

Compositeness of T_{cc} and $X(3872)$ with decay and coupled-channel effects



arXiv:2303.07038 [hep-ph]



Tomona Kinugawa

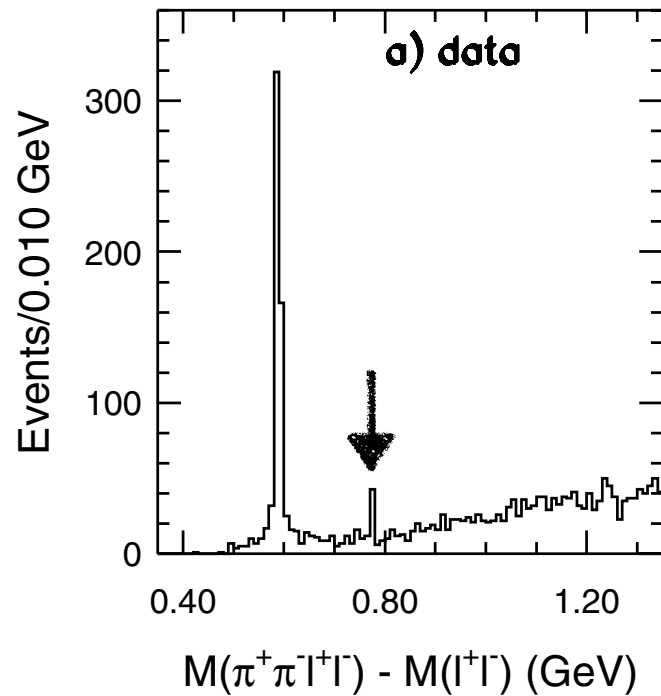
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September 14th, J-PARC Hadron 2023

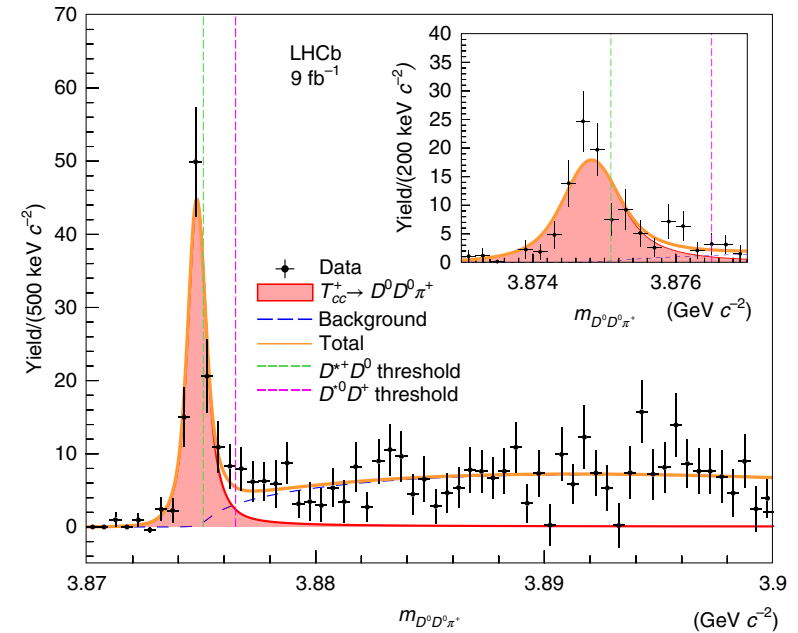
Near-threshold exotic hadrons

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$$X(3872) \rightarrow \pi^+ \pi^- J/\psi$$



$$T_{cc} \rightarrow D^0 D^0 \pi^+ (cc\bar{u}\bar{d})$$



LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

internal structure?

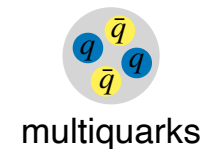
exotic hadron

$\neq qqq$ or $q\bar{q}$

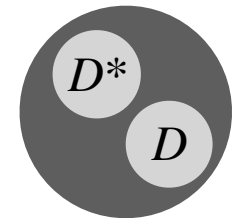


multiquarks

hadronic molecules



multiquarks



hadronic molecules

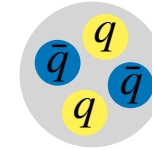
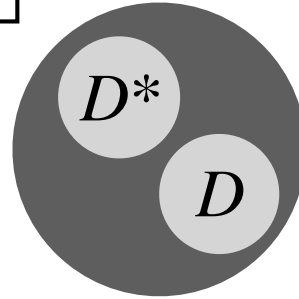
Compositeness

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **85**, 015201 (2012);
F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

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● definition

hadron wavefunction



$$|\Psi\rangle = \sqrt{X} |\text{hadronic molecule}\rangle + \sqrt{1-X} |\text{others}\rangle$$

compositeness

elementarity

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).

$$\begin{aligned} * 0 \leq X \leq 1 & \longrightarrow X > 0.5 \Leftrightarrow \text{composite dominant} \\ & X < 0.5 \Leftrightarrow \text{elementary dominant} \end{aligned}$$

- **quantitative** analysis of internal structure

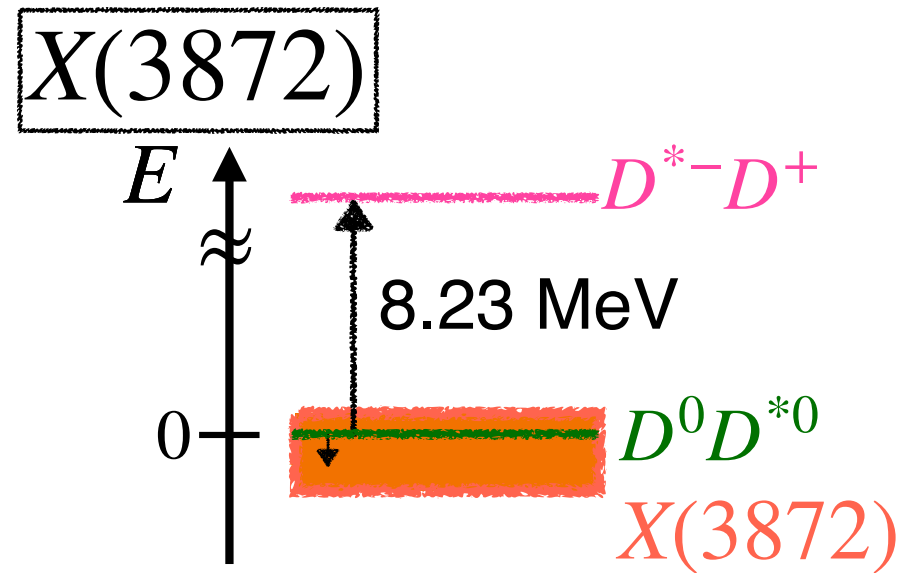
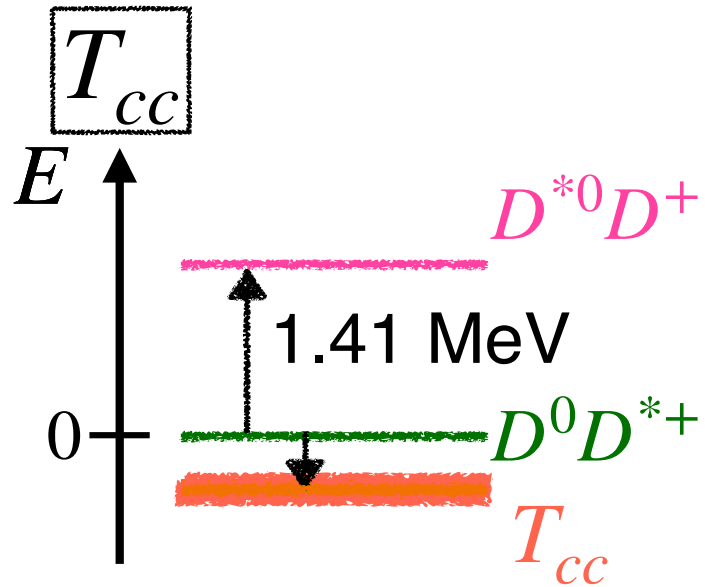
deuteron is not an elementary particle Weinberg, S. Phys. Rev. 137, 672–678 (1965).

