Compositeness of exotic hadrons with decay and coupled-channel effects

arXiv:2303.07038 [hep-ph]

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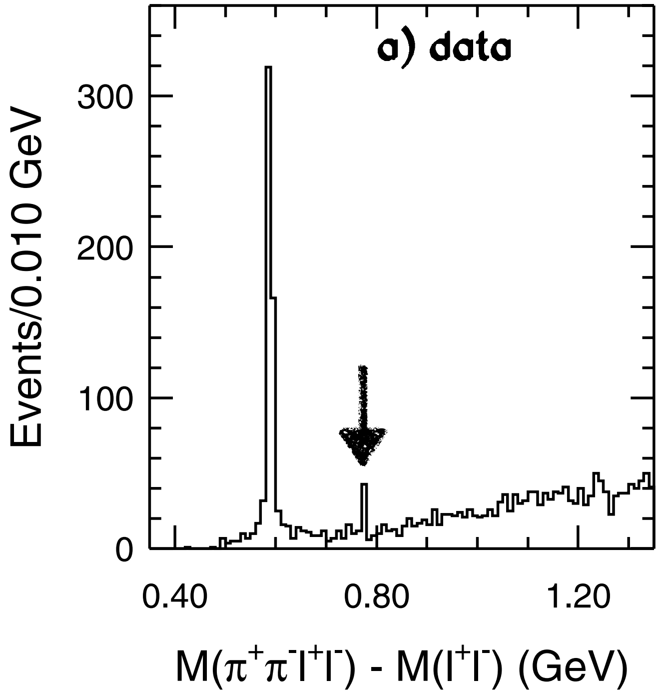
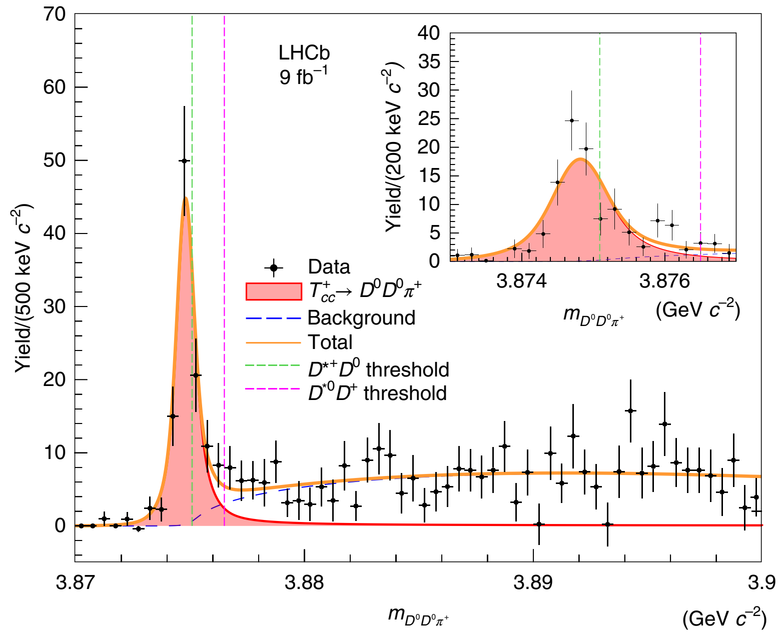
Near-threshold exotic hadrons

$X(3872) \to \pi^+\pi^- J/\psi$



S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

$T_{cc} \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$

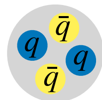


LHCb Collaboration, Nature Phys. **18** no.7, 751-754 (2022);
LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

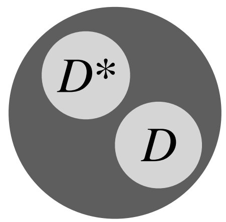
internal structure?

exotic hadron

$\neq$ qqq or $q\bar{q}$ ⟶ multiquarks, hadronic molecules



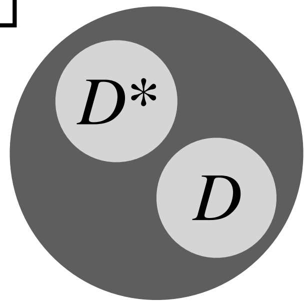
multiquarks

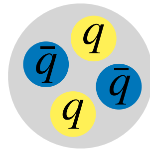


hadronic molecules

Compositeness

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **85**, 015201 (2012);
F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).





◉ definition

hadron wavefunction

$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

compositeness **elementarity**

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).

※ $0 \leq X \leq 1$ ⟶ $X > 0.5 \Leftrightarrow$ **composite dominant**

$X < 0.5 \Leftrightarrow$ **elementary dominant**

- **quantitative** analysis of internal structure

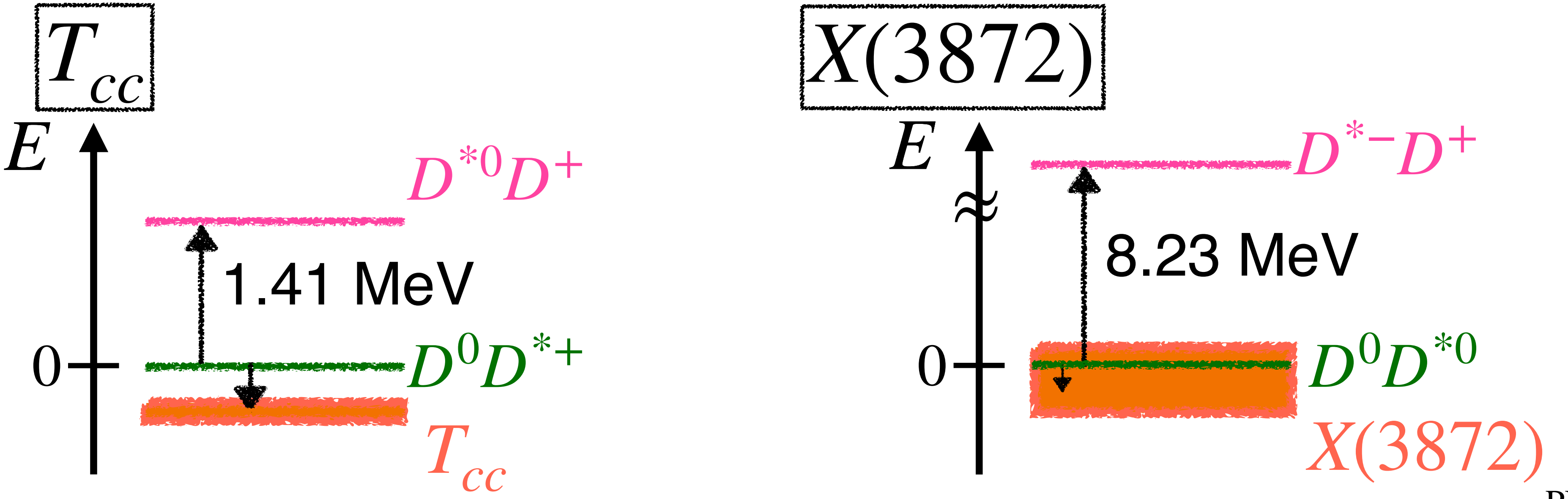
deuteron is not an elementary particle Weinberg, S. Phys. Rev. 137, 672–678 (1965).

$f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016); T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.

$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

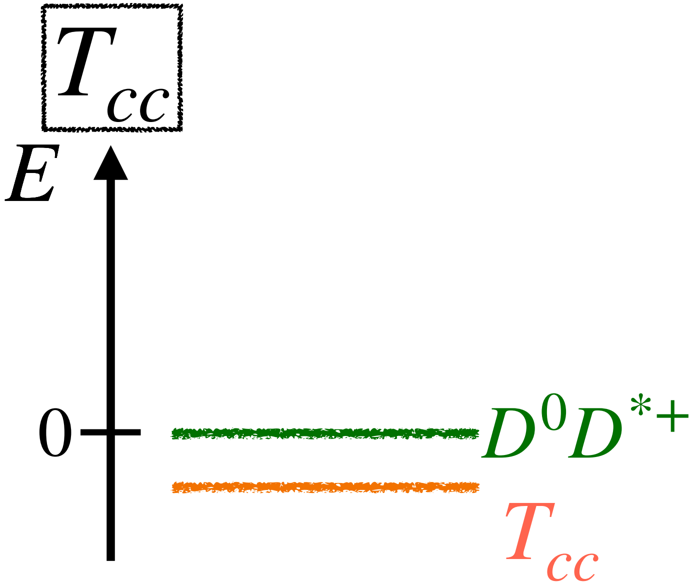
Near-threshold states



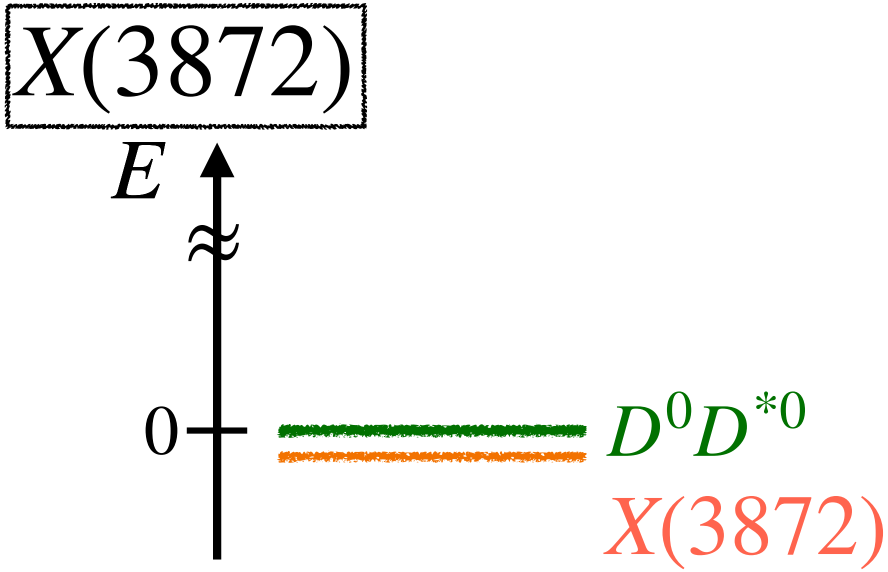
LHCb Collaboration, Nat. Commun **13** 3351 (2022).

PDG

Near-threshold states



LHCb Collaboration, Nat. Commun **13** 3351 (2022).



PDG

- compositeness $X = 1$ in $B \to 0$ limit (universality)

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

near threshold states ($B \neq 0$) is composite dominant ?

