Nature of T_{cc} with effective field theory

<u>Tomona Kinugawa</u>

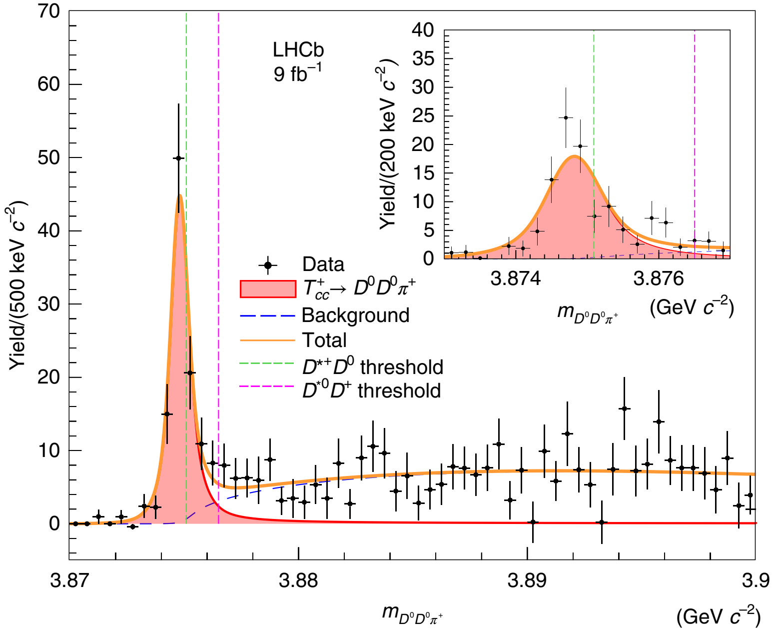
Tetsuo Hyodo

Department of Physics, Tokyo Metropolitan University

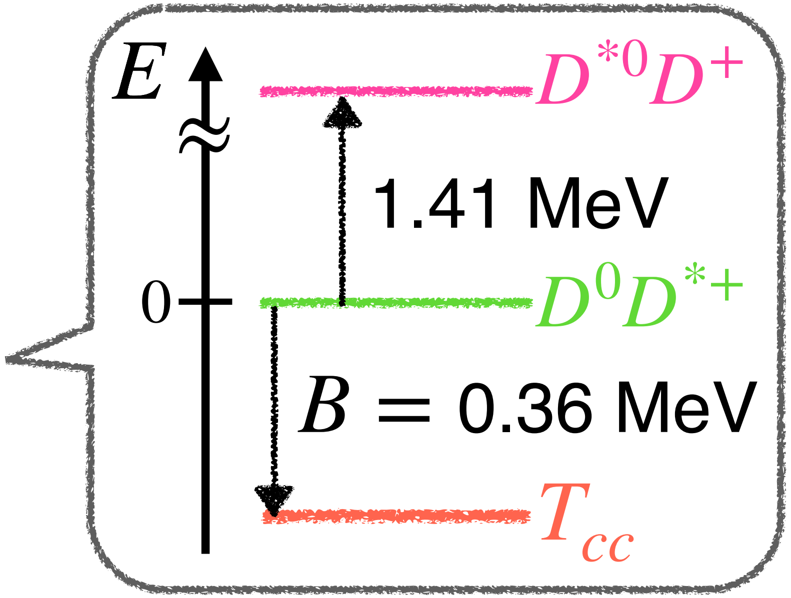
Jan. 30th, Physics of heavy quark and exotic hadrons 2023

Background

T_{cc} (in 2021)



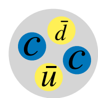
LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;
LHCb Collaboration, Nat. Commun **13** 3351 (2022).

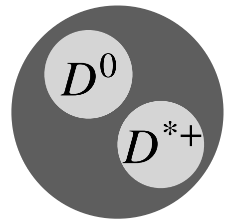


$T_{cc} \to D^0D^0\pi^+$ $(c\bar{u}c\bar{u}u\bar{d})$

→ minimum quark content is $cc\bar{u}\bar{d}$!

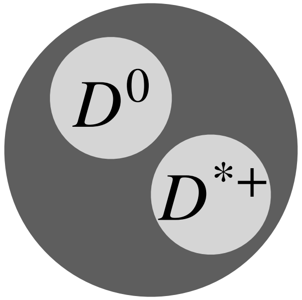
exotic hadron $\neq$ qqq or $q\bar{q}$ → multiquarks, hadronic molecules

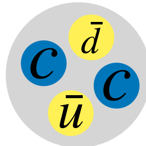




internal structure of T_{cc} ← effective field theory & compositeness

Compositeness





hadron wavefunction

$$|T_{cc}\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

compositeness **elementarity**

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).

※ $0 \leq X \leq 1$ ⟶ $X > 0.5 \Leftrightarrow$ **composite dominant**

◉ how to calculate?

S. Weinberg, Phys. Rev. 137, B672 (1965);
Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017);
T. Kinugawa and T. Hyodo, Phys. Rev. C 106, 015205 (2022).

1. weak-binding relation (model-independent)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

a_0 : scattering length

$R \equiv (2\mu B)^{-1/2}$, B : binding energy

$$R_{\rm typ} = \max\{R_{\rm int}, r_e, \cdots\}$$ ($R_{\rm int}$: interaction range, r_e : effective range)

2. model calculation❗ ⟵ this work

Model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).

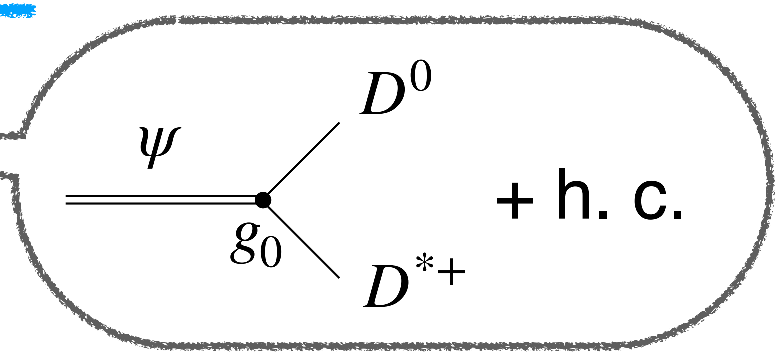
◉ single-channel resonance model

$$\mathscr{H}_{\text{free}} = \frac{1}{2m_{D^0}} \nabla D^{0\dagger} \cdot \nabla D^0 + \frac{1}{2m_{D^{*+}}} \nabla D^{*+\dagger} \cdot \nabla D^{*+} + \frac{1}{2m_\Psi} \nabla \psi^\dagger \cdot \nabla \psi + \nu_0 \psi^\dagger \psi,$$

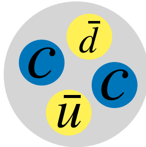
1.

$$\mathscr{H}_{\text{int}} = g_0(\psi^\dagger D^0 D^{*+} + D^{0\dagger} D^{*+\dagger} \psi) .$$

2.



