クーロン力と短距離力が共存する系の閾値近傍状態の複合性

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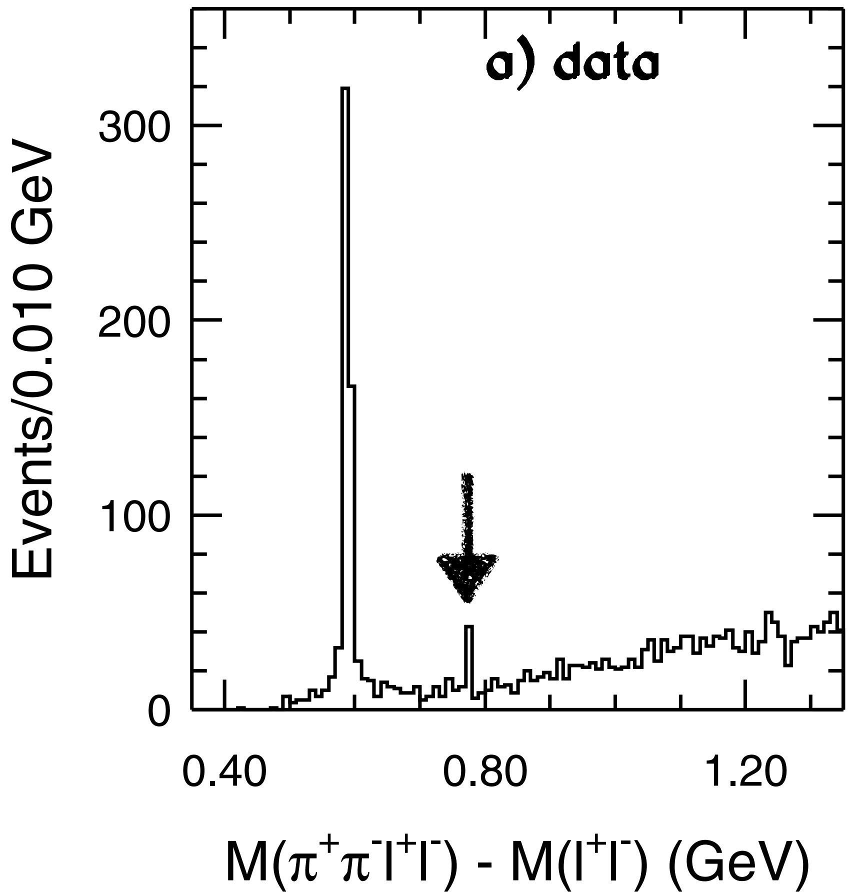
September 17th, JPS2024 秋

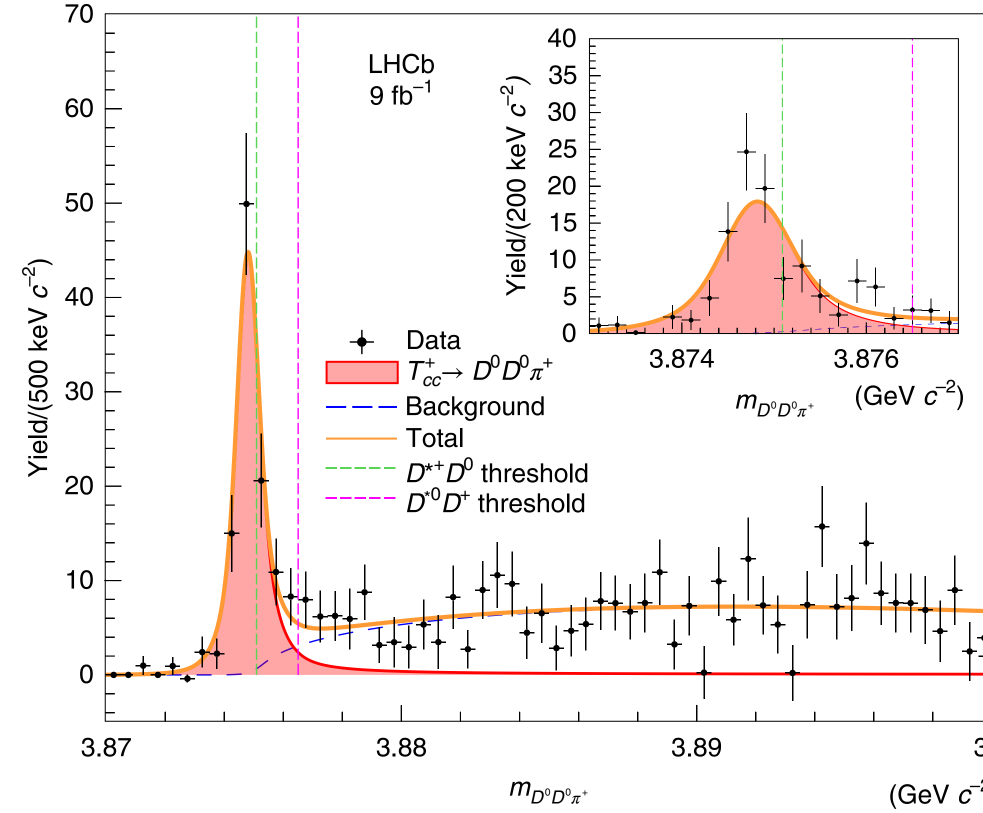
Near-threshold exotic hadrons

$X(3872) \to \pi^+\pi^- J/\psi$



S. K. Choi *et al*. (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

$T_{cc} \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$



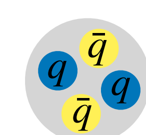
LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

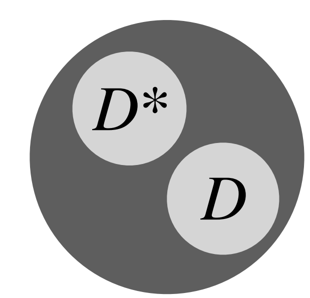
- internal structure?

exotic hadron

$\neq$ qqq or $q\bar{q}$ → multiquarks, hadronic molecules



multiquarks

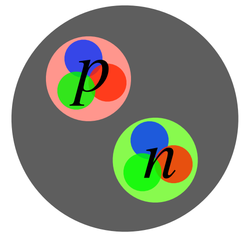


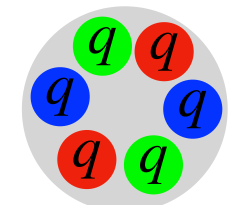
hadronic molecules

Compositeness

S. Weinberg, Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).

◉ definition





wavefunction

$$|\Psi\rangle = \underbrace{\sqrt{X}}_{\text{compositeness}}|\text{composite}\rangle + \underbrace{\sqrt{1-X}}_{\text{elementarity}}|\text{non composite}\rangle$$

※ $0 \leq X \leq 1$ ⟶ $X > 0.5 \Leftrightarrow$ **composite dominant**

$X < 0.5 \Leftrightarrow$ **elementary dominant**

- **quantitative** analysis of internal structure

deuteron is not an elementary particle S. Weinberg, Phys. Rev. 137, 672–678 (1965).

$f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016); T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.

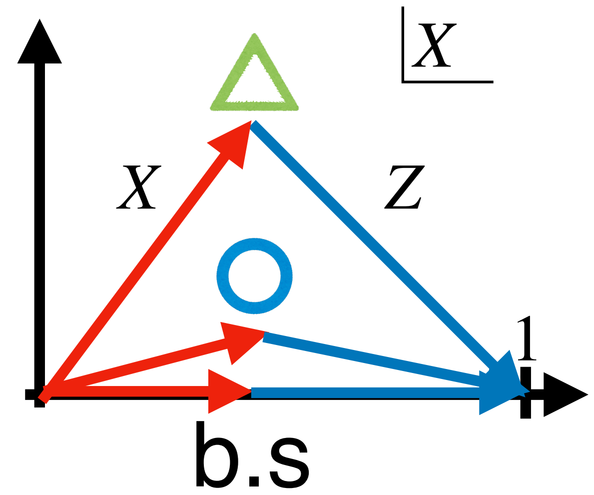
$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

Compositeness of resonances

- interpretation for complex X of resonances?

complex X plane



$X \in \mathbb{C}$ and $X+Z=1$

- If Im X is large, it seems that reasonable interpretation is impossible △

◉ our proposal

T. Berggren, Phys. Lett. B 33, 547 (1970).

i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$

ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$

iii) $\mathscr{Y}$: probability of uncertain identification

◉ conditions for sensible interpretation

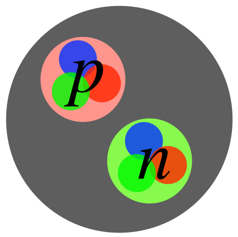
- normalization : $\mathscr{X}+\mathscr{Y}+\mathscr{Z}=1$ for probabilistic interpretation
- in bound state limit : $\mathscr{X} \to X$, $\mathscr{Z} \to Z$ and $\mathscr{Y} \to 0$

$\mathscr{Y}$ characterizes uncertainty of resonance

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

Near-threshold states

◉ near-threshold states with short range interaction



- at threshold ($E = 0$)

  completely composite ($X = 1$)

  ∵ low-energy universality $|a_s| \to \infty$

  T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

- near-threshold bound states

  ($E \neq 0$, but small **negative**)

  composite dominant ($X \sim 1$)

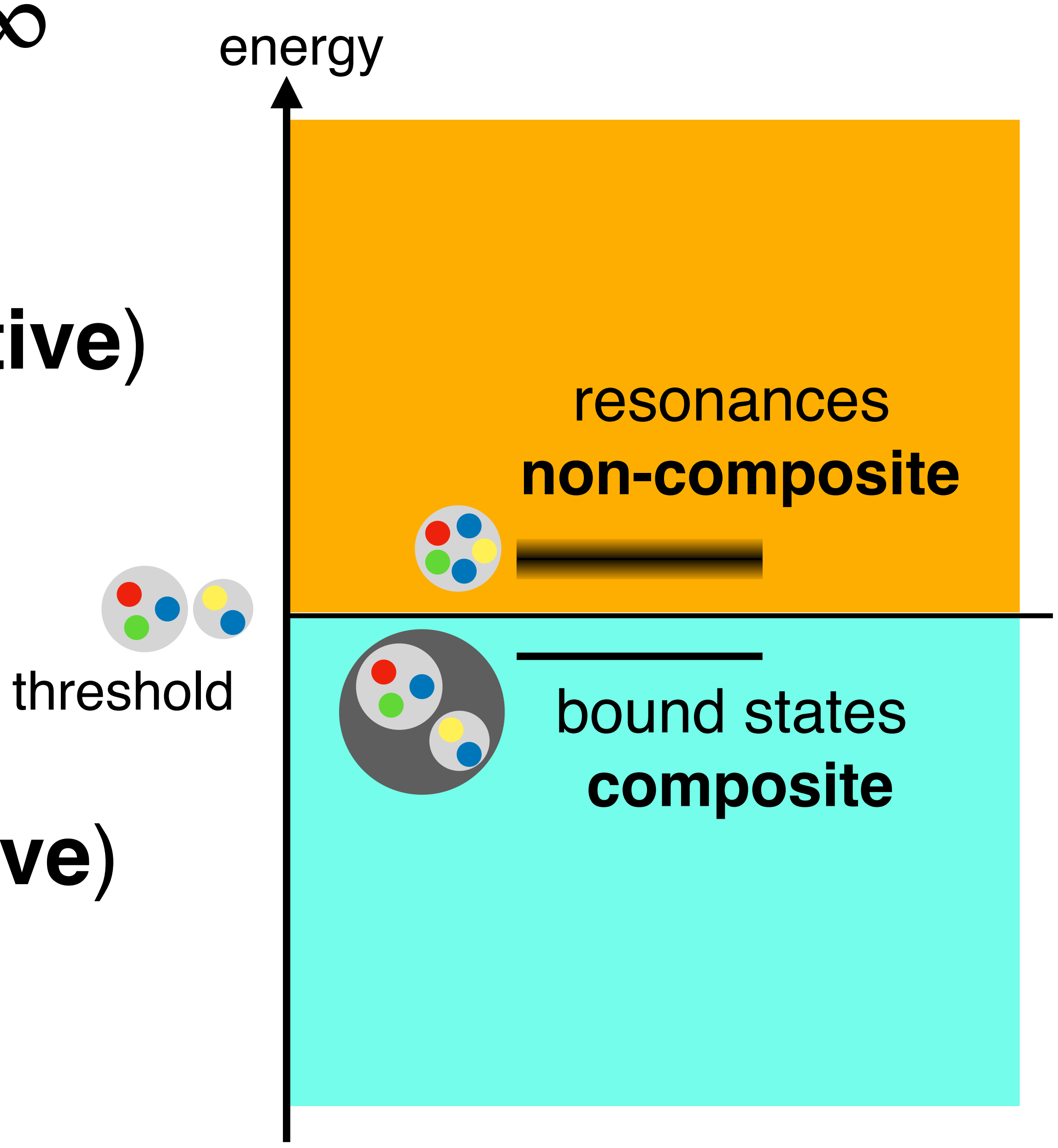
  C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014);
  T. Kinugawa and T. Hyodo Phys. Rev. C 109 , 045205 (2024).

- near-threshold resonances

  ($E \neq 0$, but small **positive**)

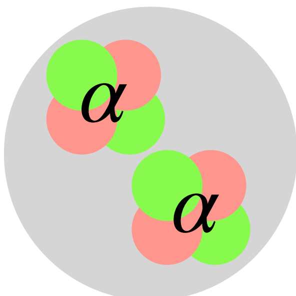
  non-composite dominant ($\mathcal{X} \sim 0$)

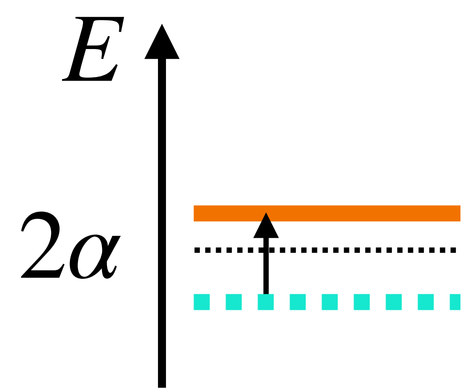
  T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].



Coulomb + short range systems

- ^{8}Be nuclei J. Hiura, and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 52, 25 (1972).

