

Internal structure of T_{cc} and $X(3872)$ using compositeness



T. Kinugawa and T. Hyodo
Phys. Rev. C 109 , 045205
(2024).

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].



Tomona Kinugawa

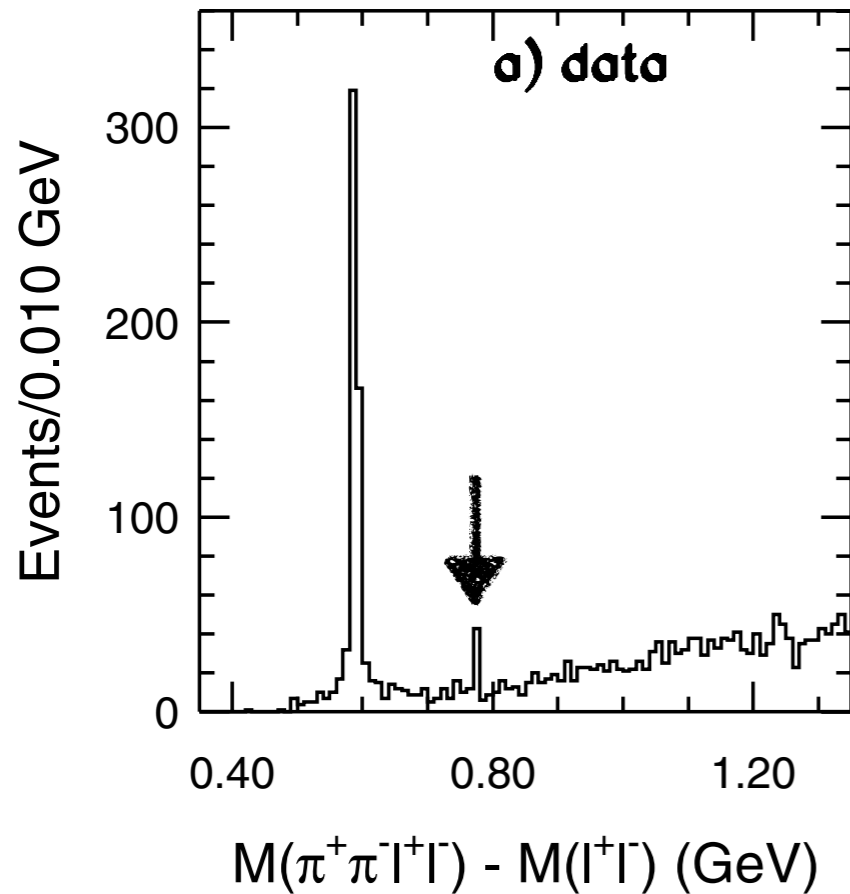
Tetsuo Hyodo

Department of Physics, Tokyo Metropolitan University

August 22nd, Confinement 2024

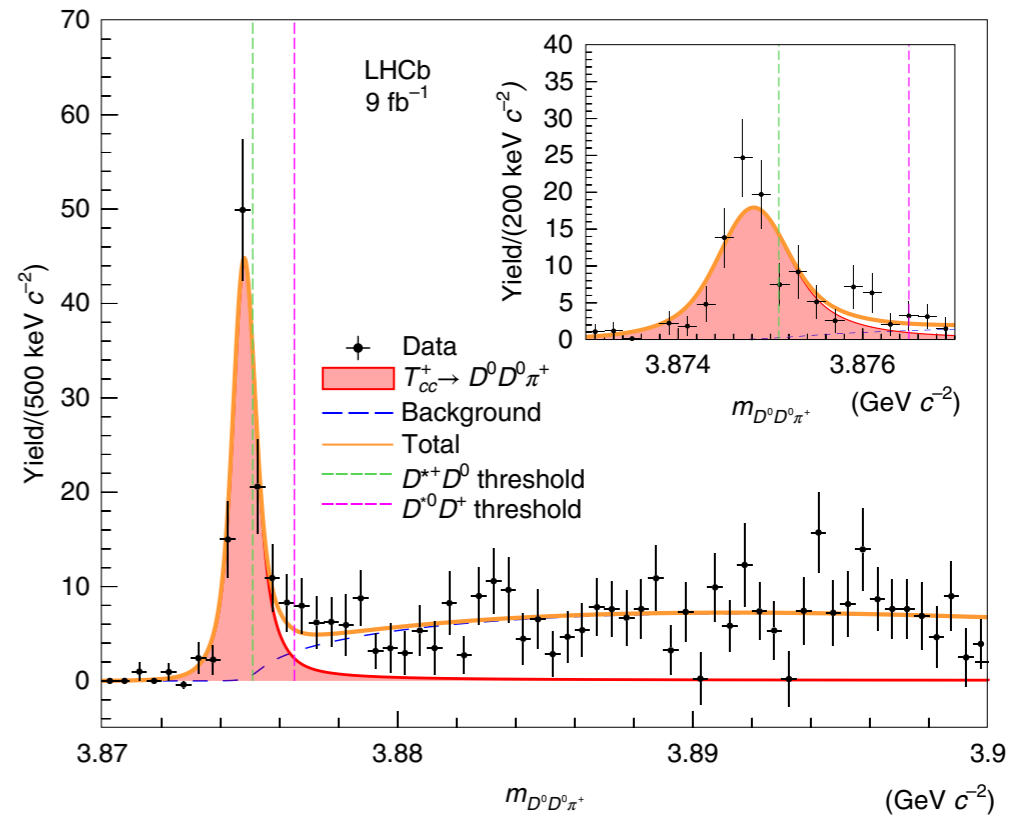
Near-threshold exotic hadrons

$$X(3872) \rightarrow \pi^+ \pi^- J/\psi$$



S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

$$T_{cc} \rightarrow D^0 D^0 \pi^+ (cc\bar{u}\bar{d})$$



LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

internal structure?