$f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016);
T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.

$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013);
Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

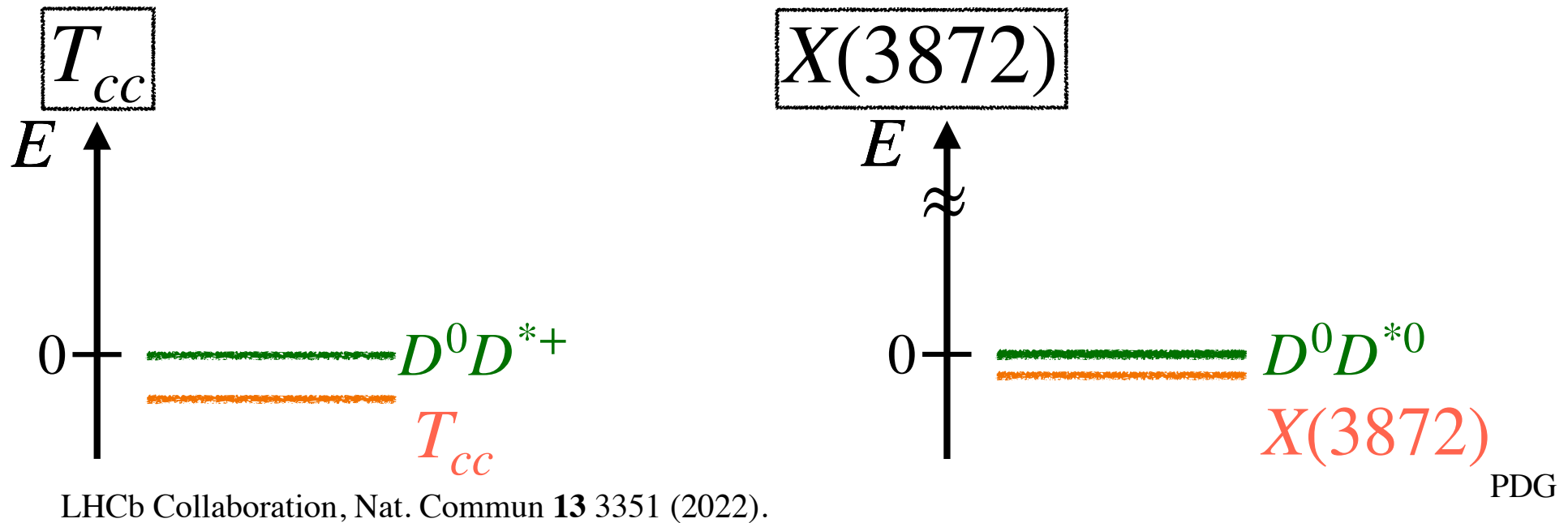
Near-threshold states



LHCb Collaboration, Nat. Commun **13** 3351 (2022).

PDG

Near-threshold states



- compositeness $X = 1$ in $B \rightarrow 0$ limit (universality)

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

Near threshold states ($B \neq 0$) is composite dominant ?

- However, elementary dominant states is realized with fine tuning

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;

C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014).

→ How finely tuning parameter?

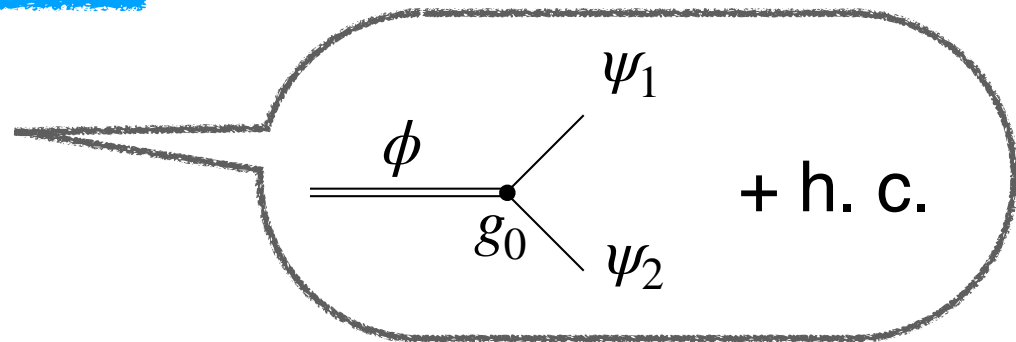
● single-channel resonance model

$$\mathcal{H}_{\text{free}} = \frac{1}{2m_1} \nabla \psi_1^\dagger \cdot \nabla \psi_1 + \frac{1}{2m_2} \nabla \psi_2^\dagger \cdot \nabla \psi_2 + \frac{1}{2m_\phi} \nabla \phi^\dagger \cdot \nabla \phi + \nu_0 \phi^\dagger \phi,$$

1.

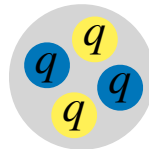
$$\mathcal{H}_{\text{int}} = g_0(\phi^\dagger \psi_1 \psi_2 + \psi_1^\dagger \psi_2^\dagger \phi).$$

2.



