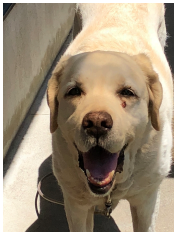


Range correction in the weak-binding relation for unstable states



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Background

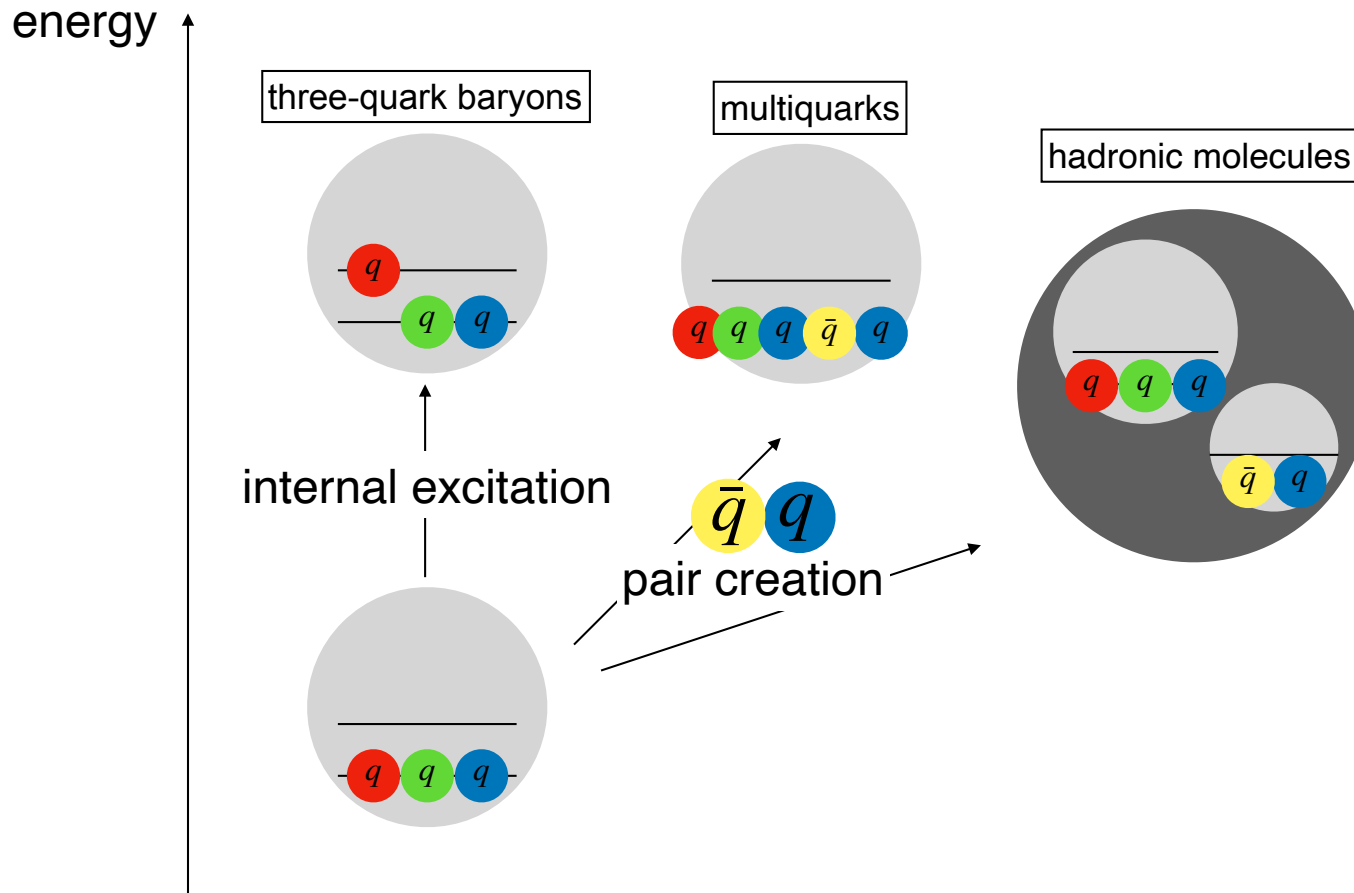
candidates for exotic hadrons

$\Lambda(1405)$, XYZ meson etc...