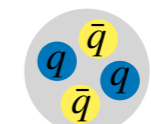
exotic hadron

$\neq qqq$ or $q\bar{q}$

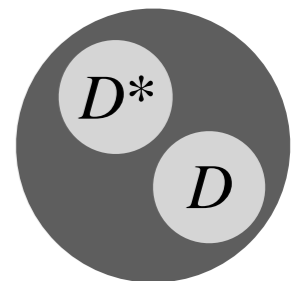


multiquarks

hadronic molecules



multiquarks



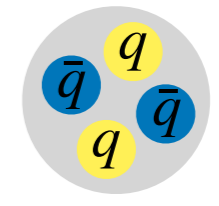
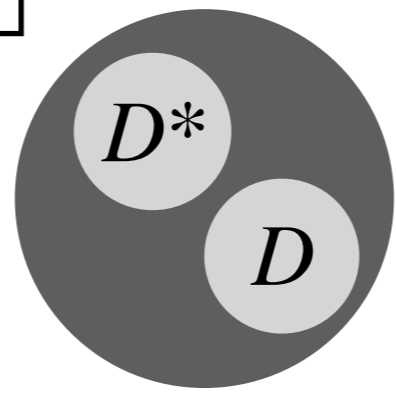
hadronic molecules

Compositeness

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).

● definition

hadron wavefunction



$$|\Psi\rangle = \sqrt{X} |\text{hadronic molecule}\rangle + \sqrt{1-X} |\text{others}\rangle$$

compositeness

elementarity

* $0 \leq X \leq 1 \longrightarrow X > 0.5 \Leftrightarrow$ **composite dominant**
 $X < 0.5 \Leftrightarrow$ **elementary dominant**

- **quantitative** analysis of internal structure

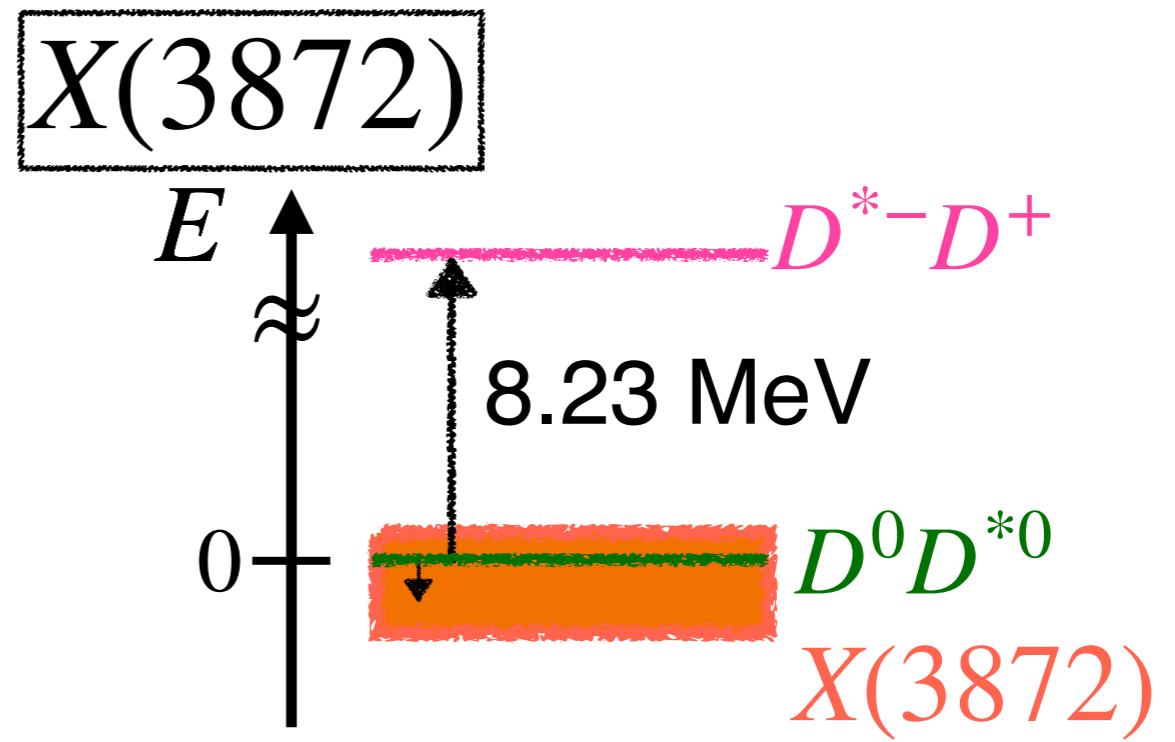
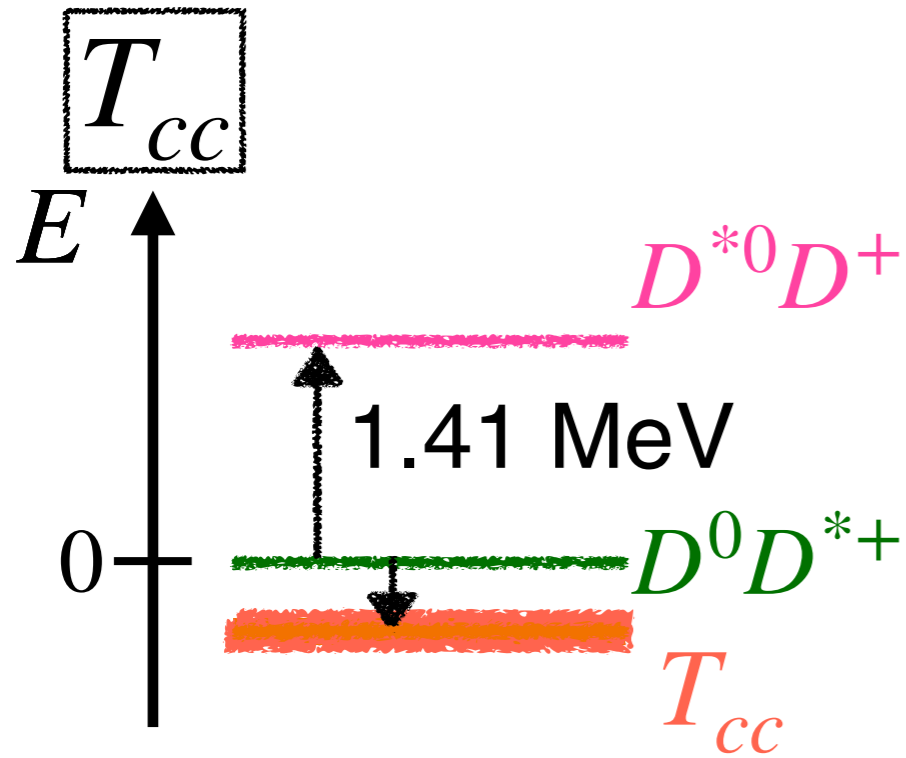
deuteron is not an elementary particle Weinberg, S. Phys. Rev. 137, 672–678 (1965).

