閾値近傍の
p波束縛状態の複合性

三宅 浩太朗, 安福 翔太, 衣川友那*, 岡真*

名古屋大学, *理研仁科センター

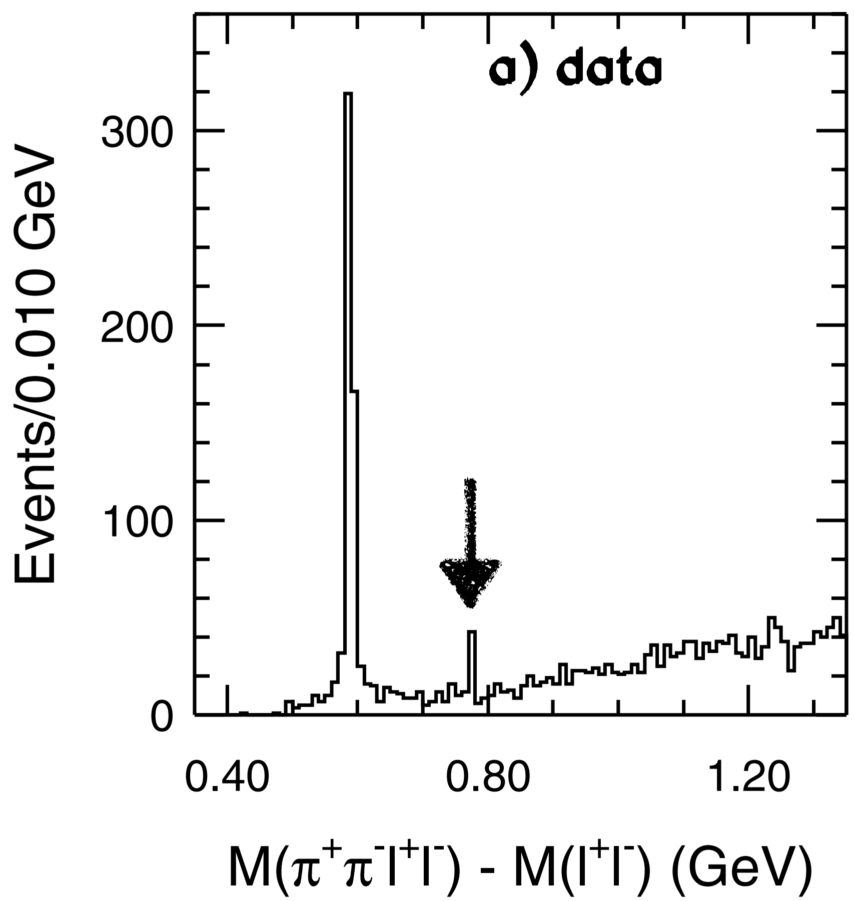
9月19日, JPS 第80回 年次大会 @広島大学

闘値近傍のエキゾチックハドロン

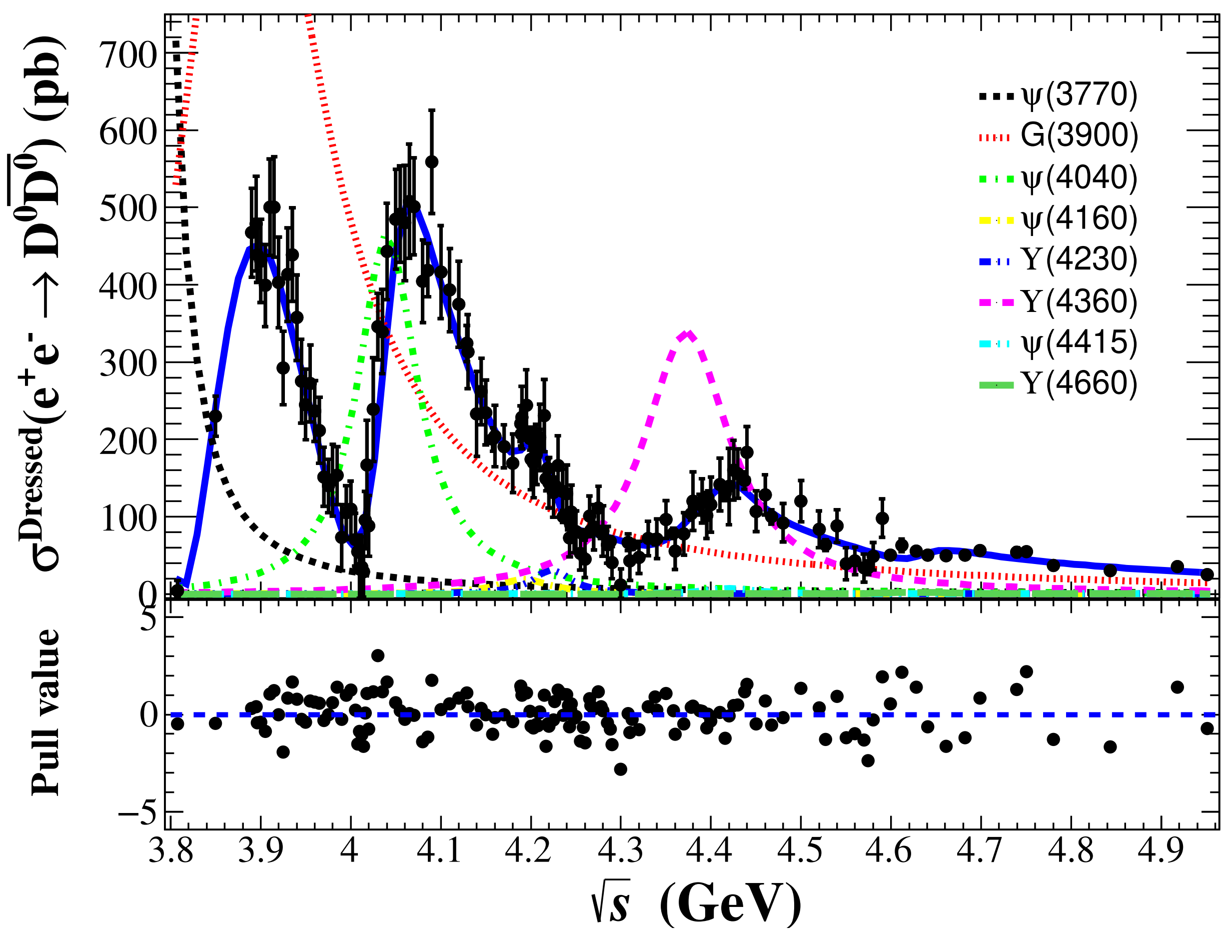
s-波

p-波

$X(3872)$



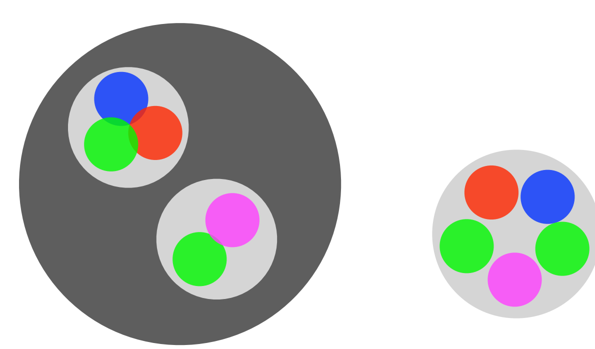
$G(3900)$



S. K. Choi *et al*. (Belle), Phys. Rev. Lett. **91**, 262001 (2003).　Medina Ablikim et al., Phys. Rev. Lett. **8**, 133 (2024).

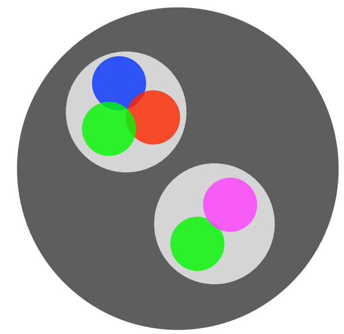
- ハドロン分子？マルチクォーク？

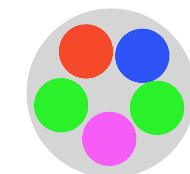
→ 闘値近傍状態の内部構造



複合性 (compositeness)

◉ 定義





$$|\mathrm{hadron}\rangle = \sqrt{X}\,|\mathrm{hadronic\ molecule}\rangle + \sqrt{Z}\,|\mathrm{others}\rangle$$

複合性 **素粒子性**

- **定量的**な指標

Weinberg, S. Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Kinugawa, T. Hyodo, Eur. Phys. J. A, 61(7):154, (2025).

$0 \leq X \leq 1$ $\longrightarrow$ $X > 0.5 \Leftrightarrow$ **複合的** (ハドロン分子的)

$X + Z = 1$ $\quad$ $X < 0.5 \Leftrightarrow$ **素粒子的** (非ハドロン分子的)

- エキゾチックハドロンに応用

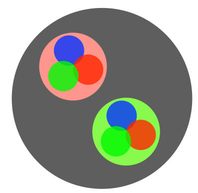
$f_0(980)$, $a_0(980)$ V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004); Y. Kamiya and T. Hyodo, PTEP 2017; Phys. Rev. C 93, 035203 (2016) etc.

$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

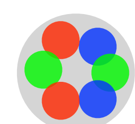
閾値近傍状態と普遍性

◉ *s*波の閾値近傍状態

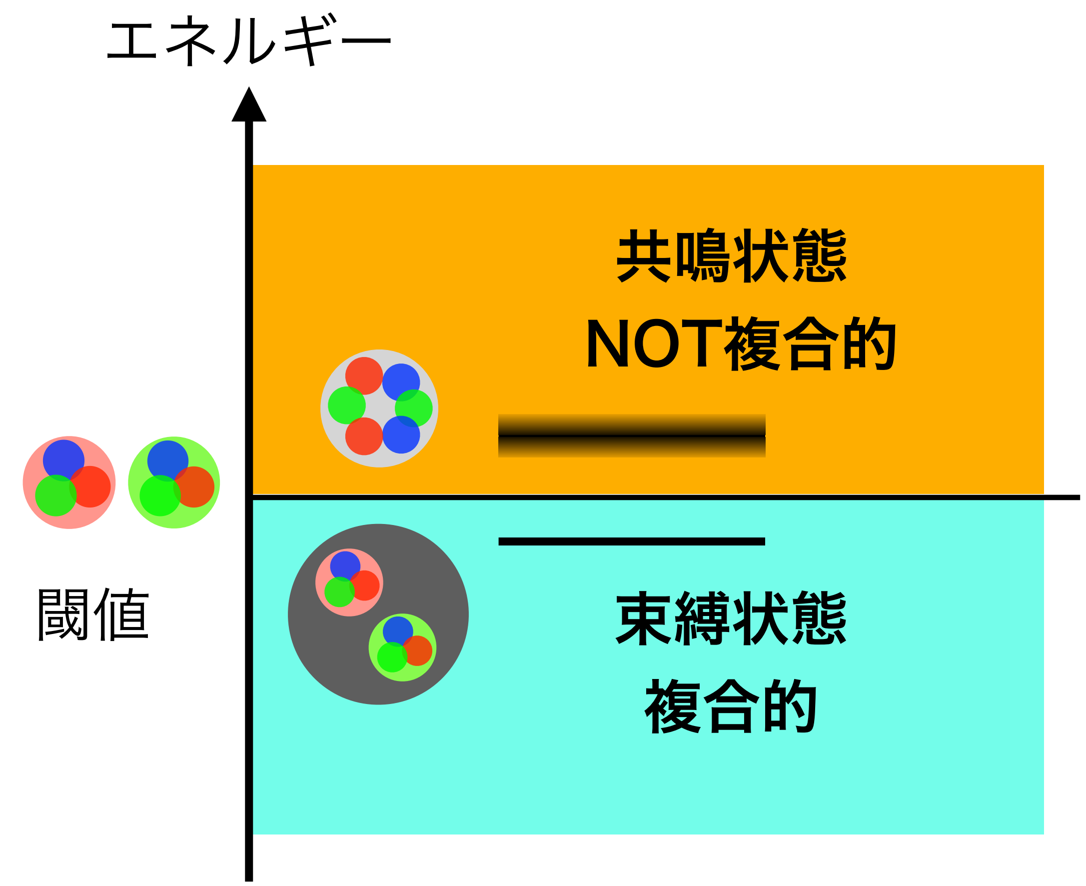
T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024);
arXiv:2403.12635 [hep-ph].