multiquarks

hadronic molecules



structure of hadrons

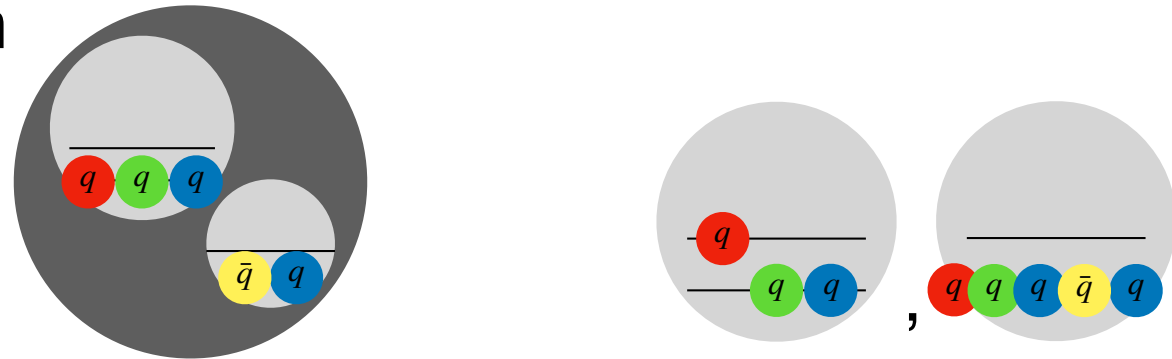


model independent

observable

Previous work

Hadron wave function



$$|\Psi\rangle = \sqrt{X} |\text{hadronic molecule}\rangle + \sqrt{1 - X} |\text{others}\rangle$$

Compositeness (weight of hadronic molecule)

Weak-binding relation

$$a_0 = R \left\{ \frac{2X}{1 + X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}$$

a_0 (scattering length)

$R \equiv (2\mu B)^{-1/2}$, B (binding energy)

R_{typ} (interaction range)

When $R \gg R_{\text{typ}}$: observable(a_0, B) \longrightarrow compositeness(X)

S. Weinberg, Phys. Rev. 137, B672 (1965); Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Motivation

Weak-binding relation $a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}$

Low-energy universality $\rightarrow a_0 = R \ (R \rightarrow \infty)$

- Deviation by contributions from other channels $\leftarrow X \neq 1$
- Deviation by interaction range $\leftarrow R_{\text{typ}} \neq 0$

\rightarrow We study the **range correction** in the weak-binding relation by introducing the effective range r_e .

Effective range model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008), 0709.0499.

Single channel scattering of identical bosons with mass m :

$$\mathcal{H}_{\text{int}} = \frac{1}{4} \lambda_0 (\psi^\dagger \psi)^2 + \frac{1}{4} \rho_0 \nabla (\psi^\dagger \psi) \cdot \nabla (\psi^\dagger \psi)$$

Off-shell T-matrix:

$$T(E, k, k') = T_1(E) + T_2(E)(k^2 + k'^2) + T_3(E)k^2 k'^2,$$

$$\begin{pmatrix} T_1 & T_2 \\ T_2 & T_3 \end{pmatrix} = -i \begin{pmatrix} \lambda_0 & \rho_0 \\ \rho_0 & 0 \end{pmatrix} - i \begin{pmatrix} \lambda_0 & \rho_0 \\ \rho_0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \int \frac{d^3 q}{(2\pi)^2} \frac{i}{E - q^2/m + i0^+} & \frac{1}{2} \int \frac{d^3 q}{(2\pi)^2} \frac{iq^2}{E - q^2/m + i0^+} \\ \frac{1}{2} \int \frac{d^3 q}{(2\pi)^2} \frac{iq^2}{E - q^2/m + i0^+} & \frac{1}{2} \int \frac{d^3 q}{(2\pi)^2} \frac{iq^4}{E - q^2/m + i0^+} \end{pmatrix} \begin{pmatrix} T_1 & T_2 \\ T_2 & T_3 \end{pmatrix}.$$

cut off at Λ

Typical range $R_{\text{typ}} \sim 1/\Lambda$

On-shell scattering amplitude:

$$f(k) = \left[-\frac{8\pi}{m} \frac{\left(1 + \frac{m}{12\pi^2} \Lambda^3 \rho_0\right)^2}{N(k)} - \frac{2}{\pi} \Lambda - ik \right]^{-1}, \quad N(k) = \left[\lambda_0 - \frac{m}{20\pi^2} \Lambda^5 \rho_0^2 \right] + 2\rho_0 \left(\frac{m}{24\pi^2} \Lambda^3 \rho_0 + 1 \right) k^2.$$

Effective range model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008), 0709.0499.

