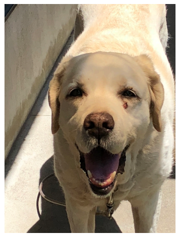
Range correction in the weak-binding relation for unstable states



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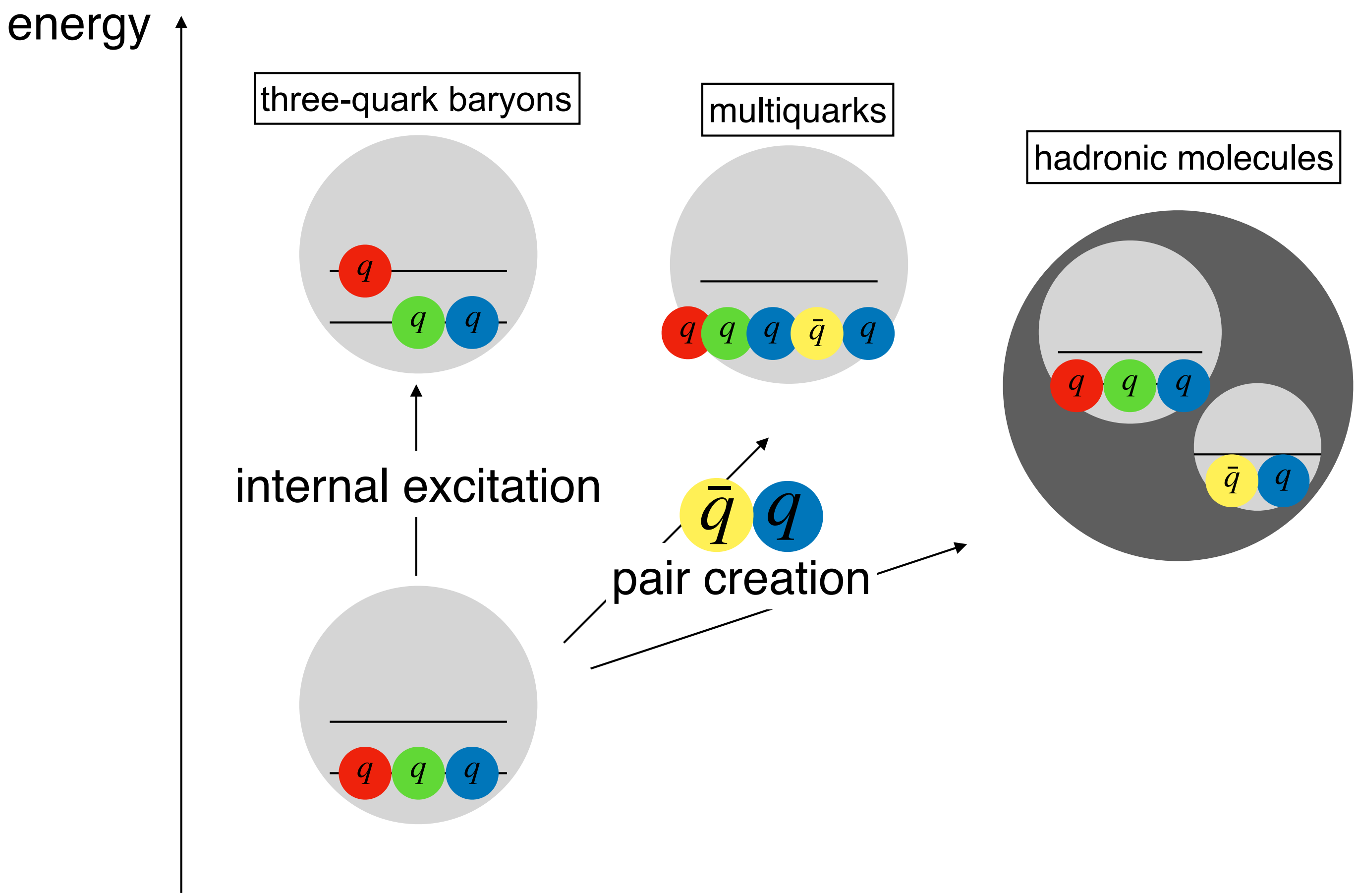
Background