$f_0(980), a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016);
 T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.

$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ;
 Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

Near-threshold states

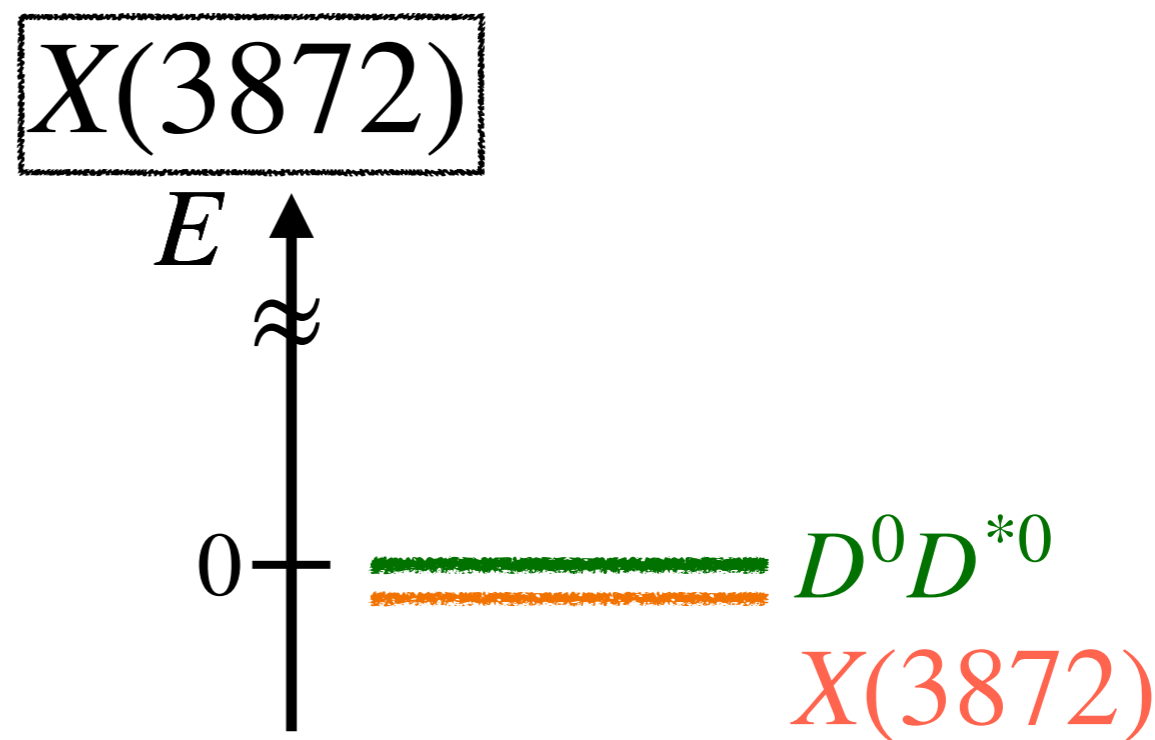
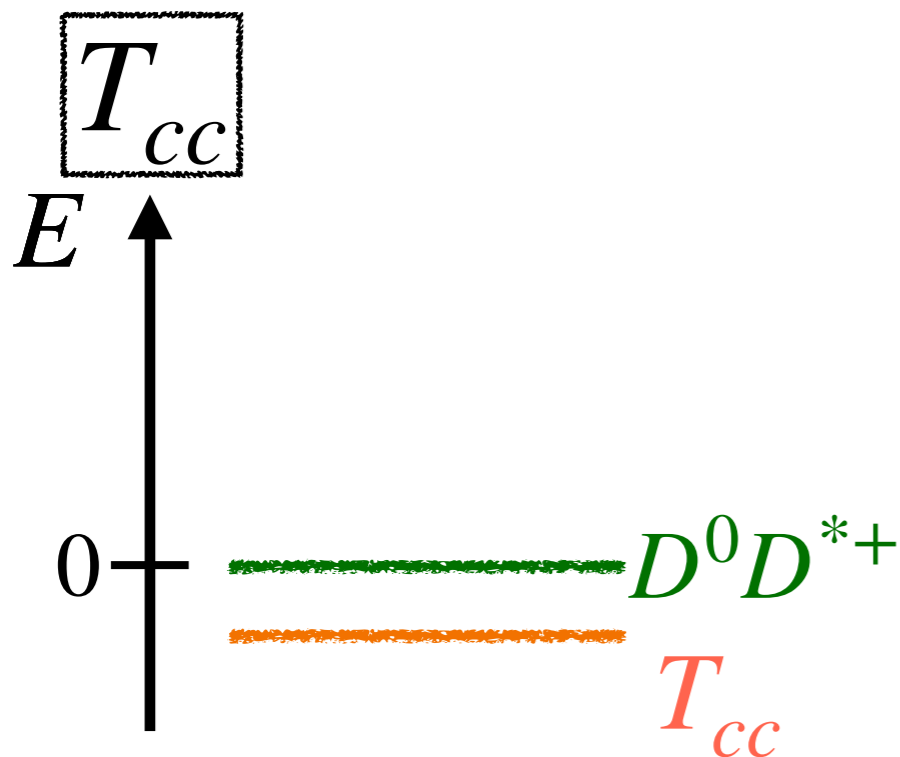