- 束縛状態は**複合的** 

  散乱長で決まる低エネルギー普遍性

- 共鳴状態は**複合的ではない！** 

  散乱長+有効レンジで決まる普遍性



◉ *p*波の閾値近傍状態

T. Sekihara, T. Arai, J. Yamagata-Sekihara, and S. Yasui, Phys. Rev. C 93, 035204 (2016).

- 低エネルギー普遍性は**あらわれない！** T. Hyodo, Phys. Rev. C 90, 055208 (2014).

→ 原理的に、内部構造はモデルに依存する！！

*p*波の閾値近傍状態に共通する普遍的な特徴は他にあるか？

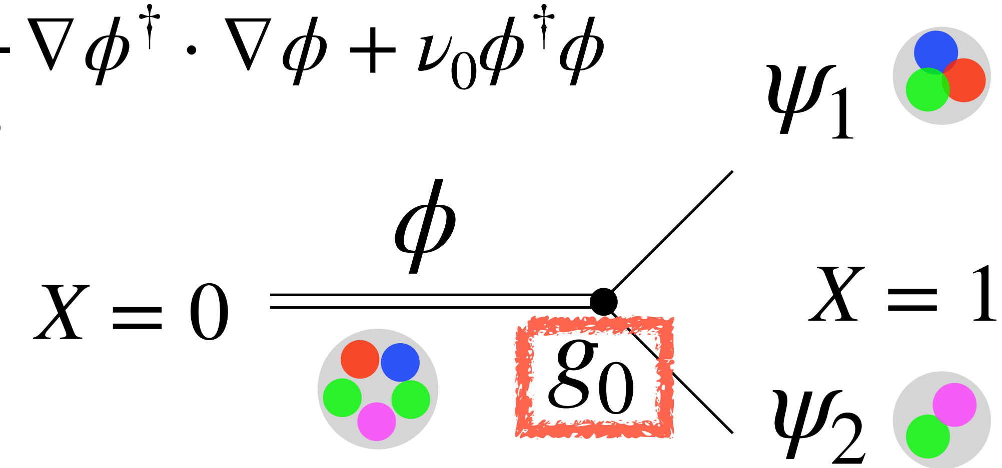
EFTモデル

◉ 共鳴モデル

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).
C. A. Bertulani, H. W. Hammer, and U. Van Kolck, Nucl. Phys. A, 712 (2002).

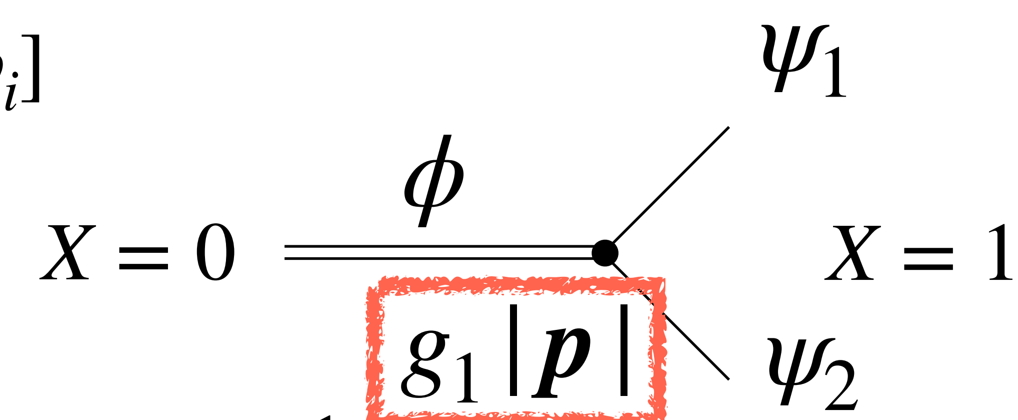
$$\mathscr{H}_{\rm free} = \frac{1}{2m_1}\nabla\psi_1^\dagger\cdot\nabla\psi_1 + \frac{1}{2m_2}\nabla\psi_2^\dagger\cdot\nabla\psi_2 + \frac{1}{2m_\phi}\nabla\phi^\dagger\cdot\nabla\phi + \nu_0\phi^\dagger\phi$$

- s波 $\mathscr{H}_{\rm int}^{\rm s-wave} = g_0(\phi^\dagger\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger\phi)$



- p波 $\mathscr{H}_{\rm int}^{\rm p-wave} = ig_1[\phi_i^\dagger(\psi_1\overleftrightarrow{\nabla}\psi_2) + (\psi_1^\dagger\overleftrightarrow{\nabla}\psi_2^\dagger)\phi_i]$

$$\psi_1\overleftrightarrow{\nabla}\psi_2 = (\nabla\psi_2)\psi_1 - \psi_2(\nabla\psi_1)$$



- Lippmann-Schwinger 方程式 $T = [V^{-1} - G]^{-1}$

$$V = \frac{g_0^2}{E-\nu_0}, \quad G = \int\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{p^{2l}}{E - p^2/(2\mu)}$$ for l-th partial wave

G の計算にsharp cutoff Λ を導入

複合性

$$X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara, and S. Yasui, Phys. Rev. C 93, 035204 (2016).

- s波

$$X_s = \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1+\left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}. \quad \kappa = \sqrt{2\mu B}$$

$$\longrightarrow X_{\mathrm{s-wave}} = \frac{1}{1-\kappa r_0} \quad (\Lambda \gg \kappa)$$

Weinberg, S. Phys. Rev. 137, 672–678 (1965).

- p波

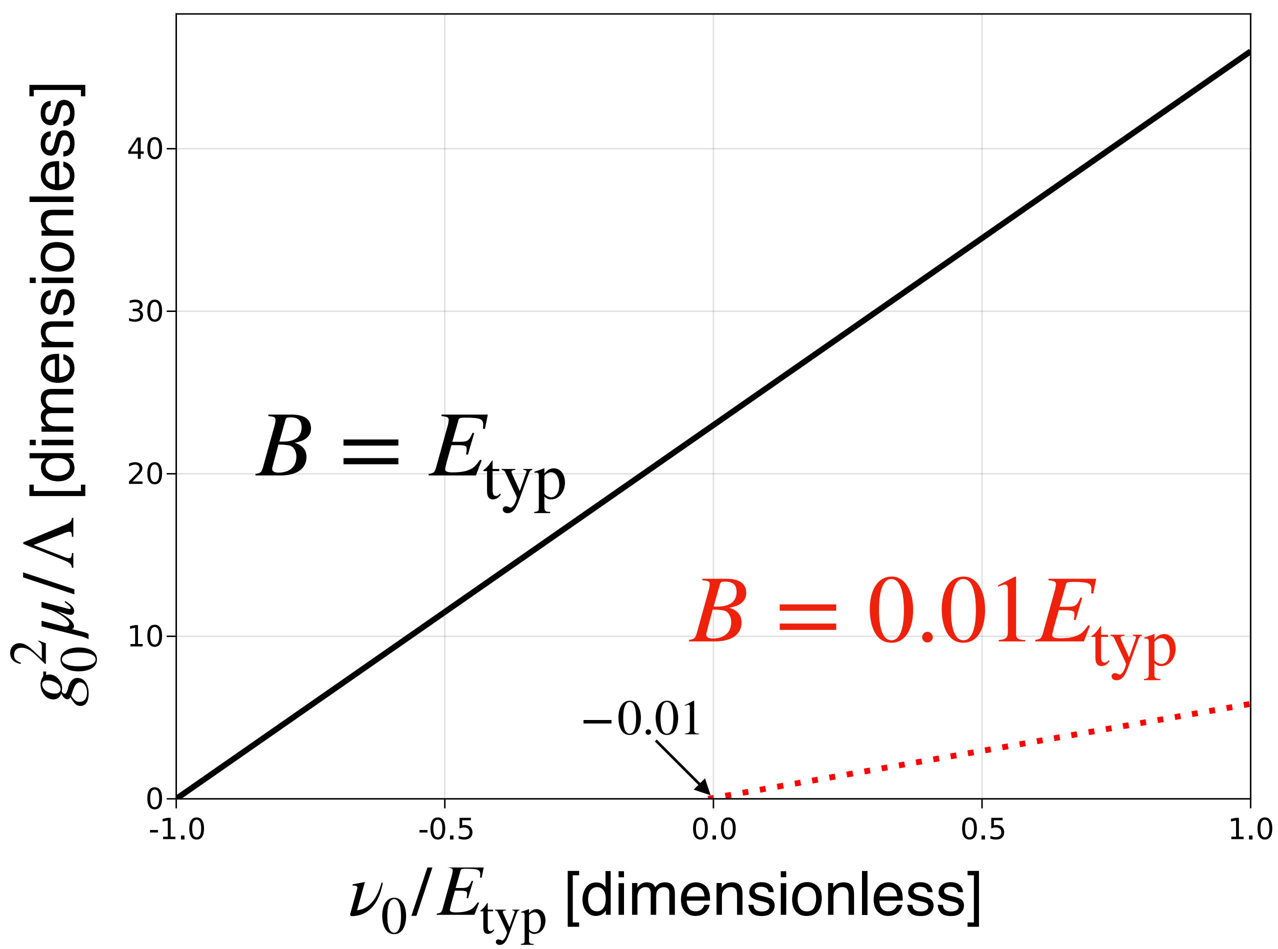
$$X_p = \left[1 + \frac{\pi^2\kappa}{g_1^2\mu^2}\left[\boxed{2\Lambda/\kappa} - \arctan(\Lambda/\kappa) + 3\frac{\Lambda/\kappa}{1+(\Lambda/\kappa)^2}\right]^{-1}\right]^{-1}.$$

$$\longrightarrow X_{\mathrm{p-wave}} = \left[1 + \frac{\pi}{2}\left(r_1 + \boxed{\frac{4\Lambda}{\pi}}\right)\left(\boxed{-2\Lambda} - \frac{3}{2}\pi\kappa\right)^{-1}\right]^{-1} \quad (\Lambda \gg \kappa)$$