candidates for exotic hadrons

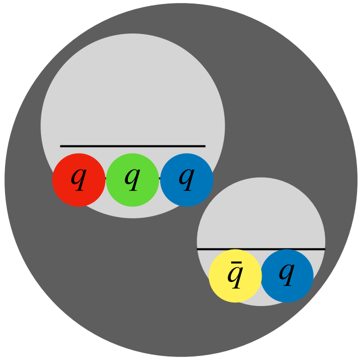
$\Lambda(1405)$, XYZ meson etc… → multiquarks, hadronic molecules

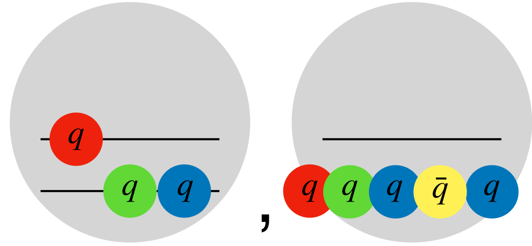


structure of hadrons ← model independent observable

Previous work

Hadron wave function



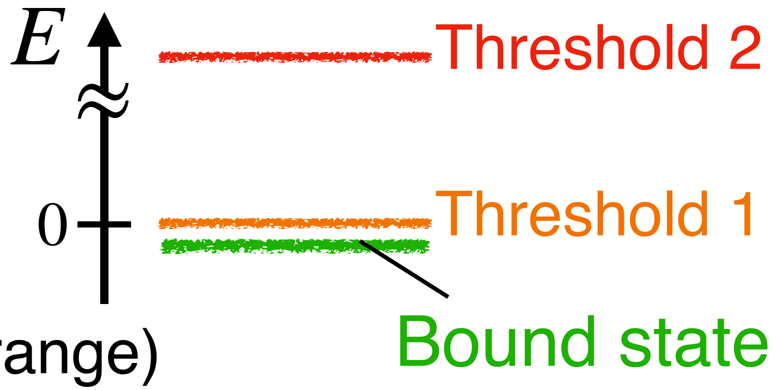


$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

Compositeness (weight of hadronic molecule)

Weak-binding relation for bound state

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$



a_0 (scattering length) $R_{\rm typ}$(interaction range)

$R \equiv (2\mu B)^{-1/2}$, B (binding energy)

When $R \gg R_{\rm typ}$: observable(a_0, B) → compositeness(X)

S. Weinberg, Phys. Rev. 137, B672 (1965); Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Motivation

Low-energy universality → $a_0 = R \ (R \to \infty)$

→ We study the range correction in the weak-binding relation by introducing the effective range r_e.

Range correction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

Apply to the following model :

Single channel: $|\text{hadronic molecule}\rangle$ only $\Rightarrow X = 1$

Zero range limit: $R_{\rm typ} \to 0 \Rightarrow \mathcal{O}(R_{\rm typ}/R) \to 0$

$\Leftrightarrow a_0 = R$?

Effective range model in the zero range limit (single channel)

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008), 0709.0499.

$$\mathcal{H}_{\rm int} = \frac{1}{4}\lambda_0(\psi^\dagger\psi)^2 + \frac{1}{4}\rho_0\nabla(\psi^\dagger\psi)\cdot\nabla(\psi^\dagger\psi) \quad \to \quad f(k) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik\right]^{-1}$$

$$a_0 = R\frac{2r_e/R}{1-(r_e/R-1)^2} = R\left[1 + \mathcal{O}\left(\left|\frac{r_e}{R}\right|\right)\right] \Rightarrow a_0 \neq R$$

→ Weak-binding relation should be improved.

