Compositeness of near-threshold s-wave resonances

T. Kinugawa and T. Hyodo
Phys. Rev. C 109 , 045205 (2024).

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

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Jun 28th

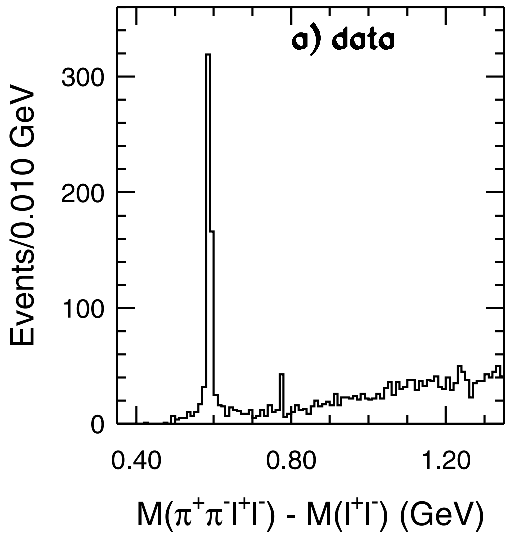
Hadron interactions with strangeness and charm

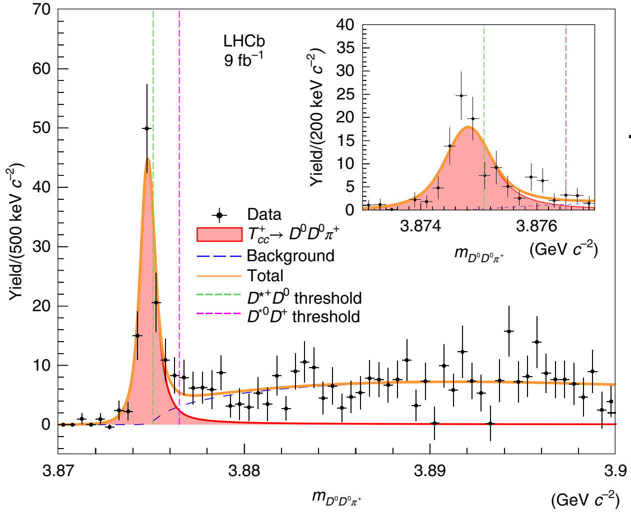
Background

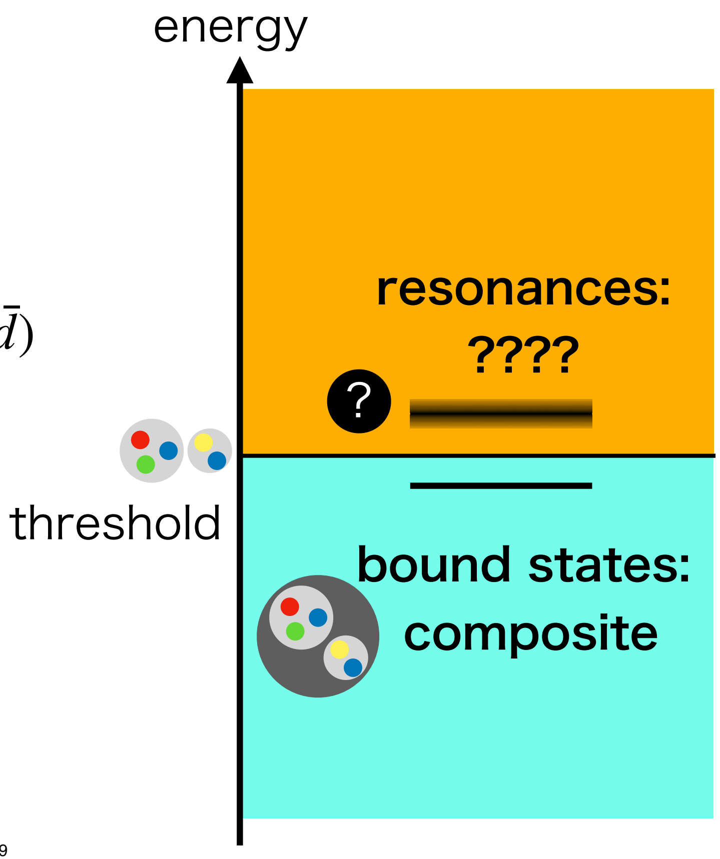
Near-threshold states are interesting!

- exotic hadrons

$X(3872) \to \pi^+\pi^- J/\psi$ $\quad$ $T_{cc}(3875)^+ \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$







S. K. Choi *et al*. (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;
LHCb Collaboration, Nat. Commun. **13**, 3351 (2022).

- shallow **bound** states are molecular dominant

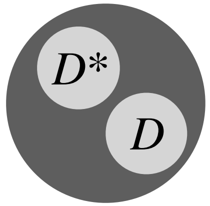
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014);
T. Kinugawa, T. Hyodo, 2303.07038 [hep-ph].

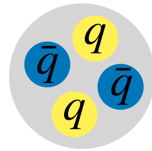
- structure of **resonances** slightly above threshold?

Compositeness & this talk

◉ definition

hadron wavefunction





$$|\Psi\rangle = \sqrt{X}\,|\text{molecule}\rangle + \sqrt{Z}\,|\text{non molecule}\rangle$$

compositeness **elementarity**

- **quantitative** analysis of internal structure of bound states

$X > 0.5 \Leftrightarrow$ **composite dominant**

$Z > 0.5 \Leftrightarrow$ **elementary dominant**

Weinberg, S. Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
Y. Li, F.-K. Guo, J.-Y. Pang, and J.-J. Wu, Phys. Rev. D 105, L071502 (2022);
T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022).

◉ this talk

internal structure of near-threshold s-wave bound/resonance?

1. structure of shallow bound states
   T. Kinugawa and T. Hyodo Phys. Rev. C 109 , 045205 (2024).
2. structure of near-threshold resonances
   T. Kinugawa and T. Hyodo arXiv:2403.12635 [hep-ph].

Bound state with EFT model

◉ single-channel resonance model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).

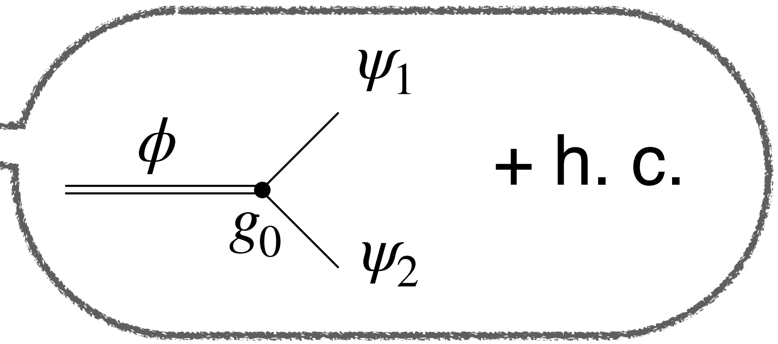
T. Kinugawa and T. Hyodo
Phys. Rev. C 109 , 045205 (2024).

$$\mathscr{H}_{\mathrm{free}} = \frac{1}{2m_1}\nabla\psi_1{}^{\dagger}\cdot\nabla\psi_1 + \frac{1}{2m_2}\nabla\psi_2{}^{\dagger}\cdot\nabla\psi_2 + \frac{1}{2m_\phi}\nabla\phi^{\dagger}\cdot\nabla\phi + \nu_0\phi^{\dagger}\phi,$$

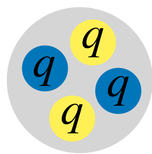
1.

$$\mathscr{H}_{\mathrm{int}} = g_0(\phi^{\dagger}\psi_1\psi_2 + \psi_1{}^{\dagger}\psi_2^{\dagger}\phi)\,.$$

2.



1. single-channel scattering
2. coupling to bare state ϕ



◉ scattering amplitude

$$V = \frac{g_0^2}{E-\nu_0}, \quad G = -\frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right].$$

Λ : cutoff

$$\xrightarrow[T=\frac{1}{V^{-1}-G}]{} \quad f(k) = -\frac{\mu}{2\pi}\left[\frac{\frac{k^2}{2\mu}-\nu_0}{g_0^2} + \frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right]\right]^{-1}.$$

Model scales and parameters

◉ calculation of compositeness with given B

- typical energy scale : $E_{\rm typ} = \Lambda^2/(2\mu)$

1. reduce d.o.f. of model parameters g_0, ν_0, Λ

coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \frac{\pi^2}{\mu}(B + \nu_0)\left[\Lambda - \kappa \arctan(\Lambda/\kappa)\right]^{-1}$

$\kappa = \sqrt{2\mu B}$.

