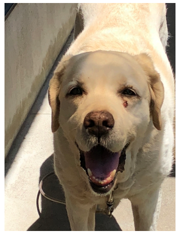
Range correction in the weak-binding relation



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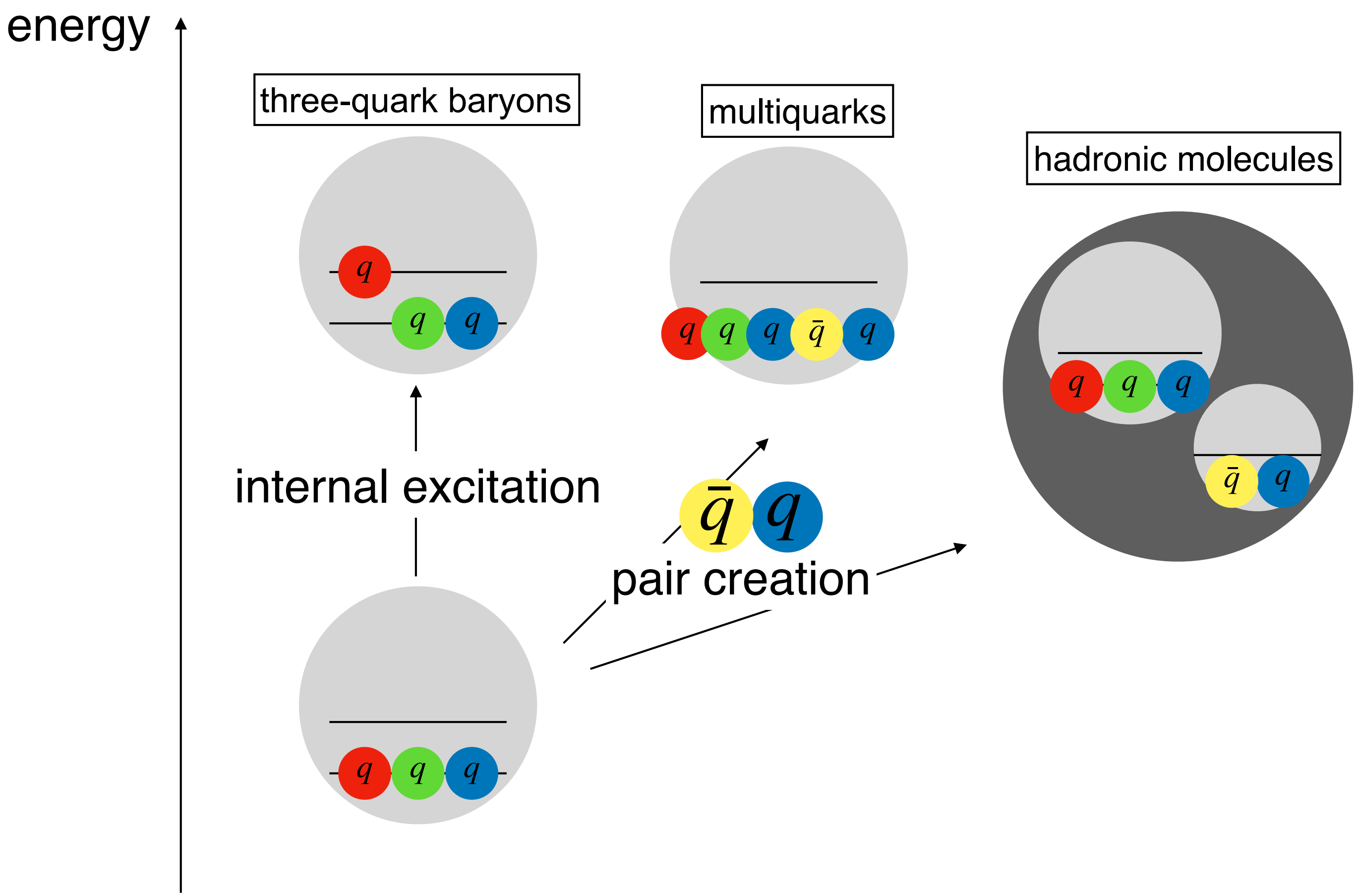
Background

