Compositeness of near-threshold states with Coulomb plus short range interaction


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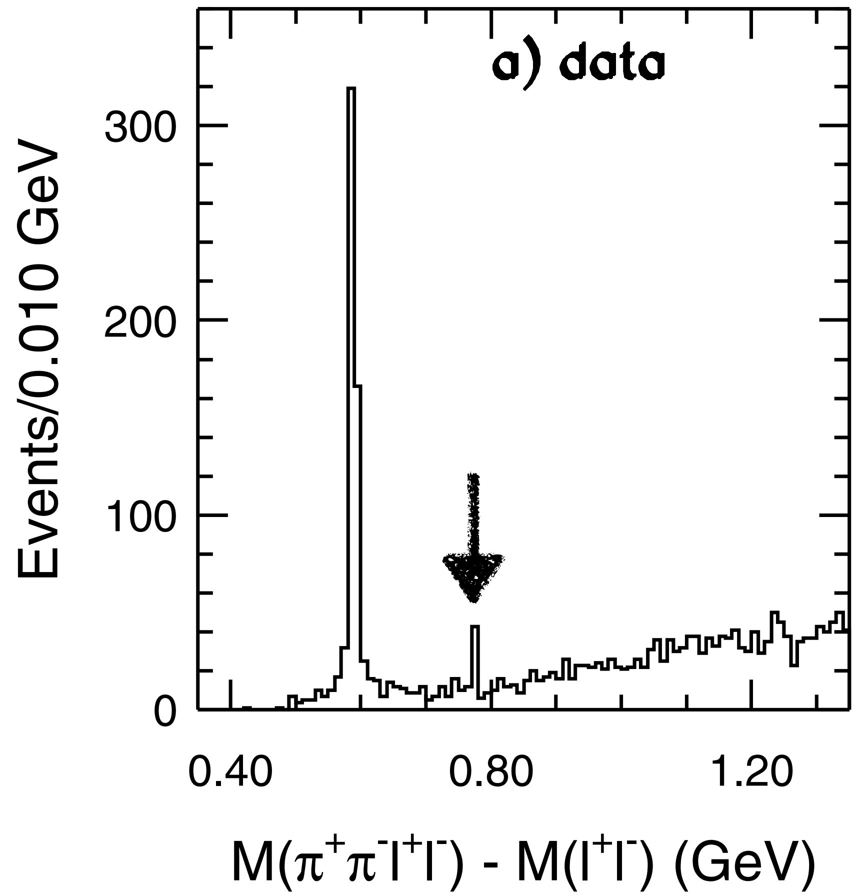
September 7th

対称性と有効模型で切り拓くクォーク・ハドロン物理の最前線

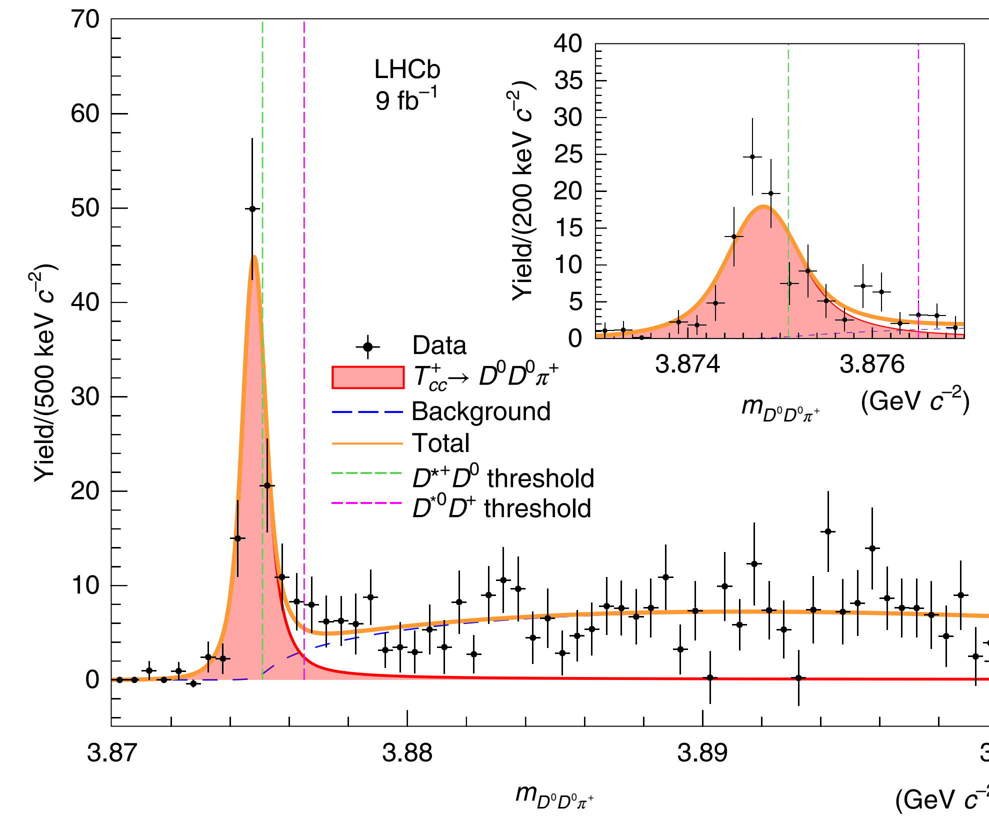
Near-threshold exotic hadrons

$X(3872) \to \pi^+\pi^- J/\psi$



S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

$T_{cc} \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$



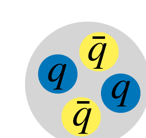
LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

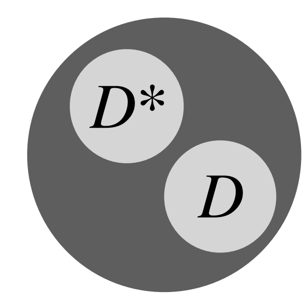
- internal structure?

exotic hadron

$\neq \ qqq$ or $q\bar{q}$ → multiquarks, hadronic molecules



multiquarks

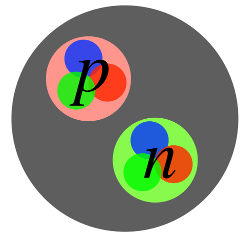


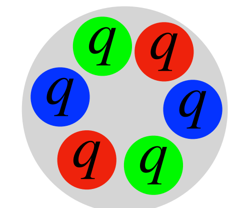
hadronic molecules

Compositeness

S. Weinberg, Phys. Rev. 137, 672–678 (1965);
T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).

◉ definition





wavefunction

$$|\Psi\rangle = \sqrt{X}\,|\text{composite}\rangle + \sqrt{1-X}\,|\text{non composite}\rangle$$

compositeness (X) **elementarity** ($1-X$)

※ $0 \leq X \leq 1$ ⟶ $X > 0.5$ ⇔ **composite dominant**

$X < 0.5$ ⇔ **elementary dominant**

- **quantitative** analysis of internal structure

deuteron is not an elementary particle S. Weinberg, Phys. Rev. 137, 672–678 (1965).

$f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016); T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.

$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

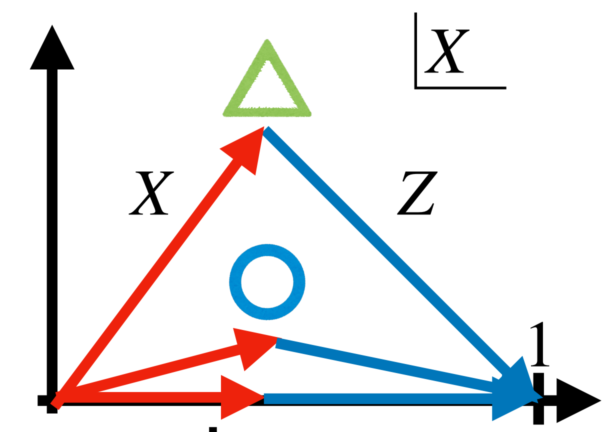
Compositeness of resonances

- interpretation for complex X of resonances?

$X \in \mathbb{C}$ and $X + Z = 1$

- If Im X is large, it seems that reasonable interpretation is impossible △

complex X plane



◉ our proposal

T. Berggren, Phys. Lett. B 33, 547 (1970).

i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$
ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$
iii) $\mathscr{Y}$: probability of uncertain identification

◉ conditions for sensible interpretation

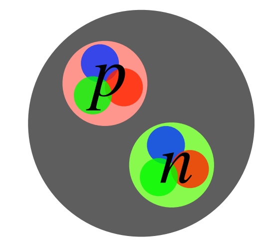
- normalization : $\mathscr{X} + \mathscr{Y} + \mathscr{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathscr{X} \to X$, $\mathscr{Z} \to Z$ and $\mathscr{Y} \to 0$

$\mathscr{Y}$ characterizes uncertainty of resonance

T. Kinugawa and T. Hyodo arXiv:2403.12635 [hep-ph].

Near-threshold states

◉ near-threshold states with short range interaction



- at threshold ($E = 0$)

  completely composite ($X = 1$)

  ∵ low-energy universality $|a_s| \to \infty$

  T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

- near-threshold bound states

  ($E \neq 0$, but small **negative**)

  composite dominant ($X \sim 1$)

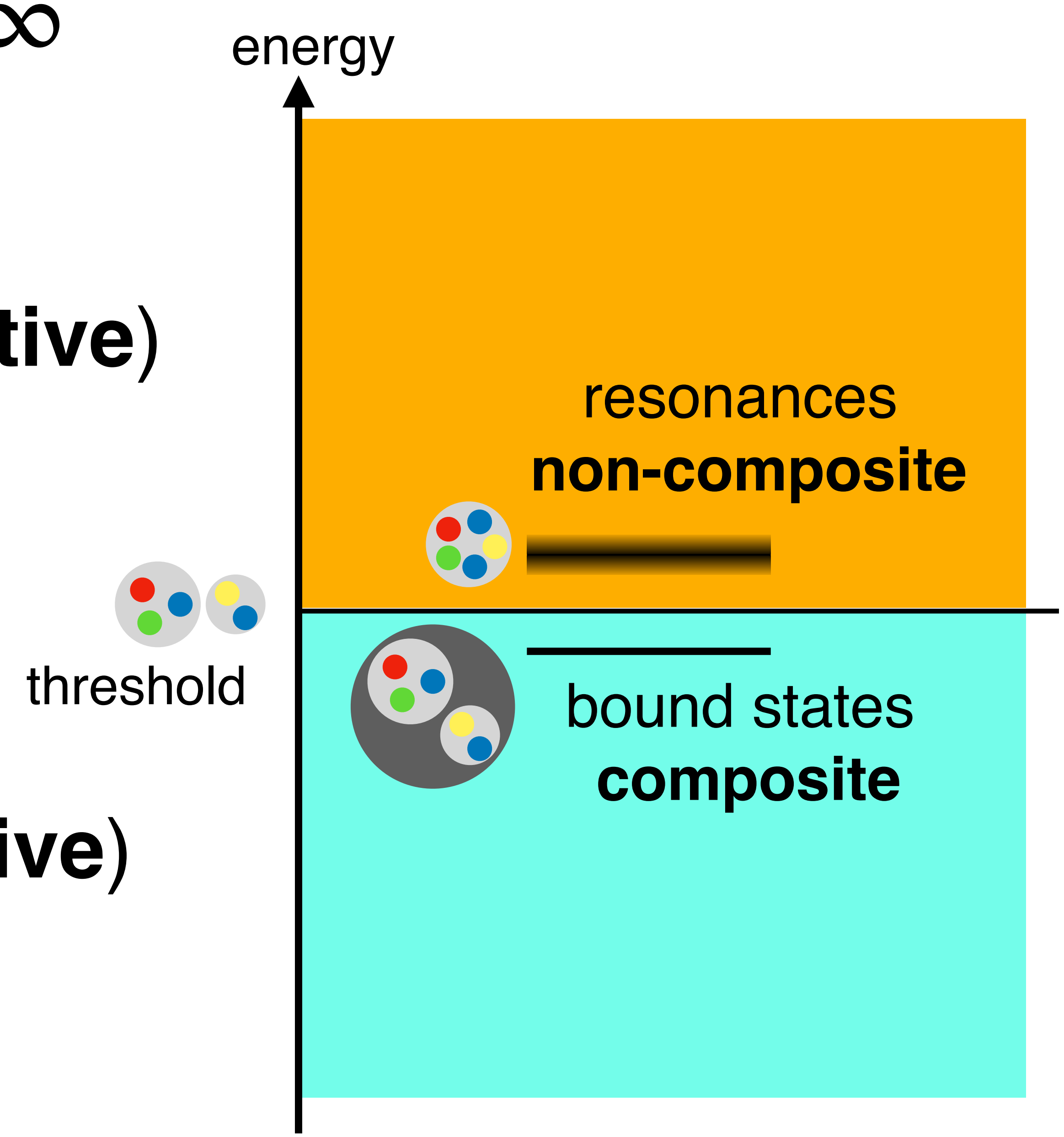
  C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014);
  T. Kinugawa and T. Hyodo Phys. Rev. C 109 , 045205 (2024).

- near-threshold resonances

  ($E \neq 0$, but small **positive**)

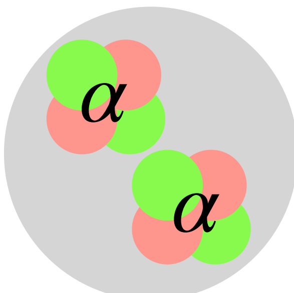
  non-composite dominant ($\mathscr{X} \sim 0$)

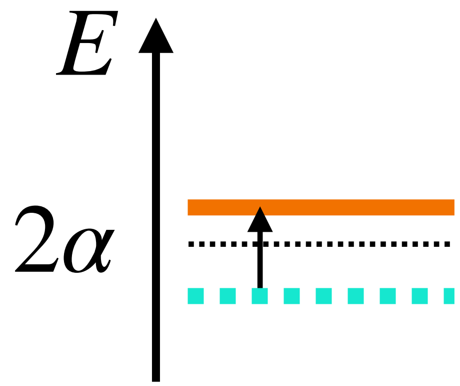
  T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].