2. use dimensionless quantities with Λ

⟶ absorb Λ dependence

3. energy of bare quark state ν_0

varied in the region : $-B/E_{\rm typ} \leq \nu_0/E_{\rm typ} \leq 1$

∵ to have $g_0^2 \geq 0$ & applicable limit of model

Compositeness in model

◉ compositeness X

scattering amplitude : $T = \dfrac{1}{V^{-1} - G}$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

$$\longrightarrow X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1+\left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}.$$

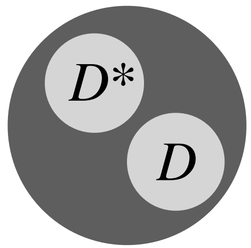
- fix B to consider typical and shallow bound states

- ν_0 region : $-B/E_{\text{typ}} \le \nu_0/E_{\text{typ}} \le 1$

⟷ model dependence of X

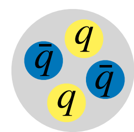
compositeness X as a function of ν_0

$X > 0.5$



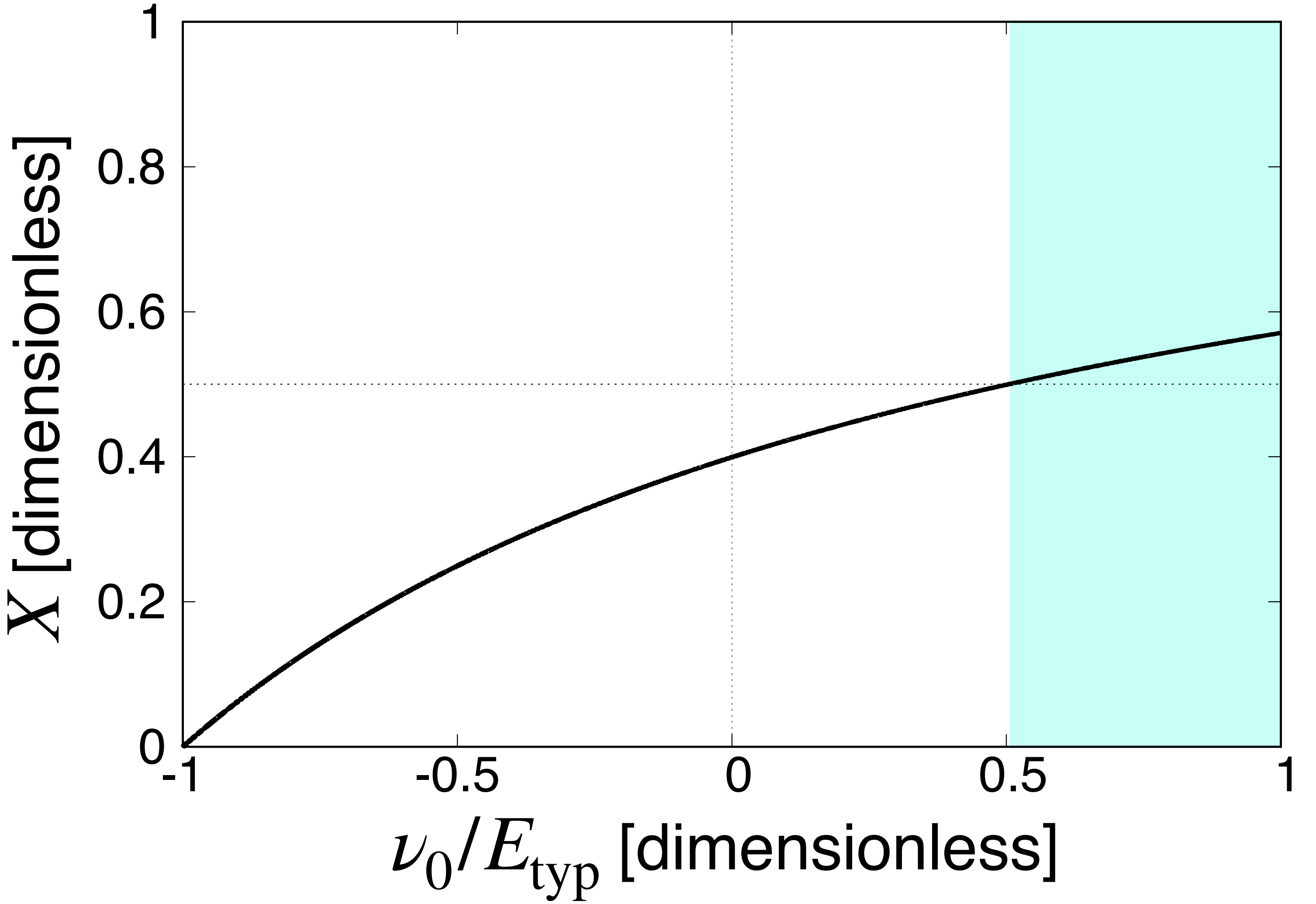
or

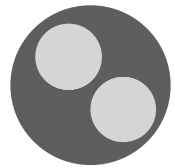
$X < 0.5$

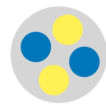


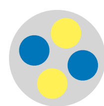
⟶ model dependence of structure of bound state?

◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = E_{\rm typ}$

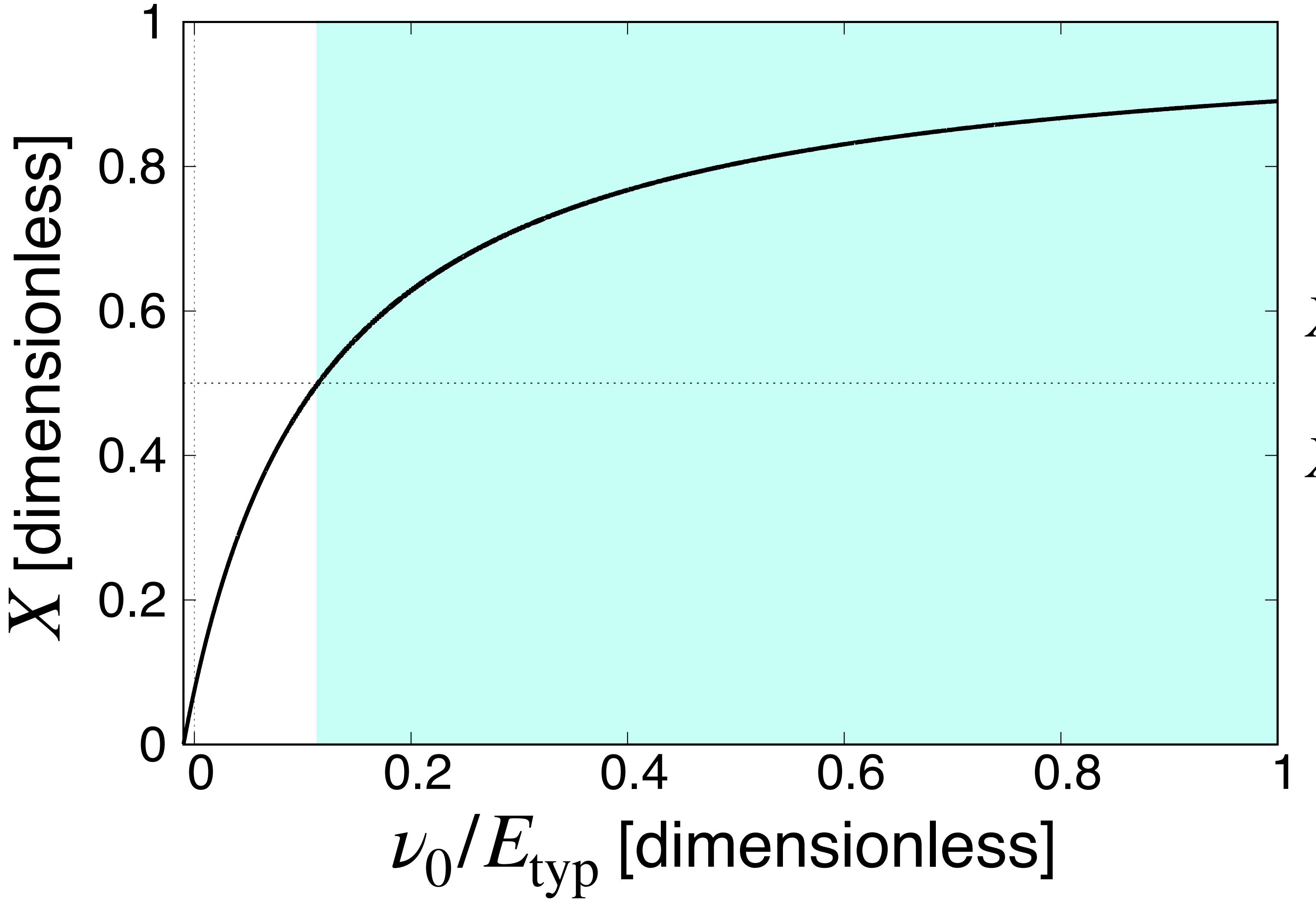


$X > 0.5$: 

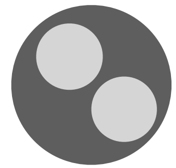
$X < 0.5$: 

- typical energy scale : $B = E_{\rm typ} = \Lambda^2/(2\mu)$
- $X > 0.5$ only for $25\ \%$ of ν_0 $\because$ **bare state origin** 

