複合性を用いたハドロン分子状態へのクーロン力の影響

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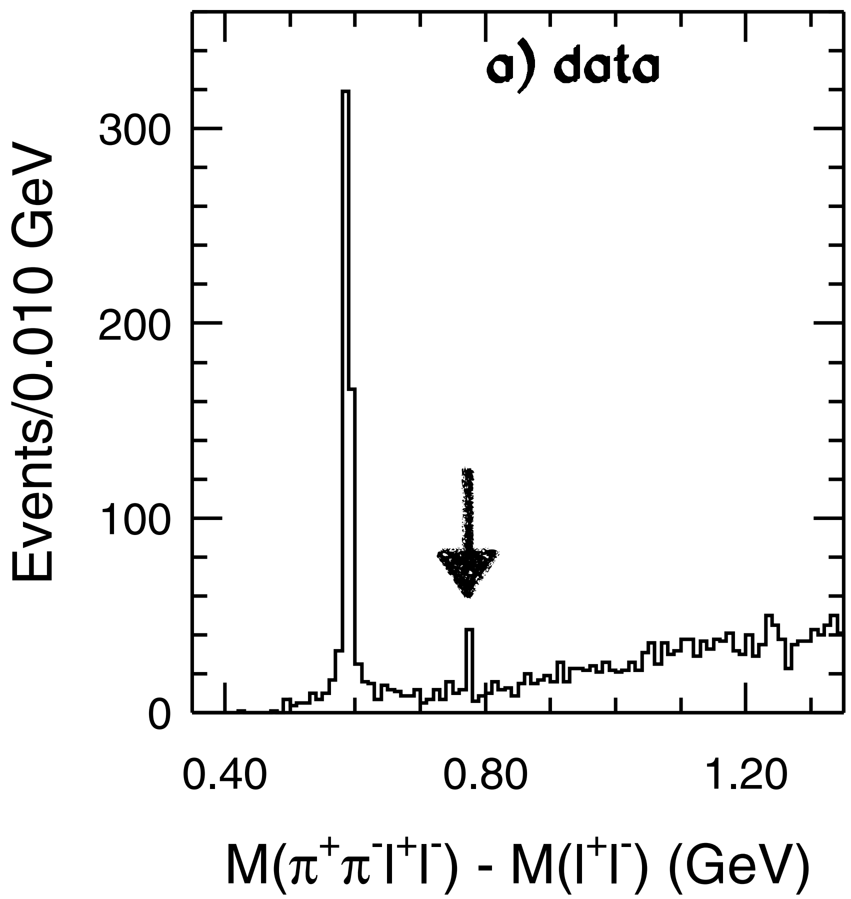
Department of Physics, Tokyo Metropolitan University

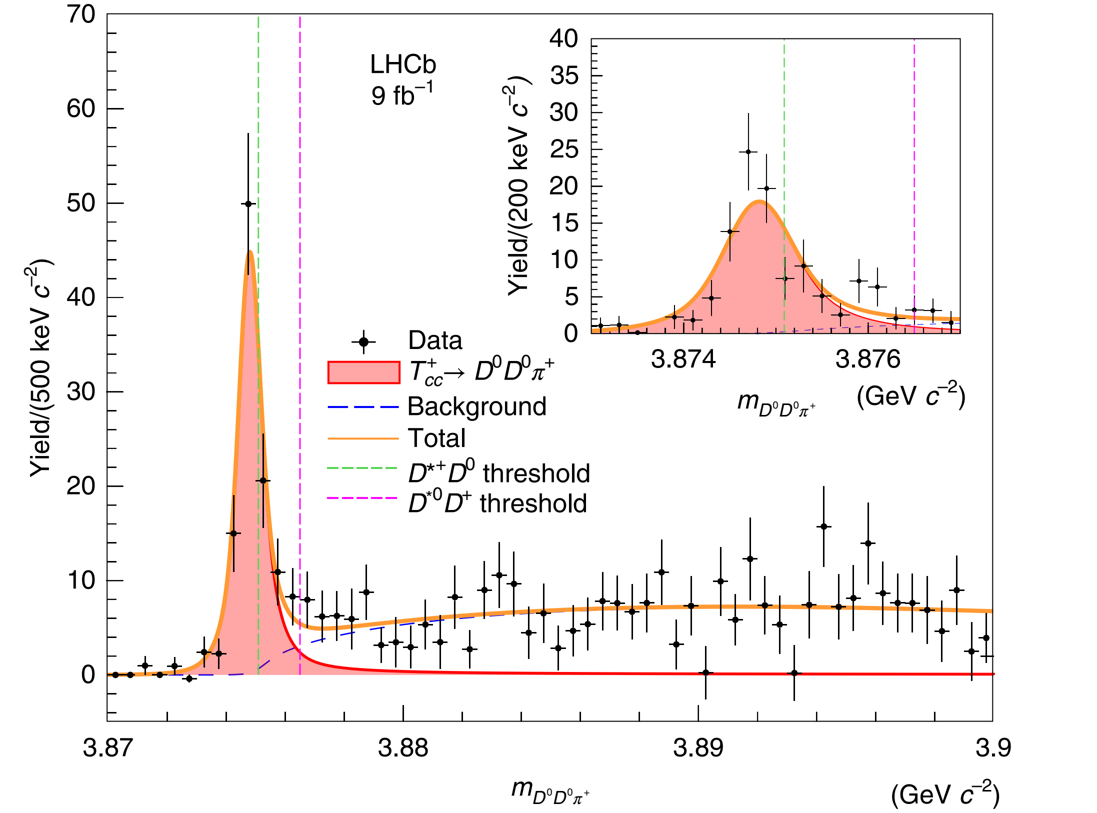
3月18日, JPS 2025 春季大会

閾値近傍のエキゾチックハドロン

$X(3872) \to \pi^+\pi^- J/\psi$ $\quad$ $T_{cc}(3875)^+ \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$



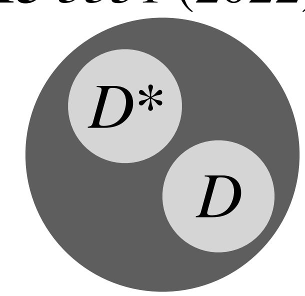
S. K. Choi *et al*. (Belle), Phys. Rev. Lett. **91**, 262001 (2003).



LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

- 閾値近傍の束縛状態はハドロン分子的になりやすい



⟵ **低エネルギー普遍性**

<u>詳しくは20日午前のtalk</u>

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;

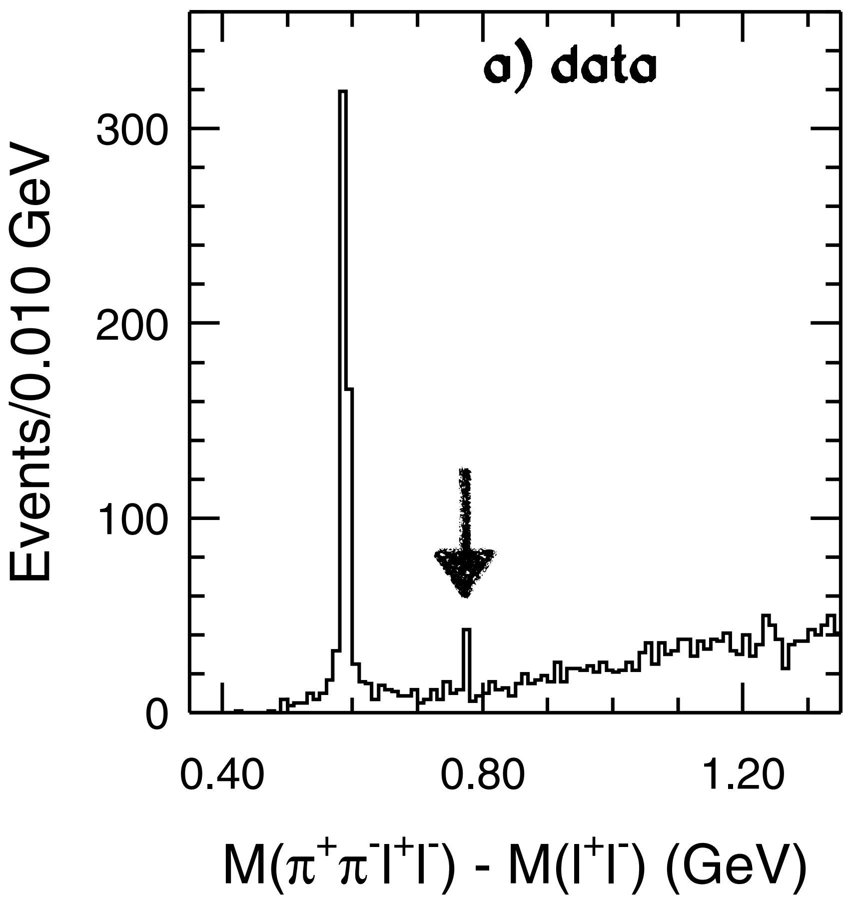
C. Hanhart and A. Nefediev, Phys. Rev. D 106, 114003 (2022);

T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024).

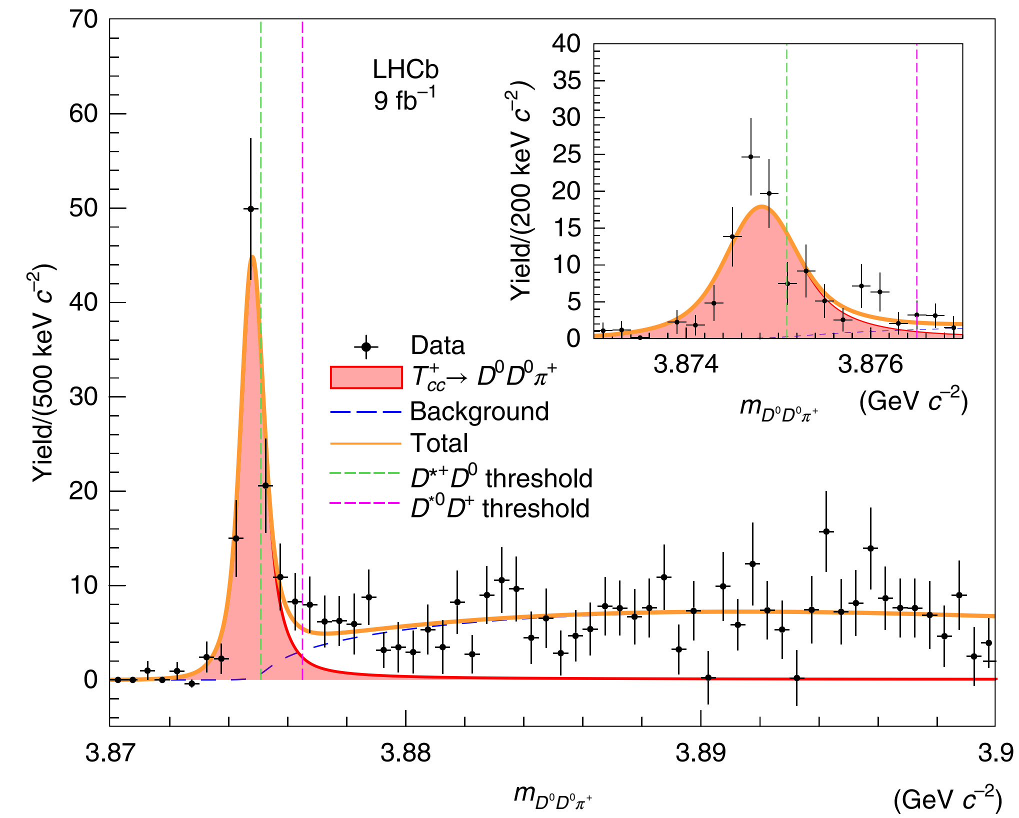
- **荷電**ハドロン散乱の閾値近傍 ⟶ **クーロン力**も寄与？

閾値近傍のエキゾチックハドロン

$X(3872) \to \pi^+\pi^- J/\psi$ $T_{cc}(3875)^+ \to D^0 D^0 \pi^+$ $(cc\bar{u}\bar{d})$



S. K. Choi *et al*. (Belle), Phys. Rev. Lett. **91**, 262001 (2003).



LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

- 閾値近傍の束縛状態はハドロン

⟵ **低エネルギー普遍性**

詳しくは20日午前のtalk

20日 V1会場 20aV1 9:00〜11:35

実験核物理領域, 理論核物理領域

実験核物理領域,理論核物理領域 日本物理学会若手奨励賞受賞記念講演

(若手奨励賞) エキゾチックハドロンの複合性に関する理論的研究 (30分)

東京都立大理

衣川友那

T. Hy

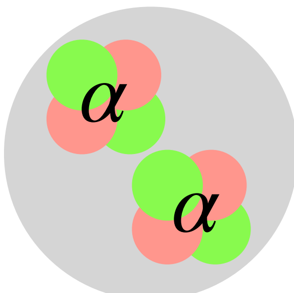
C. Hanhart and 22);

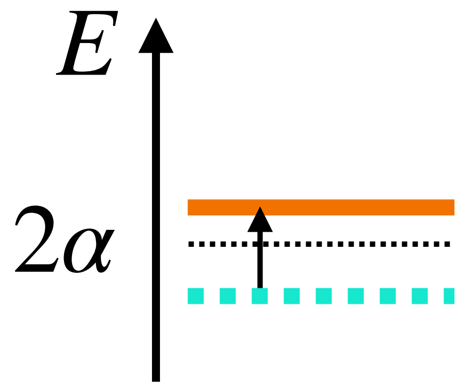
T. Kinugawa a 24).

- **荷電**ハドロン散乱の閾値近傍 ⟶ **クーロン力**も寄与？

クーロン + 短距離力の系

- ^{8}Be 原子核 ($2\alpha^{++}$) J. Hiura, and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 52, 25 (1972).

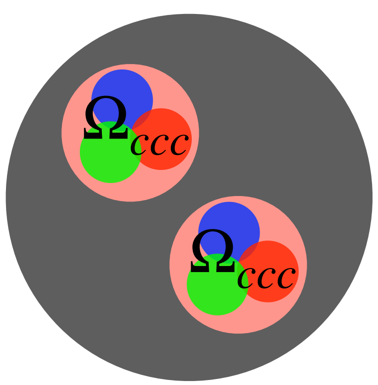


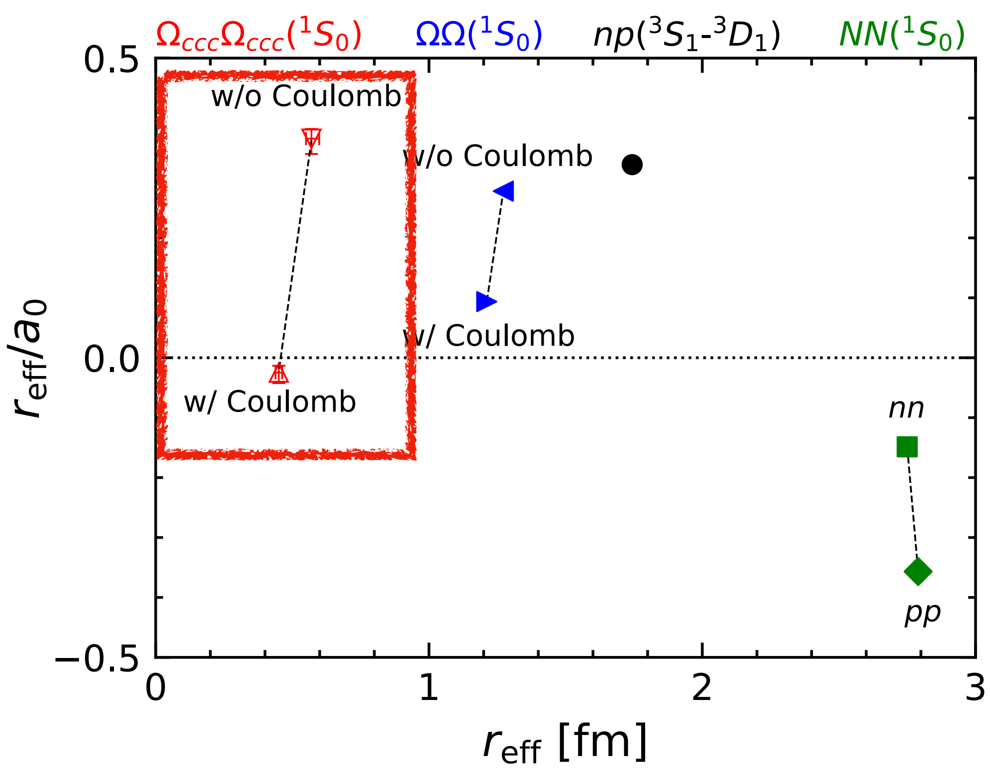


共鳴状態 (クーロン斥力あり)

束縛状態 (クーロン斥力なし)

- $\Omega_{ccc}^{++}\ \Omega_{ccc}^{++}$ (HAL QCD) Y. Lyu, H.Tong, *et al*. [HAL QCD Coll.], Phys. Rev. Lett. 127 (2021) 072003.





共鳴状態 (クーロン斥力あり)

束縛状態 (クーロン斥力なし)

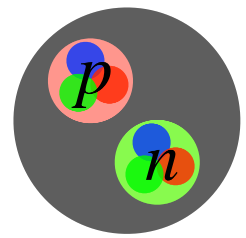
- $\Xi^{-}\alpha$: Coulomb assisted 束縛状態？ E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, Phys. Rev. C 106, 064318 (2022).

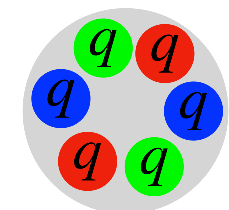
→ 閾値近傍ではクーロン力の寄与が重要！...では内部構造は？

複合性

S. Weinberg, Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).
T. Kinugawa and T. Hyodo, arXiv: 2411.12285 [hep-ph].