- However, elementary dominant states is realized with fine tuning

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014).

⟶ How finely tuning parameter?

Model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).

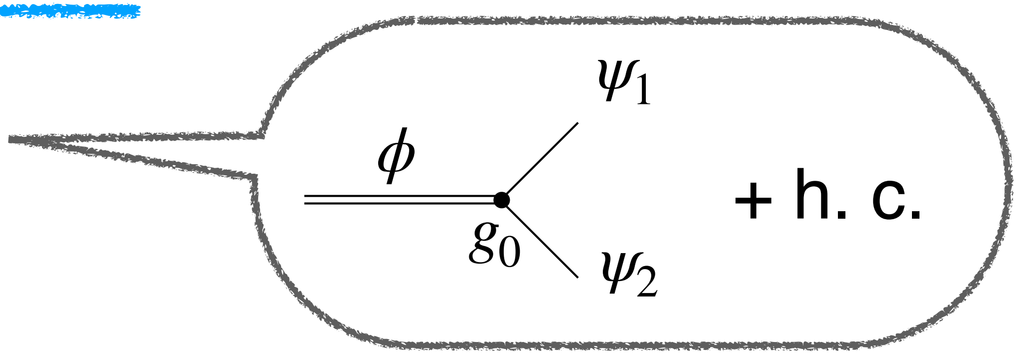
◉ single-channel resonance model

$$\mathscr{H}_{\text{free}} = \frac{1}{2m_1}\nabla\psi_1^\dagger\cdot\nabla\psi_1 + \frac{1}{2m_2}\nabla\psi_2^\dagger\cdot\nabla\psi_2 + \frac{1}{2m_\phi}\nabla\phi^\dagger\cdot\nabla\phi + \nu_0\phi^\dagger\phi,$$

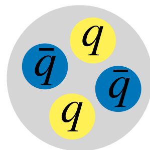
1.

$$\mathscr{H}_{\text{int}} = g_0(\phi^\dagger\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger\phi)\,.$$

2.



1. single-channel scattering
2. coupling with compact state ϕ



◉ scattering amplitude

$$V = \frac{g_0^2}{E-\nu_0},\quad G = -\frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right].$$

Λ : cutoff

$$\xrightarrow[T=\frac{1}{V^{-1}-G}]{} \quad f(k) = -\frac{\mu}{2\pi}\left[\frac{\frac{k^2}{2\mu}-\nu_0}{g_0^2} + \frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right]\right]^{-1}.$$

Model scales and parameters

- typical energy scale : $E_{\rm typ} = \Lambda^2/(2\mu)$

- three model parameters g_0, ν_0, Λ

1. calculation with given B

 coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \frac{\pi^2}{\mu}(B + \nu_0)\left[\Lambda - \kappa \arctan(\Lambda/\kappa)\right]^{-1}$

 ∵ bound state condition $f^{-1} = 0$ $\kappa = \sqrt{2\mu B}$.

2. use dimensionless quantities with Λ

 ⟶ results do not depend on cutoff Λ

3. energy of bare quark state ν_0

varied in the region : $-B/E_{\rm typ} \leq \nu_0/E_{\rm typ} \leq 1$

∵ to have $g_0^2 \geq 0$ & applicable limit of EFT

Calculation

◉ compositeness X

scattering amplitude : $T = \dfrac{1}{V^{-1} - G}$ Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

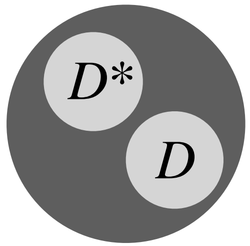
$$\longrightarrow X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1+\left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}.$$

- ν_0 region : $-B/E_{\rm typ} \leq \nu_0/E_{\rm typ} \leq 1$

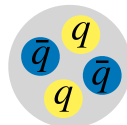
compositeness X as a function of ν_0

$X > 0.5$



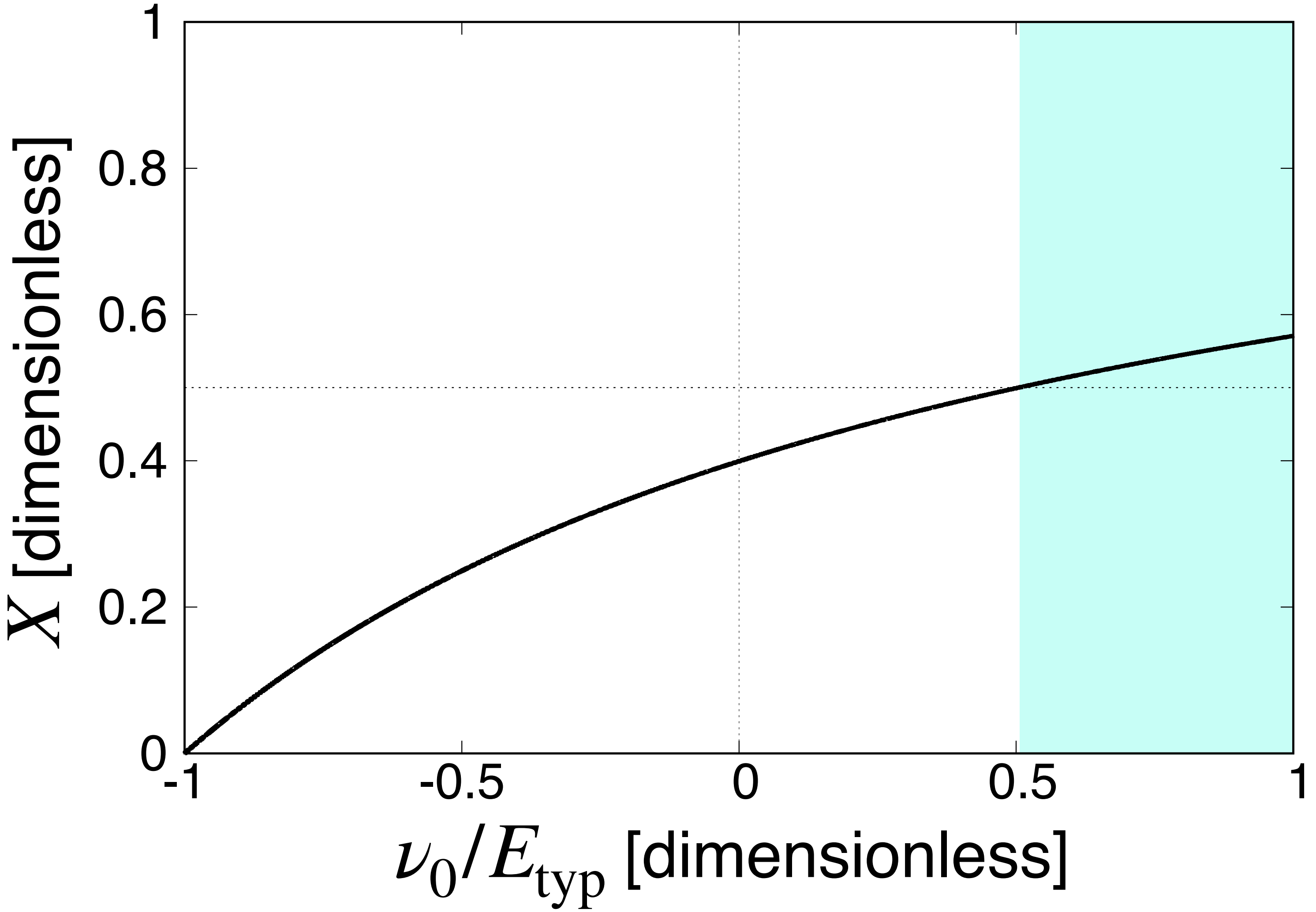
or

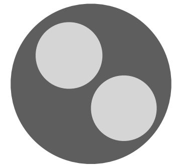
$X < 0.5$

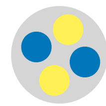


$\longrightarrow$ internal structure of bound state?

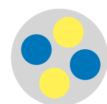
◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = E_{\rm typ}$



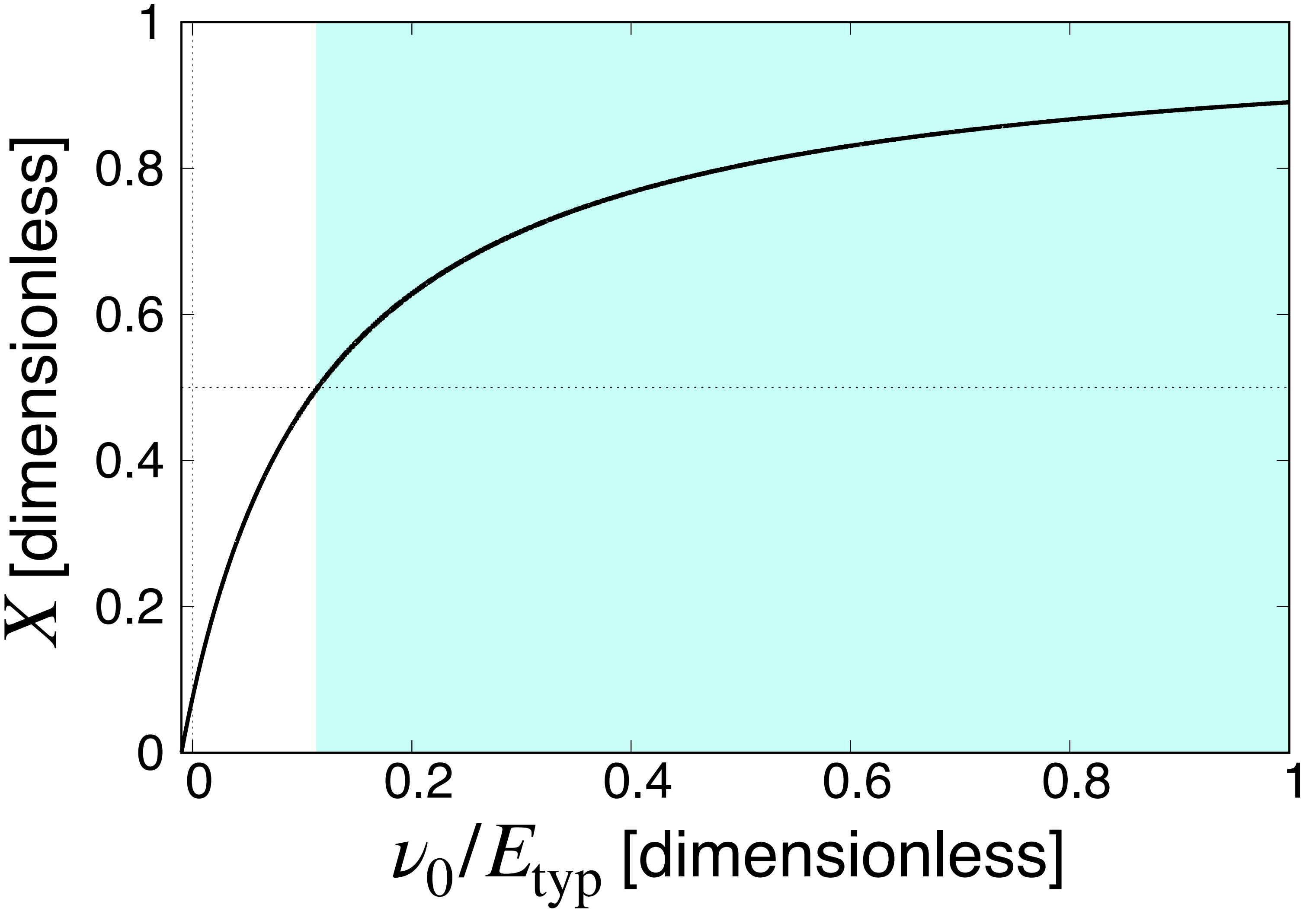
$X > 0.5$: 

