Compositeness of exotic hadrons with decay and coupled-channel effects

T. Kinugawa and T. Hyodo
arXiv:2303.07038 [hep-ph]

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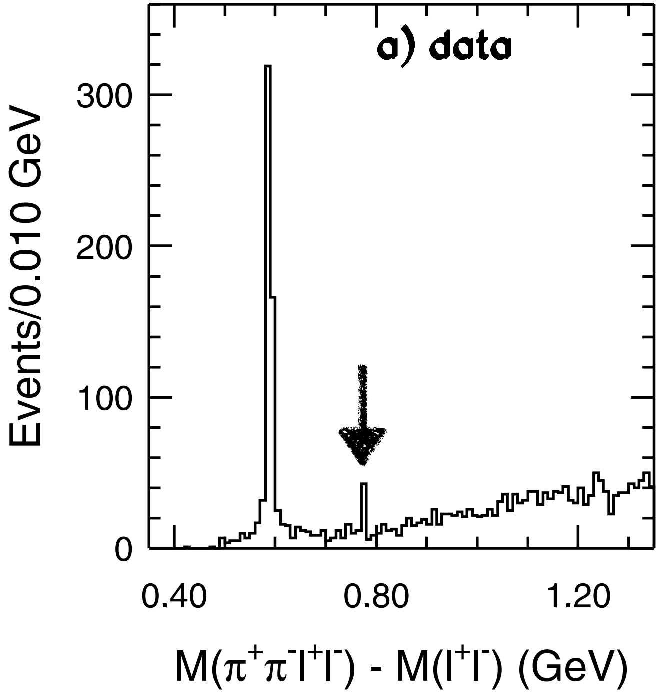
March 28th, J-PARCハドロン研究会 2023

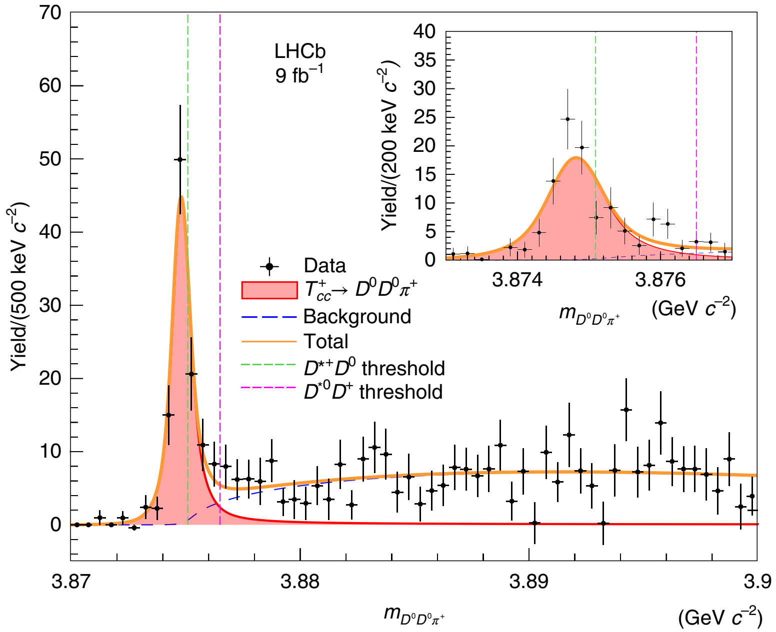
Near-threshold exotic hadrons

$X(3872) \to \pi^+\pi^- J/\psi$



S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

$T_{cc} \to D^0 D^0 \pi^+ \; (cc\bar{u}\bar{d})$



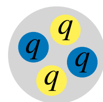
LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

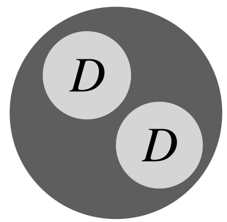
internal structure?

exotic hadron

$\neq \; qqq$ or $q\bar{q}$ ⟶ multiquarks, hadronic molecules

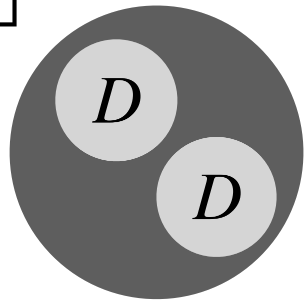


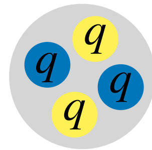
multiquarks



hadronic molecules

Compositeness





◉ definition

hadron wavefunction

$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

compositeness **elementarity**

T. Hyodo, Int. J. Mod. Phys. A **28**, 1330045 (2013);
T. Kinugawa and T. Hyodo, Phys. Rev. C **106**, 015205 (2022).

※ $0 \leq X \leq 1$ ⟶ $X > 0.5 \Leftrightarrow$ **composite dominant**

$X < 0.5 \Leftrightarrow$ **elementary dominant**

◉ model calculation

T. Hyodo, D. Jido, and A. Hosaka, Phys. Rev. C **85**, 015201 (2012);
F. Aceti and E. Oset, Phys. Rev. D **86**, 014012 (2012).

compositeness X ⟵ residue of pole of scattering amplitude

Low-energy universality

- scattering length $a_0 \gg$ typical length scale of system

low-energy universality E. Braaten and H.-W. Hammer, Phys. Rept. **428**, 259 (2006) ; F. P. Naidon and S. Endo, Rept. Prog. Phys. **80**, 056001 (2017).

→ length scales are written only by $|a_0| (\to \infty)$

for bound states ?

$$a_0 = R \qquad R = 1/\sqrt{2\mu B} \qquad a_0 \to \infty \longrightarrow B \to 0$$

→ universality holds for weakly-bound states!!

- compositeness $X = 1$ in $B \to 0$ limit T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

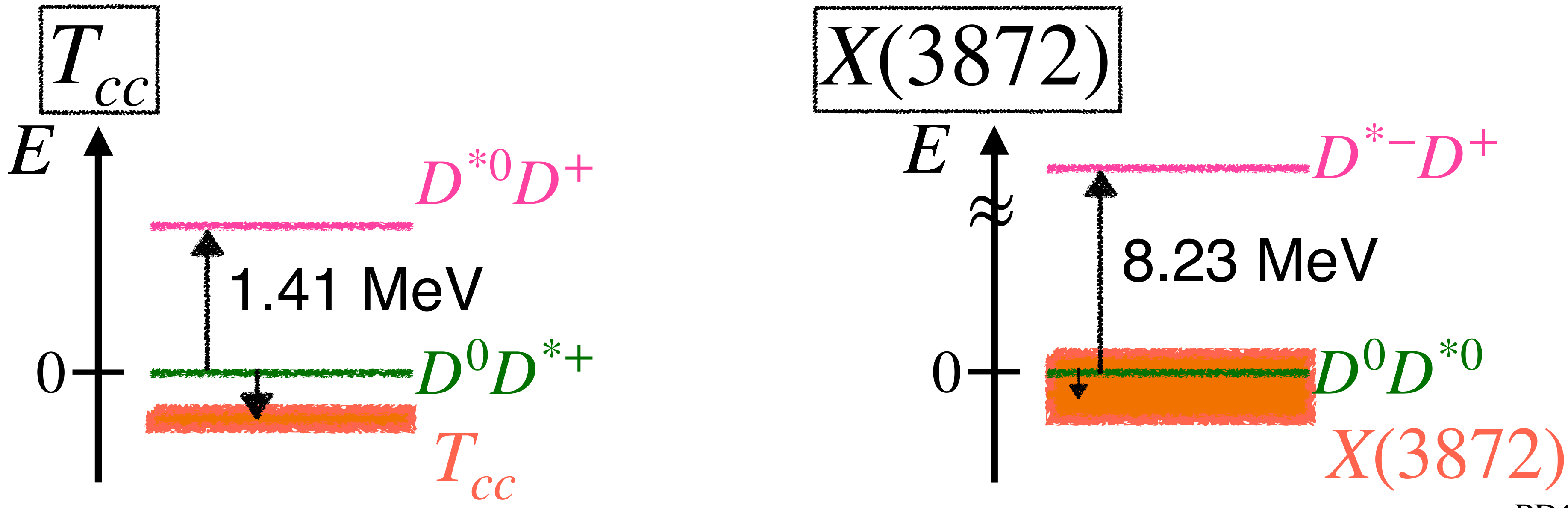
→ near threshold poles = composite dominant ?

e.g. $^{8}\mathrm{Be}$, $^{12}\mathrm{C}$ Hoyle state → α cluster? H. Horiuchi, K. Ikeda, and Y. Suzuki, Prog. Theor. Phys. Suppl. **52**, 89 (1972) .