◉ 定義





波動関数

$$|\Psi\rangle = \sqrt{X}\,|\text{composite}\rangle + \sqrt{Z}\,|\text{non composite}\rangle$$

複合性 (compositeness)　　素粒子性 (elementarity)

※ $0 \le X \le 1$ ⟶ $X > 0.5 \Leftrightarrow$ **複合的**

$X + Z = 1$　　$X < 0.5 \Leftrightarrow$ **非複合的** (素粒子的)

- 内部構造の**定量的**な指標

重陽子 S. Weinberg, Phys. Rev. 137, 672–678 (1965).

$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ;
Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

T_{cc}, $X(3872)$ T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024);
L. R. Dai, L. M. Abreu, A. Feijoo, and E. Oset, Eur. Phys. J. C 83, 983 (2023) etc.

エキゾチック原子核, 原子系 T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

閾値近傍状態

◉ 短距離力の閾値近傍状態

- 散乱の閾値 : 散乱長が発散, 複合性 $X = 1$
- 束縛状態 : **複合的**

  低エネルギー普遍性が**成り立つ**

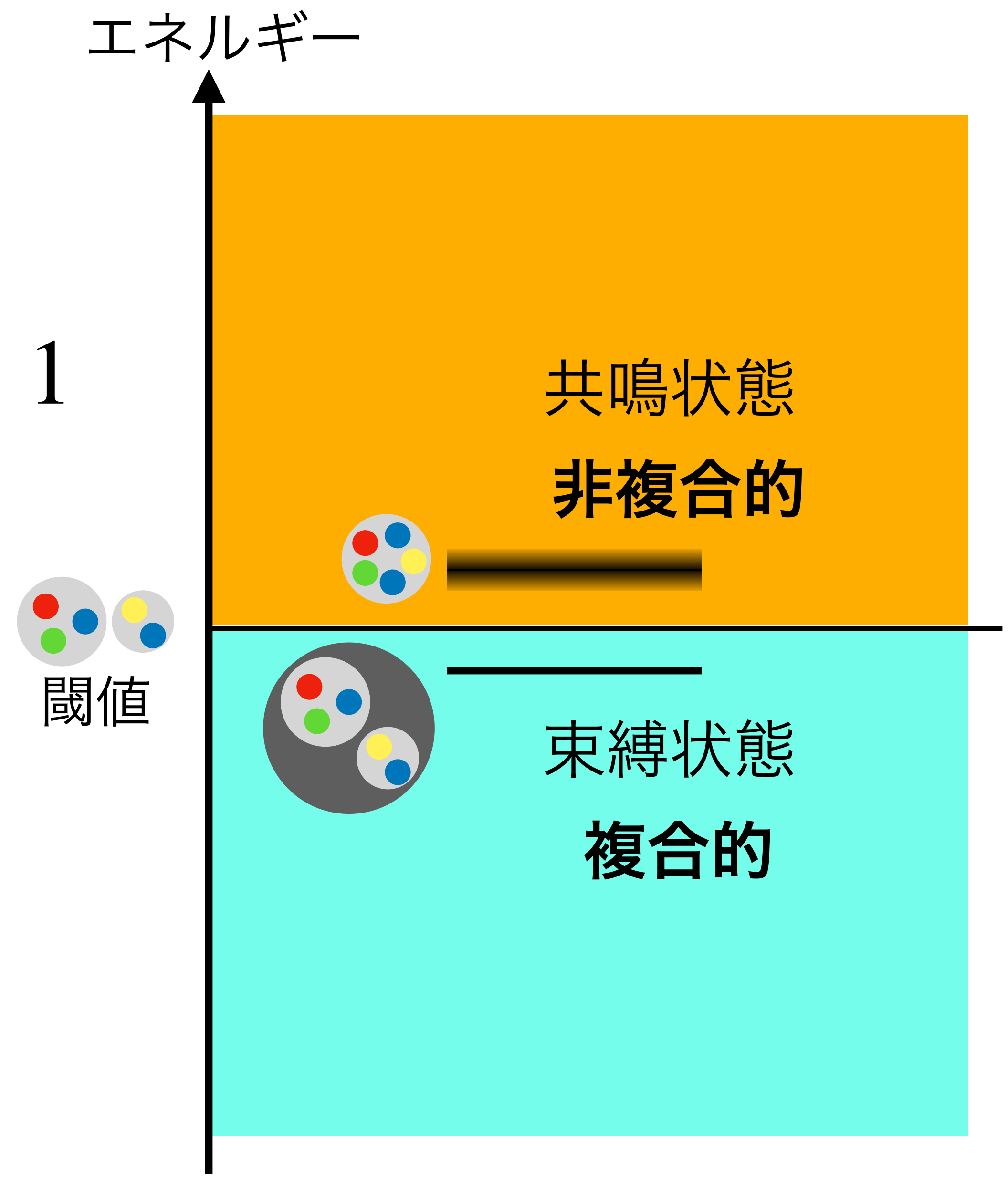
  T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;

  C. Hanhart and A. Nefediev, Phys. Rev. D 106, 114003 (2022);

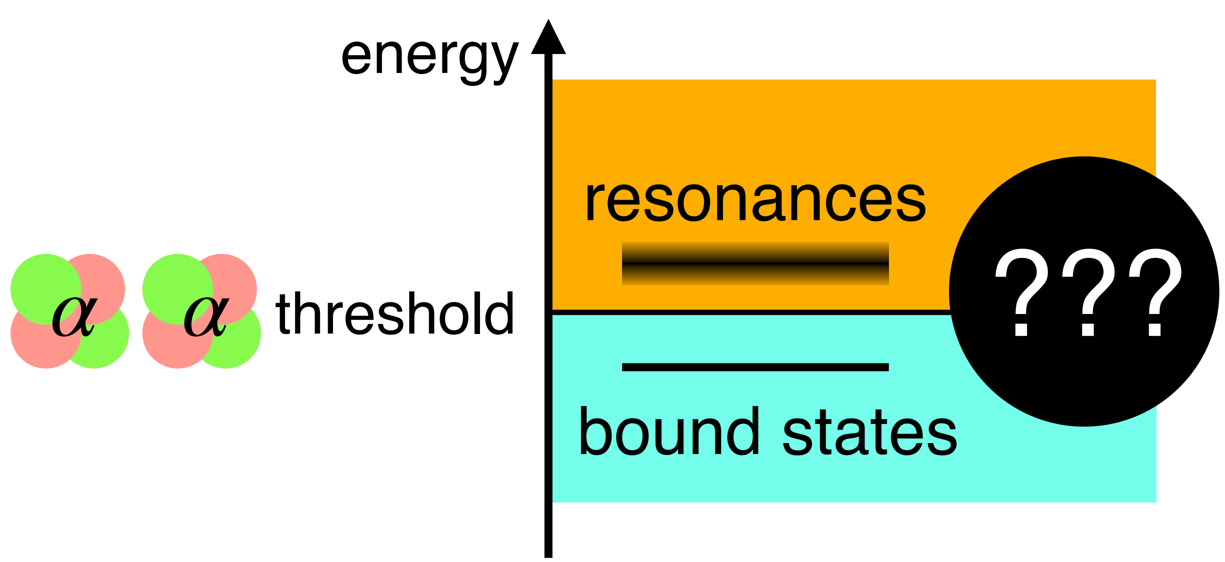
  T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024).
- 共鳴状態 : **非複合的**

  低エネルギー普遍性が**成り立たない**

  T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].



◉ 本研究でのクーロン + 短距離力の閾値近傍状態



- 2体系
- 閾値近傍 ⟶ s-波散乱
- クーロン斥力
- クーロン有効レンジ < 0

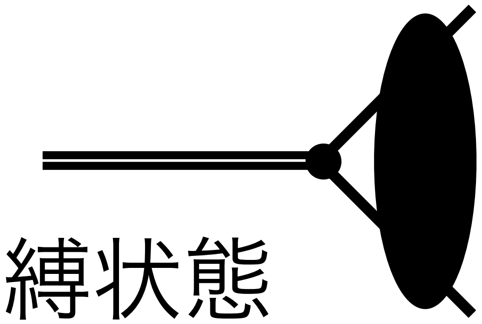
クーロン + 短距離力のモデル

◉ Hamiltonian

W. Domcke, Atom. Mol. Phys. 16 359 (1983). R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

Q channel

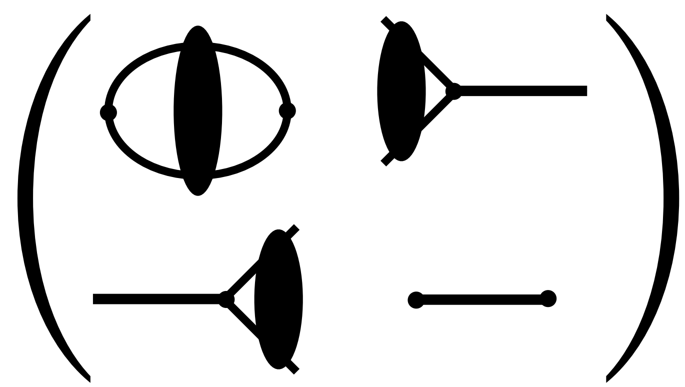
⇔ 短距離力の束縛状態



P channel

⇔ クーロン散乱

$$\hat{H} = \begin{pmatrix} \hat{H}_{PP} & \hat{H}_{PQ} \\ \hat{H}_{QP} & \hat{H}_{QQ} \end{pmatrix} =$$



H. Feshbach, Annals Phys. 19 287-313 (1962).

◉ pole の条件式

H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798 (2019).

$$-\frac{1}{\boxed{a_s}} + \frac{\boxed{r_e}}{2}k^2 - ik \pm \frac{2}{\boxed{a_B}}\left[\underbrace{\log(-ia_Bk)}_{\text{log cut}} + \psi\left(1 + \frac{i}{a_Bk}\right)\right] = 0$$

◉ 複合性 X

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

$$X = 1 - \frac{1}{1 - \frac{d}{dE}\boxed{F(E)}}$$ 自己エネルギー

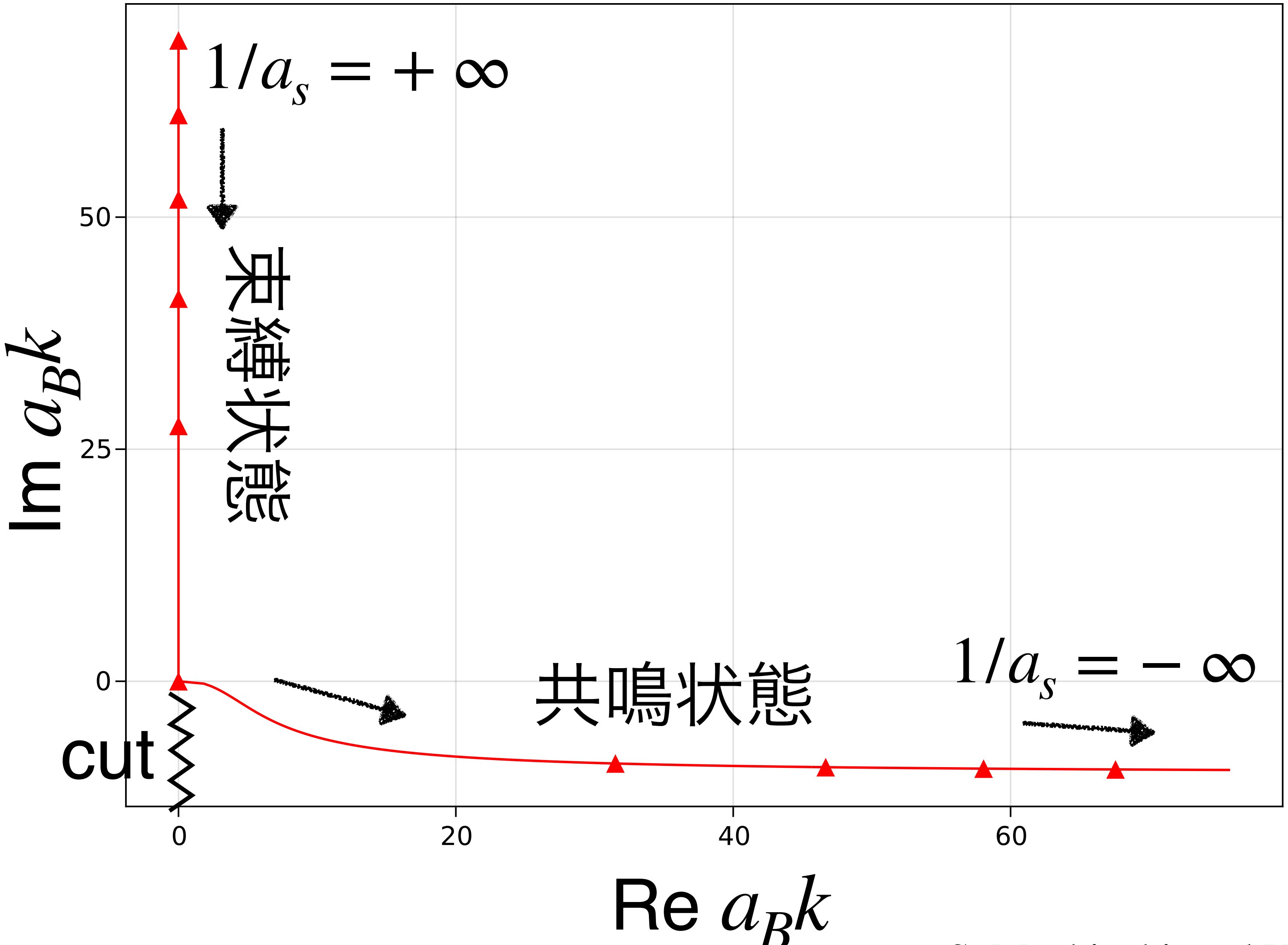
Bohr radius

$$a_B = \frac{\hbar c}{\mu c^2 Z_1 Z_2}$$

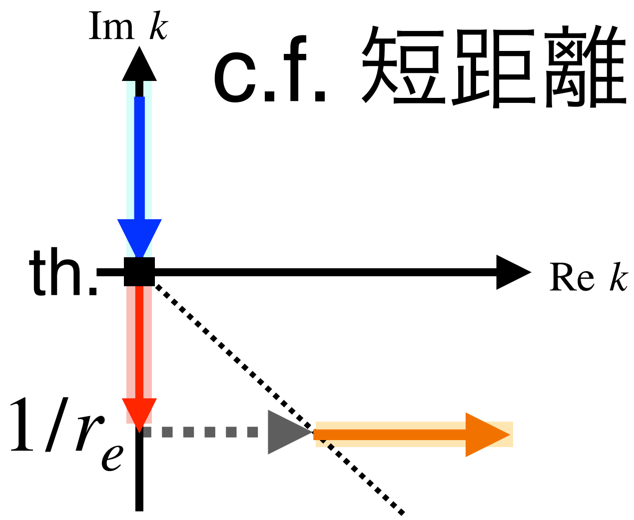
Poleの軌跡 (固有運動量の変化)