1. single-channel scattering
2. coupling with compact four-quark state ψ $(cc\bar{u}\bar{d})$



◉ scattering amplitude

$$V = \frac{g_0^2}{E - \nu_0}, \quad G = -\frac{\mu}{\pi^2}\left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right)\right].$$

Λ : cutoff

$$\xrightarrow[T = \frac{1}{V^{-1} - G}]{} \quad f(k) = -\frac{\mu}{2\pi}\left[\frac{\frac{k^2}{2\mu} - \nu_0}{g_0^2} + \frac{\mu}{\pi^2}\left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right)\right]\right]^{-1}.$$

Model parameters and scales

- cutoff Λ : e.g. 140 MeV = m_π (π exchange)

- coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \frac{\pi^2}{\mu}(B + \nu_0)\Big[\Lambda - \kappa \arctan(\Lambda/\kappa)\Big]^{-1}$

  ∵ bound state condition $f^{-1} = 0$ $\kappa = \sqrt{2\mu B}\,.$

- energy of bare 4-quark state ν_0

  varied in the region : $-B \leq \nu_0 \leq \Lambda^2/(2\mu)$

  ∵ to have $g_0^2 \geq 0$ & applicable limit of EFT

- typical energy scale : $E_{\mathrm{typ}} = \Lambda^2/(2\mu)$

→ we calculate with fixed B and dimensionless quantities with Λ and E_{typ}

Calculation

◉ compositeness X

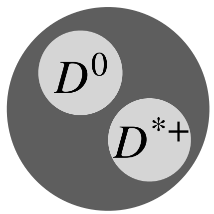
scattering amplitude : $T = \dfrac{1}{V^{-1} - G}$ Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

$$\longrightarrow X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1+\left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}.$$

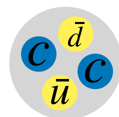
- ν_0 region : $-B/E_{\rm typ} \leq \nu_0/E_{\rm typ} \leq 1$

$X > 0.5$



or

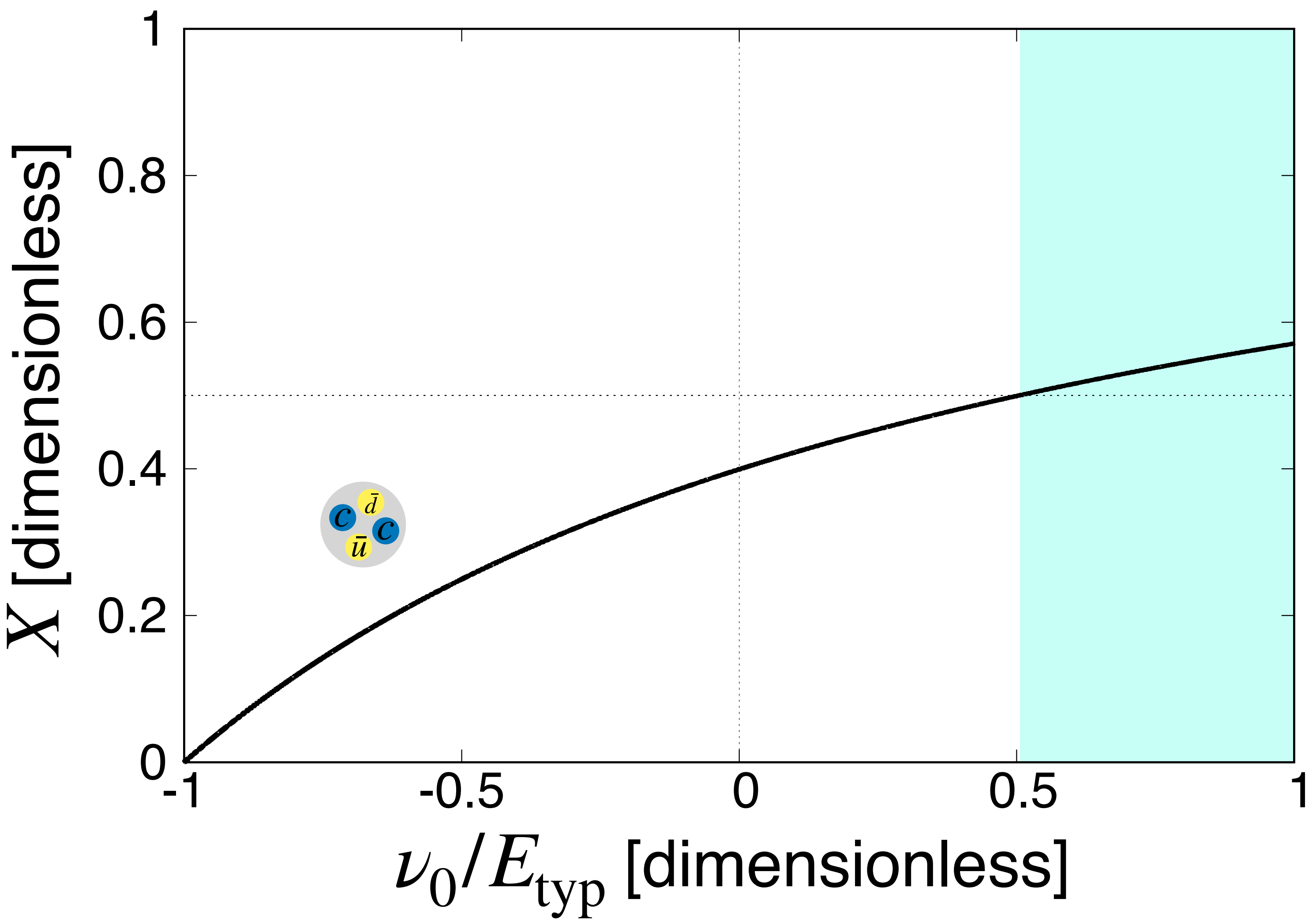
$X < 0.5$

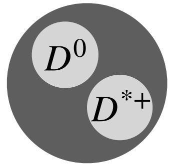


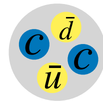
compositeness X as a function of ν_0

$\longrightarrow$ internal structure of bound state?