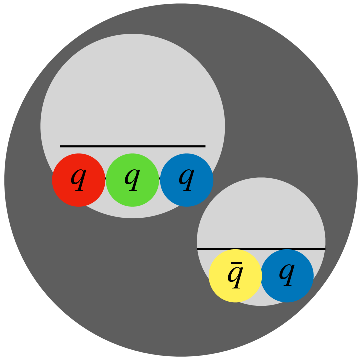
candidates for exotic hadrons

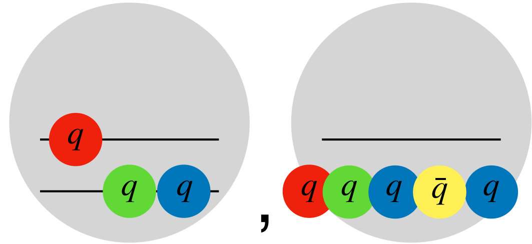
$\Lambda(1405)$, XYZ meson etc… → multiquarks / hadronic molecules



structure of hadrons ← model independent — observable

Previous work





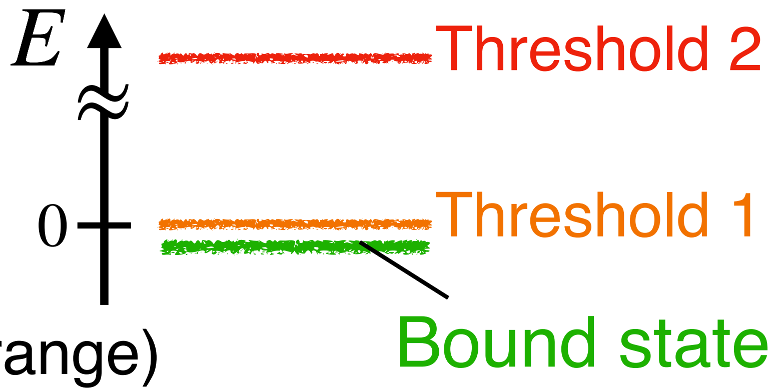
Hadron wave function

$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

Compositeness (weight of hadronic molecule)

Weak-binding relation for bound state

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$



a_0 (scattering length) $R_{\rm typ}$(interaction range)

$R \equiv (2\mu B)^{-1/2}$, B (binding energy)

When $R \gg R_{\rm typ}$: observable(a_0, B) → compositeness(X)

S. Weinberg, Phys. Rev. 137, B672 (1965); Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Motivation

Weak-binding relation $a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$

Low-energy universality → $a_0 = R \ (R \to \infty)$

-Deviation by contributions from other channels ← $X \neq 1$

-Deviation by interaction range ← $R_{\rm typ} \neq 0$

→ We study the range correction in the weak-binding relation by introducing the effective range r_e.

Range correction

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008), 0709.0499.

Effective range model $$\mathscr{H}_{\text{int}} = \frac{1}{4}\lambda_0(\psi^\dagger\psi)^2 + \frac{1}{4}\rho_0 \nabla(\psi^\dagger\psi)\cdot\nabla(\psi^\dagger\psi)$$

Properties of the effective range model:

-Single channel: $|\text{hadronic molecule}\rangle$ only $\Leftrightarrow X = 1$

-Zero range limit: $R_{\text{typ}} \to 0$

$$\Rightarrow a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\} \to R\ ?$$

Renormalized scattering amplitude ($R_{\text{typ}} \to 0$):

$$1/f(k = i/R) = 0$$

$$a_0 = R\frac{2r_e/R}{1-(r_e/R-1)^2} = R\left[1 + \mathcal{O}\left(\left|\frac{r_e}{R}\right|\right)\right] \Rightarrow a_0 \neq R$$

→ range correction in the weak-binding relation form r_e

Improved weak-binding relation

Weak-binding relation $a_0 = R\left\{ \frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right) \right\}$

interaction range: $R_{\rm typ}$ → $R_{\rm int}$

Redefinition of $R_{\rm typ}$:

$$R_{\rm typ} = \max\left\{ R_{\rm int}, R_{\rm eff} \right\},$$

$$R_{\rm eff} = \max\left\{ |r_e|, \frac{|P_s|}{R^2}, \cdots \right\}.$$

$$f(k) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 - \frac{P_s}{4}k^4 + \cdots - ik \right]^{-1}$$

Length scale in the effective range expansion except for a_0

Numerical calculation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

When dose the weak-binding relation work?