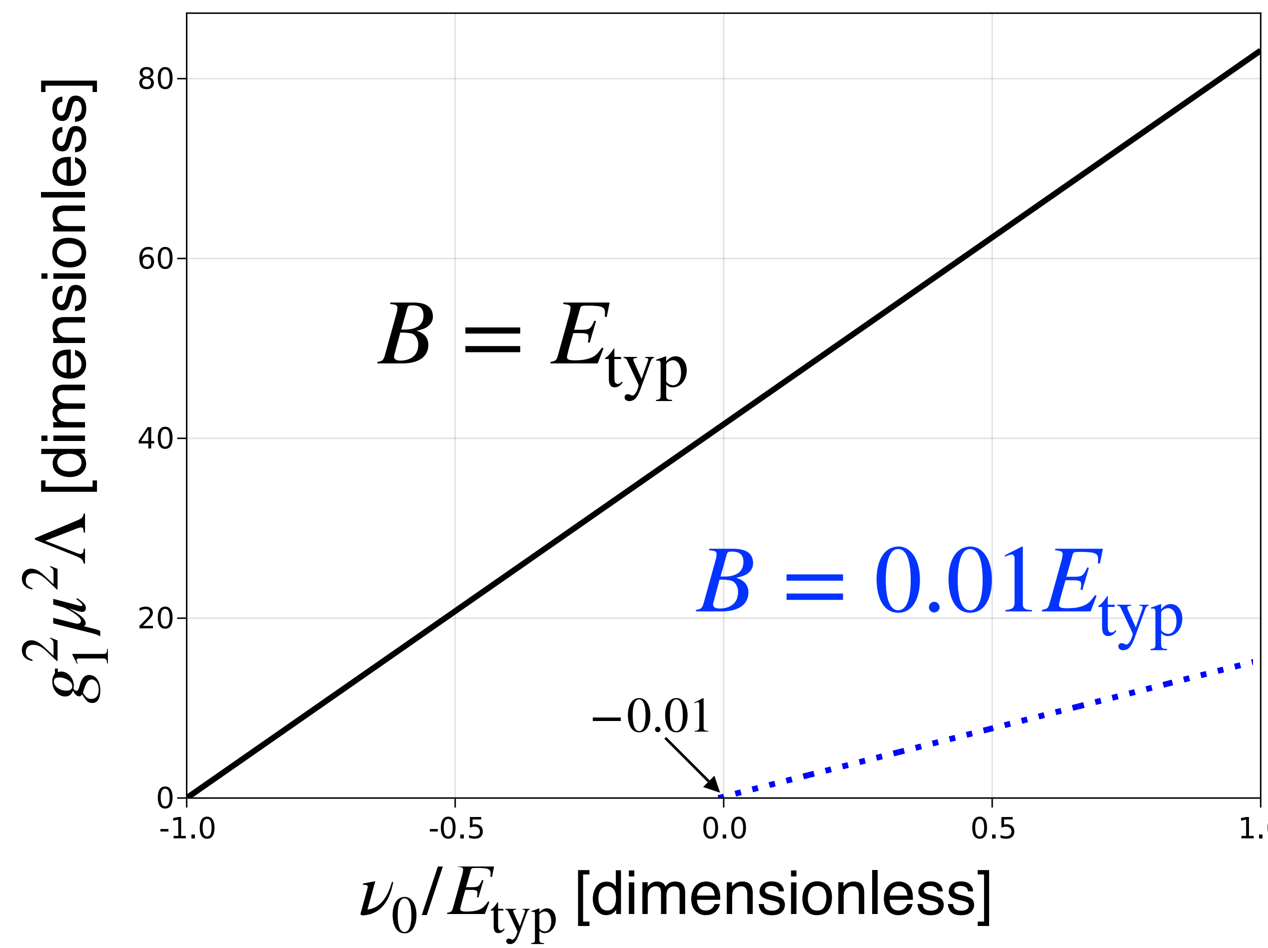
結合定数のパラメタ依存性

束縛エネルギー B を固定、$-B \leq \nu_0 \leq E_{\rm typ}$ でパラメタを動かす

- s波



- p波

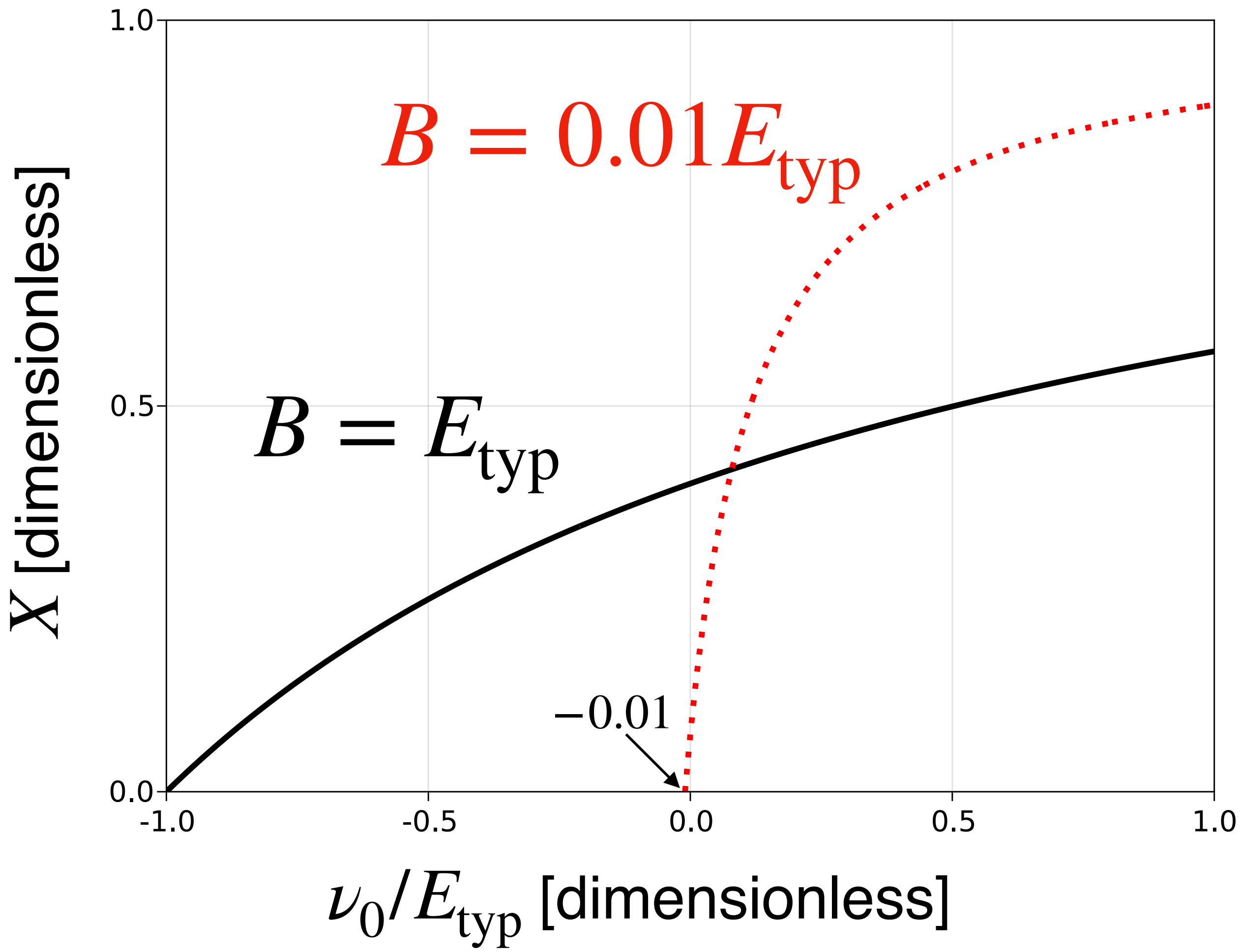


- $E_{\rm typ} = \Lambda^2/(2\mu)$: 系の典型的エネルギースケール
- s波でもp波でも、Bが大きくなるほど結合定数が大きくなる