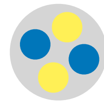
◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = 0.01E_{\rm typ}$



$X > 0.5$:



$X < 0.5$:



- weakly-bound state : $B = 0.01E_{\rm typ}$
- $X > 0.5$ for $88\ \%$ of ν_0 ⟶ **realization of universality**
- elementary dominant state can be realized with fine tuning

Near-th. resonances in ERE

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

◉ resonance pole written by effective range expansion (ERE)

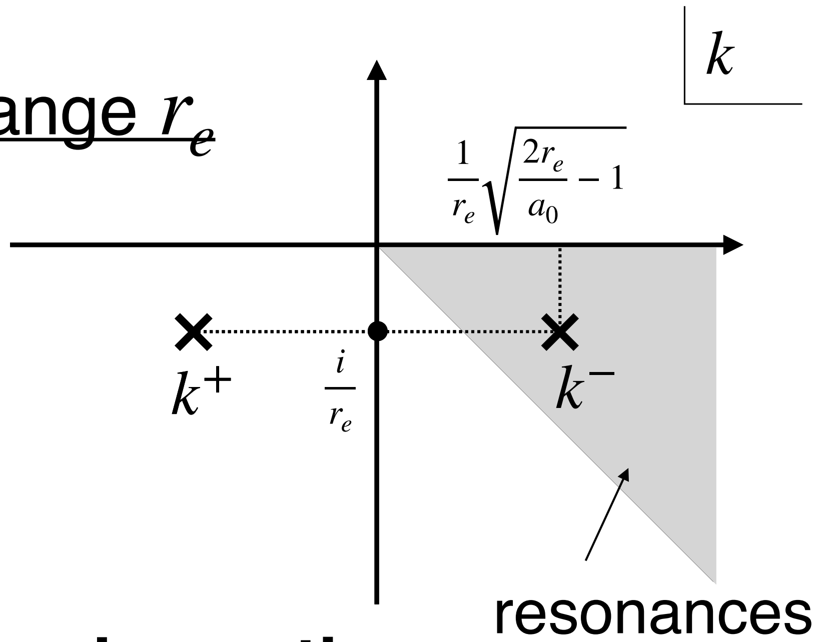
$$f(k)^{-1} = -\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik \longrightarrow k^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e}\sqrt{\frac{2r_e}{a_0} - 1}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

- pole position ($k^{\pm}$) ⟷ a_0 and r_e

◉ scattering length a_0 and effective range r_e

- r_e should be negative to obtain resonances
- $|i/r_e|$ should be small to obtain near-threshold poles (narrow width)



→ Effective range should be **large and negative** for near-threshold resonances

Universality for near-th. resonances

- near-threshold **bound** (and virtual) states

$a_0 \to \infty$ and universality holds in $B \to 0$ limit ○

→ $X \to 1$ (completely composite)

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
T. Kinugawa, T. Hyodo, 2303.07038 [hep-ph].

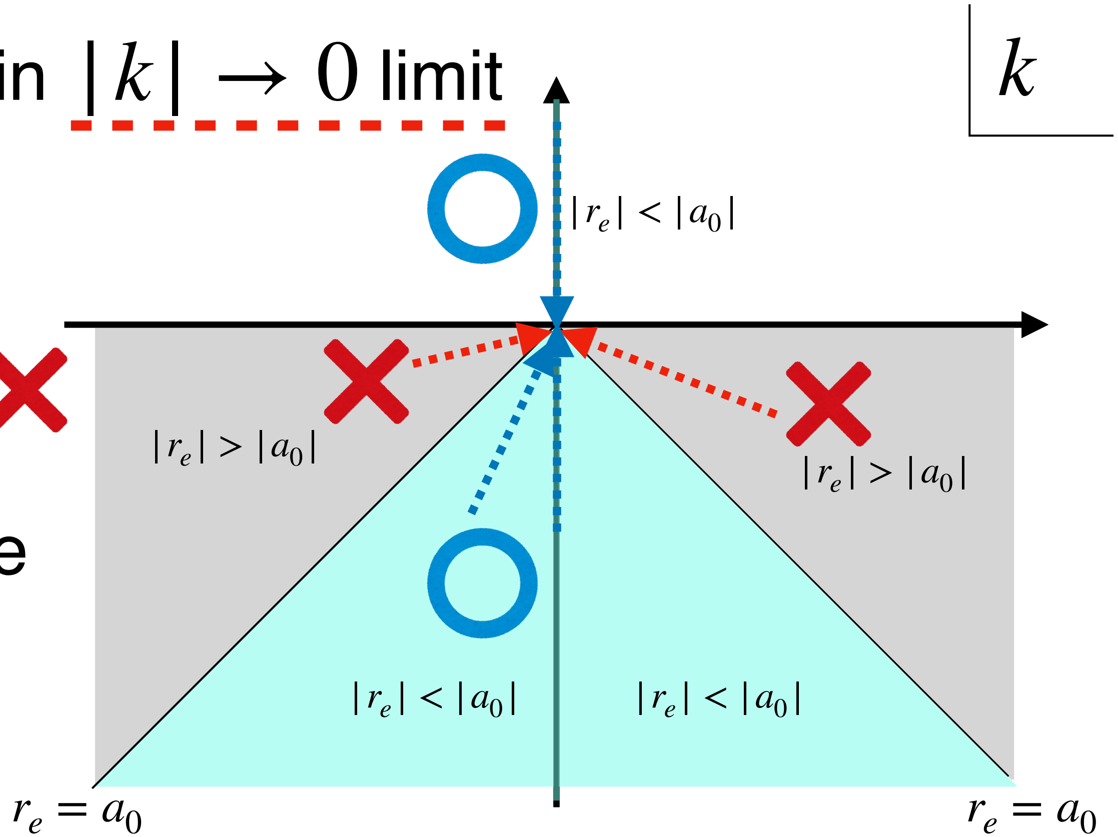
- near-threshold **resonances**

$a_0 \to \infty$ but also $|r_e| \to \infty$ in $|k| \to 0$ limit

$\because |a_0| \le |r_e|$

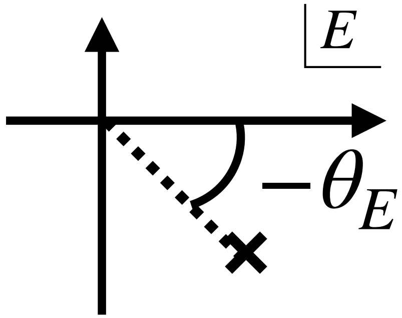
→ universality does not hold ✕

Near-threshold resonances are **not** necessarily composite dominant



Compositeness in ERE

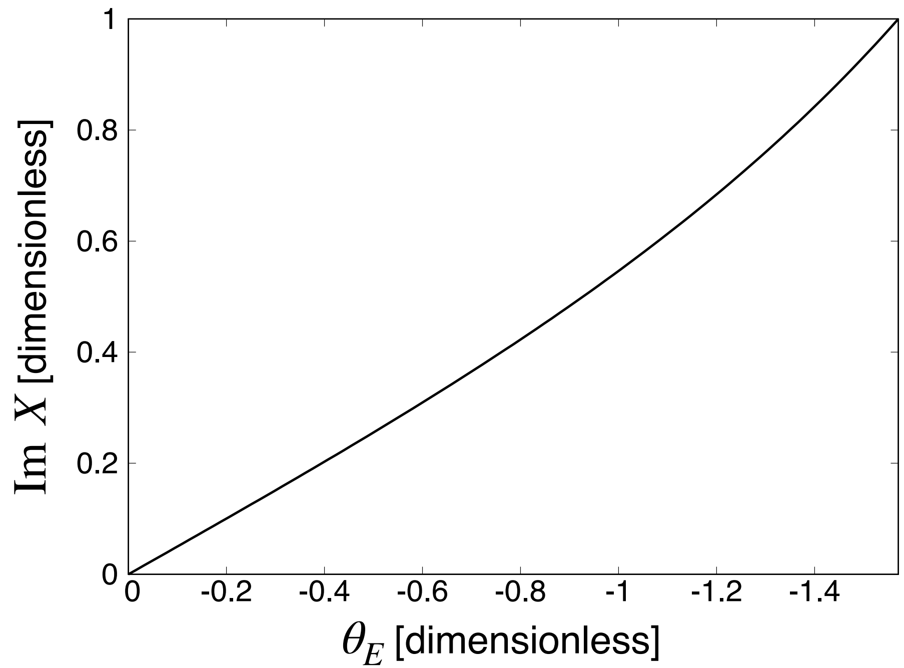
$$X = \sqrt{\frac{1}{1-\frac{2r_e}{a_0}}} = -i\tan\theta_k = -i\tan(\theta_E/2)$$



$$(k = |k|e^{i\theta_k}, E = |E|e^{i\theta_E})$$

→ X in ERE is pure imaginary

Im X as a function of θ_E



- in general, compositeness X of unstable resonances becomes **complex** by definition