Decay & coupled ch. effects

However…

actual exotic hadrons ⟶ decay and coupled channel



LHCb Collaboration, Nat. Commun **13** 3351 (2022).

PDG

other ch. than threshold ch. make deviation from $X = 1$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

This work…
study those deviations quantitatively!

Model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).

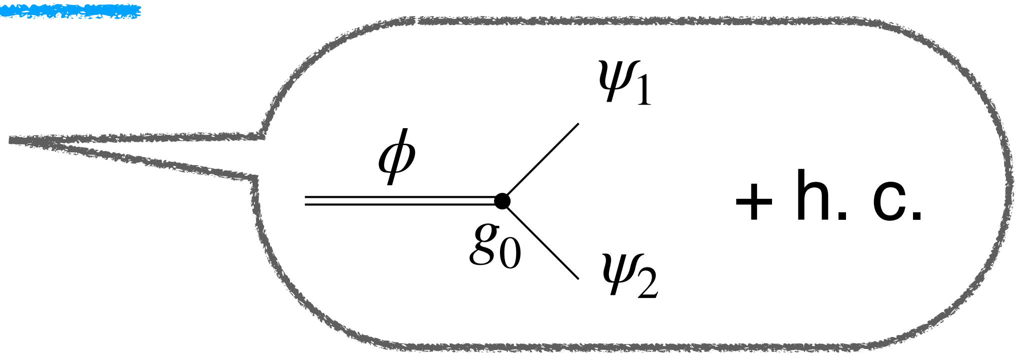
◉ single-channel resonance model

$$\mathscr{H}_{\text{free}} = \frac{1}{2m_1}\nabla\psi_1^\dagger\cdot\nabla\psi_1 + \frac{1}{2m_2}\nabla\psi_2^\dagger\cdot\nabla\psi_2 + \frac{1}{2m_\phi}\nabla\phi^\dagger\cdot\nabla\phi + \nu_0\phi^\dagger\phi,$$

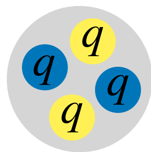
1.

$$\mathscr{H}_{\text{int}} = g_0(\phi^\dagger\psi_1\phi_2 + \phi_1{}^\dagger\psi_2^\dagger\phi)\,.$$

2.



1. single-channel scattering
2. coupling with compact state ϕ



◉ scattering amplitude

$$V = \frac{g_0^2}{E-\nu_0}, \quad G = -\frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right].$$

Λ : cutoff

$$\xrightarrow[T = \frac{1}{V^{-1}-G}]{} \quad f(k) = -\frac{\mu}{2\pi}\left[\frac{\frac{k^2}{2\mu}-\nu_0}{g_0^2} + \frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right]\right]^{-1}.$$

Model scales and parameters

- typical energy scale : $E_{\text{typ}} = \Lambda^2/(2\mu)$

- three model parameters g_0, ν_0, Λ

1. calculation with given B

coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \dfrac{\pi^2}{\mu}(B + \nu_0)\left[\Lambda - \kappa \arctan(\Lambda/\kappa)\right]^{-1}$

∵ bound state condition $f^{-1} = 0$ $\kappa = \sqrt{2\mu B}\,.$

2. use dimensionless quantities with Λ

⟶ results do not depend on cutoff Λ

3. energy of bare state ν_0

varied in the region : $-B/E_{\text{typ}} \leq \nu_0/E_{\text{typ}} \leq 1$

∵ to have $g_0^2 \geq 0$ & applicable limit of EFT

Calculation

◉ compositeness X

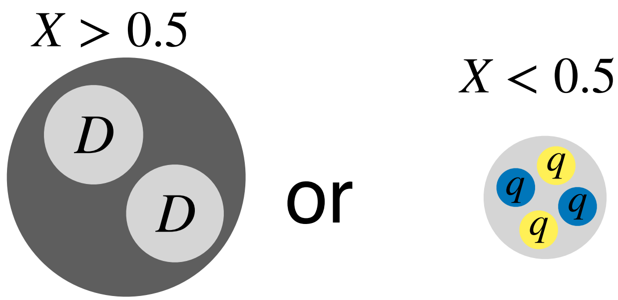
scattering amplitude : $T = \dfrac{1}{V^{-1} - G}$ Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

$$\longrightarrow X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1 + \left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}.$$

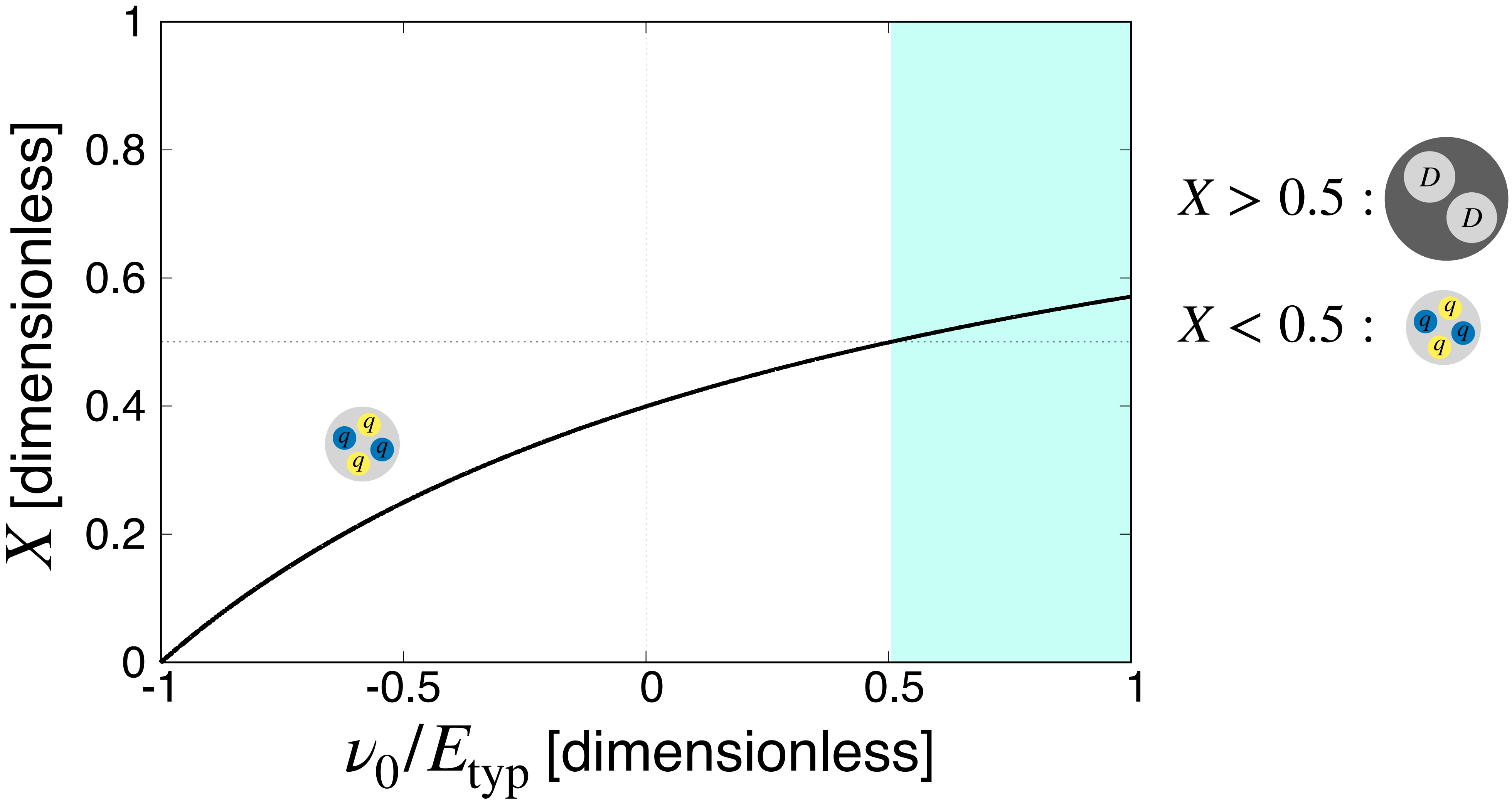
- ν_0 region : $-B/E_{\rm typ} \leq \nu_0/E_{\rm typ} \leq 1$

compositeness X as a function of ν_0 with fixed B



$\longrightarrow$ internal structure of bound state?