Coulomb + short range systems

- ^{8}Be nuclei J. Hiura, and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 52, 25 (1972).



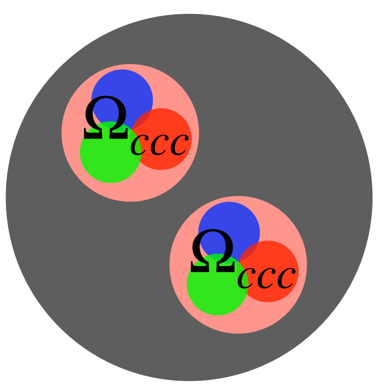


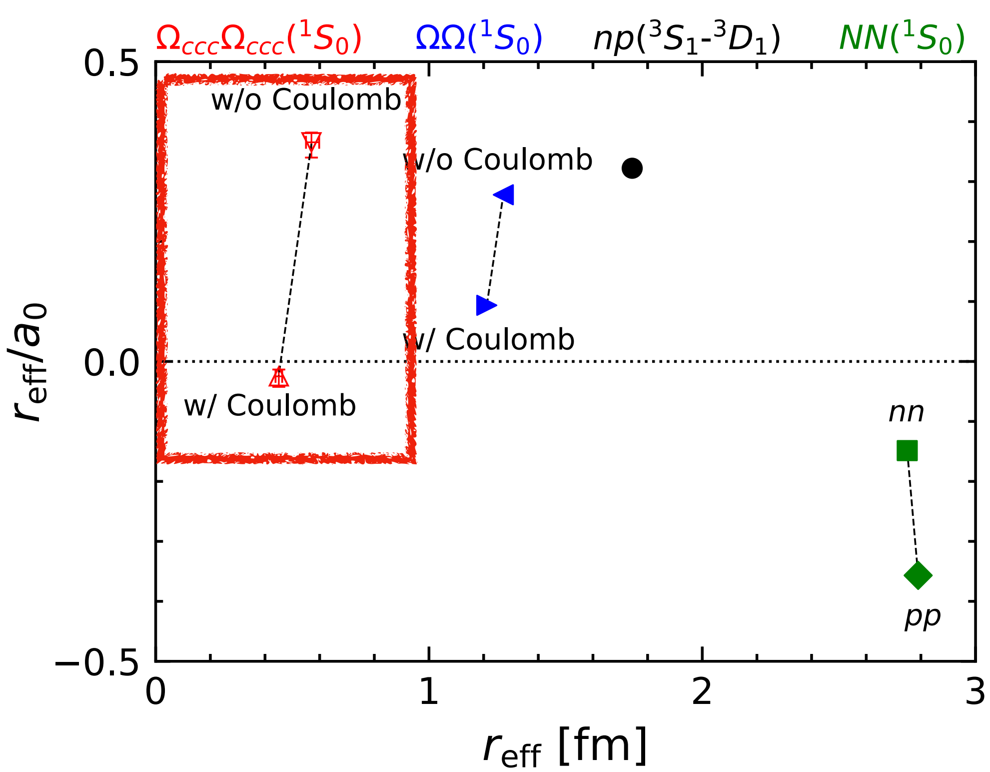
resonance (w/ Coulomb)

↑

bound (w/o Coulomb)

- $\Omega_{ccc}\,\Omega_{ccc}$ (HAL QCD) Y. Lyu, H.Tong, *et al*. [HAL QCD Coll.], Phys. Rev. Lett. 127 (2021) 072003.





resonance (w/ Coulomb)

↑

bound (w/o Coulomb)

- $\Xi^{-}\alpha$: Coulomb assisted bound state ? E. Hiyama, M. Isaka, T. Doi, and T. Hatsuda, Phys. Rev. C 106, 064318 (2022).

→ Coulomb is important for near-threshold states!

Coulomb + short range systems

◉ Coulomb + short range interaction

H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

R. Oppenheim Berger and Larry Spruch, Phys. Rev. 138, B1106-B1115 (1965).

W. Domcke, Atom. Mol. Phys. 16, 359 (1983).

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809, 171 (2008).

C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798, 135016 (2019).

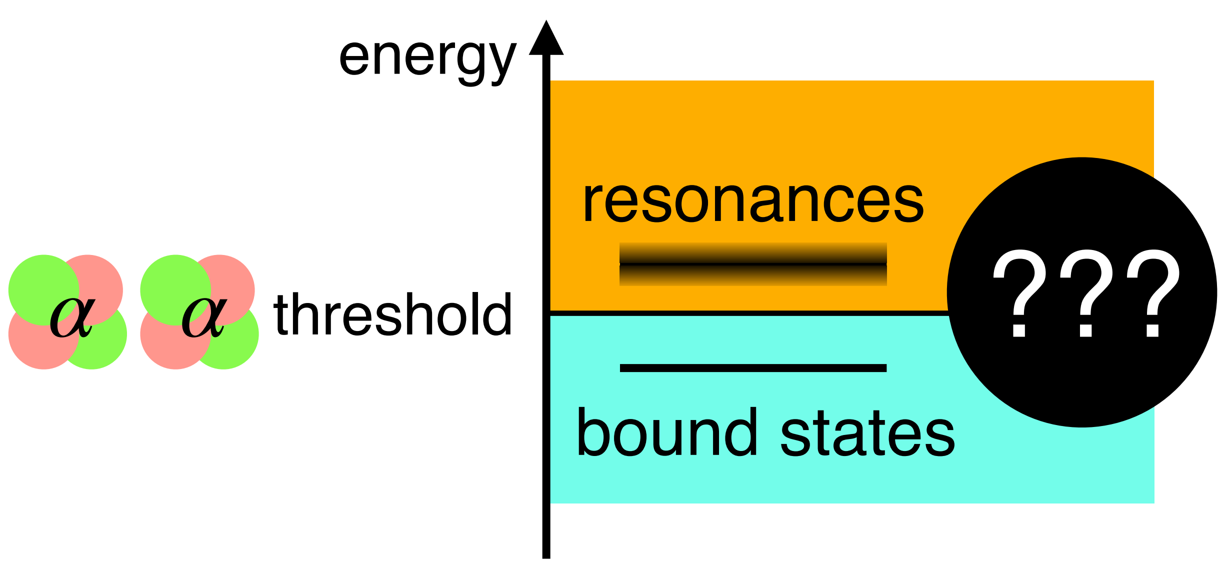
S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

- we **cannot** expand scattering amplitude $f(k)$ in terms of k^2

$$\frac{1}{f(k)} = -\frac{1}{a_{\mathrm{s.r.}}} + \frac{r_{\mathrm{s.r.}}}{2}k^2 + \mathcal{O}(k^4) - ik$$

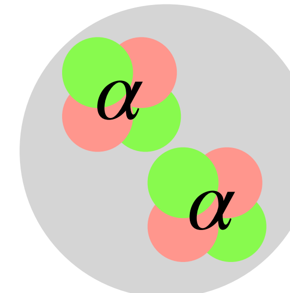
→ different from short range interaction

◉ nature of near-threshold state with Coulomb + short range interaction?

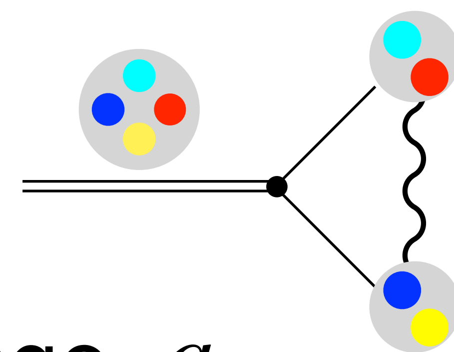


This work

near-threshold bound states & resonances
with **Coulomb + short range** interaction



<u>framework : model with Feshbach method</u>



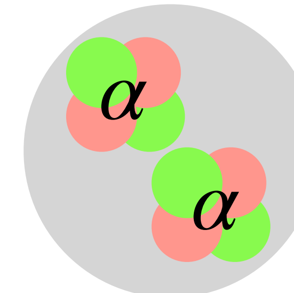
- bare state which couples to Coulomb scattering
- Coulomb scattering length, Coulomb effective range, a_B

<u>numerical calculations & discussion</u>

- investigate pole trajectory
- analyze internal structure with compositeness
- study universal nature of near-threshold states

This work

near-threshold bound states & resonances
with **Coulomb + short range** interaction



<u>framework : model with Feshbach method</u>

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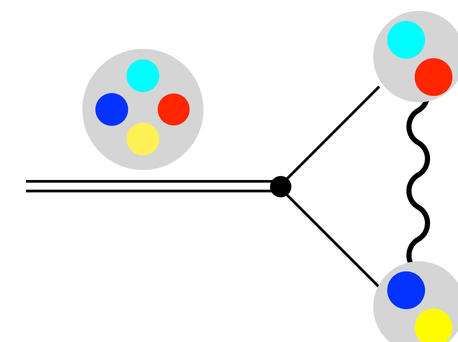

- Coulomb scattering length, Coulomb effective range, a_B

<u>numerical calculations & discussion</u>

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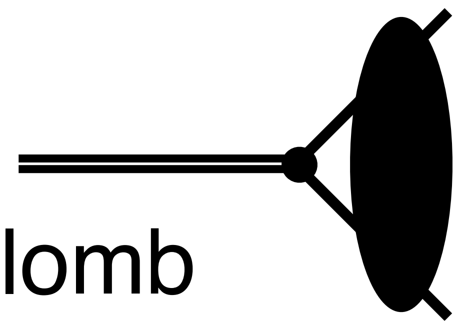
- study universal nature of near-threshold states

Coulomb+short range model

◉ Hamiltonian

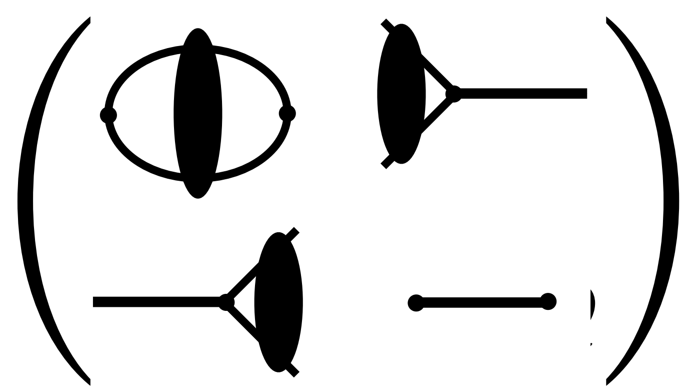
W. Domcke, Atom. Mol. Phys. 16 359 (1983). C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798, 135016

Q channel
⇔ b.s. w/o Coulomb



P channel
⇔ scattering w/ Coulomb

$$\hat{H} = \begin{pmatrix} \hat{H}_{PP} & \hat{H}_{PQ} \\ \hat{H}_{QP} & \hat{H}_{QQ} \end{pmatrix} =$$



H. Feshbach, Annals Phys. 19 287-313 (1962).

◉ pole condition

H. A. Bethe, Phys. Rev. 76, 38-50 (1949).

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B}\left[\log(-ia_B k) + \psi\left(1 + \frac{i}{a_B k}\right)\right] = 0$$

◉ compositeness X

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

$$X = 1 - \frac{1}{1 - \frac{d}{dE}F(E)}$$

self energy

Coulomb+short range model

◉ short range limit $a_B \to \infty$

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B}\left[\log(-ia_Bk) + \psi\left(1 + \frac{i}{a_Bk}\right)\right] = 0$$

$$\to 0$$

$$\longrightarrow -\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik = 0$$ short range interaction

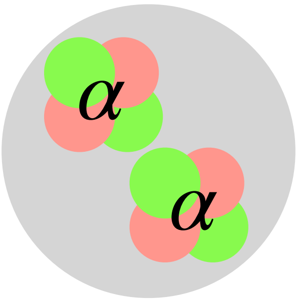
◉ further low-energy limit $r_e \to 0$

- zero-range theory S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

$$\frac{ia_Bk}{2} \mp \log(-ia_Bk) + \psi\left(1 + \frac{i}{a_Bk}\right) + \frac{a_B}{2a_s} = 0$$