LHCb Collaboration, Nat. Commun **13** 3351 (2022).

PDG

Near-threshold states

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LHCb Collaboration, Nat. Commun **13** 3351 (2022).

PDG

- compositeness $X = 1$ in $B \rightarrow 0$ limit (universality)

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

shallow b.s. ($B \neq 0$) seems to be composite dominant

- however, elementary dominant states is realized with fine tuning

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;

C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014).

→ How finely tuning parameter?

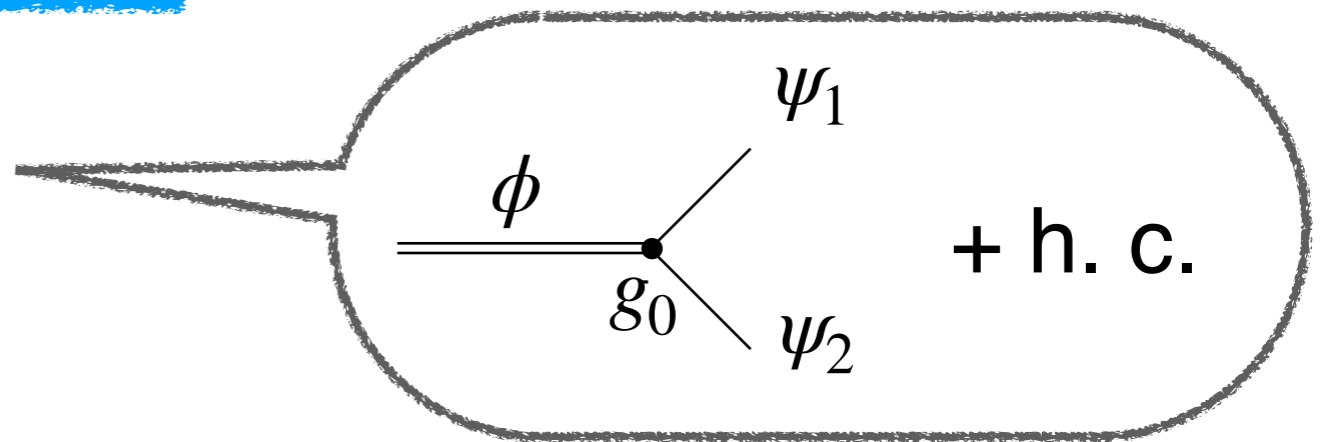
● single-channel resonance model

$$\mathcal{H}_{\text{free}} = \frac{1}{2m_1} \nabla \psi_1^\dagger \cdot \nabla \psi_1 + \frac{1}{2m_2} \nabla \psi_2^\dagger \cdot \nabla \psi_2 + \frac{1}{2m_\phi} \nabla \phi^\dagger \cdot \nabla \phi + \nu_0 \phi^\dagger \phi,$$

1.

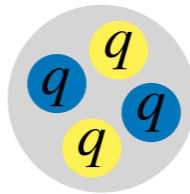
$$\mathcal{H}_{\text{int}} = g_0 (\phi^\dagger \psi_1 \psi_2 + \psi_1^\dagger \psi_2^\dagger \phi).$$

2.