Improved weak-binding relation

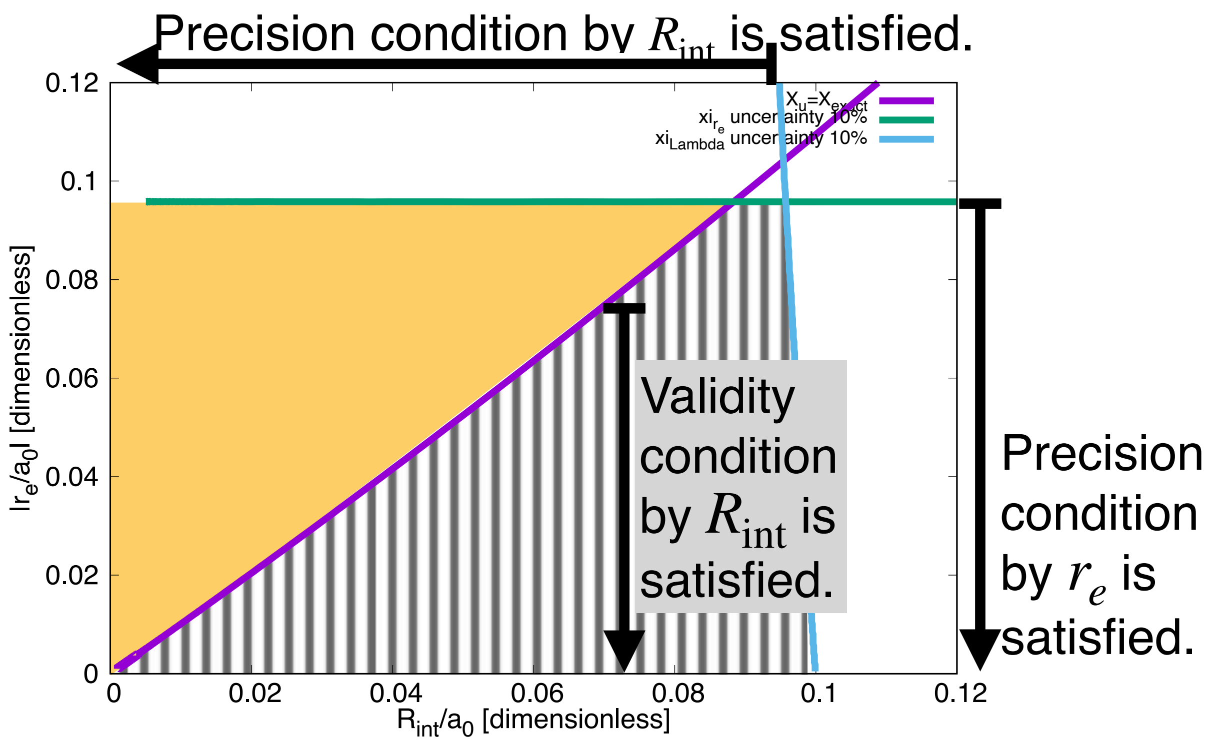
Redefinition of $R_{\rm typ}$ $\quad R_{\rm int}$: interaction range

$$R_{\rm typ} = \max\left\{ R_{\rm int}, R_{\rm eff} \right\},\ R_{\rm eff} = \max\left\{ |r_e|, \frac{|P_s|}{R^2}, \cdots \right\}.$$

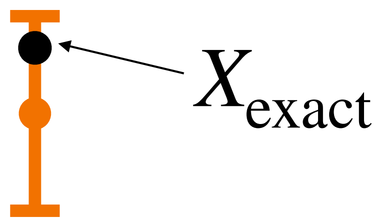
Numerical calculation

Effective range model ($R_{\rm int} \neq 0$)

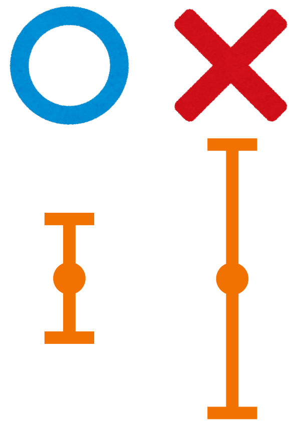
Validity and precision conditions in $R_{\rm int}/a_0$-$|r_e/a_0|$ plane



Validity condition



Precision condition



Only the improved weak-binding relation can be applied.