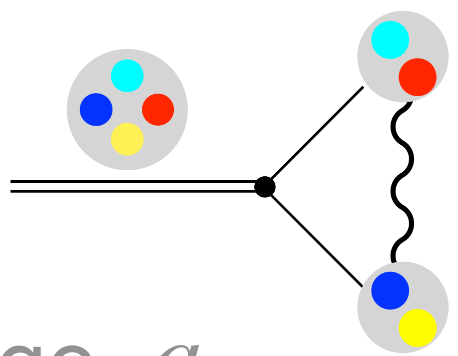
This work

near-threshold bound states & resonances
with **Coulomb + short range** interaction



framework : model with Feshbach method

- bare state which couples to Coulomb scattering



- lomb scattering length, Coulomb effective range, a_B

numerical calculations & discussion

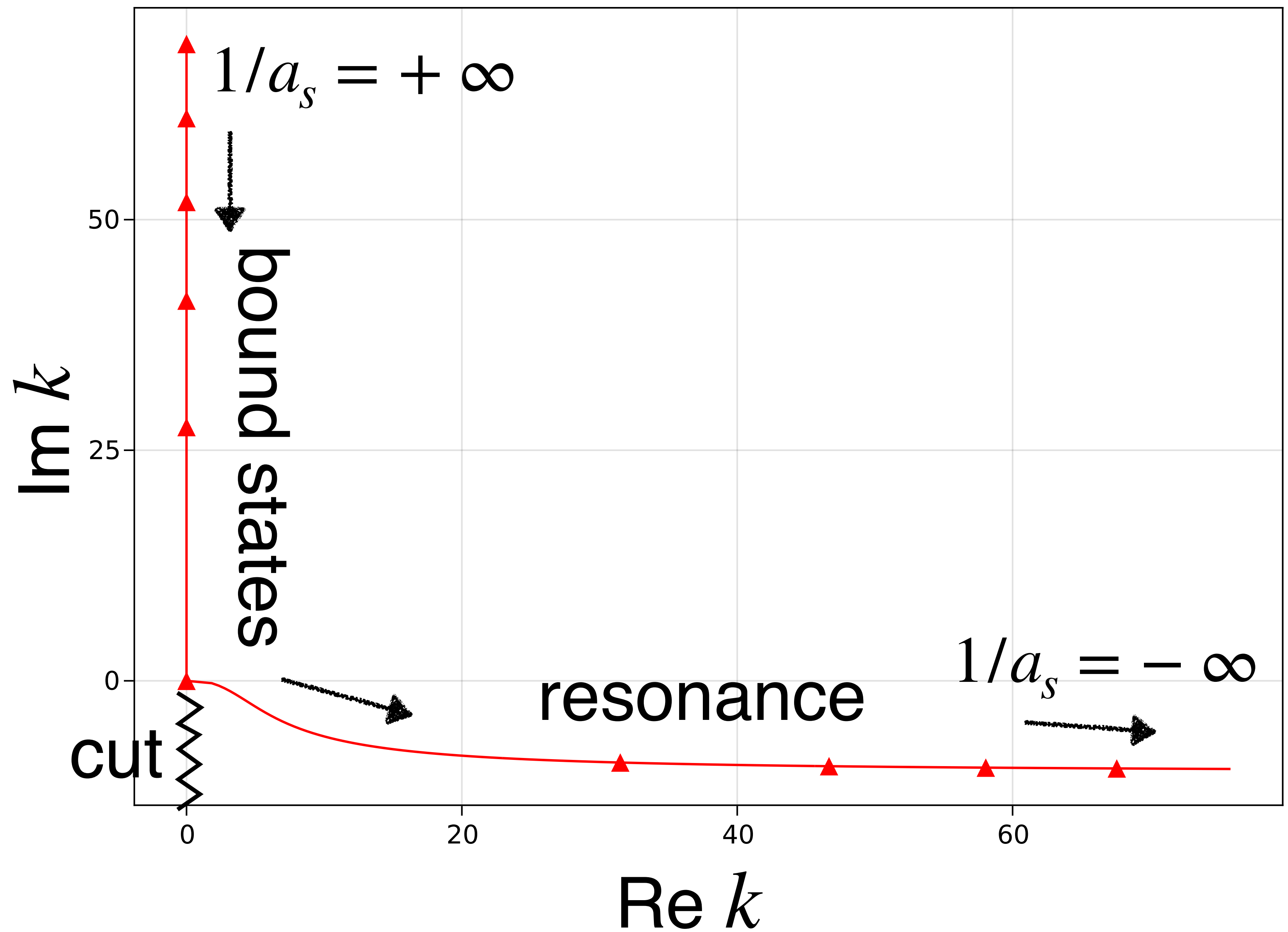
- investigate pole trajectory
- analyze internal structure with compositeness
- study universal nature of near-threshold states

Pole trajectory (repulsive Coulomb)

◉ pole trajectory in complex momentum k plane

- varying Coulomb scattering length a_s with fixed r_e and a_B

→ pole position (eigenmomentum) moves



- b.s directory goes to resonance
- $a_s \to \infty$ at threshold
- but no universality

  ∵ radius of w.f. $< \infty$

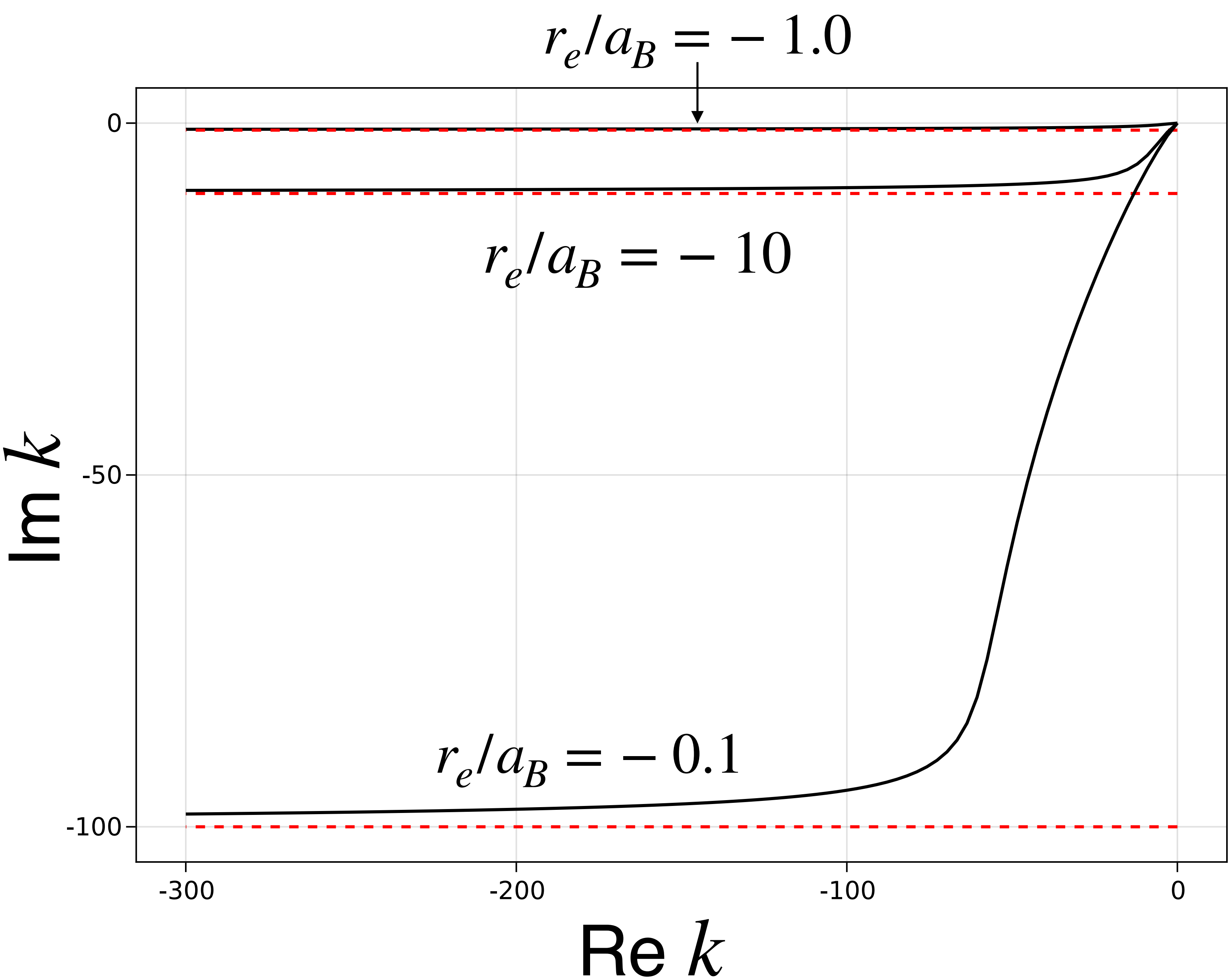
S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

Note : virtual states exist near Im $k < 0$ axis

far from threshold (repulsive Coulomb)

◉ imaginary part of eigenenergy in complex momentum k plane

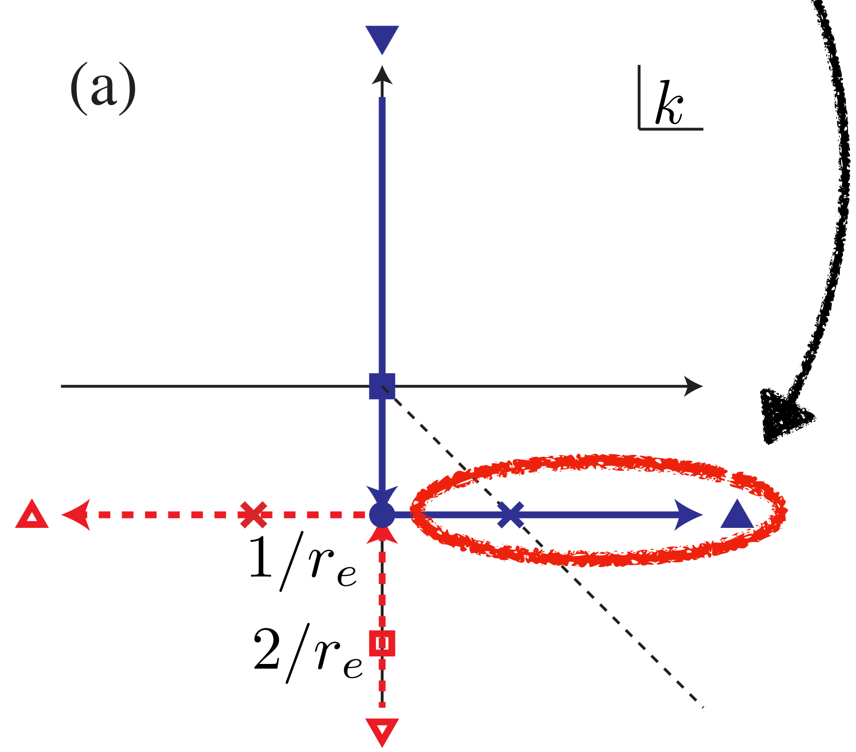
- far from threshold in $1/a_s \to -\infty$ limit



