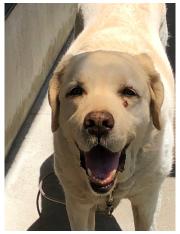
Range correction in the weak-binding relation for unstable states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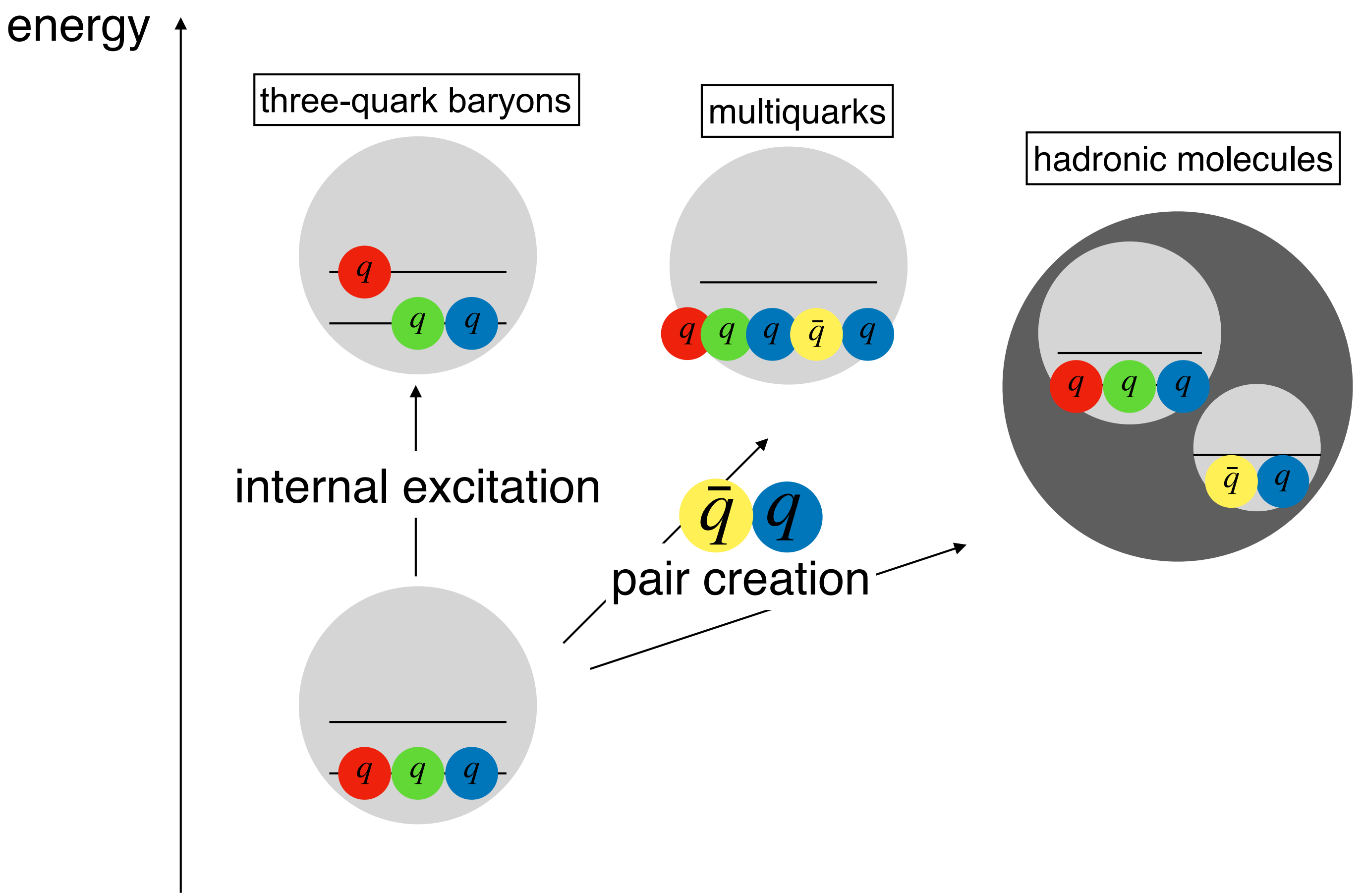
Background

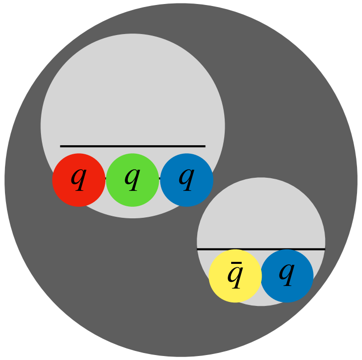
candidates for exotic hadrons
$\Lambda(1405)$, XYZ meson etc… → multiquarks, hadronic molecules