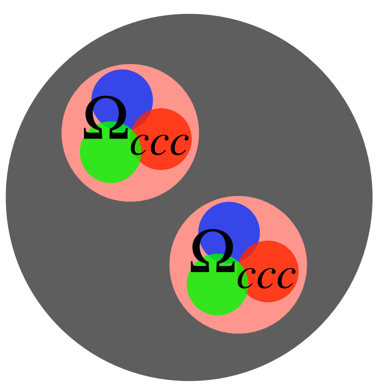


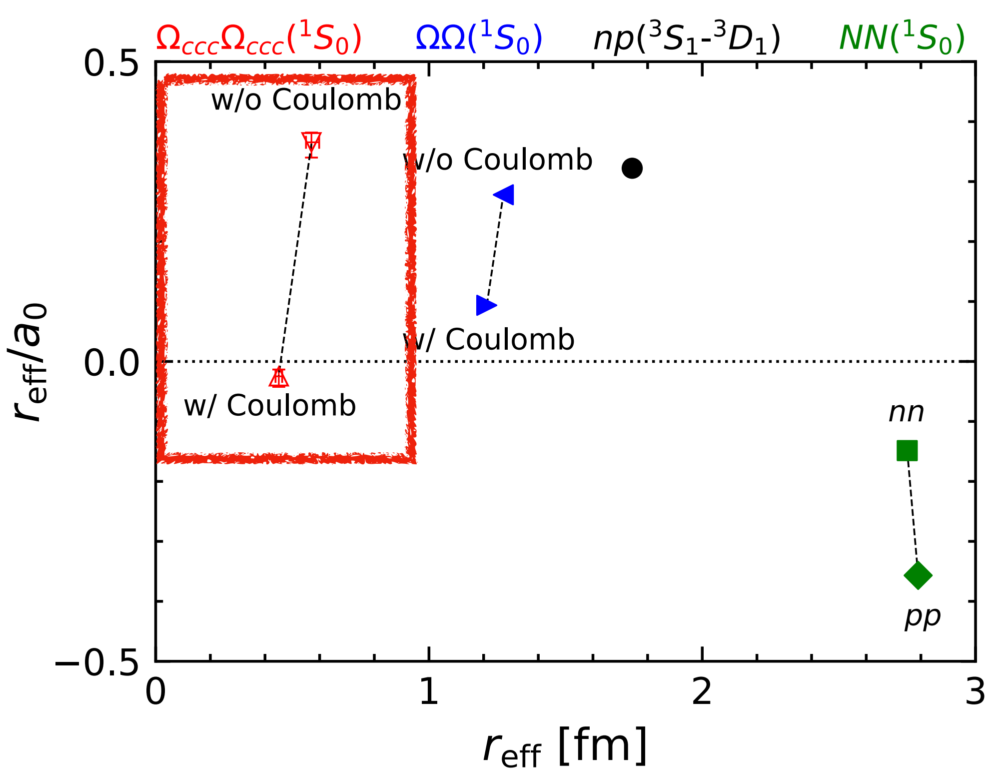


resonance (w/ Coulomb)

bound (w/o Coulomb)

- $\Omega_{ccc}\,\Omega_{ccc}$ (HAL QCD) Y. Lyu, H.Tong, *et al*. [HAL QCD Coll.], Phys. Rev. Lett. 127 (2021) 072003.





resonance (w/ Coulomb)

bound (w/o Coulomb)

- $\Xi^{-}\alpha$: Coulomb assisted bound state ? E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, Phys. Rev. C 106, 064318 (2022).

Coulomb is important for near-threshold states!

Coulomb + short range systems

◉ Coulomb + short range interaction

H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

R. Oppenheim Berger and Larry Spruch, Phys. Rev. 138, B1106-B1115 (1965).

W. Domcke, Atom. Mol. Phys. 16, 359 (1983).

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809, 171 (2008).

C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798, 135016 (2019).

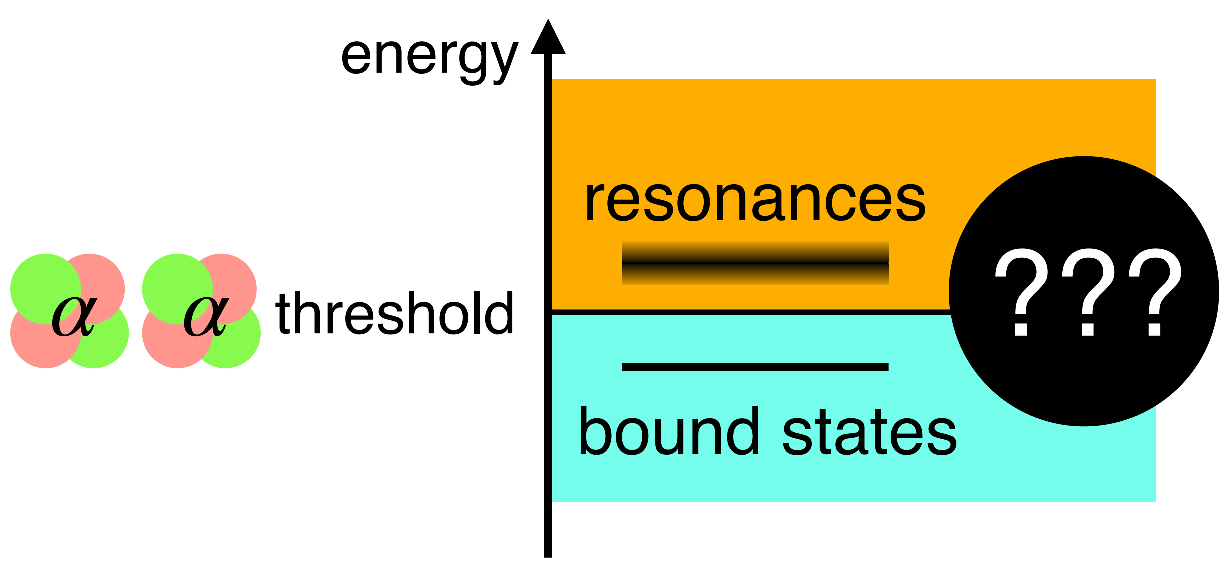
S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

- we **cannot** expand scattering amplitude $f(k)$ in terms of k^2

$$\frac{1}{f(k)} = -\frac{1}{a_{\mathrm{s.r.}}} + \frac{r_{\mathrm{s.r.}}}{2}k^2 + \mathcal{O}(k^4) - ik$$

→ different from short range interaction

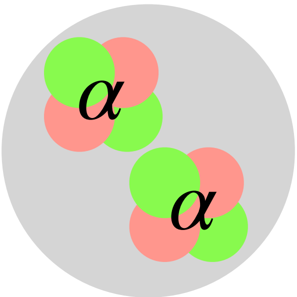
◉ nature of near-threshold state with Coulomb + short range interaction?



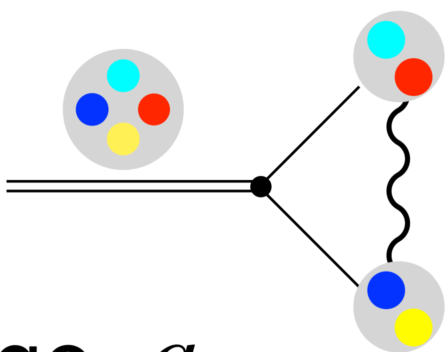
- two body
- small k region → s-wave
- Coulomb repulsive (& attractive)

This work

near-threshold bound states & resonances
with **Coulomb + short range** interaction



<u>framework : model with Feshbach method</u>



- bare state which couples to Coulomb scattering

- Coulomb scattering length, Coulomb effective range, a_B

<u>numerical calculations & discussion</u>

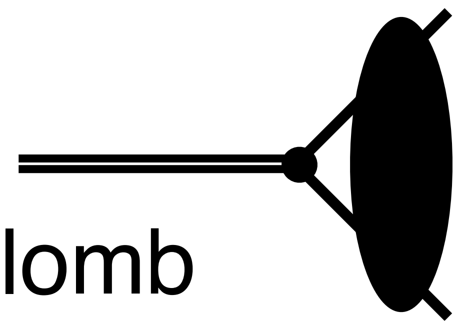
- investigate pole trajectory
- analyze internal structure with compositeness
- study universal nature of near-threshold states

Coulomb+short range model

◉ Hamiltonian

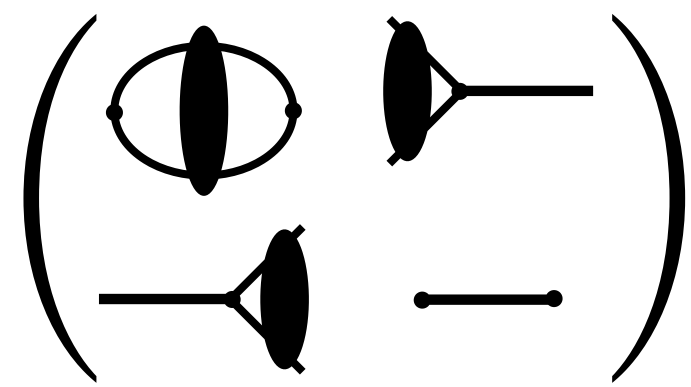
W. Domcke, Atom. Mol. Phys. 16 359 (1983). C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798, 135016

Q channel
⇔ b.s. w/o Coulomb



P channel
⇔ scattering w/ Coulomb

$$\hat{H} = \begin{pmatrix} \hat{H}_{PP} & \hat{H}_{PQ} \\ \hat{H}_{QP} & \hat{H}_{QQ} \end{pmatrix} =$$



H. Feshbach, Annals Phys. 19 287-313 (1962).

◉ pole condition

C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798 (2019).

$$-\frac{1}{\boxed{a_s}} + \frac{\boxed{r_e}}{2}k^2 - ik \pm \frac{2}{\boxed{a_B}}\left[\log(-ia_B k) + \psi\left(1 + \frac{i}{a_B k}\right)\right] = 0$$

◉ compositeness X

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

$$X = 1 - Z = 1 - \frac{1}{1 - \frac{d}{dE}\boxed{F(E)}}$$

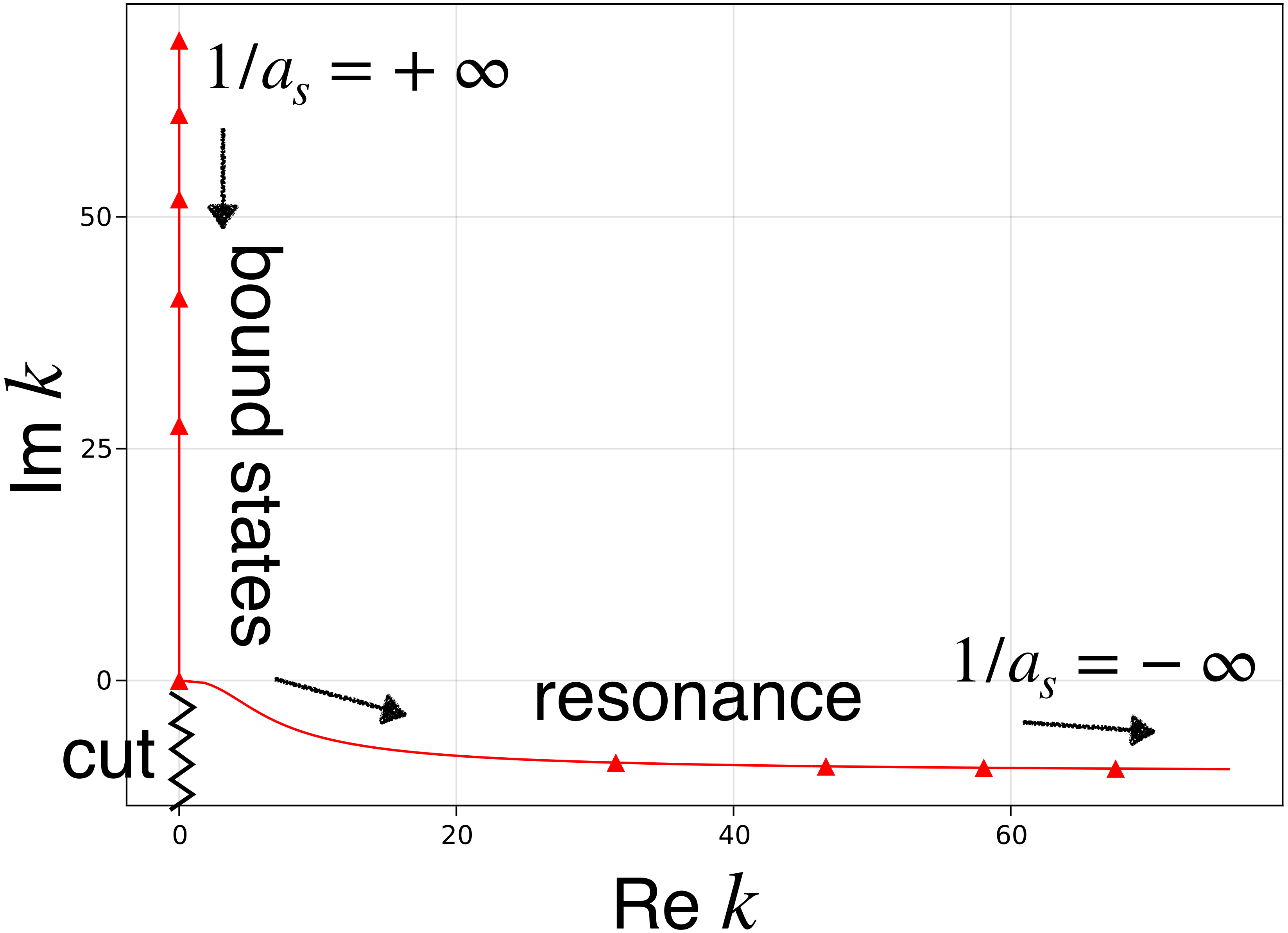
self energy

Pole trajectory (repulsive Coulomb)

◉ pole trajectory in complex momentum k plane

- varying Coulomb scattering length a_s with fixed r_e and a_B

→ pole position (eigenmomentum) moves

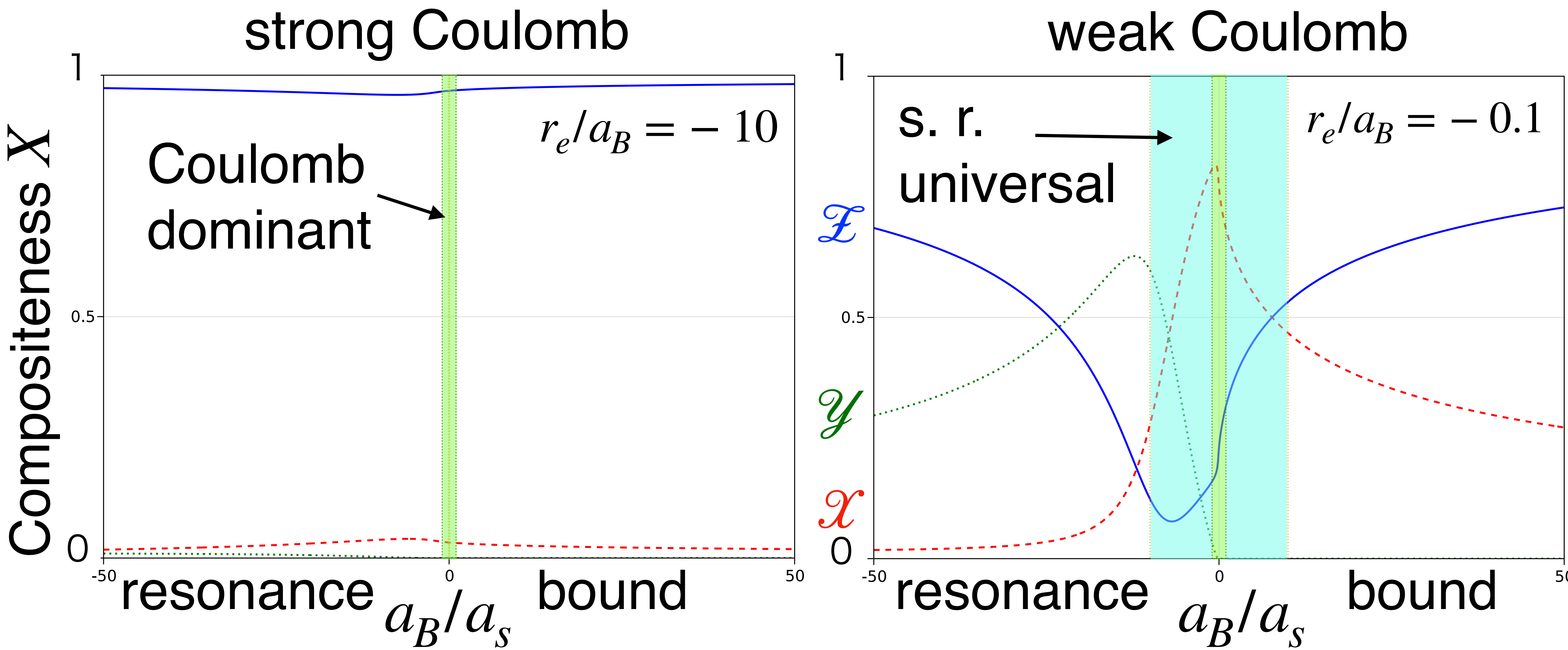


- b.s directory goes to resonance
- $a_s \to \infty$ at threshold
- but no universality