dotted lines :$1/r_e$

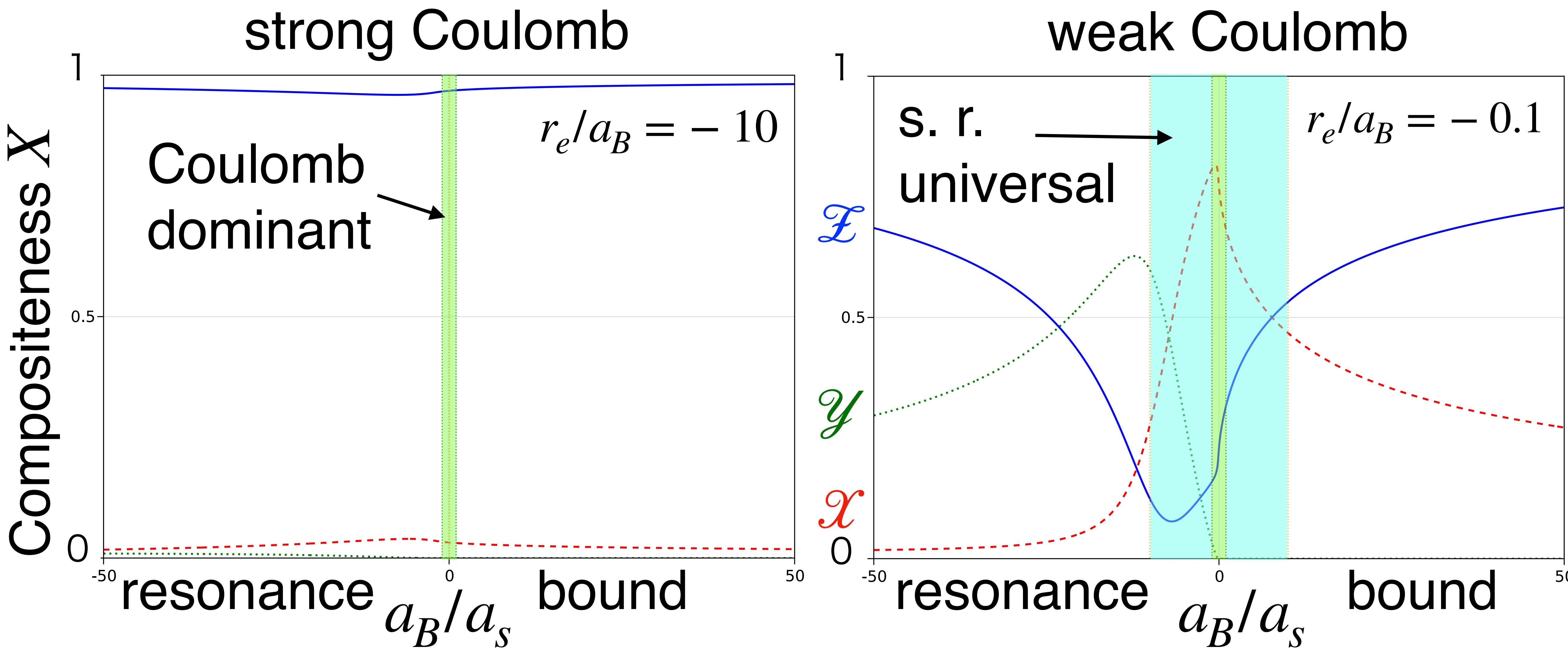
- Im $k \to 1/r_e$

→ trajectory close to that of resonance in ERE



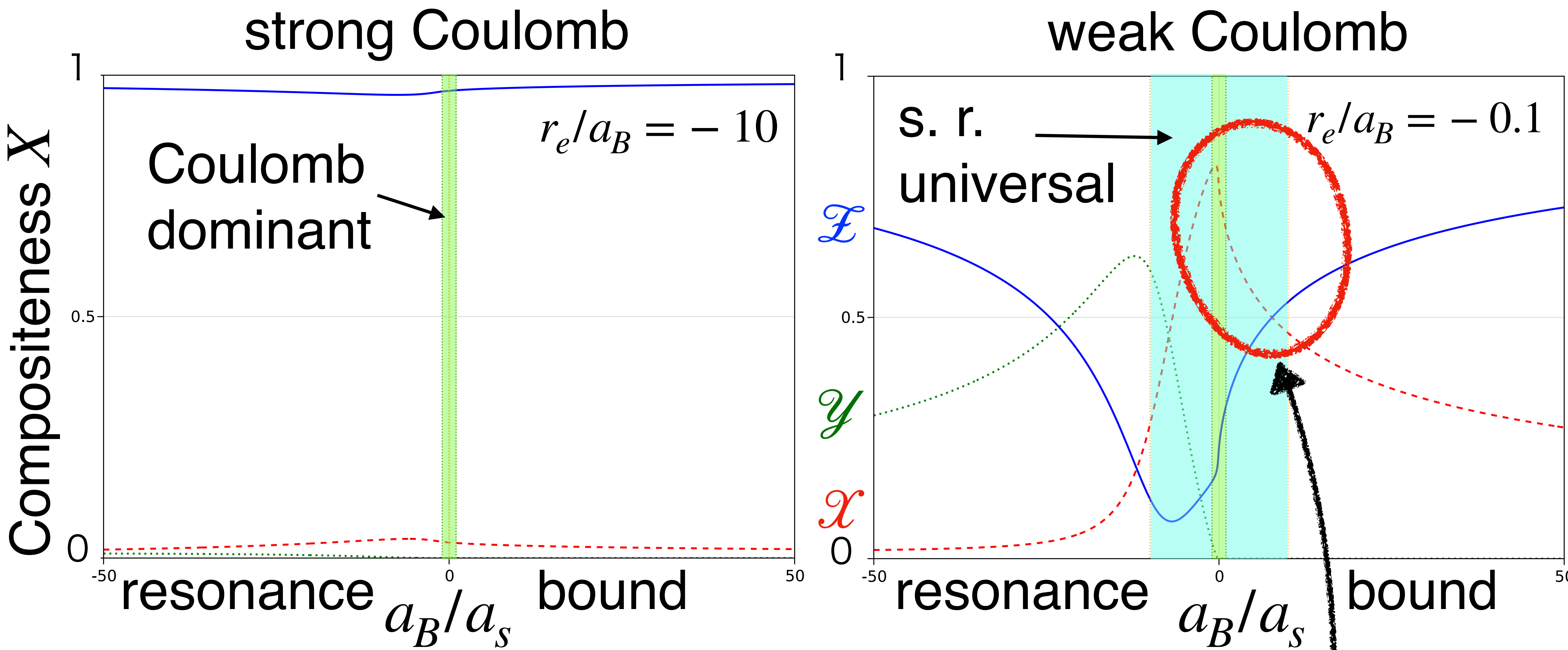
T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) .

Compositeness (repulsive Coulomb)



- complex compositeness ⟵ $\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].
- states with large $|1/a_s|$ are elementary $\mathscr{Z}$ dominant
- structure of bound states ≈ resonances ∵ continuous X

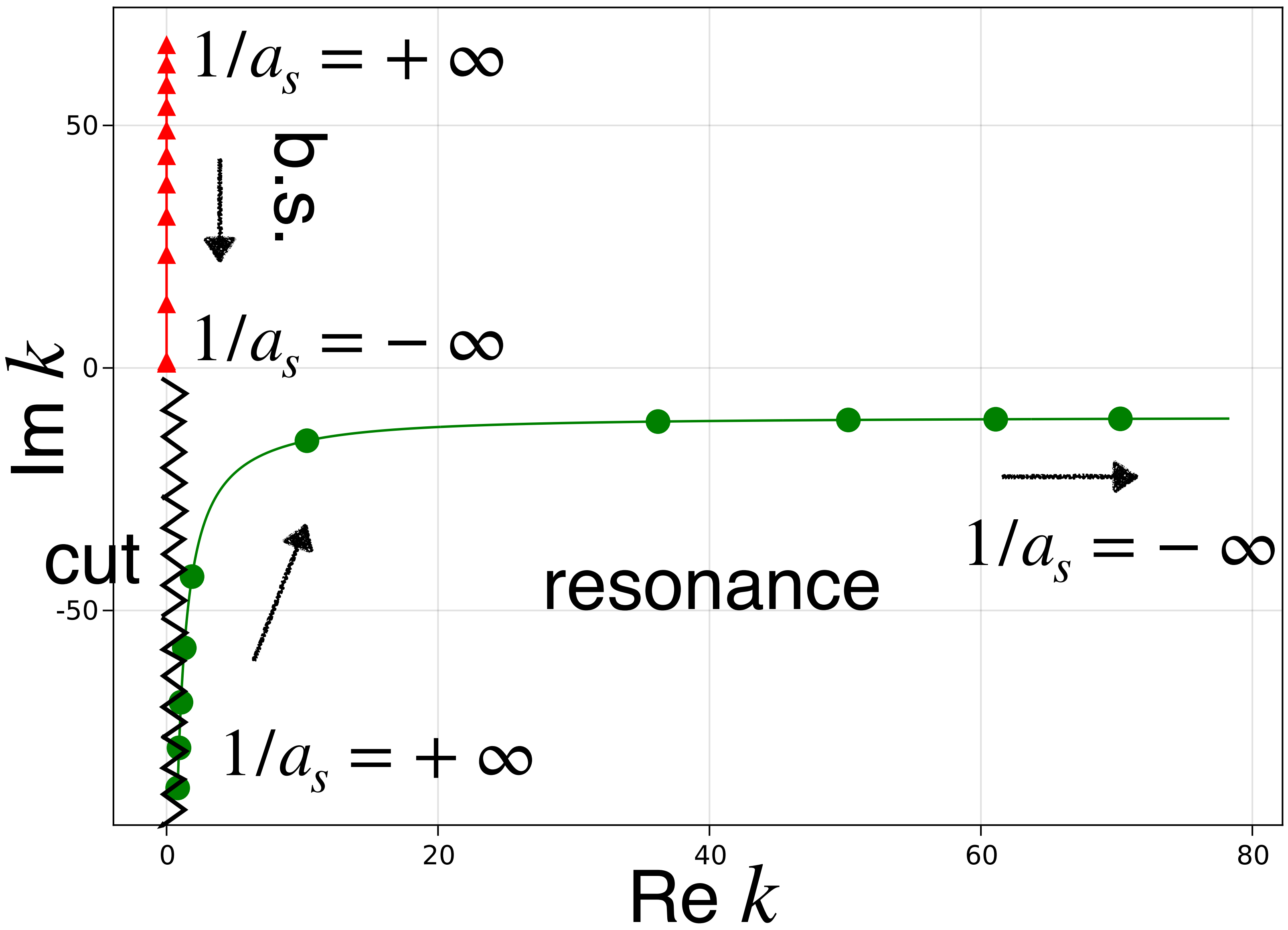
Compositeness (repulsive Coulomb)



- complex compositeness ← $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].
- states with large $|1/a_s|$ are elementary $\mathcal{Z}$ dominant
- structure of bound states ≈ resonances ∵ continuous X
- remnant of short range universality in $|r_e| \ll |a_B|$ case
  $X \to 1$ in $B \to 0$ limit in short range

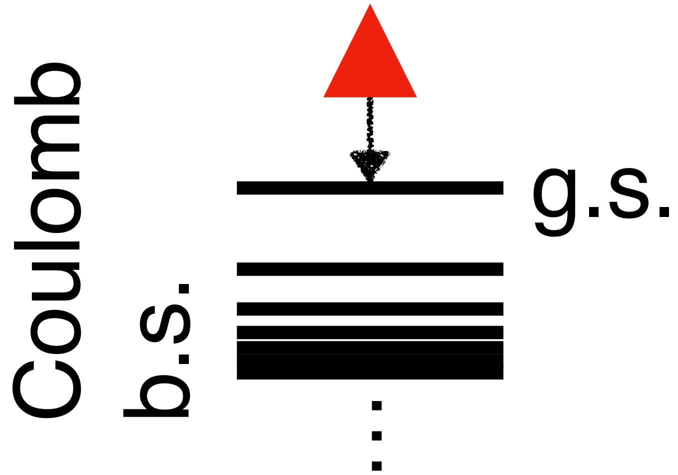
Pole trajectory (attractive Coulomb)

◉ pole trajectory in complex momentum k plane



- bound ≠ resonance

- bound pole
 → Coulomb g.s.



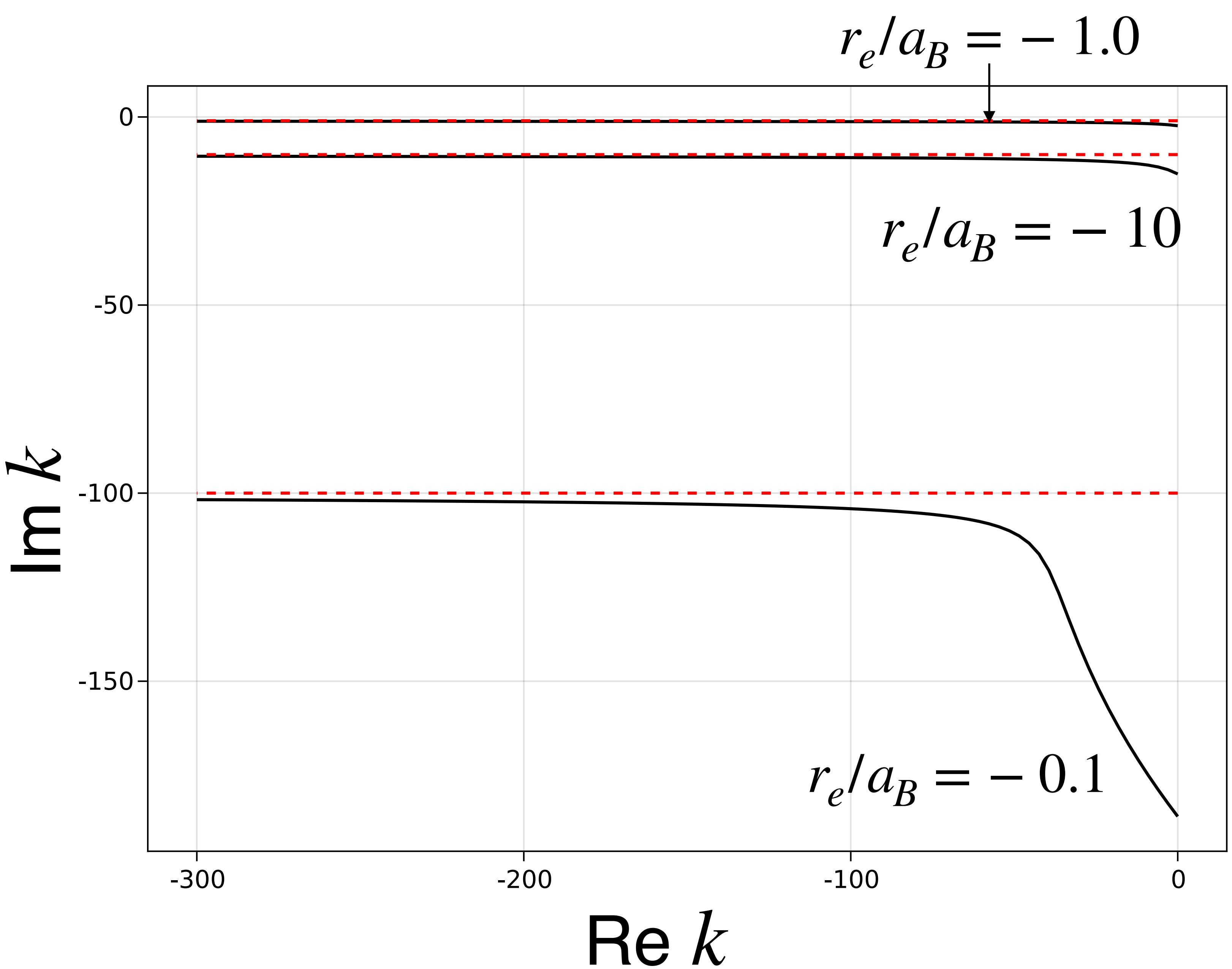
- pole cannot go to $k = 0$

W. Domcke, Atom. Mol. Phys. 16 359 (1983);
S. Mochizuki, and Y. Nishida, arXiv:2408.06011 [nucl-th].

far from threshold (attractive Coulomb)