◉ Poleの軌跡 in 運動量 k 平面

- クーロン有効レンジ r_e と ボーア半径 a_B を固定
- クーロン散乱長 a_s を変化



- 束縛 → 共鳴



c.f. 短距離力のみ

束縛 → virtual → 共鳴

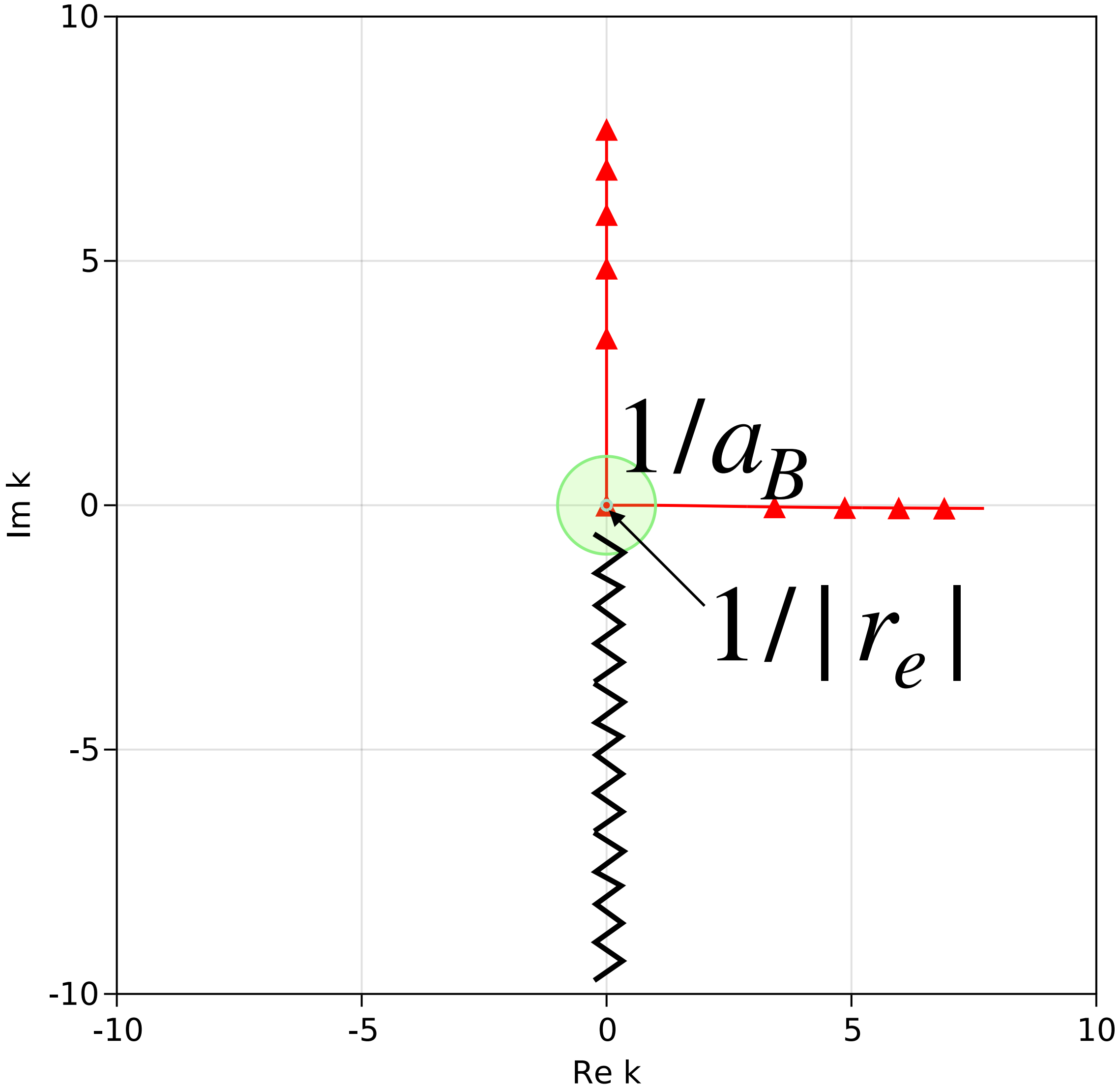
- 閾値で $a_s \to \infty$
- but 普遍性は**ない**

∵ w.f. の広がり $< \infty$

S. Mochizuki, and Y. Nishida, Phys. Rev. C 110 , 064001 (2024).

Poleの軌跡 - 2種類のクーロン有効レンジ r_e

$$a_B = 1,\ r_e = -10$$

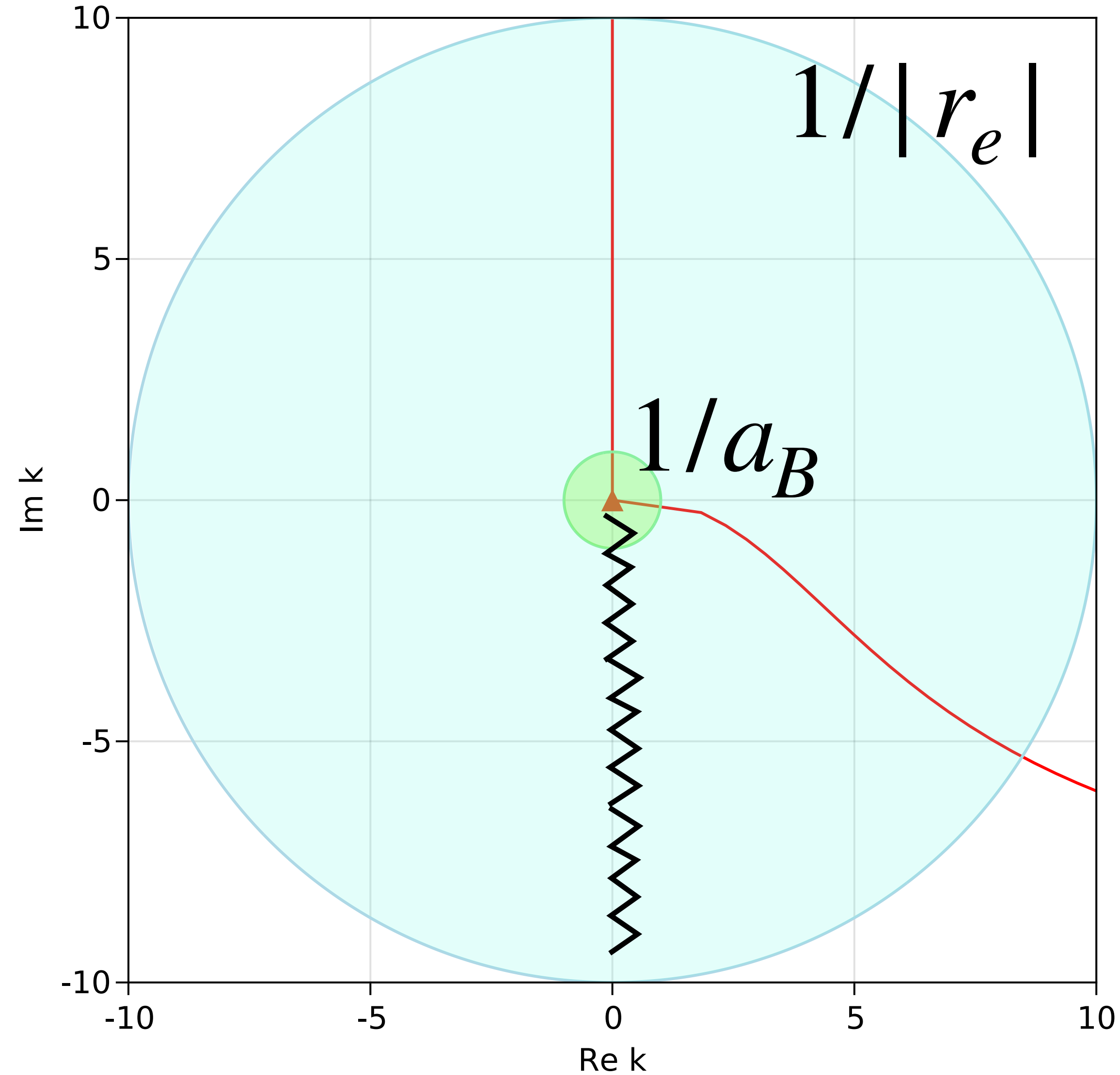


短距離力の普遍性の領域

∧

クーロン力が支配的な領域

$$a_B = 1,\ r_e = -0.1$$



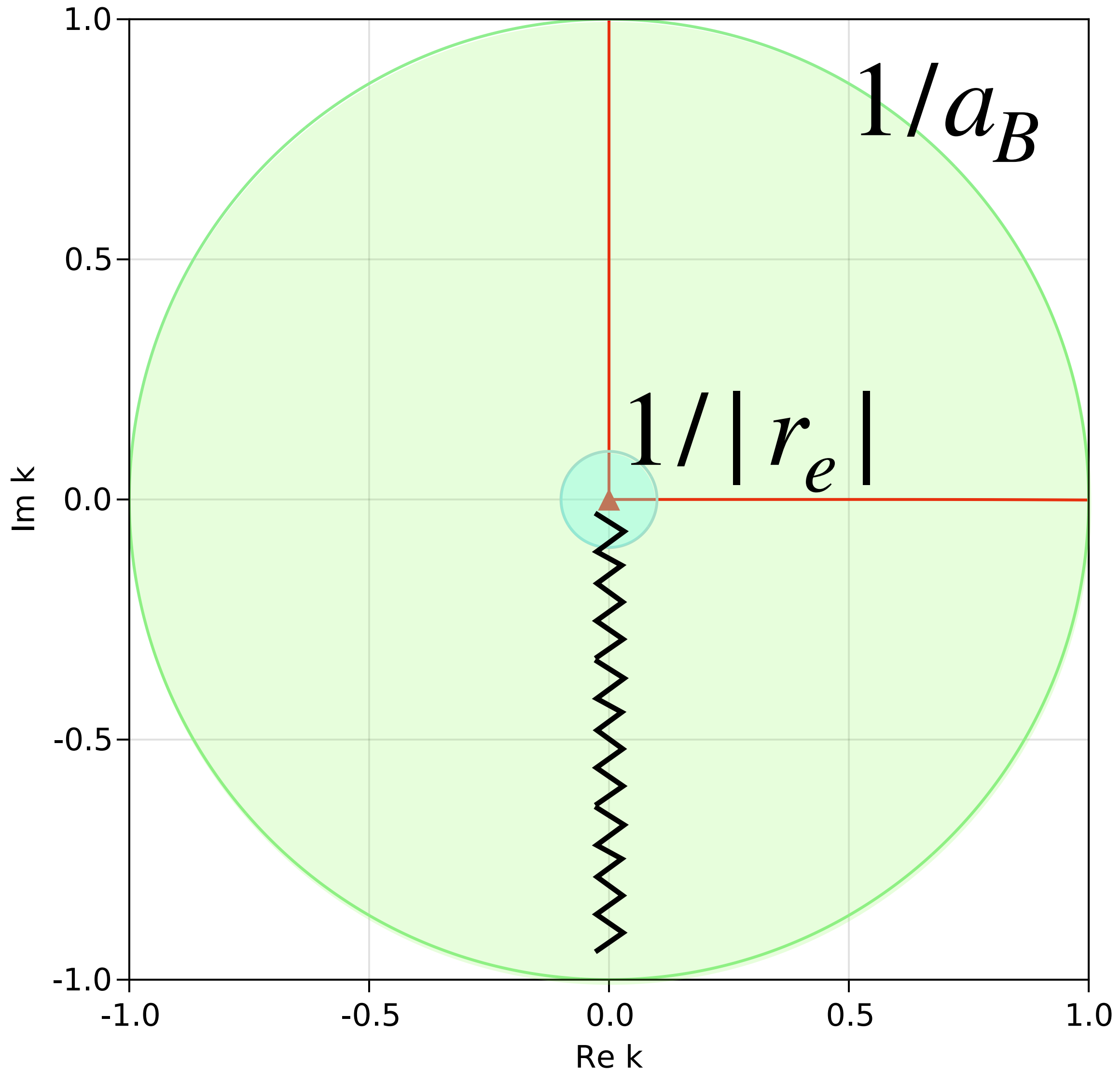
短距離力の普遍性の領域

∨

クーロン力が支配的な領域

Poleの軌跡 - 2種類のクーロン有効レンジ r_e

$a_B = 1, r_e = -10$

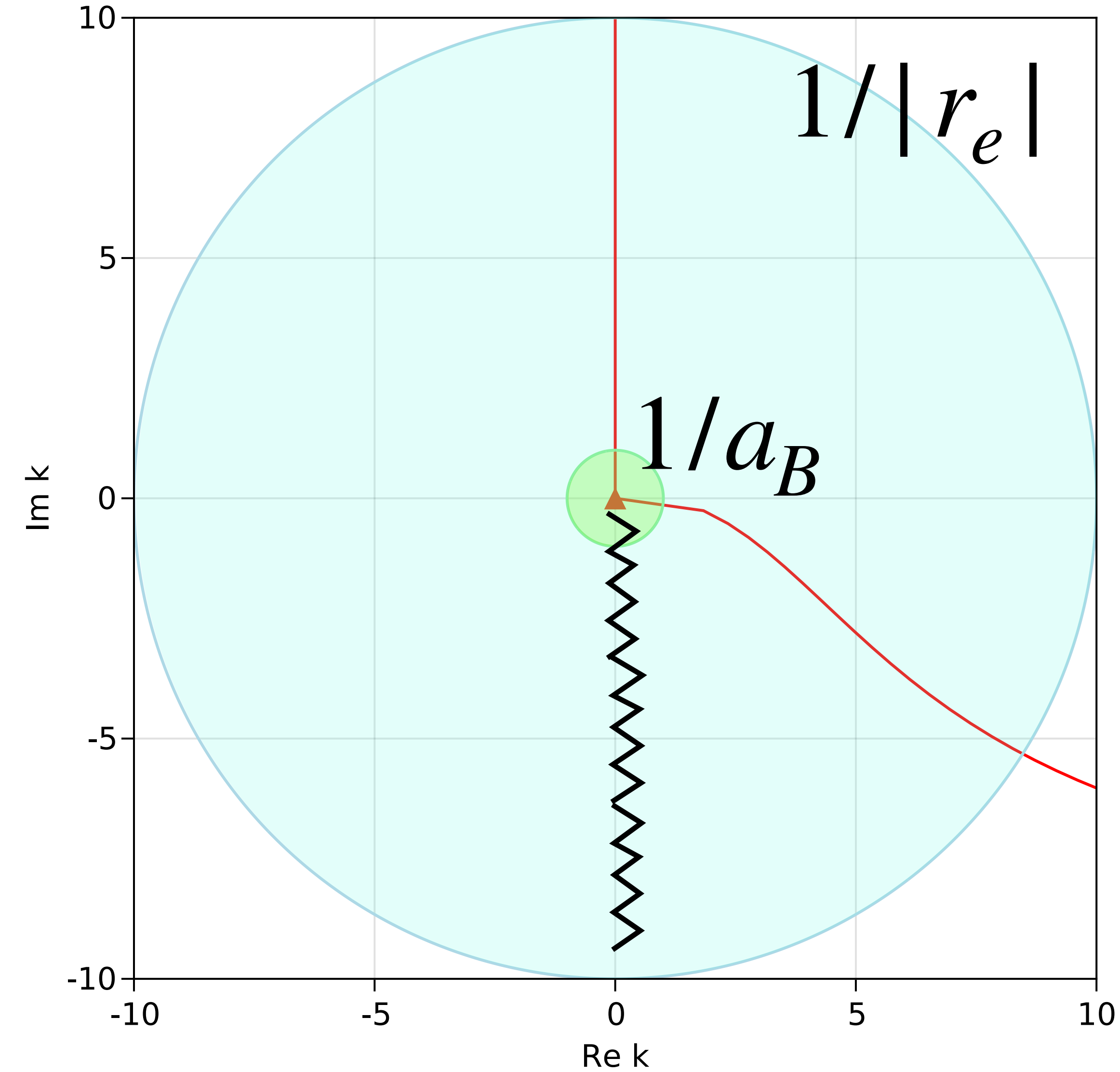


短距離力の普遍性の領域

$\wedge$

クーロン力が支配的な領域

$a_B = 1, r_e = -0.1$



短距離力の普遍性の領域

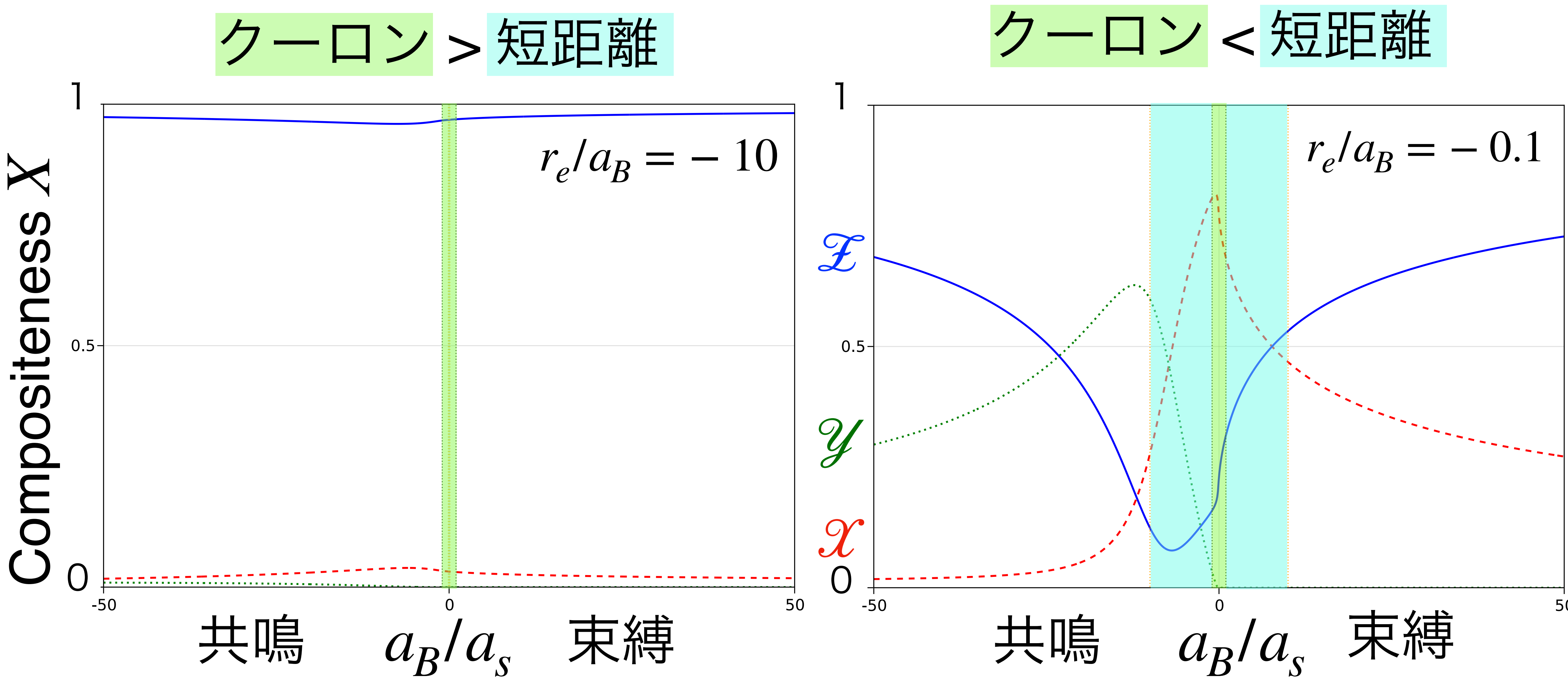
$\vee$

クーロン力が支配的な領域

複合性 X

複素複合性の解釈 ⟶ $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$

T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].