We obtain the scattering length a_0 and effective range r_e from low-energy behavior of $f(k; \lambda_0, \rho_0, \Lambda)$.

$$\longrightarrow a_0 = a_0(\lambda_0, \rho_0, \Lambda), \quad r_e = r_e(\lambda_0, \rho_0, \Lambda).$$

a_0 and r_e are the functions of the bare parameters and Λ .

Renormalization:

the bare parameters λ_0, ρ_0 are adjusted as functions of Λ so that a_0 and r_e are independent of Λ .

$$\begin{aligned} \longrightarrow f(k) &= \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 + \mathcal{O}\left(\frac{1}{\Lambda}\right) - ik \right]^{-1} \\ &\rightarrow \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik \right]^{-1} \quad (\Lambda \rightarrow \infty) \\ &\quad \text{Zero range limit} \end{aligned}$$

Effective range model

Properties of the effective range model:

-Single channel: | hadronic molecule \rangle only $\Leftrightarrow X = 1$

-Zero range limit: $\Lambda \rightarrow \infty \Leftrightarrow R_{\text{typ}} = 1/\Lambda \rightarrow 0$

$$\Rightarrow a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\} \rightarrow R ?$$

Renormalized scattering amplitude ($\Lambda \rightarrow \infty$):

$$1/f(k = i/R) = 0$$

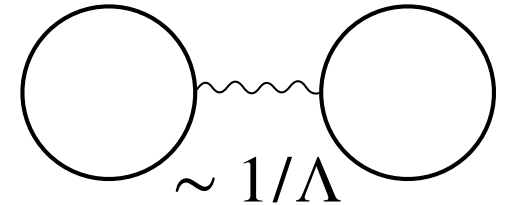
$$a_0 = R \frac{2r_e/R}{1 - (r_e/R - 1)^2} = R \left[1 + \mathcal{O}\left(\left|\frac{r_e}{R}\right|\right) \right] \Rightarrow a_0 \neq R$$

→ range correction in the weak-binding relation form r_e

Improved weak-binding relation

Weak-binding relation $a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}$

interaction range: $R_{\text{typ}} \longrightarrow R_{\text{int}} \sim 1/\Lambda$



Redefinition of R_{typ} :

$$R_{\text{typ}} = \max \left\{ R_{\text{int}}, R_{\text{eff}} \right\},$$

$$R_{\text{eff}} = \max \left\{ |r_e|, \frac{|P_s|}{R^2}, \dots \right\}.$$

$$f(k) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - \frac{P_s}{4}k^4 + \dots - ik \right]^{-1}$$

Length scale in the effective range expansion except for a_0

Numerical calculation 1

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}$$

When dose the weak-binding relation work?

Estimation with correction terms ($\xi \equiv R_{\text{typ}}/R$): Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

$$\text{Central value: } X_c = \frac{a_0/R}{2 - a_0/R}$$

$$X_{\text{upper}}(\xi) = \frac{a_0/R}{2 - a_0/R} + \xi, \quad X_{\text{lower}}(\xi) = \frac{a_0/R}{2 - a_0/R} - \xi.$$

Weak-binding relation works when...

$$\left\{ \begin{array}{l} X_{\text{lower}} < X_{\text{exact}} < X_{\text{upper}} \\ \longrightarrow \text{Validity condition} \\ (X_{\text{upper}} - X_c)/X_c < 0.1 \text{ and } (X_c - X_{\text{lower}})/X_c < 0.1 \\ \longrightarrow \text{Precision condition} \end{array} \right.$$

Numerical calculation 1

Effective range model ($\Lambda < \infty$)

$$f(k; \lambda_0, \rho_0, \Lambda) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 + \mathcal{O}(R_{\text{int}}) - ik \right]^{-1} \text{ (two length scales } r_e \text{ and } R_{\text{int}})$$

$$1/f(k = i/R) = 0$$

- $r_e \neq 0$ (range correction): $\xi_{r_e} = |r_e/R| \longrightarrow$ Uncertainty from r_e

$r_e < 0$ (effective range model)