◉ imaginary part of eigenenergy in complex momentum k plane

- far from threshold in $1/a_s \to -\infty$ limit



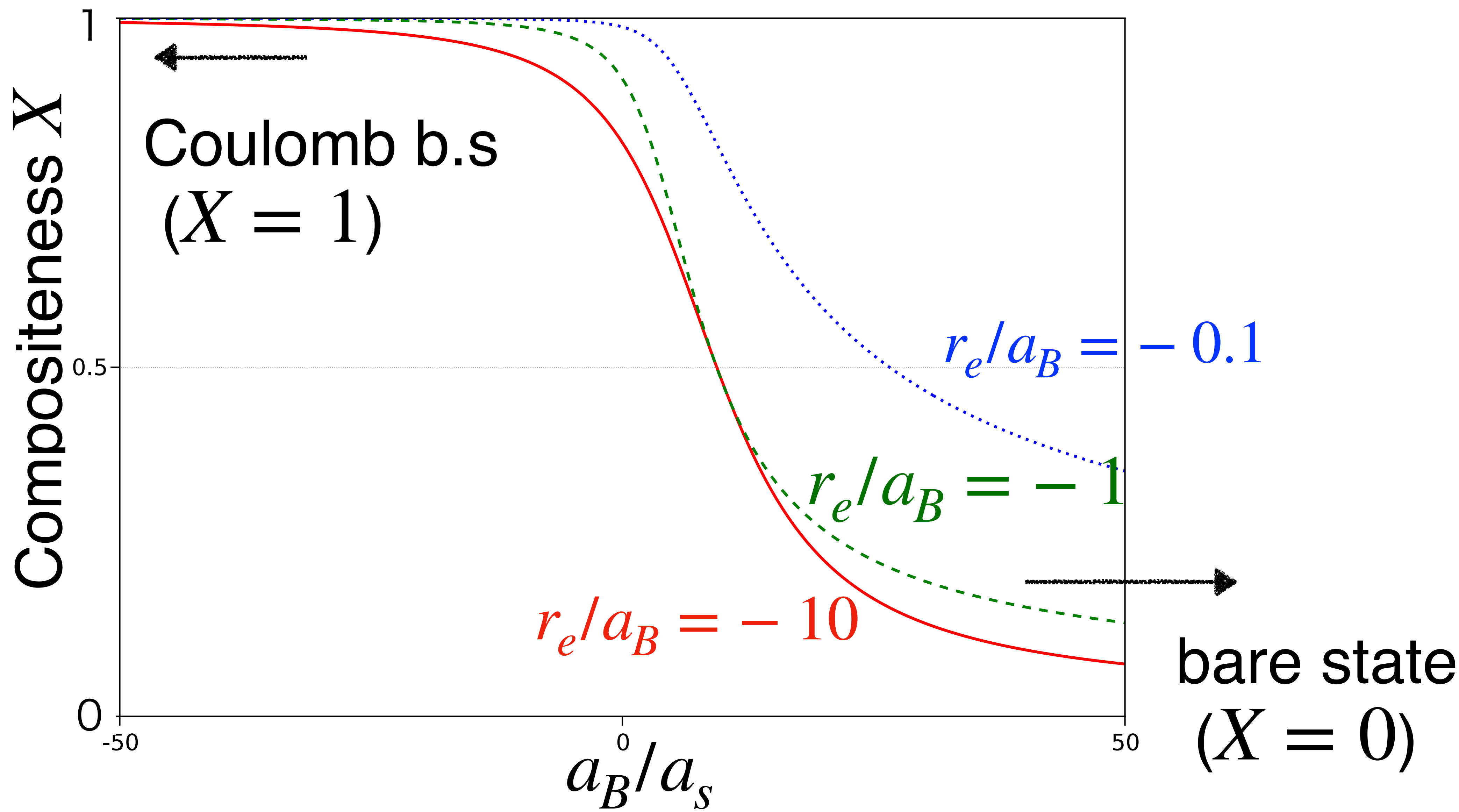
- Im k of resonance pole
- Im $k \to 1/r_e$

→ trajectory close to that of resonance in ERE

- same as repulsive case

dotted lines : $1/r_e$

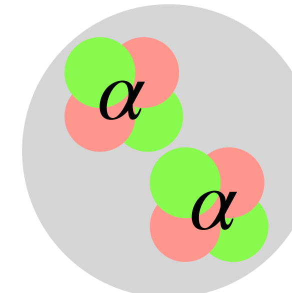
Compositeness (att. Coulomb b.s.)

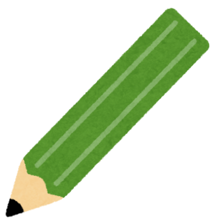


- $1/a_s \to +\infty$: states becomes elementary dominant ($X \to 0$)
- no short range universality but $X \to 1$ in $B \to B_{\text{Coulomb g.s.}}$ limit
  ∵ Coulomb g.s. has no bare state contribution (i.e. $X = 1$)
- Coulomb < short range ($r_e = -0.1$) : remnant of universality

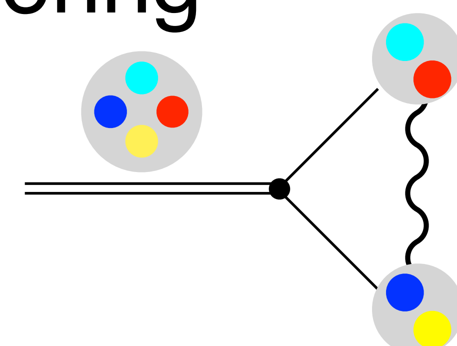
Summary

near-threshold bound states & resonances
with **Coulomb + short range** interaction





- bare state which couples to Coulomb scattering
- pole condition ← $a_s,\ r_e,\ a_B$



- repulsive Coulomb

  bound → resonance (does not become virtual states)

  X is not necessary to be unity at threshold

  if Coulomb < s.r., remnant of s.r. universality can be seen

  nature of b.s. ≈ nature of resonance

- attractive Coulomb

  bound → Coulomb g.s. & virtual → resonance

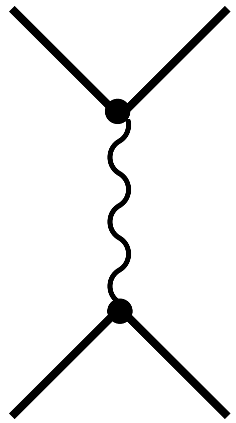
  $X \to 1$ for bound state

Back up

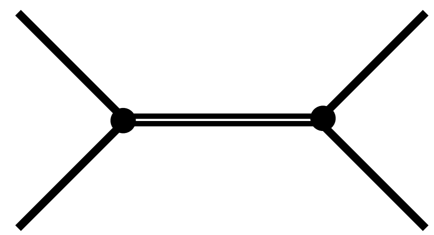
Coulomb+short range model

◉ model

Coulomb



short range (s.r.)



Weinberg, S. Phys. Rev. 137, 672–678 (1965);
V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova, A.E. Kudryavtsev, Phys. Lett. B 586, 53–61 (2004);
T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

◉ Hamiltonian

W. Domcke, Atom. Mol. Phys. 16 359 (1983);
H. Feshbach, Annals Phys. 19 287-313 (1962).

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

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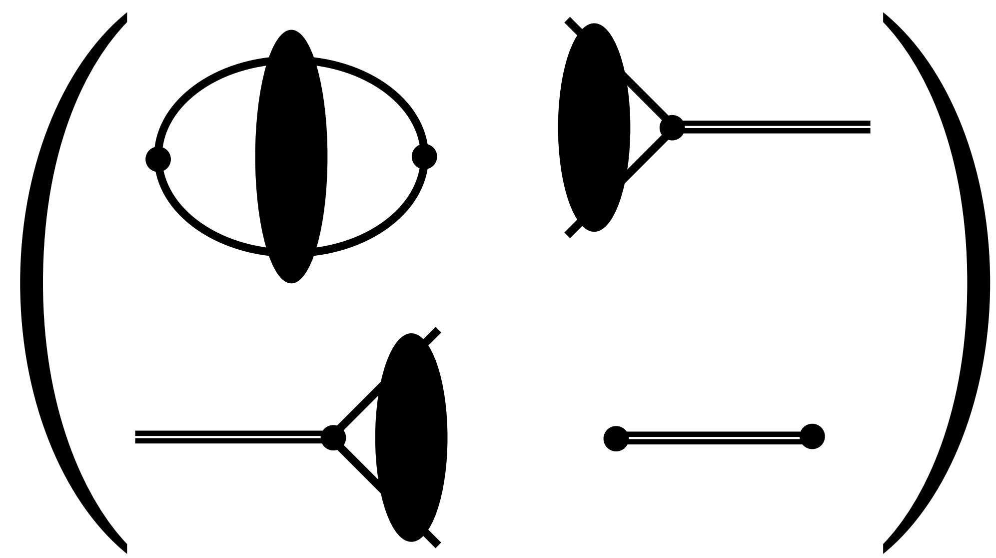
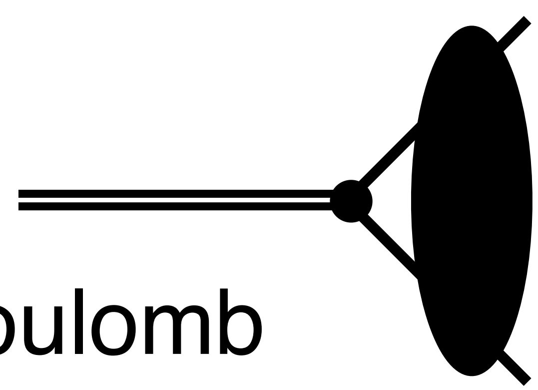


Q channel
⇔ b.s. w/o Coulomb



P channel
⇔ scattering w/ Coulomb + s.r.

Coulomb+short range model

◉ Schrödinger equation $\hat{H}|\Psi\rangle = E|\Psi\rangle$ $|\Psi\rangle = \begin{pmatrix} |P\rangle \\ |Q\rangle \end{pmatrix}$

$$\hat{H}_{PP}|P\rangle + \hat{H}_{PQ}|Q\rangle = E|P\rangle$$

$$\hat{H}_{QQ}|Q\rangle + \hat{H}_{QP}|P\rangle = E|Q\rangle$$

◉ effective Hamiltonian (channel eliminating)

$$\hat{H}_{P\mathrm{ch}}|P\rangle = E|P\rangle \quad \hat{H}_{P\mathrm{ch}} = \hat{H}_{PP} + \hat{H}_{PQ}(E - \hat{H}_{QQ})^{-1}\hat{H}_{QP}$$

Coulomb+short range model

◉ Schrödinger equation $\hat{H}|\Psi\rangle = E|\Psi\rangle \quad |\Psi\rangle = \begin{pmatrix} |P\rangle \\ |Q\rangle \end{pmatrix}$

$$\hat{H}_{PP}|P\rangle + \hat{H}_{PQ}|Q\rangle = E|P\rangle$$
$$\hat{H}_{QQ}|Q\rangle + \hat{H}_{QP}|P\rangle = E|Q\rangle$$

◉ effective Hamiltonian (channel eliminating)

$$\hat{H}_{P\mathrm{ch}}|P\rangle = E|P\rangle \quad \hat{H}_{P\mathrm{ch}} = \hat{H}_{PP} + \hat{H}_{PQ}(E - \hat{H}_{QQ})^{-1}\hat{H}_{QP}$$

$$\hat{H}_{PP} = \hat{H}^0 + \hat{V}_P \qquad \hat{H}_{PQ}(E - \hat{H}_{QQ})^{-1}\hat{H}_{QP} = \hat{V}_Q$$

$$\longrightarrow \hat{H}_{P\mathrm{ch}} = \hat{H}^0 + (\hat{V}_P + \hat{V}_Q)$$

$\hat{H}^0$: free Hamiltonian

$\hat{V}_P$: pure Coulomb interaction

$\hat{V}_Q$: short range interaction

Coulomb+short range model

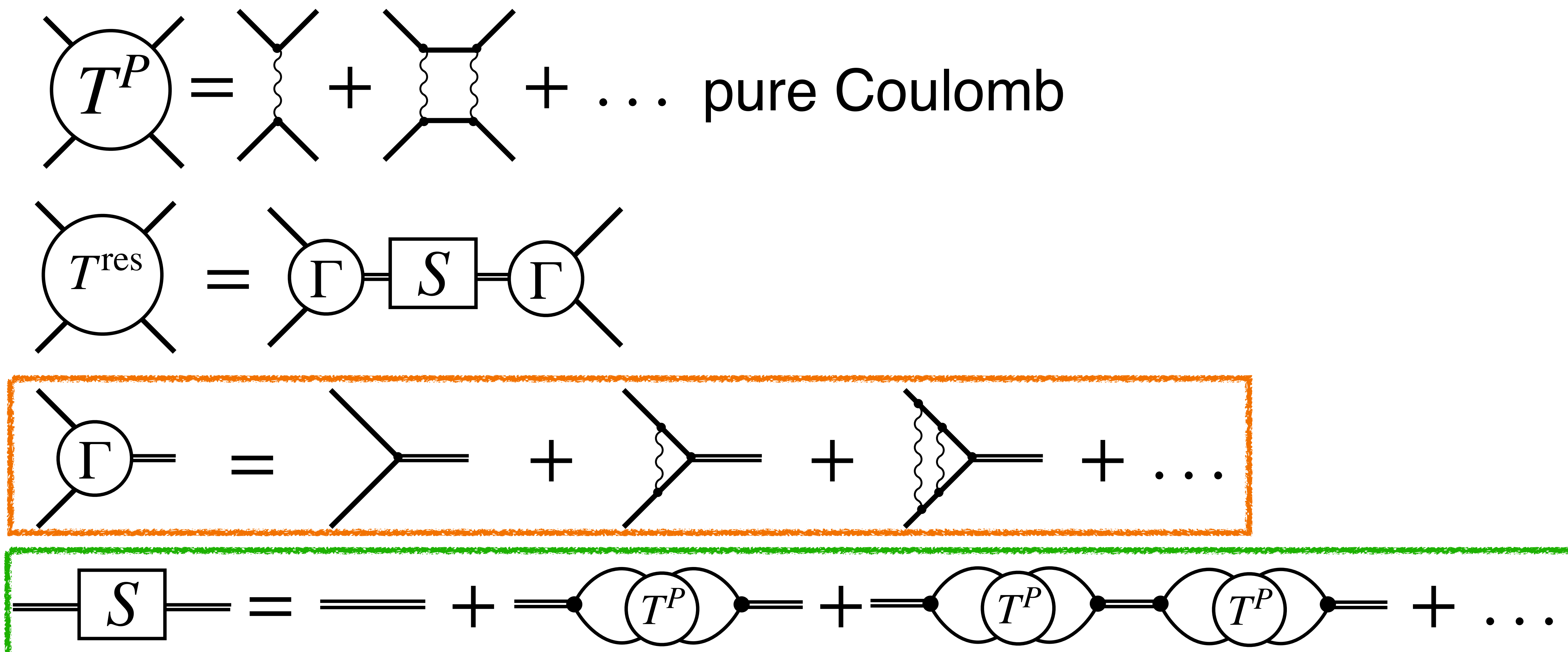
◉ *T*-matrix

H. Feshbach, Annals Phys. 19 287-313 (1962); W. Domcke, Atom. Mol. Phys. 16 359 (1983); R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