1. single-channel scattering

2. coupling to bare state ϕ



● scattering amplitude

$$V = \frac{g_0^2}{E - \nu_0}, \quad G = -\frac{\mu}{\pi^2} \left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right) \right]. \quad \Lambda : \text{cutoff}$$

$$T = \frac{1}{V^{-1} - G} \longrightarrow f(k) = -\frac{\mu}{2\pi} \left[\frac{\frac{k^2}{2\mu} - \nu_0}{g_0^2} + \frac{\mu}{\pi^2} \left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right) \right] \right]^{-1}.$$

Model scales and parameters

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- typical energy scale : $E_{\text{typ}} = \Lambda^2/(2\mu)$

- three model parameters g_0, ν_0, Λ

1. calculation with given B

coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \frac{\pi^2}{\mu}(B + \nu_0) \left[\Lambda - \kappa \arctan(\Lambda/\kappa) \right]^{-1}$

\therefore bound state condition $f^{-1} = 0$ $\kappa = \sqrt{2\mu B}$.

2. use dimensionless quantities with Λ

→ results do not depend on cutoff Λ

3. energy of bare quark state ν_0

varied in the region : $-B/E_{\text{typ}} \leq \nu_0/E_{\text{typ}} \leq 1$

\therefore to have $g_0^2 \geq 0$ & applicable limit of model

Calculation

● compositeness X

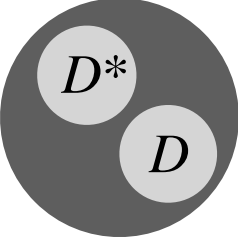
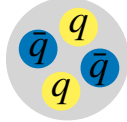
scattering amplitude : $T = \frac{1}{V^{-1} - G}$ Y. Kamiya and T. Hyodo,
PTEP 2017, 023D02 (2017).

$$\longrightarrow X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2 \kappa}{g_0^2 \mu^2} \left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1 + (\Lambda/\kappa)^2} \right)^{-1} \right]^{-1}.$$

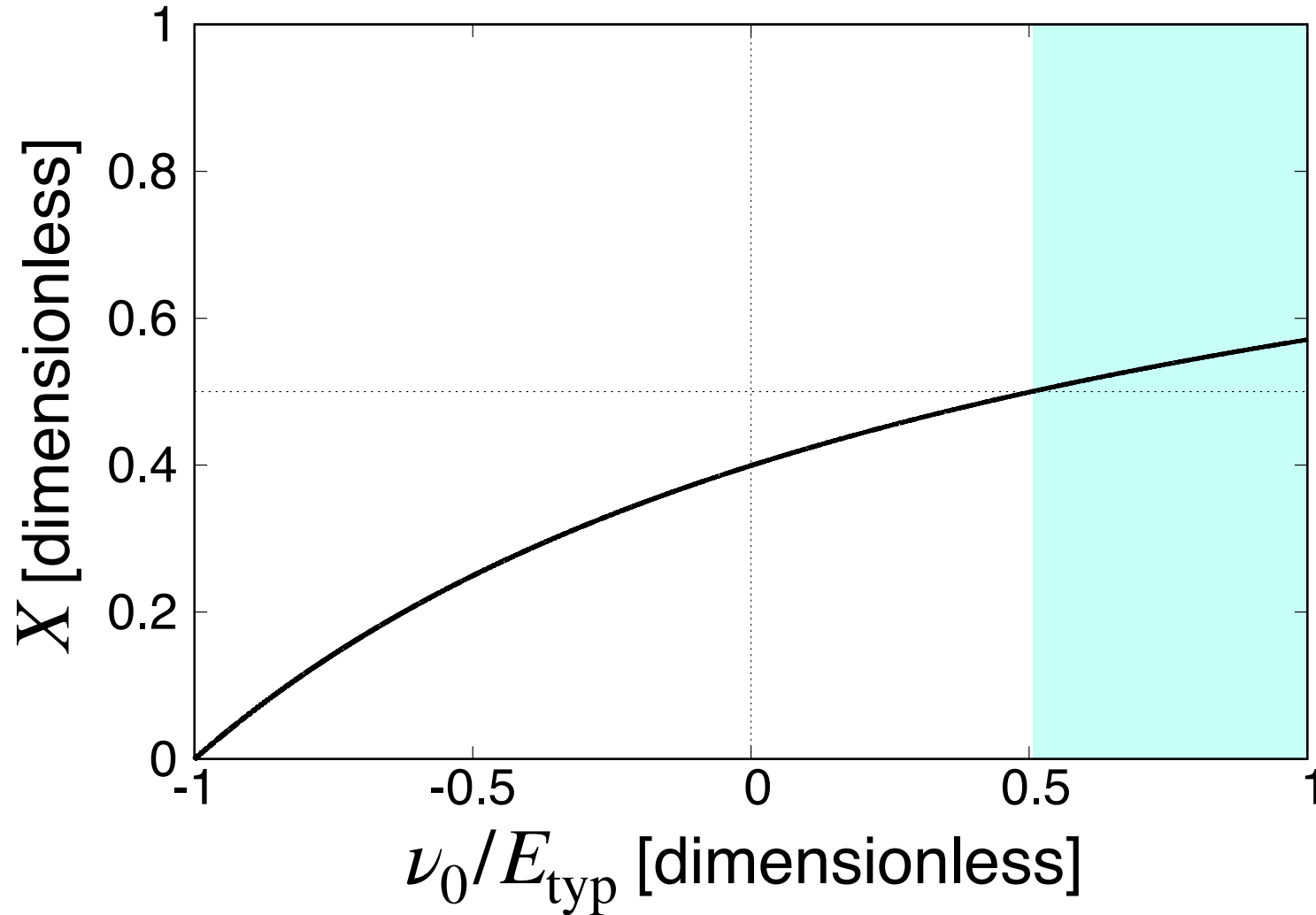
- ν_0 region : $-B/E_{\text{typ}} \leq \nu_0/E_{\text{typ}} \leq 1$

ν_0 dependence \longleftrightarrow model dependence

compositeness X as a function of ν_0 $X > 0.5$  or $X < 0.5$ 

\longrightarrow internal structure of bound state?