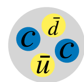
◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = E_{\rm typ}$



$X > 0.5$: 

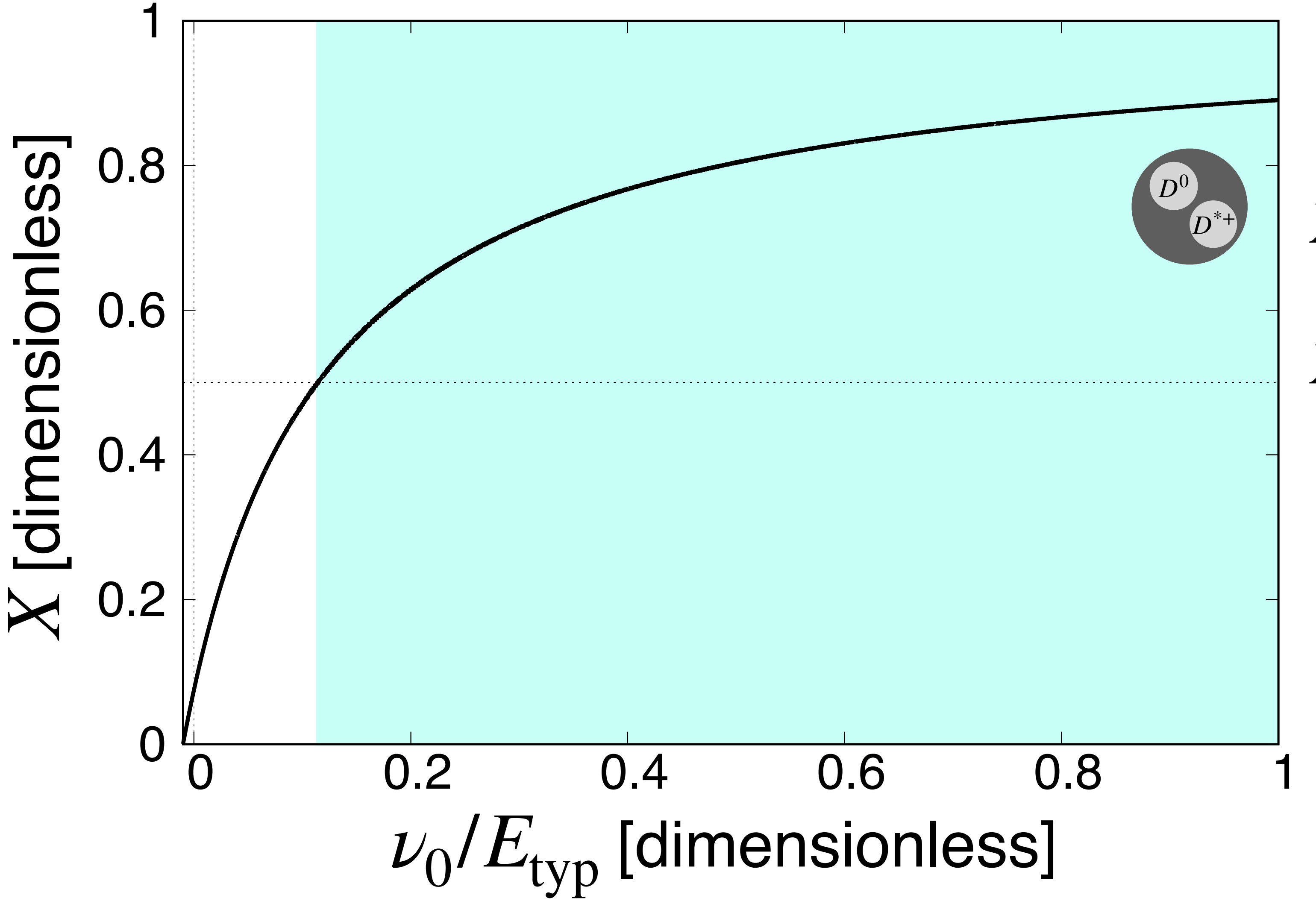
$X < 0.5$: 

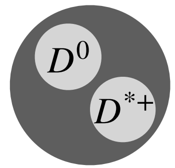
- typical energy scale : $B = E_{\rm typ} = \Lambda^2/(2\mu)$

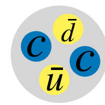
- $X > 0.5$ only for 25 % of ν_0 = elementary dominant 

∵ bare state origin

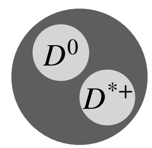
◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = 0.01E_{\rm typ}$



$X > 0.5$: 

$X < 0.5$: 

- weakly-bound state : $B = 0.01E_{\rm typ}$
- $X > 0.5$ for 88 % of ν_0 = composite dominant



∵ **low-energy universality !**

Effect of decay

◉ introducing decay effect

- formally : introducing decay channel in lower energy region than threshold ch.

  ⟶ eigenenergy becomes complex

- effectively : coupling const. $g_0 \in \mathbb{C}$! ⟵ this work

$$E = -B \longrightarrow E = -B - i\Gamma/2$$

compositeness

$$X \in \mathbb{R} \longrightarrow X \in \mathbb{C}$$

$$\tilde{X} = \frac{|X|}{|X| + |1 - X|}$$

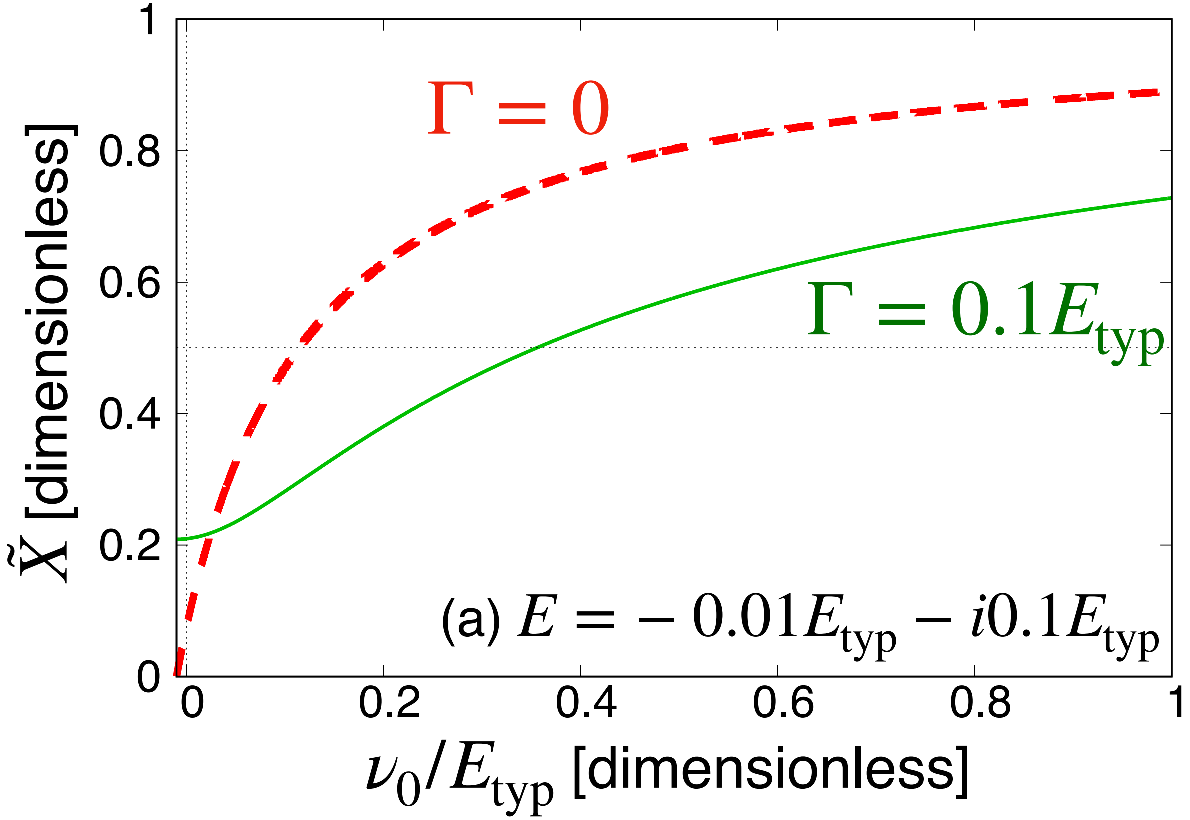
T. Sekihara, *et. al.*, PRC 93, 035204 (2016).