∵ radius of w.f. $< \infty$

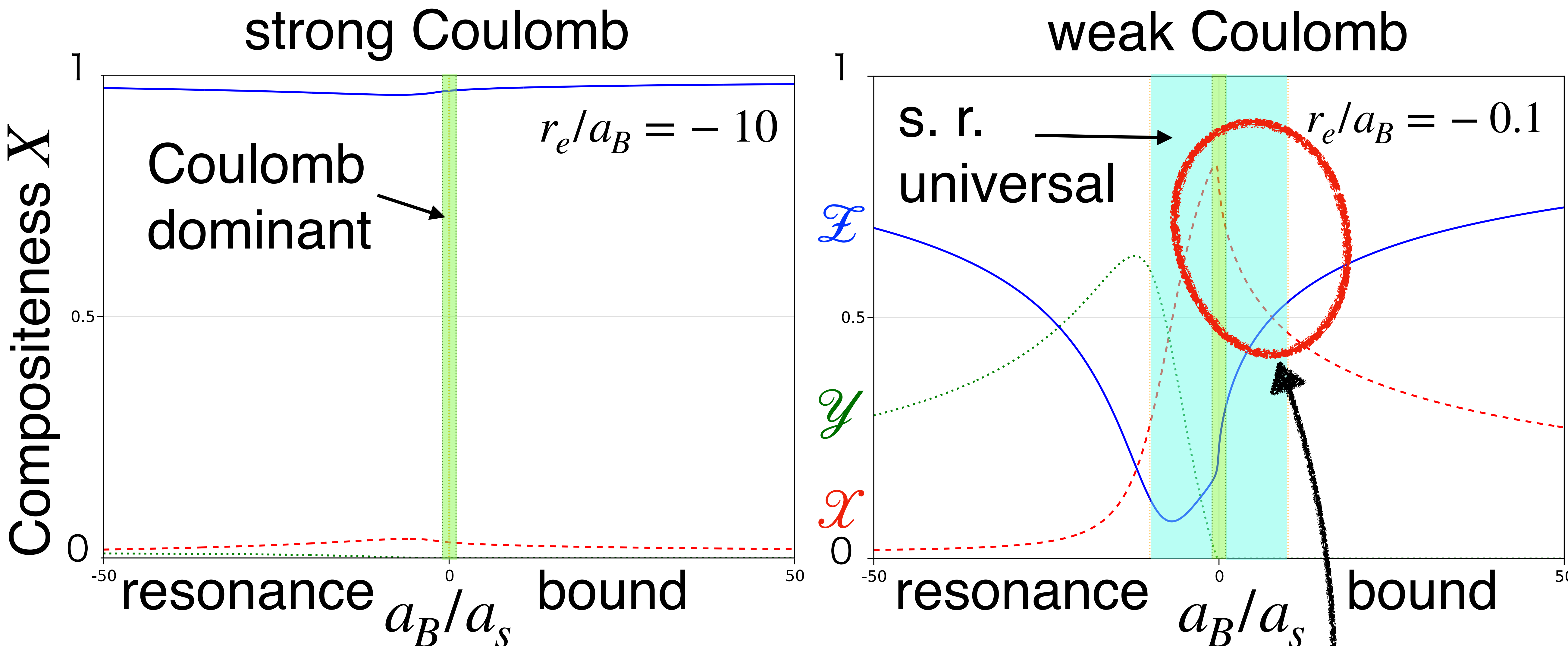
S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

Compositeness (repulsive Coulomb)



- complex compositeness ⟵ $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].
- states with large $|1/a_s|$ are elementary $\mathcal{Z}$ dominant
- structure of bound states ≈ resonances ∵ continuous X

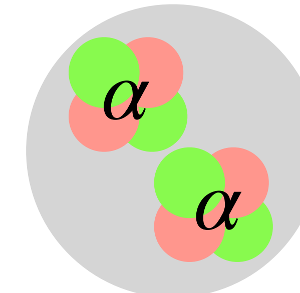
Compositeness (repulsive Coulomb)

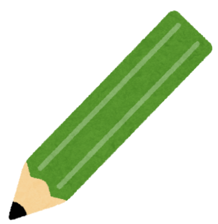


- complex compositeness ← $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].
- states with large $|1/a_s|$ are elementary $\mathscr{Z}$ dominant
- structure of bound states ≈ resonances ∵ continuous X
- remnant of short range universality in $|r_e| \ll |a_B|$ case
  $X \to 1$ in $B \to 0$ limit in short range

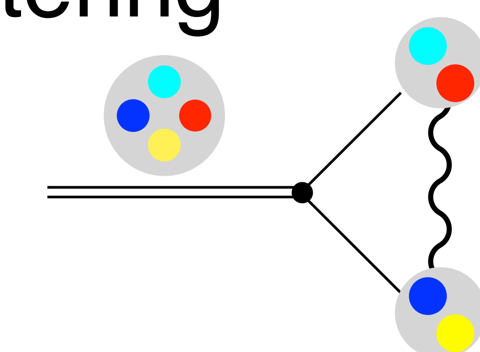
Summary

near-threshold bound states & resonances
with **Coulomb + short range** interaction





- bare state which couples to Coulomb scattering
- pole condition ← $a_s,\ r_e,\ a_B$



- repulsive Coulomb

bound → resonance (does not become virtual states)

X is not necessary to be unity at threshold

if Coulomb is weak, remnant of s.r. universality can be seen

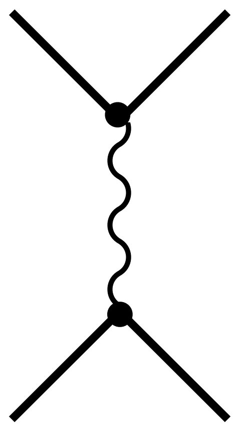
nature of b.s. ≈ nature of resonance

Back up

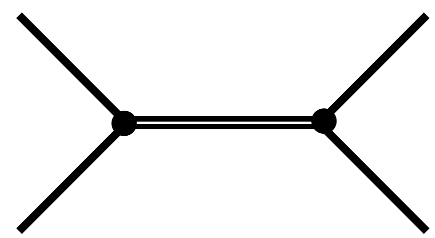
Coulomb+short range model

◉ model

Coulomb



short range (s.r.)



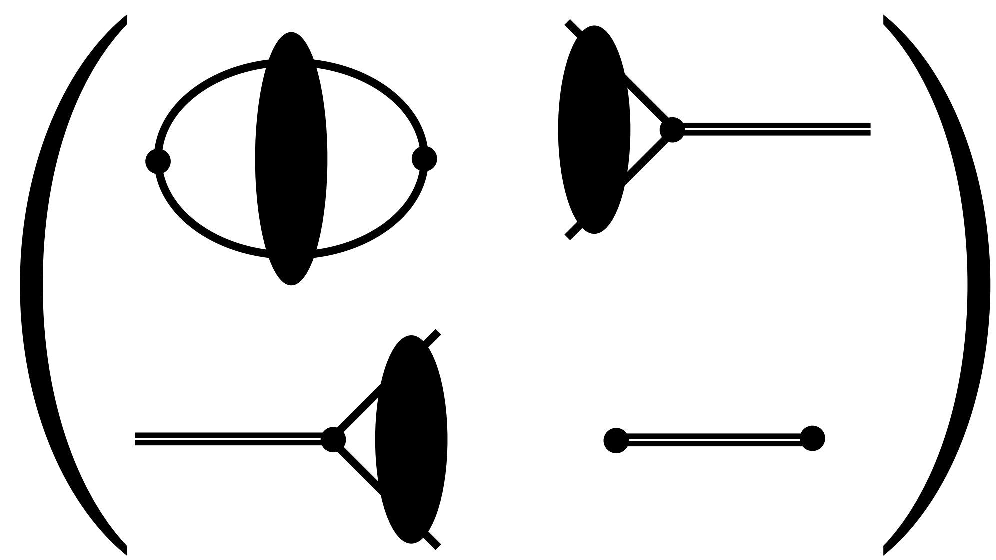
Weinberg, S. Phys. Rev. 137, 672–678 (1965);
V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004);
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

◉ Hamiltonian

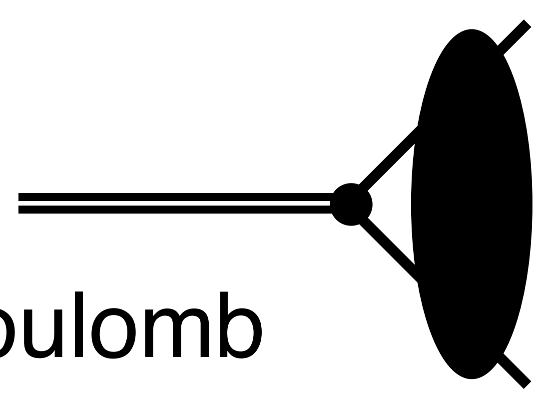
W. Domcke, Atom. Mol. Phys. 16 359 (1983);
H. Feshbach, Annals Phys. 19 287-313 (1962).

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

$$\hat{H} = \begin{pmatrix} \hat{H}_{PP} & \hat{H}_{PQ} \\ \hat{H}_{QP} & \hat{H}_{QQ} \end{pmatrix} =$$



Q channel
⇔ b.s. w/o Coulomb



P channel
⇔ scattering w/ Coulomb + s.r.

Coulomb+short range model

◉ Schrödinger equation $\hat{H}|\Psi\rangle = E|\Psi\rangle \quad |\Psi\rangle = \begin{pmatrix} |P\rangle \\ |Q\rangle \end{pmatrix}$

$$\hat{H}_{PP}|P\rangle + \hat{H}_{PQ}|Q\rangle = E|P\rangle$$

$$\hat{H}_{QQ}|Q\rangle + \hat{H}_{QP}|P\rangle = E|Q\rangle$$

◉ effective Hamiltonian (channel eliminating)

$$\hat{H}_{P\mathrm{ch}}|P\rangle = E|P\rangle \quad \hat{H}_{P\mathrm{ch}} = \hat{H}_{PP} + \hat{H}_{PQ}(E - \hat{H}_{QQ})^{-1}\hat{H}_{QP}$$

Coulomb+short range model

◉ Schrödinger equation $\hat{H}|\Psi\rangle = E|\Psi\rangle \quad |\Psi\rangle = \begin{pmatrix} |P\rangle \\ |Q\rangle \end{pmatrix}$

$$\hat{H}_{PP}|P\rangle + \hat{H}_{PQ}|Q\rangle = E|P\rangle$$

$$\hat{H}_{QQ}|Q\rangle + \hat{H}_{QP}|P\rangle = E|Q\rangle$$

◉ effective Hamiltonian (channel eliminating)

$$\hat{H}_{P\mathrm{ch}}|P\rangle = E|P\rangle \quad \hat{H}_{P\mathrm{ch}} = \underbrace{\boxed{\hat{H}_{PP}}}_{=\hat{H}^0 + \hat{V}_P} + \underbrace{\boxed{\hat{H}_{PQ}(E-\hat{H}_{QQ})^{-1}\hat{H}_{QP}}}_{=\hat{V}_Q}$$

$$\longrightarrow \hat{H}_{P\mathrm{ch}} = \hat{H}^0 + (\hat{V}_P + \hat{V}_Q)$$

$\hat{H}^0$: free Hamiltonian

$\hat{V}_P$: pure Coulomb interaction

$\hat{V}_Q$: short range interaction

Coulomb+short range model

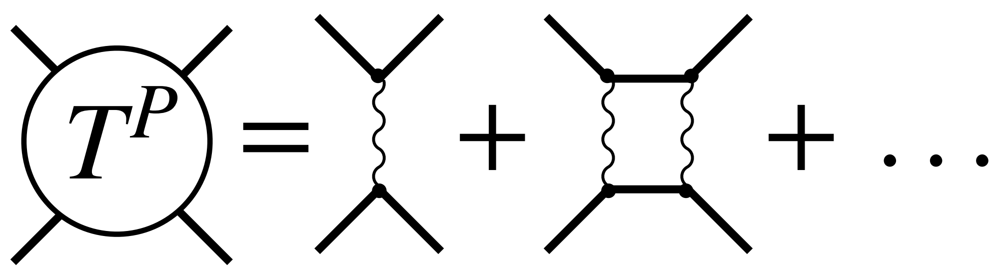
◉ T-matrix

H. Feshbach, Annals Phys. 19 287-313 (1962); R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).
W. Domcke, Atom. Mol. Phys. 16 359 (1983);

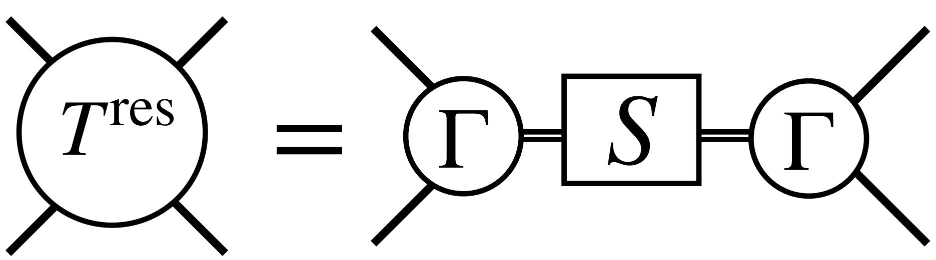
Lippmann-Schwinger eq. : $\hat{T} = [(\hat{V}_P + \hat{V}_Q)^{-1} - \hat{G}^0]^{-1}$

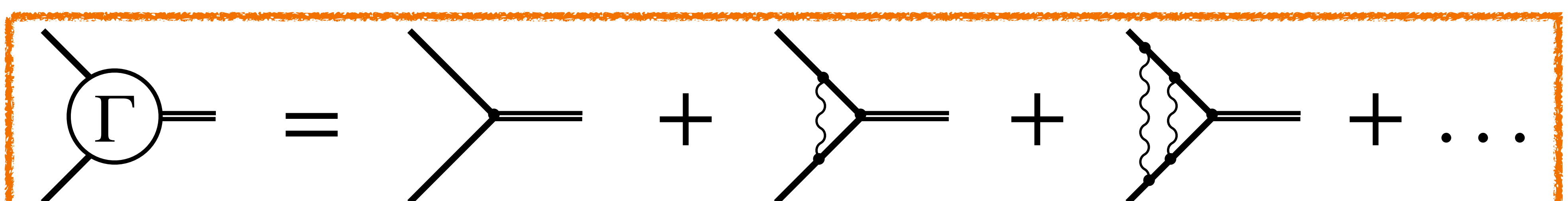
⇔ Feshbach method : $\hat{T} = \hat{T}^P + \hat{T}^{\rm res}$