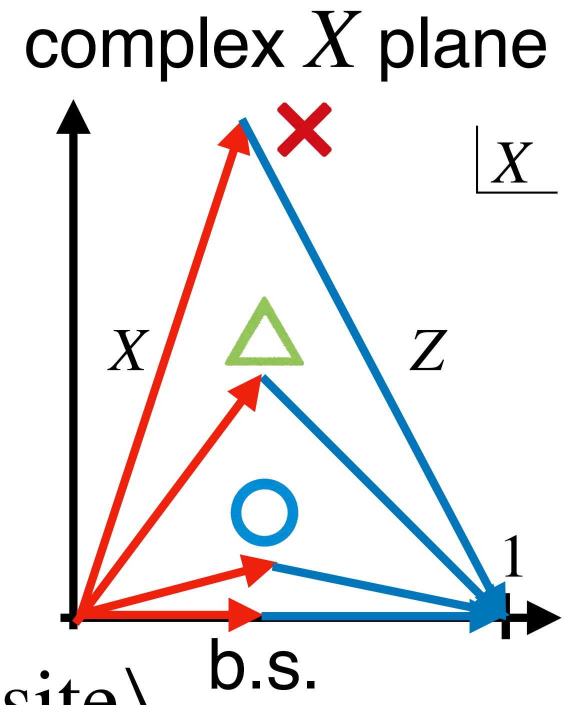
- complex X **cannot** be directly interpreted as a probability

Complex compositeness

- probabilistic interpretation?

  $X \in \mathbb{C}$ and $X + Z = 1$

- If Im X is large, it seems that reasonable interpretation is impossible ✕ △

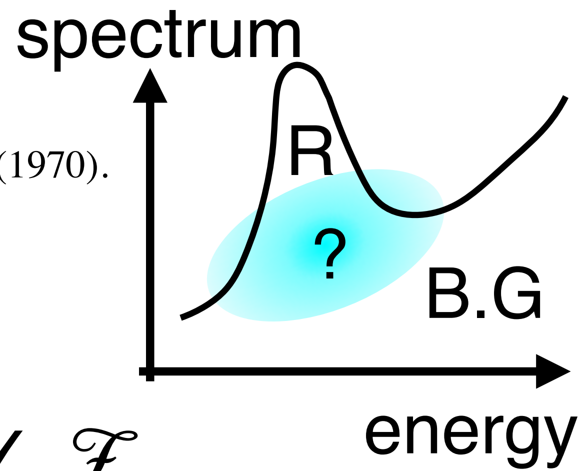


- our proposal

i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$
ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$
iii) $\mathscr{Y}$: probability of uncertain identification

uncertain appears from T. Berggren, Phys. Lett. B 33, 547 (1970).

- finite lifetime (uncertainty in energy)
- separation from B.G.



complex compositeness $X \in \mathbb{C} \longrightarrow \mathscr{X}, \mathscr{Y}, \mathscr{Z}$

Definition

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

◉ conditions for sensible interpretation

- normalization : $\mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathcal{X} \to X$, $\mathcal{Z} \to Z$ and $\mathcal{Y} \to 0$

$\mathcal{Y}$ characterizes uncertainty of resonance

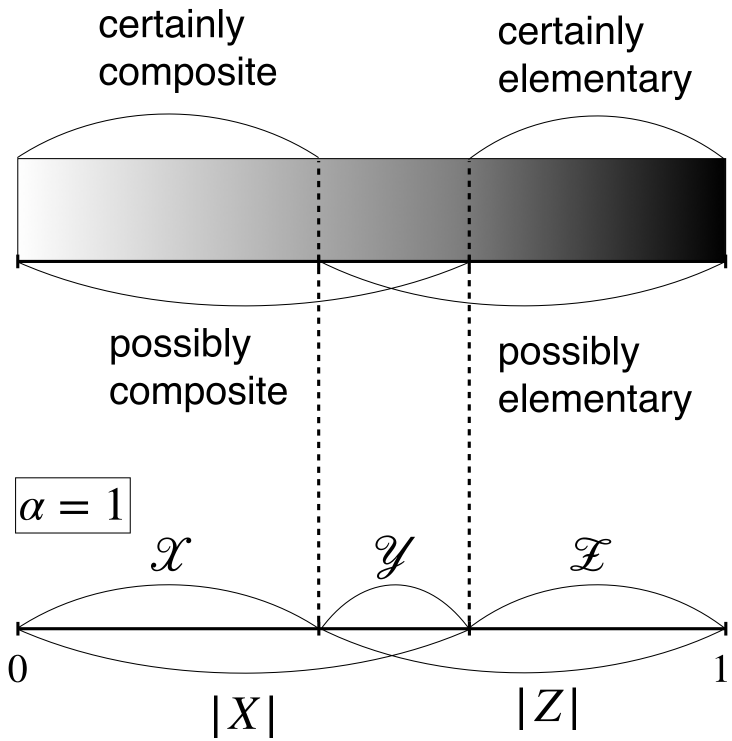
◉ new interpretation

$$\mathcal{X} + \alpha\mathcal{Y} = |X|, \quad \mathcal{Z} + \alpha\mathcal{Y} = |Z|$$

$$\mathcal{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathcal{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$\mathcal{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

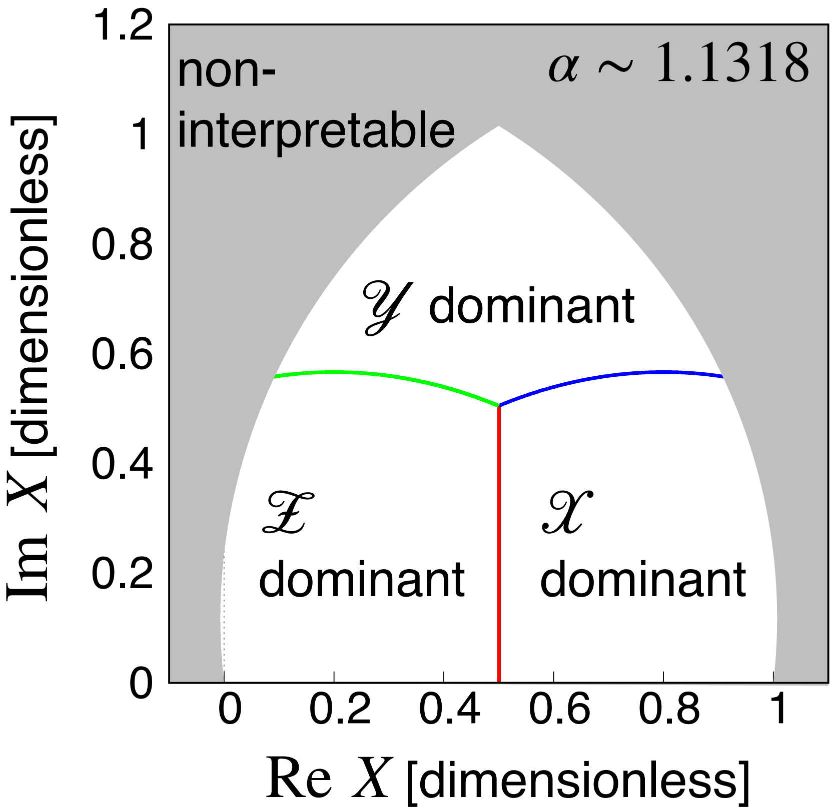


α reflects uncertain nature of resonances

Definition

- if $\alpha > 1/2$, $\mathscr{Y}$ is always positive but $\mathscr{X}$, $\mathscr{Z}$ can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		non-interpretable