- low-energy universality with decay effect

  $X \sim 1$ (threshold channel)

Effect of decay

◉ compositeness $\tilde{X}$



- $\tilde{X}$ is suppressed by $\Gamma \neq 0$

  ∵ threshold ch. component (X) decreases with inclusion of decay ch. component ($1-X$)

- finite Γ induces deviation from expectation of low-energy universality ($X \sim 1$)

Effect of coupled channel

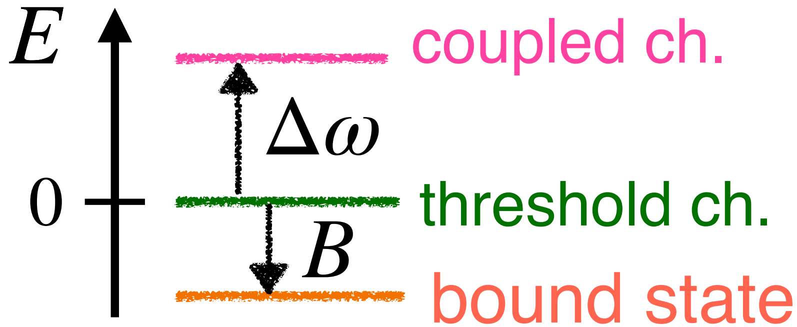
◉ introducing coupled channel $\Psi_1 \Psi_2$

$$|\Psi\rangle = \sqrt{X_1}\,|\text{threshold ch}\rangle + \sqrt{X_2}\,|\text{coupled ch}\rangle + \sqrt{1-(X_1+X_2)}\,|\text{others}\rangle$$

$X_1 \longleftrightarrow X$ for single ch. case (threshold ch component)

$X_2 \longleftrightarrow 1-X$ for single ch. case (other ch component)

- threshold energy difference $\Delta\omega$



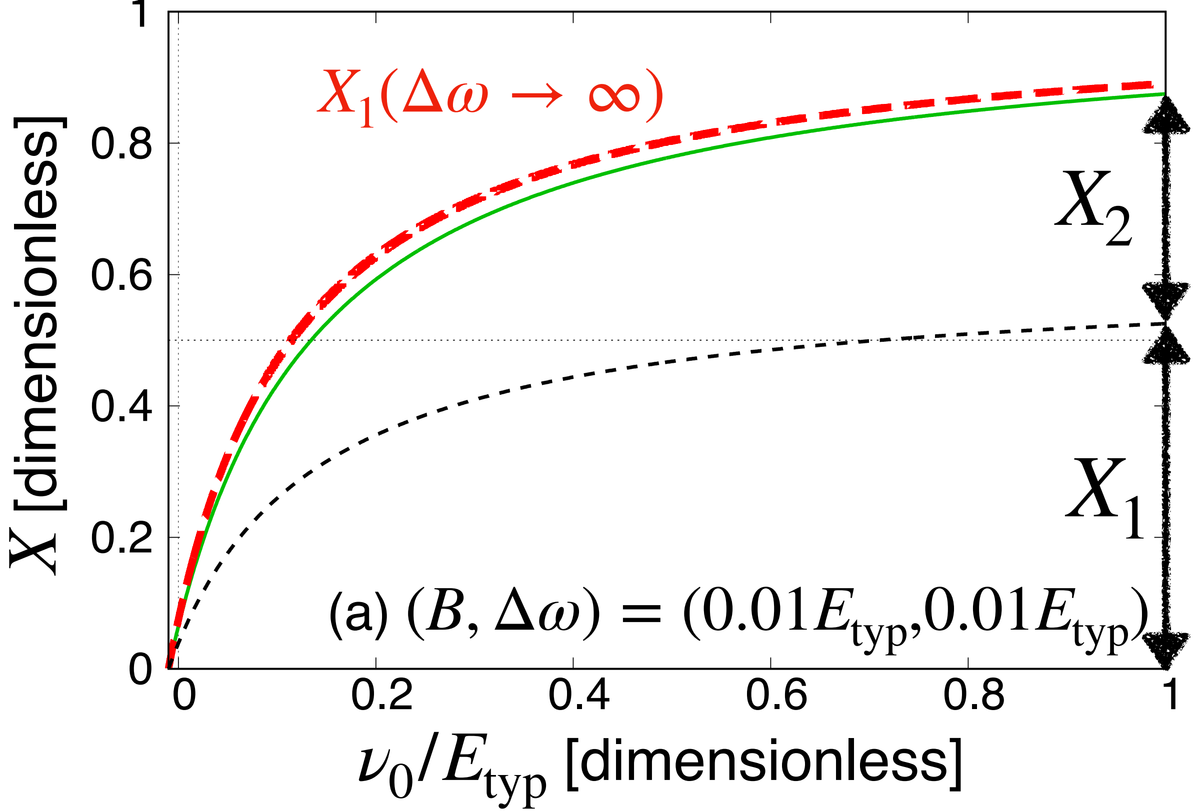
- low-energy universality for multichannel case

$X_1 \sim 1$ (threshold channel)

$X_2 \sim 0$ (other channel)

Effect of coupled channel

◉ compositeness X_1 and X_2



- X_1 is suppressed by $\Delta\omega$

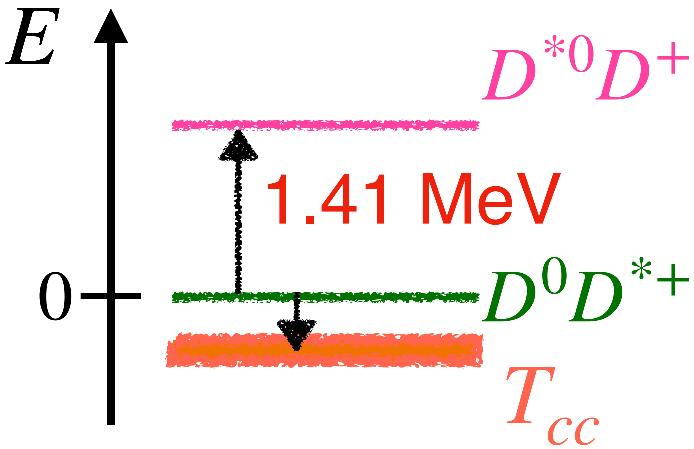
  ∵ threshold ch. component (X_1) decreases with inclusion of coupled ch. component (X_2)