Lippmann-Schwinger eq. : $\hat{T} = [(\hat{V}_P + \hat{V}_Q)^{-1} - \hat{G}^0]^{-1}$

⟺ Feshbach method : $\hat{T} = \hat{T}^P + \hat{T}^{\rm res}$

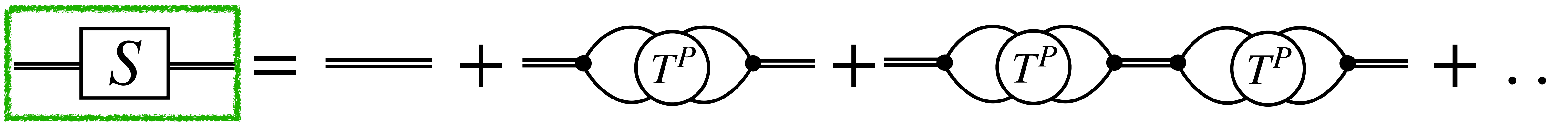
$$\hat{T}^P = [\hat{V}_P^{-1} - \hat{G}^0]^{-1}, \quad \hat{T}^{\rm res} = \hat{T}^P \hat{V}_P^{-1} [\hat{V}_Q^{-1} - \hat{G}_P]^{-1} \hat{V}_P^{-1} \hat{T}^P$$

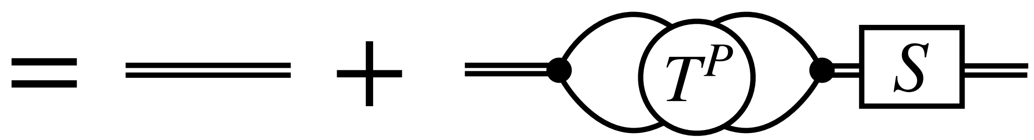


Coulomb+short range model

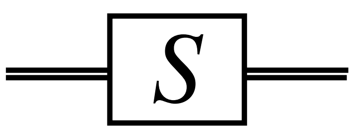
◉ pole condition $\hat{T} = \hat{T}^P + \hat{T}^{\mathrm{res}}$

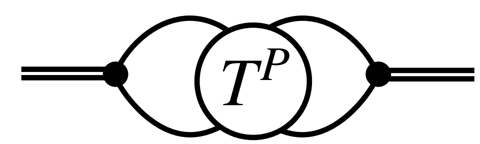
pole of $T(k,k')$ $\Leftrightarrow$ pole of $T^{\mathrm{res}}(k,k')$ $\Leftrightarrow$ $[V_Q^{-1} - G_P]^{-1} = \infty$





$H_{QP}G_PH_{PQ}$



$= S(E)$, ═══ $= (E - \varepsilon_d)^{-1}$,


$= F(E)$

$\longrightarrow S(E) = (E - \varepsilon_d)^{-1} + (E - \varepsilon_d)^{-1}F(E)S(E)$

$= [E - \varepsilon_d - F(E)]^{-1}$

$\longrightarrow$ pole condition : $E - \varepsilon_d - F(E) = 0$

bare state energy (ε_d)   self energy ($F(E)$)

Coulomb+short range model

◉ self energy $F(E)$ **in low-energy limit**

W. Domcke, Atom. Mol. Phys. 16 359 (1983).

- attractive Coulomb

$$F(k) = \frac{A}{2\pi}\left[c - \frac{1}{2}ia_Bk + \log(-ia_Bk) + \psi\left(1 - \frac{i}{a_Bk}\right)\right]$$

- repulsive Coulomb

$$F(k) = -\frac{A}{2\pi}\left[c + \frac{1}{2}ia_Bk + \log(-ia_Bk) + \psi\left(1 + \frac{i}{a_Bk}\right)\right]$$

A : constant with dimension of energy

c : dimensionless constant

$\psi(x) = \frac{d}{dx}\log(\Gamma(x))$: digamma function

Coulomb+short range model

◉ pole condition **in low-energy limit**

- Coulomb scattering length a_s and effective range r_e

$$(\text{amplitude})^{-1} = -\frac{1}{a_s} + \frac{r_e}{2}k^2 + \mathcal{O}(k^4) - ik + 2\log(-ik) + 2\psi\left(1+\frac{i}{k}\right) + \ldots,$$

R. Higa, H.-W. Hammer, and U. van Kolck, Nuclear Physics A 809 (2008).

$$\longrightarrow a_s = -a_B\left[\frac{4\pi}{A}\varepsilon_d \pm 2c\right]^{-1}, \quad r_e = -\frac{4\pi}{Aa_B\mu}$$

- pole condition with a_s and r_e

C. H. Schmickler, H.-W. Hammer, and A.G. Volosniev, Physics Letters B 798 (2019).

$$-\frac{1}{a_s} + \frac{r_e}{2}k^2 - ik \pm \frac{2}{a_B}\left[\log(-ik) + \psi\left(1+\frac{i}{k}\right)\right] = 0$$

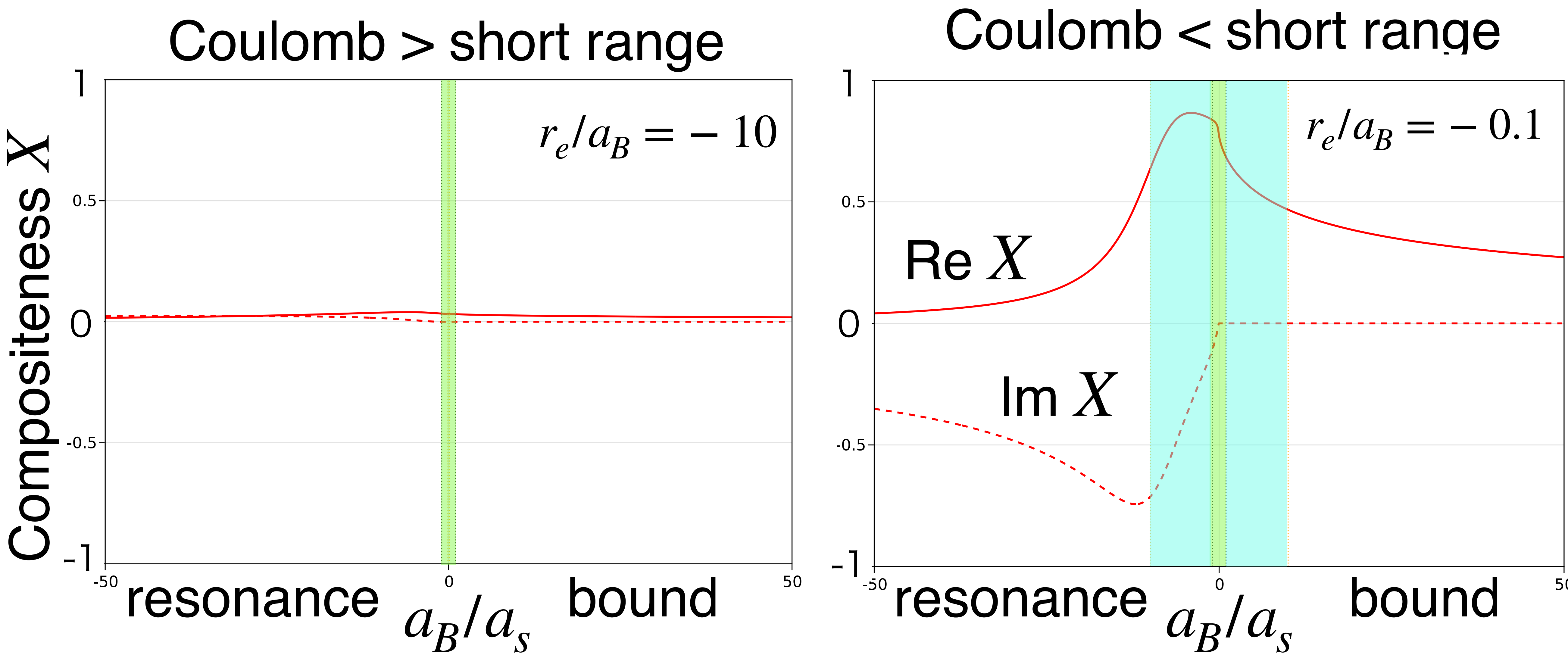
◉ compositeness X

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

$$X = 1 - \frac{1}{1 - \frac{d}{dE}F(E)}$$

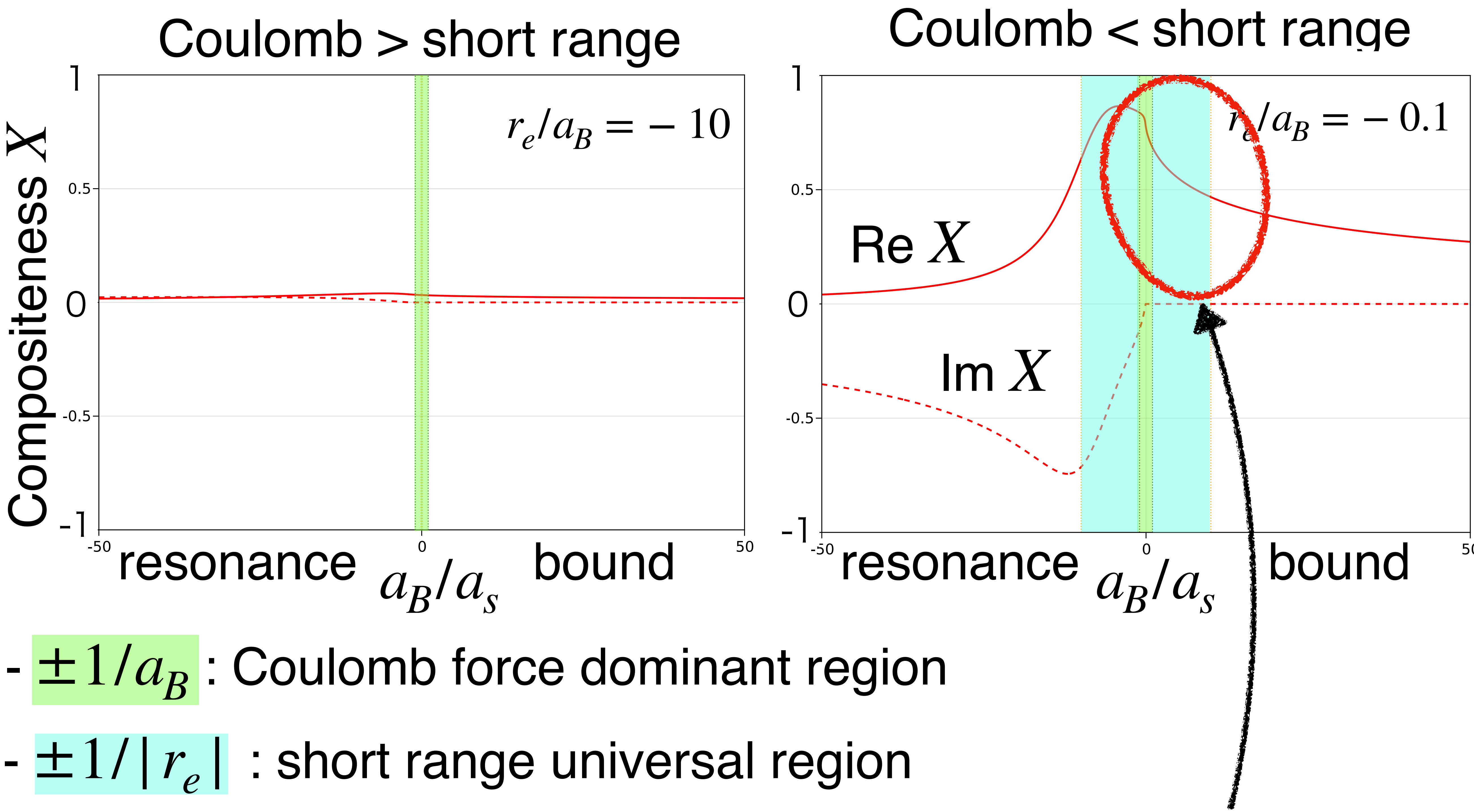
self energy

Compositeness (repulsive)