1. single-channel scattering

2. coupling to bare state ϕ



● scattering amplitude

$$V = \frac{g_0^2}{E - \nu_0}, \quad G = -\frac{\mu}{\pi^2} \left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right) \right]. \quad \Lambda : \text{cutoff}$$

$$\xrightarrow{T = \frac{1}{V^{-1} - G}} f(k) = -\frac{\mu}{2\pi} \left[\frac{\frac{k^2}{2\mu} - \nu_0}{g_0^2} + \frac{\mu}{\pi^2} \left[\Lambda + ik \arctan\left(\frac{\Lambda}{-ik}\right) \right] \right]^{-1}.$$

Model scales and parameters

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● calculation of compositeness with given B

- typical energy scale : $E_{\text{typ}} = \Lambda^2 / (2\mu)$

1. reduce d.o.f. of model parameters g_0, ν_0, Λ

coupling const. g_0 : $g_0^2(B, \nu_0, \Lambda) = \frac{\pi^2}{\mu} (B + \nu_0) \left[\Lambda - \kappa \arctan(\Lambda/\kappa) \right]^{-1}$

$$\kappa = \sqrt{2\mu B}.$$

2. use dimensionless quantities with Λ

→ absorb Λ dependence

3. energy of bare quark state ν_0

varied in the region : $-B/E_{\text{typ}} \leq \nu_0/E_{\text{typ}} \leq 1$

∴ to have $g_0^2 \geq 0$ & applicable limit of model

Compositeness in model

● compositeness X

scattering amplitude : $T = \frac{1}{V^{-1} - G}$ Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

$$\begin{aligned} \longrightarrow X &= \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, & \alpha'(E) &= d\alpha/dE \\ &= \left[1 + \frac{\pi^2 \kappa}{g_0^2 \mu^2} \left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1 + (\Lambda/\kappa)^2} \right)^{-1} \right]^{-1}. \end{aligned}$$

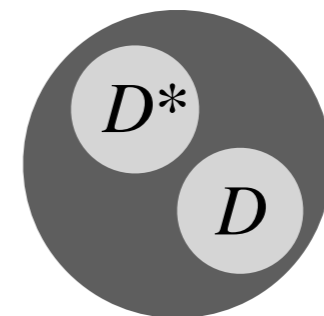
- fix B to consider typical and shallow bound states