- finite $\Delta\omega$ induces deviation from expectation low-energy universality ($X_1 \sim 1$)

Application to T_{cc} and $X(3872)$

◉ exotic hadron ⟵ decay and coupled channel

T_{cc}

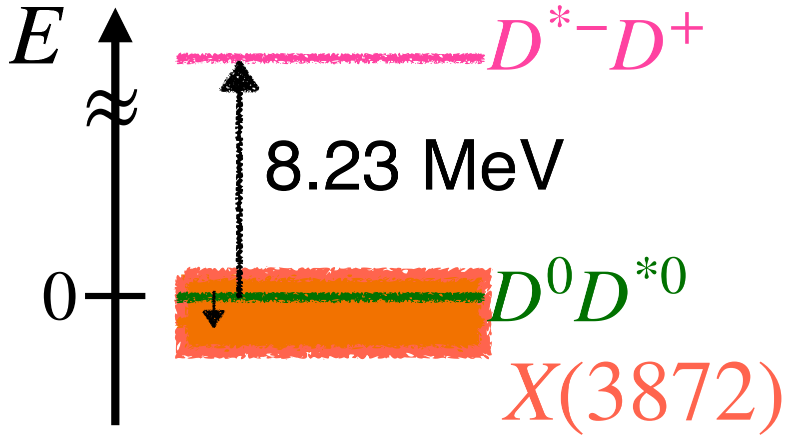


$B = 0.36$ MeV

$\Gamma = 0.048$ MeV

LHCb Collaboration, Nat. Commun **13** 3351 (2022).

$X(3872)$



$B = 0.04$ MeV

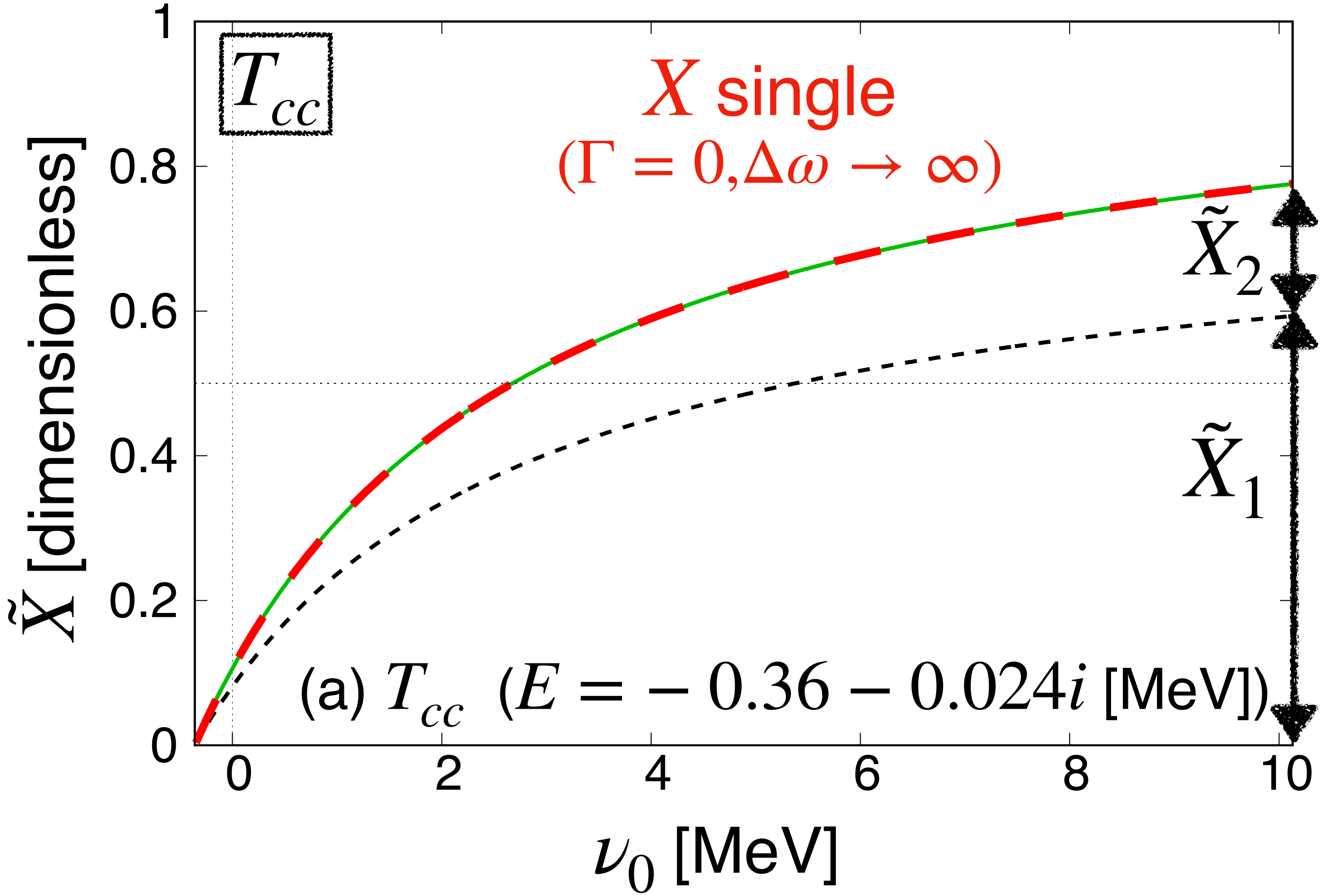
$\Gamma = 1.19$ MeV

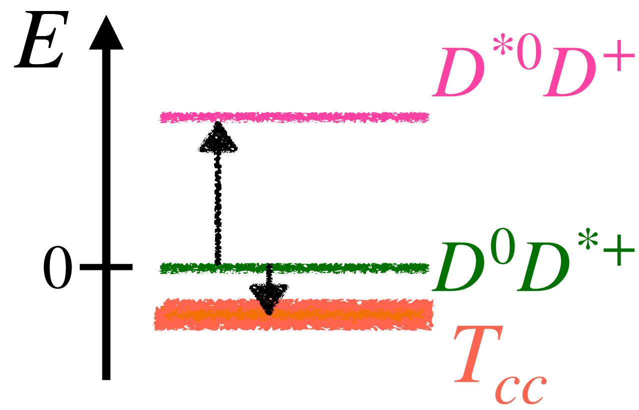
PDG

◉ compositeness T. Sekihara, *et. al.*, PRC 93, 035204 (2016).