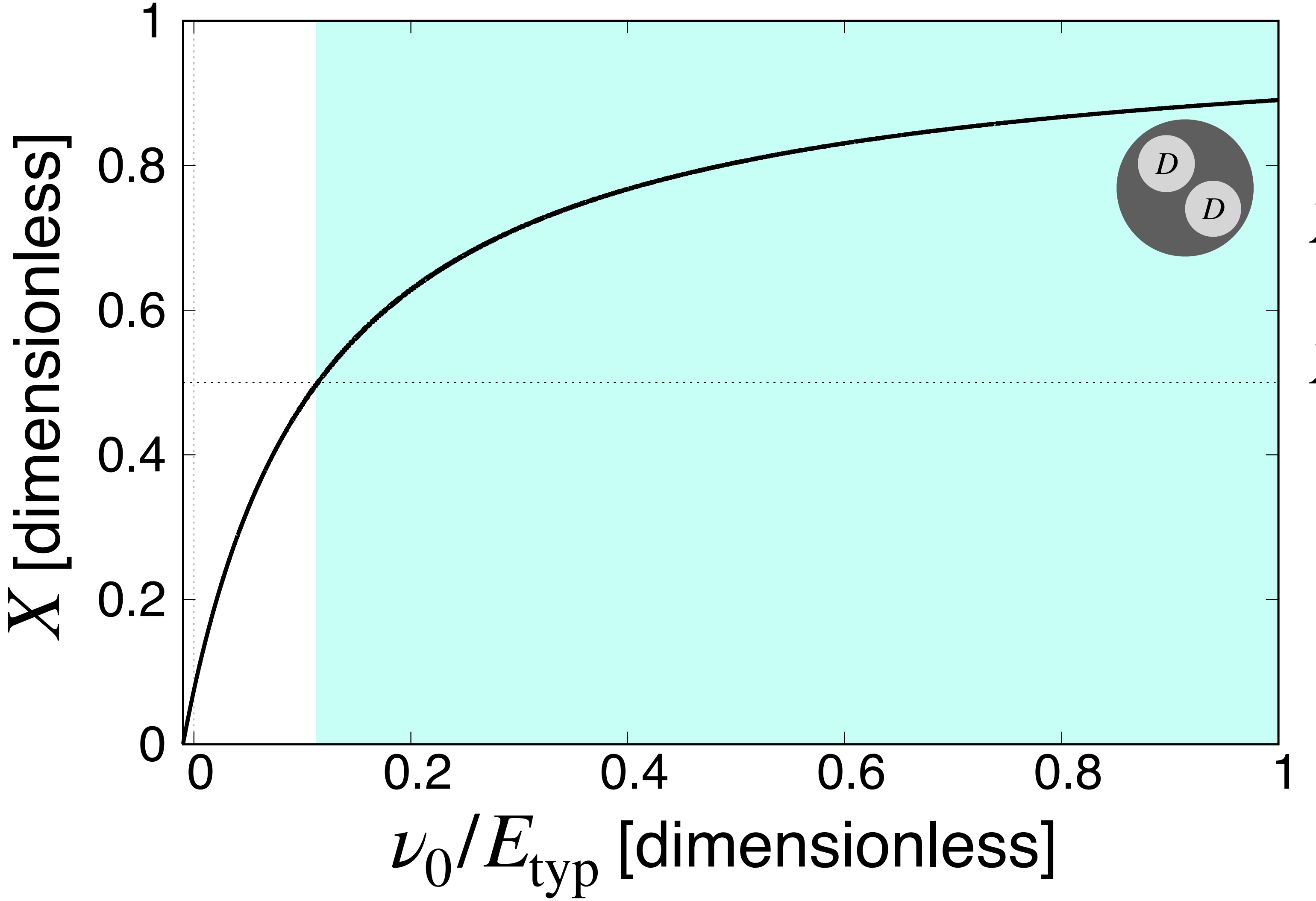
◉ X as a function of ν_0/E_{typ} of bound state $B = E_{\mathrm{typ}}$

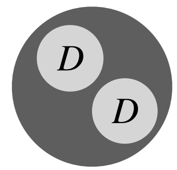


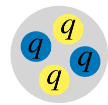
- typical energy scale : $B = E_{\mathrm{typ}} = \Lambda^2/(2\mu)$
- $X > 0.5$ only for $25\ \%$ of ν_0 = elementary dominant

∵ **bare state origin**

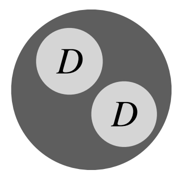
◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = 0.01E_{\rm typ}$



$X > 0.5$: 

$X < 0.5$: 

- weakly-bound state : $B = 0.01E_{\rm typ}$
- $X > 0.5$ for $88\ \%$ of ν_0 = composite dominant



∵ **low-energy universality !**

Effect of decay

◉ introducing decay effect

- formally : introducing decay channel in lower energy region than binding energy