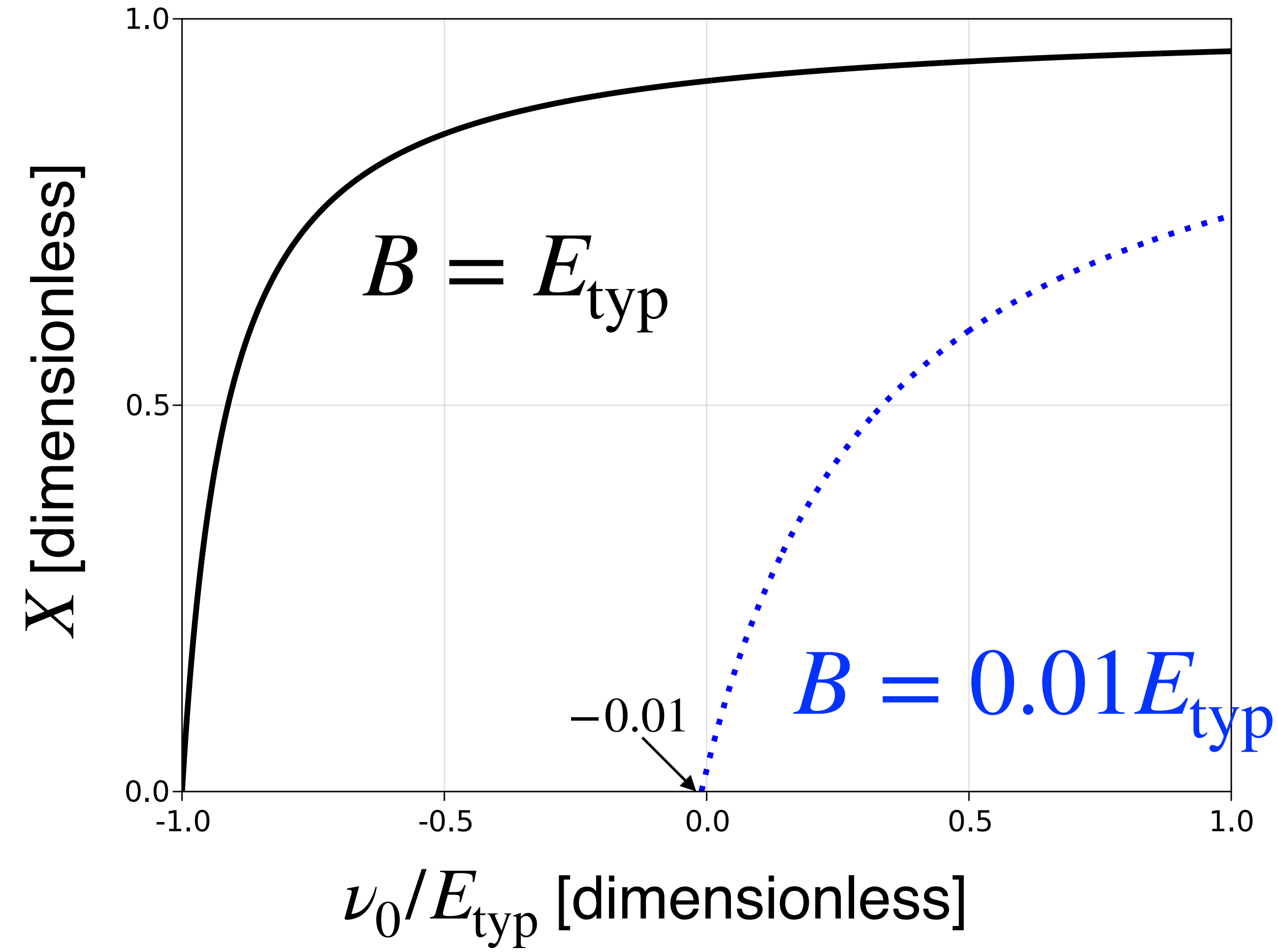
  B が大きくなるほど ν_0 からズレる ⟶ 結合が大きくなる

複合性のパラメタ依存性

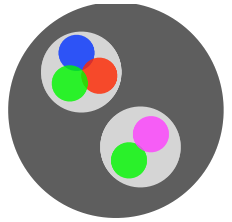
- s波



- p波



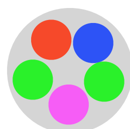
- s波: Bが小さい方が複合性が大きい



⟶ 低エネルギー普遍性の帰結

T. Kinugawa and T. Hyodo, Phys. Rev. C 109 , 045205 (2024).

- p波: Bが小さい方が複合性が小さい

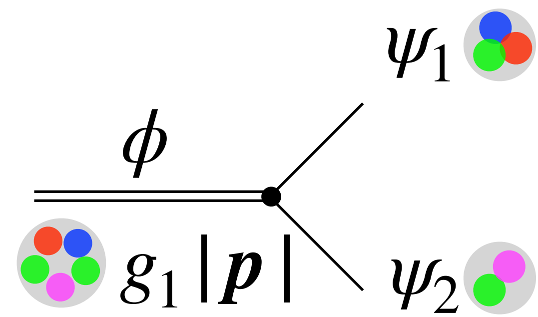


⟶ 結合定数が大きい方が散乱の寄与が多く含まれているから

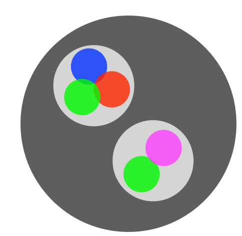
まとめ

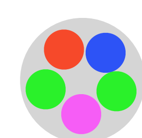
- 閾値近傍の p波エキゾチックハドロン ← 複合性

- 非相対論的有効場の理論



- p波: 閾値近傍でも複合性のモデル (カットオフ)依存性がある

- s波: 束縛エネルギーが小さい方が  ∵ 低エネルギー普遍性

- p波: 束縛エネルギーが小さい方が  ∵ 結合定数が小さい

展望

- 閾値近傍の共鳴状態の内部構造？

- $G(3900)$ への応用

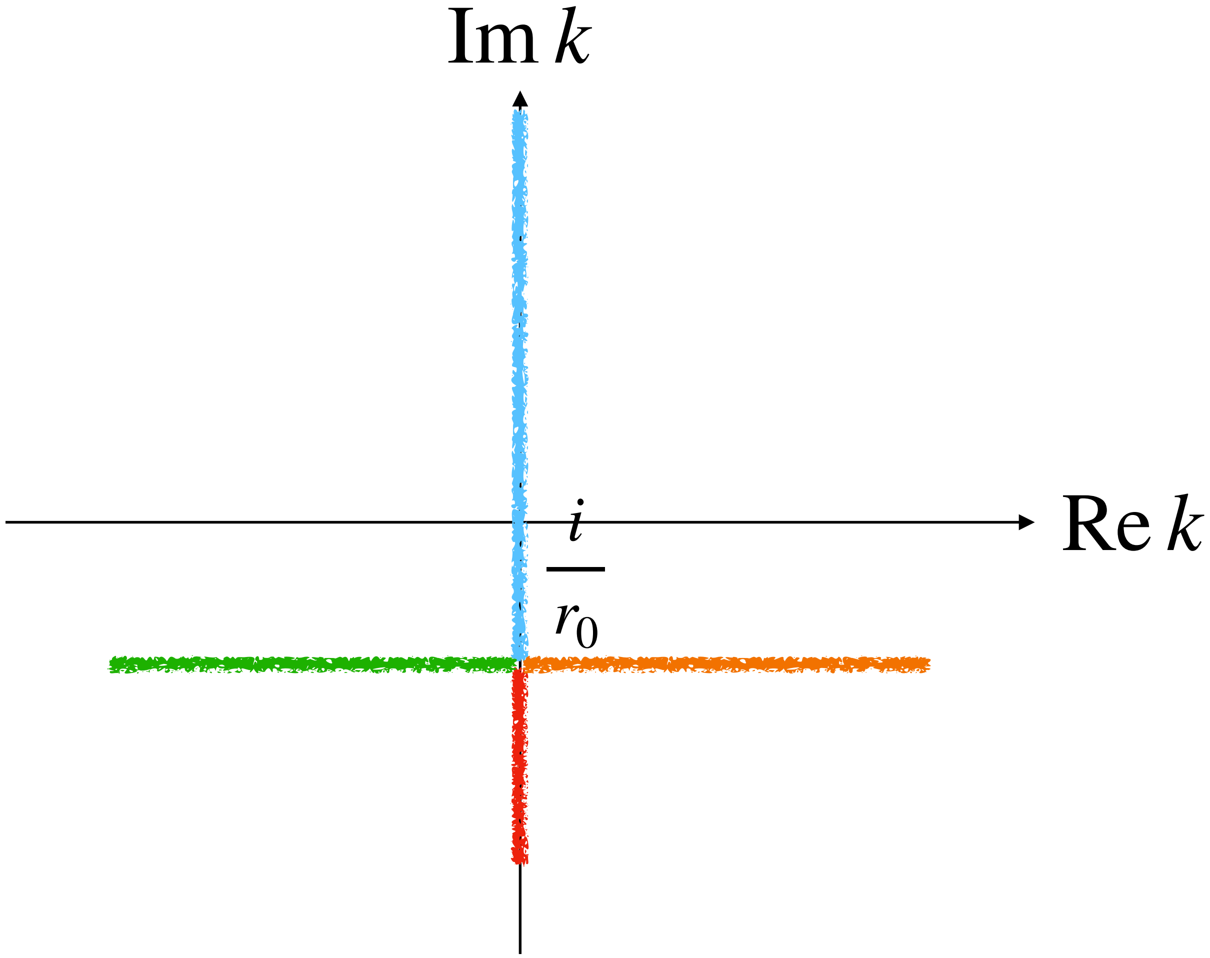
Back up

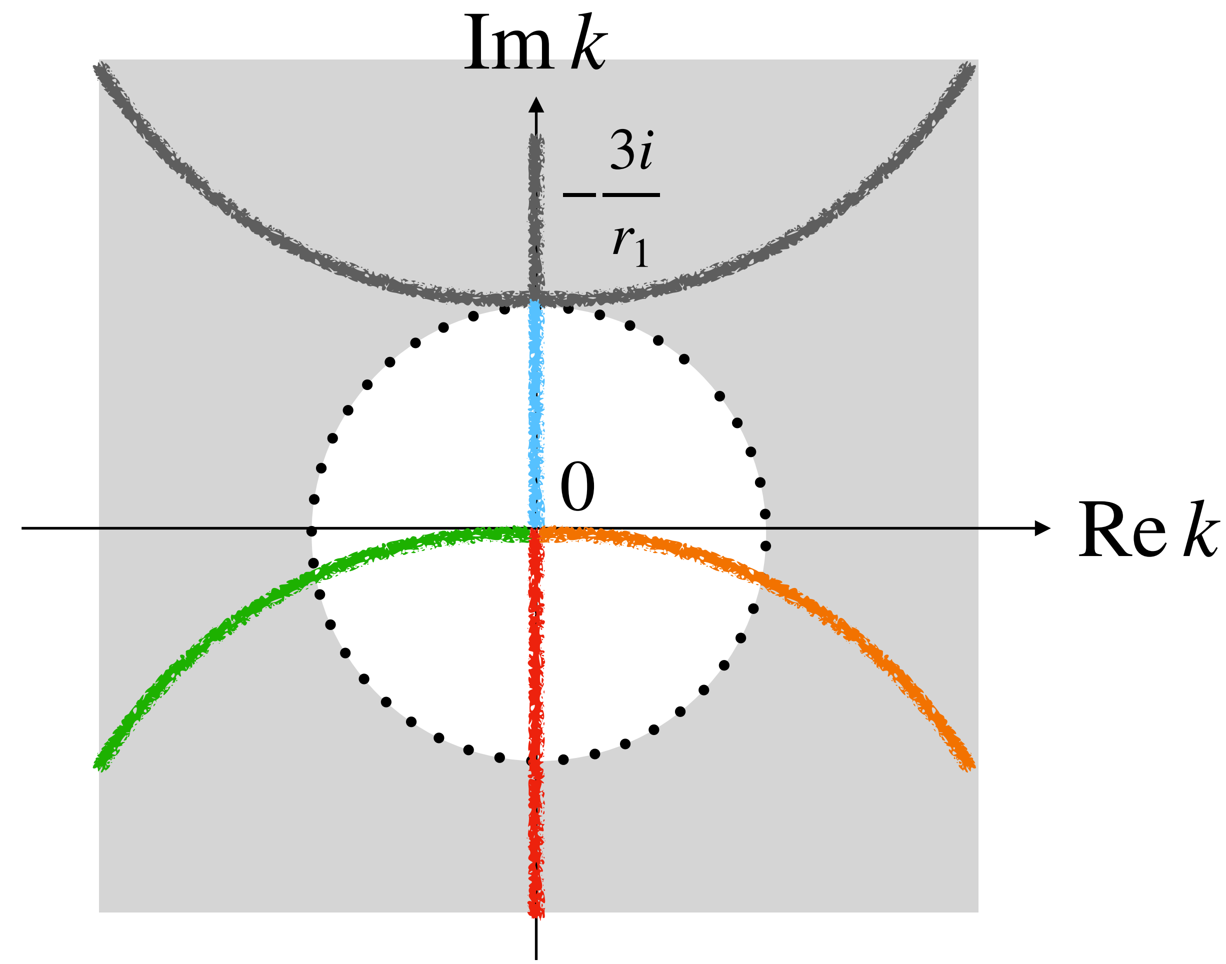
閾値近傍での散乱振幅

$\Lambda \to \infty$で有効レンジ展開 (ERE) の散乱振幅

$$f_{\mathrm{s-wave}}(k) = \frac{1}{-\frac{1}{a_0} + \frac{r_0}{2}k^2 - ik}$$

$$f_{\mathrm{p-wave}}(k,\theta) = \frac{k^2\cos\theta}{-\frac{1}{a_1} + \frac{r_1}{2}k^2 - ik^3}$$