$$\tilde{X}_j = \frac{|X_j|}{\sum_j |X_j| + |Z|}, \quad (j = 1,2)$$

Application to T_{cc} and $X(3872)$





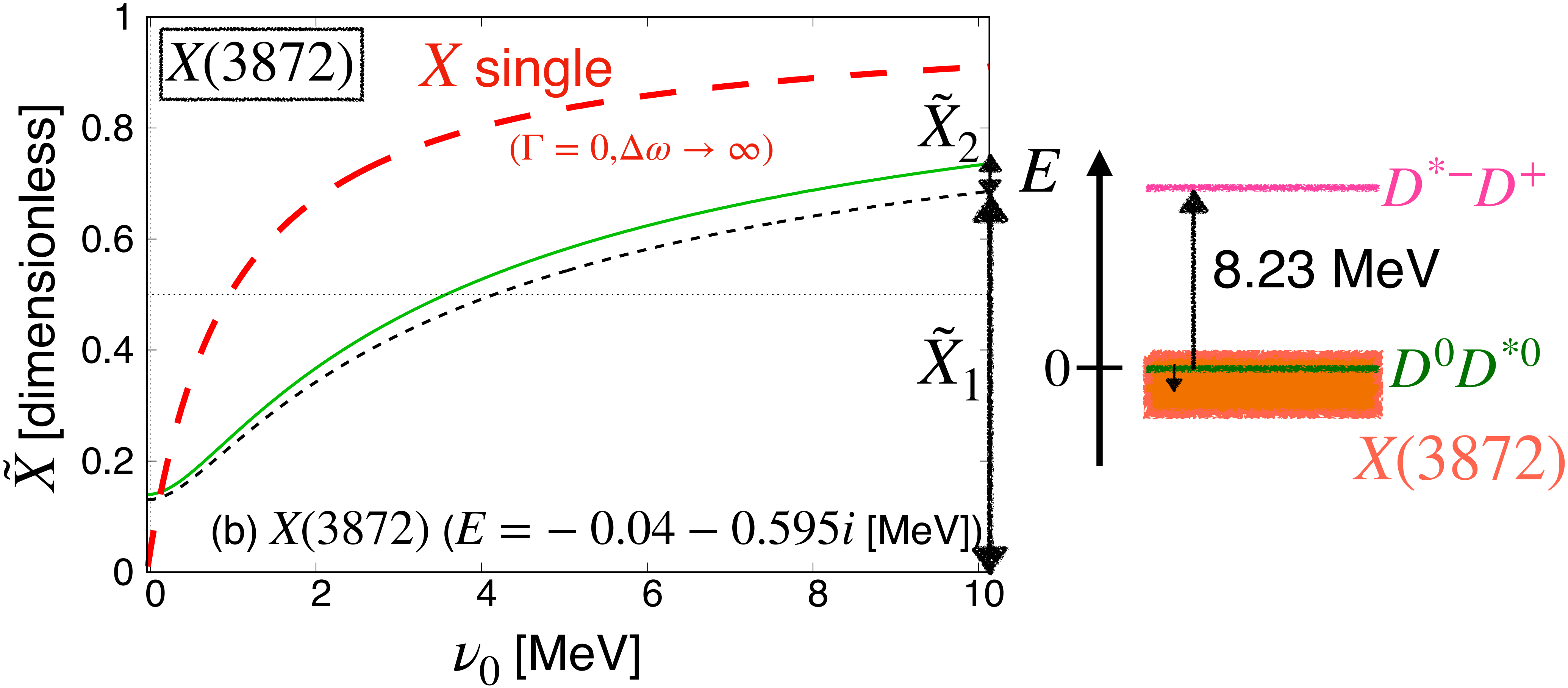
- $\tilde{X}_2$ is not negligible

  $\because$ coupled ch. effect contribution (small $\Delta\omega$)

- difference of X(single) and $\tilde{X}_1 + \tilde{X}_2$ is too small $\because \Gamma \ll B$

  → We can neglect decay contribution

Application to T_{cc} and $X(3872)$



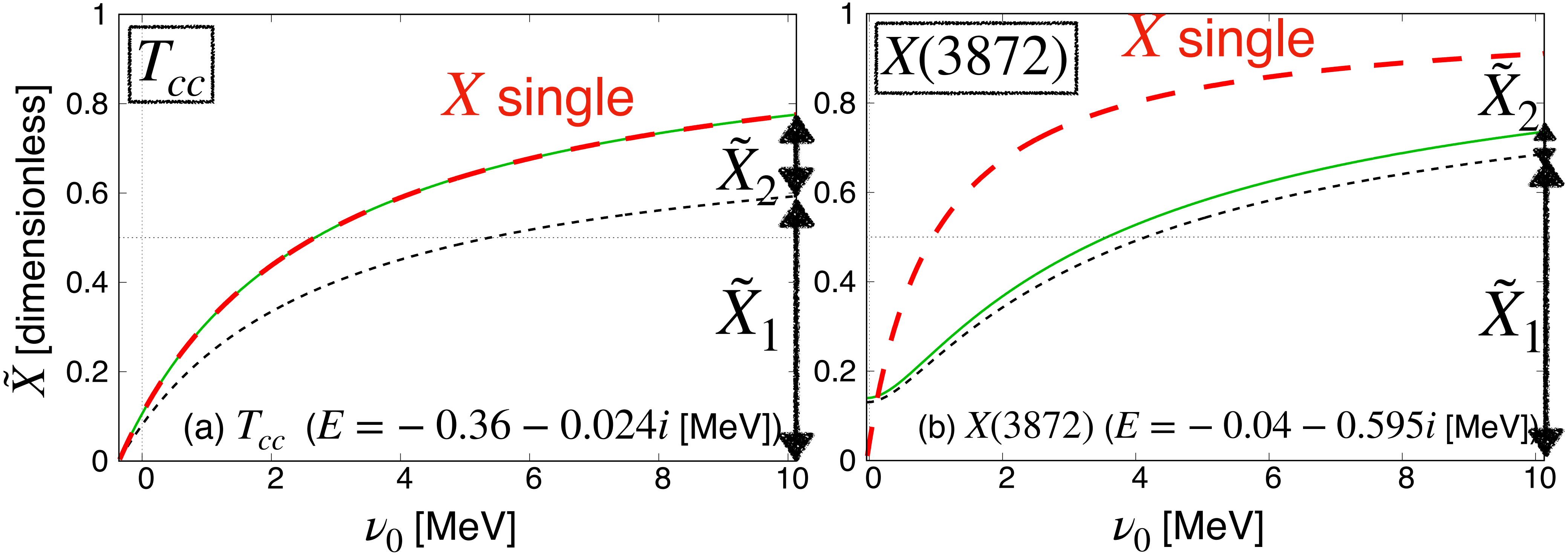
- difference of X(single) and $\tilde{X}_1 + \tilde{X}_2$ is large