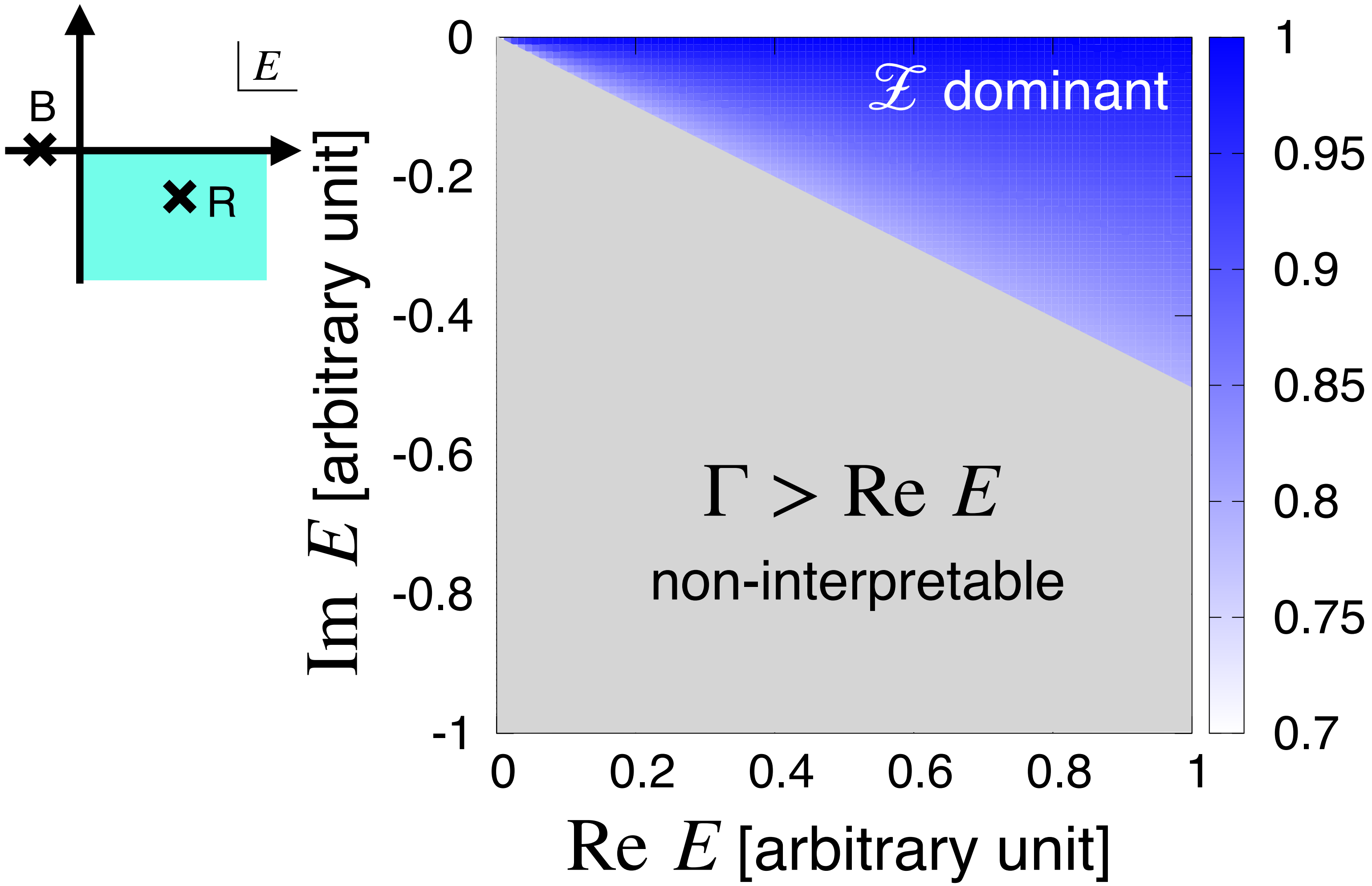
<u>our criterion for physical “state”</u>

$$\Gamma \leq \text{Re } E$$

- exclude poles which we cannot regard as physical “state” from probabilistic interpretation

$\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ dominant regions and non-interpretable region

Structure of near-th. resonances



- resonances are **not composite dominant state** ($\mathcal{Z} \gtrsim 0.8$)
- different from near-threshold bound states (composite dominant $X \sim 1$ and $Z \sim 0$)

resonances with previous works

$$\bar{Z} = 1 - \sqrt{\left|\frac{1}{1-2r_e/a_0}\right|}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

$$\tilde{Z}_{\rm KH} = \frac{1+|Z|-|X|}{2}$$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

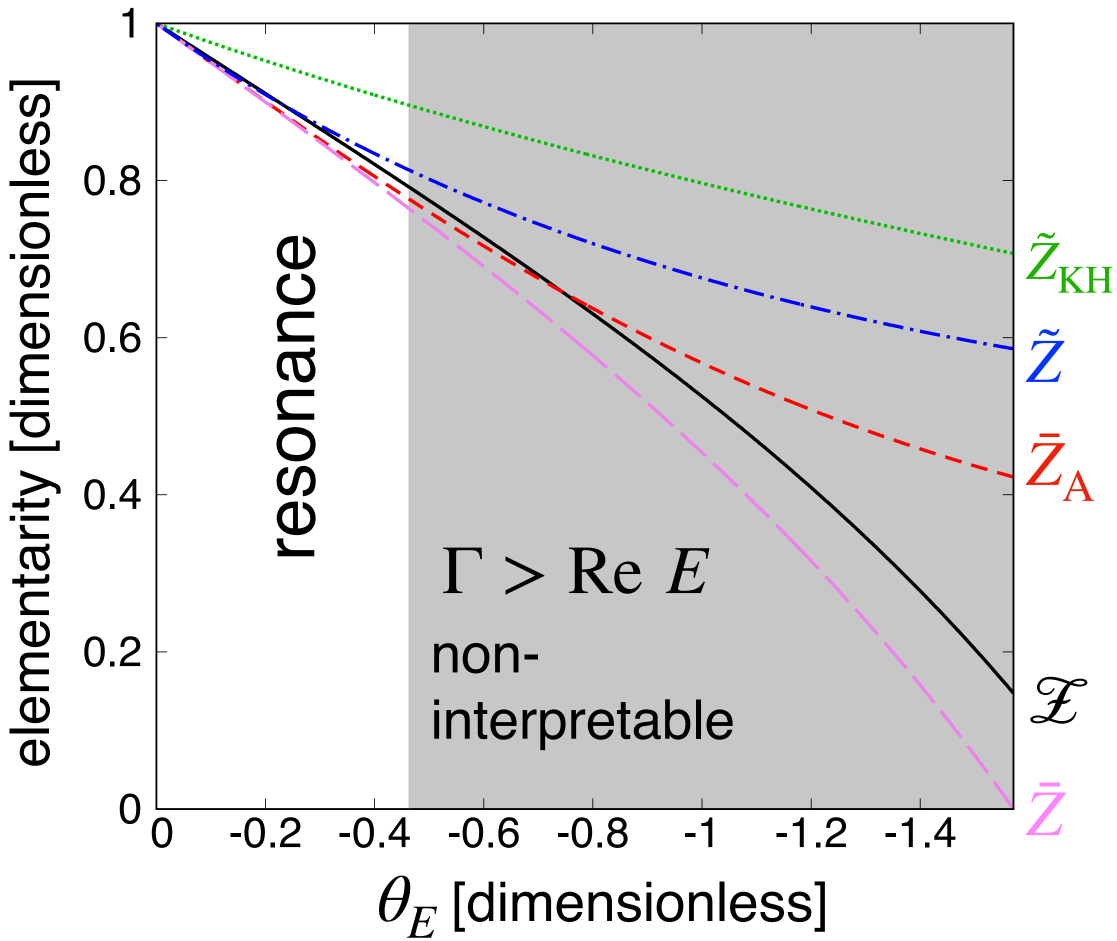
$$\tilde{Z} = \frac{|Z|}{|X|+|Z|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC **93**, 035204 (2016).

$$\bar{Z}_A = 1 - \sqrt{\frac{1}{1+|2r_e/a_0|}}$$

I. Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A **57**, 101 (2021).

interpretations as a function of θ_E



- all interpretations show resonances are elementary dominant

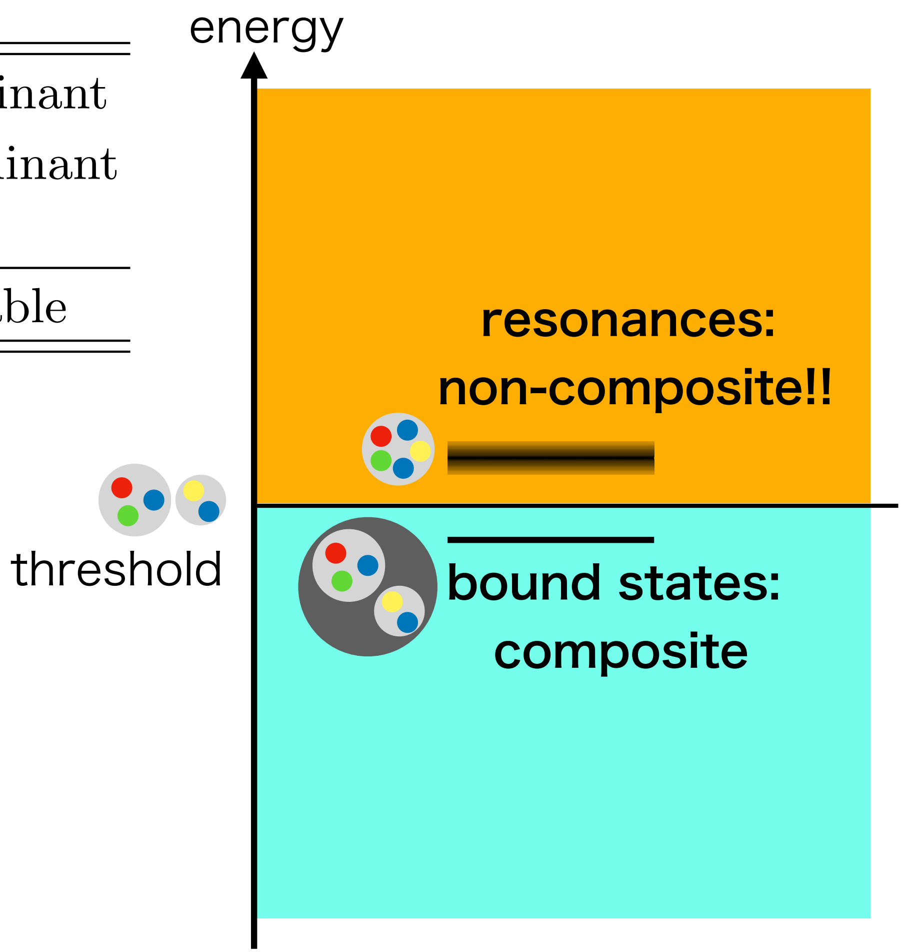
Summary

T. Kinugawa and T. Hyodo
Phys. Rev. C 109 , 045205 (2024).

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

- near-threshold s-wave bound states are composite dominant
- near-threshold s-wave resonances ← ERE
- new interpretation of complex compositeness and elementarity