- ν_0 region : $-B/E_{\text{typ}} \leq \nu_0/E_{\text{typ}} \leq 1$

↔ model dependence of X

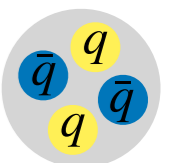
compositeness X as a function of ν_0

$X > 0.5$



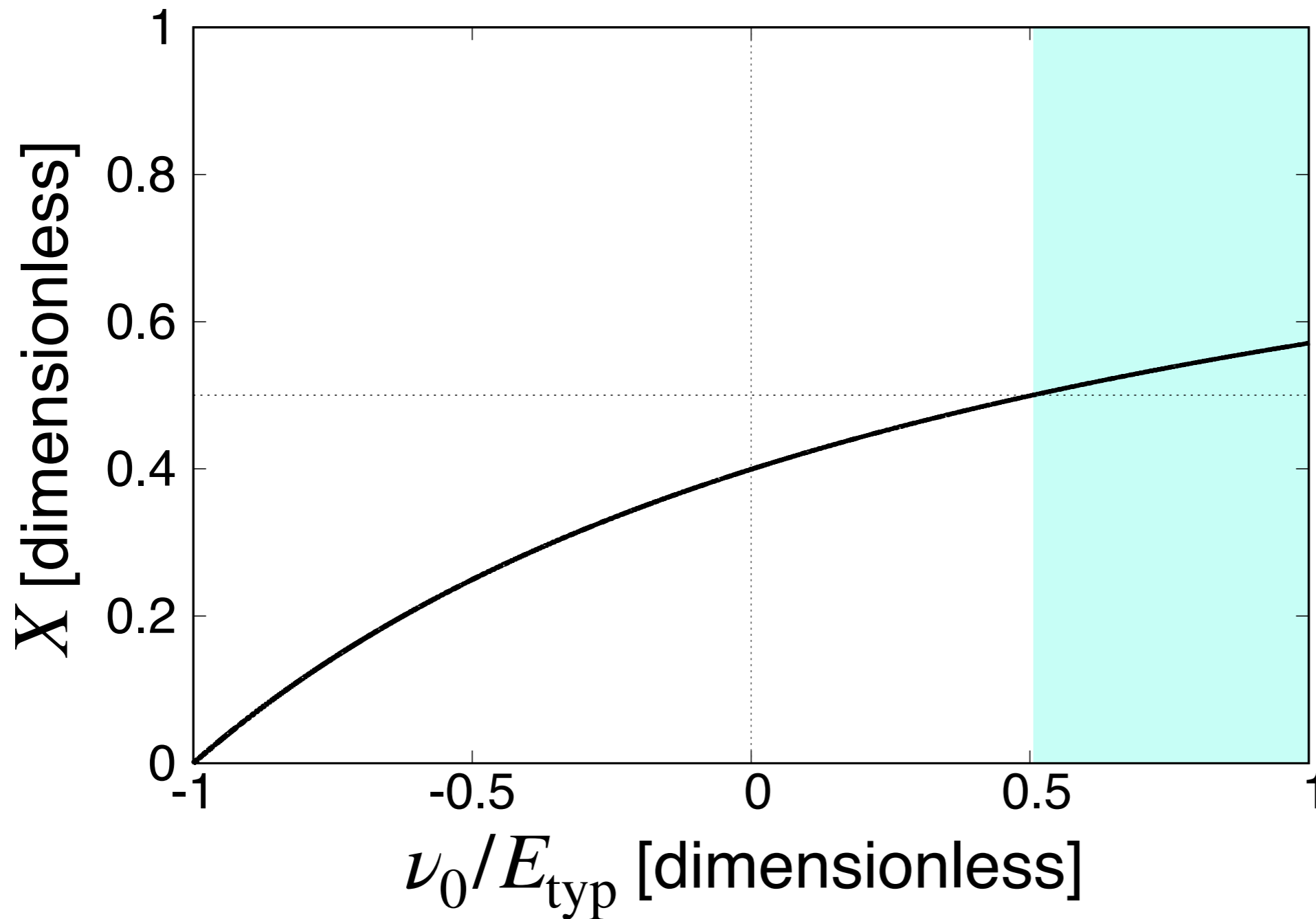
or

$X < 0.5$



→ model dependence of structure of bound state?

● X as a function of ν_0/E_{typ} of bound state $B = E_{\text{typ}}$



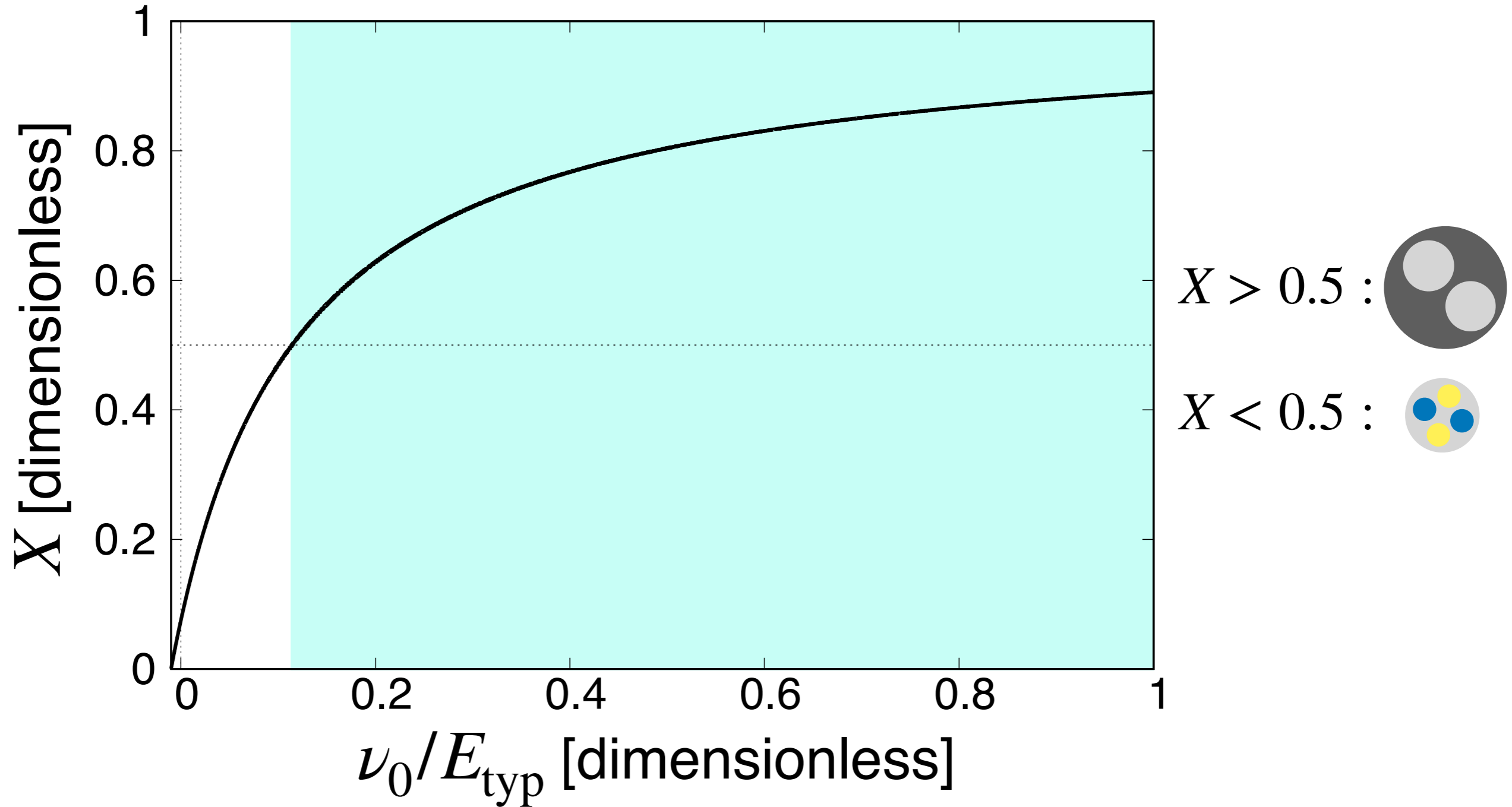
$X > 0.5$:

$X < 0.5$:

- typical energy scale : $B = E_{\text{typ}} = \Lambda^2/(2\mu)$

- $X > 0.5$ only for 25 % of ν_0 \therefore **bare state origin**

● X as a function of ν_0/E_{typ} of bound state $B = 0.01E_{\text{typ}}$

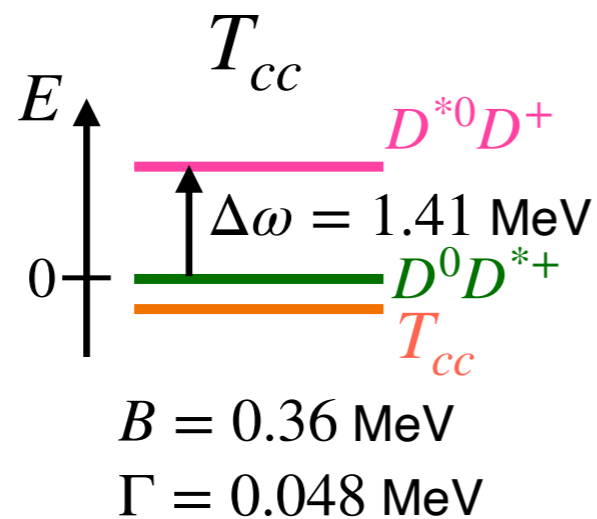


- weakly-bound state : $B = 0.01E_{\text{typ}}$
- $X > 0.5$ for 88 % of ν_0 \longrightarrow **realization of universality**
- elementary dominant state can be realized with fine tuning

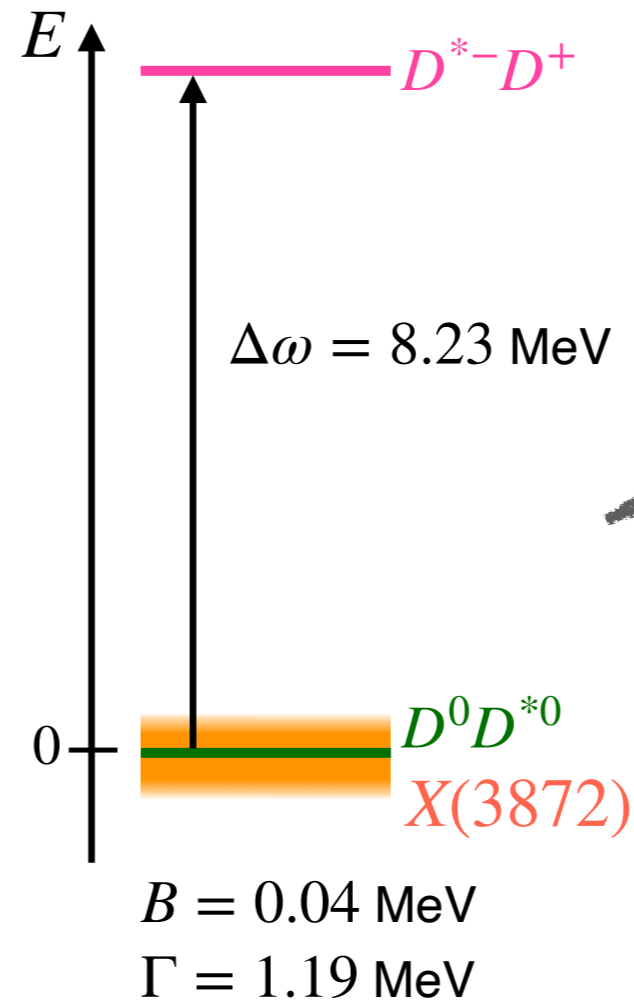
Application to T_{cc} and $X(3872)$

● exotic hadron ← decay and coupled channel
 $X(3872)$

small
 Γ and $\Delta\omega$



LHCb Collaboration, Nat. Commun **13** 3351 (2022).