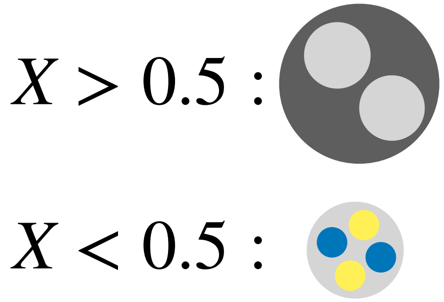
$X < 0.5$: 

- typical energy scale : $B = E_{\rm typ} = \Lambda^2/(2\mu)$

- $X > 0.5$ only for $25\ \%$ of ν_0 $\because$ **bare state origin** 

◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = 0.01E_{\rm typ}$

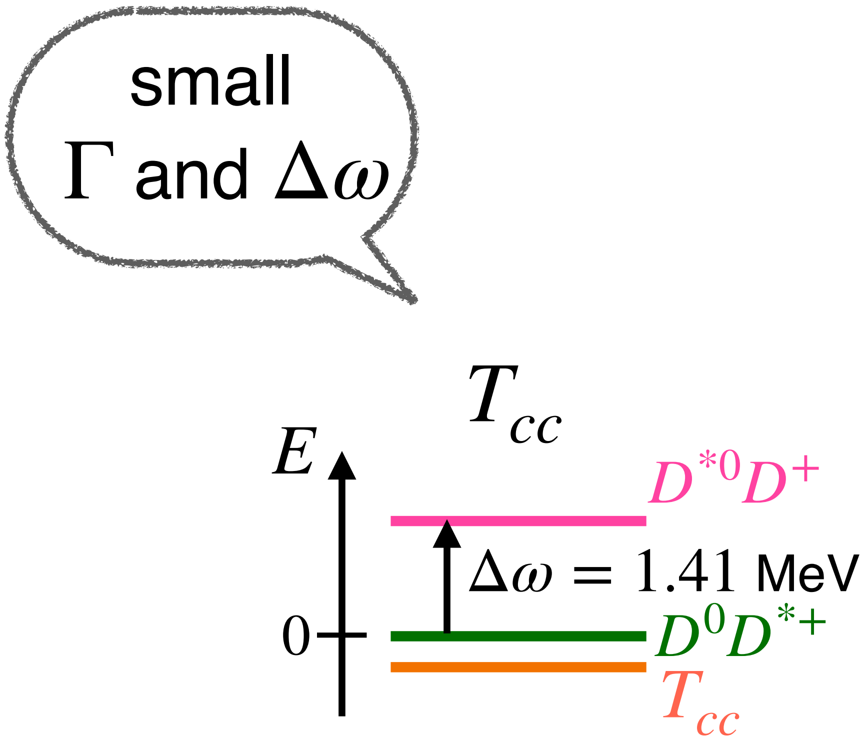




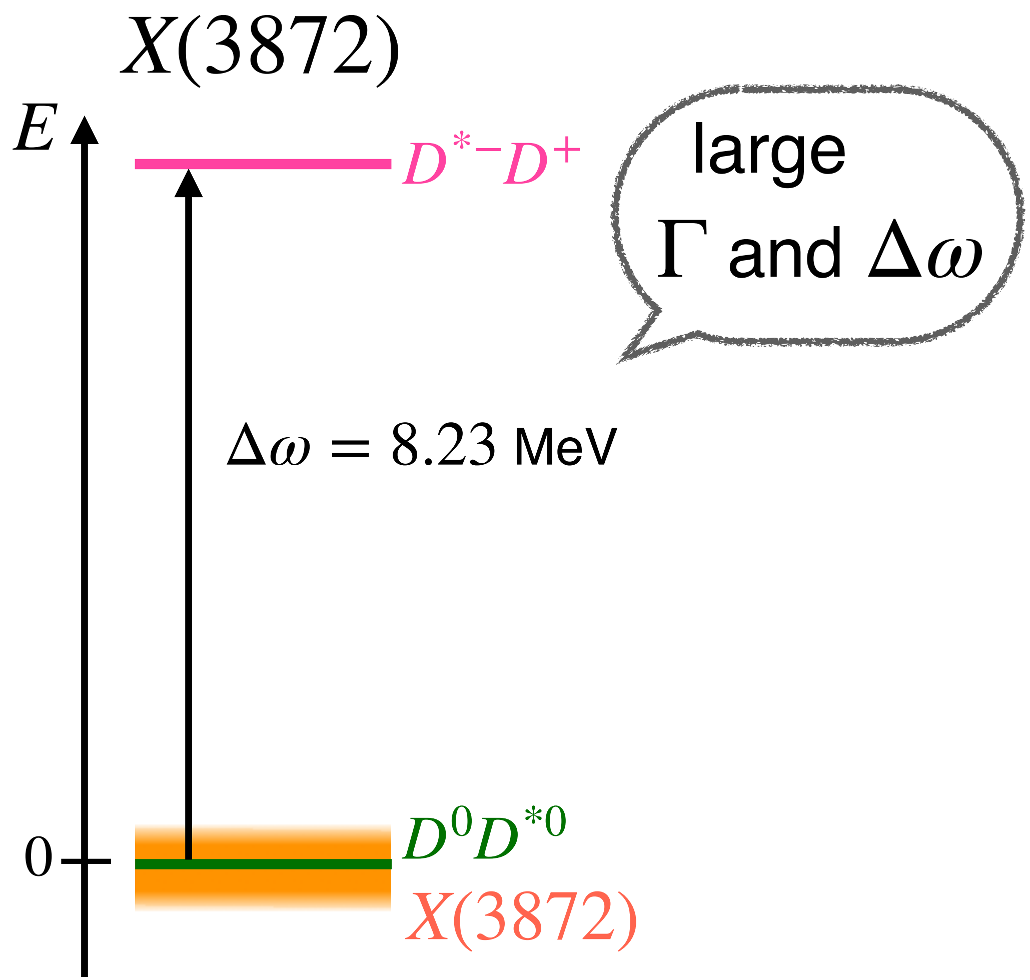
- weakly-bound state : $B = 0.01E_{\rm typ}$
- $X > 0.5$ for $88\ \%$ of ν_0 ⟶ **realization of universality !**

Application to T_{cc} and $X(3872)$

◉ exotic hadron ← decay and coupled channel



$B = 0.36$ MeV
$\Gamma = 0.048$ MeV



$B = 0.04$ MeV
$\Gamma = 1.19$ MeV

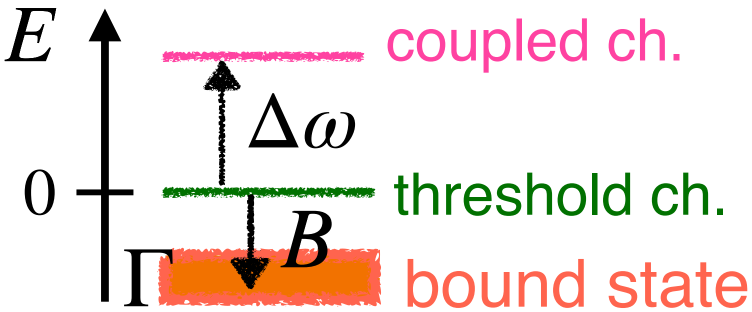
LHCb Collaboration, Nat. Commun **13** 3351 (2022).

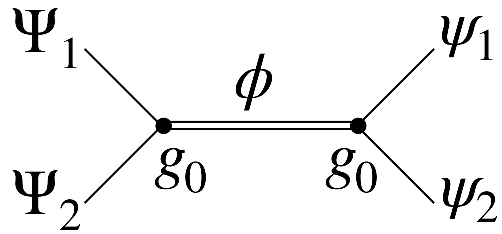
PDG

Effect of decay & coupled channel

$$|\Psi\rangle = \sqrt{X_1}\,|\text{threshold ch}\rangle + \sqrt{X_2}\,|\text{coupled ch}\rangle + \sqrt{1-(X_1+X_2)}\,|\text{others}\rangle$$

- threshold energy difference $\Delta\omega$
- ch. 1 couples to ch. 2 through ϕ with same coupling const.





- decay width $E = -B - i\Gamma/2$
- effectively introduced : coupling const. $g_0 \in \mathbb{C}$

◉ compositeness T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC 93, 035204 (2016).