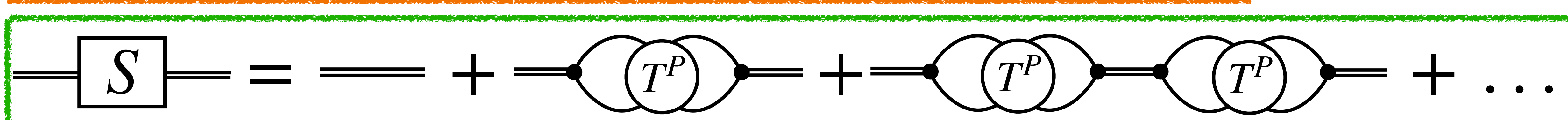
$$\hat{T}^P = [\hat{V}_P^{-1} - \hat{G}^0]^{-1}, \quad \hat{T}^{\rm res} = \boxed{\hat{T}^P \hat{V}_P^{-1}}\boxed{[\hat{V}_Q^{-1} - \hat{G}_P]^{-1}}\boxed{\hat{V}_P^{-1}\hat{T}^P}$$



pure Coulomb



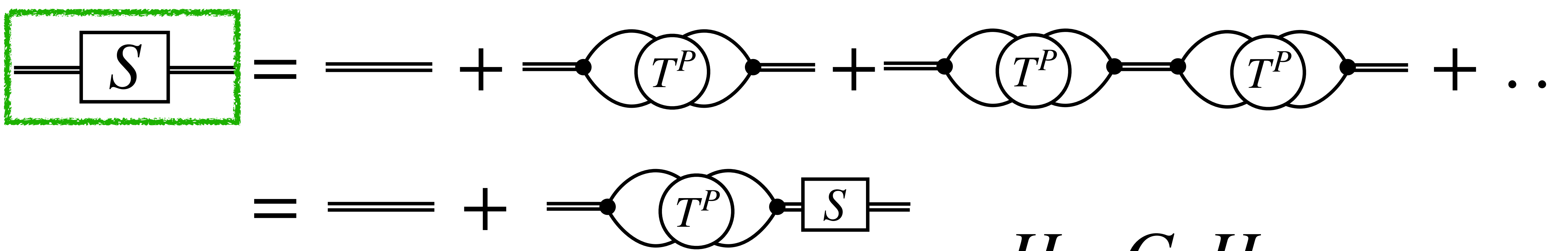




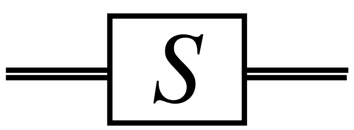
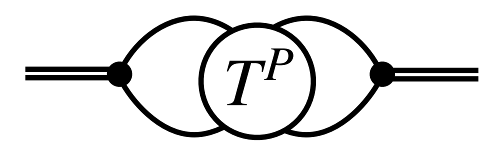
Coulomb+short range model

◉ pole condition $\hat{T} = \hat{T}^P + \hat{T}^{\mathrm{res}}$

pole of $T(k,k')$ $\Leftrightarrow$ pole of $T^{\mathrm{res}}(k,k')$ $\Leftrightarrow$ $\boxed{[V_Q^{-1} - G_P]^{-1}} = \infty$



$H_{QP}G_PH_{PQ}$

 $= S(E)$, $= (E-\varepsilon_d)^{-1}$,  $= F(E)$

$\longrightarrow$ $S(E) = (E-\varepsilon_d)^{-1} + (E-\varepsilon_d)^{-1}F(E)S(E)$

$= [E - \varepsilon_d - F(E)]^{-1}$

$\longrightarrow$ pole condition : $E - \boxed{\varepsilon_d} - \boxed{F(E)} = 0$

bare state energy self energy

Coulomb+short range model

◉ self energy $F(E)$ **in low-energy limit**

W. Domcke, Atom. Mol. Phys. 16 359 (1983).

- attractive Coulomb

$$F(k) = \frac{A}{2\pi}\left[c - \frac{1}{2}ia_B k + \log(-ia_B k) + \psi\left(1 - \frac{i}{a_B k}\right)\right]$$

- repulsive Coulomb

$$F(k) = -\frac{A}{2\pi}\left[c + \frac{1}{2}ia_B k + \log(-ia_B k) + \psi\left(1 + \frac{i}{a_B k}\right)\right]$$

A : constant with dimension of energy

c : dimensionless constant

$\psi(x) = \dfrac{d}{dx}\log(\Gamma(x))$: digamma function

Coulomb+short range model

◉ pole condition **in low-energy limit**

- Coulomb scattering length a_s and effective range r_e

$$(\text{amplitude})^{-1} = -\frac{1}{a_s} + \frac{r_e}{2}k^2 + \mathcal{O}(k^4) - ik + 2\log(-ik) + 2\psi\left(1+\frac{i}{k}\right) + \ldots,$$

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

$$\longrightarrow a_s = -a_B\left[\frac{4\pi}{A}\varepsilon_d \pm 2c\right]^{-1}, \quad r_e = -\frac{4\pi}{A a_B \mu}$$

- pole condition with a_s and r_e C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798 (2019).

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B}\left[\log(-ik) + \psi\left(1+\frac{i}{k}\right)\right] = 0$$

◉ compositeness X

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

$$X = 1 - \frac{1}{1 - \frac{d}{dE}\boxed{F(E)}}$$

self energy

Coulomb+short range model

● short range limit $a_B \to \infty$

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B}\left[\underbrace{\log(-ia_Bk) + \psi\left(1 + \frac{i}{a_Bk}\right)}\right] = 0$$

$\to 0$

$$\longrightarrow -\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik = 0$$ ERE in s.r. interaction

- larger a_B ⟶ weaker Coulomb $a_B = \dfrac{\hbar c}{\mu c^2 \alpha Z_1 Z_2}$

● further low-energy limit $r_e \to 0$

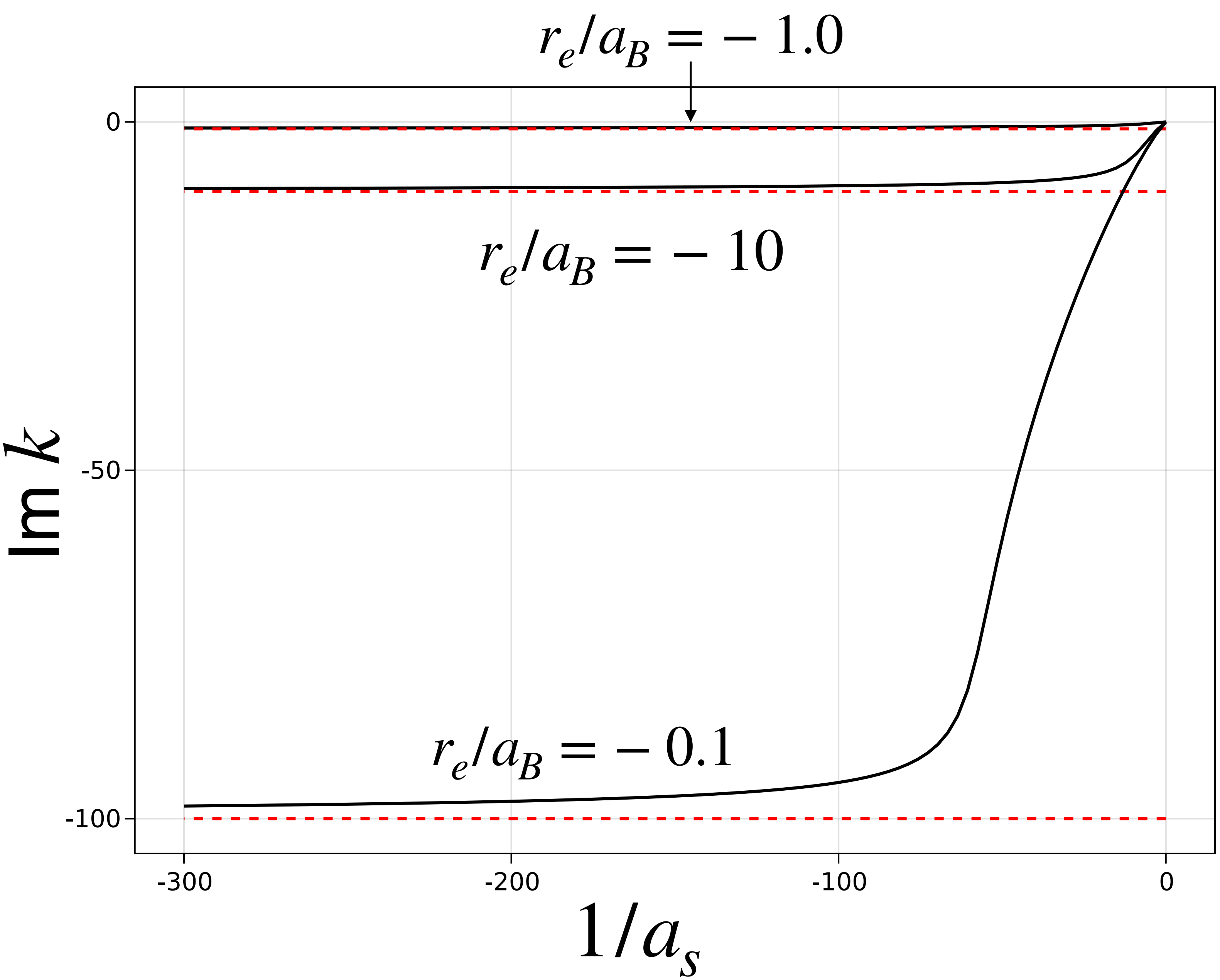
- zero-range theory S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

$$\frac{ia_Bk}{2} \mp \log(-ia_Bk) + \psi\left(1 + \frac{i}{a_Bk}\right) + \frac{a_B}{2a_s} = 0$$

far from threshold (repulsive Coulomb)

◉ imaginary part of eigenenergy in complex momentum k plane

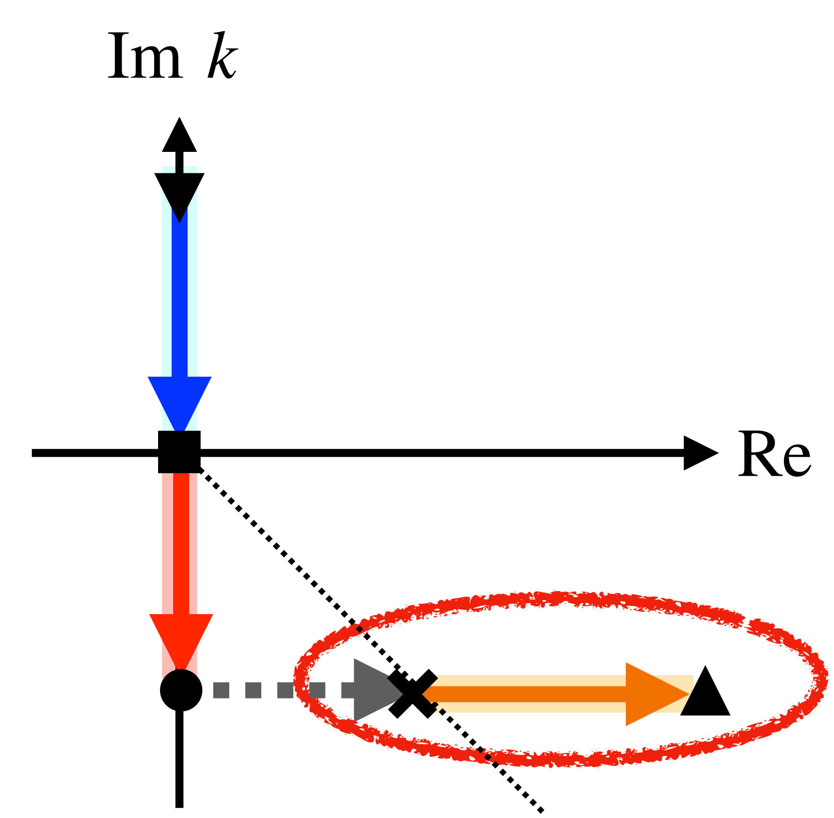
- far from threshold in $1/a_s \to -\infty$ limit