$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		non-interpretable



- near-threshold resonances are **not composite dominant!**

  qualitatively different from near-threshold bound states

Back up

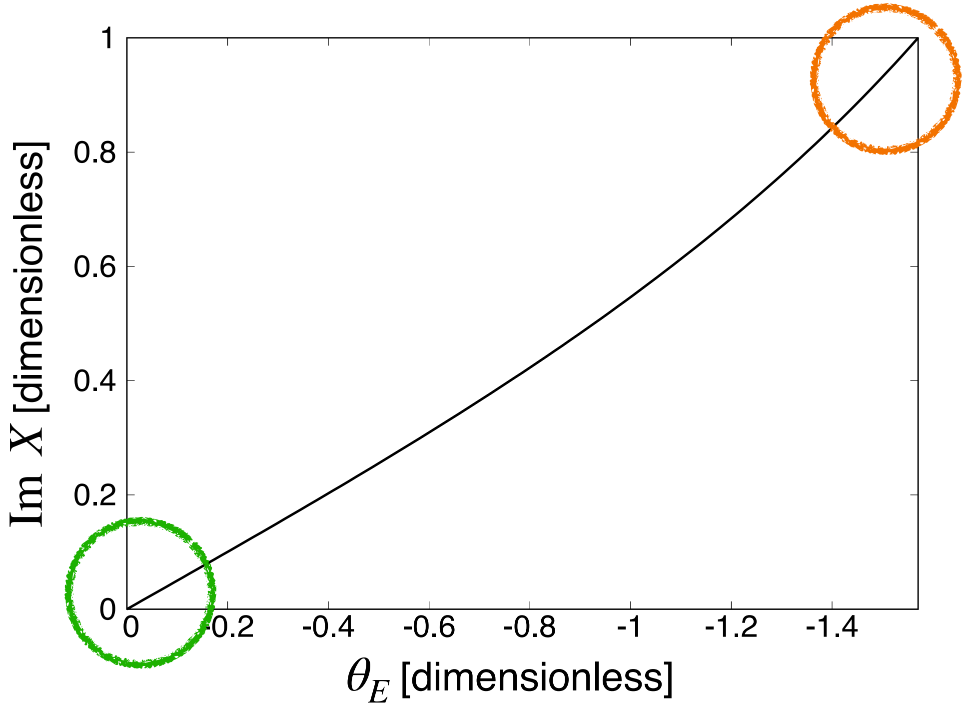
Compositeness in ERE

$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i\tan\theta_k$$

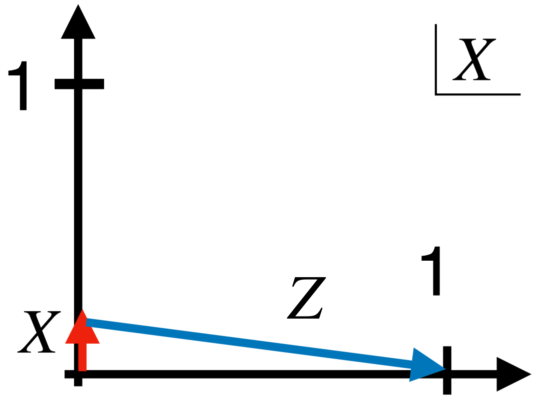
$$(k = |k|e^{i\theta_k})$$

→ X in ERE is pure imaginary

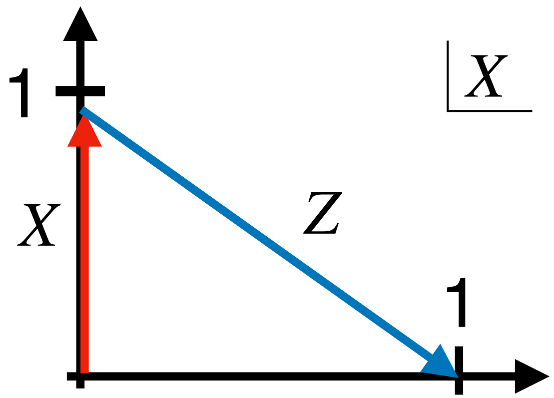
X as a function of θ_E $(E = |E|e^{i\theta_E})$



small width ($\theta_E \sim 0$)

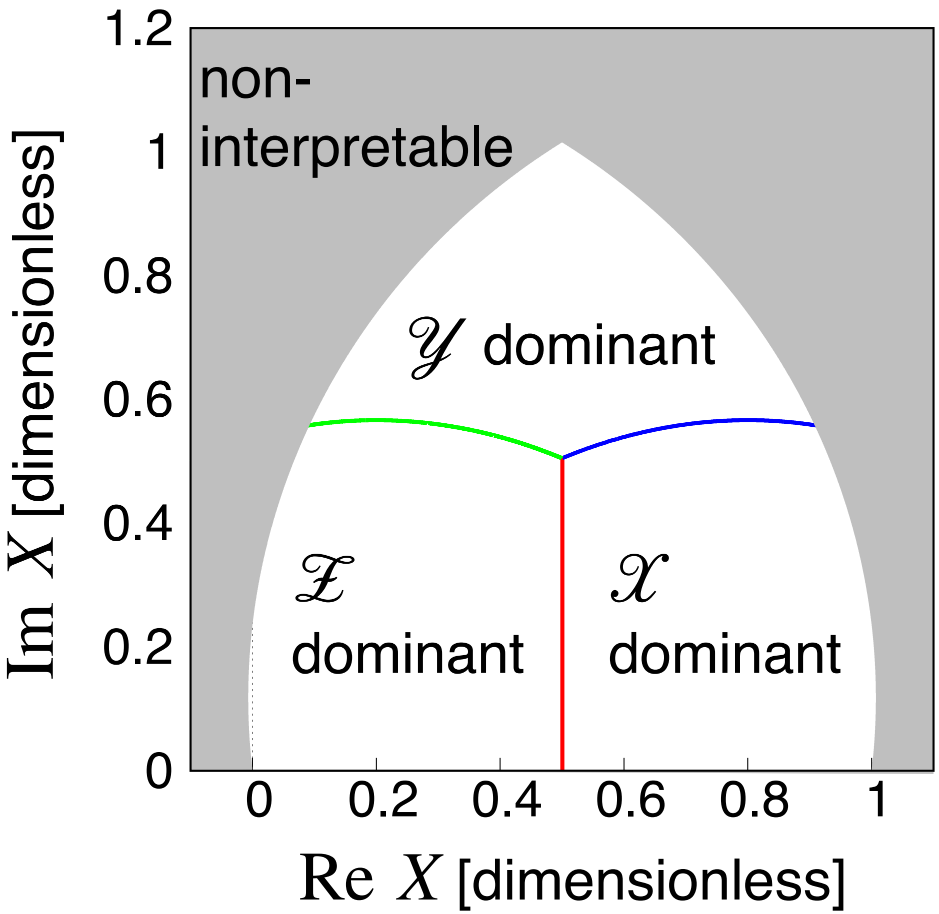


large width ($\theta_E \sim -\pi/2$)



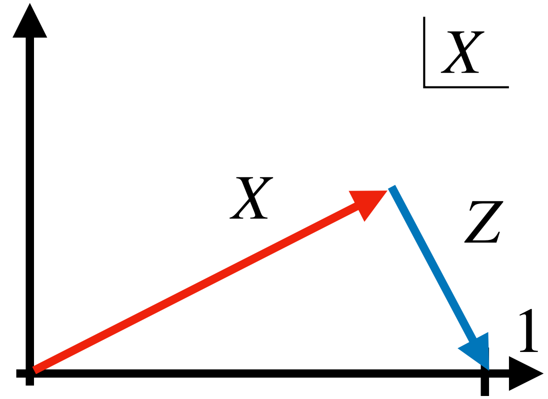
$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region

- $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region in complex X plane