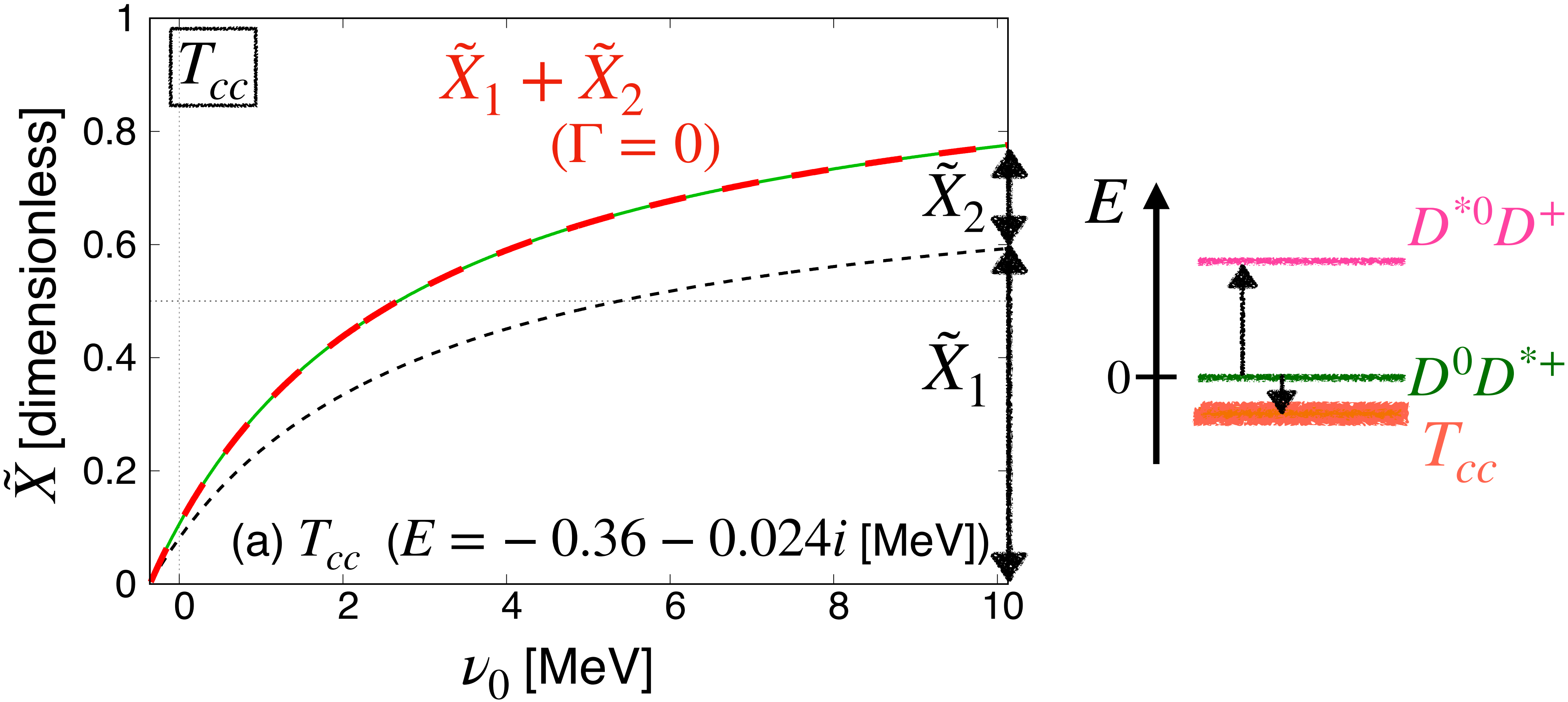
$$\tilde{X}_j = \frac{|X_j|}{\sum_j |X_j| + |Z|}, \quad (j = 1,2)$$

$\tilde{X}_1$: threshold ch. compositeness

$\tilde{X}_2$: coupled ch. compositeness

Application to T_{cc}

$\Lambda = 140$ MeV (π meson)



- $\tilde{X}_2$ is not negligible

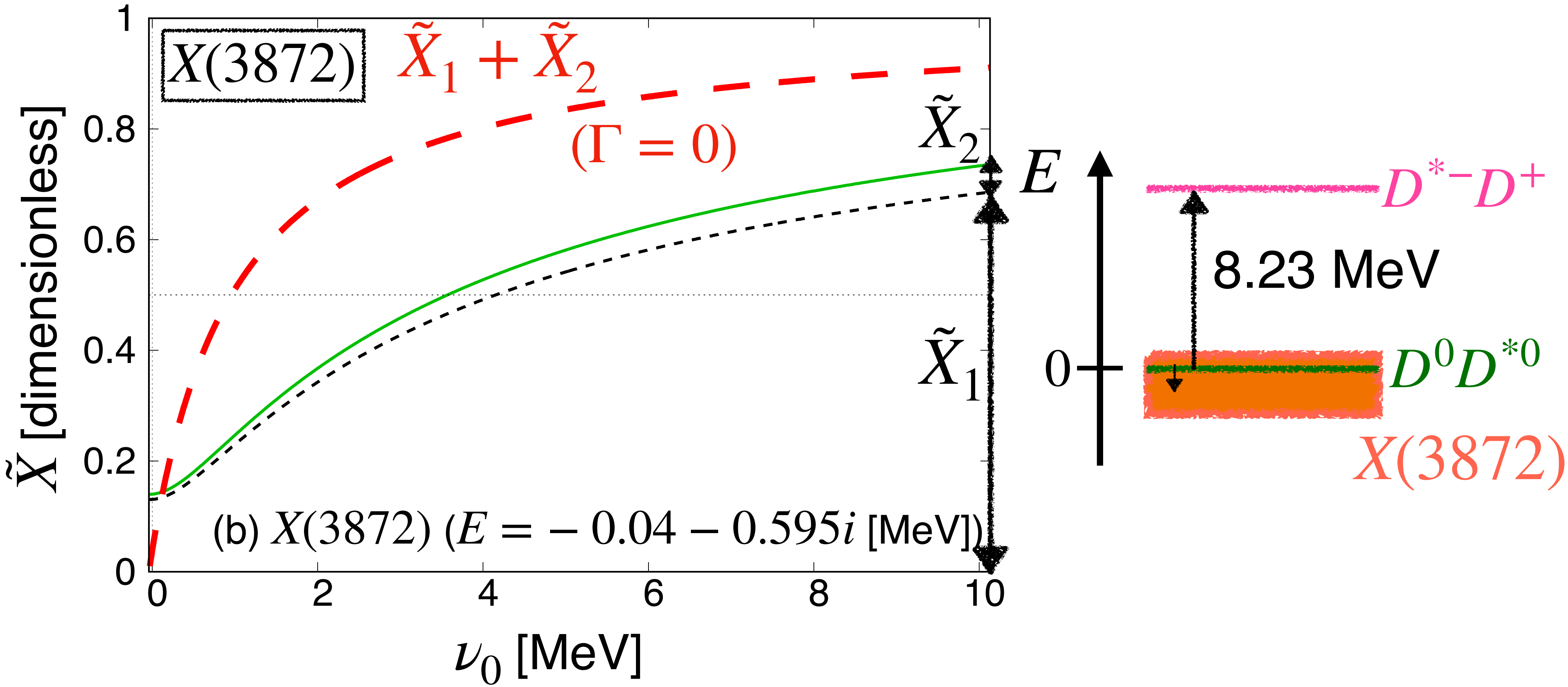
  $\because$ coupled ch. contribution (small $\Delta\omega$)

- difference of $\tilde{X}_1 + \tilde{X}_2(\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is too small

  → We can neglect decay contribution $\because \Gamma \ll B$

Application to $X(3872)$

$\Lambda = 140$ MeV (π meson)



- difference of $\tilde{X}_1 + \tilde{X}_2(\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is large

  ∵ large decay width contribution

- $\tilde{X}_2$ is much smaller than $\tilde{X}_1$

  ⟶ coupled ch. effect is small

Summary

T. Kinugawa and T. Hyodo, arXiv:2303.07038 [hep-ph]

- internal structure of exotic hadrons ⟵ EFT & compositeness

- shallow bound state

⟶ composite dominant even from bare state

fine tuning is necessary to realize elementary dominant state

- decay and coupled channel effects are introduced

⟶ both decay and coupled ch. effects suppress compositeness

- T_{cc} and $X(3872)$ with decay and coupled ch. effects

T_{cc} : important coupled ch. effect with negligible decay effect

$X(3872)$: important decay effect with negligible coupled ch. effect

Compositeness of T_{cc} by other work

paper by L. R. Dai, J. Son and E. Oset

L. R. Dai, J. Son and E. Oset, arXiv: 2306.01607 [hep-ph].

- relativistic model

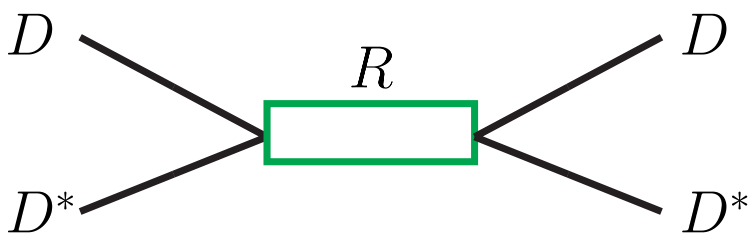


FIG. 1: DD^* amplitude based on the genuine resonance R.

- Both molecular and non-molecular states are realized by tuning of parameters

- However, case with small compositeness is excluded by experimental data (a_0 and r_e)

History of compositeness

- Weinberg's work (1960s) Weinberg, S. Phys. Rev. 137, 672–678 (1965) etc.

  deuteron is not an elementary particle ⟵ weak-binding relation

- application to exotic hadrons (2000s-)

  "compositeness"

  generalization to unstable states

  with spectral function V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004) etc.

  with effective range expansion T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) etc.

  with effective field theory Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017) etc.

  application to …

  $f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016); T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.

  $\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

  nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

Compositeness

◉ model calculation

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **85**, 015201 (2012);
F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

$$T = \frac{1}{V^{-1} - G}$$

V : effective interaction

G : loop function

residue of scattering amplitude g

$$X = -g^2 G'(E)\Big|_{E=-B} = \frac{G'(E)}{G'(E) - [V^{-1}(E)]'}\Bigg|_{E=-B}$$

$$\alpha'(E) = d\alpha/dE$$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

g^2 : model independent ⟵ $T_{\rm on}(-B)$ (observable)

$G(E)$: model dependent ⟵ cutoff dependent

Weak-binding relation

S. Weinberg, Phys. Rev. 137, 672–678 (1965).

$$X = \frac{a_0}{2R - a_0} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)$$

a_0 : scattering length

$R_{\rm typ}$: typical length scale in system

$R = 1/\sqrt{2\mu B}$

- for weakly bound states, $R \gg R_{\rm typ}$

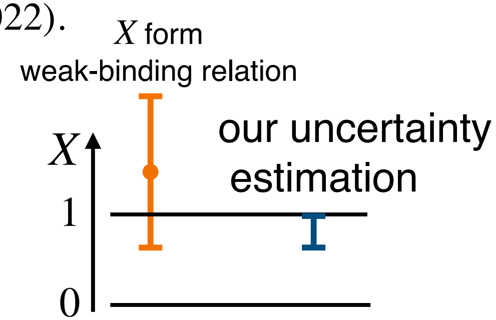
compositeness ⟵ observables (a_0, B)

Y. Li, F.-K. Guo, J.-Y. Pang, and J.-J. Wu, Phys. Rev. D 105, L071502 (2022);
J. Song, L. R. Dai, and E. Oset, Eur. Phys. J. A 58, 133 (2022);
M. Albaladejo, J. Nieves, Eur. Phys. J. C 82, 724 (2022);
T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022).