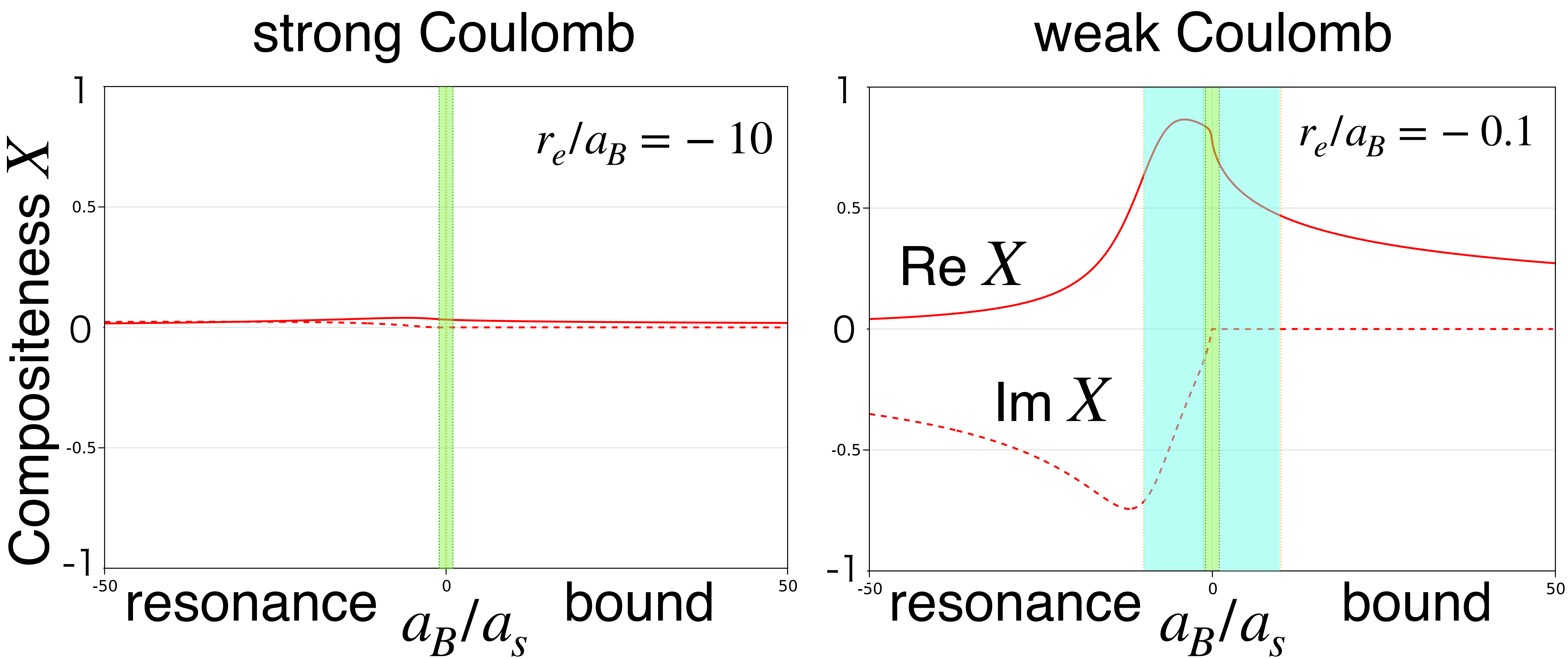
dotted lines :$1/r_e$

- Im $k \to 1/r_e$

→ trajectory close to that of resonance in ERE

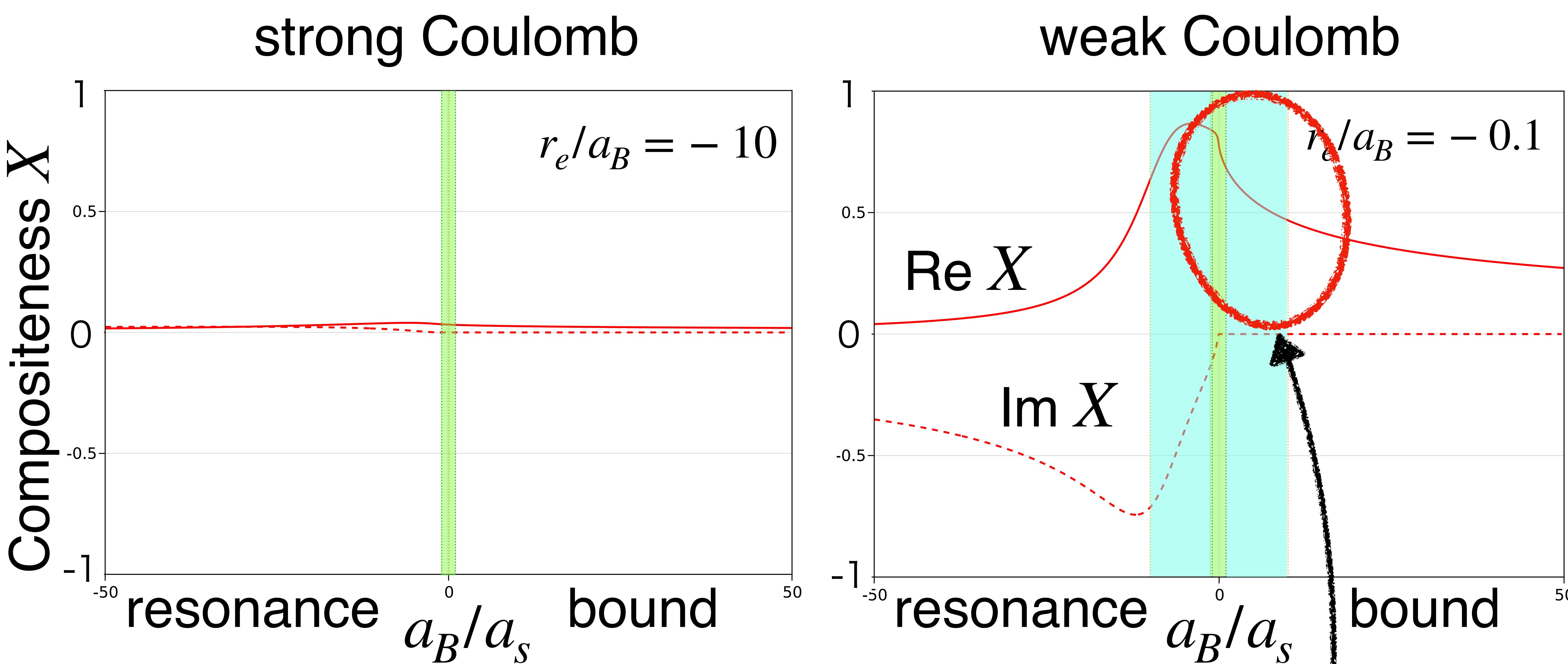


Compositeness (repulsive Coulomb)



- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region

Compositeness (repulsive Coulomb)



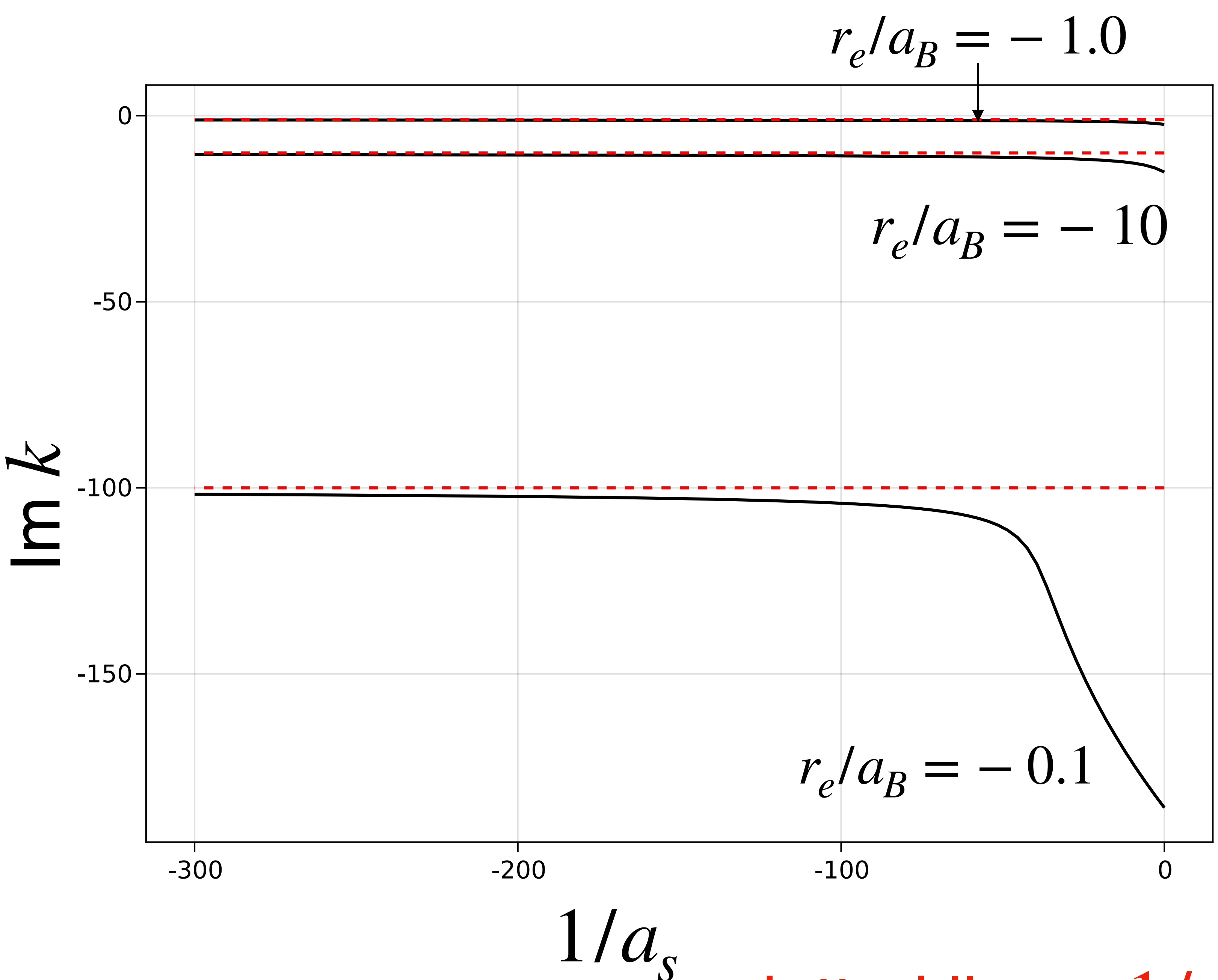
- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- remnant of short range universality in $|r_e| \ll |a_B|$ case

  $X \to 1$ in $B \to 0$ limit in short range

far from threshold (attractive Coulomb)

◉ imaginary part of eigenenergy in complex momentum k plane

- far from threshold in $1/a_s \to -\infty$ limit



dotted lines :$1/r_e$

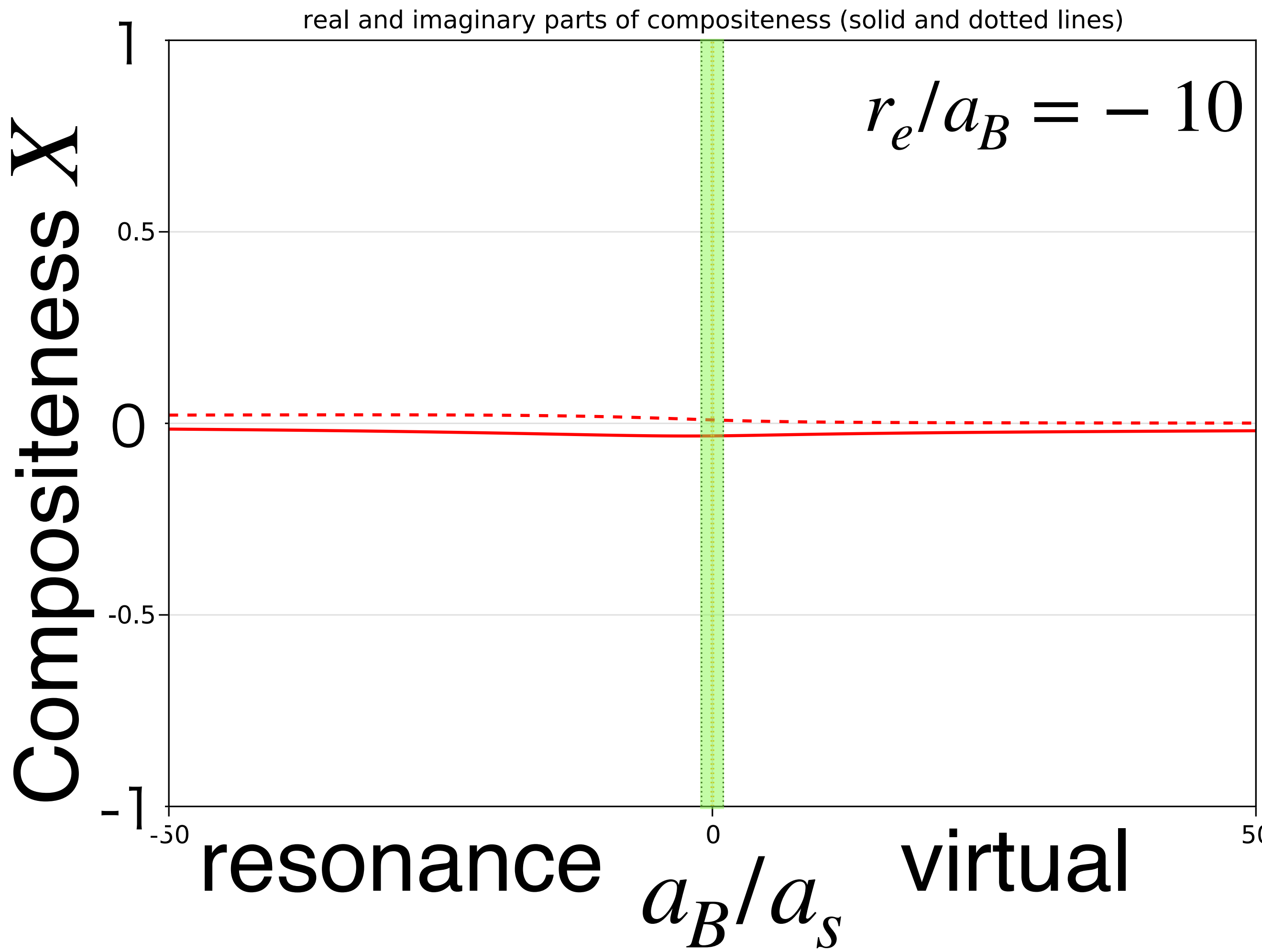
- Im k of resonance pole
- Im $k \to 1/r_e$

→ trajectory close to that of resonance in ERE

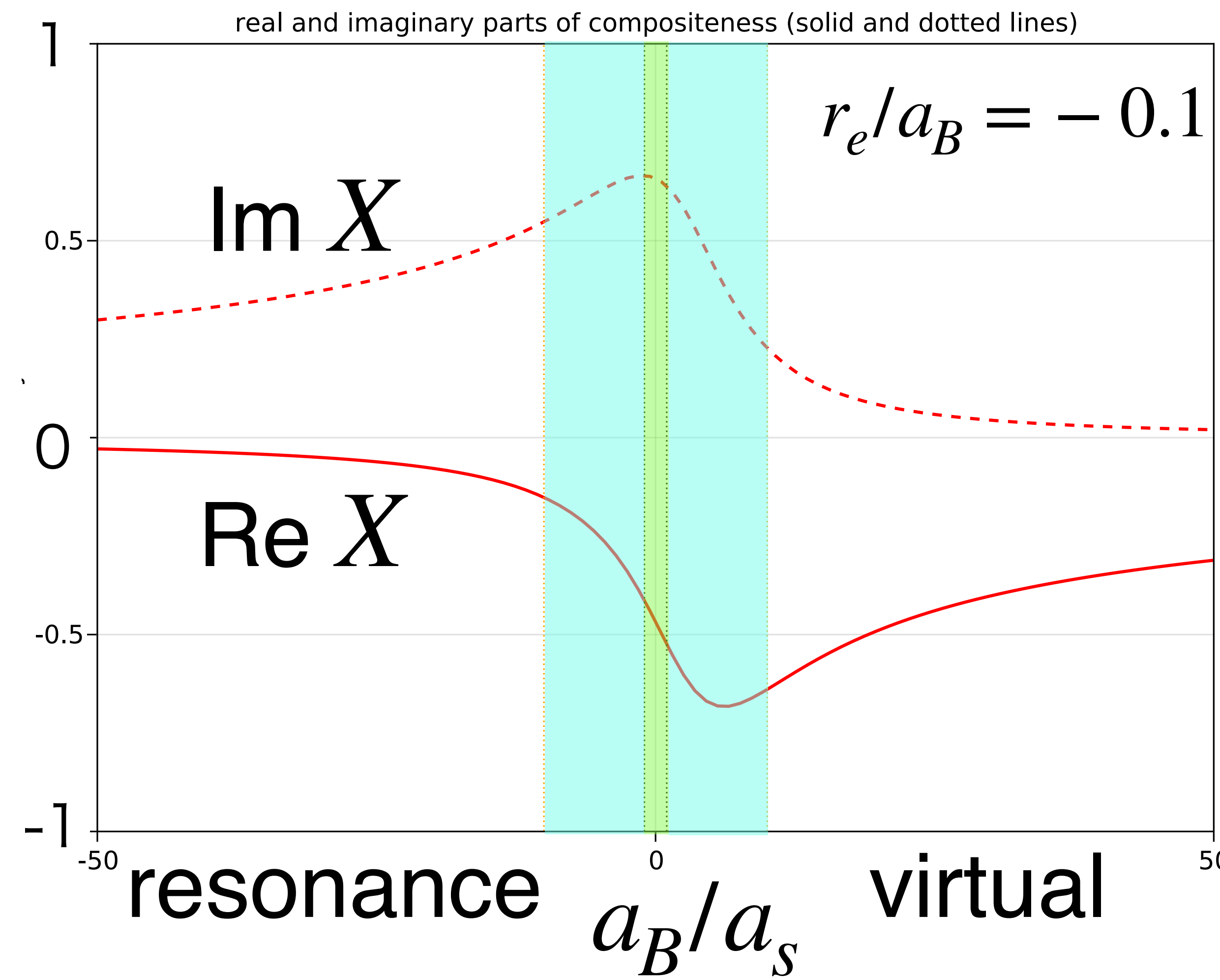
- same as repulsive case

Compositeness (att. Coulomb resonance)