  ⟶ eigenenergy becomes complex

- effectively : coupling const. $g_0 \in \mathbb{C}$! ⟵ this work

$$E = -B \longrightarrow E = -B - i\Gamma/2$$

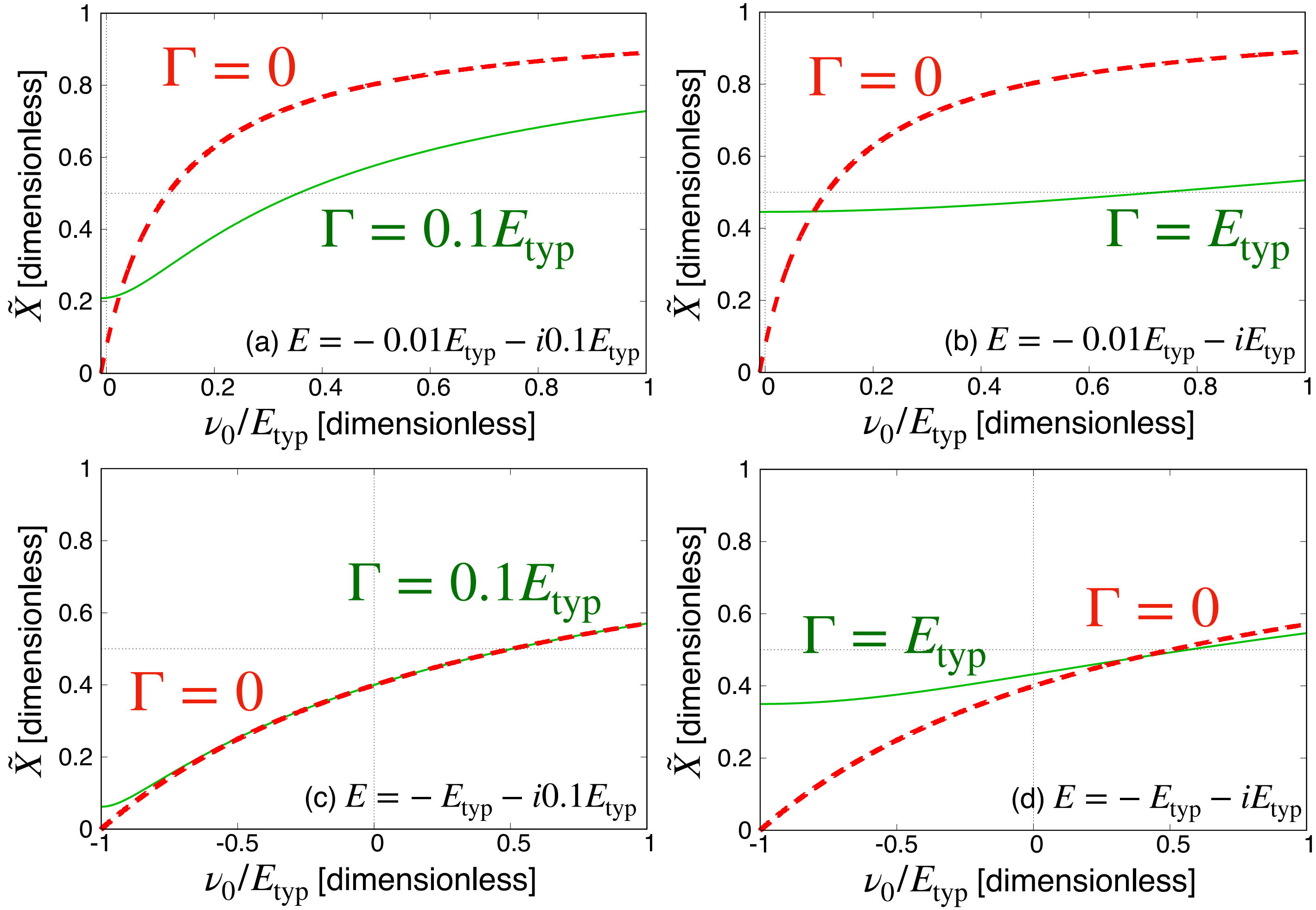
compositeness

$$X \in \mathbb{R} \longrightarrow X \in \mathbb{C}$$

$$\tilde{X} = \frac{|X|}{|X| + |1 - X|}$$

T. Sekihara, *et. al*., PRC 93, 035204 (2016).

Effect of decay



- $\tilde{X}$ is suppressed by decay effect

  ∵ threshold ch. component ($\tilde{X}$) decreases with inclusion of decay ch. component ($1-\tilde{X}$)

- $\tilde{X}$ is determined by ratio of B to Γ

Effect of coupled channel

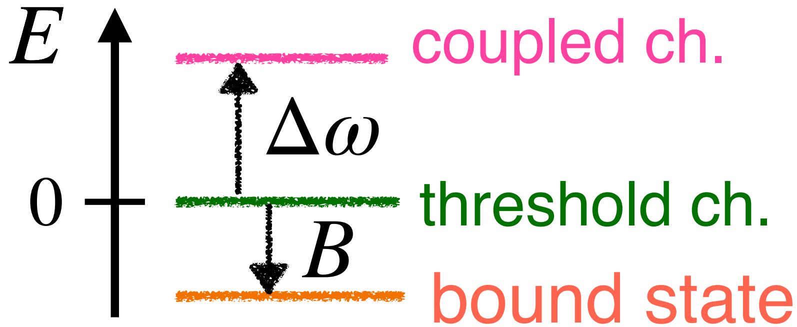
◉ introducing coupled channel $\Psi_1 \Psi_2$

$$|\Psi\rangle = \sqrt{X_1}\,|\text{threshold ch}\rangle + \sqrt{X_2}\,|\text{coupled ch}\rangle + \sqrt{1-(X_1+X_2)}\,|\text{others}\rangle$$

X_1 : threshold ch. compositeness

X_2 : coupled ch. compositeness

- threshold energy difference $\Delta\omega$

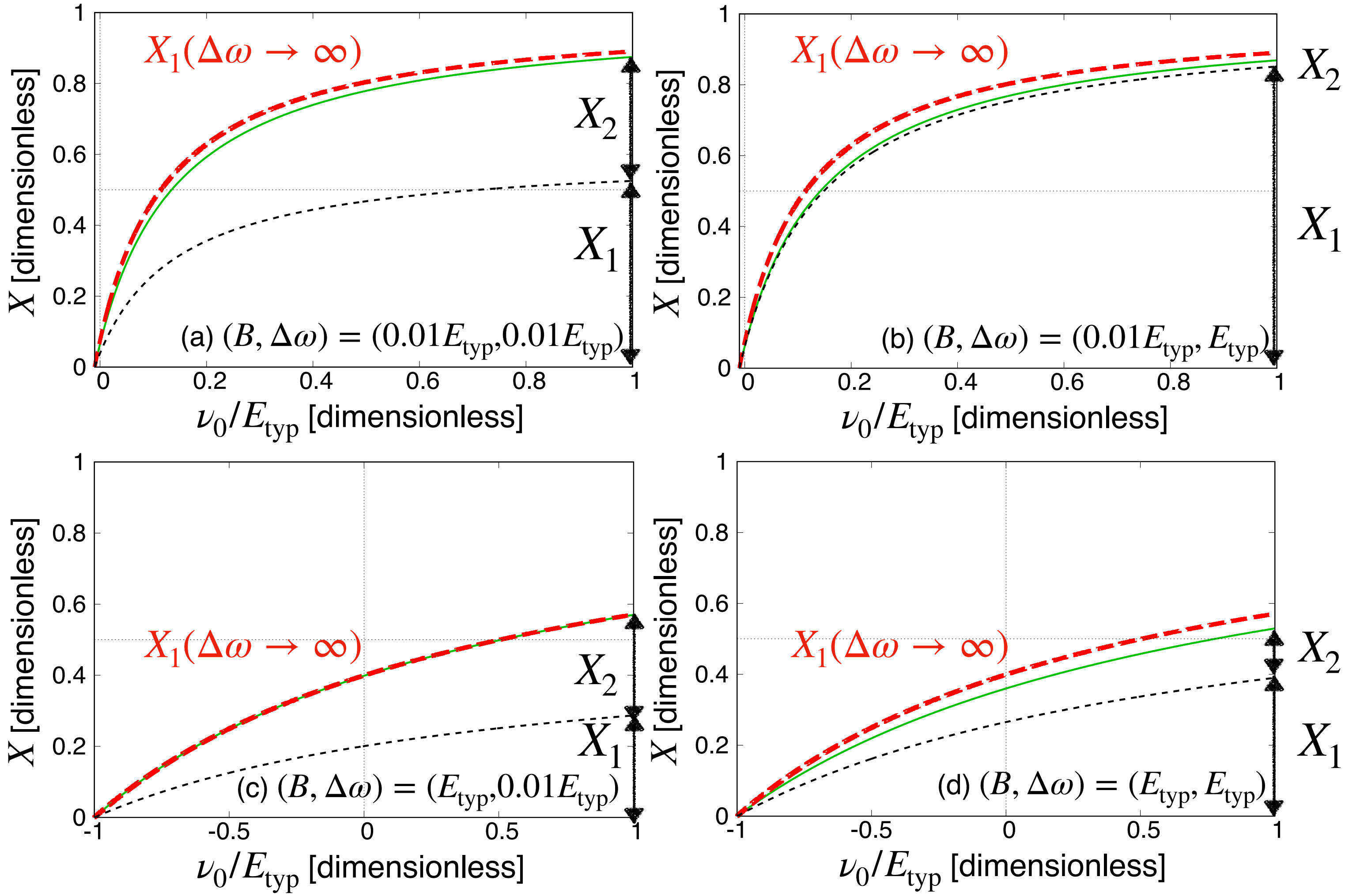


- low-energy universality with coupled-channel effect

$X_1 \sim 1$ (threshold channel)

$X_2 \sim 0$ (other channel) $Z \sim 0$ (other channel)