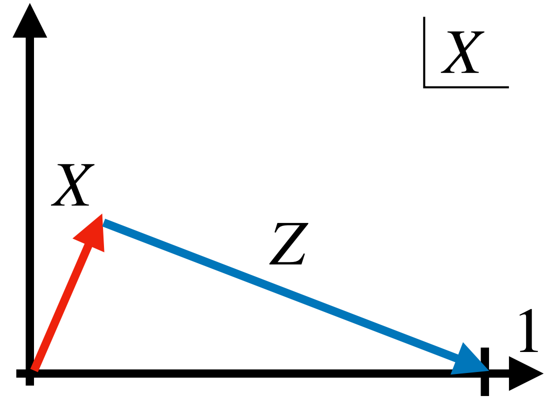


large Im X is assigned to non-interpretable cases

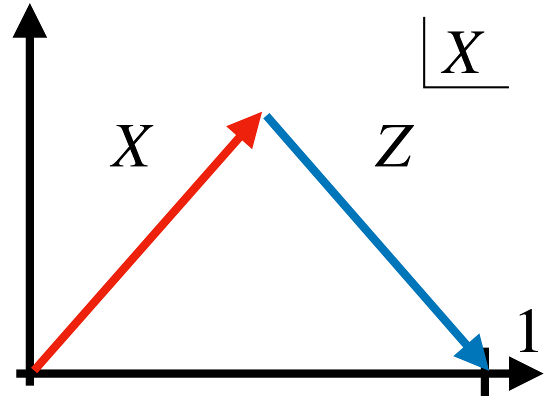
composite dominant



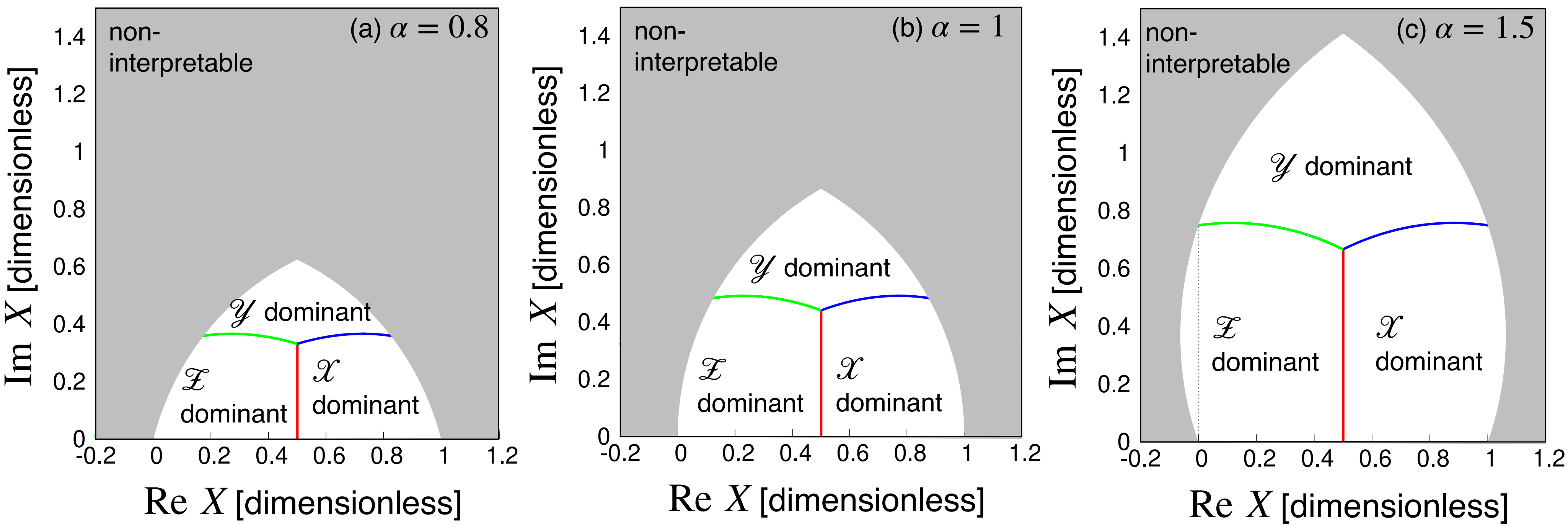
elementary dominant



uncertain



$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region



- Interpretable regions become large with increase of α

$\alpha \to \infty$ ⟶ $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ reduce to interpretation in previous work

$\mathscr{X} \to \tilde{X}, \mathscr{Z} \to \tilde{Z}, \mathscr{Y} \to 0$

Y. Kamiya and T. Hyodo,
Phys. Rev. C **93**, 035203 (2016).

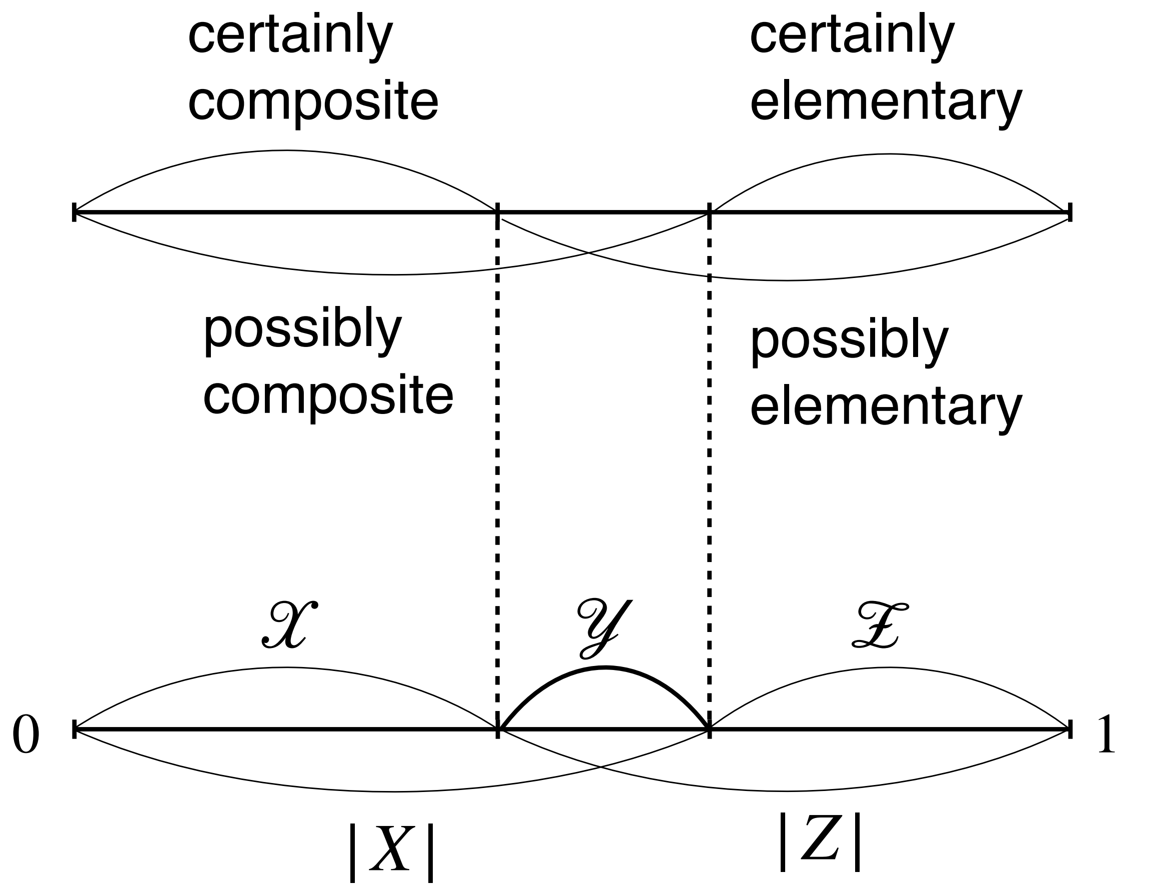
Definition

◉ new interpretation of complex compositeness & elementarity

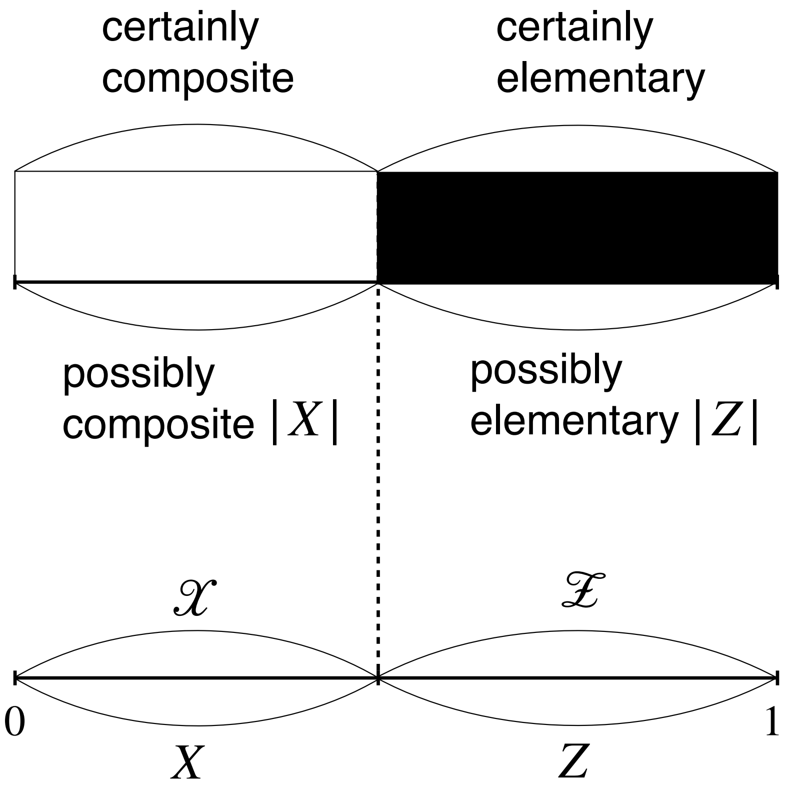
⟶ from Berggren's idea T. Berggren, Phys. Lett. B 33, 547 (1970).