◉ range correction

compositeness of deuteron $X \sim 1.7 > 1$

⟶ important to consider effective range



- our work : range correction ⟵ uncertainty estimation

compositeness of deuteron : $0.74 \leq X \leq 1$

Low-energy universality

scattering length $a_0(\to \infty)$

$\gg$ typical length scale of system $R_{\rm typ}$

low-energy universality E. Braaten and H.-W. Hammer, Phys. Rept. **428**, 259 (2006) ; F. P. Naidon and S. Endo, Rept. Prog. Phys. **80**, 056001 (2017).

⟶ length scales are written only by $|a_0|$

- for bound states ?

$$R = 1/\sqrt{2\mu B} : a_0 = R \to \infty \longrightarrow B \to 0$$

⟶ universality holds for **weakly**-bound states!

- compositeness $X = 1$ in $B \to 0$ limit T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

⟶ near threshold states ($B \sim 0$) = composite dominant ?

e.g. ^{8}Be, ^{12}C Hoyle state ⟶ α cluster?

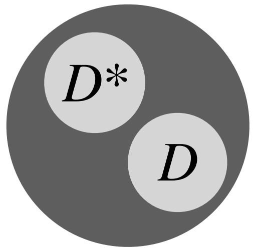
H. Horiuchi, K. Ikeda, and Y. Suzuki, Prog. Theor. Phys. Suppl. **52**, 89 (1972) .

universality

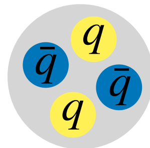
T_{cc} and $X(3872)$ are shallow-bound states

→ low-energy universality is important!

1. naive expectation : near-threshold states are composite dominant



2. However, elementary dominant states is realized with fine tuning



T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014).

↓

How finely tuning parameter?

In this work, we study fine tuning quantitatively!

Effect of decay

◉ introducing decay effect

- formally : introducing decay channel in lower energy region than binding energy

→ eigenenergy becomes complex

- effectively : coupling const. $g_0 \in \mathbb{C}$! ← this work

$$\mathscr{H}_{\rm int} = g_0(\phi^\dagger \psi_1 \phi_2 + \phi_1^\dagger \psi_2^\dagger \phi) .$$

$$E = -B \longrightarrow E = -B - i\Gamma/2$$

compositeness

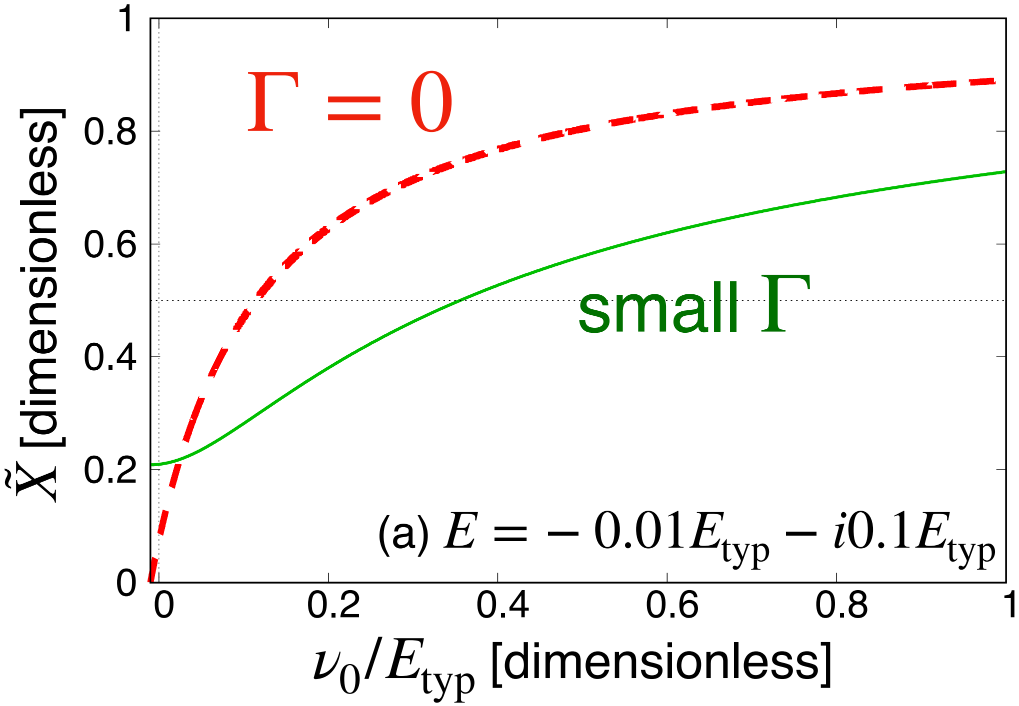
$$X \in \mathbb{R} \longrightarrow X \in \mathbb{C}$$

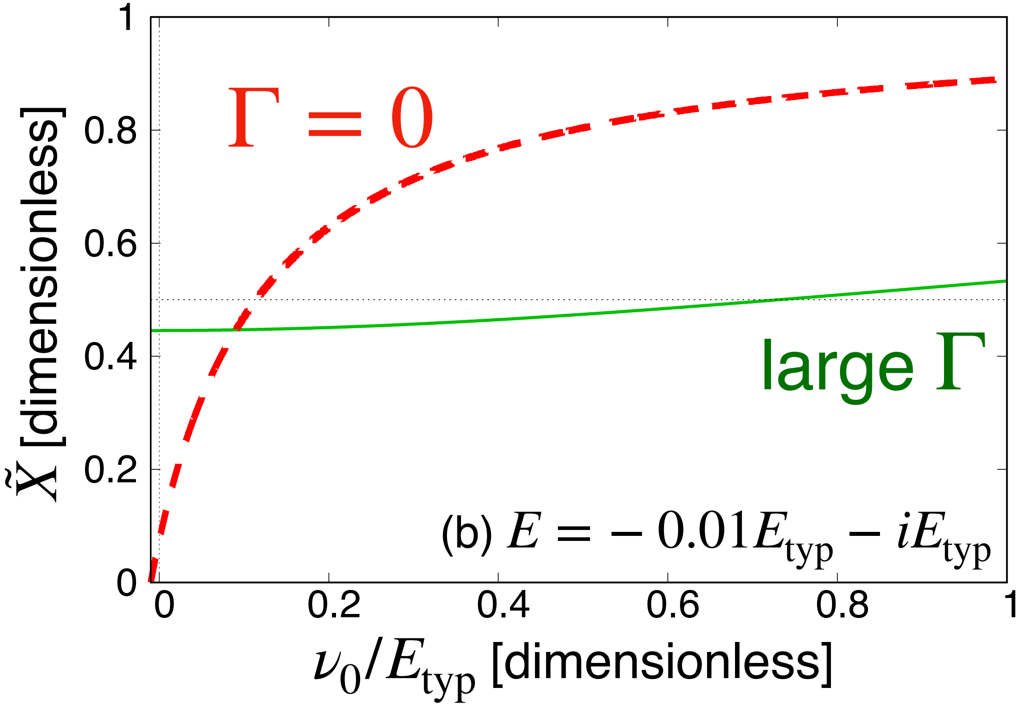
$$\tilde{X} = \frac{|X|}{|X| + |1 - X|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC 93, 035204 (2016).