● X as a function of ν_0/E_{typ} of bound state $B = E_{\text{typ}}$



$X > 0.5$:

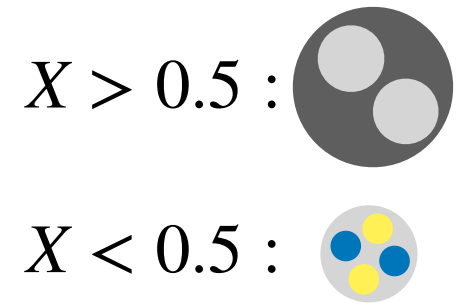
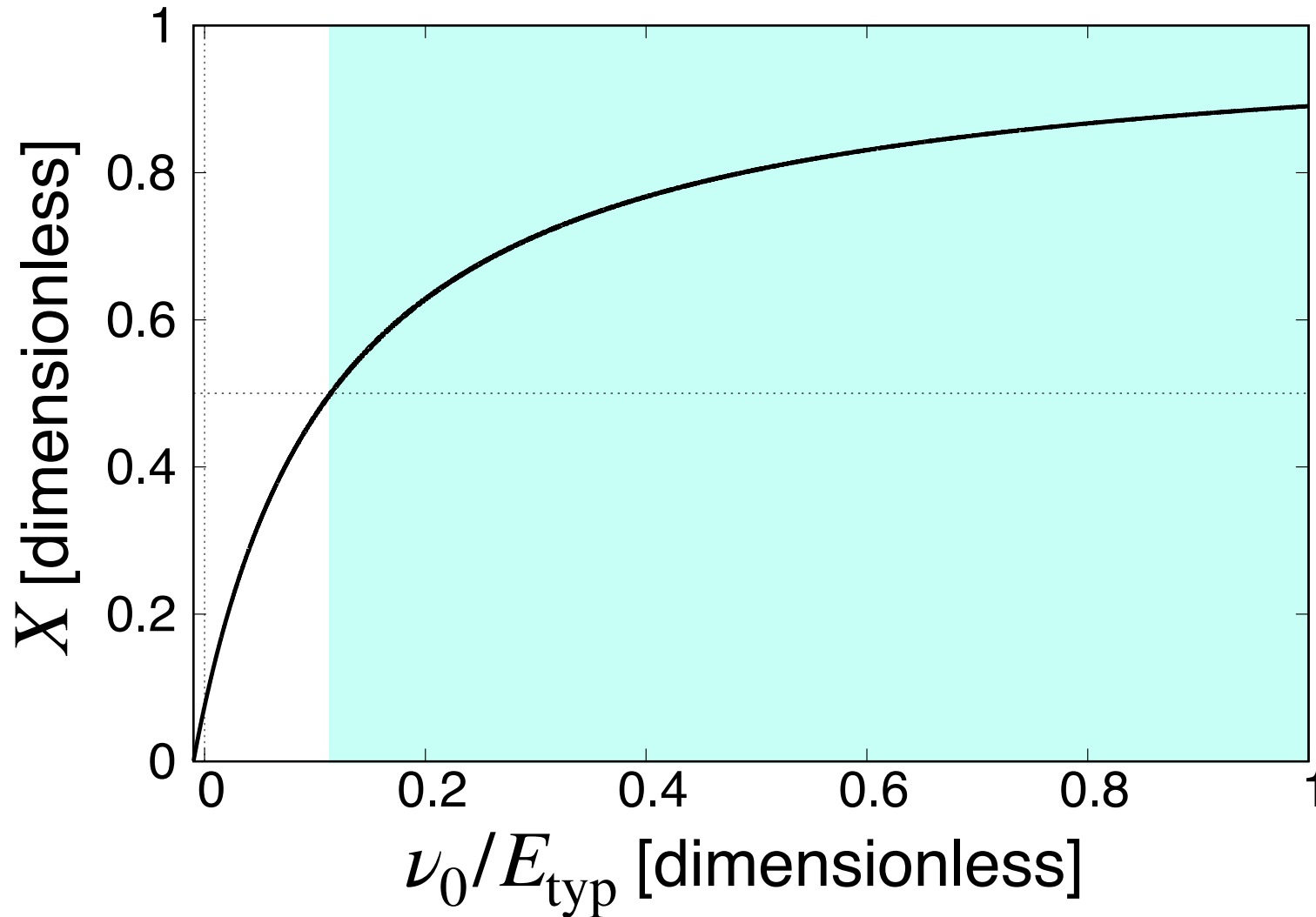
$X < 0.5$:

- typical energy scale : $B = E_{\text{typ}} = \Lambda^2/(2\mu)$

- $X > 0.5$ only for **25 %** of ν_0 \therefore **bare state origin**

● X as a function of ν_0/E_{typ} of bound state $B = 0.01E_{\text{typ}}$

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- weakly-bound state : $B = 0.01E_{\text{typ}}$

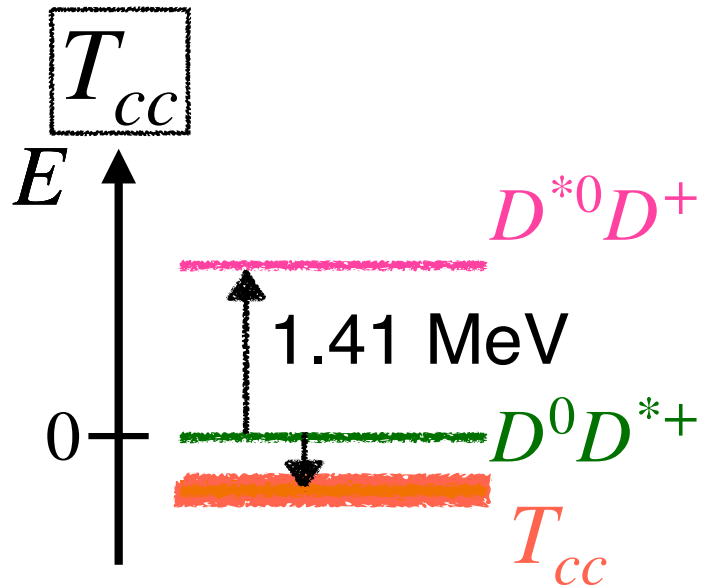
- $X > 0.5$ for 88 % of ν_0 \longrightarrow **realization of universality**

- elementary dominant state can be realized with fine tuning

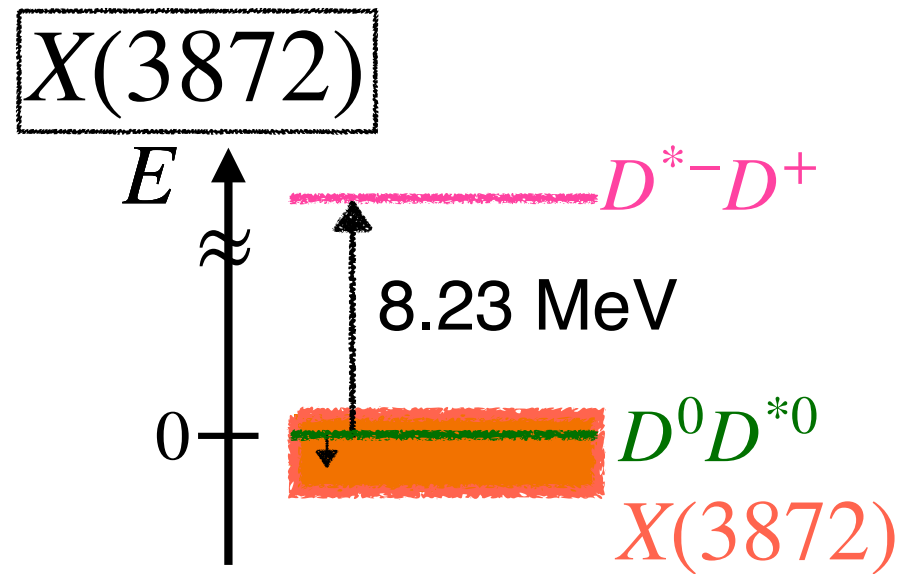
Decay & coupled ch. effects

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actual exotic hadrons \longrightarrow decay and coupled channels



LHCb Collaboration, Nat. Commun **13** 3351 (2022).