Effect of coupled channel

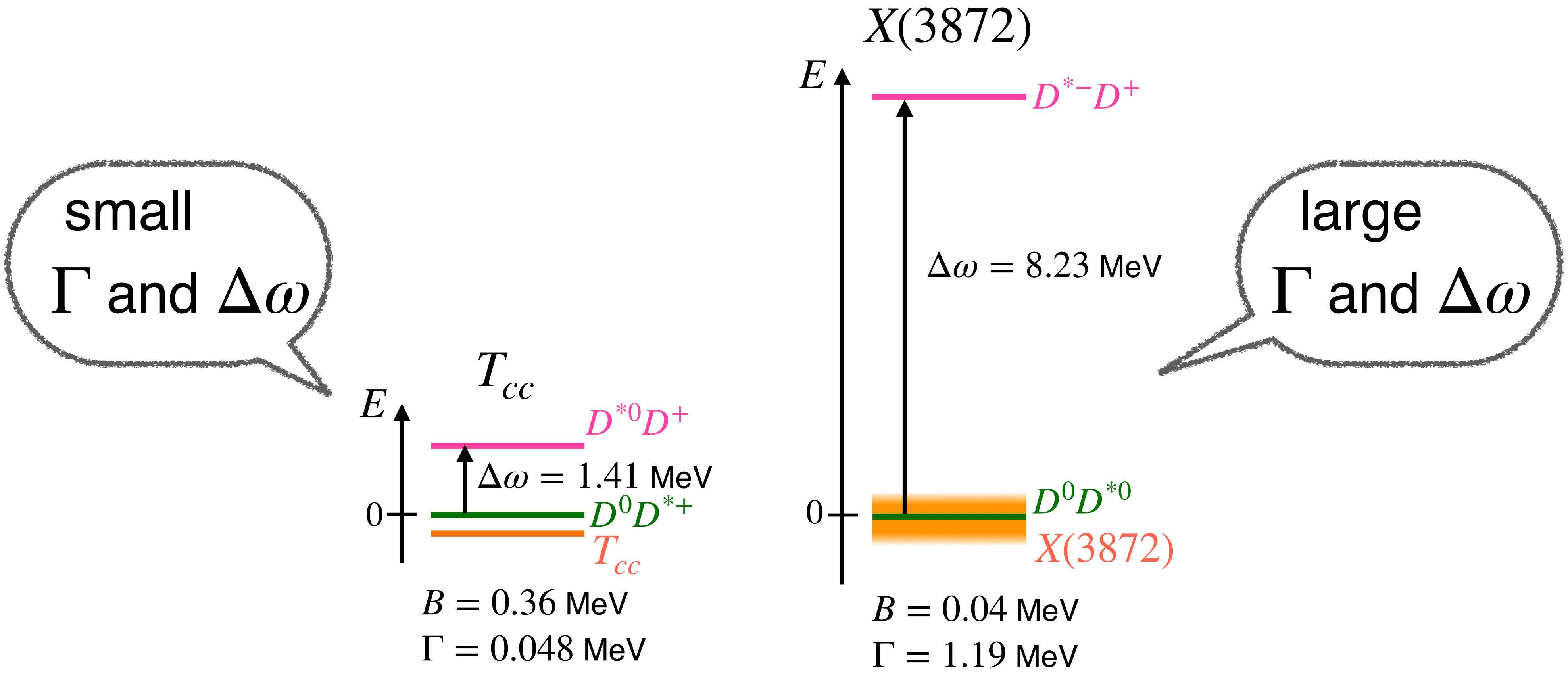


- X_1 is suppressed by channel coupling

∵ threshold ch. component (X_1) decreases with inclusion of coupled ch. component (X_2)

Application to T_{cc} and $X(3872)$

◉ exotic hadron ⟵ decay and coupled channel



LHCb Collaboration, Nat. Commun **13** 3351 (2022).　　　PDG

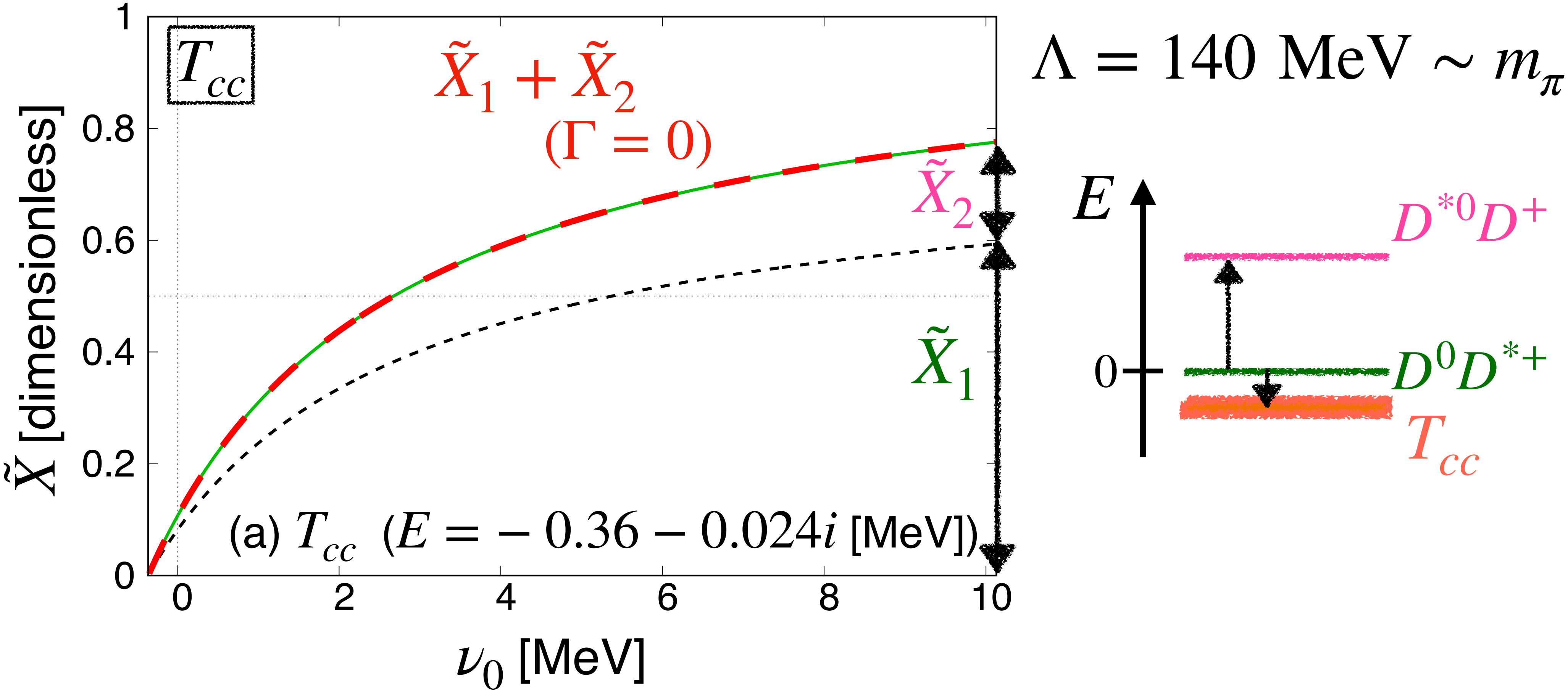
◉ compositeness T. Sekihara, *et. al.*, PRC 93, 035204 (2016).