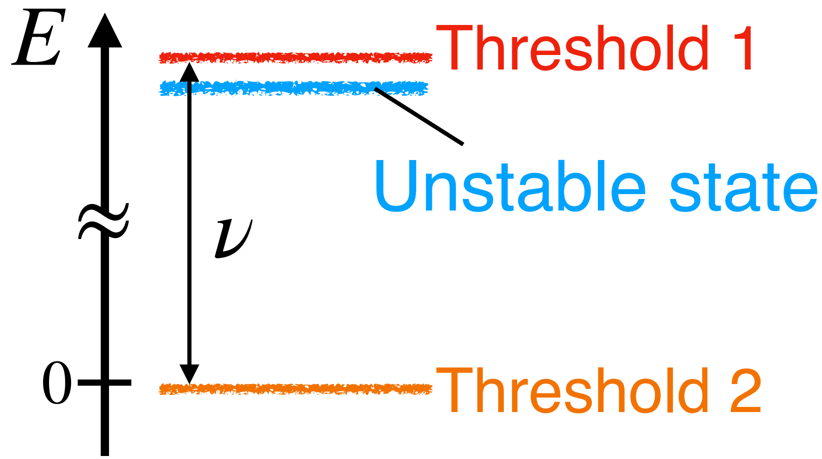
Extension to unstable states

Candidates for exotic hadrons are unstable states!

Unstable systems → $E, a_0, r_e \in \mathbb{C}$

Weak-binding relation for unstable states

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}$$



$l \equiv (2\mu\nu)^{-1/2}$

ν (deference between the threshold energies)

$R \equiv (2\mu E)^{-1/2}$, E (eigenenergy)

When $|R| \gg R_{\rm typ}$ and $|R| \gg l$: observables (a_0, E) → compositeness(X)

Interpretation of X → $\tilde{X} \equiv \dfrac{1 - |1-X| + |X|}{2} \in \mathbb{R} \quad \because X \in \mathbb{C}$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Extension to unstable states

What are the features of weak-binding relation in the effective range model with zero range limit ($R_{\rm typ} \to 0$)?

$$f(k) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik \right]^{-1}$$

One length scale r_e except for a_0

→ $R_{\rm typ} = |r_e|$

- unstable states in the **effective single channel description**

$a_0, r_e \in \mathbb{C}$ → pole of $f(k) \in \mathbb{C}$ → eigenenergy $E \in \mathbb{C}$