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other channels than threshold channel make deviation from $X = 1$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

In this work, we study those deviations quantitatively!

Effect of decay

● introducing decay effect

- formally : introducing decay channel in lower energy region than binding energy

→ eigenenergy becomes complex

- effectively : coupling const. $g_0 \in \mathbb{C}$! ← this work

$$\mathcal{H}_{\text{int}} = \underline{g_0}(\phi^\dagger \psi_1 \phi_2 + \phi_1^\dagger \psi_2^\dagger \phi).$$

$$E = -B \rightarrow E = -B - \underline{i\Gamma/2}$$

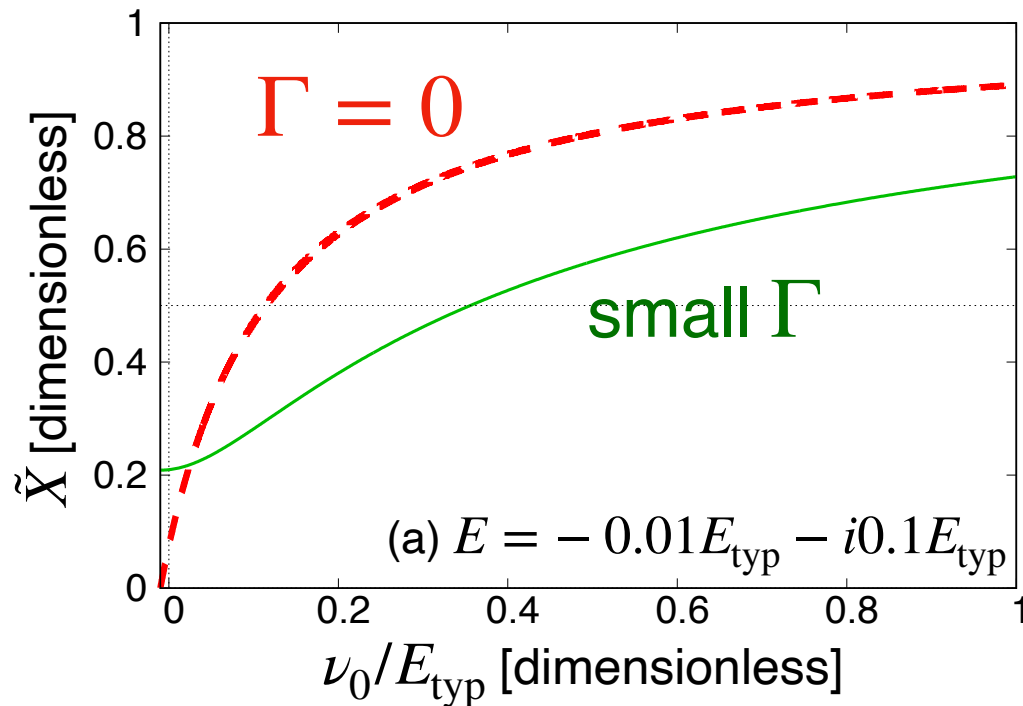
compositeness T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC 93, 035204 (2016).

$$X \in \mathbb{R} \rightarrow X \in \mathbb{C}$$

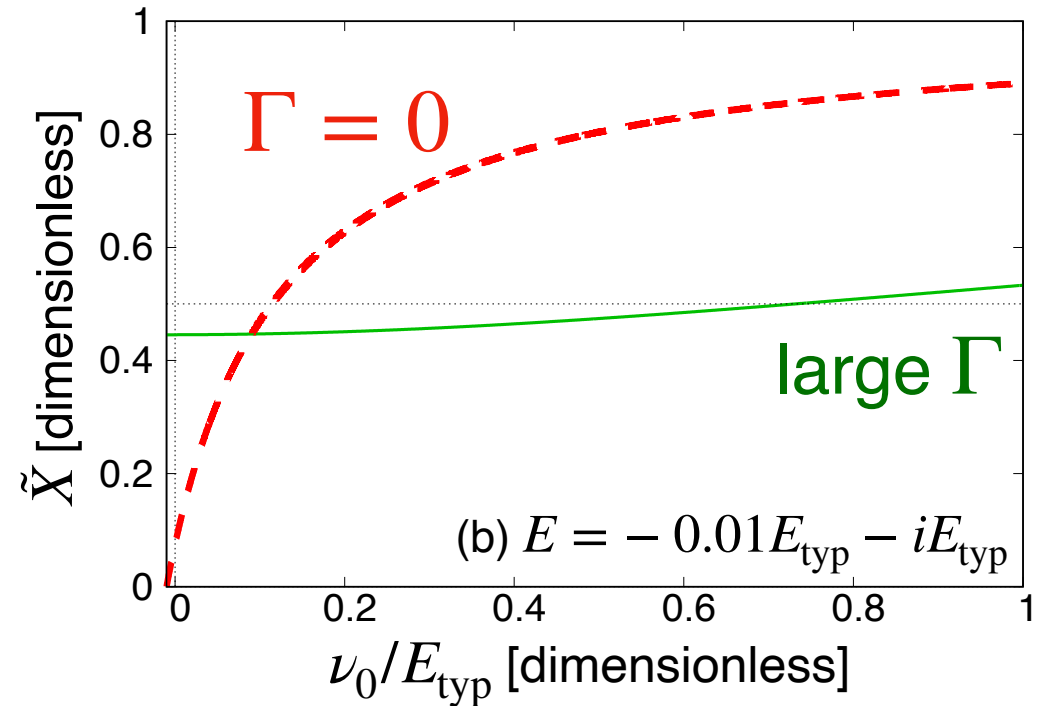
$$\tilde{X} = \frac{|X|}{|X| + |1 - X|}$$