$$\tilde{X}_j = \frac{|X_j|}{\sum_j |X_j| + |Z|}, \quad (j = 1,2)$$

$\tilde{X}_1$: threshold ch. compositeness

$\tilde{X}_2$: coupled ch. compositeness

Application to T_{cc} and $X(3872)$



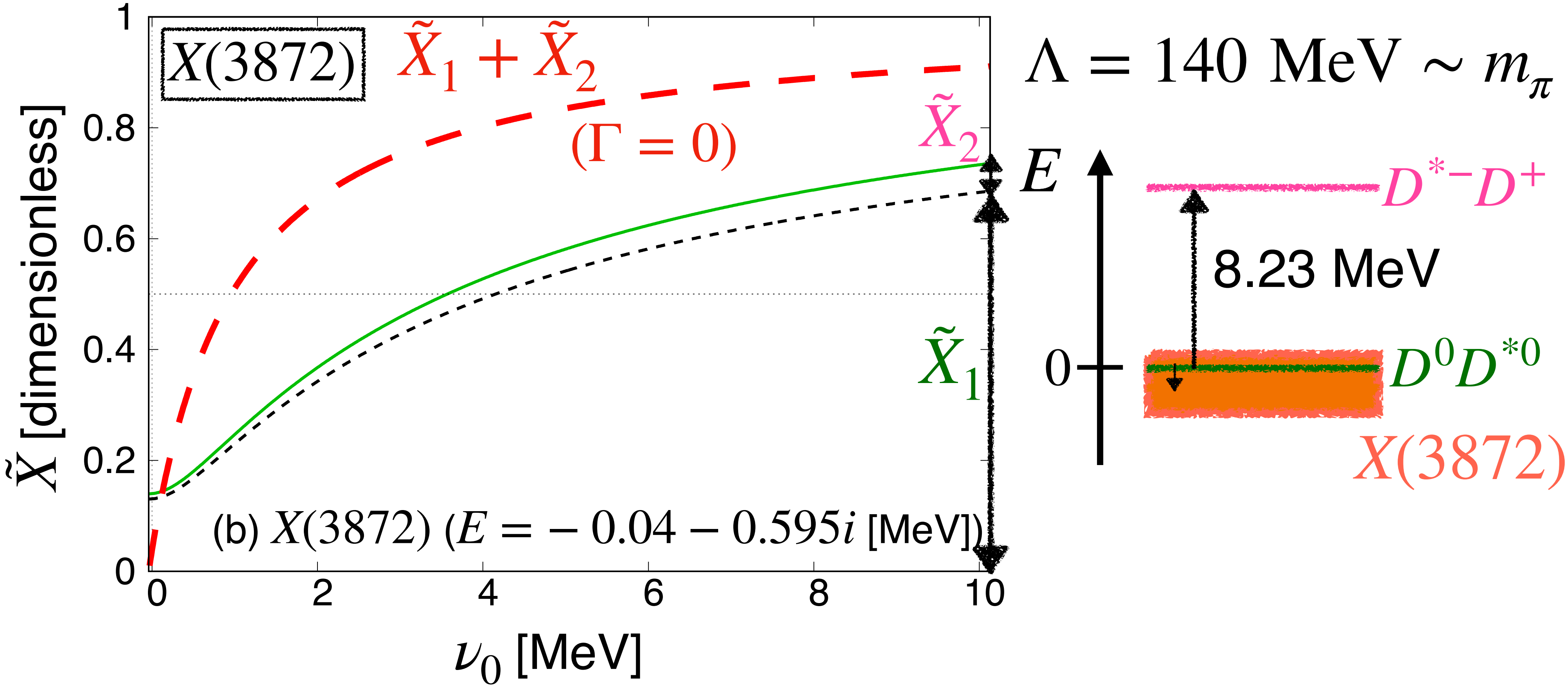
- $\tilde{X}_2$ is not negligible

  $\because$ coupled ch. contribution (small $\Delta\omega$)

- difference of $\tilde{X}_1 + \tilde{X}_2(\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is too small

  ⟶ We can neglect decay contribution $\because \Gamma \ll B$

Application to T_{cc} and $X(3872)$



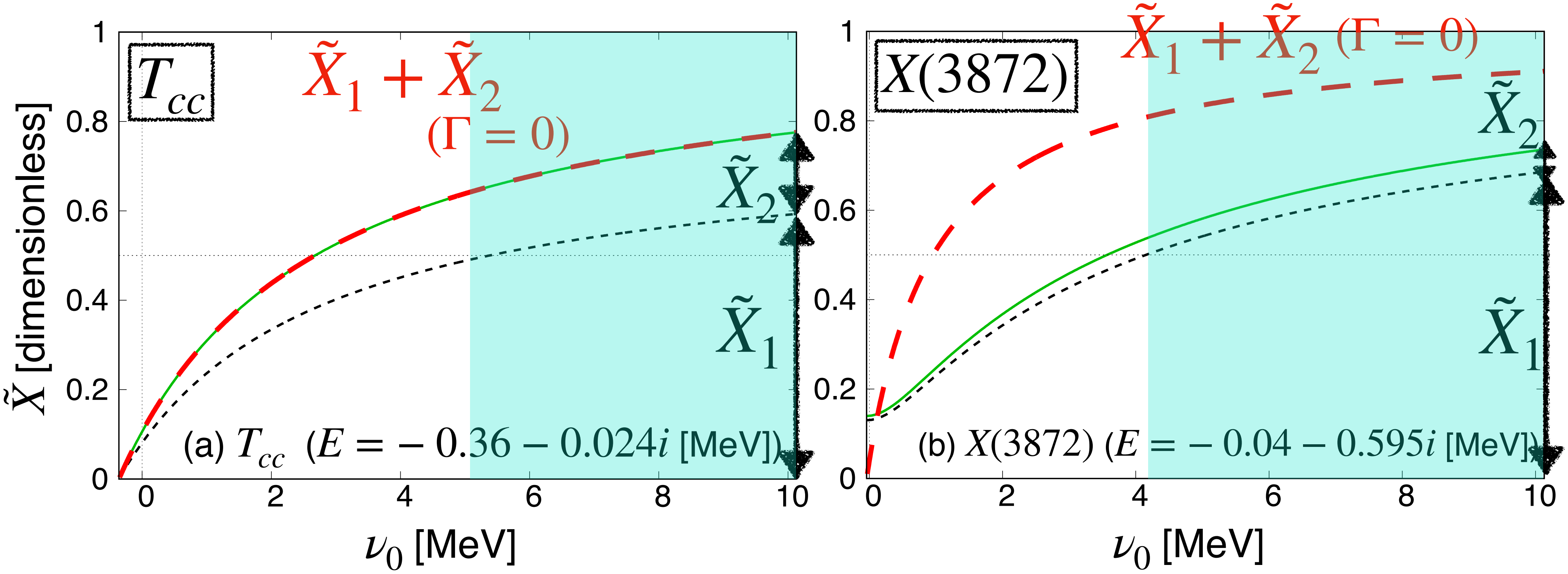
- difference of $\tilde{X}_1 + \tilde{X}_2 (\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is large

  ∵ large decay width contribution

- $\tilde{X}_2$ is much smaller than $\tilde{X}_1$

  ⟶ coupled ch. effect is small

Application to T_{cc} and $X(3872)$



- T_{cc} : $\tilde{X}_1 > 0.5$ for 45 % of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for 59 % of ν_0 region
- coupled ch. effect is more important for T_{cc} than $X(3872)$
- decay effect is more important for $X(3872)$ than T_{cc}

Summary

T. Kinugawa and T. Hyodo arXiv:2303.07038 [hep-ph]

- internal structure of exotic hadrons ⟵ compositeness
- model with bare state coupled to the scattering state
- shallow bound state is composite dominant even from bare state

  ∵ low-energy universality
- decay and coupled channel effects are introduced

⟶ both decay and coupled ch. effect suppress compositeness

- X of T_{cc} and $X(3872)$ are calculated with decay and coupled ch. effects

T_{cc} : important coupled ch. effect with negligible decay effect
$X(3872)$: important decay effect with negligible coupled ch. effect

⟶ non-composite state is realized without significant fine tuning

Compositeness of exotic hadrons with decay and coupled-channel effects

arXiv:2303.07038 [hep-ph]