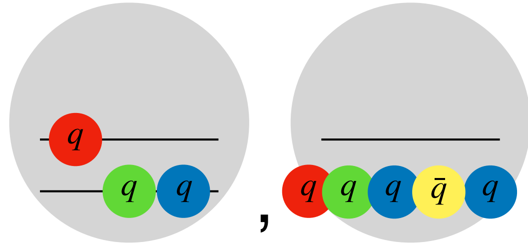


structure of hadrons ← model independent — observable

Previous work

Hadron wave function



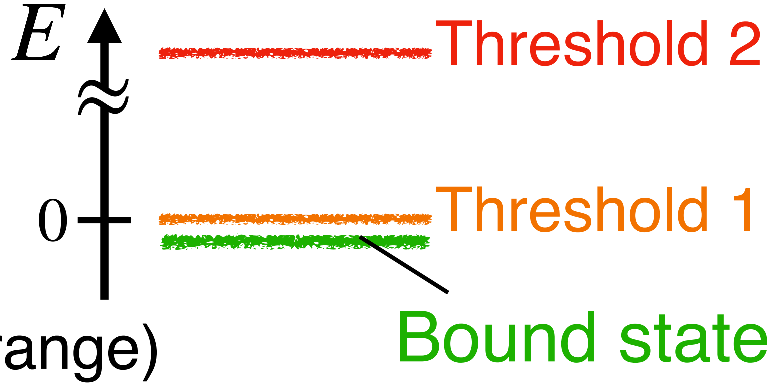


$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

Compositeness (weight of hadronic molecule)

Weak-binding relation for bound state

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}$$



a_0 (scattering length) R_{typ}(interaction range)

$R \equiv (2\mu B)^{-1/2}$, B (binding energy)

When $R \gg R_{\text{typ}}$: observable(a_0, B) → compositeness(X)

S. Weinberg, Phys. Rev. 137, B672 (1965); Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Motivation

Low-energy universality → $a_0 = R \ (R \to \infty)$

→ We study the range correction in the weak-binding relation by introducing the effective range r_e.

Range correction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

Apply to the following model :

Single channel: $|\text{hadronic molecule}\rangle$ only $\Rightarrow X = 1$

Zero range limit: $R_{\rm typ} \to 0 \Rightarrow \mathcal{O}(R_{\rm typ}/R) \to 0$

$\Leftrightarrow a_0 = R$?

Effective range model in the zero range limit (single channel)

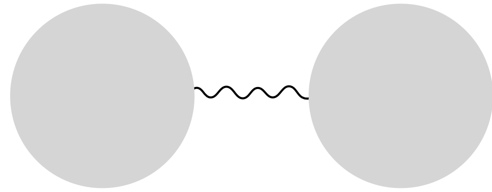
E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008), 0709.0499.

$$\mathcal{H}_{\rm int} = \frac{1}{4}\lambda_0(\psi^\dagger\psi)^2 + \frac{1}{4}\rho_0 \nabla(\psi^\dagger\psi)\cdot\nabla(\psi^\dagger\psi) \rightarrow f(k) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - ik\right]^{-1}$$

$$a_0 = R\frac{2r_e/R}{1-(r_e/R-1)^2} = R\left[1 + \mathcal{O}\left(\left|\frac{r_e}{R}\right|\right)\right] \Rightarrow a_0 \neq R$$

→ Weak-binding relation should be improved.

Improved weak-binding relation



$R_{\rm int} \sim 1/\Lambda$

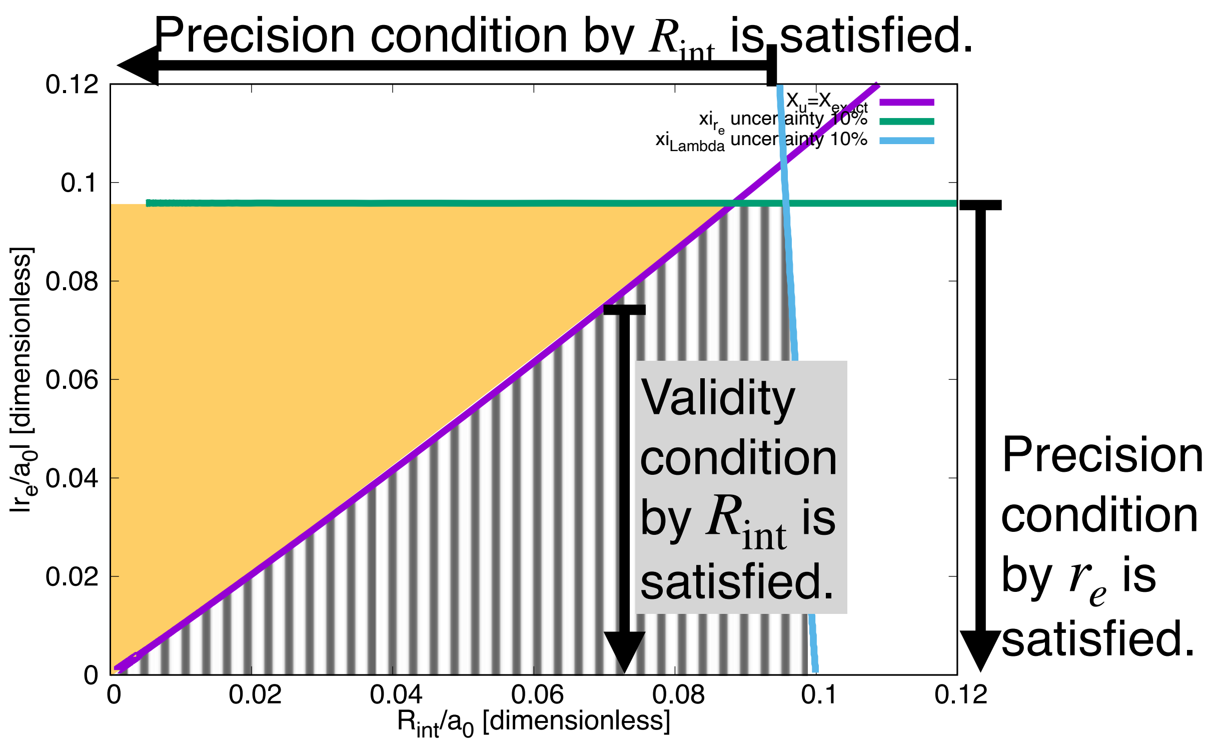
Redefinition of $R_{\rm typ}$ $\quad$ $R_{\rm int}$: interaction range