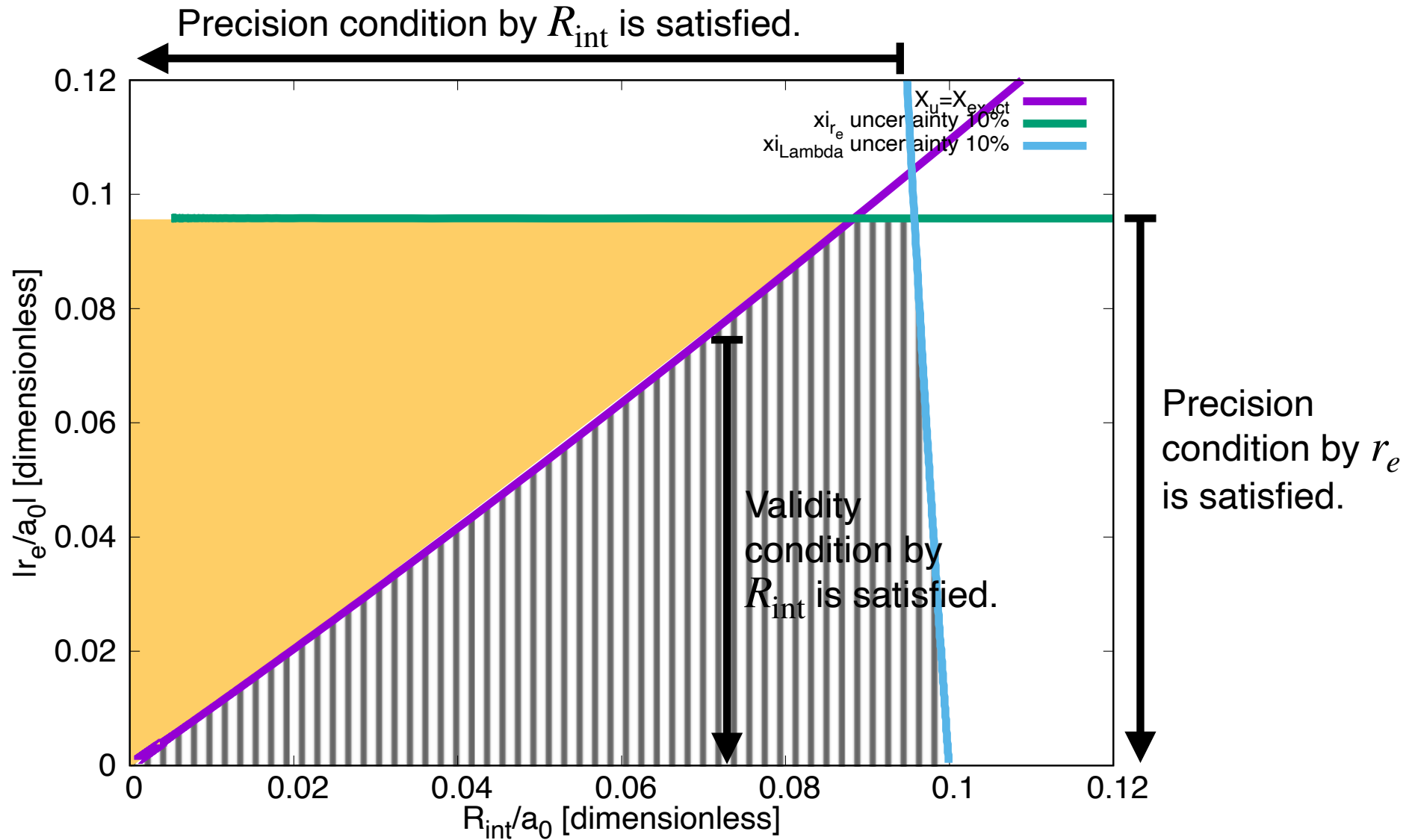
- $R_{\text{int}} = 1/\Lambda \neq 0$: $\xi_{\text{int}} = R_{\text{int}}/R. \longrightarrow$ Uncertainty from R_{int}

- $X_{\text{exact}} = 1$

\longrightarrow We search for the regions of r_e and R_{int} in which validity and precision conditions are satisfied.

Numerical calculation 1

Validity and precision conditions in $R_{\text{int}}/a_0 - |r_e/a_0|$ plane



Only the improved weak-binding relation can be applied.

Numerical calculation 2

What are the effects of the improvement of the central value?