strong Coulomb

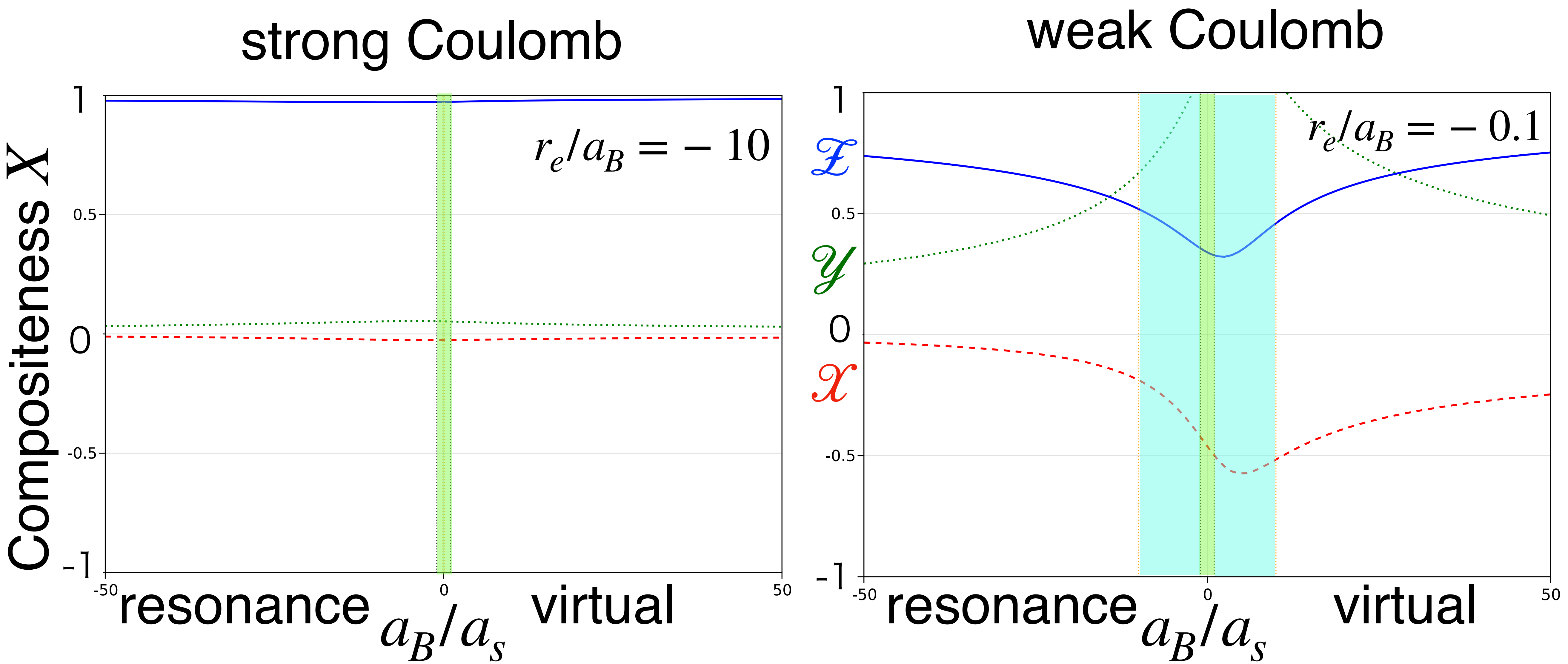


weak Coulomb



- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- compositeness of unstable resonances are complex $X \in \mathbb{C}$

Compositeness (att. resonance)

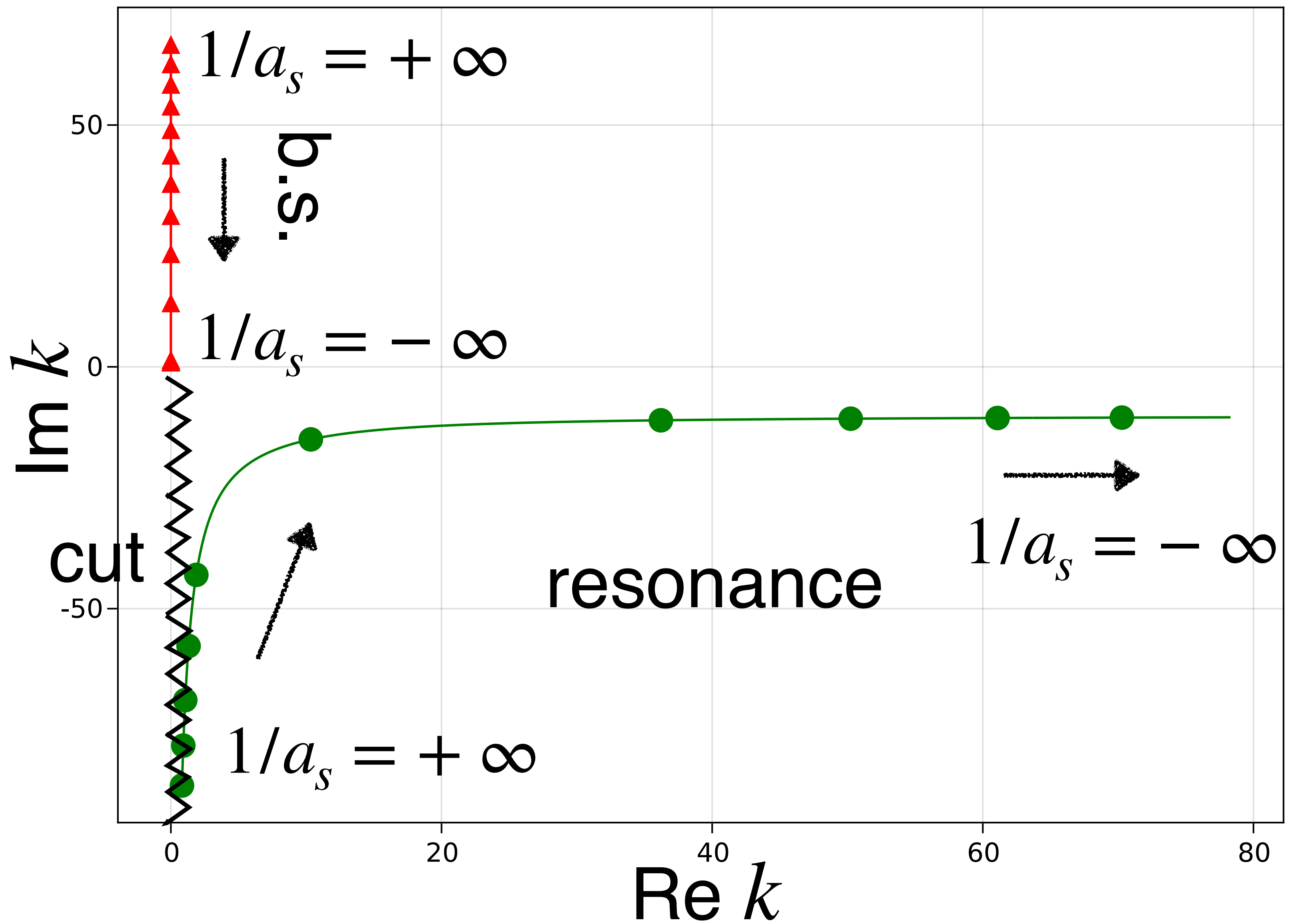


- $\mathscr{X} < 0$ ➞ non-interpretable in this region
- but $\mathscr{X} \geq 0$ in far-threshold region with large $|1/a_s|$

  ➞ states are $\mathscr{Z}$ dominant with large bare state contribution

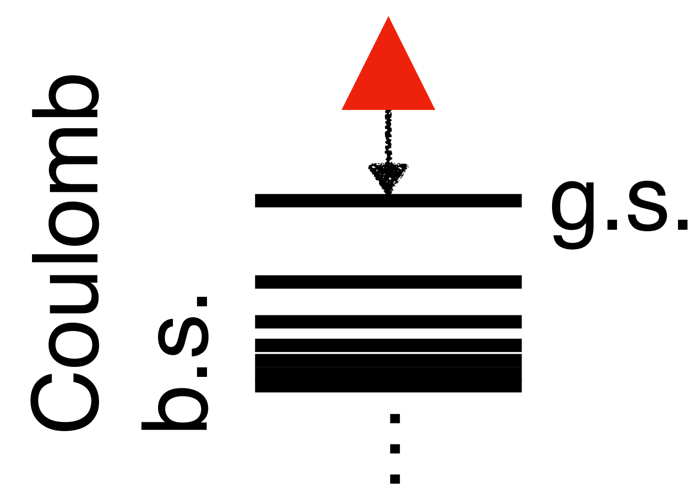
Pole trajectory (attractive Coulomb)

◉ pole trajectory in complex momentum k plane



- bound ≠ resonance

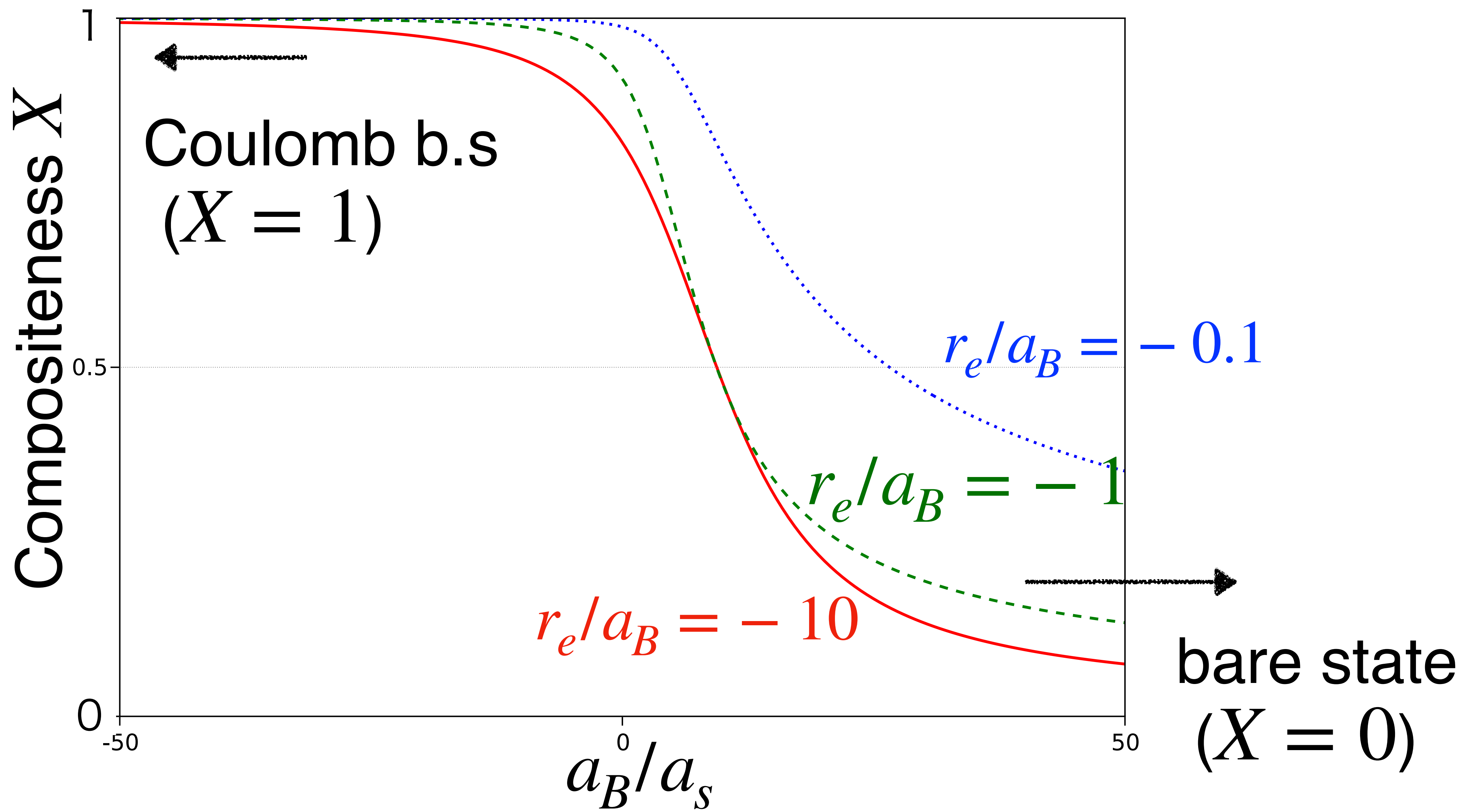
- bound pole
  → Coulomb g.s.



- pole cannot go to $k = 0$

W. Domcke, Atom. Mol. Phys. 16 359 (1983);
S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

Compositeness (att. Coulomb b.s.)

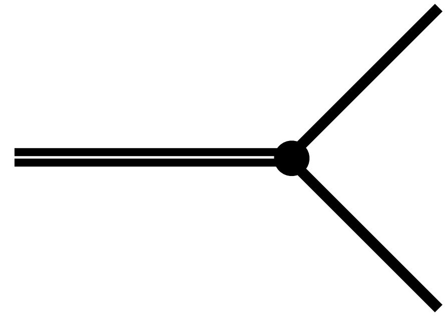


- $1/a_s \to +\infty$: states becomes elementary dominant ($X \to 0$)
- no short range universality but $X \to 1$ in $B \to B_{\text{Coulomb g.s.}}$ limit
  ∵ Coulomb g.s. has no bare state contribution (i.e. $X = 1$)
- Coulomb < short range ($r_e = -0.1$) : remnant of universality

Pole trajectory (only w/ s.r.)