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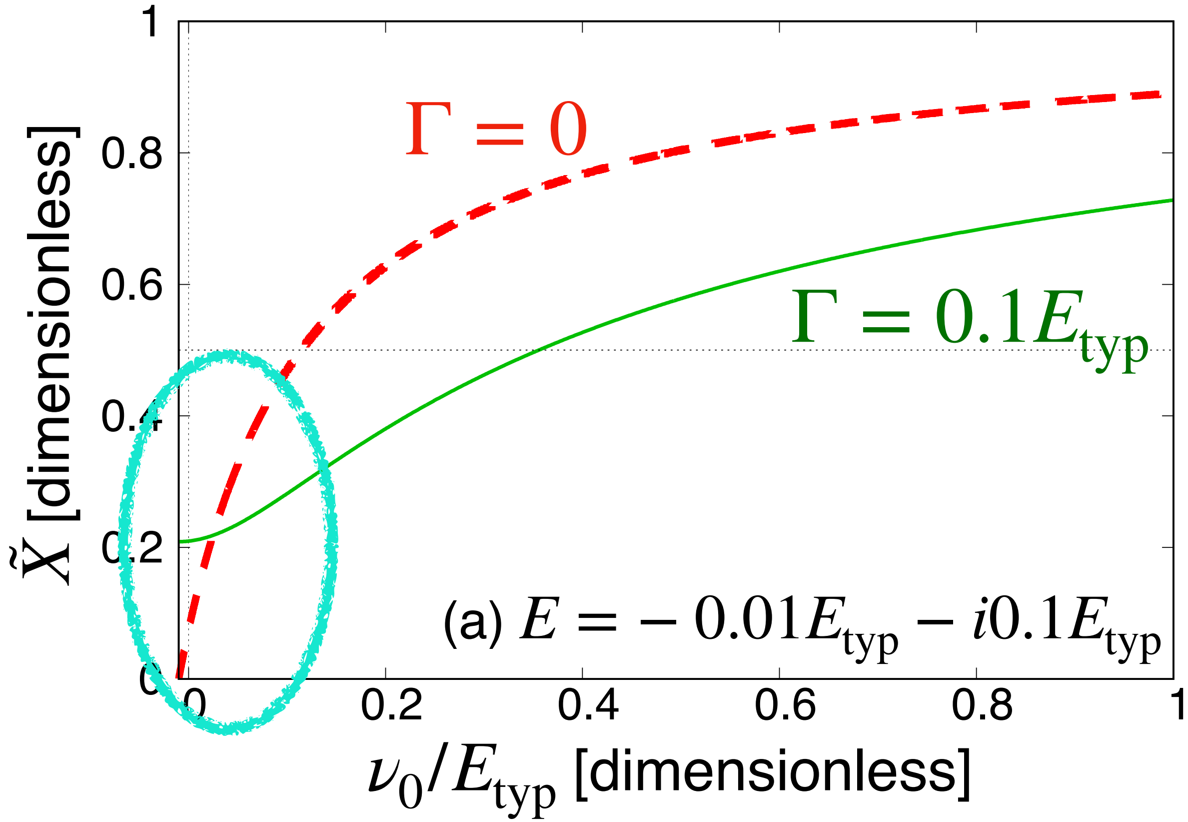
March. 16th, 3rd J-PARC HEF-ex WS

Effect of decay



- $X \neq 0$ with $\Gamma \neq 0$
  $\because\ g_0 \neq 0$ at $\nu_0 = -B$

c.f. $g_0 = 0$ at $\nu_0 = -B$ with $\Gamma = 0$

$$g_0^2\left(-\nu_0 + i\frac{\Gamma}{2}; \nu_0, \Lambda\right) = \frac{\pi^2}{\mu}\left(-i\frac{\Gamma}{2}\right)\left[\Lambda - \kappa \arctan\left(\frac{\Lambda}{\kappa}\right)\right]^{-1} \neq 0$$

$$X = \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1+(\Lambda/\kappa)^2}\right)^{-1}\right]^{-1}$$

Compositeness for two-channel case

$$V(k) = \begin{pmatrix} v(k) & v(k) \\ v(k) & v(k) \end{pmatrix}, \; v(k) = \frac{g_0^2}{\frac{k^2}{2\mu_1} - \nu_0}.$$

$$G(k) = \begin{pmatrix} G_1(k) & 0 \\ 0 & G_2(k) \end{pmatrix},$$

$$G_1(k) = -\frac{\mu_1}{\pi^2}\left[\Lambda + ik\arctan\left(-\frac{\Lambda}{ik}\right)\right],$$

$$G_2(k') = -\frac{\mu_2}{\pi^2}\left[\Lambda + ik'\arctan\left(-\frac{\Lambda}{ik'}\right)\right].$$

$$k = \sqrt{2\mu_1 E}, \quad k'(k) = \sqrt{2\mu_2(E - \Delta\omega)} = \sqrt{\frac{\mu_2}{\mu_1}k^2 - 2\mu_2\Delta\omega}.$$

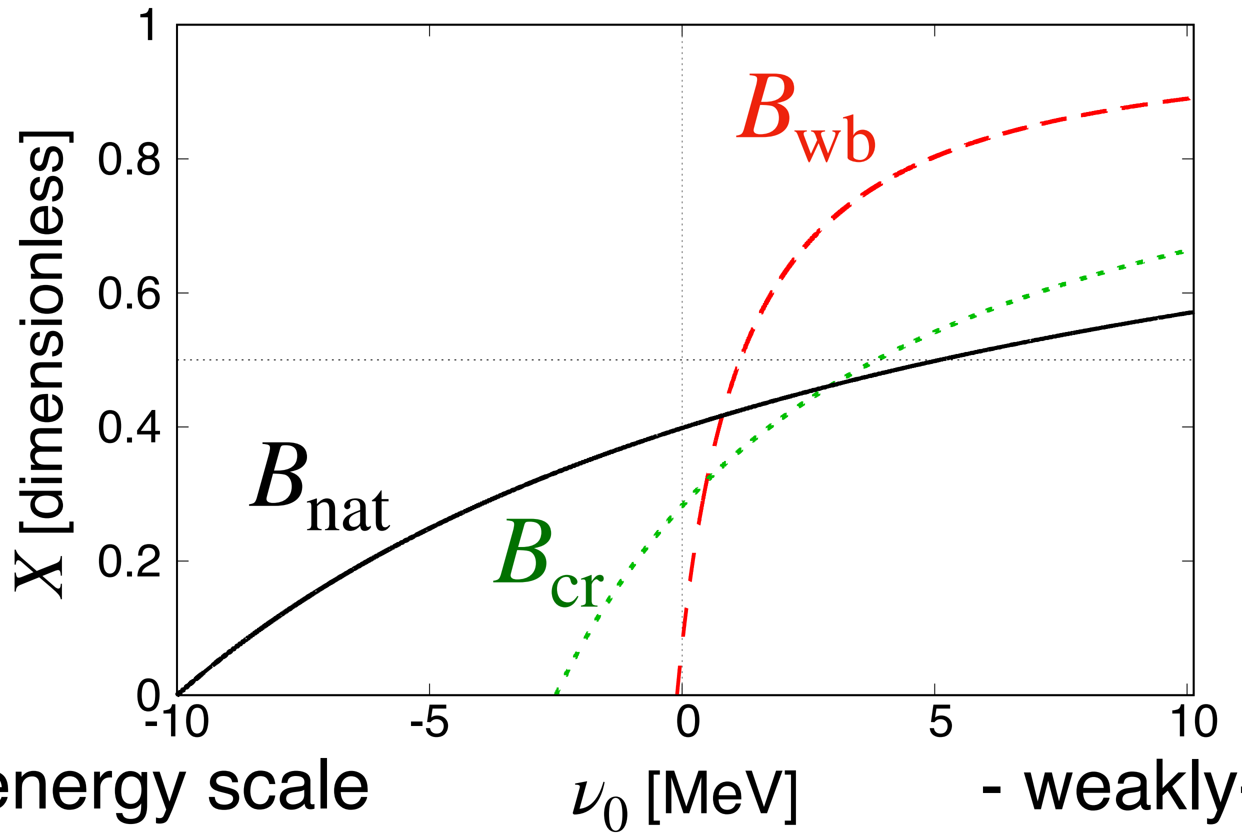
$\longrightarrow$

$$X_1 = \frac{G_1'}{(G_1' + G_2') - [v^{-1}]'},$$

$$X_2 = \frac{G_2'}{(G_1' + G_2') - [v^{-1}]'}.$$

Calculation

X & low-energy universality



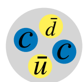
- natural energy scale

$$B_{\rm nat} = \Lambda^2/(2\mu)$$
$$\sim 10\ \text{MeV}$$

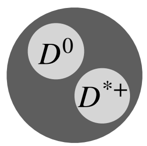
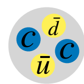
$X > 0.5$ for 25 % of ν_0