閾値近傍は束縛状態のみ

$k \to 0$ で$a_0 \to \infty$

閾値近傍に対になるpole

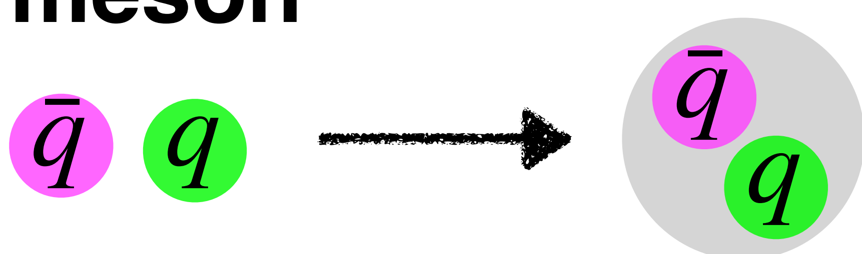
$k \to 0$ で$a_1 \to \infty$ かつ $r_1 \to \infty$

Exotic hadron

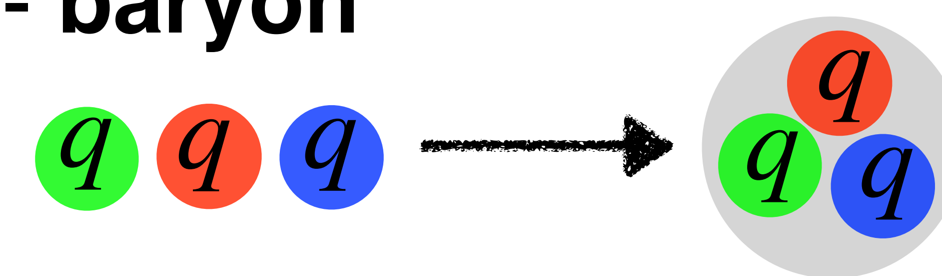
quark q, gluon $\xrightarrow[\text{strong interaction}]{}$ hadron

◉ ordinary hadron

- **meson**



- **baryon**

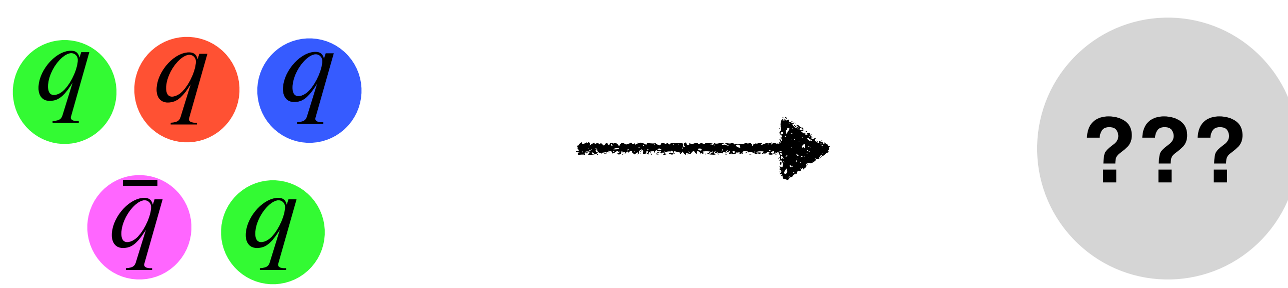


- ~400 kinds have been observed so far

Particle Data Group, S. Navas et al., Phys. Rev. D 110, 030001 (2024).

◉ exotic hadron

- composed of four or more quarks



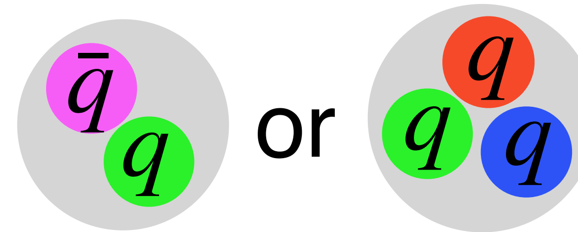
internal structure?

- **rarely** observed ← Why?
- hint for understanding low-energy phenomena in QCD

Exotic hadrons

A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, PTEP 2016, 062C01 (2016)

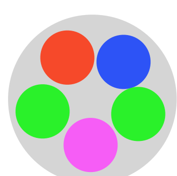
- ordinaly hadron: $q\bar{q}$ (meson) or qqq (baryon)



- other than $q\bar{q}$ or qqq ⟶ **exotic hadron** ⟶ structure?

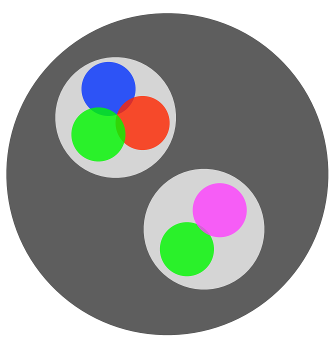
◉ possible structure F-K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, and B-S. Zou, RevModPhys.90.015004 (2018).

- **multiquarks**: quarks ⟶ hadron



size $\lesssim$ 1 fm (**compact**)

- **hadronic molecule**: quarks ⟶ hadrons (subunit) ⟶ hadron



size $\gtrsim$ 1 fm (spatially large radious)

e.g. deuteron (pn molecule), $R = 4.32$ fm

⟶ **superposition** of possible components

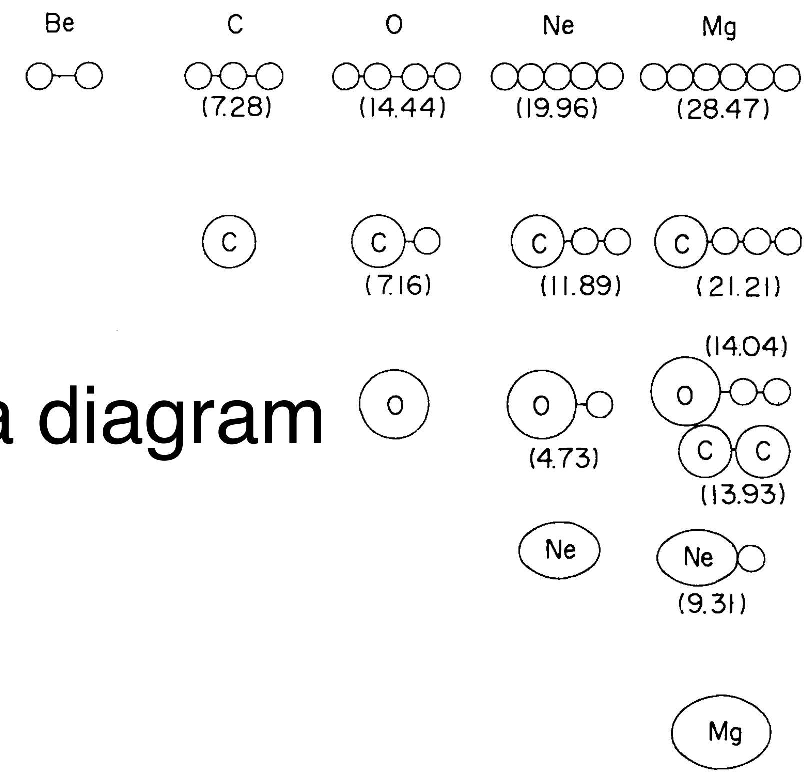
internal structure ⟵ investigate weight of each component

Clustering phenomena

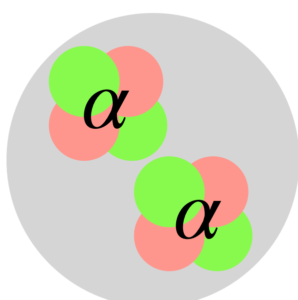
- near-threshold states ⟶ clustering phenomena

- α clustering phenomena

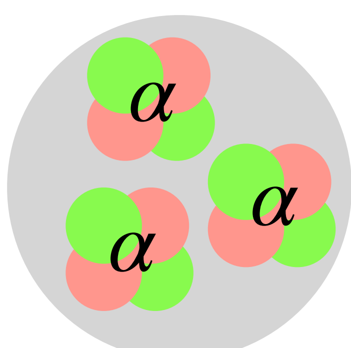
K. Ikeda, and N. Takizawa, and H. Horiuchi, Prog. Theor. Phys. Suppl. E68, 464-475 (1968).