◉ pole trajectory in complex momentum k plane (No Coulomb)

Q channel
⇔ bare state

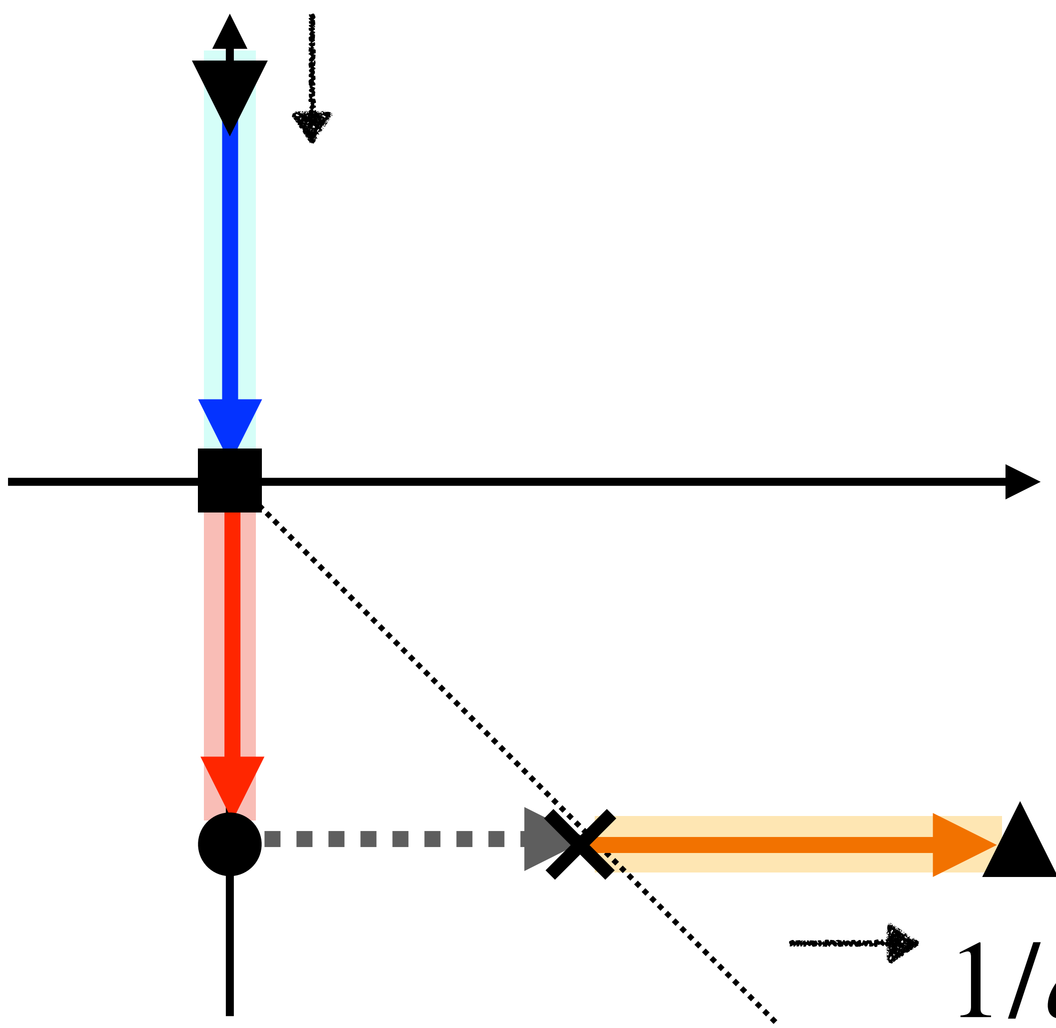


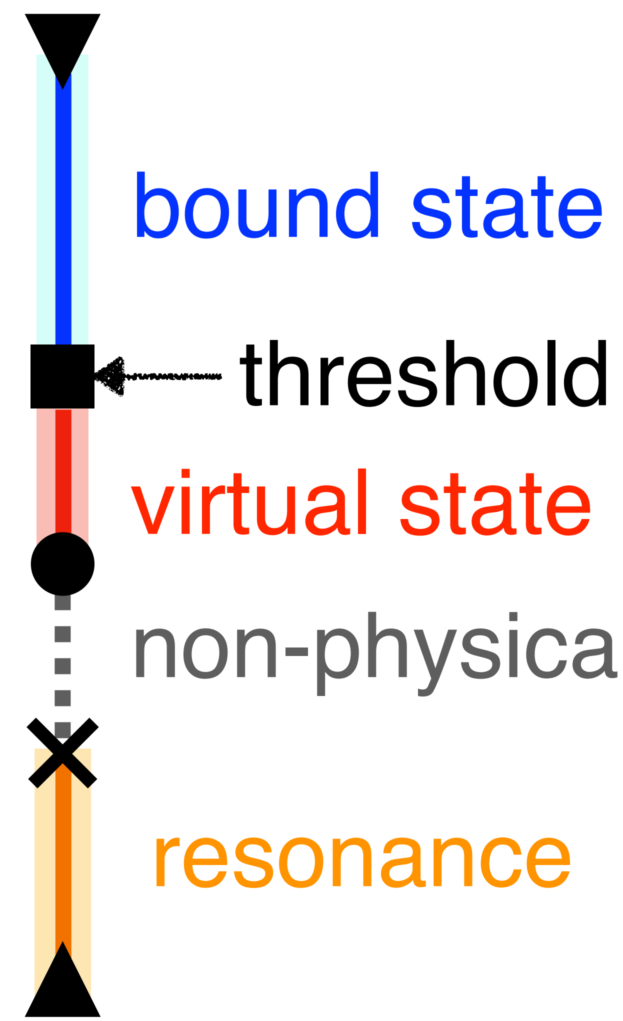
P channel

⇔ free scattering

$G_P \to G^0$ free Green's function

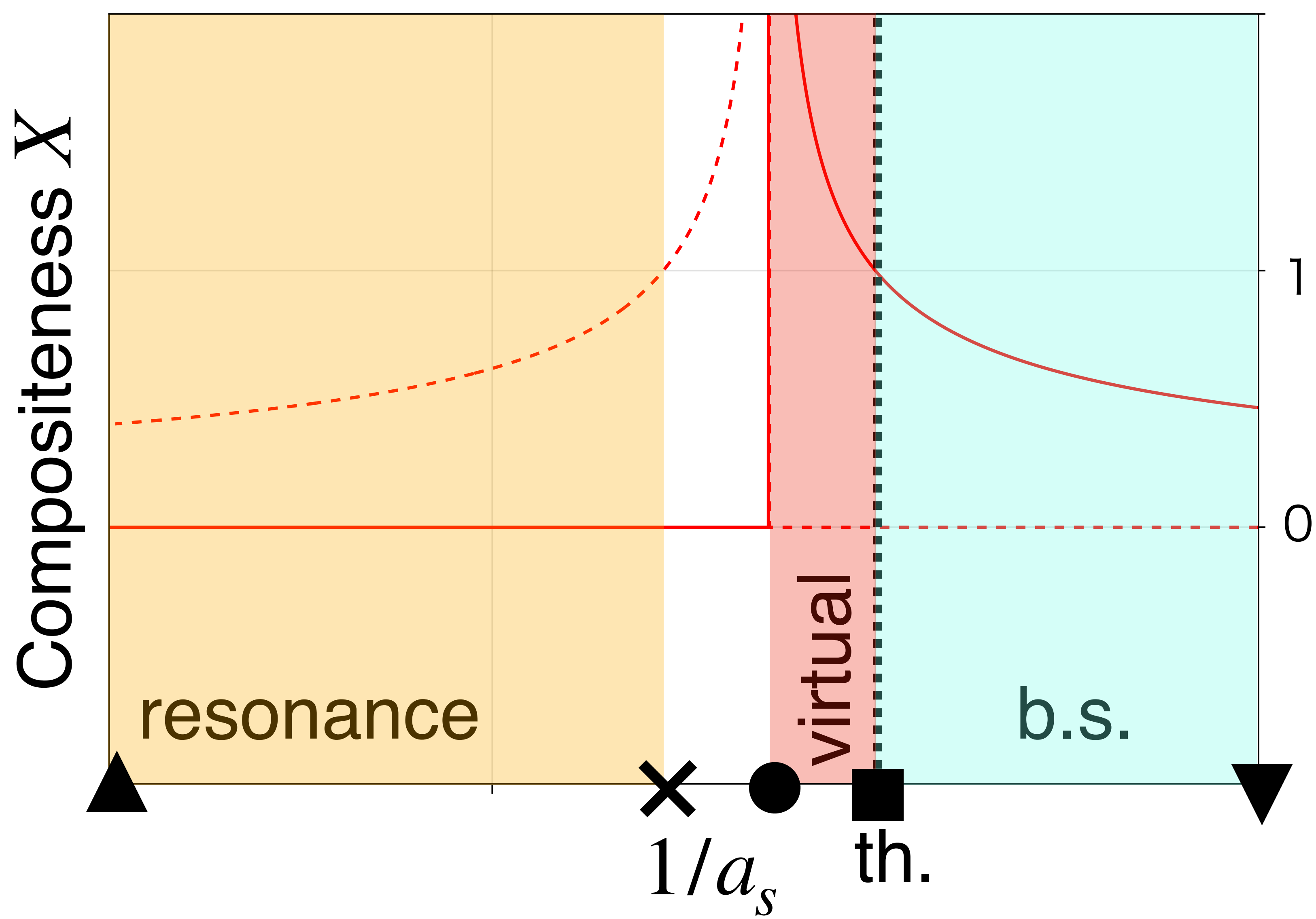
$\mathrm{Im}\ k$ $1/a_{\mathrm{s.r.}} = +\infty$

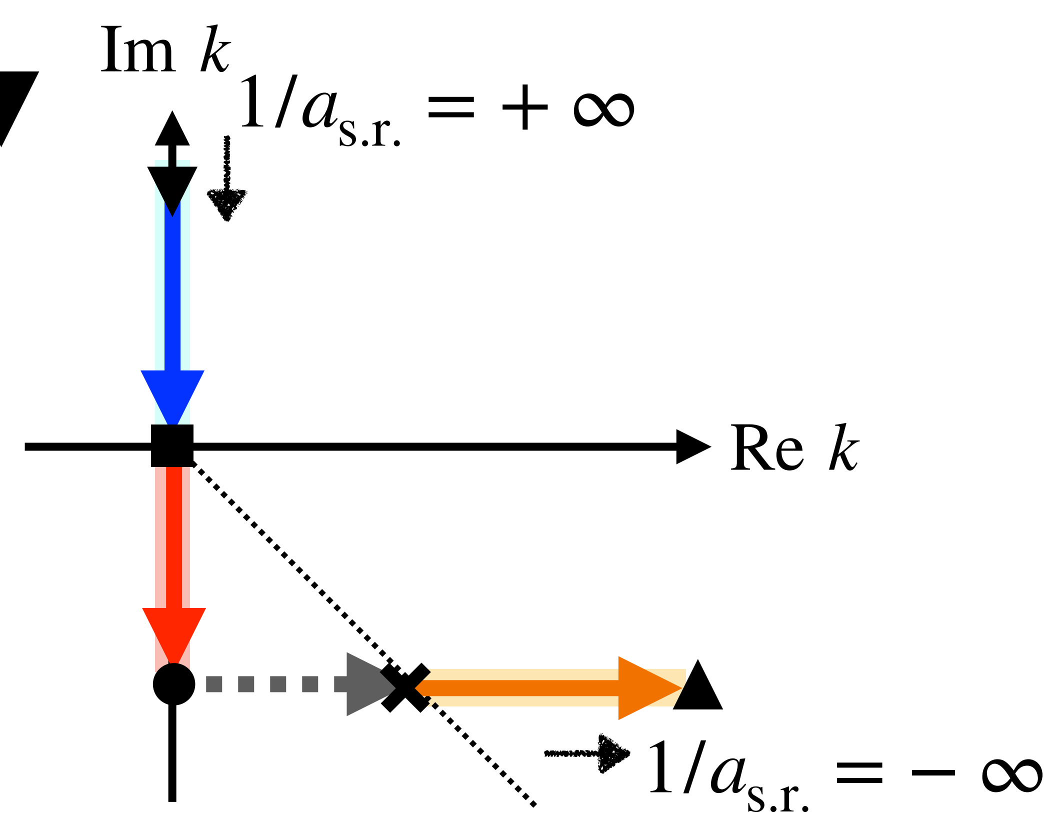




T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) .

Compositeness (only w/ s.r.)



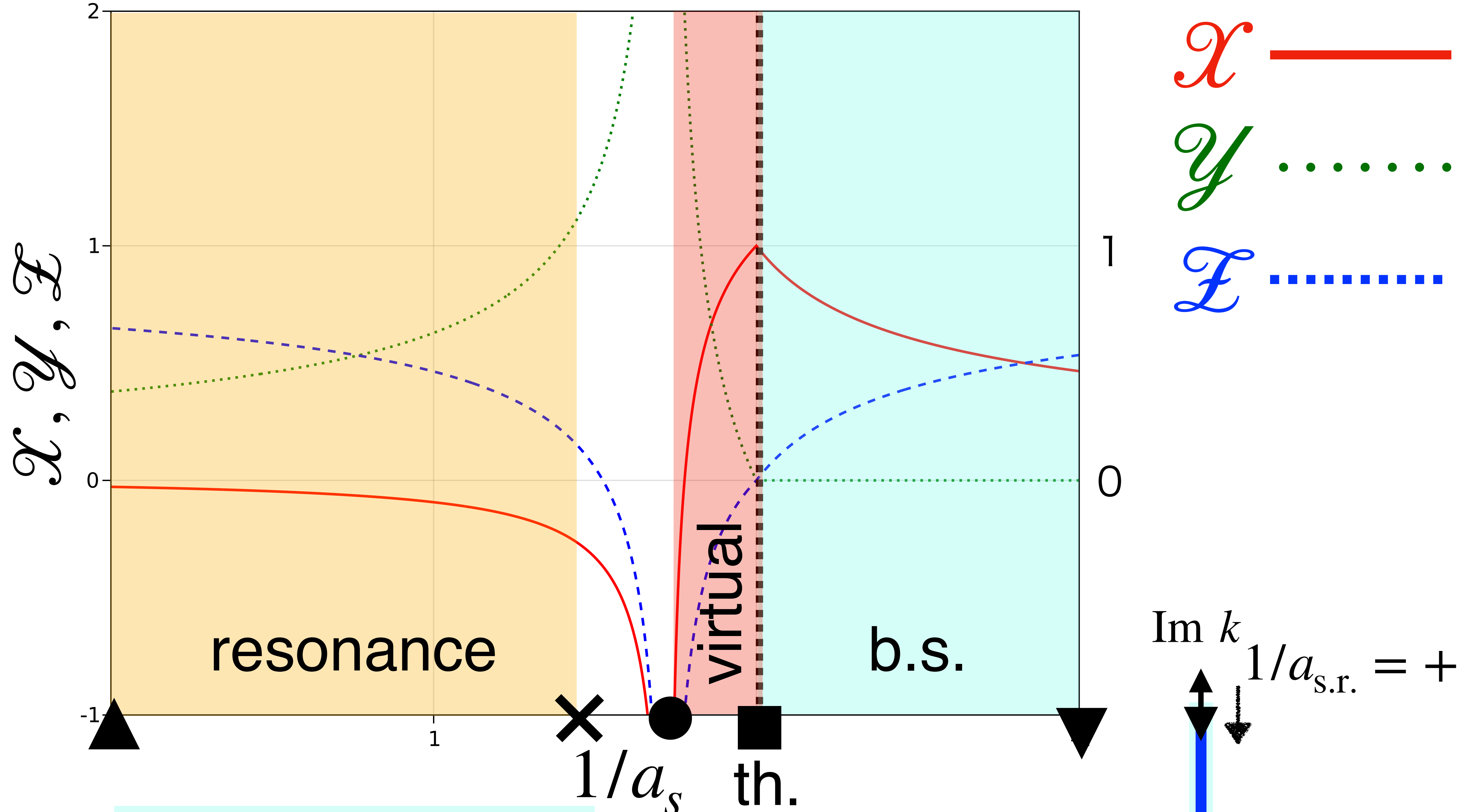


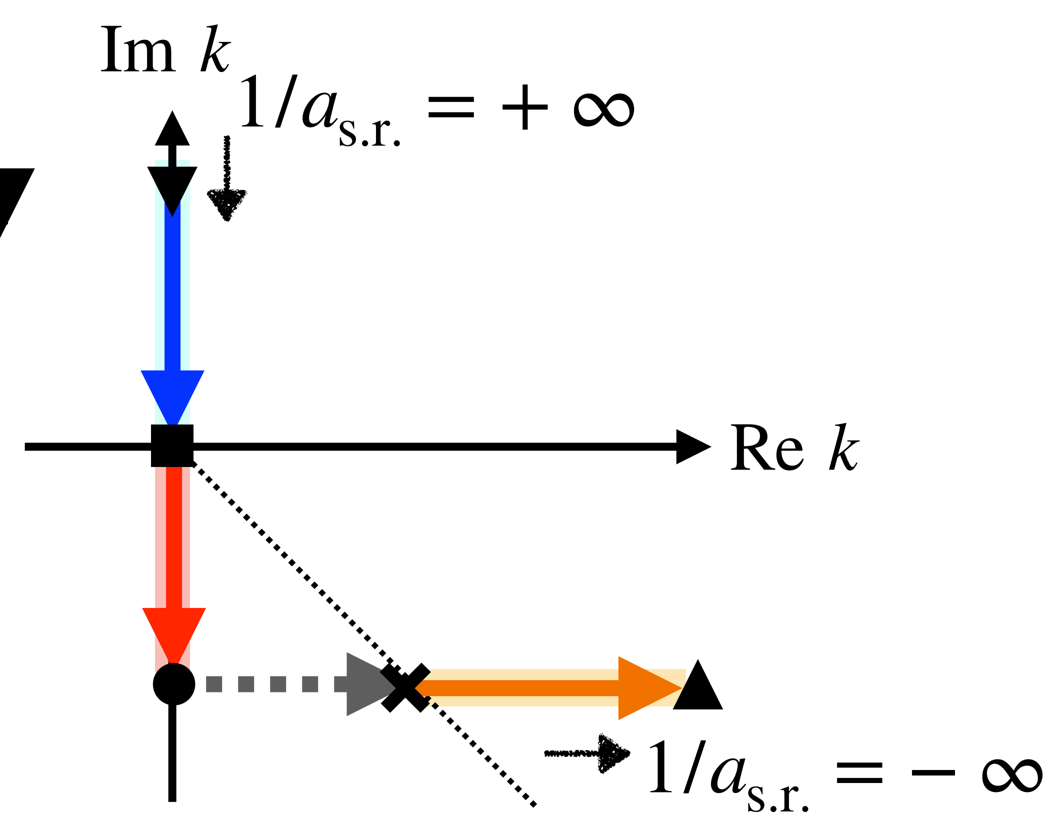
- b.s. : $0 \leq X \leq 1$
- virtual : $1 < X$

● divergence $X \to \infty$

- resonance : $\mathrm{Im}\ X \leq 1$

$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ (only w/ s.r.)