= elementary dominant

∵ **bare state origin**



$B_{\rm cr} \sim 2.5$ MeV

 :  = 1:1

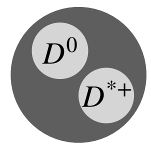
- weakly-bound state

$B_{\rm wb} = 0.1$ MeV

$X > 0.5$ for 88 % of ν_0

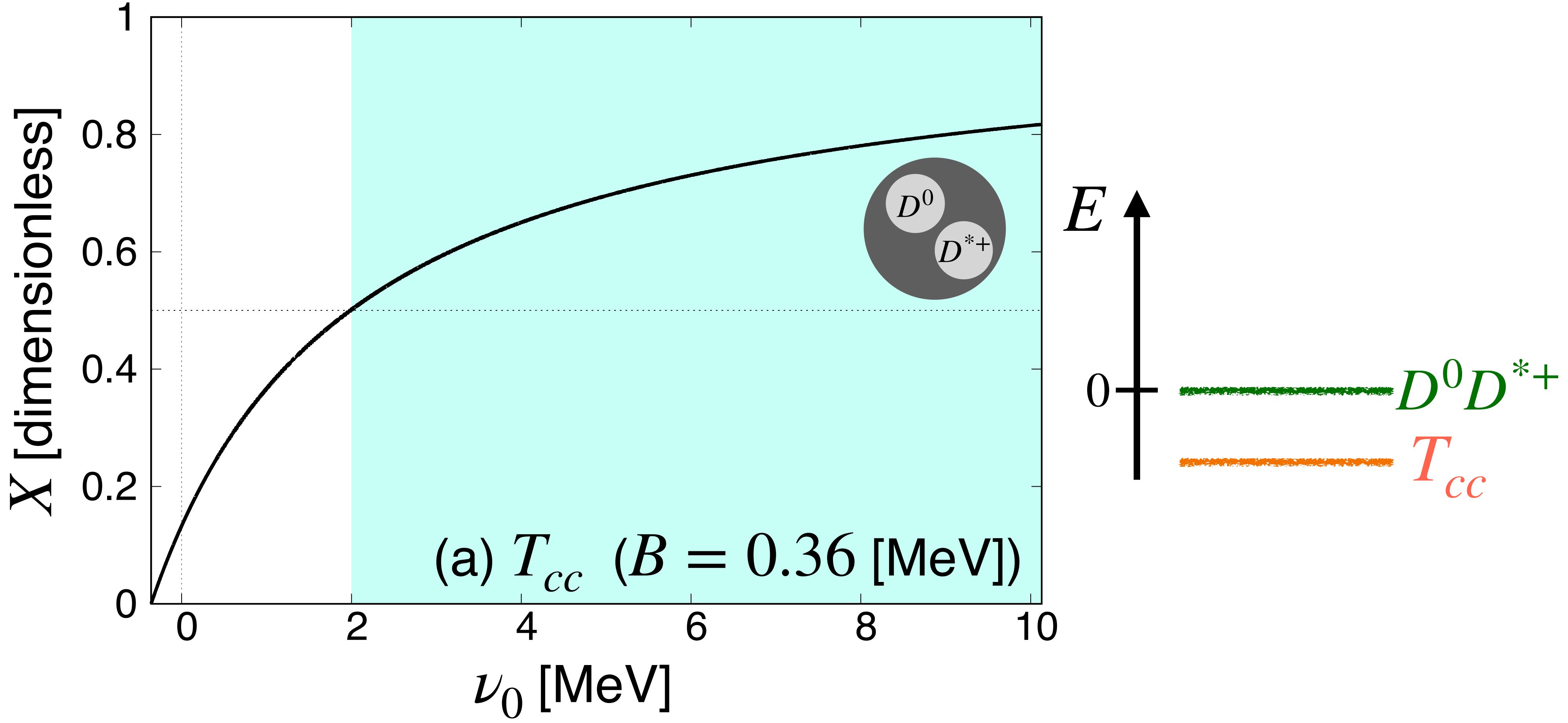
= composite dominant

∵ **low-energy universality !**



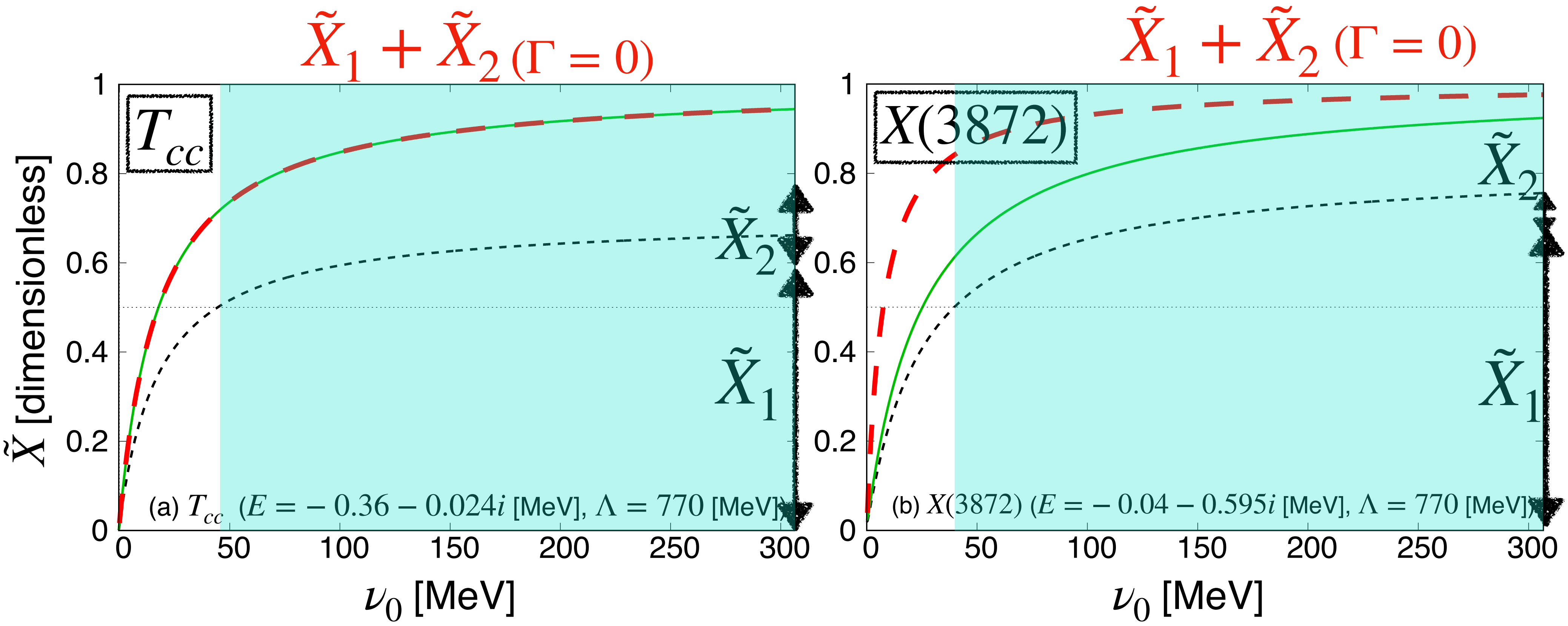
Application to T_{cc}

◉ single-channel



- $X > 0.5$ for 78 % of ν_0 = composite dominant
- fine tuning is necessary to realize $X < 0.5$

T_{cc} and $X(3872)$ with $\Gamma = 770$ MeV



- T_{cc} : $\tilde{X}_1 > 0.5$ for 85 % of ν_0 region
- $X(3872)$: $\tilde{X}_1 > 0.5$ for 87 % of ν_0 region