unstable states

$$a_0 = |a_0| e^{i\theta_{a_0}}, r_e = -|r_e| e^{i\theta_{r_e}}$$

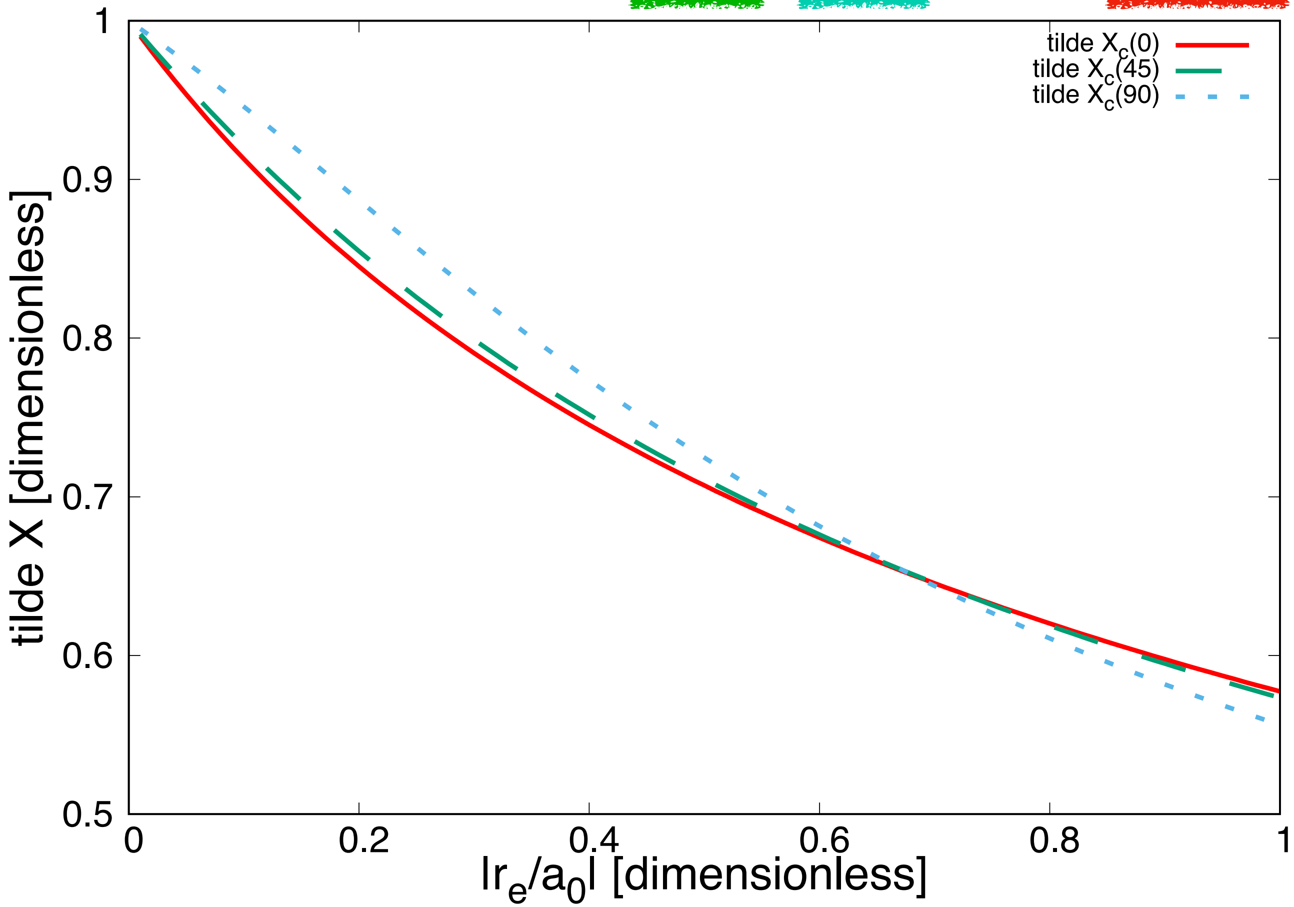
- $X(a_0, r_e) = X(a_0/r_e)$ in the zero range limit

→ X depends only on $\theta \equiv \theta_{a_0} - \theta_{r_e}$.

$$a_0 = |a_0| e^{i\theta},\ r_e = -|r_e|$$

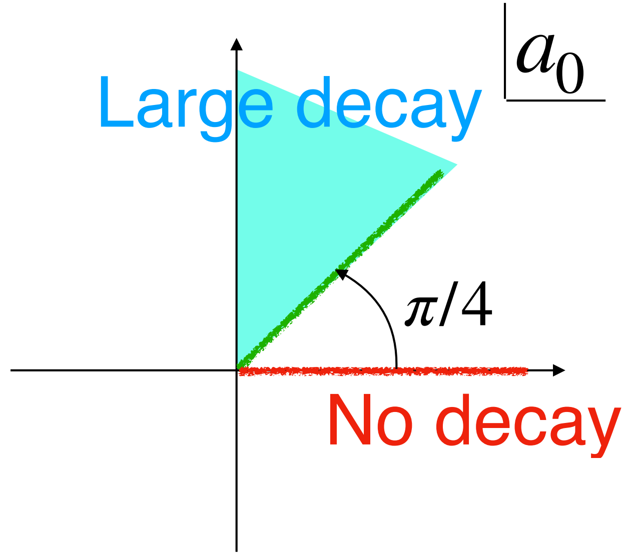
Extension to unstable states

$\tilde{X}$ for unstable ($\theta = \pi/4,\ \pi/2$) and stable ($\theta = 0$) states



$\theta = \pi/4,\ \pi/2$

→ Far from the stable state



There are almost no differences between $\tilde{X}$ for unstable and stable states.

→ Is this a feature of effective range model in the zero range limit?