- b.s. : $0 \leq X \leq 1$
- virtual : $\mathscr{Z} < 0$ (non-interpretable)

● divergence $\mathscr{X}, \mathscr{Y}, \mathscr{Z} \to \infty$

- resonance : $\mathscr{Z}$ dominant

Application

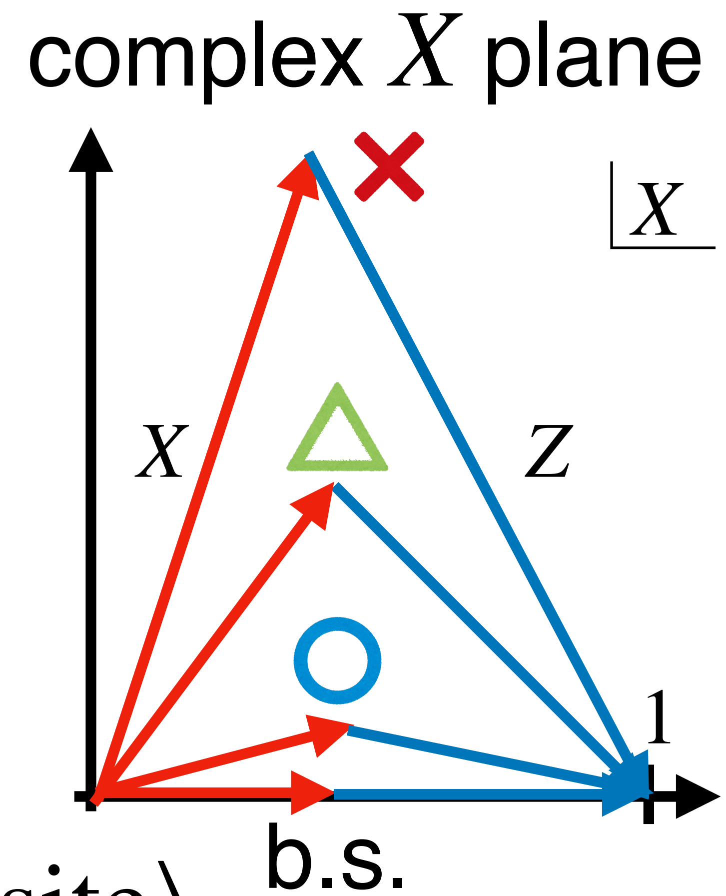
System	種類	変化	decay	m_1	m_2	μ	ボーア半径
^{8}Be ($\alpha^{++}\alpha^{++}$)	斥力	B $\to$ R	なし	4 u	4 u	1863 MeV	3.63 fm
$\Omega^-\Omega^-$	斥力	B $\to$ B	なし	1712 MeV (lattice [2])	1712 MeV (lattice [2])	856 MeV	31.6 fm
$\Omega_{ccc}^{++}\Omega_{ccc}^{++}$	斥力	B $\to$ R	なし	4837 MeV (Set 1)	4837 MeV (Set 1)	2418.5 MeV	2.80 fm
$\Omega^- p^+$	引力	B $\to$ B	d-wave で $Y\Xi$	955 MeV (lattice)	1712 MeV (lattice)	613.0 MeV	44.1 fm
$\Xi^- p^+$	引力	V $\to$ V	あり	1322 MeV	938.2 MeV	548.8 MeV	49.3 fm
$\Xi^- \alpha^{++}$	引力	V $\to$ B	あり	1322 MeV	4 u	975.8 MeV	13.9 fm
$\Xi^{-\,12}$C	引力			1322 MeV	12 u	1182 MeV	3.81 fm
Ξ^- Br	引力			1322 MeV	79.90 u	1299 MeV	0.600 fm
Ξ^- Ag	引力			1322 MeV	107.9 u	1304 MeV	0.441 fm

Complex compositeness

- probabilistic interpretation?

  $X \in \mathbb{C}$ and $X + Z = 1$

- If Im X is large, it seems that reasonable interpretation is impossible ✗ △

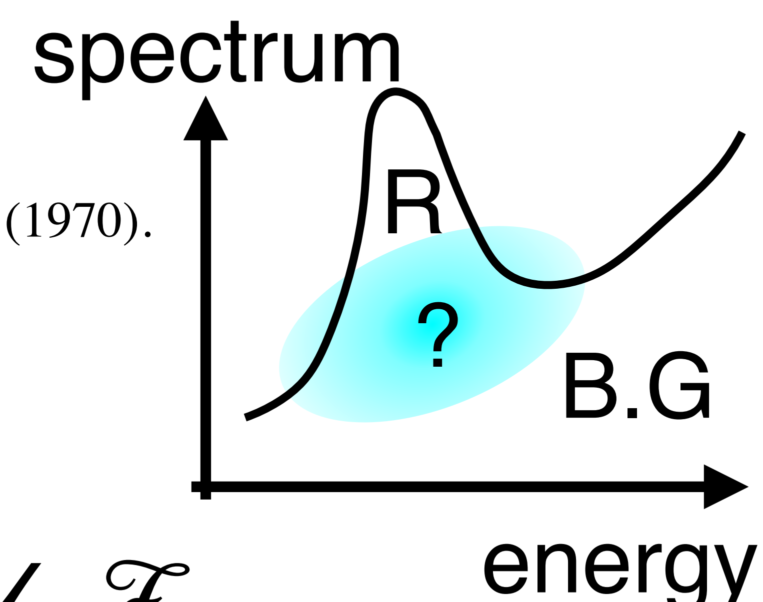


- our proposal

i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$
ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$
iii) $\mathscr{Y}$: probability of uncertain identification

uncertain appears from T. Berggren, Phys. Lett. B 33, 547 (1970).

- finite lifetime (uncertainty in energy)
- separation from B.G.



complex compositeness $X \in \mathbb{C} \longrightarrow \mathscr{X}, \mathscr{Y}, \mathscr{Z}$

Definition

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

◉ conditions for sensible interpretation

- normalization : $\mathscr{X} + \mathscr{Y} + \mathscr{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathscr{X} \to X$, $\mathscr{Z} \to Z$ and $\mathscr{Y} \to 0$

$\mathscr{Y}$ characterizes uncertainty of resonance

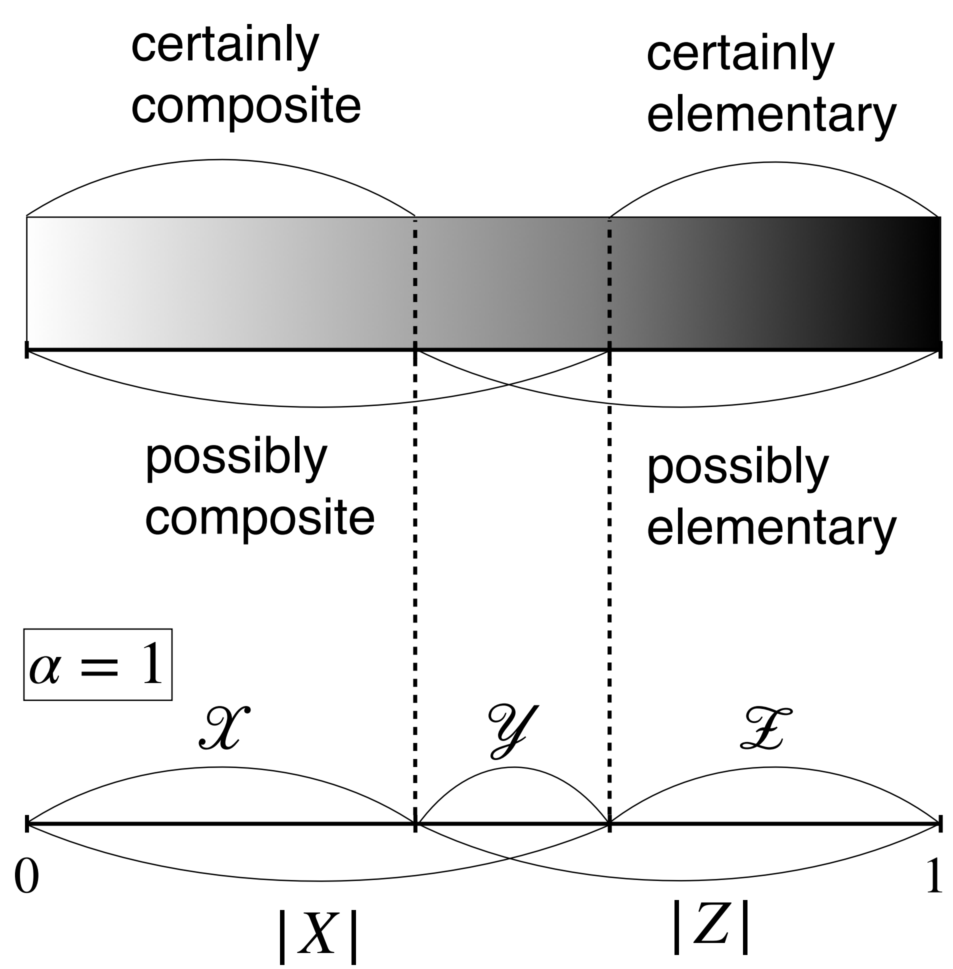
◉ new interpretation

$$\mathscr{X} + \alpha\mathscr{Y} = |X|, \quad \mathscr{Z} + \alpha\mathscr{Y} = |Z|$$

$$\mathscr{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathscr{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$\mathscr{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

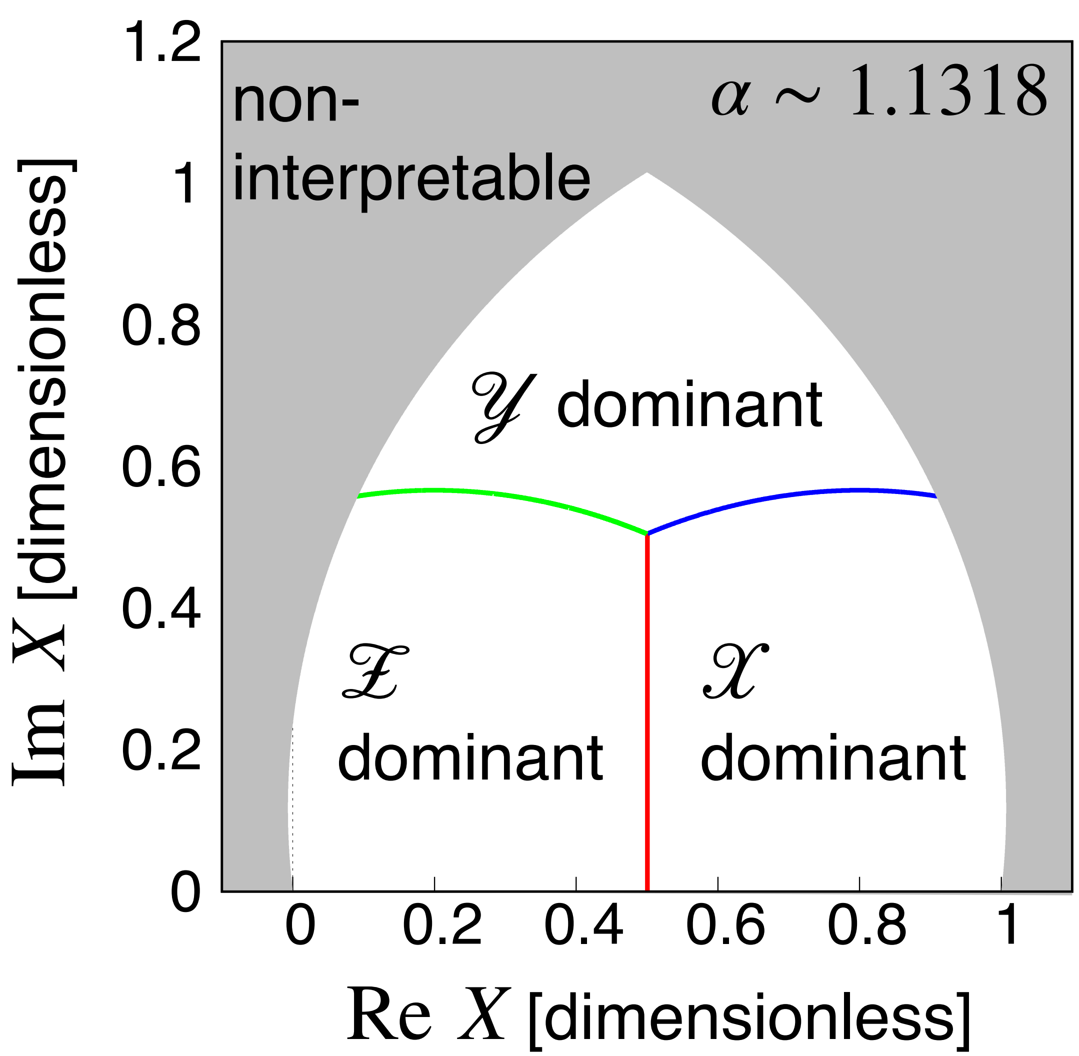


α reflects uncertain nature of resonances

Definition

- if $\alpha > 1/2$, $\mathscr{Y}$ is always positive but $\mathscr{X}$, $\mathscr{Z}$ can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		$\boxed{\text{non-interpretable}}$



our criterion for physical "state"

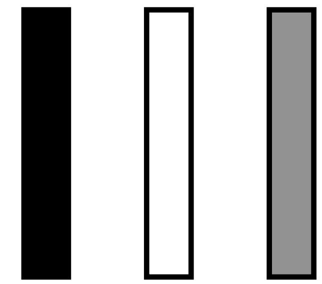
$$\Gamma \leq \mathrm{Re}\ E$$

- exclude poles which we cannot regard as physical "state" from probabilistic interpretation

$\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ dominant regions
and
non-interpretable region

uncertainty in resonances

a single
measurement



sum of measurements of a bound states / resonances