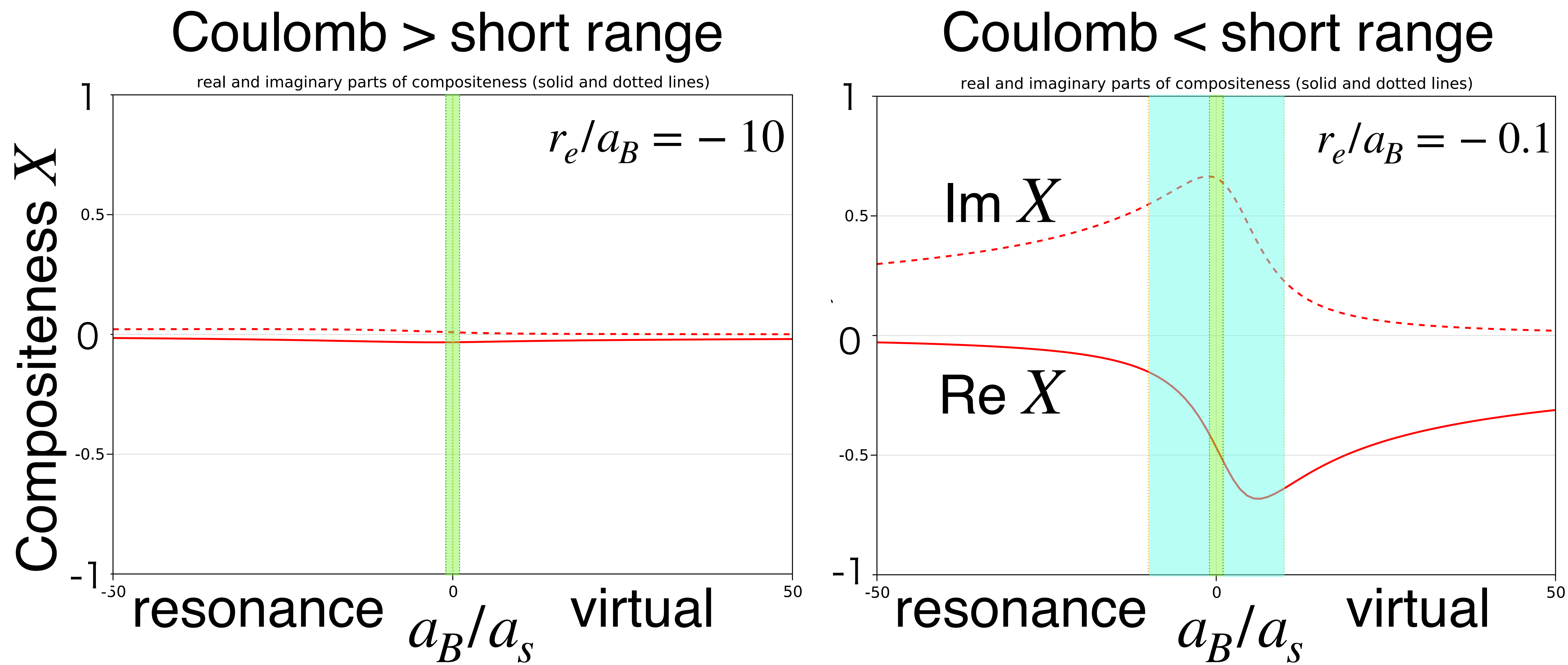
- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region

Compositeness (repulsive)



- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- remnant of short range universality in $|r_e| \ll |a_B|$ case
  $X \to 1$ in $B \to 0$ limit in short range

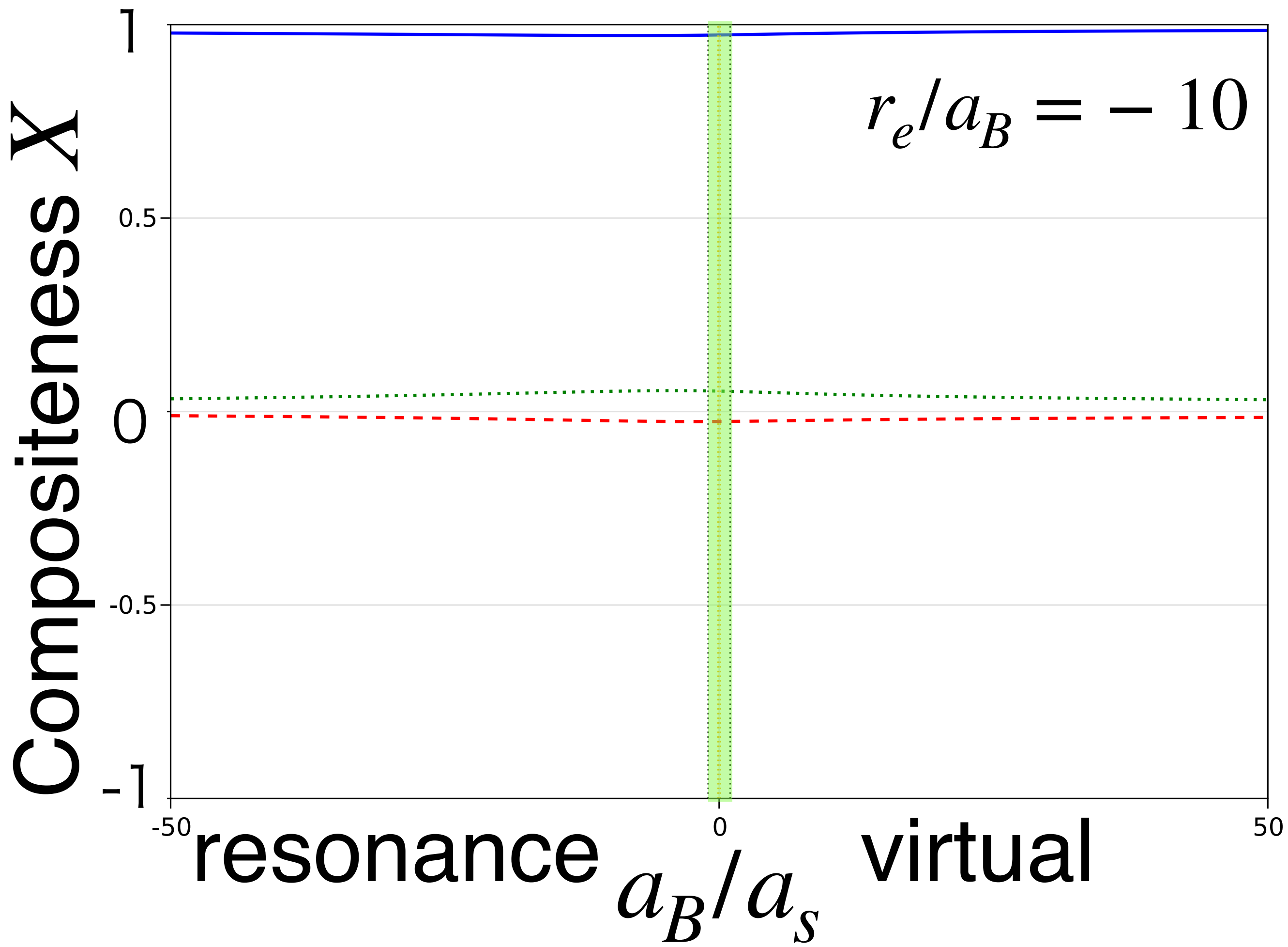
Compositeness (att. resonance)



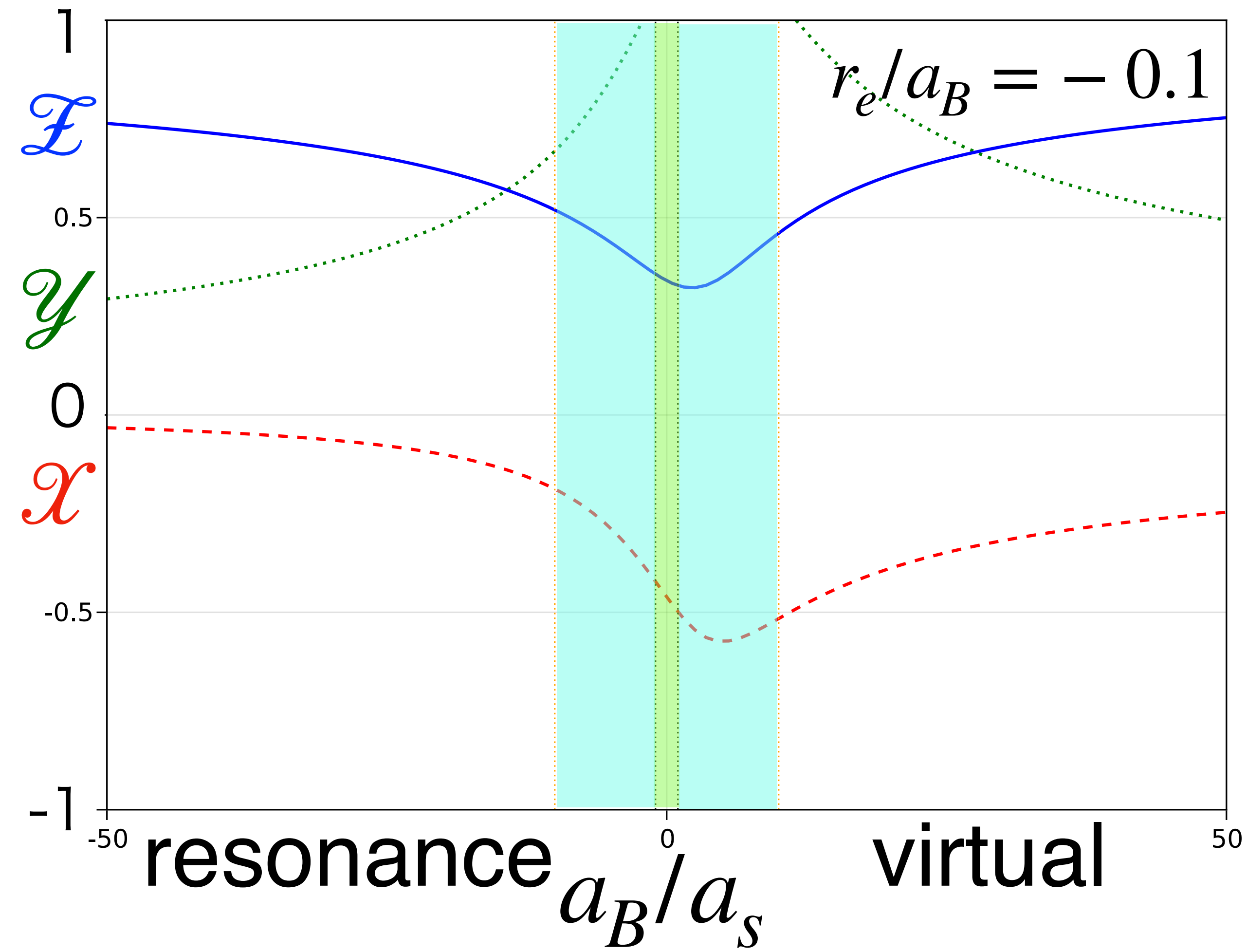
- $\pm 1/a_B$: Coulomb force dominant region
- $\pm 1/|r_e|$: short range universal region
- compositeness of unstable resonances are complex $X \in \mathbb{C}$

Compositeness (att. resonance)

Coulomb > short range



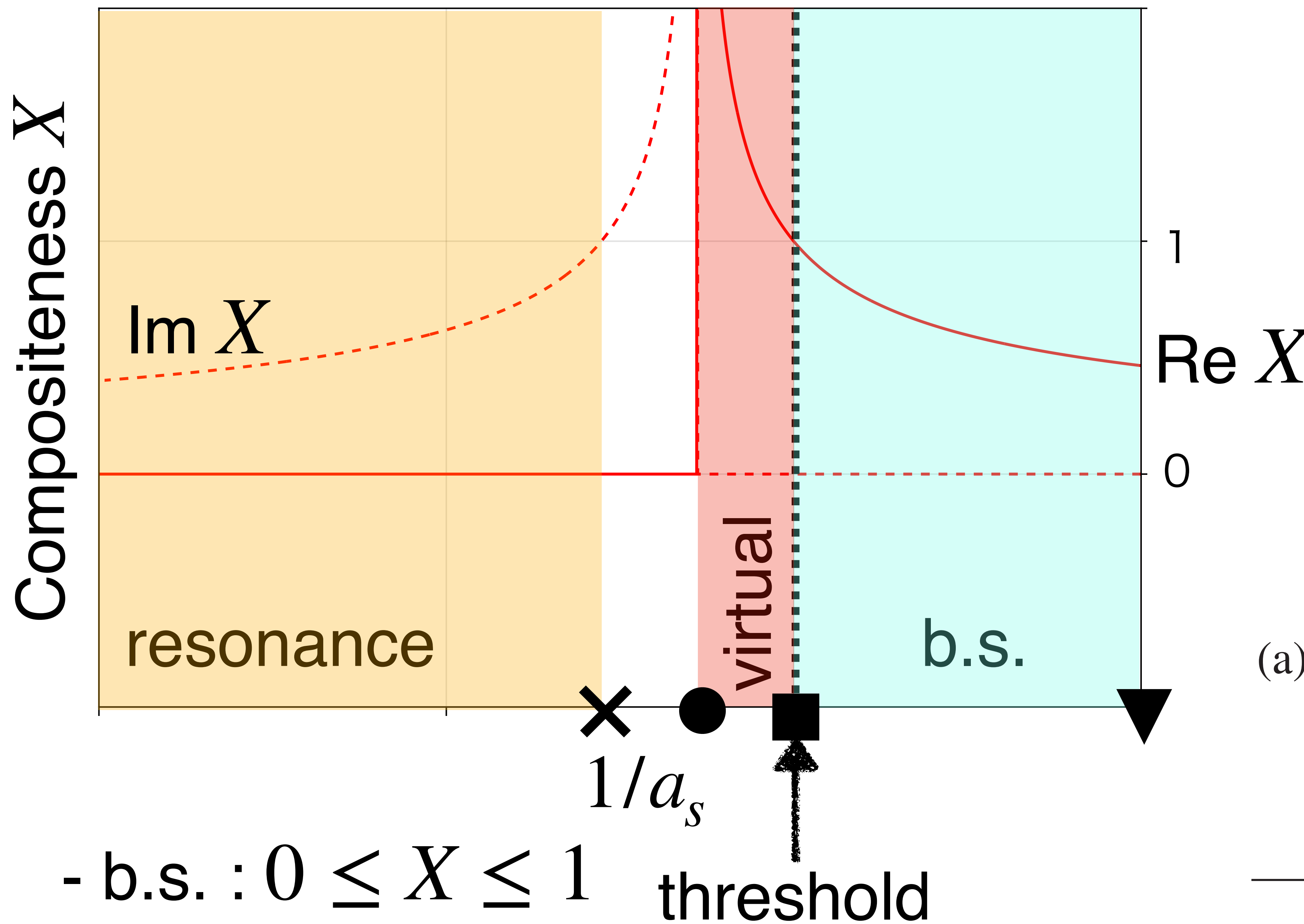
Coulomb < short range



- $\mathscr{X} < 0$ ➞ non-interpretable in this region
- but $\mathscr{X} \geq 0$ in far-threshold region with large $|1/a_s|$