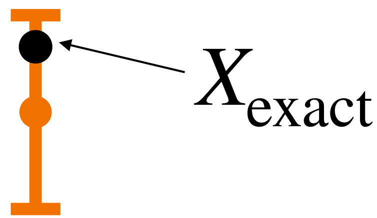
Estimation with correction terms ($\xi \equiv R_{\rm typ}/R$): Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Central value: $X_c = \dfrac{a_0/R}{2 - a_0/R}$

$$X_{\rm upper}(\xi) = \frac{a_0/R}{2-a_0/R} + \xi,\ X_{\rm lower}(\xi) = \frac{a_0/R}{2-a_0/R} - \xi\,.$$

Weak-binding relation works when…

$$X_{\rm lower} < X_{\rm exact} < X_{\rm upper}$$



→ Validity condition

Numerical calculation

Effective range model ($R_{\rm int} \neq 0$)

$f(k; \lambda_0, \rho_0, \Lambda) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 + \mathcal{O}\left(\frac{R_{\rm int}}{R}\right) - ik\right]^{-1}$ (two length scales r_e and $R_{\rm int}$)

$$1/f(k = i/R) = 0$$

- $r_e \neq 0$ (range correction): $\xi_{r_e} = |r_e/R|$ → Uncertainty from r_e

  $r_e < 0$ (effective range model)

- $R_{\rm int} \neq 0$: $\xi_{\rm int} = R_{\rm int}/R$. → Uncertainty from $R_{\rm int}$