Ikeda diagram

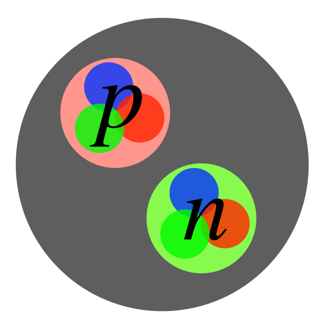


^{8}Be nuclei

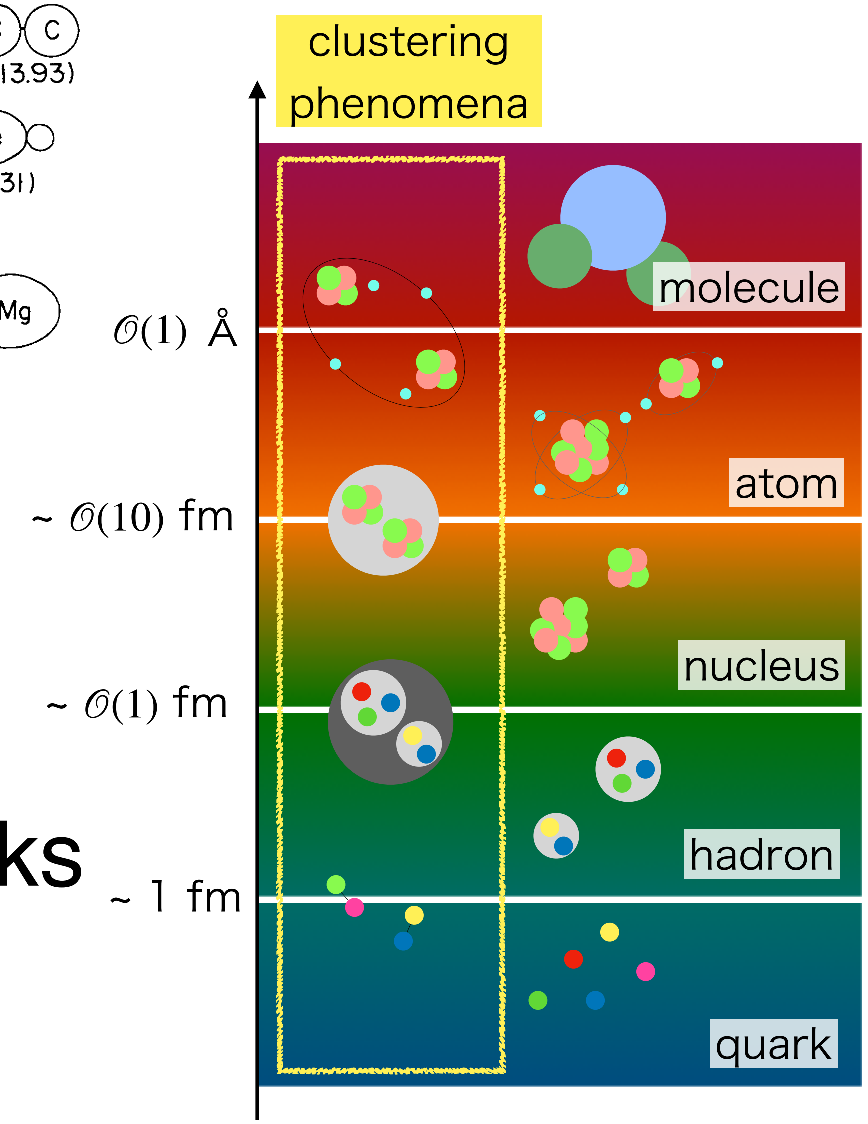


^{12}C Hoyle state

- deuteron $B = 2.22$ MeV



hadronic molecule ⟶ clustering of quarks

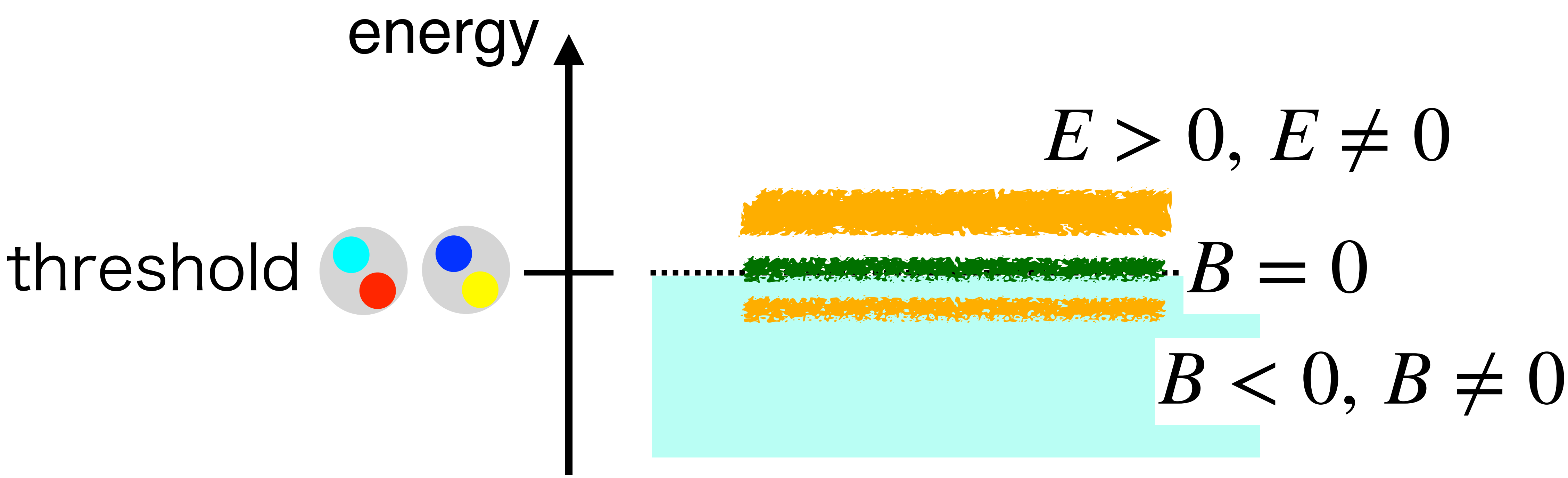


- clustering phenomena: **universal**!

Low-energy universality

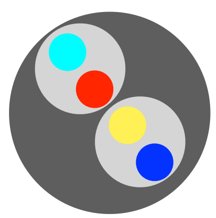
- near-threshold energy region ← universal phenomena

E. Braaten and H.-W. Hammer, Phys. Rept. **428**, 259 (2006); P. Naidon and S. Endo, Rept. Prog. Phys. **80**, 056001 (2017).



- binding energy $B = 0$ (exactly at threshold)

→ that state is **always** completely composite ($X = 1$)



compositeness is model-**independently** determined

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

- structure of **near**-threshold states ($B \neq 0$)? ← this work

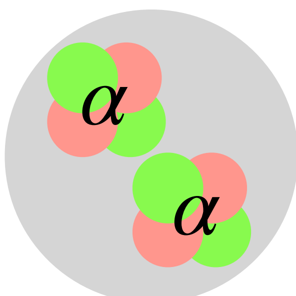
similar to $B = 0$ state? $X \sim 1$?

Near-threshold states

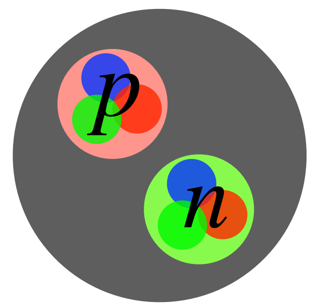
◉ Are near-threshold states composite dominant ($X \sim 1$)?

- **empiricaly** : Yes! “threshold energy rule”

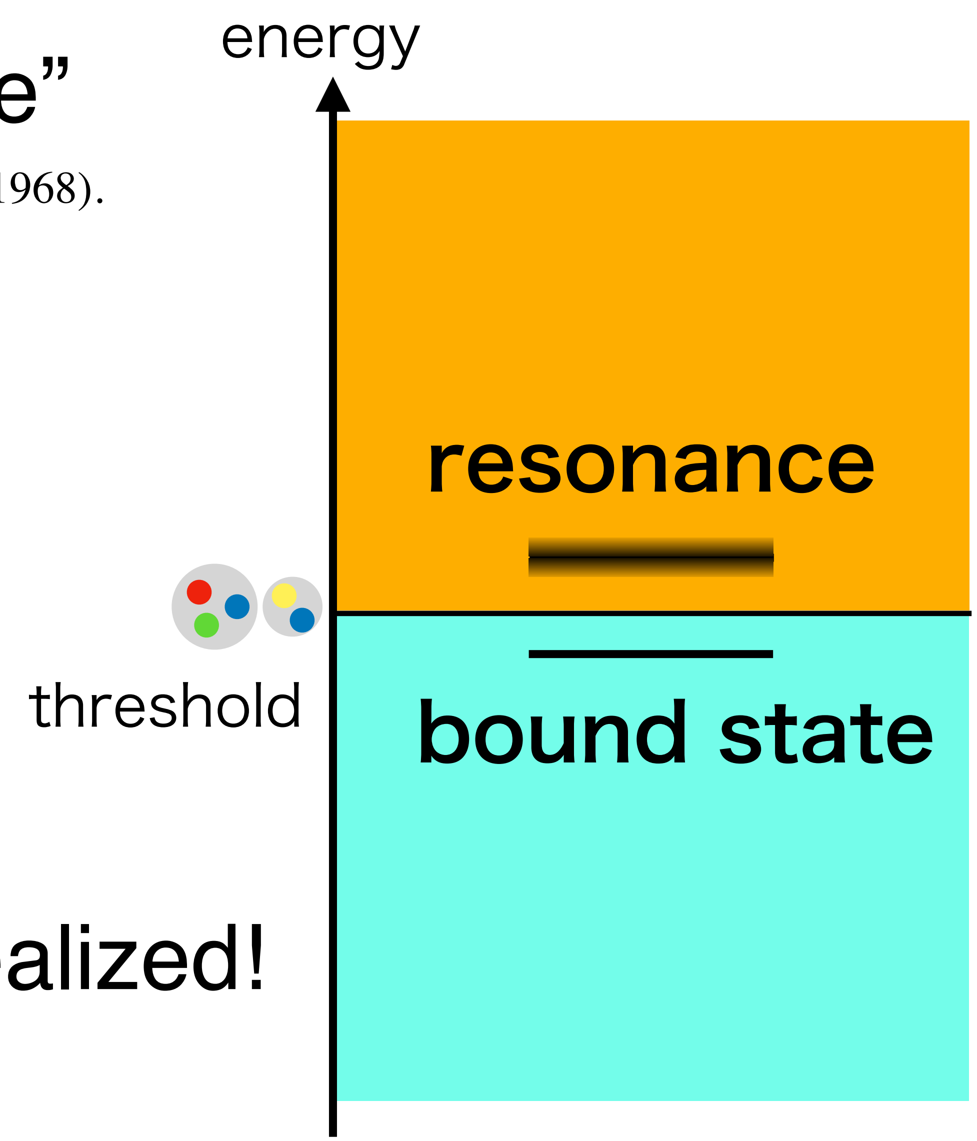
K. Ikeda, and N. Takizawa, and H. Horiuchi, Prog. Theor. Phys. Suppl. E68, 464-475 (1968).



^{8}Be nuclei



deuteron



- **theoretically**

Even non-composite dominant shallow bound states ($X \sim 0$) can be realized!

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;

C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014).

→ Why do we rarely observe such a state?

structure of **resonances** slightly above threshold?

→ provide theoretical foundation of threshold energy rule

Compositeness

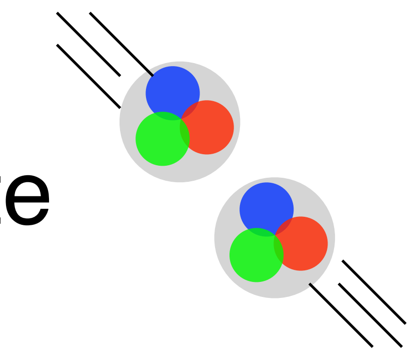
Weinberg, S. Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013);
T. Kinugawa, T. Hyodo, arXiv:2411.12285 [hep-ph] (accepted in EPJ A).