Extension to unstable states

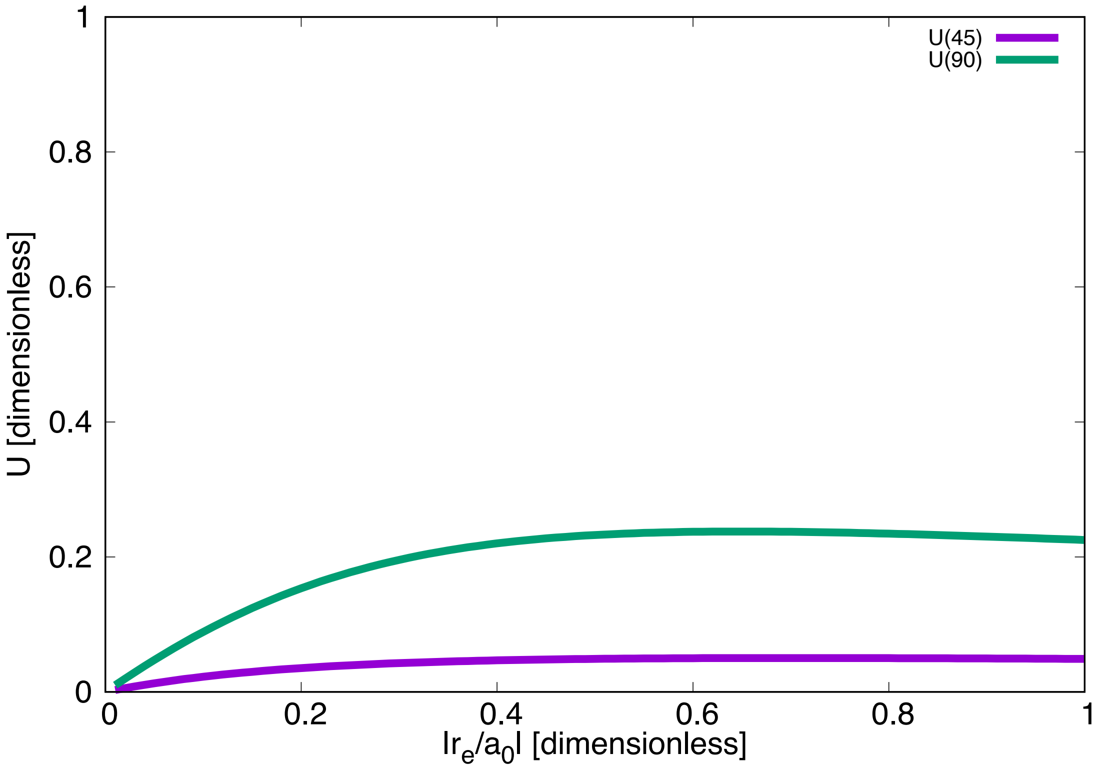
Estimation of uncertainty of interpretation (U):

$$U \equiv |1 - X| + |X| - 1.$$ Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

-When U is small, we can interpret $\tilde{X}$ as compositeness and the system is similar to the stable state.

-When $0 \leq X \leq 1$ ($X \in \mathbb{R}$), $U = 0$.

U ($\theta = \pi/2$ and $\pi/4$)



$U \lesssim 0.25$ in $\theta = \pi/2$ case

Far from the stable state

$\because a_0 = |a_0| e^{i\theta}$ is pure imaginary

The wave function of this system has similar features to the stable state.

$\because X$ depends only on a_0/r_e.

Conclusion and future prospect

- Weak-binding relation : observable → compositeness (X)

$$a_0 = R\left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R} \right) \right\}$$

- We study the range correction in weak-binding relation from r_e.

- Improved weak-binding relation by redefinition of R_{typ} :

$$R_{\mathrm{typ}} = \max\left\{ R_{\mathrm{int}}, |r_e|, \cdots \right\}$$

- We find the region where only the improved weak-binding relation can be applied.

- In effective range model in the zero range limit, the unstable state is similar to the stable state.

- Future prospect: Apply the improved relation to hadron systems.