Effect of decay

$E = -0.01E_{\rm typ} - i0.1E_{\rm typ}$ $E = -0.01E_{\rm typ} - iE_{\rm typ}$



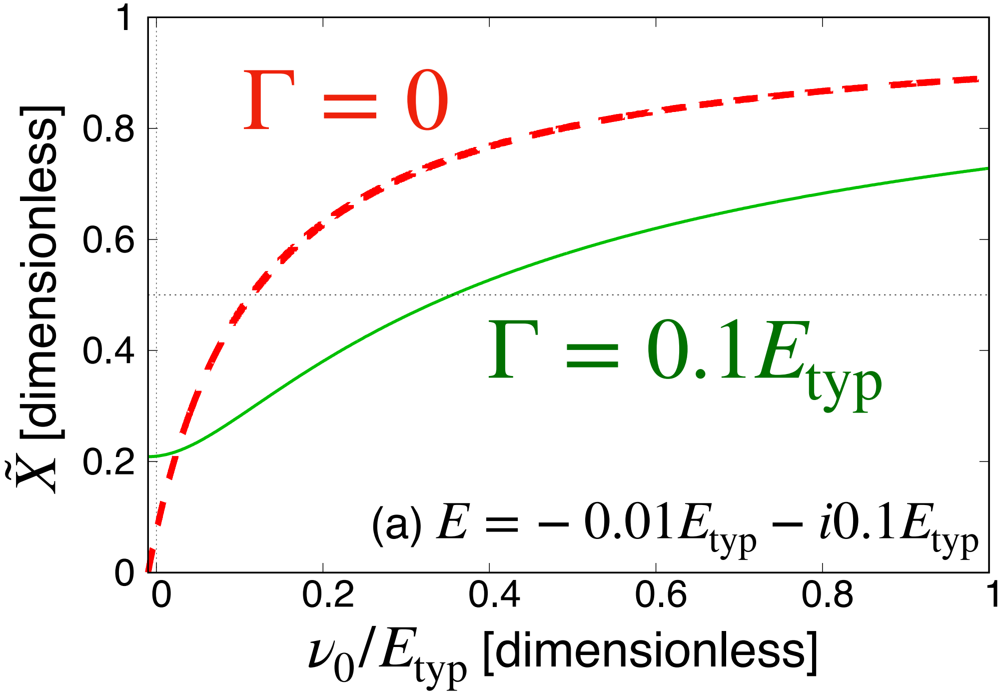


- $\tilde{X}$ is suppressed by decay effect

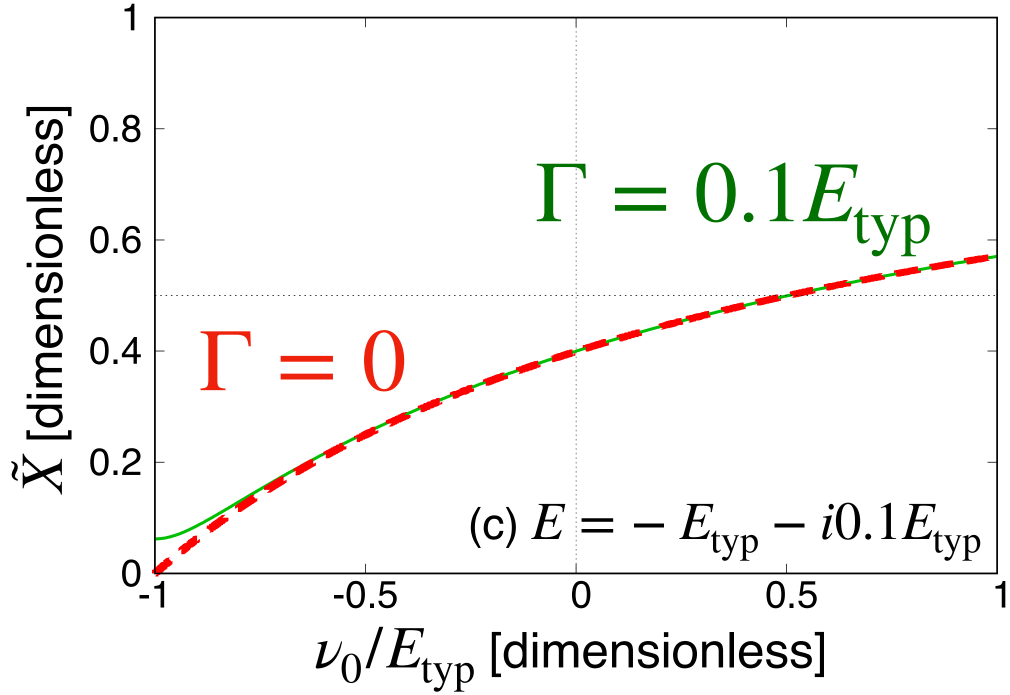
∵ threshold ch. component ($\tilde{X}$) decreases with inclusion of decay ch. component ($1-\tilde{X}$)

Effect of decay

$E = -0.01E_{\mathrm{typ}} - i0.1E_{\mathrm{typ}}$



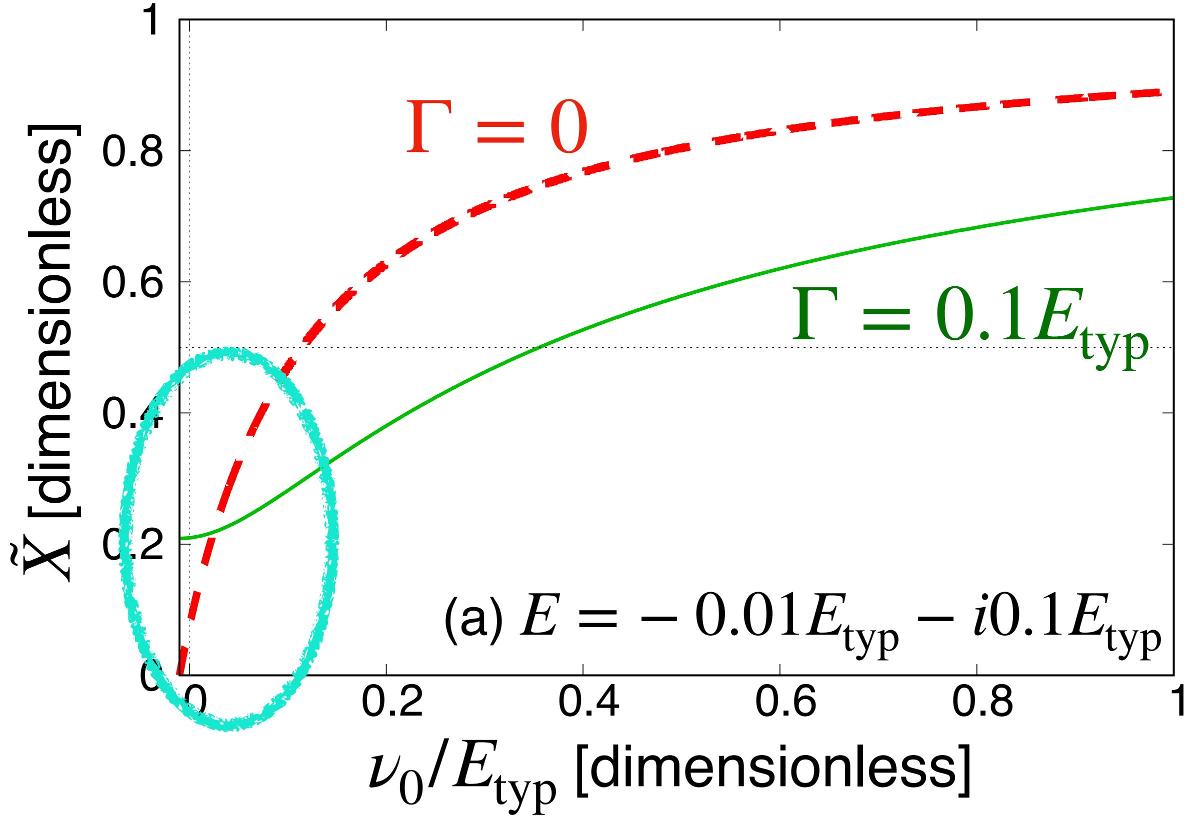
$E = -E_{\mathrm{typ}} - i0.1E_{\mathrm{typ}}$



compositeness is more suppressed when B is small

- suppression of $\tilde{X}$ is determined by ratio of B to Γ

Effect of decay



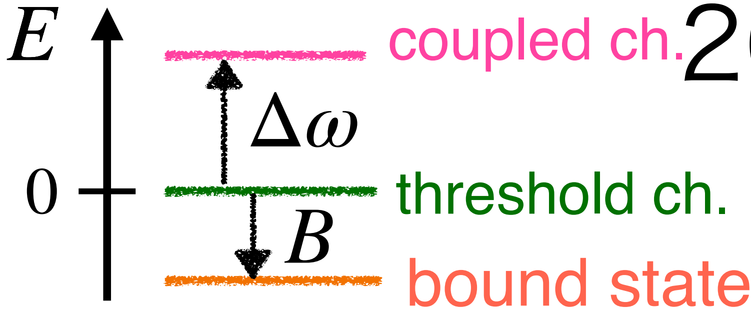
- $X \neq 0$ with $\Gamma \neq 0$
  $\because$ $g_0 \neq 0$ at $\nu_0 = -B$

c.f. $g_0 = 0$ at $\nu_0 = -B$
with $\Gamma = 0$

$$g_0^2\left(-\nu_0 + i\frac{\Gamma}{2};\nu_0,\Lambda\right) = \frac{\pi^2}{\mu}\left(-i\frac{\Gamma}{2}\right)\left[\Lambda - \kappa\arctan\left(\frac{\Lambda}{\kappa}\right)\right]^{-1} \neq 0$$

$$X = \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1+\left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}$$

Effect of coupled channel



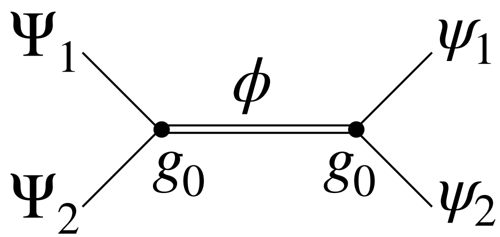
◉ introducing coupled channel $\Psi_1\Psi_2$

$$|\Psi\rangle = \sqrt{X_1}\,|\text{threshold ch}\rangle + \sqrt{X_2}\,|\text{coupled ch}\rangle + \sqrt{1-(X_1+X_2)}\,|\text{others}\rangle$$

$$\mathscr{H}_{\text{free}} = (\text{kinetic terms of } \psi_{1,2}, \Psi_{1,2}, \phi) + \omega_1\Psi_1\Psi_1^\dagger + \omega_2\Psi_2\Psi_2^\dagger + \nu_0\phi^\dagger\phi,$$

$$\mathscr{H}_{\text{int}} = g_0(\phi^\dagger\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger\phi + \phi^\dagger\Psi_1\Psi_2 + \Psi_1^\dagger\Psi_2^\dagger\phi)\,.$$

- threshold energy difference $\Delta\omega = \omega_1 + \omega_2$
- ch. 1 couples to ch. 2 through ϕ with same coupling const.



- low-energy universality with coupled-channel effect

$X_1 \sim 1$ (threshold channel)

$X_2 \sim 0$ and $Z \sim 0$ (other channel)

Compositeness for two-channel case

$$V(k) = \begin{pmatrix} v(k) & v(k) \\ v(k) & v(k) \end{pmatrix}, \quad v(k) = \frac{g_0^2}{\frac{k^2}{2\mu_1} - \nu_0} .$$

$$G(k) = \begin{pmatrix} G_1(k) & 0 \\ 0 & G_2(k) \end{pmatrix},$$

$$G_1(k) = -\frac{\mu_1}{\pi^2}\left[\Lambda + ik \arctan\left(-\frac{\Lambda}{ik}\right)\right],$$

$$G_2(k') = -\frac{\mu_2}{\pi^2}\left[\Lambda + ik' \arctan\left(-\frac{\Lambda}{ik'}\right)\right].$$

$$k = \sqrt{2\mu_1 E}, \quad k'(k) = \sqrt{2\mu_2(E - \Delta\omega)} = \sqrt{\frac{\mu_2}{\mu_1}k^2 - 2\mu_2\Delta\omega} .$$