➞ states are $\mathscr{Z}$ dominant with large bare state contribution

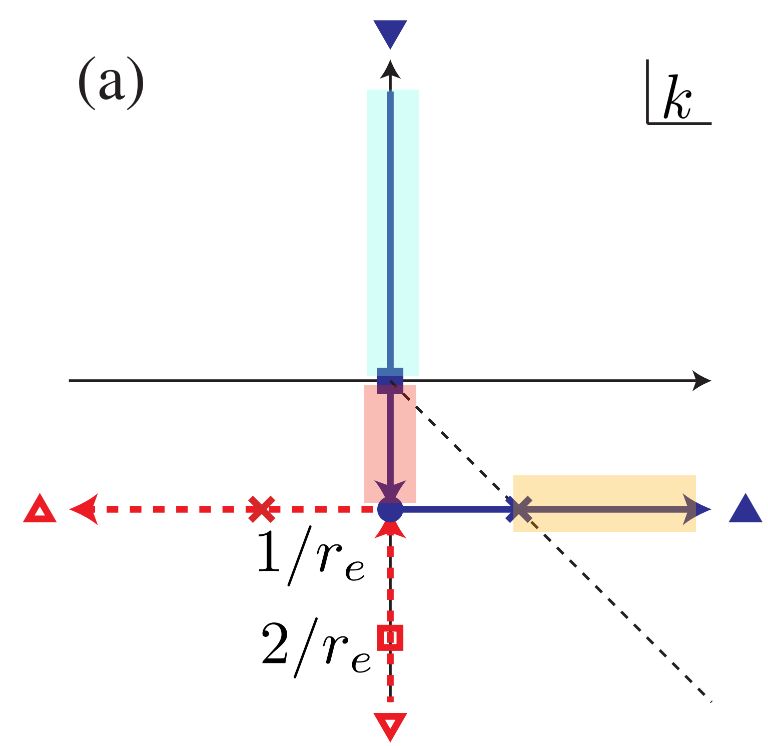
Compositeness (only w/ s.r.)



- b.s. : $0 \leq X \leq 1$
- virtual : $1 < X$

● divergence $X \to \infty$

- resonance : $\text{Im } X \leq 1$



T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013) .

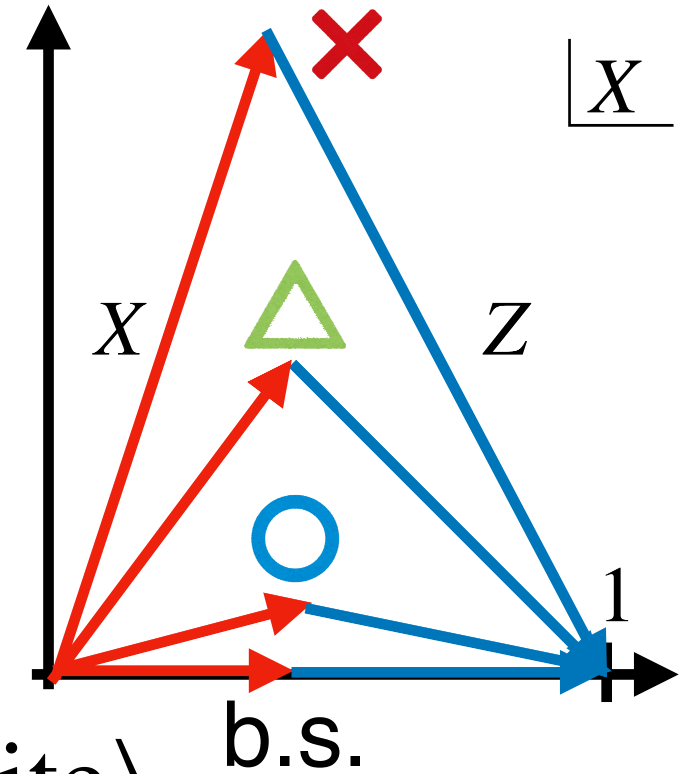
Complex compositeness

- probabilistic interpretation?

  $X \in \mathbb{C}$ and $X + Z = 1$

- If Im X is large, it seems that reasonable interpretation is impossible ✕ △

complex X plane



- our proposal

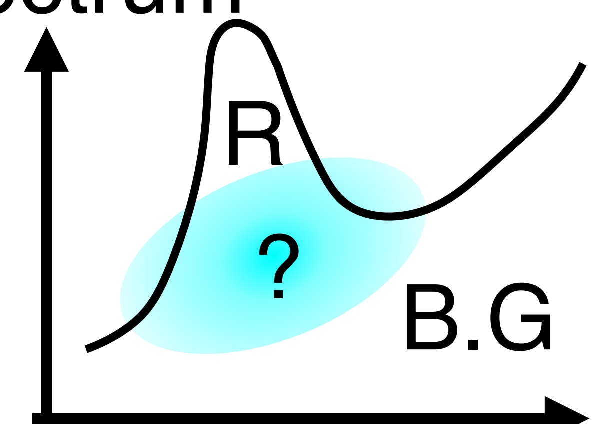
i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$

ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$

iii) $\mathscr{Y}$: probability of uncertain identification

uncertain appears from T. Berggren, Phys. Lett. B 33, 547 (1970).

- finite lifetime (uncertainty in energy)
- separation from B.G.



complex compositeness $X \in \mathbb{C} \longrightarrow \mathscr{X}, \mathscr{Y}, \mathscr{Z}$

Definition

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

◉ conditions for sensible interpretation

- normalization : $\mathcal{X}+\mathcal{Y}+\mathcal{Z}=1$ for probabilistic interpretation
- in bound state limit : $\mathcal{X}\to X$, $\mathcal{Z}\to Z$ and $\mathcal{Y}\to 0$

$\mathcal{Y}$ characterizes uncertainty of resonance

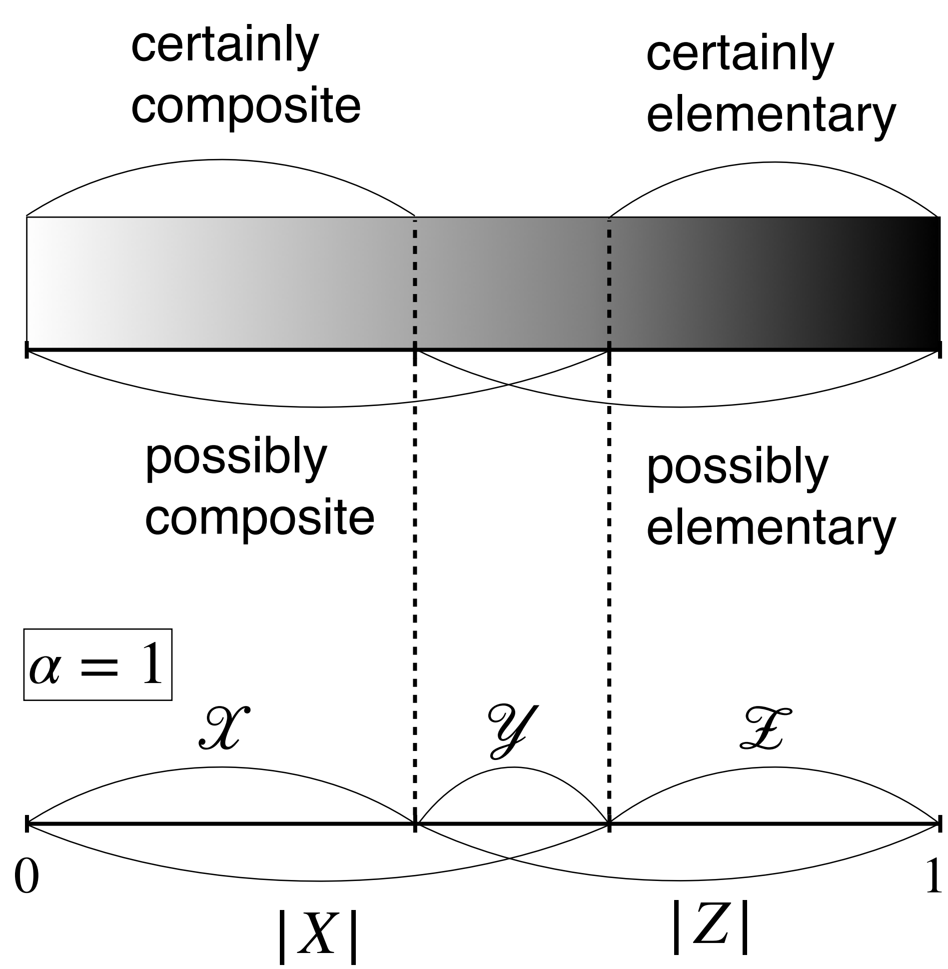
◉ new interpretation

$$\mathcal{X}+\alpha\mathcal{Y}=|X|,\quad \mathcal{Z}+\alpha\mathcal{Y}=|Z|$$

$$\mathcal{X}=\frac{(\alpha-1)|X|-\alpha|Z|+\alpha}{2\alpha-1}$$

$$\mathcal{Z}=\frac{(\alpha-1)|Z|-\alpha|X|+\alpha}{2\alpha-1}$$

$$\mathcal{Y}=\frac{|X|+|Z|-1}{2\alpha-1}$$

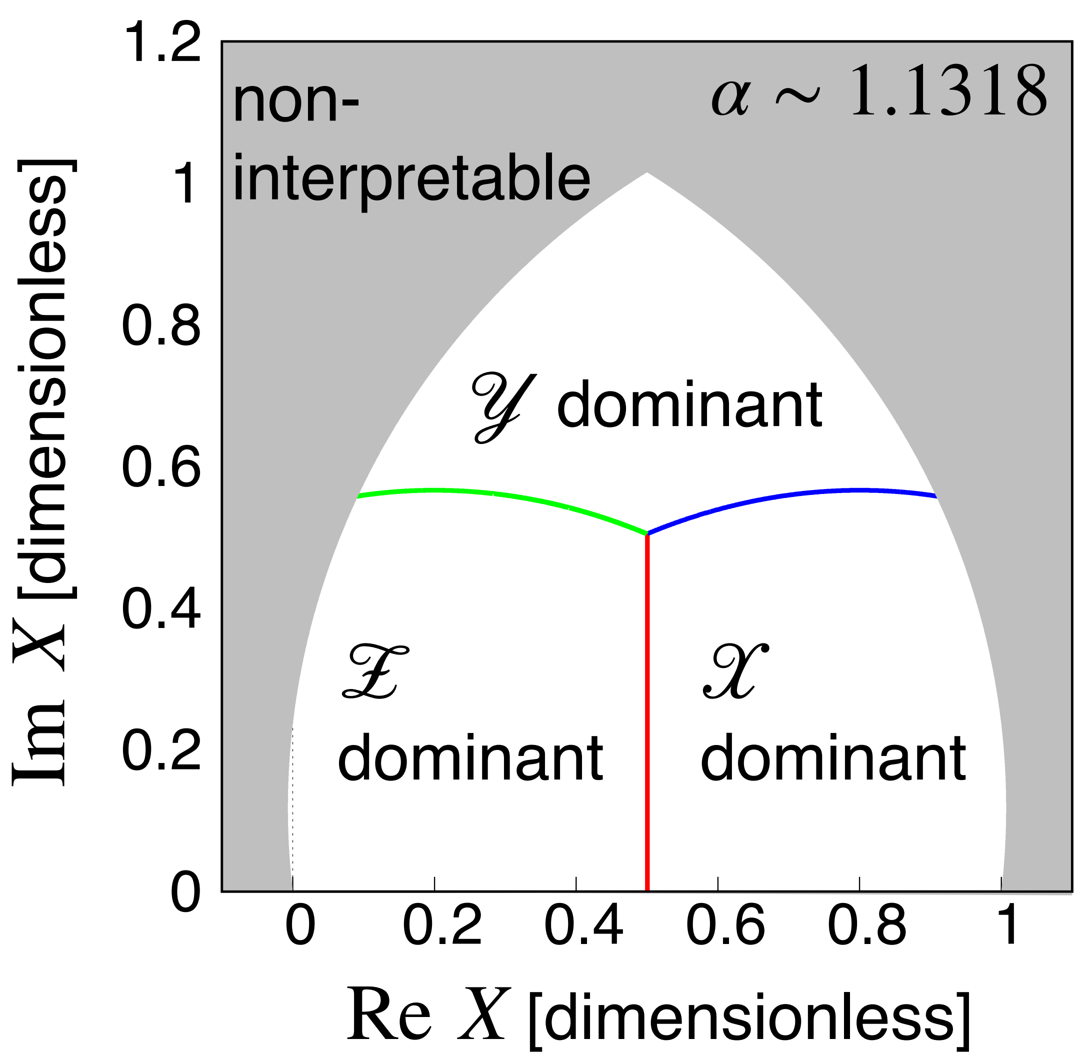


α reflects uncertain nature of resonances

Definition

- if $\alpha > 1/2$, $\mathscr{Y}$ is always positive but $\mathscr{X}$, $\mathscr{Z}$ can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		non-interpretable



our criterion for physical "state"

$$\Gamma \leq \mathrm{Re}\ E$$

- exclude poles which we cannot regard as physical "state" from probabilistic interpretation

$\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ dominant regions
and
non-interpretable region

uncertainty in resonances