Original weak-binding relation:

$$X(a_0, R) = \frac{a_0}{2R - a_0} \left[1 + \mathcal{O}\left(\frac{R_{\text{int}}}{R}\right) \right].$$



Use the bound state condition

$$X(r_e, R) = \frac{R}{R - r_e} \left[1 + \mathcal{O}\left(\frac{R_{\text{int}}}{R}\right) \right]$$

Y. Kamiya and T. Hyodo,
PTEP 2017, 023D02 (2017).



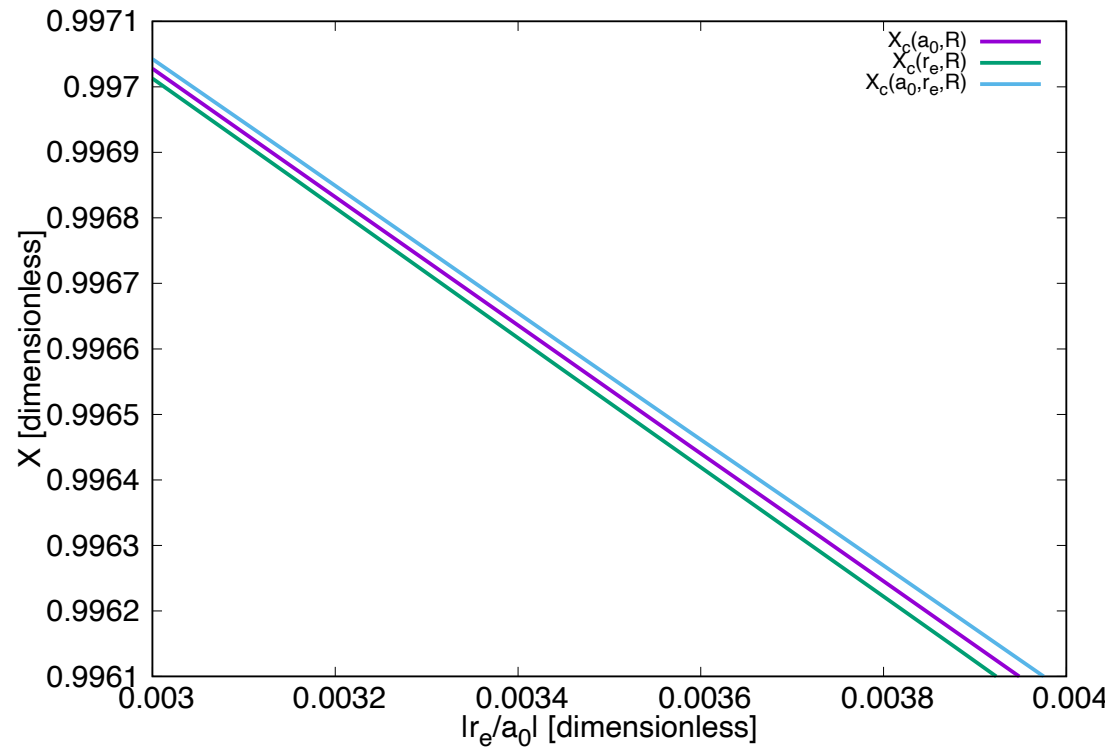
Considering the higher order term in
the effective range expansion

$$X(a_0, r_e, R) = \left[\frac{4R}{a_0} + \frac{r_e}{R} - 3 + \mathcal{O}\left(\frac{R_{\text{int}}}{R}\right) \right]^{-1}$$

Numerical calculation with the effective range model ($\Lambda < \infty$)

Numerical calculation 2

X_c from improved relations in X - $|r_e/a_0|$ plane at $\Lambda = 1.1/a_0$



There are almost no differences among X_c from the improved relations.

➔ The central-value-improved relation does not increase the parameter region in the R_{int}/a_0 - $|r_e/a_0|$ plane compared to the original one.

Conclusion and future prospect

- Weak-binding relation : observable  compositeness (X)

$$a_0 = R \left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right) \right\}$$

- We study the range correction in weak-binding relation from r_e .
- Improved weak-binding relation by redefinition of R_{typ} :
$$R_{\text{typ}} = \max \left\{ R_{\text{int}}, |r_e|, \dots \right\}$$
- We find the region where only the improved weak-binding relation can be applied.
- Improvement of the central value dose not increase the region.
- Future prospect: Apply the improved relation to hadron systems.