$$R_{\rm typ} = \max\left\{ R_{\rm int}, R_{\rm eff} \right\},\ R_{\rm eff} = \max\left\{ |r_e|, \frac{|P_s|}{R^2}, \cdots \right\}.$$

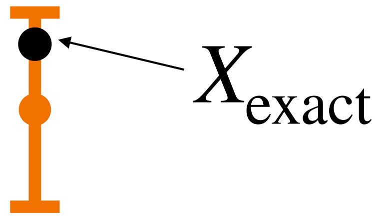
Numerical calculation

Effective range model ($R_{\rm typ} \neq 0$)

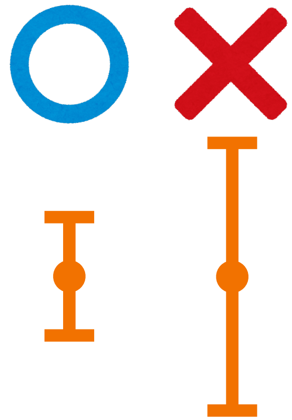
Validity and precision conditions in $R_{\rm int}/a_0$-$|r_e/a_0|$ plane



Validity condition



Precision condition



Only the improved weak-binding relation can be applied.

Extension to unstable states

Weak-binding relation for unstable states

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

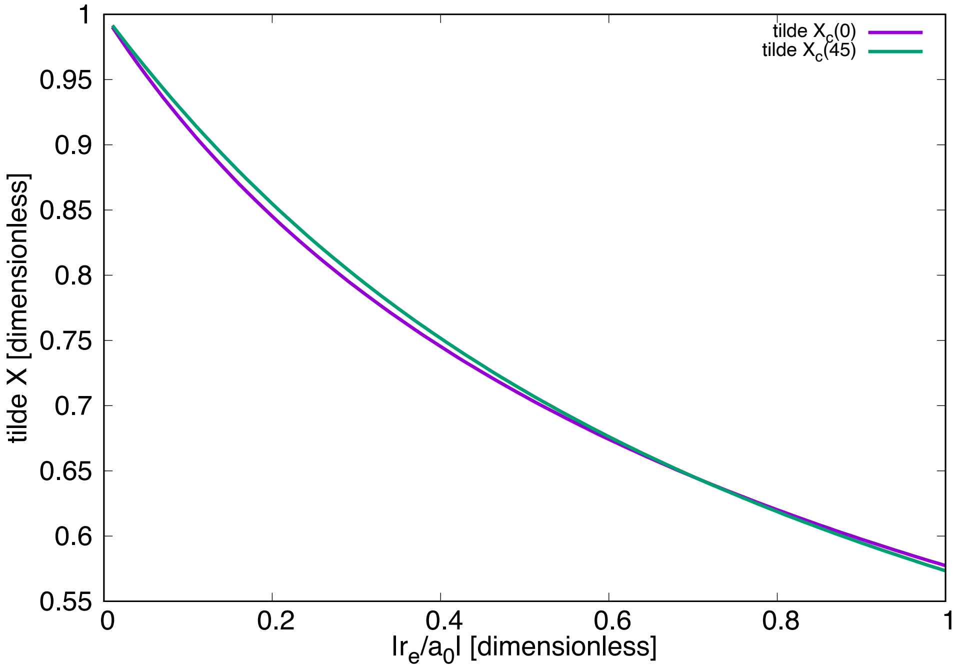
$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right) + \mathcal{O}\left(\left|\frac{l}{R}\right|^3\right)\right\}$$

$l \equiv (2\mu\nu)^{-1/2}$

ν (deference between the threshold energies)

Interpretation of X → $\tilde{X} \equiv \dfrac{1 - |1-X| + |X|}{2} \in \mathbb{R} \quad \because X \in \mathbb{C}$

$\tilde{X}$ for unstable and stable states



Application to the effective range model in the zero range limit

There are almost no differences between $\tilde{X}$ for unstable and stable states.

→ It is the feature of effective range model in the zero range limit.

Future prospect: Apply the improved relation to hadron systems.