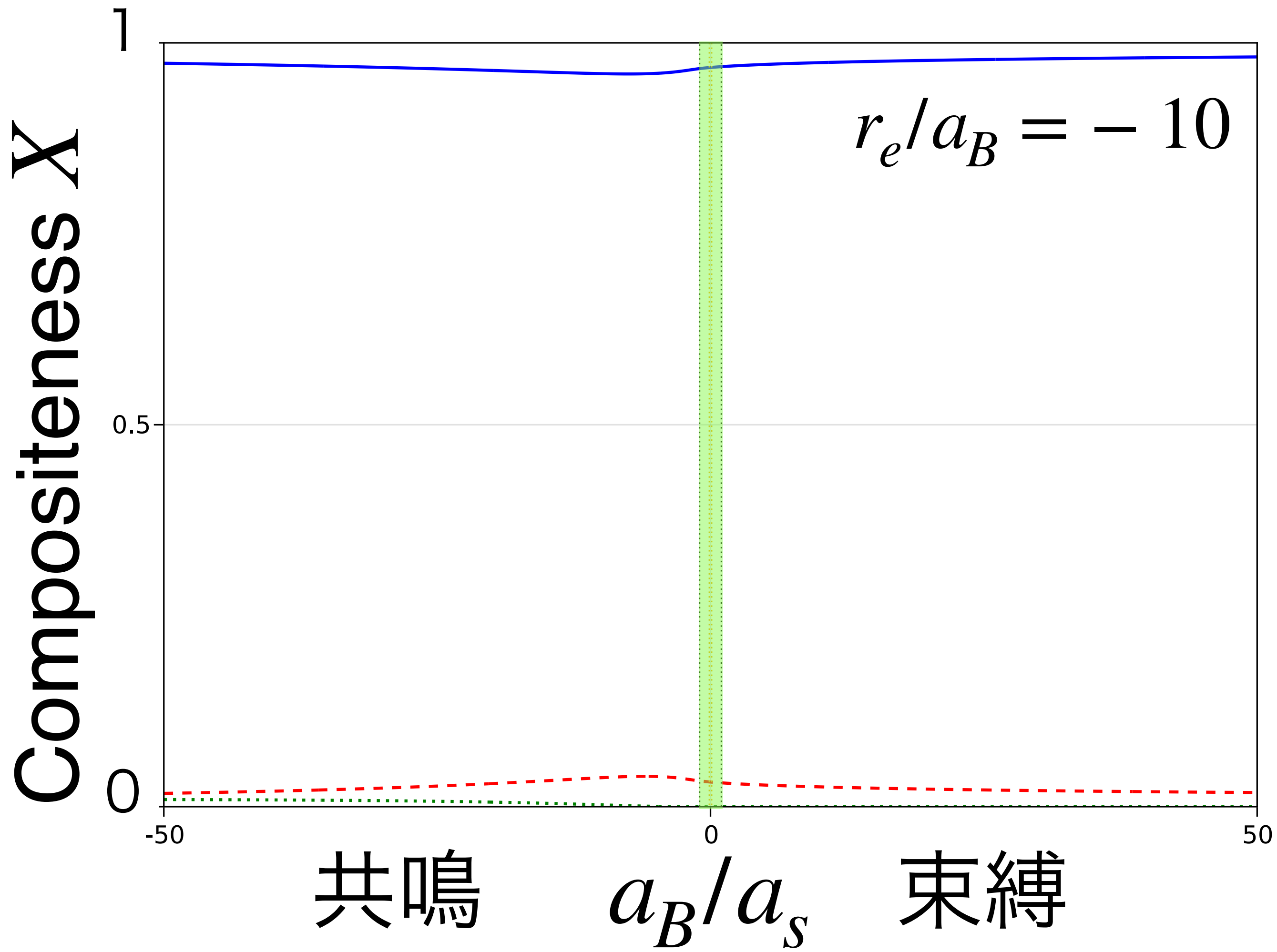
- 閾値から遠いと非複合的
- 束縛状態の構造 ≈ 共鳴状態の構造 ∵ 複合性が閾値で連続

複合性 X

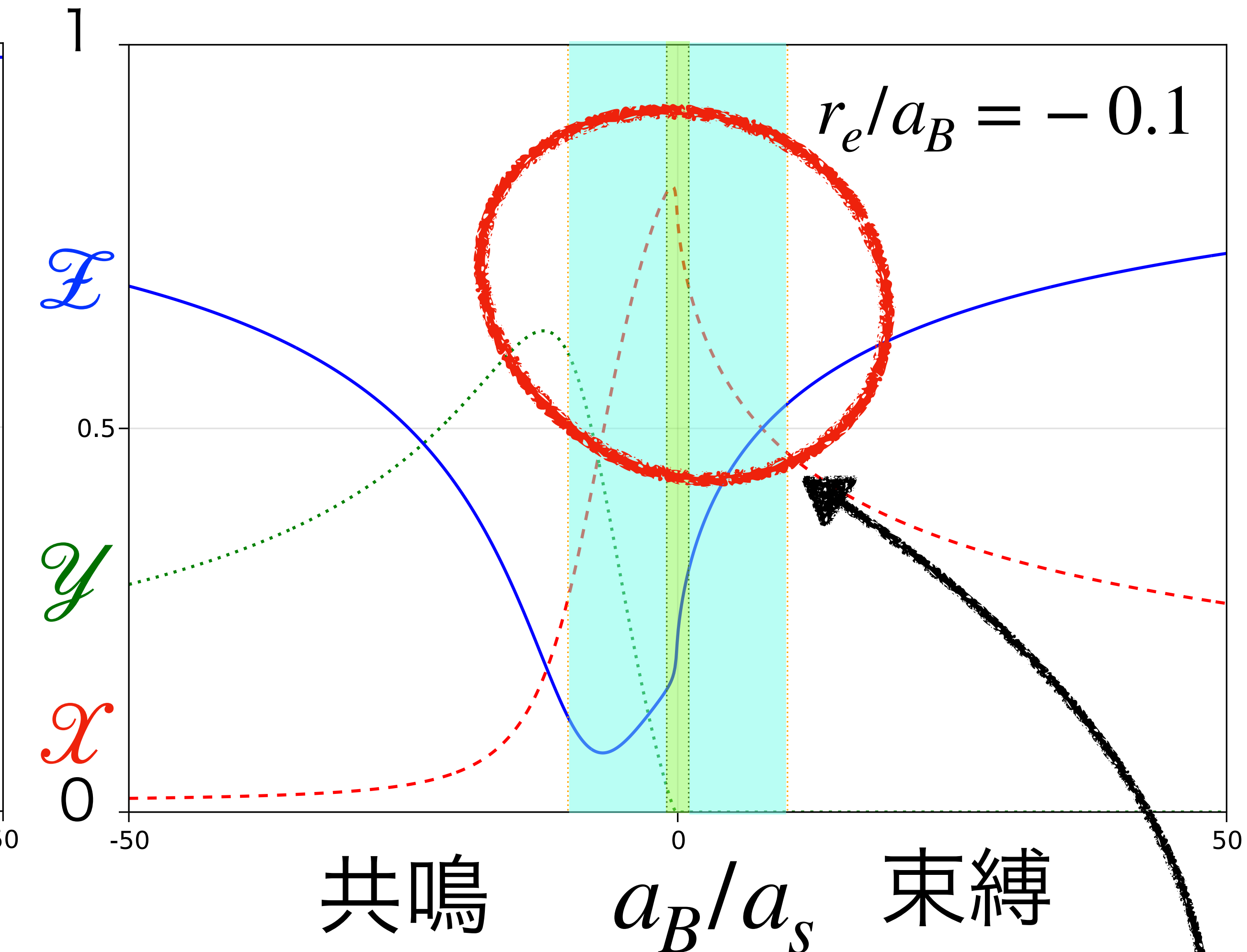
複素複合性の解釈 → $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$

T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

クーロン > 短距離



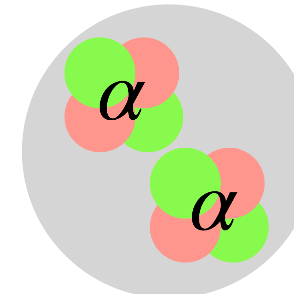
クーロン < 短距離

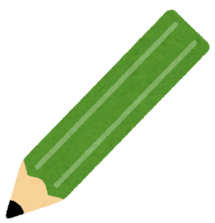


- 閾値から遠いと非複合的
- 束縛状態の構造 ≈ 共鳴状態の構造 ∵ 複合性が閾値で連続
- クーロンが弱い場合 ($|r_e| \ll |a_B|$) 短距離力の普遍性 ($B \to 0$ 極限で $X \to 1$) の名残がある

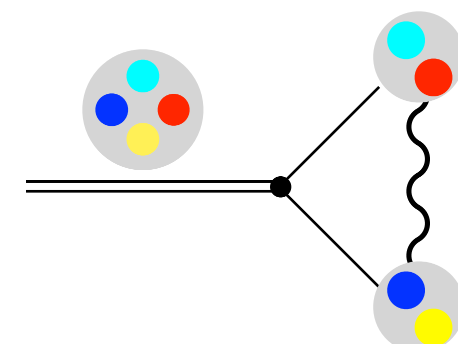
まとめ

クーロン+短距離力の系の閾値近傍状態





- クーロン散乱と短距離力の束縛状態が結合したモデル
- poleの条件式 ⟵ $a_s,\ r_e,\ a_B$



- クーロン斥力 + 短距離力

poleの軌跡: 束縛 → 共鳴

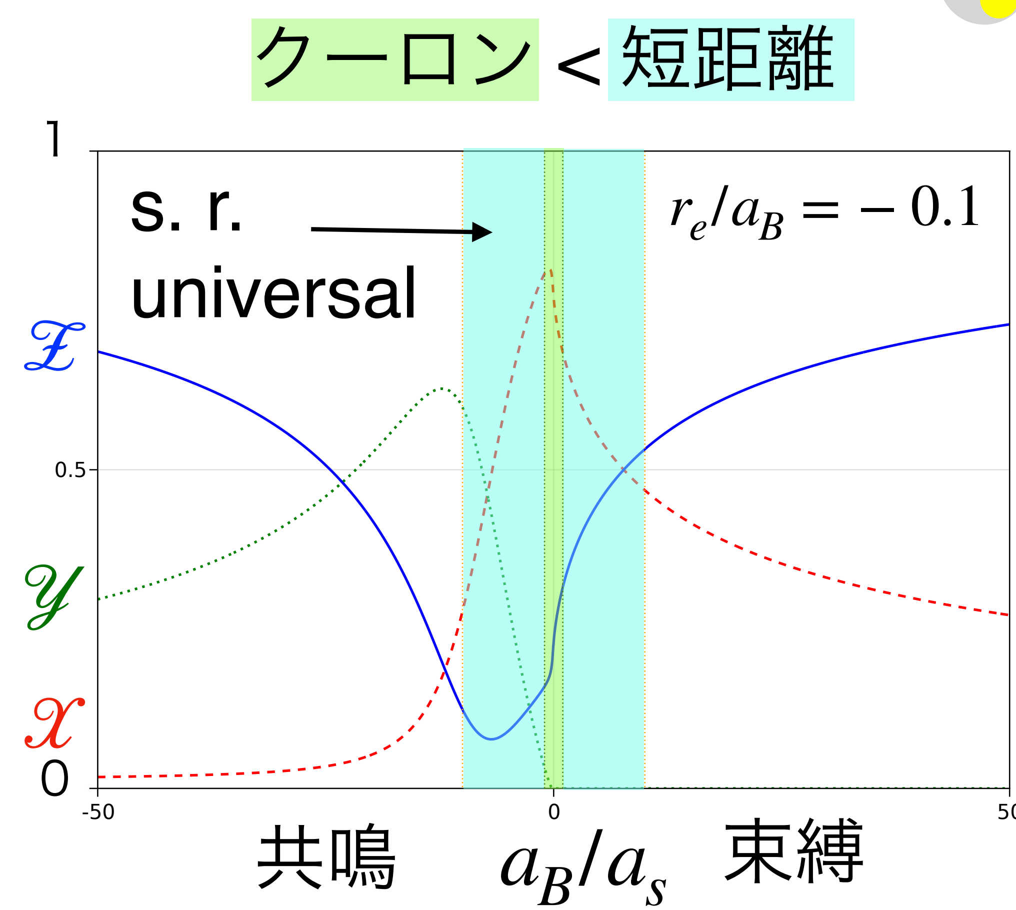
閾値近傍 ⟷ 複合的とは

限らない

but…

クーロン力と短距離力の競合

によっては普遍性の名残あり

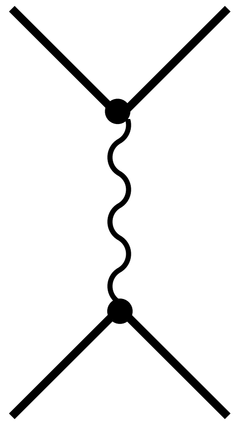


Back up

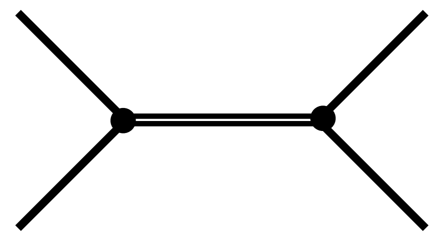
Coulomb+short range model

◉ model

Coulomb



short range (s.r.)



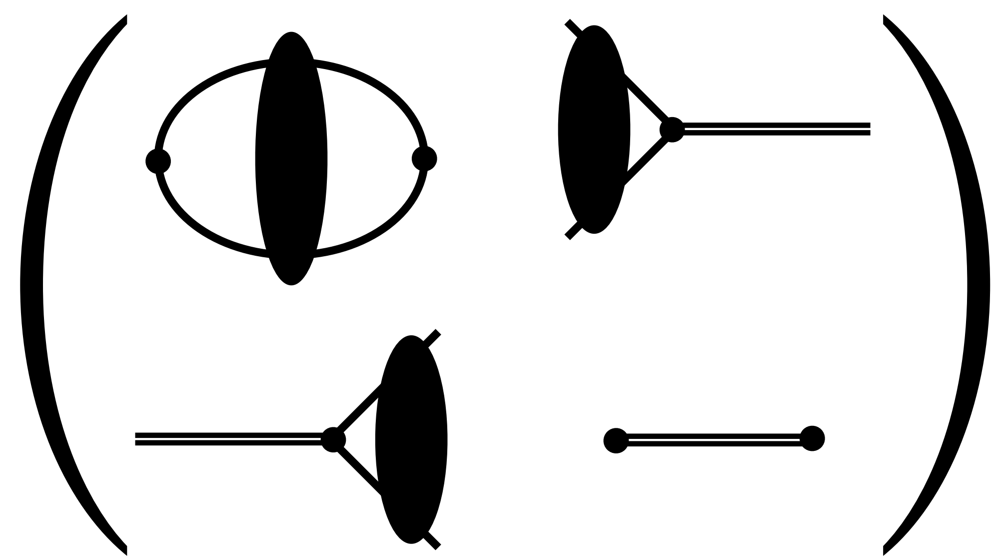
Weinberg, S. Phys. Rev. 137, 672–678 (1965);
V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004);
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

◉ Hamiltonian

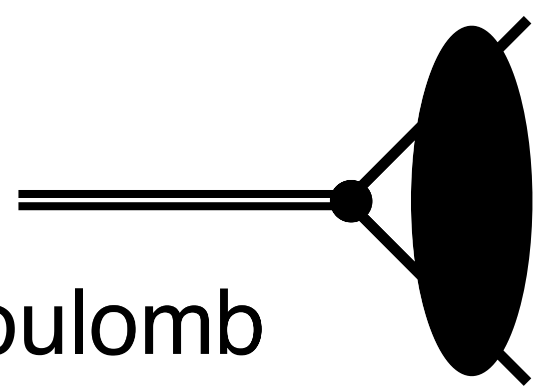
W. Domcke, Atom. Mol. Phys. 16 359 (1983);
H. Feshbach, Annals Phys. 19 287-313 (1962).

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

$$\hat{H} = \begin{pmatrix} \hat{H}_{PP} & \hat{H}_{PQ} \\ \hat{H}_{QP} & \hat{H}_{QQ} \end{pmatrix} =$$



Q channel
⇔ b.s. w/o Coulomb



P channel
⇔ scattering w/ Coulomb

Coulomb+short range model

◉ Schrödinger equation $\hat{H}|\Psi\rangle = E|\Psi\rangle \quad |\Psi\rangle = \begin{pmatrix} |P\rangle \\ |Q\rangle \end{pmatrix}$

$$\hat{H}_{PP}|P\rangle + \hat{H}_{PQ}|Q\rangle = E|P\rangle$$
$$\hat{H}_{QQ}|Q\rangle + \hat{H}_{QP}|P\rangle = E|Q\rangle$$

◉ effective Hamiltonian (channel eliminating)

$$\hat{H}_{P\mathrm{ch}}|P\rangle = E|P\rangle \quad \hat{H}_{P\mathrm{ch}} = \hat{H}_{PP} + \hat{H}_{PQ}(E - \hat{H}_{QQ})^{-1}\hat{H}_{QP}$$

Coulomb+short range model

◉ Schrödinger equation $\hat{H}|\Psi\rangle = E|\Psi\rangle \quad |\Psi\rangle = \begin{pmatrix} |P\rangle \\ |Q\rangle \end{pmatrix}$

$$\hat{H}_{PP}|P\rangle + \hat{H}_{PQ}|Q\rangle = E|P\rangle$$

$$\hat{H}_{QQ}|Q\rangle + \hat{H}_{QP}|P\rangle = E|Q\rangle$$

◉ effective Hamiltonian (channel eliminating)

$$\hat{H}_{P\mathrm{ch}}|P\rangle = E|P\rangle \quad \hat{H}_{P\mathrm{ch}} = \underbrace{\boxed{\hat{H}_{PP}}}_{= \hat{H}^0 + \hat{V}_P} + \underbrace{\boxed{\hat{H}_{PQ}(E - \hat{H}_{QQ})^{-1}\hat{H}_{QP}}}_{= \hat{V}_Q}$$

$$\longrightarrow \hat{H}_{P\mathrm{ch}} = \hat{H}^0 + (\hat{V}_P + \hat{V}_Q)$$

$\hat{H}^0$: free Hamiltonian

$\hat{V}_P$: pure Coulomb interaction

$\hat{V}_Q$: short range interaction

Coulomb+short range model

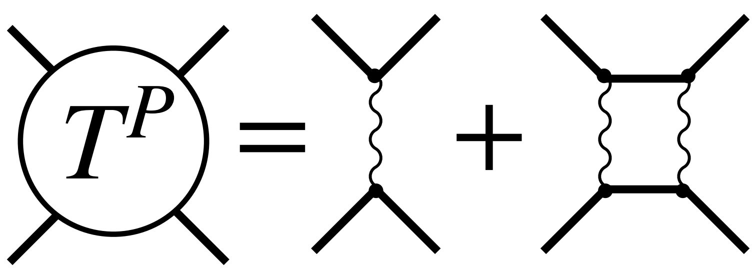
◉ T-matrix

H. Feshbach, Annals Phys. 19 287-313 (1962); W. Domcke, Atom. Mol. Phys. 16 359 (1983); R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