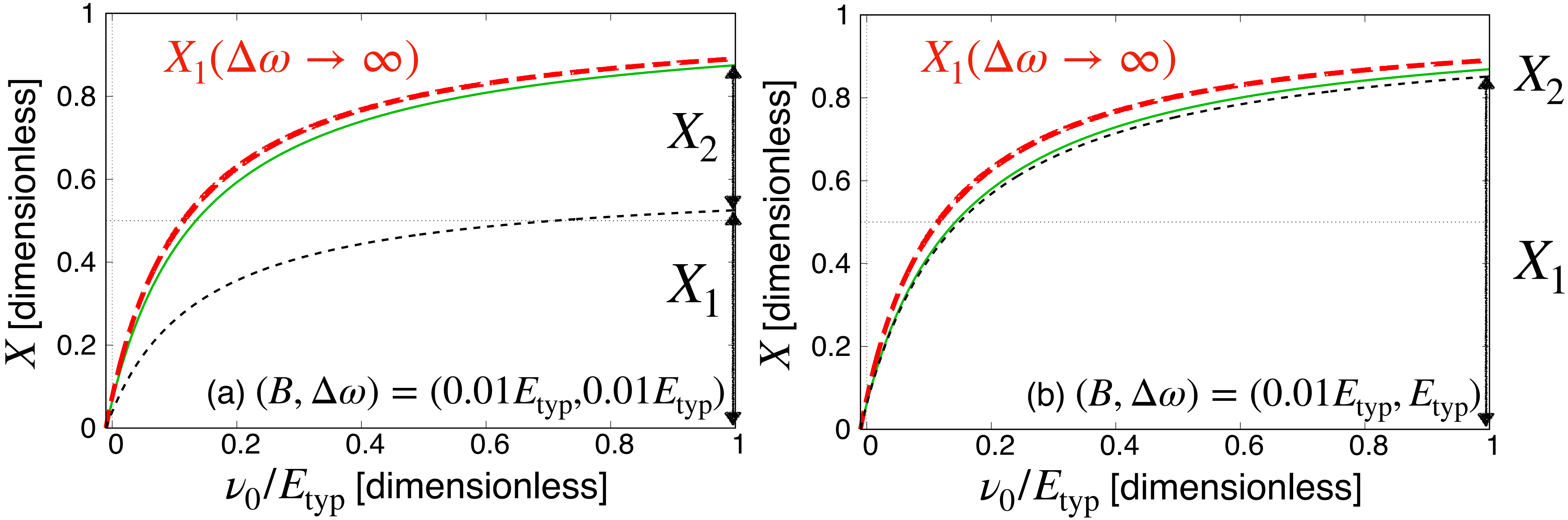
$\longrightarrow$

$$X_1 = \frac{G_1'}{(G_1' + G_2') - [v^{-1}]'},$$

$$X_2 = \frac{G_2'}{(G_1' + G_2') - [v^{-1}]'} .$$

Effect of coupled channel

$(B, \Delta\omega) = (0.01E_{\rm typ}, 0.01E_{\rm typ})$ $(B, \Delta\omega) = (0.01E_{\rm typ}, E_{\rm typ})$

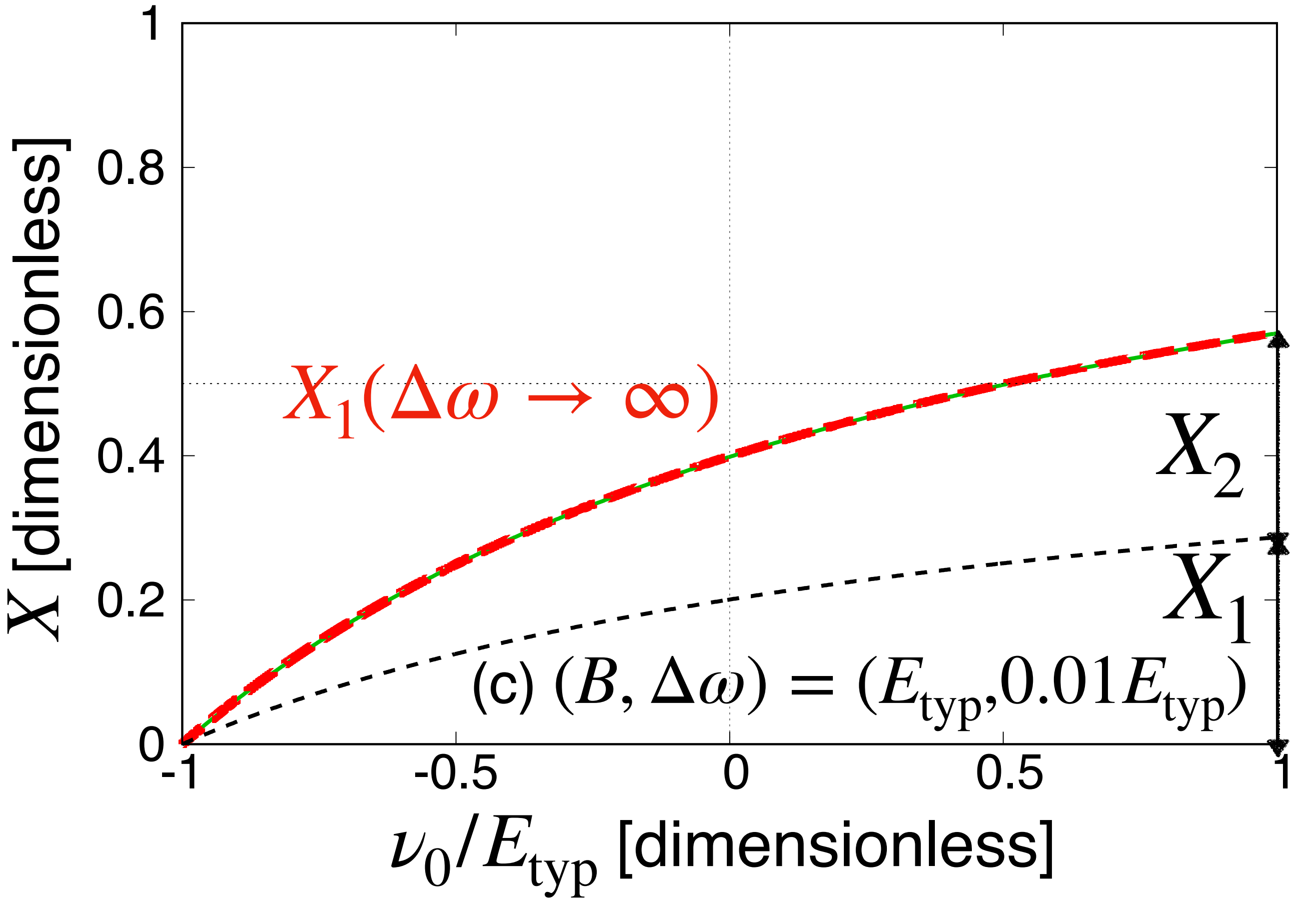


- X_1 is suppressed by channel coupling

  ∵ threshold ch. component (X_1) decreases with inclusion of coupled ch. component (X_2)

- $Z = 1 - (X_1 + X_2)$ is stable

Effect of coupled channel

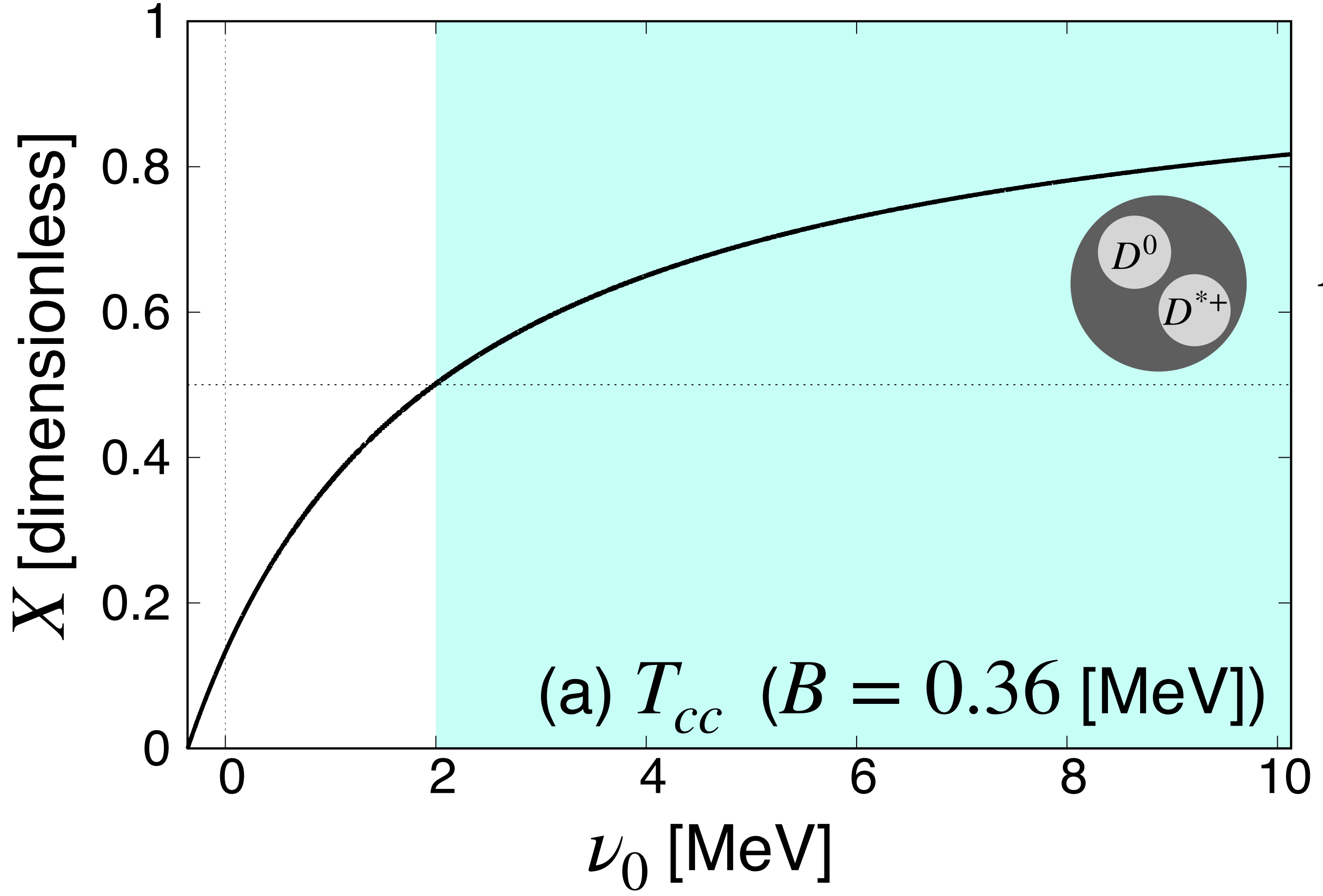


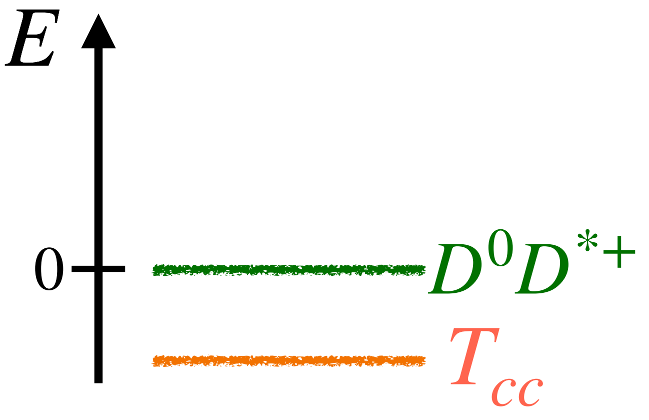
this calculation corresponds to $\Delta\omega \to 0$ case

back up??