Effect of decay

$$E = -0.01E_{\text{typ}} - \underline{i0.1E_{\text{typ}}}$$



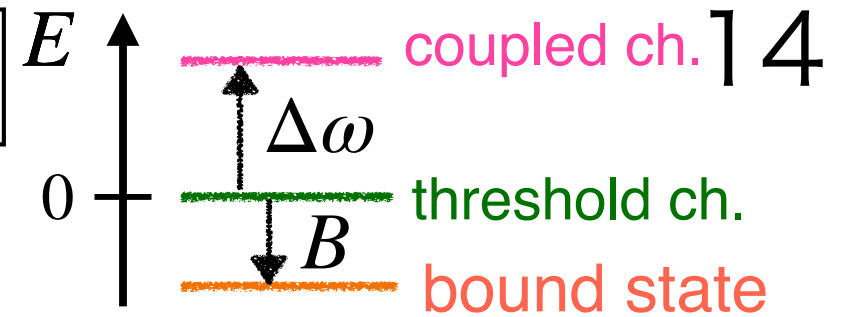
$$E = -0.01E_{\text{typ}} - \underline{iE_{\text{typ}}}$$



- \tilde{X} is suppressed by decay effect

\therefore threshold ch. component (\tilde{X}) decreases with inclusion of decay ch. component ($1 - \tilde{X}$)

Effect of coupled channel



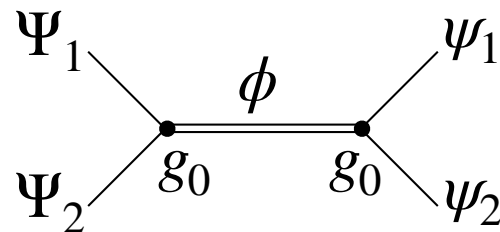
● introducing coupled channel $\Psi_1 \Psi_2$

$$|\Psi\rangle = \sqrt{X_1} |\text{threshold ch}\rangle + \sqrt{X_2} |\text{coupled ch}\rangle + \sqrt{1 - (X_1 + X_2)} |\text{others}\rangle$$

$$\mathcal{H}_{\text{free}} = (\text{kinetic terms of } \psi_{1,2}, \Psi_{1,2}, \phi) + \omega_1 \Psi_1 \Psi_1^\dagger + \omega_2 \Psi_2 \Psi_2^\dagger + \nu_0 \phi^\dagger \phi,$$

$$\mathcal{H}_{\text{int}} = g_0 (\phi^\dagger \psi_1 \psi_2 + \psi_1^\dagger \psi_2^\dagger \phi + \phi^\dagger \Psi_1 \Psi_2 + \Psi_1^\dagger \Psi_2^\dagger \phi).$$

- threshold energy difference $\Delta\omega = \omega_1 + \omega_2$
- ch. 1 couples to ch. 2 through ϕ with same coupling const.



- low-energy universality with coupled-channel effect ($B \rightarrow 0$)

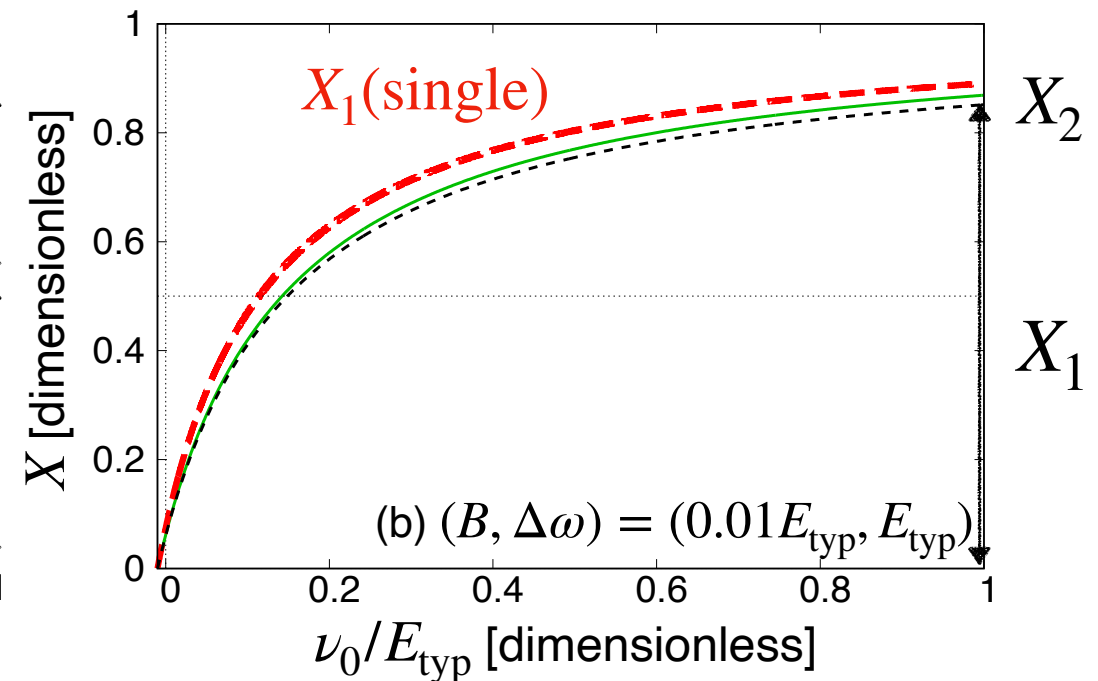
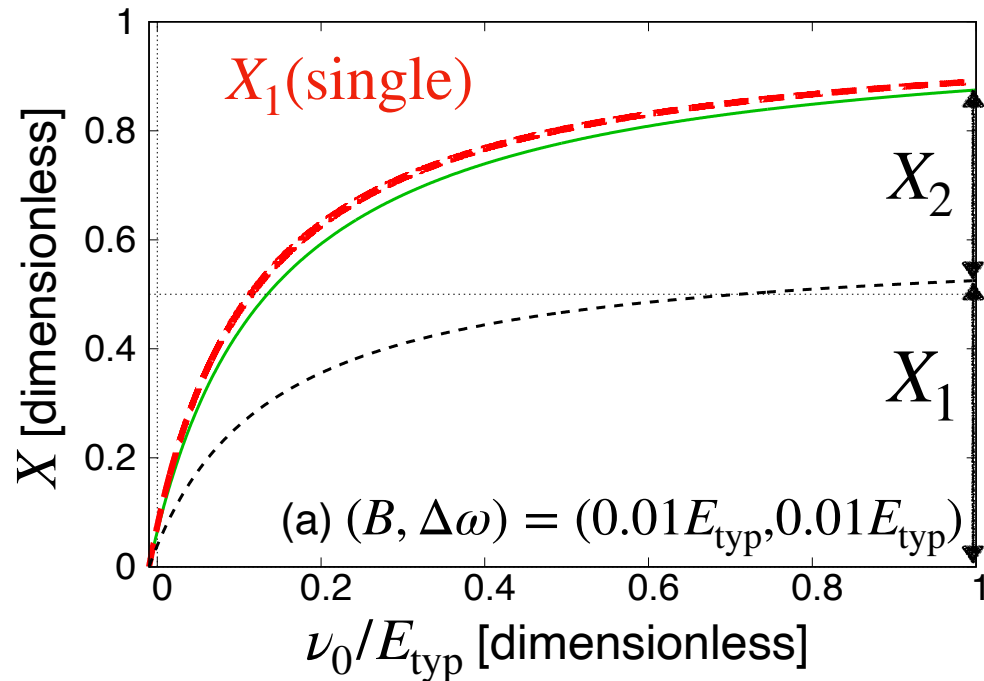
$$X_1 \rightarrow 1 \text{ (threshold channel)}$$