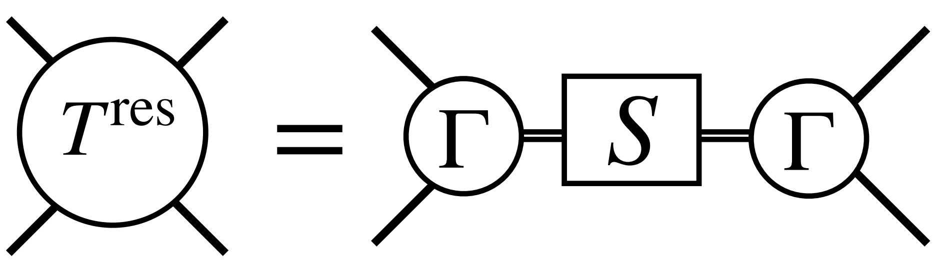
Lippmann-Schwinger eq. : $\hat{T} = [(\hat{V}_P + \hat{V}_Q)^{-1} - \hat{G}^0]^{-1}$

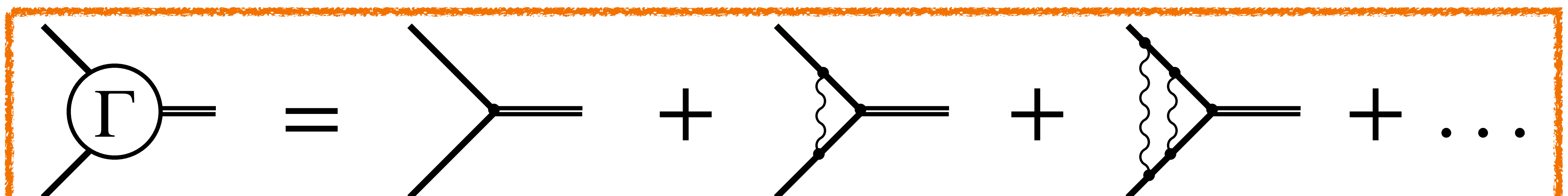
⇔ Feshbach method : $\hat{T} = \hat{T}^P + \hat{T}^{\rm res}$

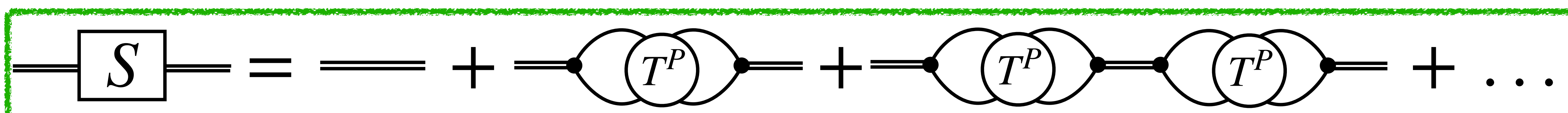
$$\hat{T}^P = [\hat{V}_P^{-1} - \hat{G}^0]^{-1}, \quad \hat{T}^{\rm res} = \boxed{\hat{T}^P \hat{V}_P^{-1}} \boxed{[\hat{V}_Q^{-1} - \hat{G}_P]^{-1}} \boxed{\hat{V}_P^{-1} \hat{T}^P}$$



pure Coulomb



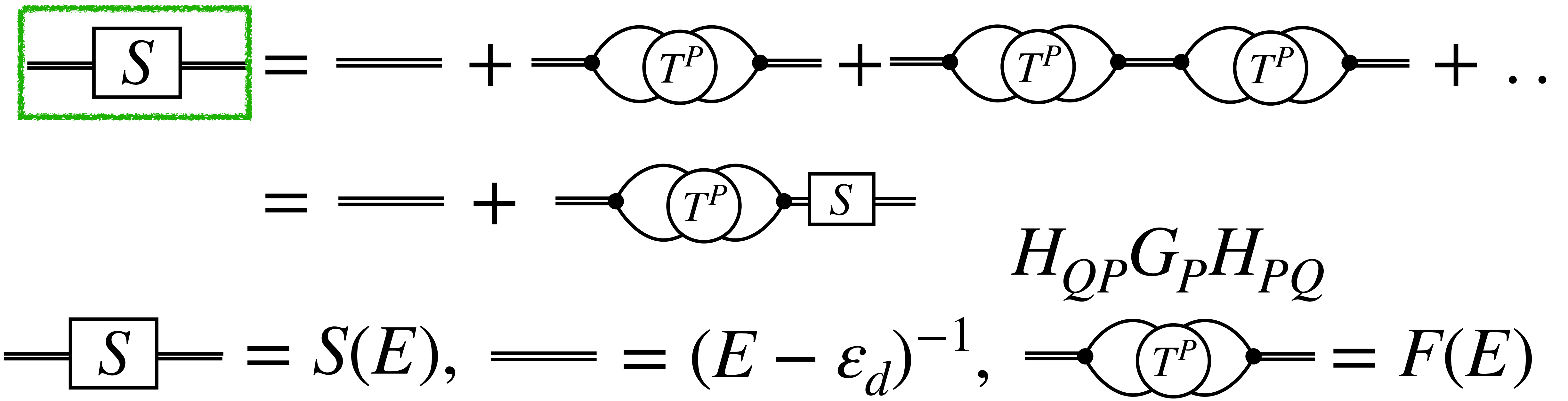




Coulomb+short range model

◉ pole condition $\hat{T} = \hat{T}^P + \hat{T}^{\rm res}$

pole of $T(k,k')$ $\Leftrightarrow$ pole of $T^{\rm res}(k,k')$ $\Leftrightarrow \boxed{[V_Q^{-1} - G_P]^{-1}} = \infty$



$\longrightarrow S(E) = (E-\varepsilon_d)^{-1} + (E-\varepsilon_d)^{-1}F(E)S(E)$

$= [E - \varepsilon_d - F(E)]^{-1}$

$\longrightarrow$ pole condition : $E - \boxed{\varepsilon_d} - \boxed{F(E)} = 0$

bare state energy self energy

Coulomb+short range model

◉ self energy $F(E)$ **in low-energy limit**

W. Domcke, Atom. Mol. Phys. 16 359 (1983).

- attractive Coulomb

$$F(k) = \frac{A}{2\pi}\left[c - \frac{1}{2} i a_B k + \log(-i a_B k) + \psi\left(1 - \frac{i}{a_B k}\right)\right]$$

- repulsive Coulomb

$$F(k) = -\frac{A}{2\pi}\left[c + \frac{1}{2} i a_B k + \log(-i a_B k) + \psi\left(1 + \frac{i}{a_B k}\right)\right]$$

A : constant with dimension of energy

c : dimensionless constant

$\psi(x) = \frac{d}{dx}\log(\Gamma(x))$: digamma function

Coulomb+short range model

◉ pole condition **in low-energy limit**

- Coulomb scattering length a_s and effective range r_e

$$(\mathrm{amplitude})^{-1} = -\frac{1}{a_s} + \frac{r_e}{2}k^2 + \mathcal{O}(k^4) - ik + 2\log(-ik) + 2\psi\left(1 + \frac{i}{k}\right) + \ldots,$$

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

$$\longrightarrow a_s = -a_B\left[\frac{4\pi}{A}\varepsilon_d \pm 2c\right]^{-1}, \quad r_e = -\frac{4\pi}{Aa_B\mu}$$

- pole condition with a_s and r_e H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B}\left[\log(-ia_Bk) + \psi\left(1 + \frac{i}{a_Bk}\right)\right] = 0$$

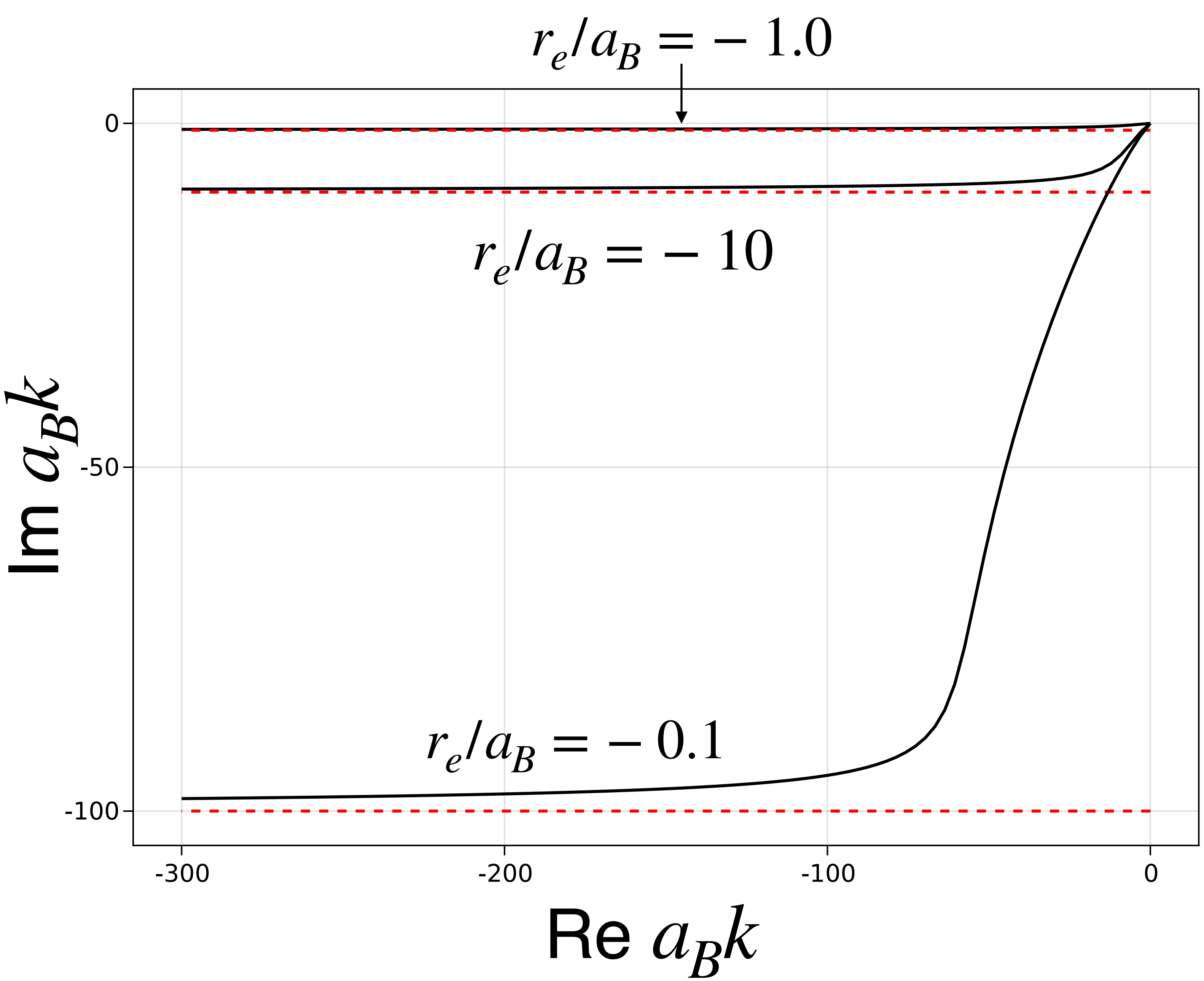
◉ compositeness X T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

$$X = 1 - \frac{1}{1 - \frac{d}{dE}\boxed{F(E)}}$$ self energy

far from threshold (repulsive Coulomb)

◉ imaginary part of eigenenergy in complex momentum k plane

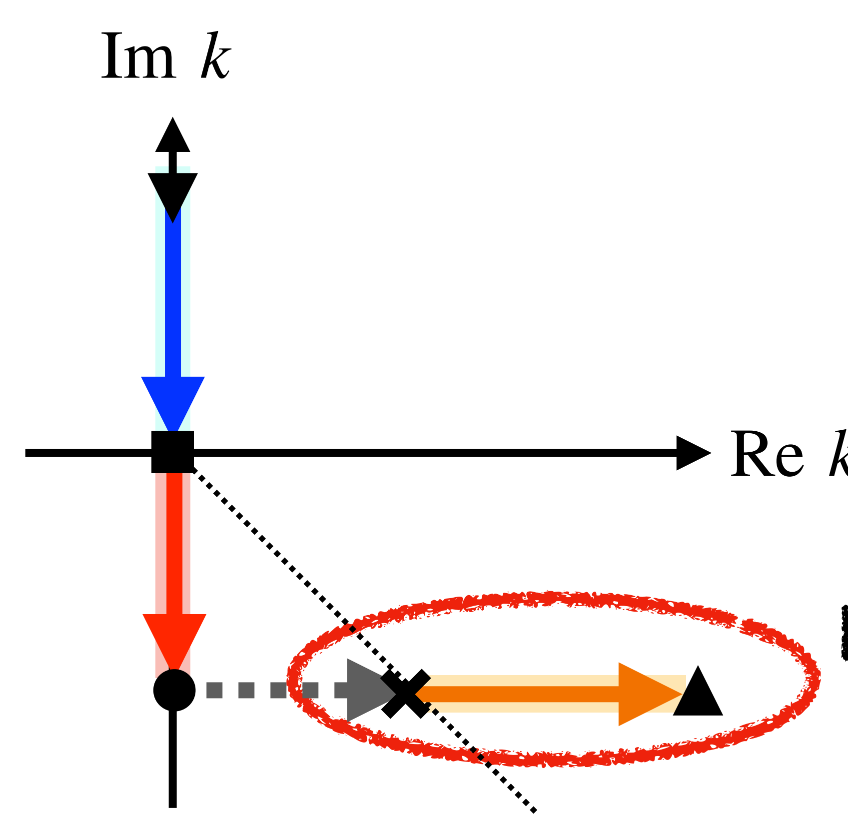
- far from threshold in $1/a_s \to -\infty$ limit



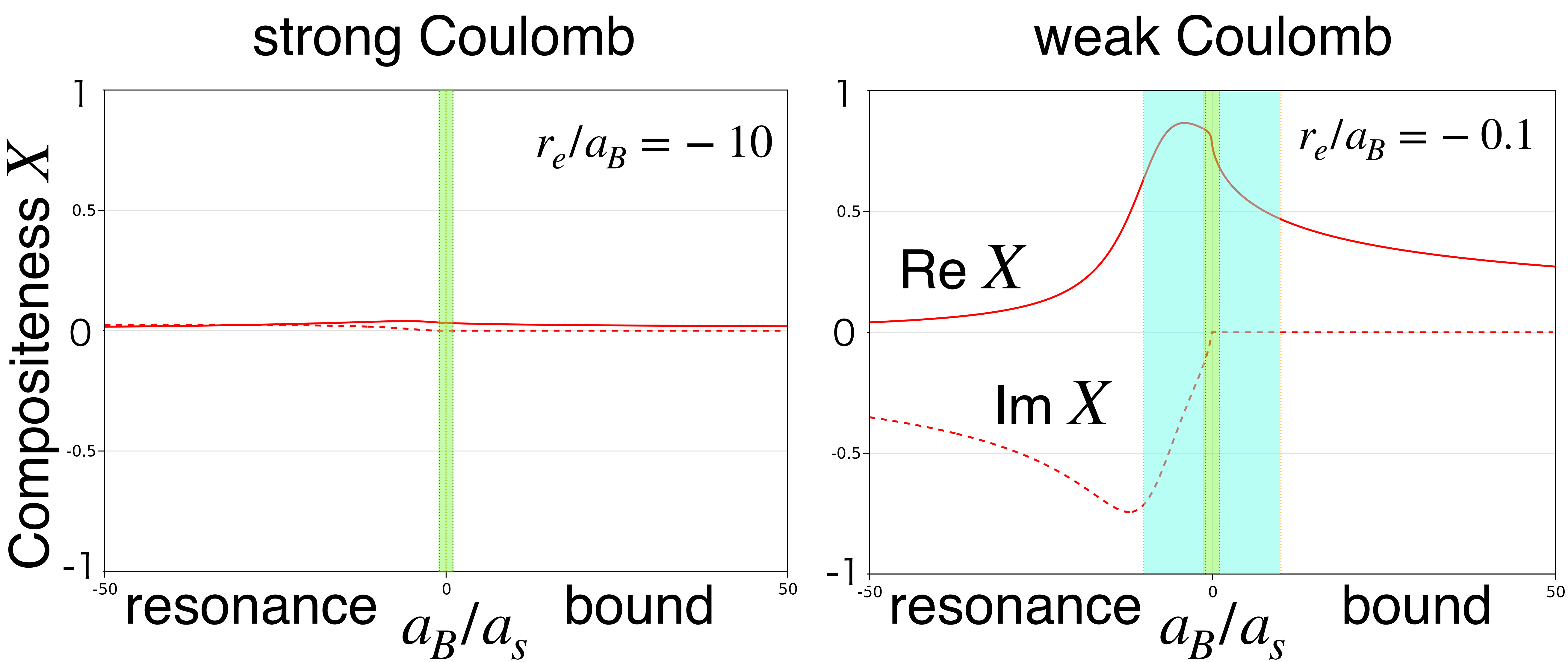
dotted lines : a_B/r_e

- Im $k \to 1/r_e$

→ trajectory close to that of resonance in ERE

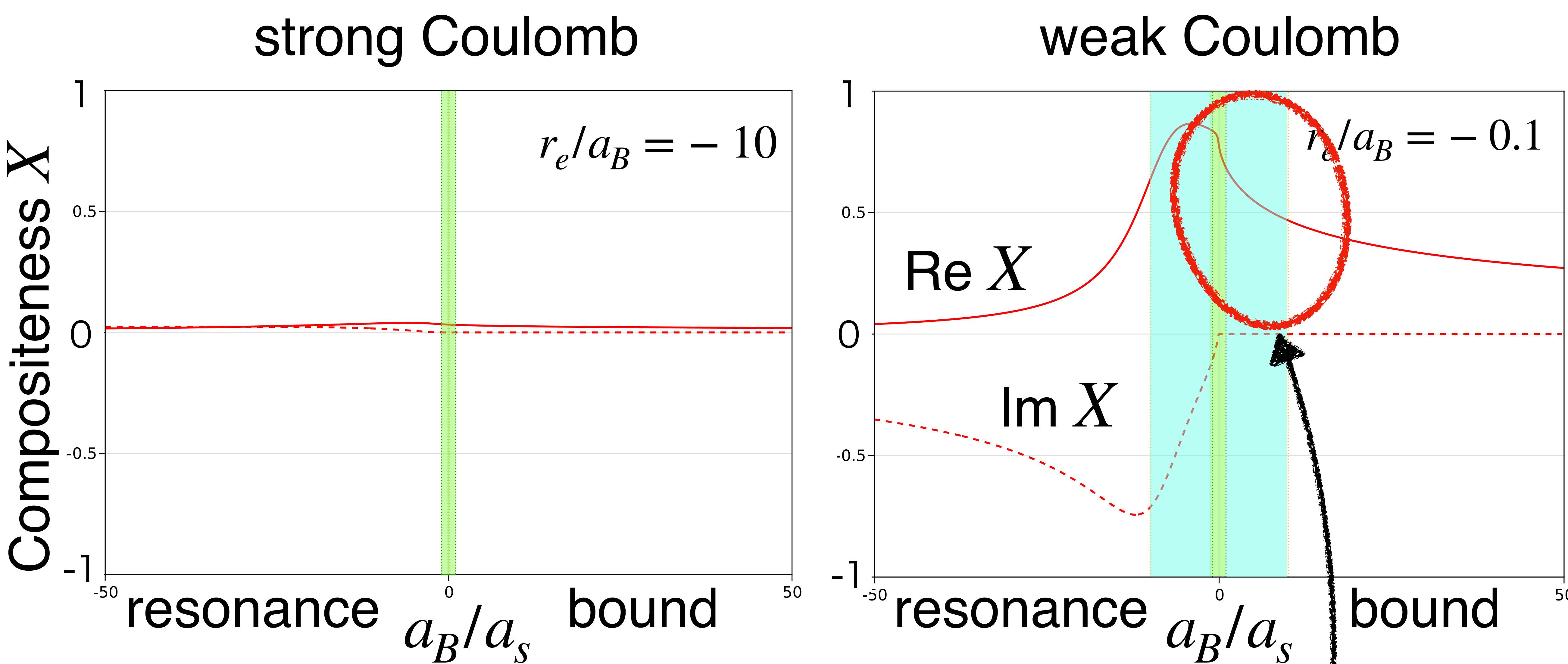


Compositeness (repulsive Coulomb)



- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region

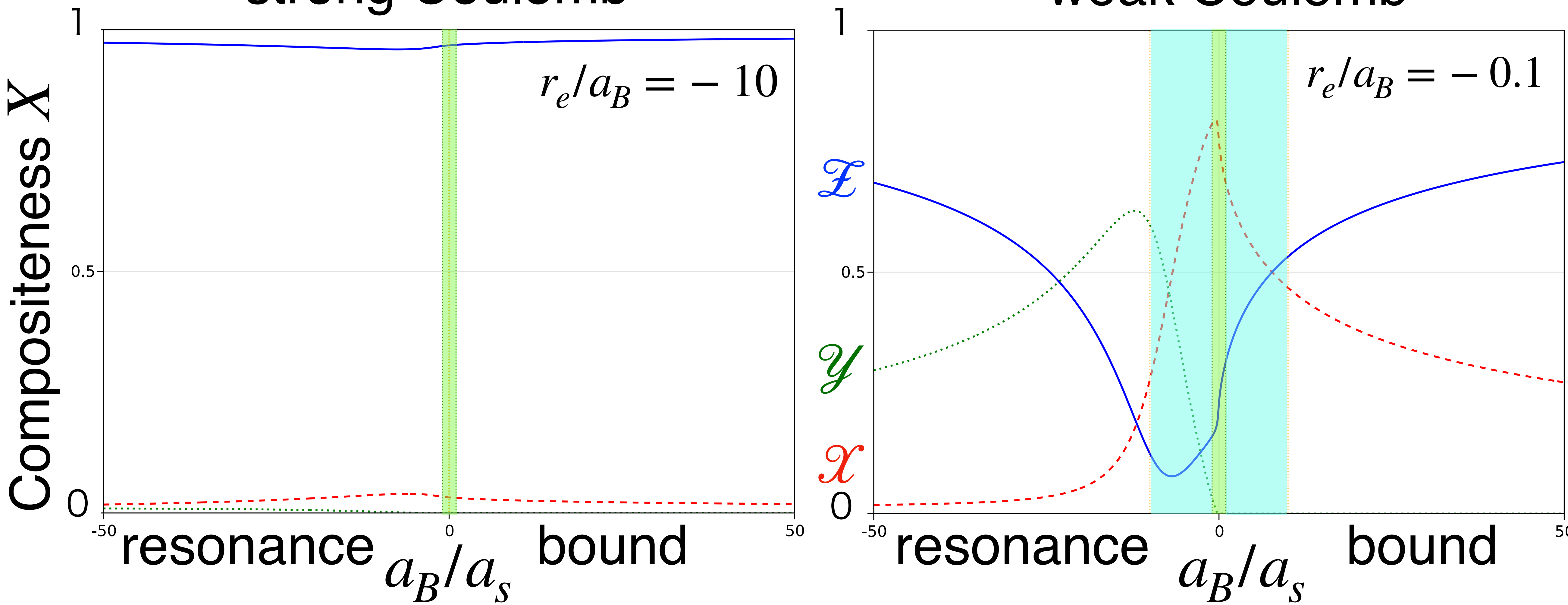
Compositeness (repulsive Coulomb)



- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- remnant of short range universality in $|r_e| \ll |a_B|$ case
  $X \to 1$ in $B \to 0$ limit in short range

Compositeness (repulsive Coulomb)

strong Coulomb | weak Coulomb

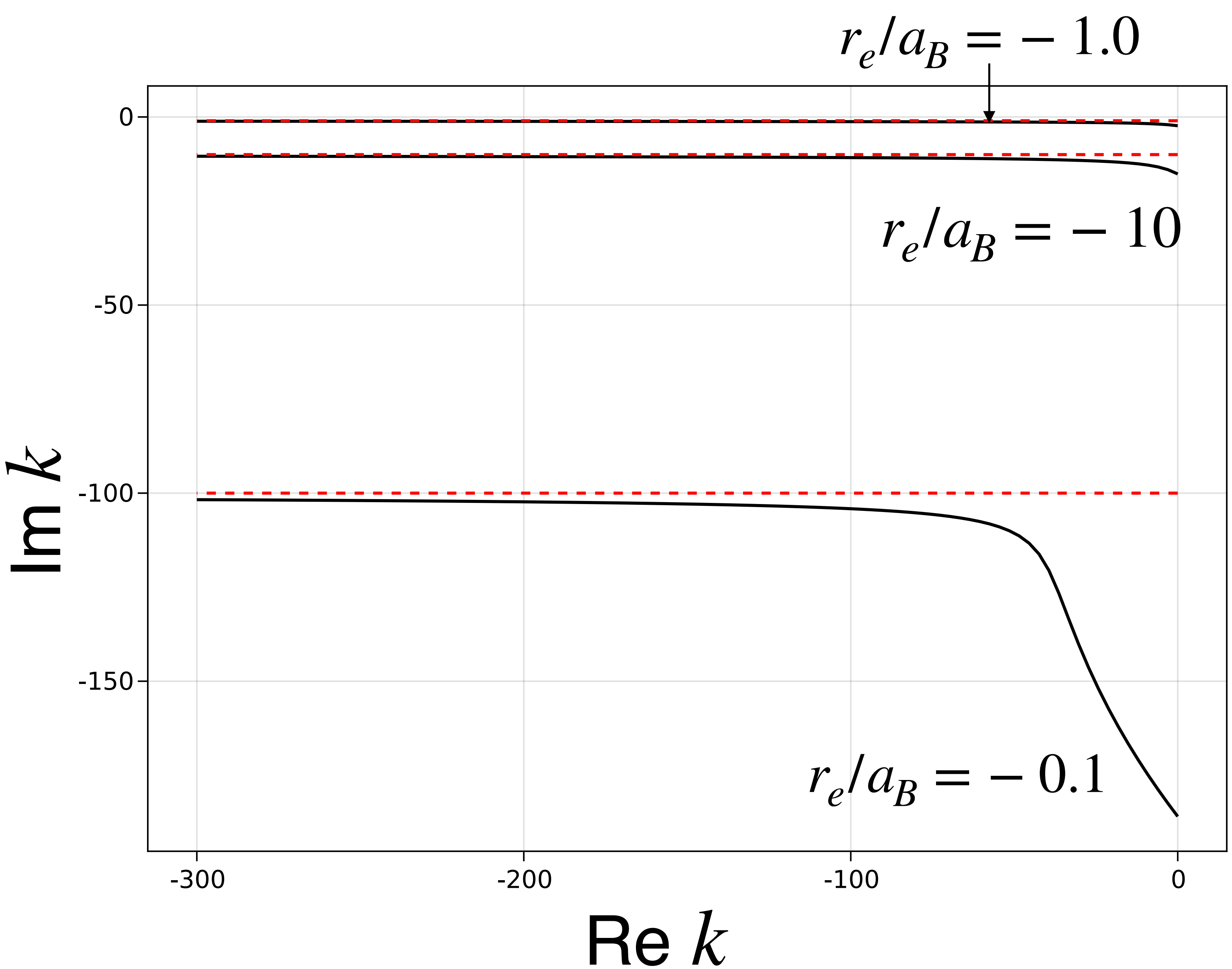


- compositeness of resonances ⟵ $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].
- all states are interpretable ∵ no virtual states
- states with large $|1/a_s|$ are elementary $\mathscr{Z}$ dominant
- nature of bound states = nature of resonances
  ∵ X is continuous across threshold

far from threshold (attractive Coulomb)