- decomposing Hamiltonian into free part and remainder

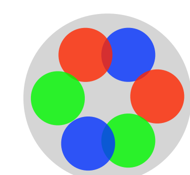
$$\hat{H} = \hat{H}_0 + \hat{V}$$

free Hamiltonian

$$\hat{H}_0 |\boldsymbol{p}\rangle = \frac{\boldsymbol{p}^2}{2\mu} |\boldsymbol{p}\rangle$$ free scattering state



$$\hat{H}_0 |\phi\rangle = \nu_0 |\phi\rangle$$ bare discrete state



- compositeness X, elementarity Z

$$X = \int d\boldsymbol{p}\ |\langle \boldsymbol{p} | B \rangle|^2, \quad Z = |\langle \phi | B \rangle|^2$$

overlaps between bound state $|B\rangle$ and $|\boldsymbol{p}\rangle$ or $|\phi\rangle$

- decomposition is not unique! $\longrightarrow$ **model dependence** of X

S. Weinberg, Phys. Rev. 130, 776 (1963); D. Luri´e and A. J. Macfarlane, Phys. Rev. 136, B816 (1964).

Compositeness

◉ model calculation

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **85**, 015201 (2012);
F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

$$T = \frac{1}{V^{-1} - G}$$

V : effective interaction

G : loop function

residue of scattering amplitude g

$$X = -g^2 G'(E)\Big|_{E=-B} = \frac{G'(E)}{G'(E) - [V^{-1}(E)]'}\Bigg|_{E=-B}$$

$$\alpha'(E) = d\alpha/dE$$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

g^2 : model independent ← $T_{\mathrm{on}}(-B)$ (observable)

$G(E)$: model dependent ← cutoff dependent

History of compositeness

- Weinberg's work (1960s) Weinberg, S. Phys. Rev. 137, 672–678 (1965) etc.

  deuteron is not an elementary particle ← weak-binding relation

  Z : field renormalization factor

- application to exotic hadrons (2000s-)

  X: "compositeness"

  generalization to unstable states

  - with spectral function V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004) etc.
  - with effective range expansion T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) etc.
  - with effective field theory Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017) etc.

  application to …

  - $f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016); T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.
  - $\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.
  - nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

Weak-binding relation

S. Weinberg, Phys. Rev. 137, 672–678 (1965).

$$X = \frac{a_0}{2R - a_0} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)$$

a_0 : scattering length

R_{typ} : typical length scale in system

$R = 1/\sqrt{2\mu B}$

- for weakly bound states, $R \gg R_{\mathrm{typ}}$

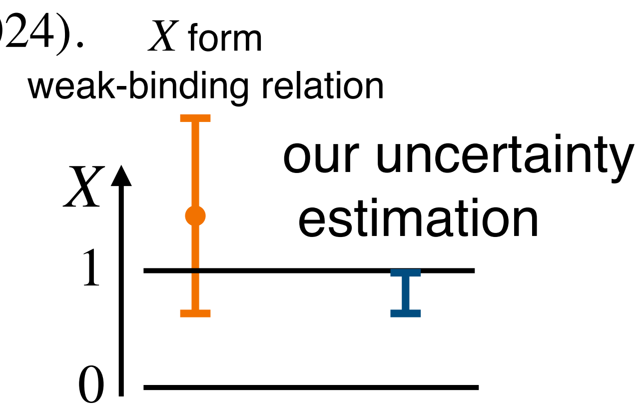
compositeness ⟵ observables (a_0, B)

Y. Li, F.-K. Guo, J.-Y. Pang, and J.-J. Wu, Phys. Rev. D 105, L071502 (2022);
J. Song, L. R. Dai, and E. Oset, Eur. Phys. J. A 58, 133 (2022);
M. Albaladejo, J. Nieves, Eur. Phys. J. C 82, 724 (2022);
T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022);
Z. Yin and D. Jido, Phys. Rev. C 110, no.5, 055202 (2024).

◉ range correction

compositeness of deuteron $X \sim 1.7 > 1$

⟶ important to consider effective range



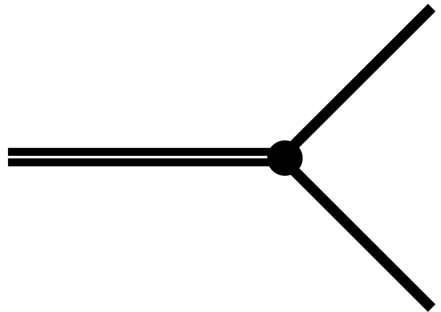
- our work : range correction ⟵ uncertainty estimation

compositeness of deuteron : $0.74 \leq X \leq 1$

Pole trajectory (only w/ s.r.)

◉ pole trajectory in complex momentum k plane (No Coulomb)