$$X_2 \rightarrow 0 \text{ and } Z \rightarrow 0 \text{ (other channel)}$$

Effect of coupled channel

$$(B, \Delta\omega) = (0.01E_{\text{typ}}, 0.01E_{\text{typ}})$$

$$(B, \Delta\omega) = (0.01E_{\text{typ}}, E_{\text{typ}})$$



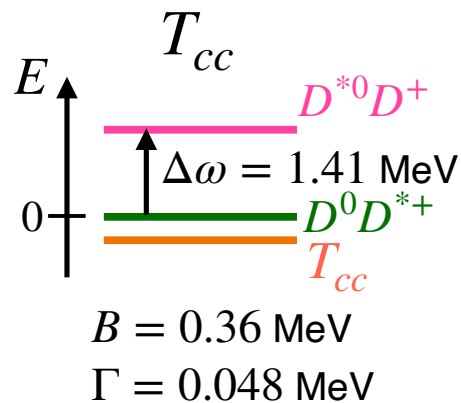
- X_1 is suppressed by channel coupling

\therefore threshold ch. component (X_1) decreases with inclusion of coupled ch. component (X_2)

Application to T_{cc} and $X(3872)$

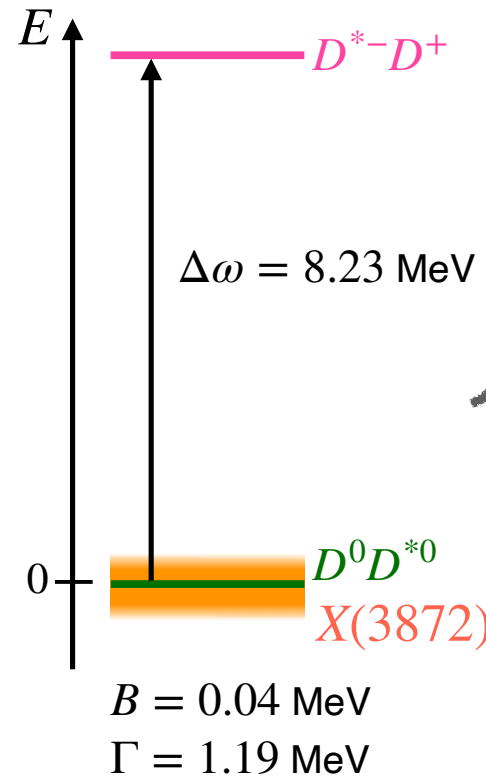
● exotic hadron ← decay and coupled channel
 $X(3872)$

small
 Γ and $\Delta\omega$



LHCb Collaboration, Nat. Commun **13** 3351 (2022).

large
 Γ and $\Delta\omega$



PDG

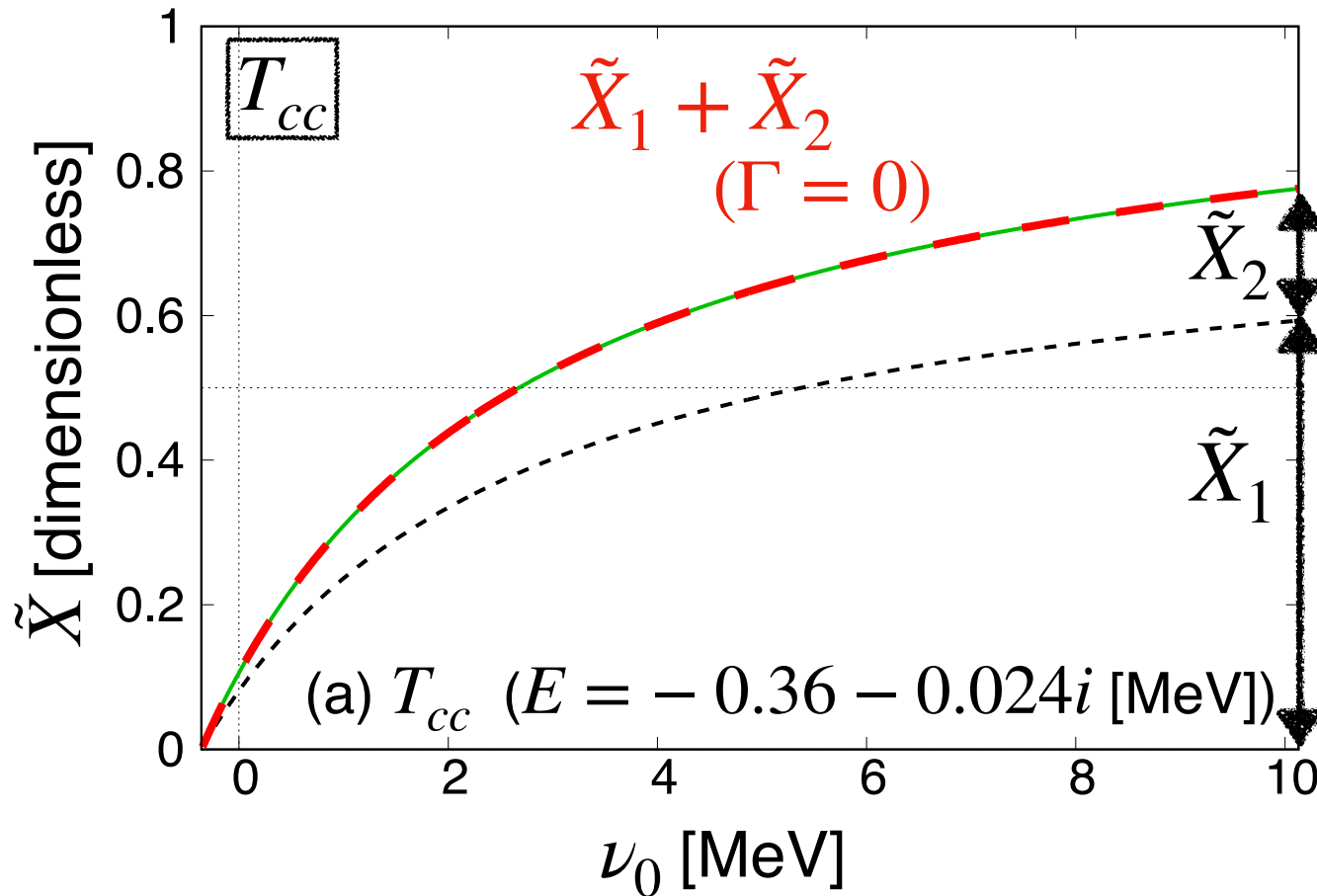
● compositeness T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC 93, 035204 (2016).