◉ imaginary part of eigenenergy in complex momentum k plane

- far from threshold in $1/a_s \to -\infty$ limit



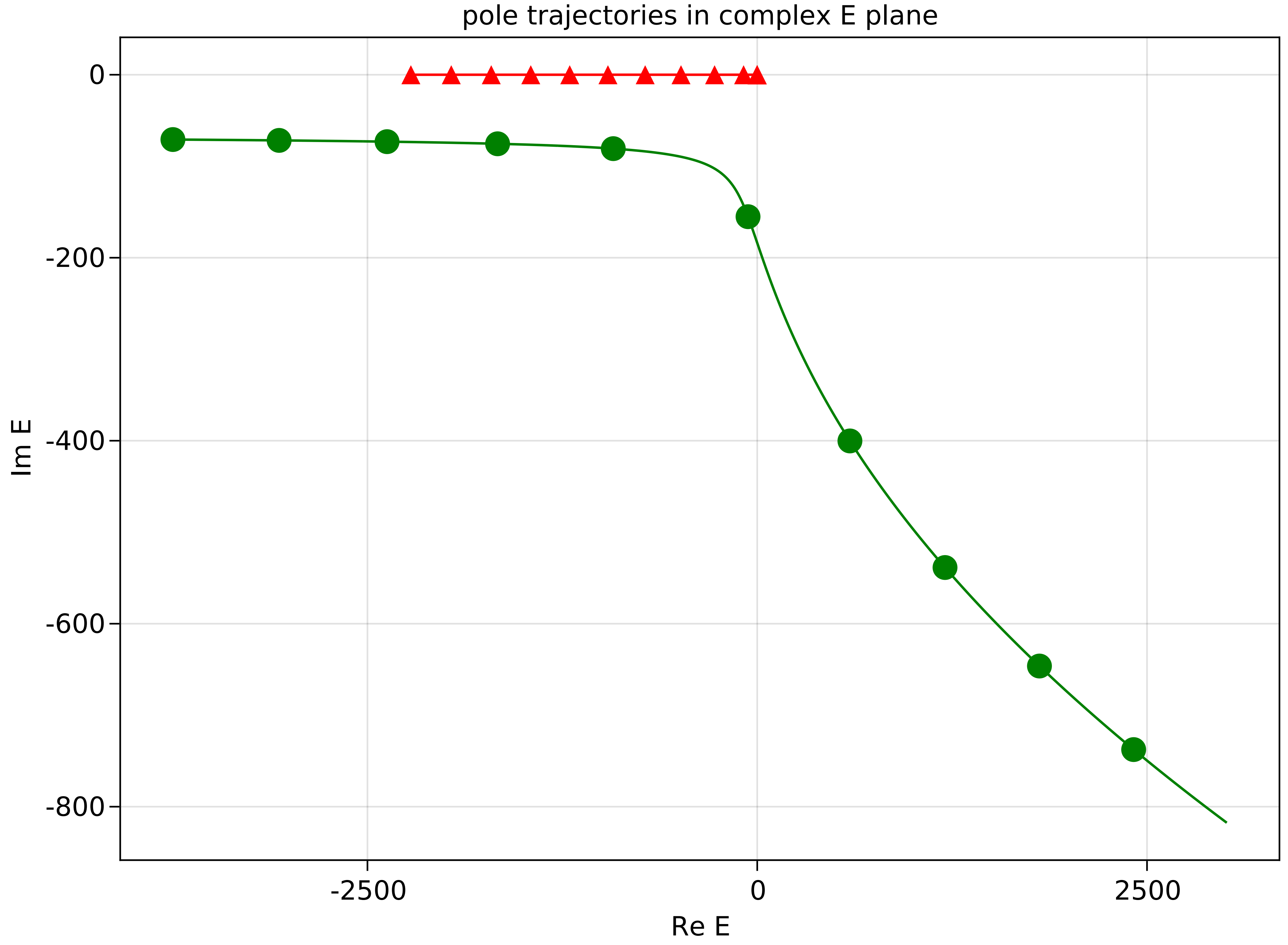
dotted lines :$1/r_e$

- Im k of resonance pole
- Im $k \to 1/r_e$

→ trajectory close to that of resonance in ERE

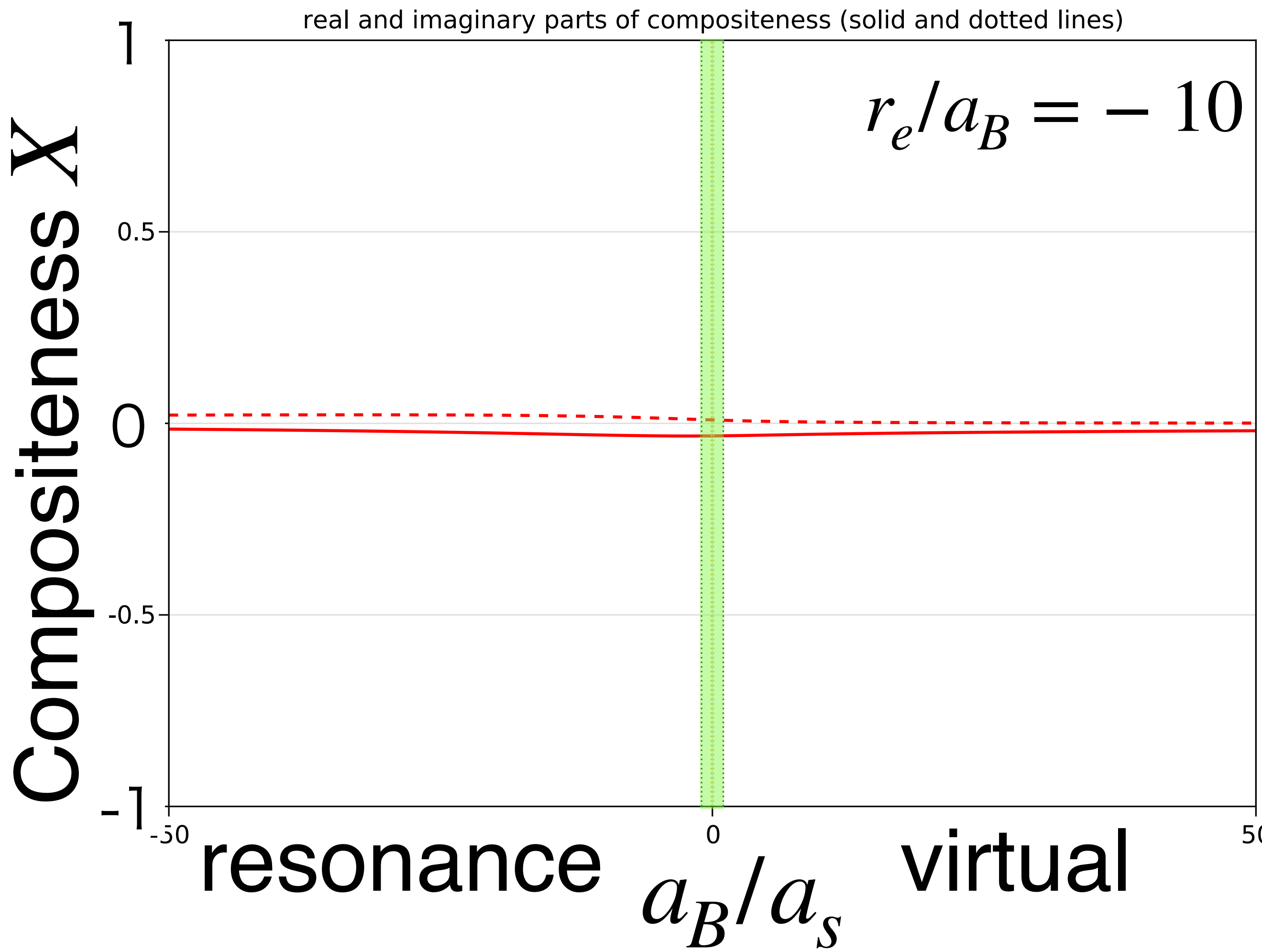
- same as repulsive case

Attractive

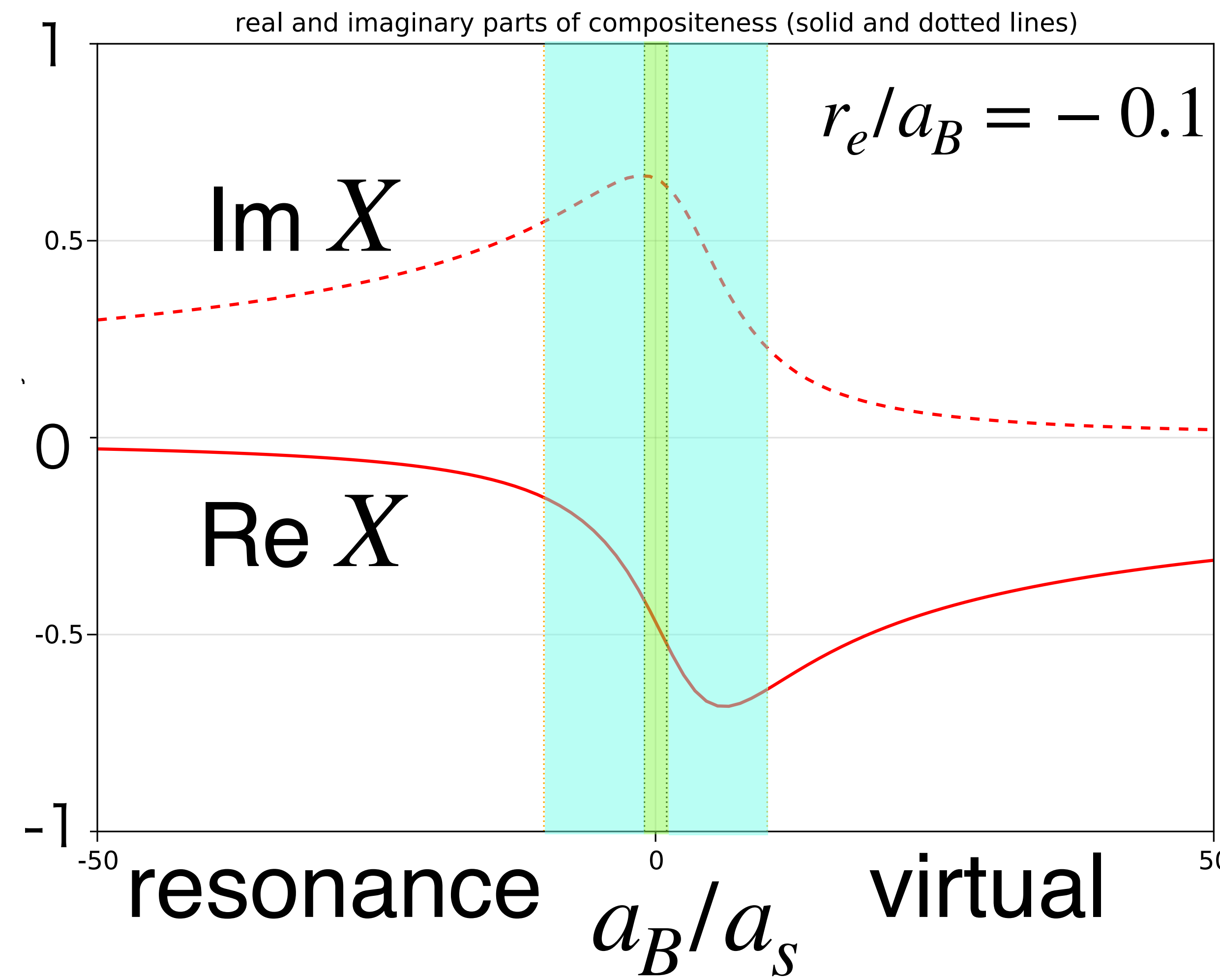

pole trajectories in complex E plane
0
-200
-400
-600
-800
Im E
-2500
0
2500
Re E

Compositeness (att. Coulomb **resonance**)

strong Coulomb

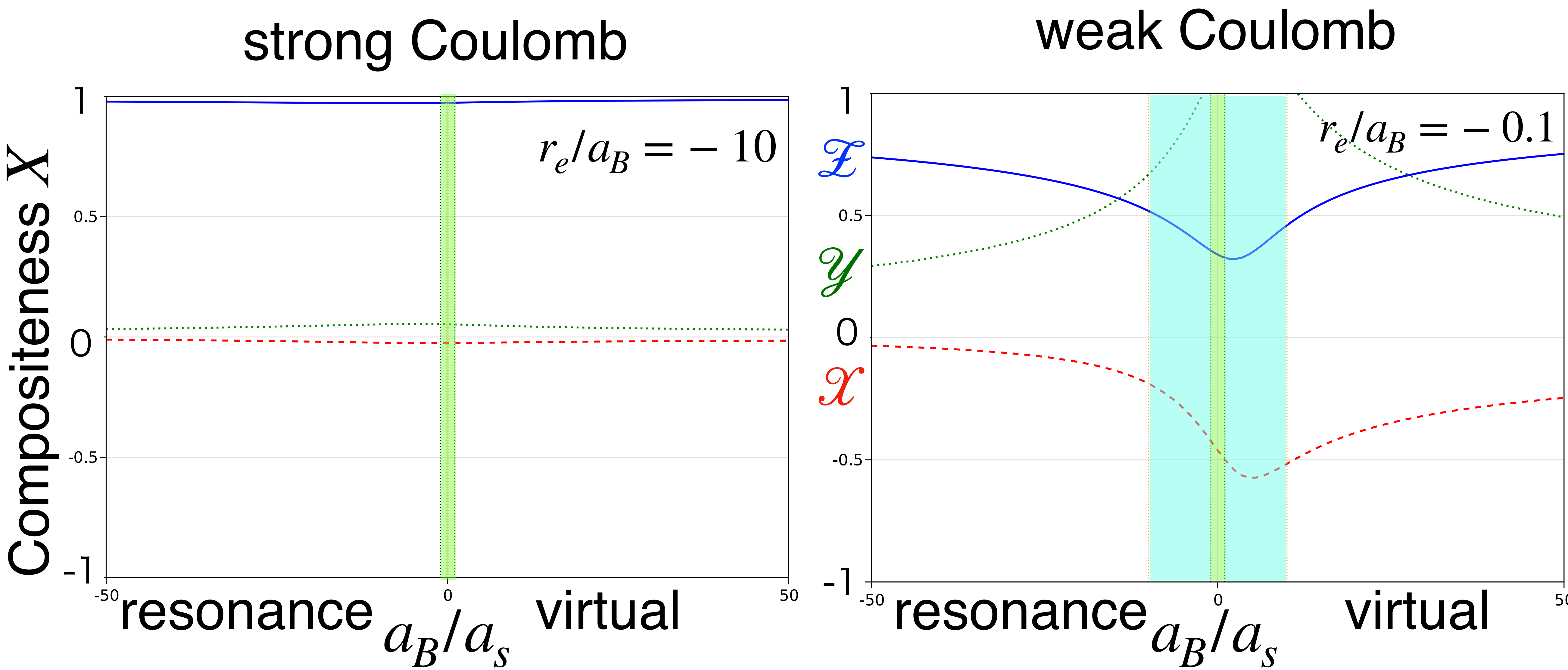


weak Coulomb



- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- compositeness of unstable resonances are complex $X \in \mathbb{C}$

Compositeness (att. Coulomb resonance)



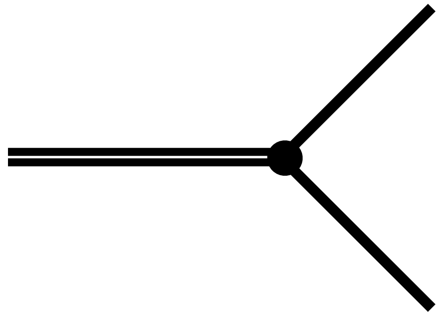
- $\mathscr{X} < 0$ ➔ non-interpretable in this region
- but $\mathscr{X} \geq 0$ in far-threshold region with large $|1/a_s|$
  ➔ states are $\mathscr{Z}$ dominant with large bare state contribution

Pole trajectory (only w/ s.r.)

◉ pole trajectory in complex momentum k plane (No Coulomb)

Q channel

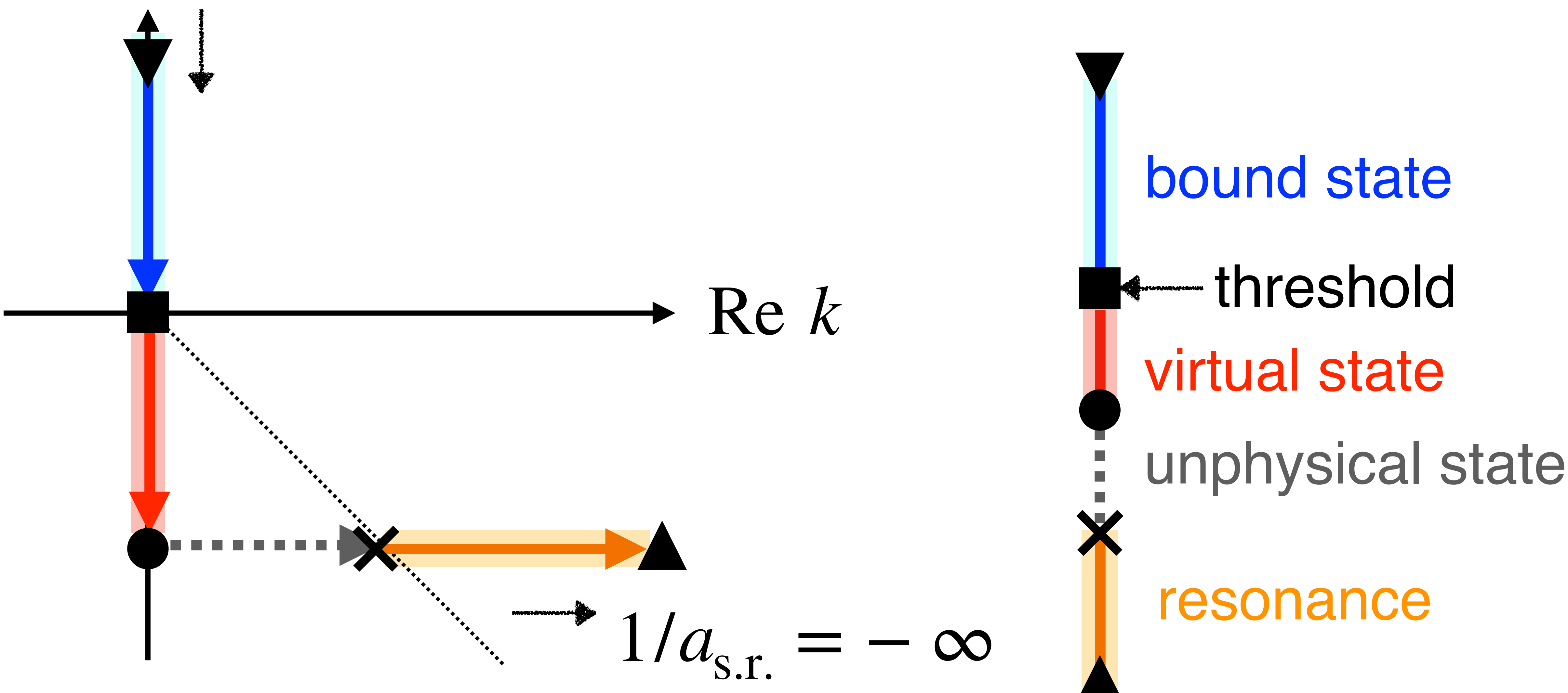
⇔ bare state



P channel

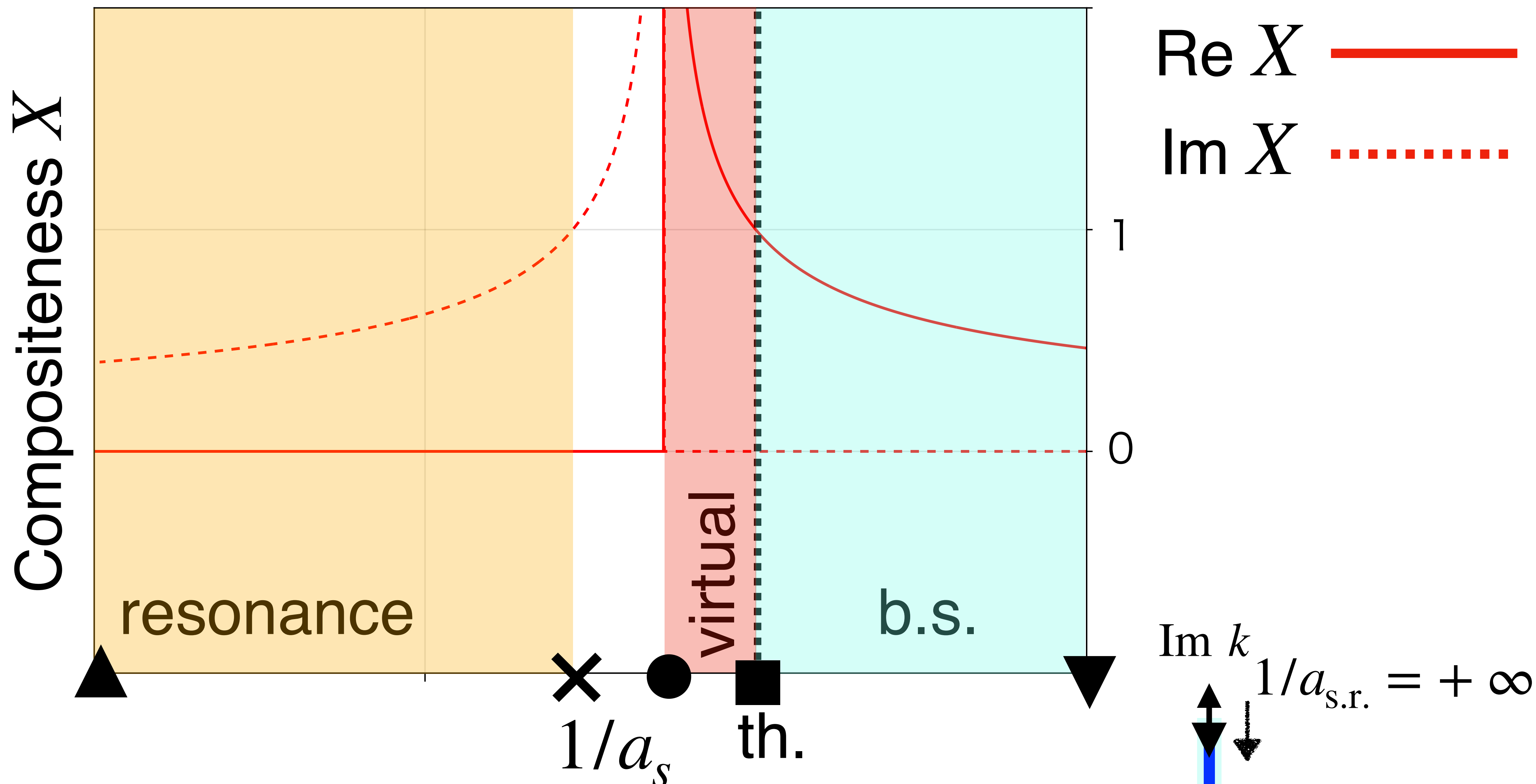
⇔ free scattering

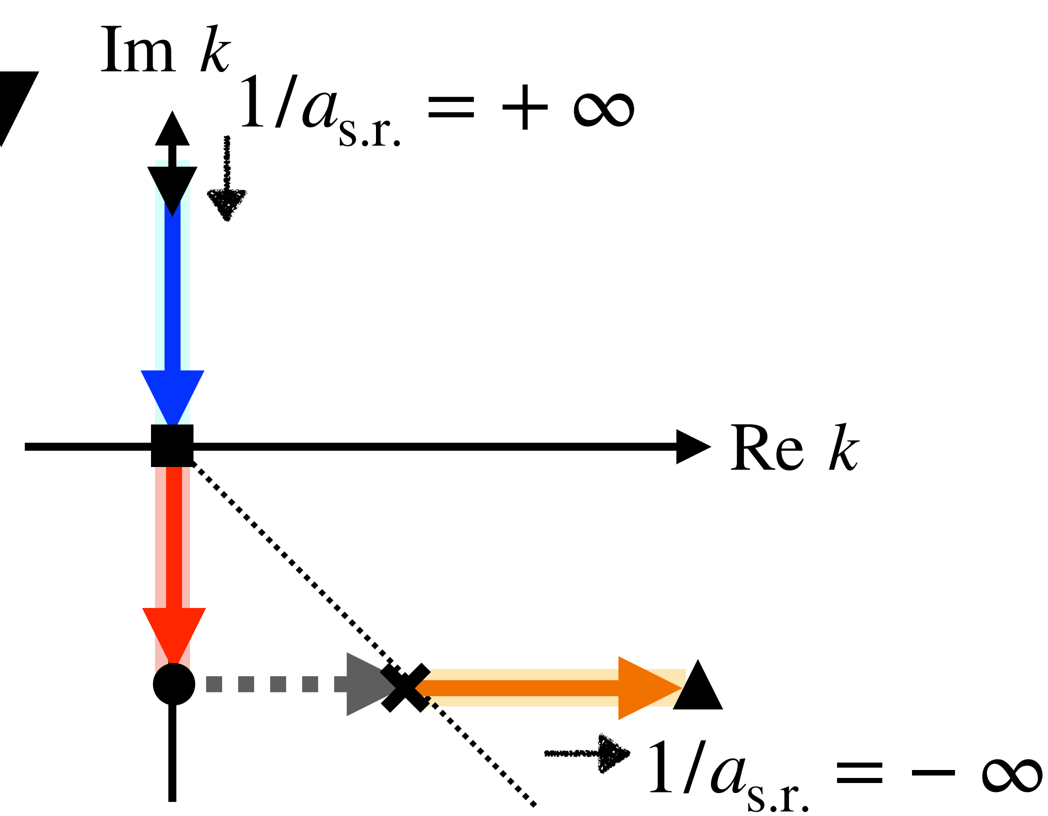
$\mathrm{Im}\ k$ $\quad 1/a_{\mathrm{s.r.}} = +\infty$ $\qquad G_P \to G^0$ free Green's function



T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) .

Compositeness (only w/ s.r.)