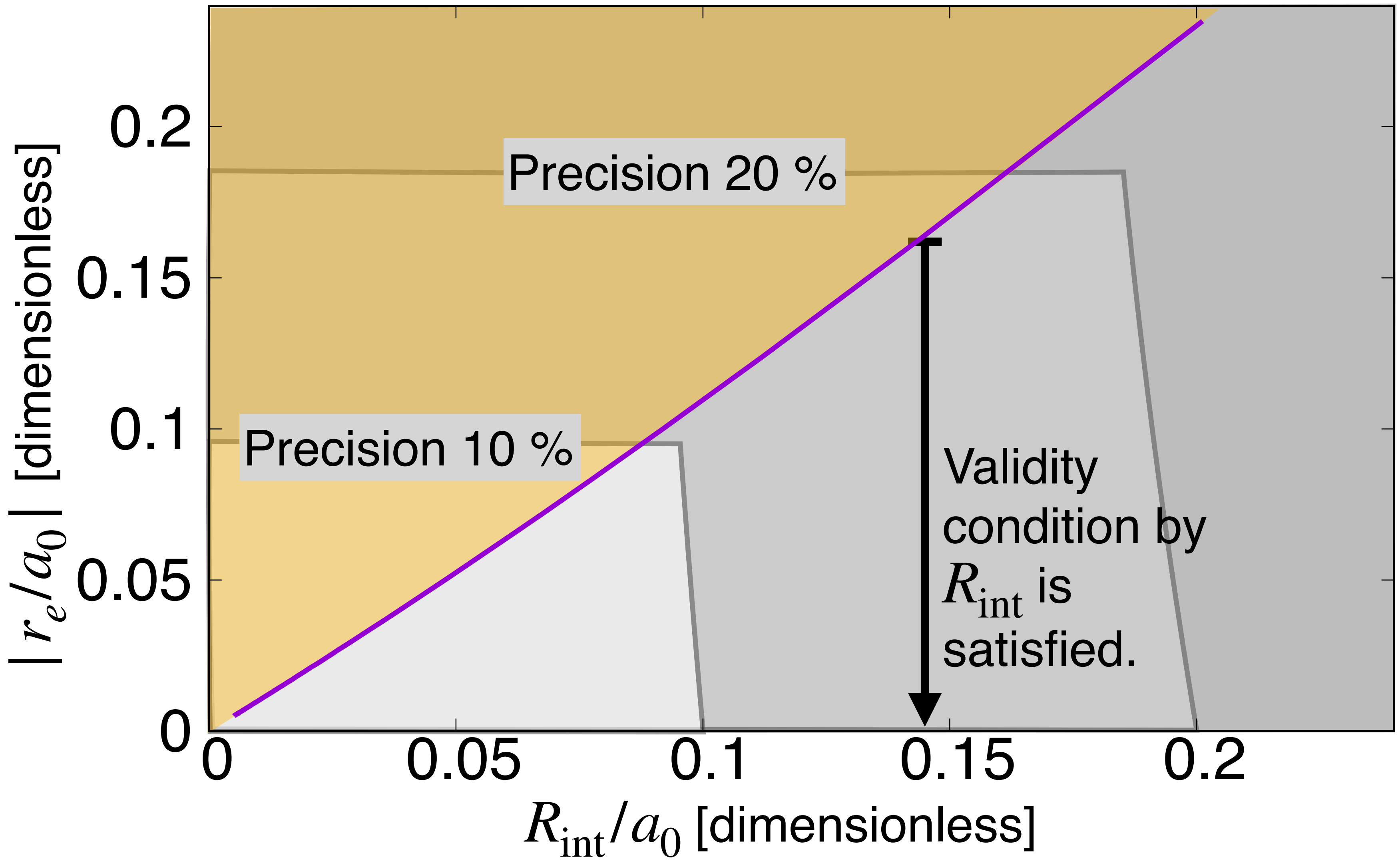
- $X_{\rm exact} = 1$

→ We search for the region of r_e and $R_{\rm int}$ in which validity condition are satisfied.

Numerical calculation

Validity condition by r_e is satisfied in all region of this plot.

Validity condition in R_{int}/a_0-$|r_e/a_0|$ plane



Only the improved weak-binding relation can be applied.

Discussion of the range correction

What is the range correction for the central value ?

From effective range model ($X = 1$)

$$a_0 = R\frac{2r_e/R}{1-(r_e/R-1)^2} = R\left[1+\mathcal{O}\left(\left|\frac{r_e}{R}\right|\right)\right] \quad \Rightarrow a_0 \neq R$$

$$\rightarrow \quad \bar{R} \equiv R\frac{2r_e/R}{1-(r_e/R-1)^2}$$

$$\rightarrow \quad a_0 = \bar{R} \quad \text{and} \quad X = 1$$

$$\rightarrow \quad a_0 = \bar{R}\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm int}}{\bar{R}}\right)\right\} \ ?$$

↑ Range correction for the central value ?

Discussion of the range correction

What are the origins of the effective range?

$$\mathscr{H}_{\text{int}} = \frac{1}{4}\lambda_0(\psi^\dagger\psi)^2 + \frac{1}{4}\rho_0\nabla(\psi^\dagger\psi)\cdot\nabla(\psi^\dagger\psi) + \frac{1}{2}g_0(\phi^\dagger\psi^2 + \psi^{\dagger 2}\phi)$$

derivative couplings — $X = 1$

Bare field ϕ — $X \neq 1$

One of the origins of the deviation of a_0 form R

$$r_e = \frac{16\pi}{m}\frac{[1+(m/12\pi^2)\rho_0\Lambda^3]^2\{2\rho_0[1+(m/24\pi^2)\rho_0\Lambda^3] - g_0{}^2/(m\nu_0{}^2)\}}{\left[\lambda_0 - (m/20\pi^2)\rho_0^2\Lambda^5 - \frac{g_0^2}{\nu_0}\right]^2}$$

ν_0 : bare mass Λ: cut off

- Both derivative coupling ($X = 1$, ρ_0 term) and contribution from bare field ($X \neq 1$, g_0 term) can be the origins of r_e

Discussion of the range correction

If these terms were separable…

$$r_e = \text{Contribution of derivative couplings} + \text{Contribution of bare field } \phi$$

we would consider the range correction for X_c

$$a_0 = \bar{R}\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{\bar{R}}\right)\right\}$$

Contribution of derivative couplings ($\bar{R}$)

Contribution of bare field ϕ ($\frac{2X}{1+X}$)

However we can not consider the range correction for the central value because r_e is not separable.

$$r_e = \frac{16\pi}{m}\frac{[1+(m/12\pi^2)\rho_0\Lambda^3]^2\{2\rho_0[1+(m/24\pi^2)\rho_0\Lambda^3] - g_0{}^2/(mv_0{}^2)\}}{\left[\lambda_0 - (m/20\pi^2)\rho_0^2\Lambda^5 - \frac{g_0^2}{v_0}\right]^2}$$

Conclusion and future prospect

- Weak-binding relation : observable → compositeness (X)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\text{typ}}}{R}\right)\right\}$$

- We study the range correction in weak-binding relation from r_e.

- Improved weak-binding relation by redefinition of R_{typ} :

$$R_{\text{typ}} = \max\left\{R_{\text{int}}, |r_e|, \cdots\right\}$$

- We find the region where only the improved weak-binding relation can be applied.

- There are two origins of the range correction; one related to X and the other unrelated to X.

- Future prospect: Apply the improved relation to hadron systems.