$$\tilde{X}_j = \frac{|X_j|}{\sum_j |X_j| + |Z|}, \quad (j = 1, 2)$$

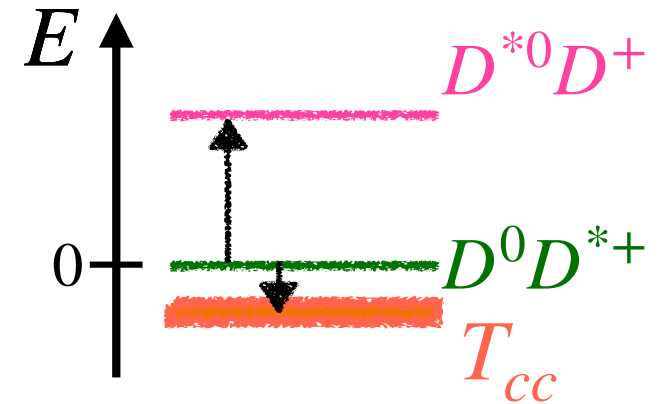
\tilde{X}_1 : threshold ch. compositeness
 \tilde{X}_2 : coupled ch. compositeness

Application to T_{cc} and $X(3872)$

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$\Lambda = 140$ MeV
(π meson)



- \tilde{X}_2 is not negligible

\because coupled ch. contribution (small $\Delta\omega$)

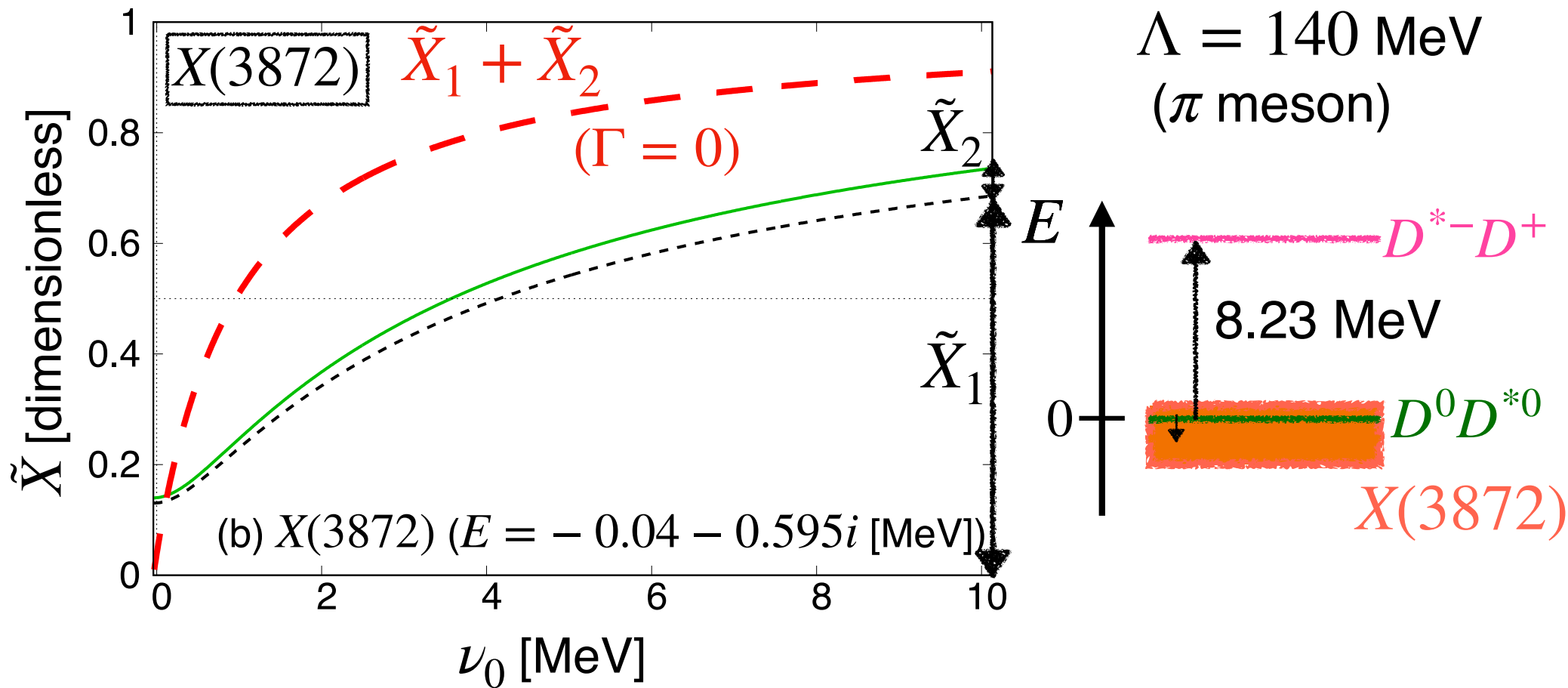
- difference of $\tilde{X}_1 + \tilde{X}_2(\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is too small

\longrightarrow We can neglect decay contribution

$\because \Gamma \ll B$

Application to T_{cc} and $X(3872)$

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- difference of $\tilde{X}_1 + \tilde{X}_2(\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is large
 - \therefore large decay width contribution
- \tilde{X}_2 is much smaller than \tilde{X}_1
 - \rightarrow coupled ch. effect is small

- internal structure of exotic hadrons ← compositeness
 - shallow bound state
 - fine tuning is necessary to realize elementary dominant state
 - decay and coupled channel effects are introduced
 - both decay and coupled ch. effects suppress compositeness
 - T_{cc} and $X(3872)$ with decay and coupled ch. effects
- T_{cc} : important coupled ch. effect with negligible decay effect
- $X(3872)$: important decay effect with negligible coupled ch. effect