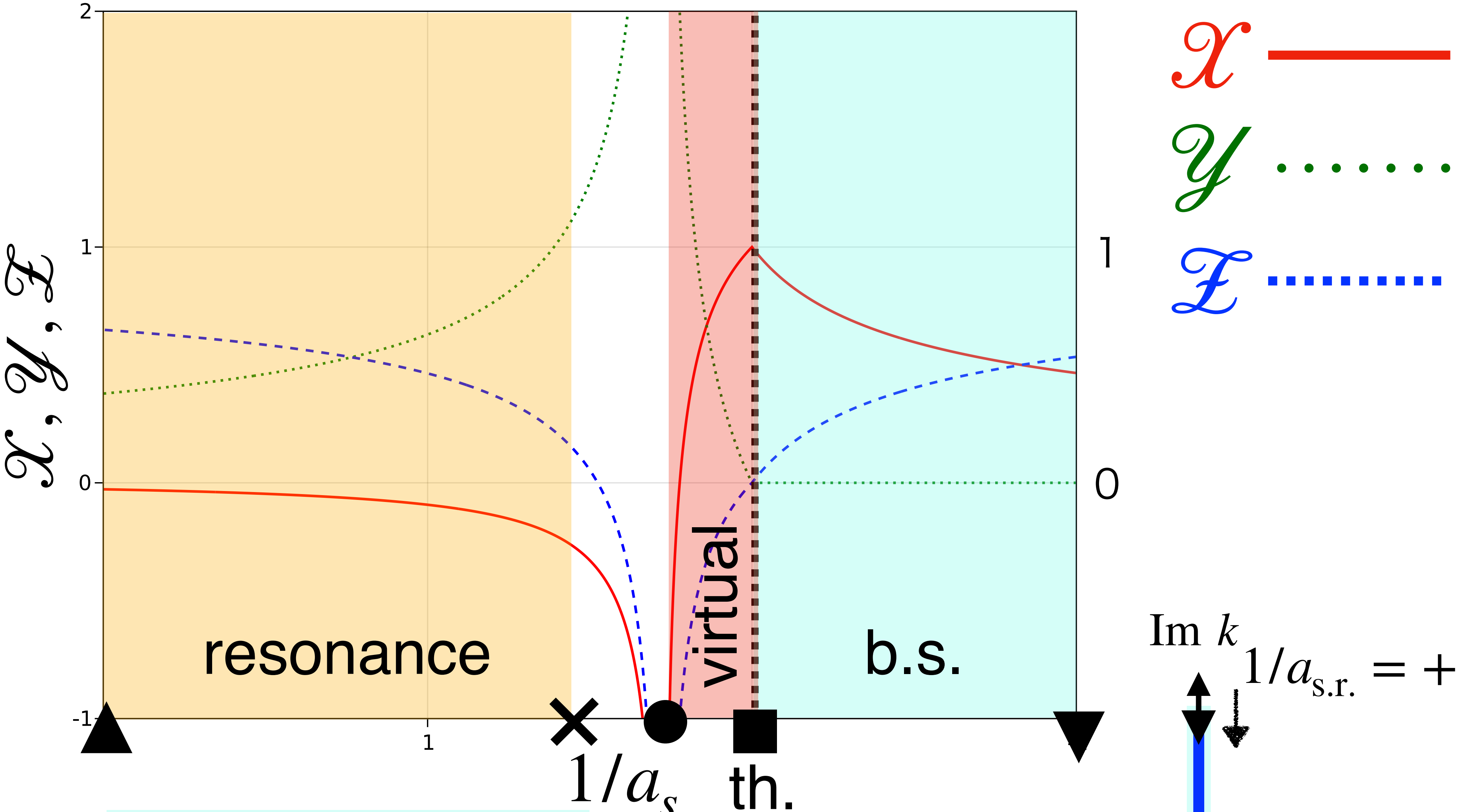


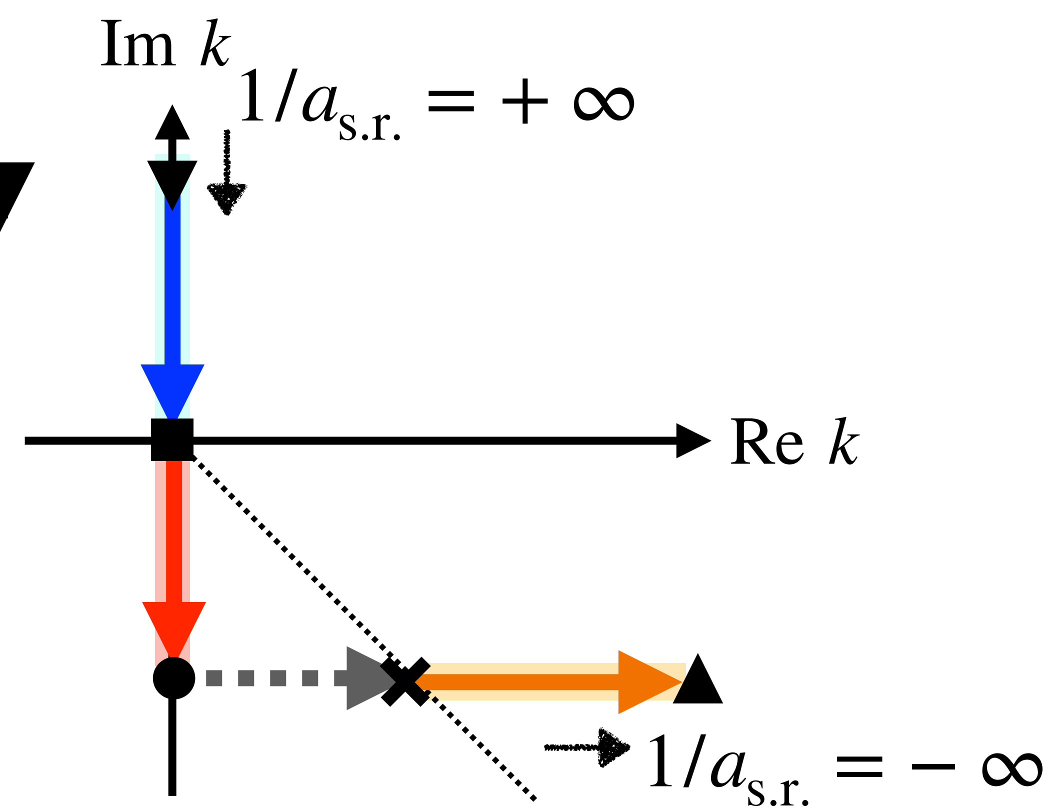
- b.s. : $0 \leq X \leq 1$
- virtual : $1 < X$

● divergence $X \to \infty$

- resonance : $\mathrm{Im}\ X \leq 1$

$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ (only w/ s.r.)





- b.s. : $0 \leq X \leq 1$
- virtual : $\mathscr{Z} < 0$ (non-interpretable)

● divergence $\mathscr{X}, \mathscr{Y}, \mathscr{Z} \to \infty$

- resonance : $\mathscr{Z}$ dominant

Complex compositeness

- probabilistic interpretation?

$X \in \mathbb{C}$ and $X + Z = 1$

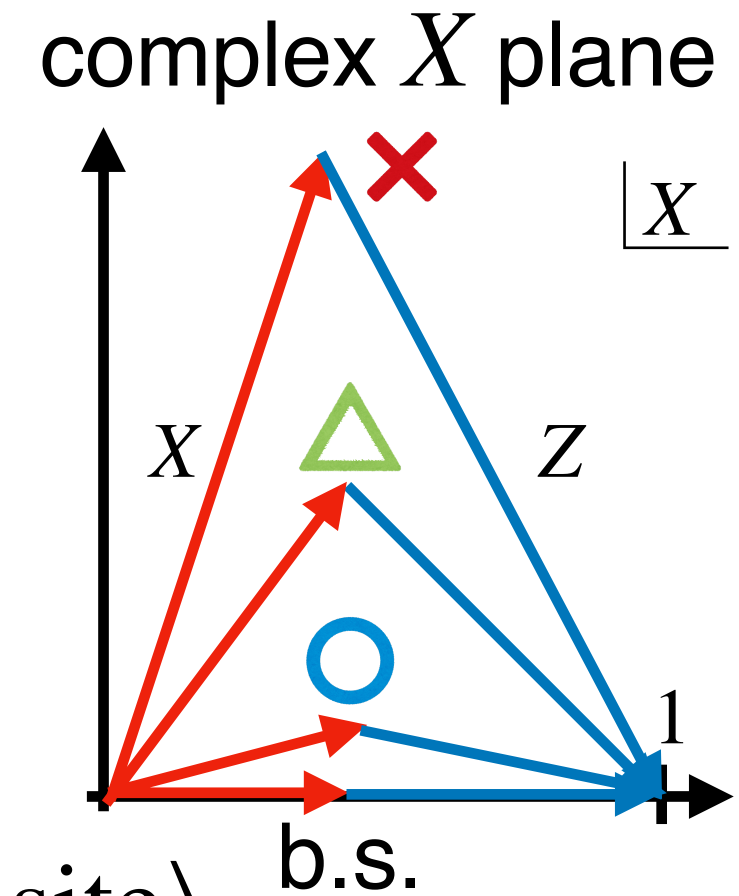
- If Im X is large, it seems that reasonable interpretation is impossible × △

- our proposal

i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$

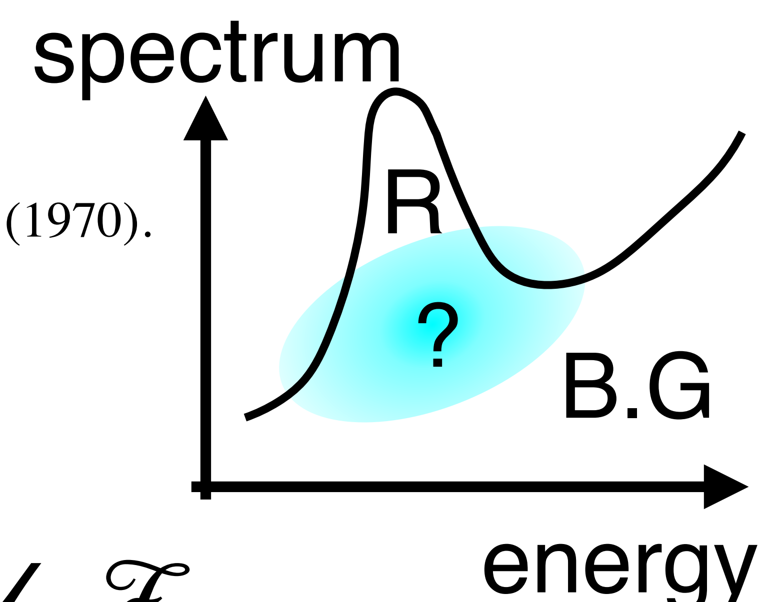
ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$

iii) $\mathscr{Y}$: probability of uncertain identification



uncertain appears from T. Berggren, Phys. Lett. B 33, 547 (1970).

- finite lifetime (uncertainty in energy)
- separation from B.G.



complex compositeness $X \in \mathbb{C} \longrightarrow \mathscr{X}, \mathscr{Y}, \mathscr{Z}$

Definition

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

◉ conditions for sensible interpretation

- normalization : $\mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathcal{X} \to X$, $\mathcal{Z} \to Z$ and $\mathcal{Y} \to 0$

$\mathcal{Y}$ characterizes uncertainty of resonance

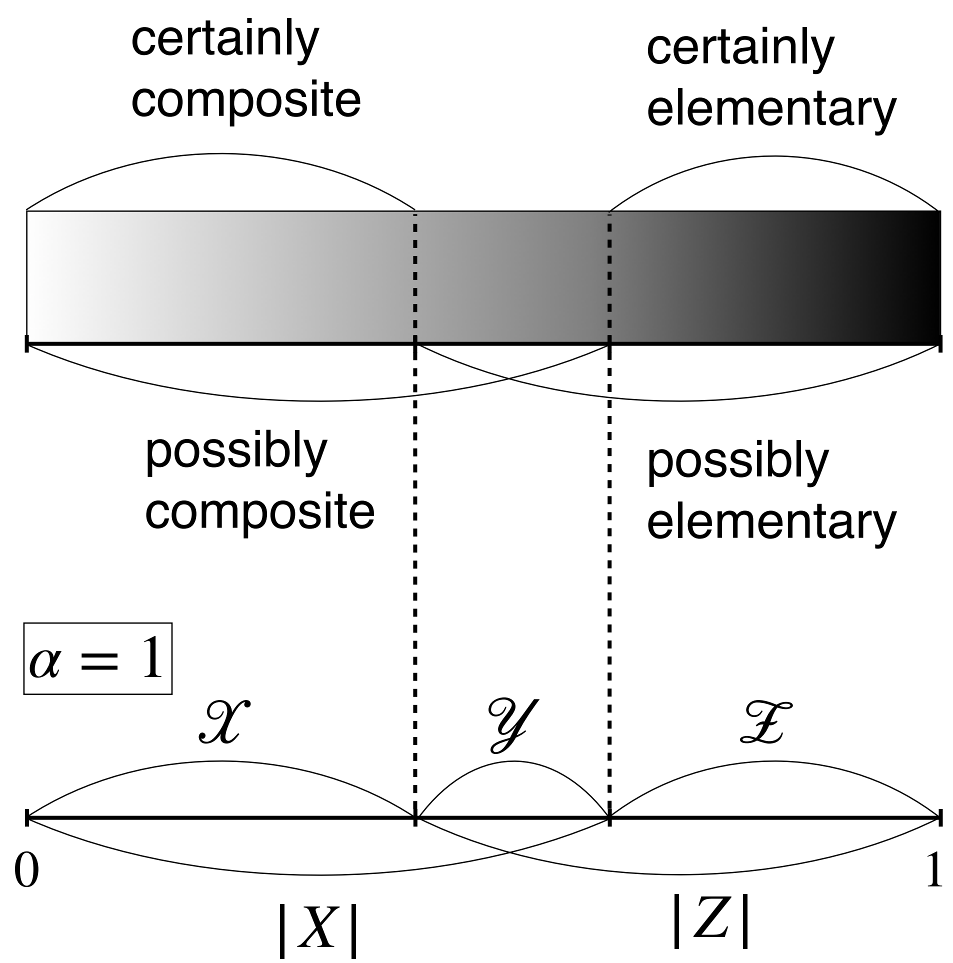
◉ new interpretation

$$\mathcal{X}+\alpha\mathcal{Y} = |X|,\ \mathcal{Z}+\alpha\mathcal{Y} = |Z|$$

$$\mathcal{X} = \frac{(\alpha-1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathcal{Z} = \frac{(\alpha-1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$\mathcal{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

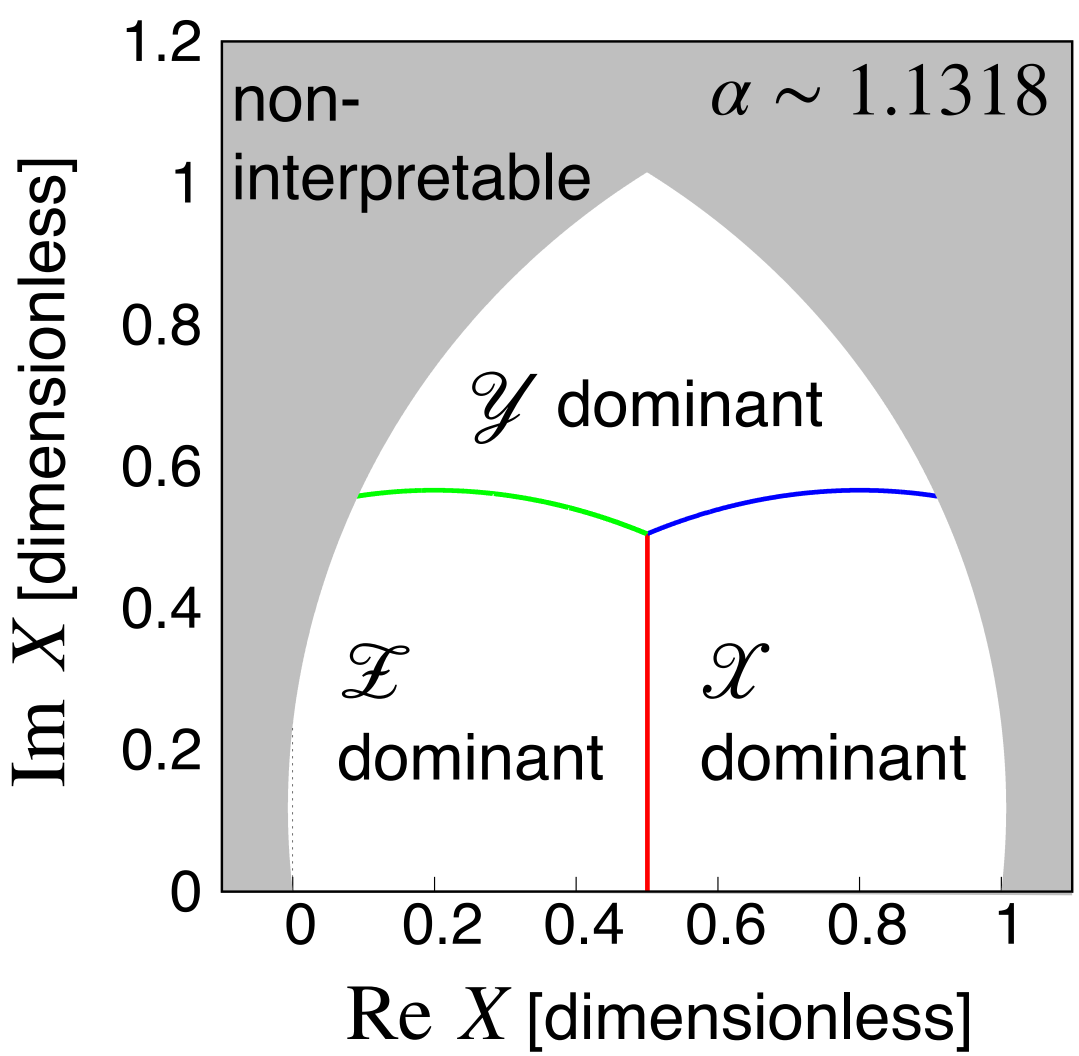


α reflects uncertain nature of resonances

Definition

- if $\alpha > 1/2$, $\mathscr{Y}$ is always positive but $\mathscr{X}$, $\mathscr{Z}$ can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		non-interpretable



our criterion for physical "state"

$$\Gamma \leq \mathrm{Re}\ E$$

- exclude poles which we cannot regard as physical "state" from probabilistic interpretation

$\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ dominant regions and non-interpretable region

uncertainty in resonances