$$\mathscr{X} + \mathscr{Y} = |X| \ \& \ \mathscr{Z} + \mathscr{Y} = |Z|$$

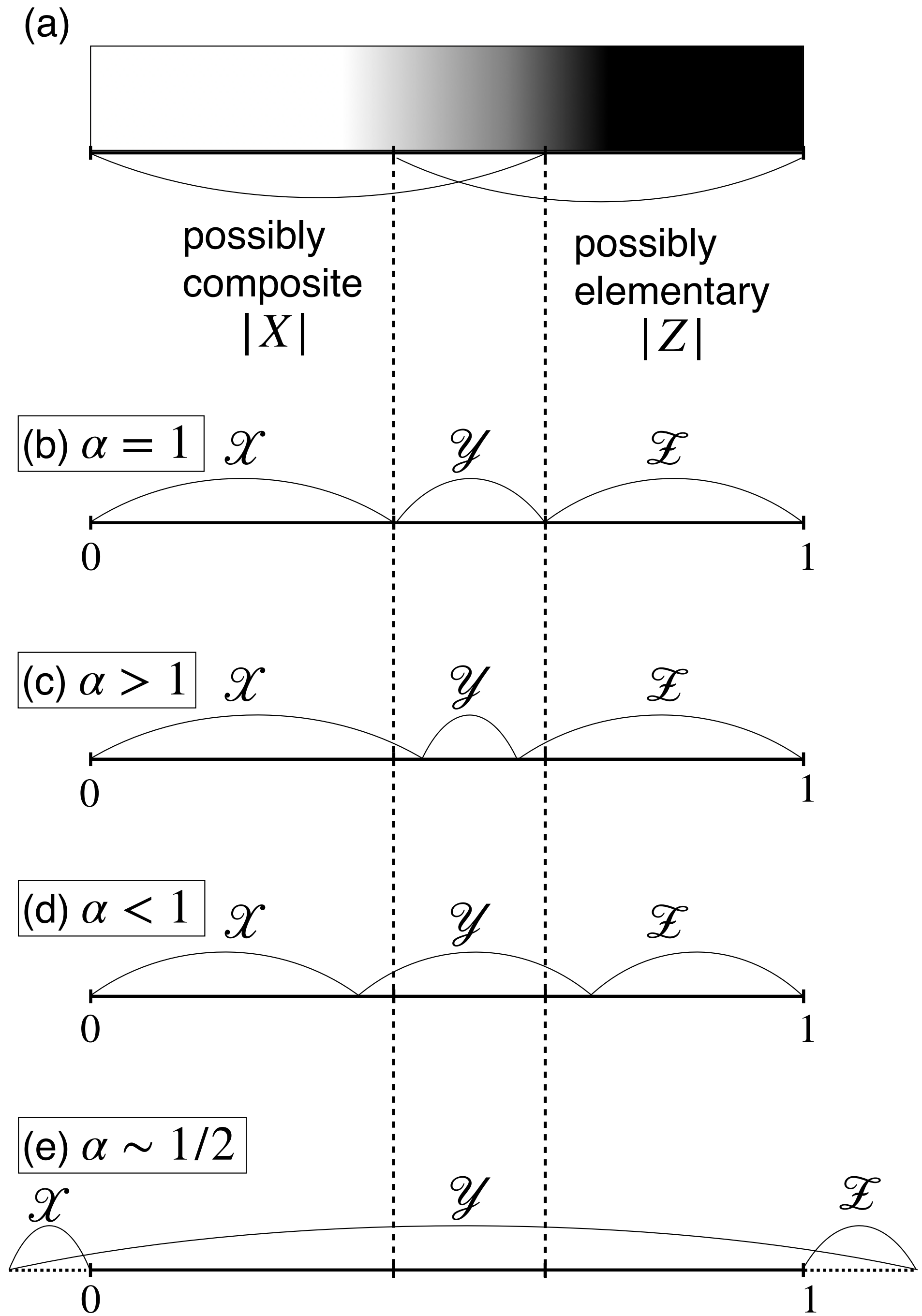
$$\mathscr{X} = 1 - |Z|$$
$$\mathscr{Z} = 1 - |X|$$
$$\mathscr{Y} = |X| + |Z| - 1$$



interpretation



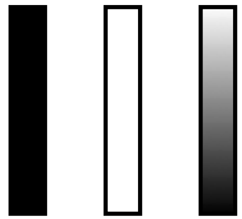
bound state



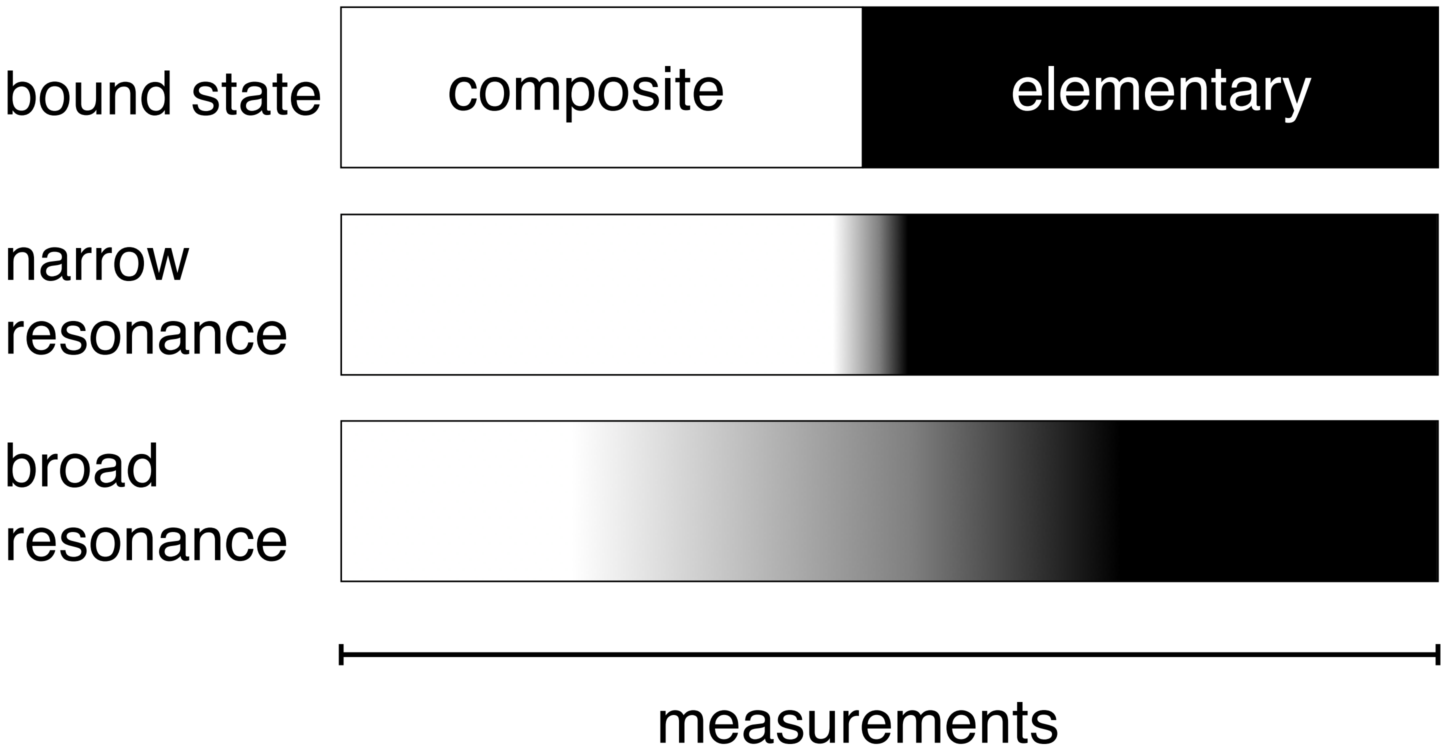
resonance

uncertainty in resonances

a single
measurement



sum of measurements of a bound states / resonances



Structure of near-th. resonances