  ∵ large decay width contribution

- $\tilde{X}_2$ is much smaller than $\tilde{X}_1$

  ⟶ coupled ch. effect is negligible

Application to T_{cc} and $X(3872)$



- coupled ch. effect is more important for T_{cc} than $X(3872)$
- decay effect is more important for $X(3872)$ than T_{cc}

Summary

- internal structure of exotic hadrons ← EFT & compositeness
- model with bare 4-quark state coupled to the scattering state
- shallow bound state is composite dominant
  even from bare state

  ∵ low-energy universality
- decay and coupled channel effects are introduced

  → both decay and coupled ch. effects contribute to deviation from low-energy universality

- X of T_{cc} and $X(3872)$ are calculated with decay and coupled ch. effects

T_{cc} : important coupled ch. effect with negligible decay effect
$X(3872)$: important decay effect with negligible coupled ch. effect

Nature of T_{cc} with effective field theory

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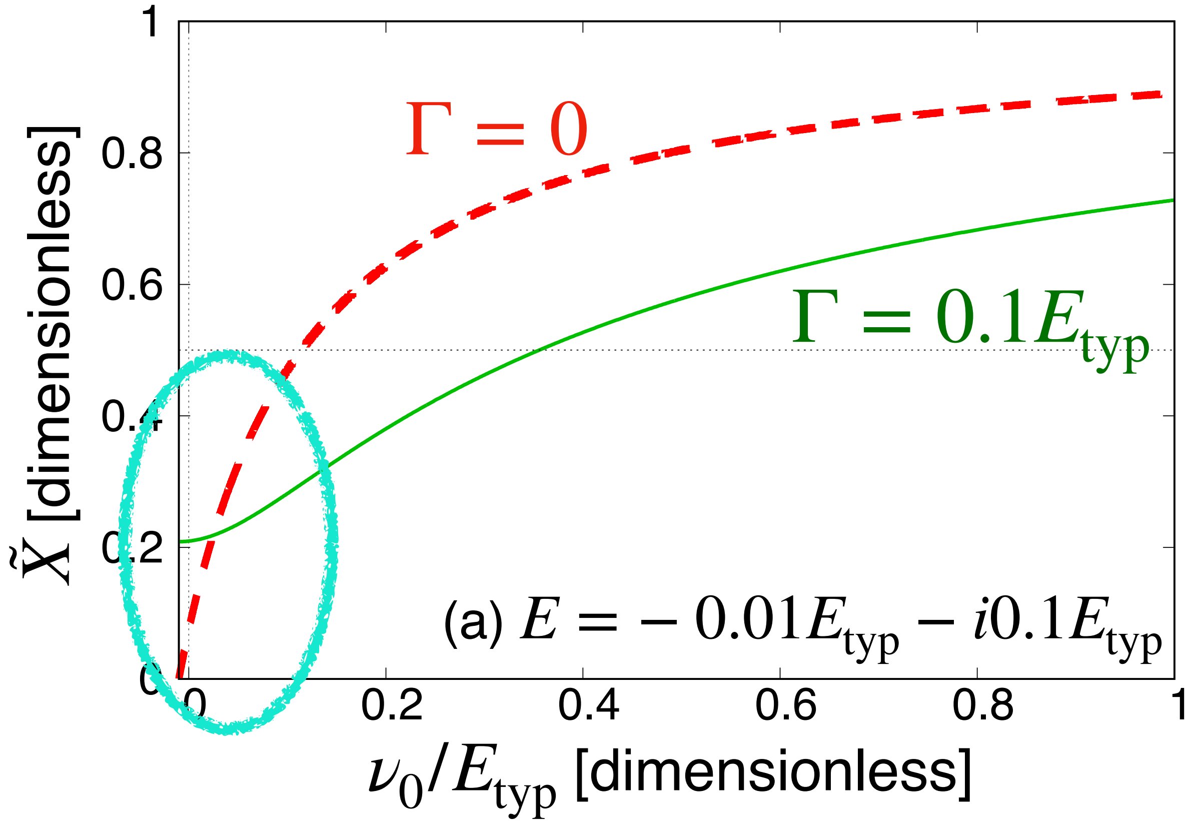
Effect of decay



- $X \neq 0$ with $\Gamma \neq 0$
  $\because\ g_0 \neq 0$ at $\nu_0 = -B$

c.f. $g_0 = 0$ at $\nu_0 = -B$
with $\Gamma = 0$

$$g_0^2\left(-\nu_0 + i\frac{\Gamma}{2};\nu_0,\Lambda\right) = \frac{\pi^2}{\mu}\left(-i\frac{\Gamma}{2}\right)\left[\Lambda - \kappa \arctan\left(\frac{\Lambda}{\kappa}\right)\right]^{-1} \neq 0$$

$$X = \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1+\left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}$$

Compositeness for two-channel case

$$V(k) = \begin{pmatrix} v(k) & v(k) \\ v(k) & v(k) \end{pmatrix}, \; v(k) = \frac{g_0^2}{\frac{k^2}{2\mu_1} - \nu_0} .$$

$$G(k) = \begin{pmatrix} G_1(k) & 0 \\ 0 & G_2(k) \end{pmatrix},$$

$$G_1(k) = -\frac{\mu_1}{\pi^2}\left[\Lambda + ik\arctan\left(-\frac{\Lambda}{ik}\right)\right],$$

$$G_2(k') = -\frac{\mu_2}{\pi^2}\left[\Lambda + ik'\arctan\left(-\frac{\Lambda}{ik'}\right)\right].$$

$$k = \sqrt{2\mu_1 E}, \quad k'(k) = \sqrt{2\mu_2(E - \Delta\omega)} = \sqrt{\frac{\mu_2}{\mu_1}k^2 - 2\mu_2\Delta\omega} .$$

