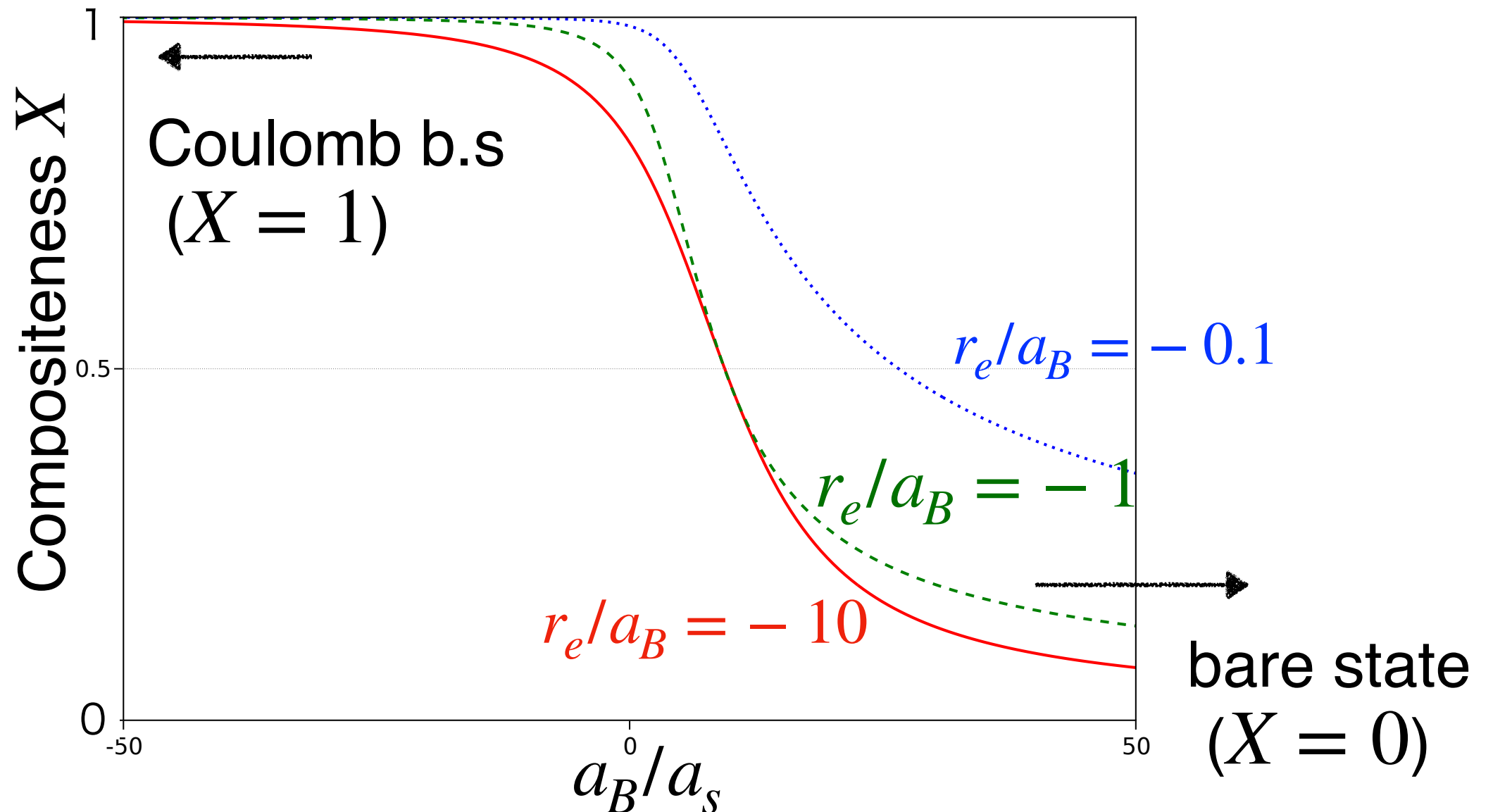


- pole cannot go to $k = 0$

W. Domcke, Atom. Mol. Phys. 16 359 (1983);

S. Mochizuki, and Y. Nishida,
Phys. Rev. C 110 , 064001 (2024).

Compositeness (att. Coulomb **b.s.**) 58



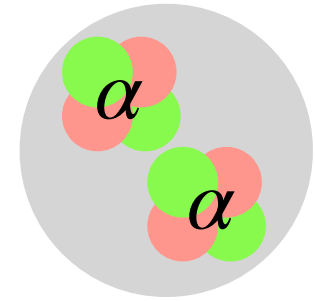
- $1/a_s \rightarrow +\infty$: states becomes elementary dominant ($X \rightarrow 0$)
- no short range universality but $X \rightarrow 1$ in $B \rightarrow B_{\text{Coulomb g.s.}}$ limit
 $\because$ Coulomb g.s. has no bare state contribution (i.e. $X = 1$)
- Coulomb < short range ($r_e = -0.1$) : remnant of universality

Summary so far

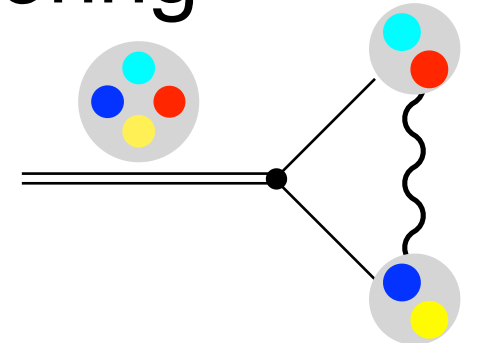
59



near-threshold bound states & resonances
with **Coulomb + short range** interaction



- bare state which couples to Coulomb scattering
- pole condition $\leftarrow a_s, r_e, a_B$



- repulsive Coulomb
bound $\rightarrow$ resonance (does not become virtual states)
 X is not necessary to be unity at threshold
if Coulomb $<$ s.r., remnant of s.r. universality can be seen
structure of b.s. $\approx$ nature of resonance

- attractive Coulomb
bound $\rightarrow$ Coulomb g.s. & virtual $\rightarrow$ resonance
 $X \rightarrow 1$ for bound state