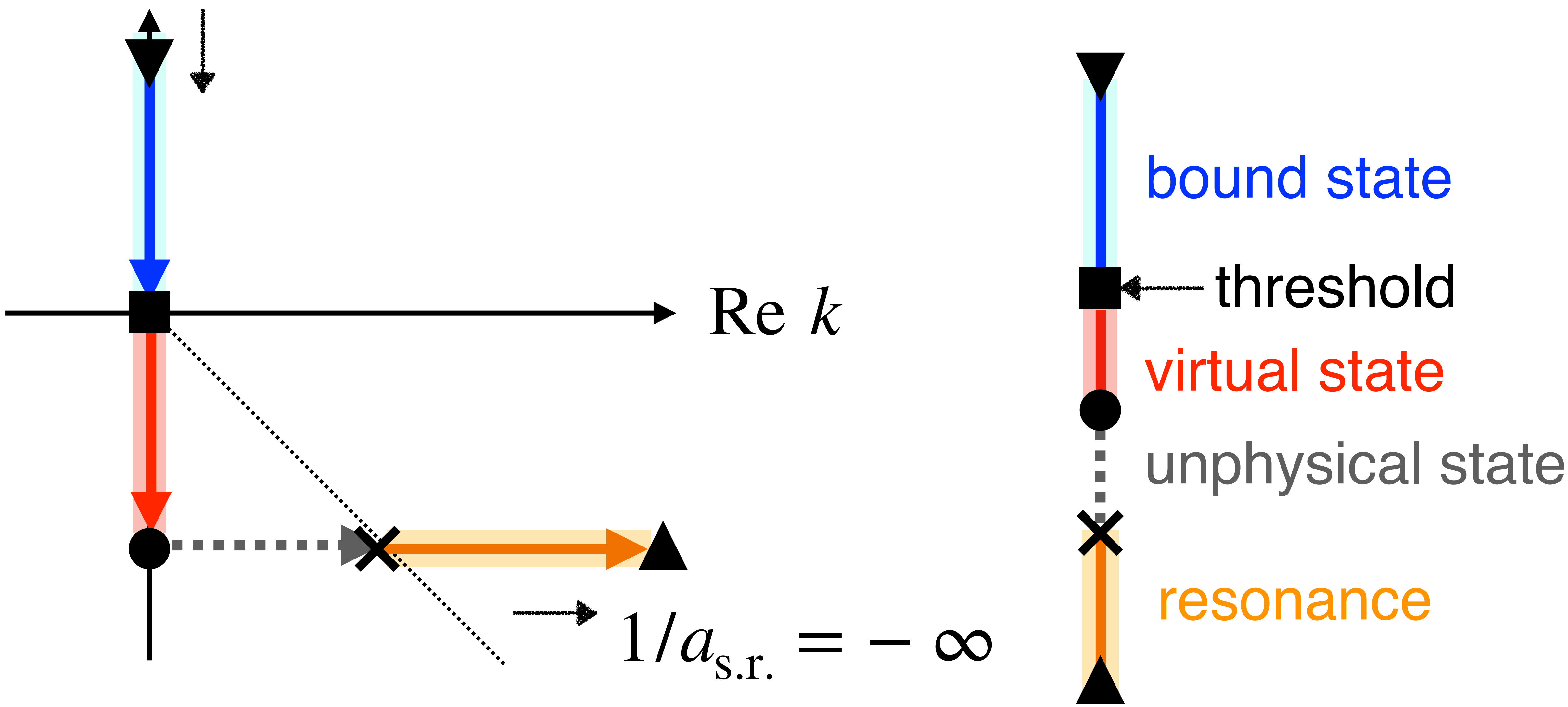
Q channel
⇔ bare state



P channel
⇔ free scattering

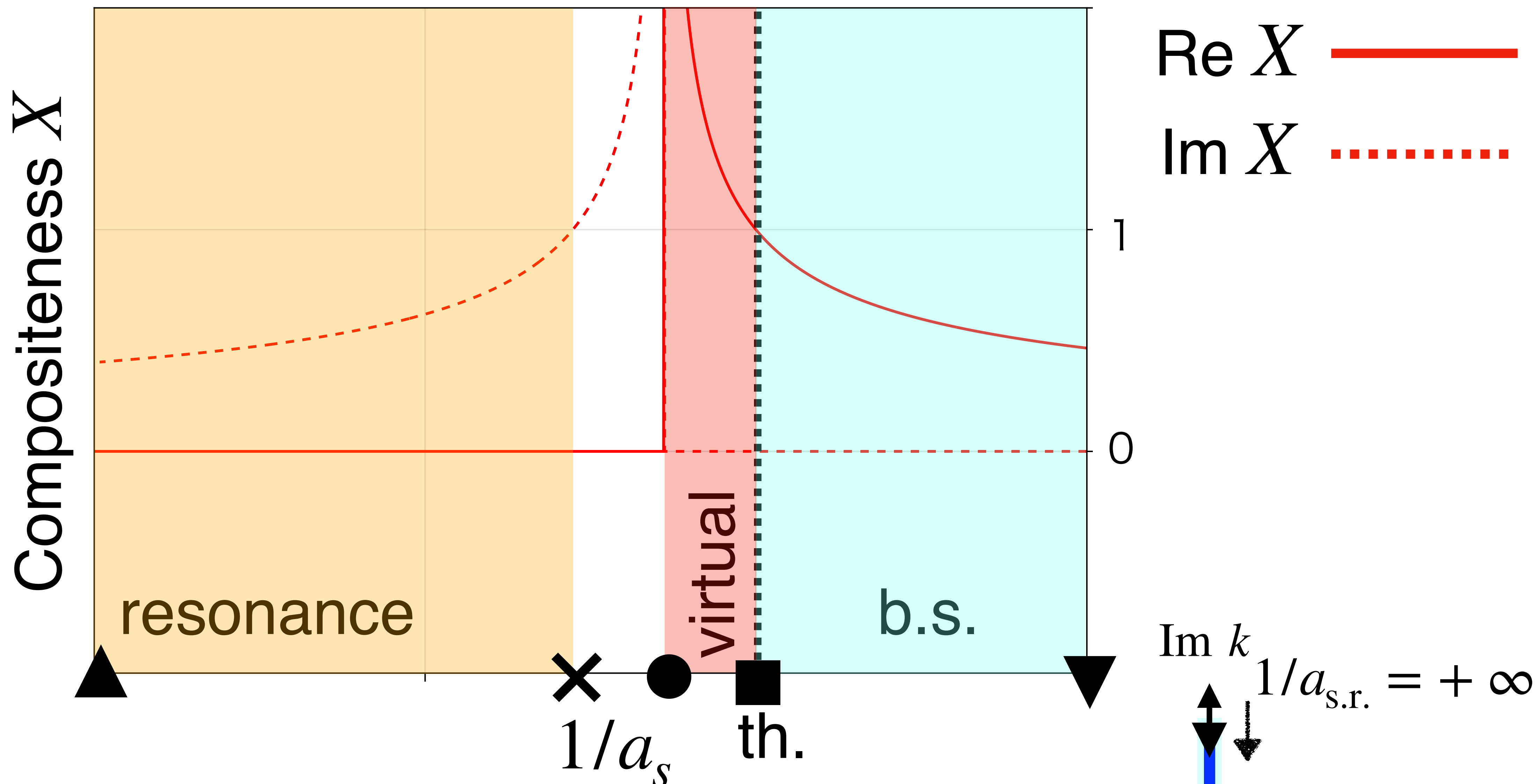
$G_P \to G^0$ free Green's function

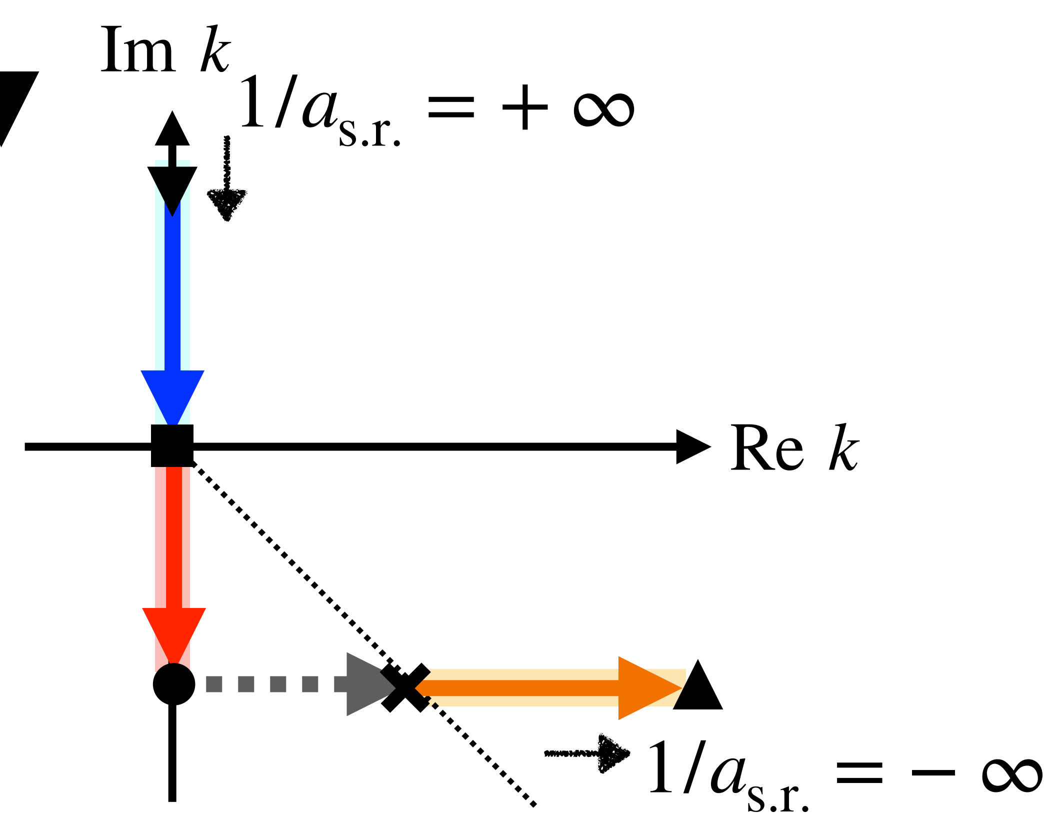
$\mathrm{Im}\ k$ $1/a_{\mathrm{s.r.}} = +\infty$



T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) .

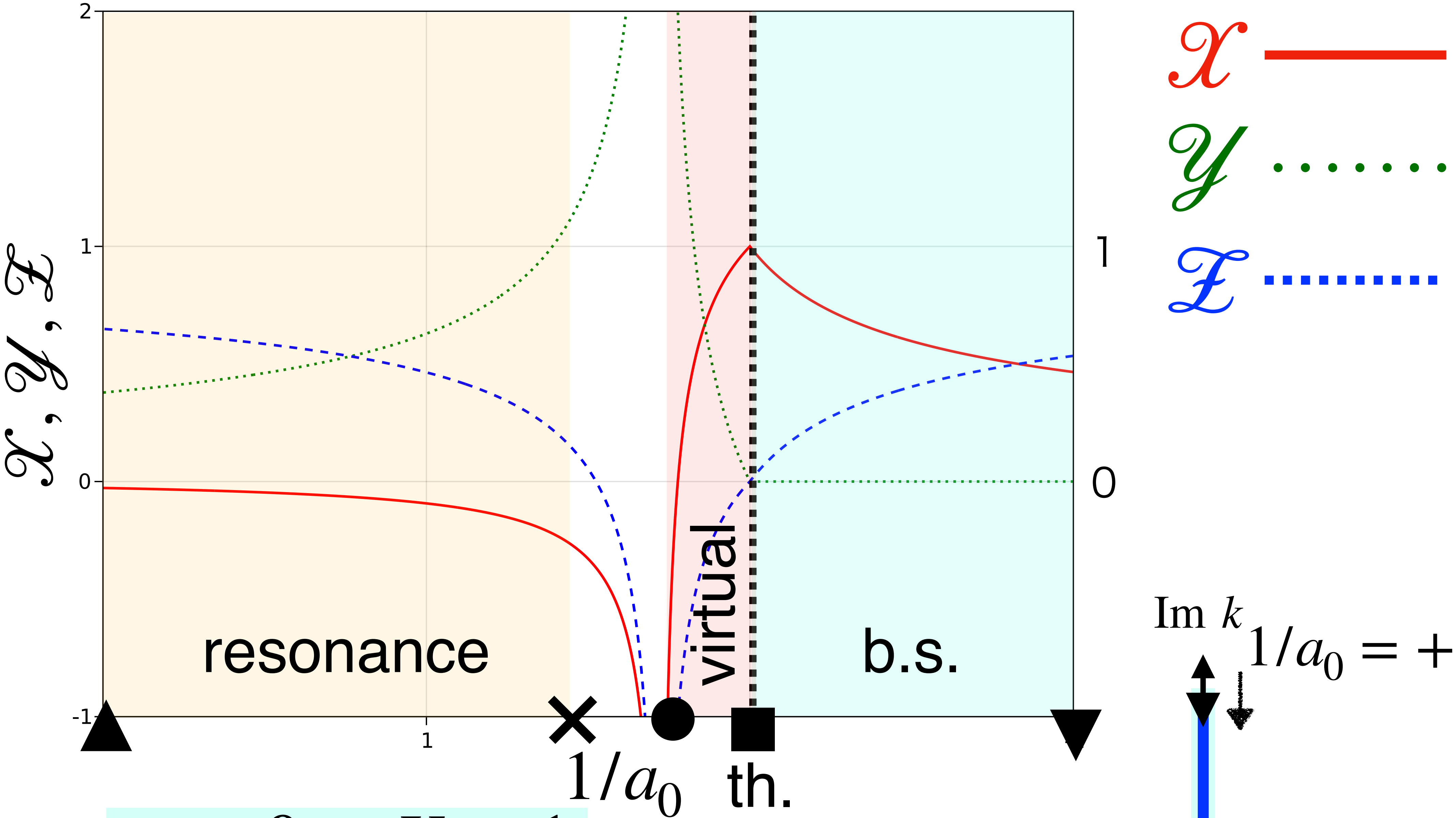
Compositeness (only w/ s.r.)

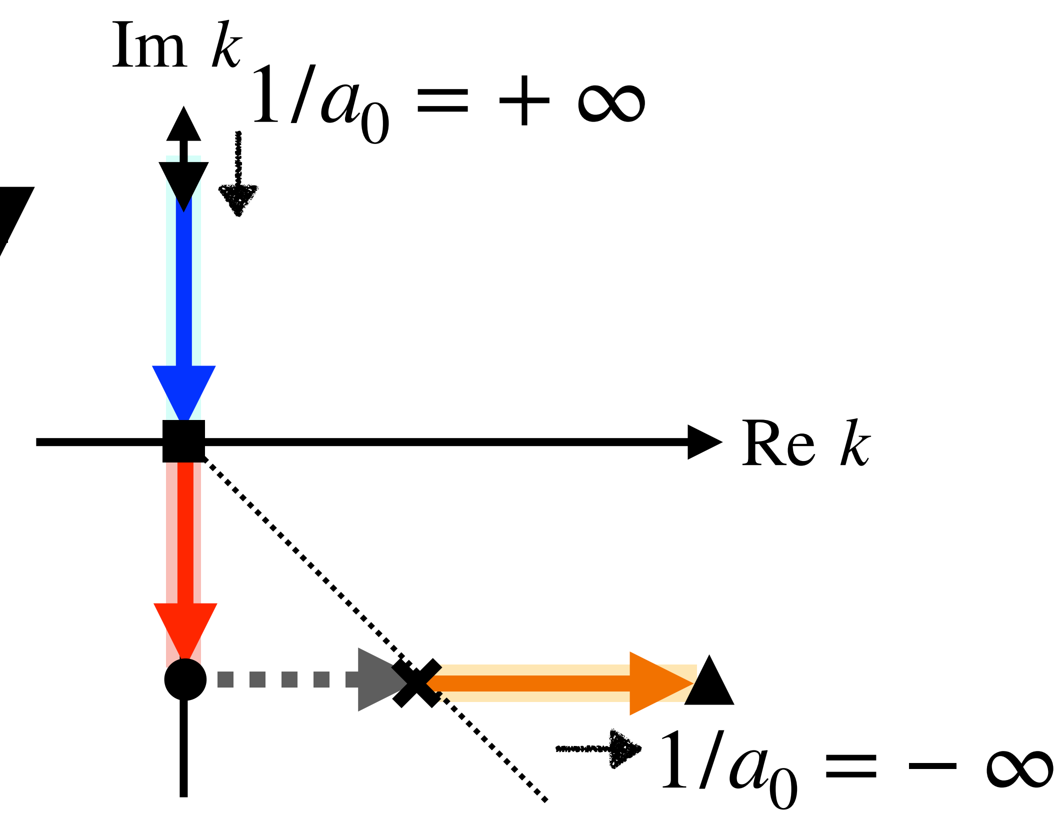




- b.s. : $0 \leq X \leq 1$
- virtual : $1 < X$
- ● divergence $X \to \infty$
- resonance : $\mathrm{Im}\ X \leq 1$

$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ from b.s. to resonance





- b.s. : $0 \leq X \leq 1$
- virtual : $\mathscr{Z} < 0$ (non-interpretable)

● divergence $\mathscr{X}, \mathscr{Y}, \mathscr{Z} \to \infty$

- resonance : $\mathscr{Z}$ dominant