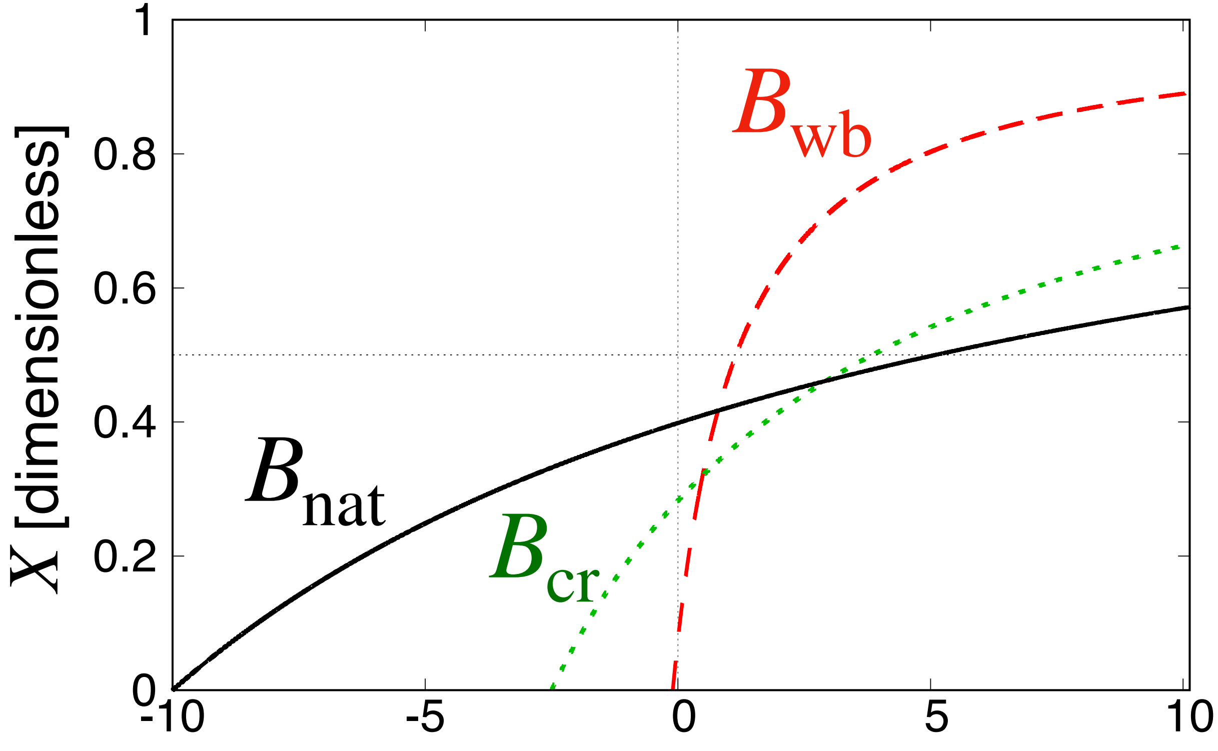
$\longrightarrow$

$$X_1 = \frac{G_1'}{(G_1' + G_2') - [v^{-1}]'},$$

$$X_2 = \frac{G_2'}{(G_1' + G_2') - [v^{-1}]'} .$$

Calculation

X & low-energy universality



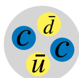
- natural energy scale

$$B_{\rm nat} = \Lambda^2/(2\mu)$$
$$\sim 10 \text{ MeV}$$

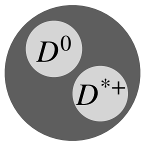
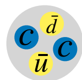
$X > 0.5$ for 25 % of ν_0

= elementary dominant

∵ **bare state origin**



$B_{\rm cr} \sim 2.5$ MeV

 :  = 1:1

⟷

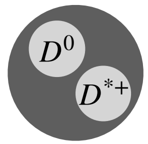
- weakly-bound state

$B_{\rm wb} = 0.1$ MeV

$X > 0.5$ for 88 % of ν_0

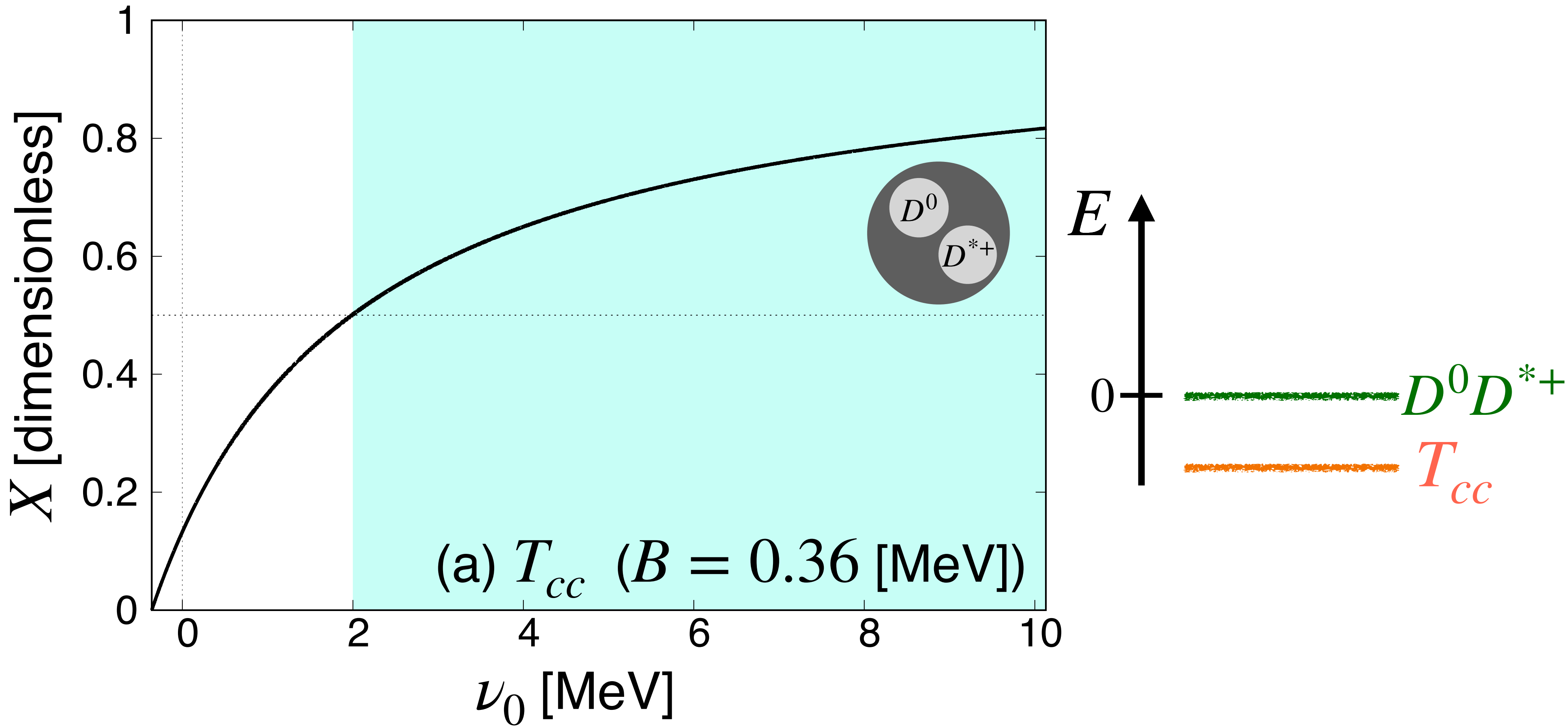
= composite dominant

∵ **low-energy universality !**



Application to T_{cc}

◉ single-channel



- $X > 0.5$ for 78 % of ν_0 = composite dominant
- fine tuning is necessary to realize $X < 0.5$