Application to T_{cc} (single ch. model)

◉ single-channel

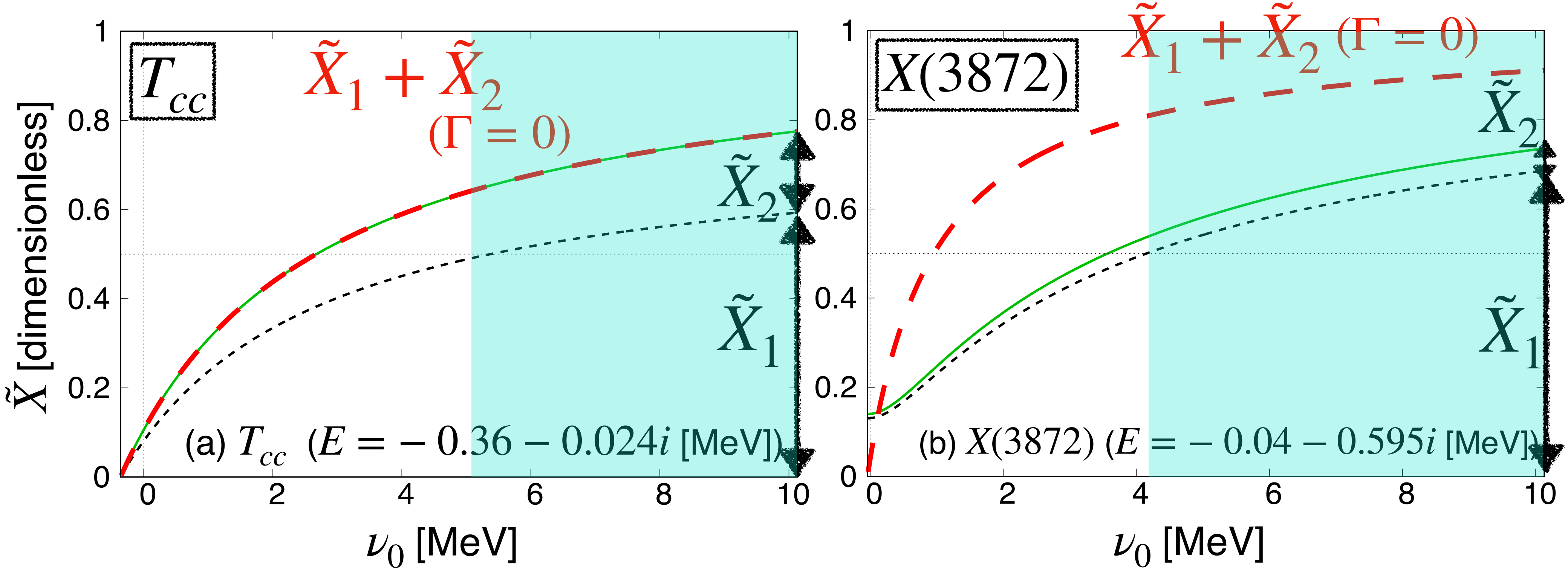




- $X > 0.5$ for 78 % of ν_0 = composite dominant
- fine tuning is necessary to realize $X < 0.5$

Application to T_{cc} and $X(3872)$

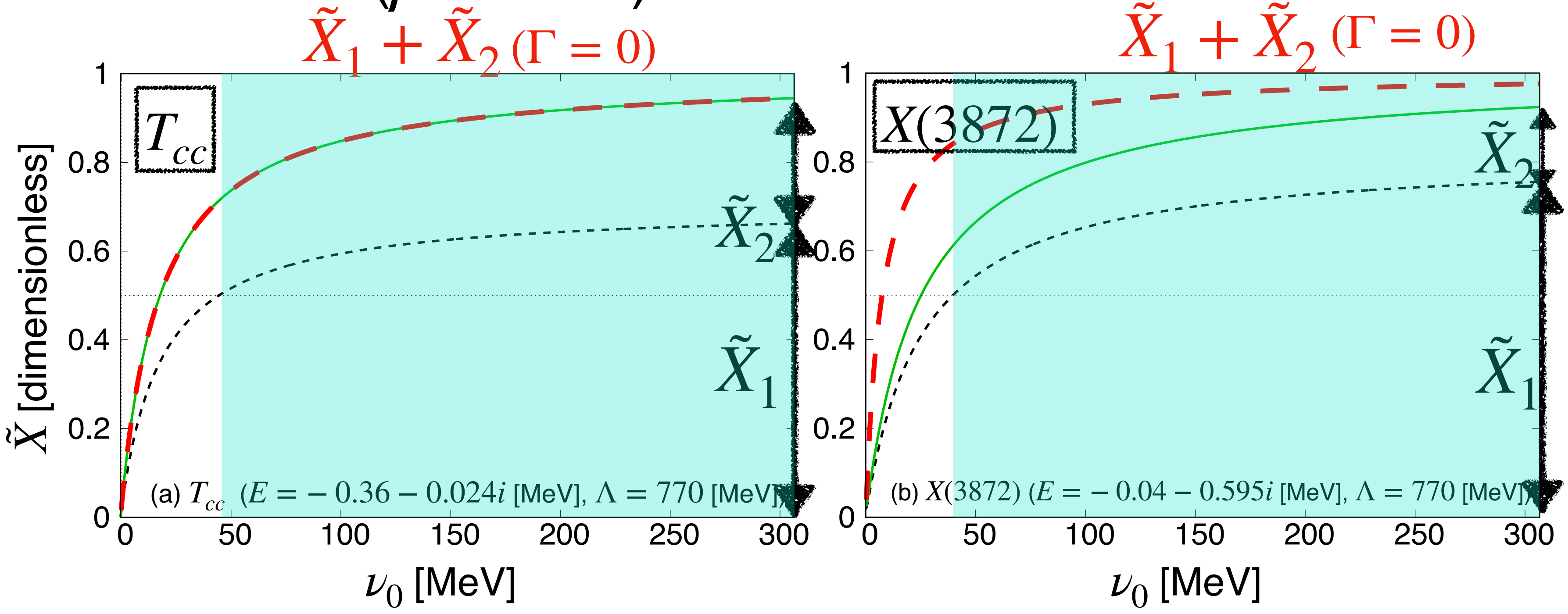
- $\Lambda = 140$ MeV (π meson)



- T_{cc} : $\tilde{X}_1 > 0.5$ for 45 % of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for 59 % of ν_0 region
- coupled ch. effect is more important for T_{cc} than $X(3872)$
- decay effect is more important for $X(3872)$ than T_{cc}

Application to T_{cc} and $X(3872)$

- $\Lambda = 770$ MeV (ρ meson)



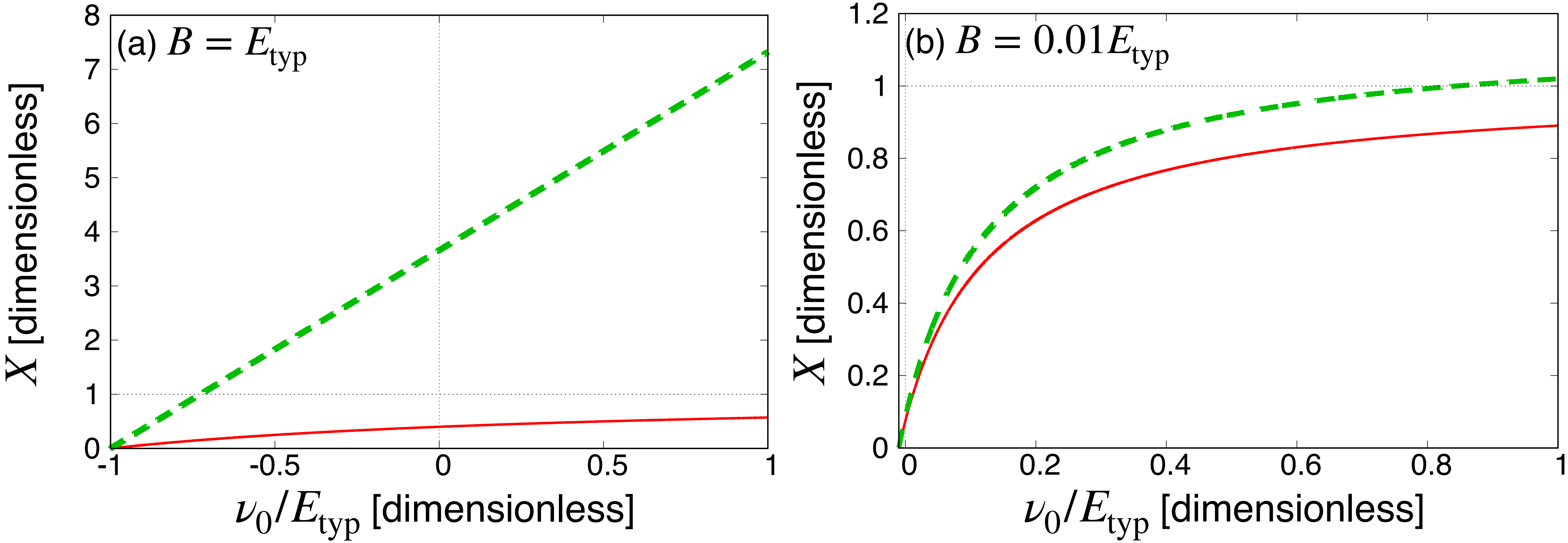
- T_{cc} : $\tilde{X}_1 > 0.5$ for $85\ \%$ of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for $87\ \%$ of ν_0 region
- typical energy scale $E_{\rm typ}$ is larger

→ states becomes close to universality limit $X \to 1$

decay effect : suppressed coupled ch. effect : enhanced

validity of weak-biding relation

single-channel scattering model



comparison of central value of weak-binding relation with model

(a) typical scale binding energy : weak-binding relation ×

(b) weak-binding energy : weak-binding relation ◯

even for elementary dominant state with small ν_0