PDG

large
 Γ and $\Delta\omega$

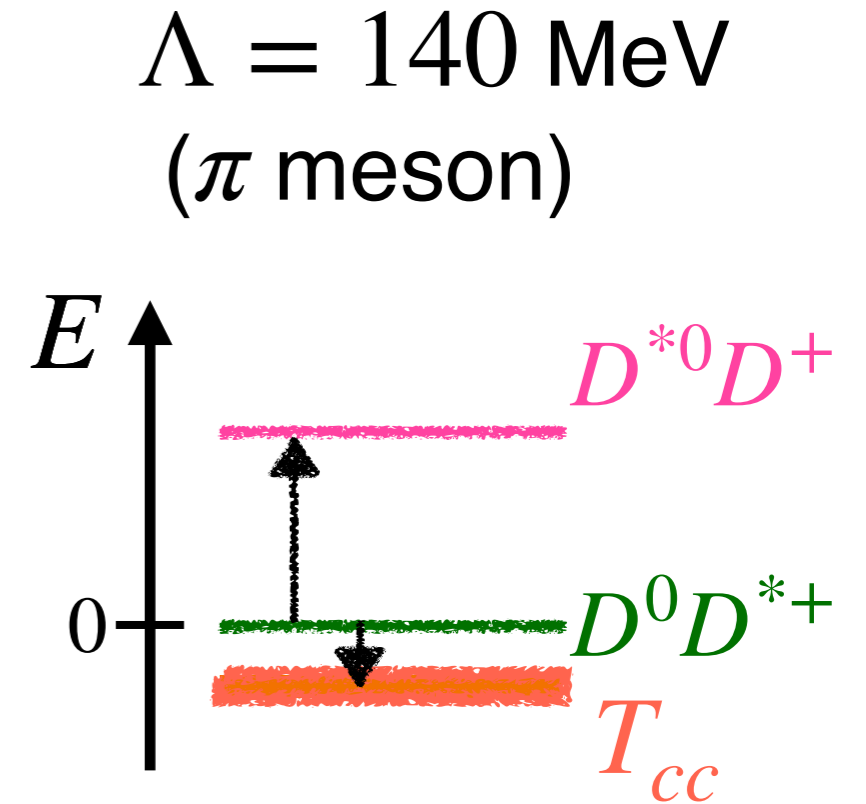
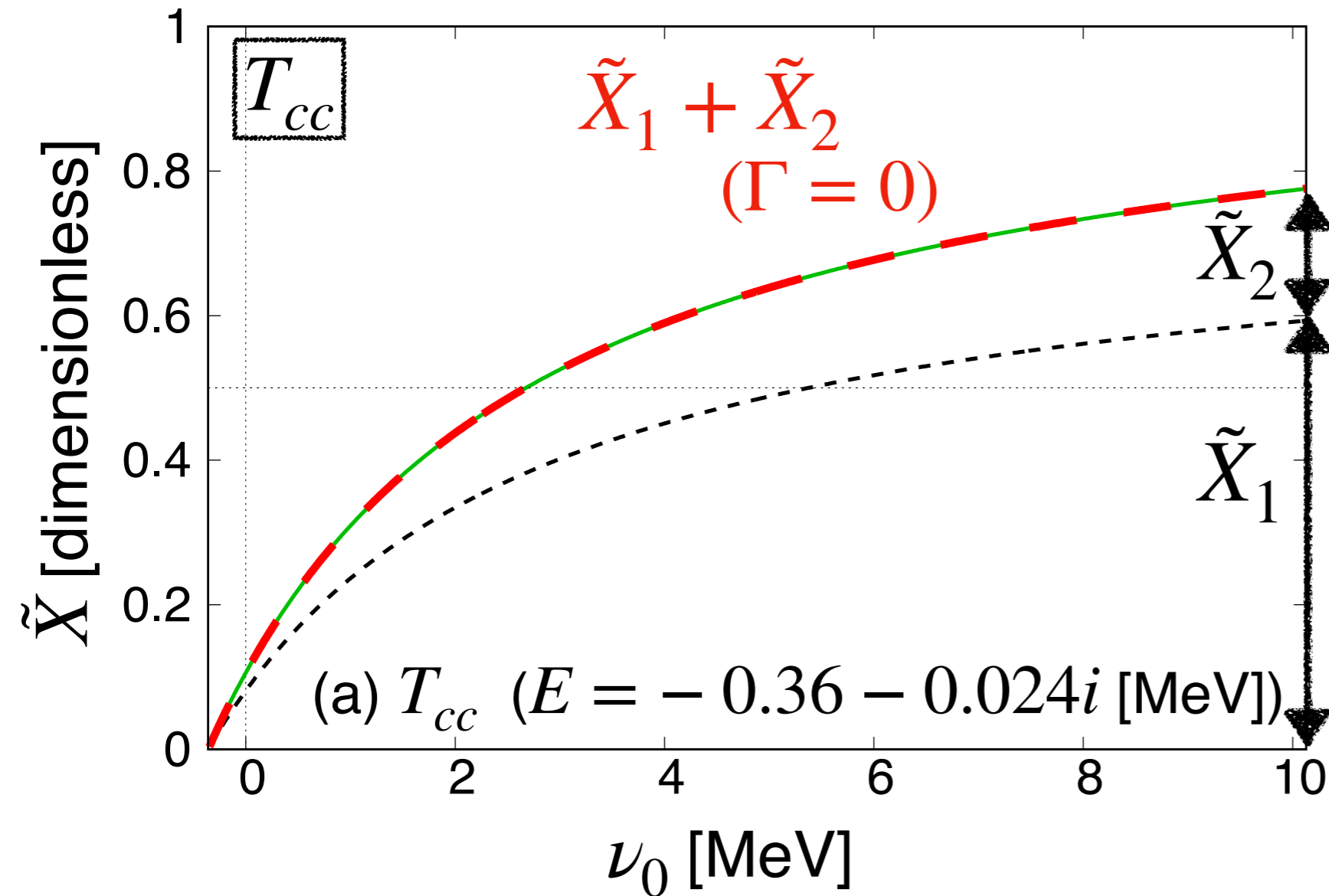
● compositeness T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC 93, 035204 (2016).

$$\tilde{X}_j = \frac{|X_j|}{\sum_j |X_j| + |Z|}, \quad (j = 1, 2)$$

\tilde{X}_1 : threshold ch. compositeness

\tilde{X}_2 : coupled ch. compositeness

Application to T_{cc}



- \tilde{X}_2 is not negligible

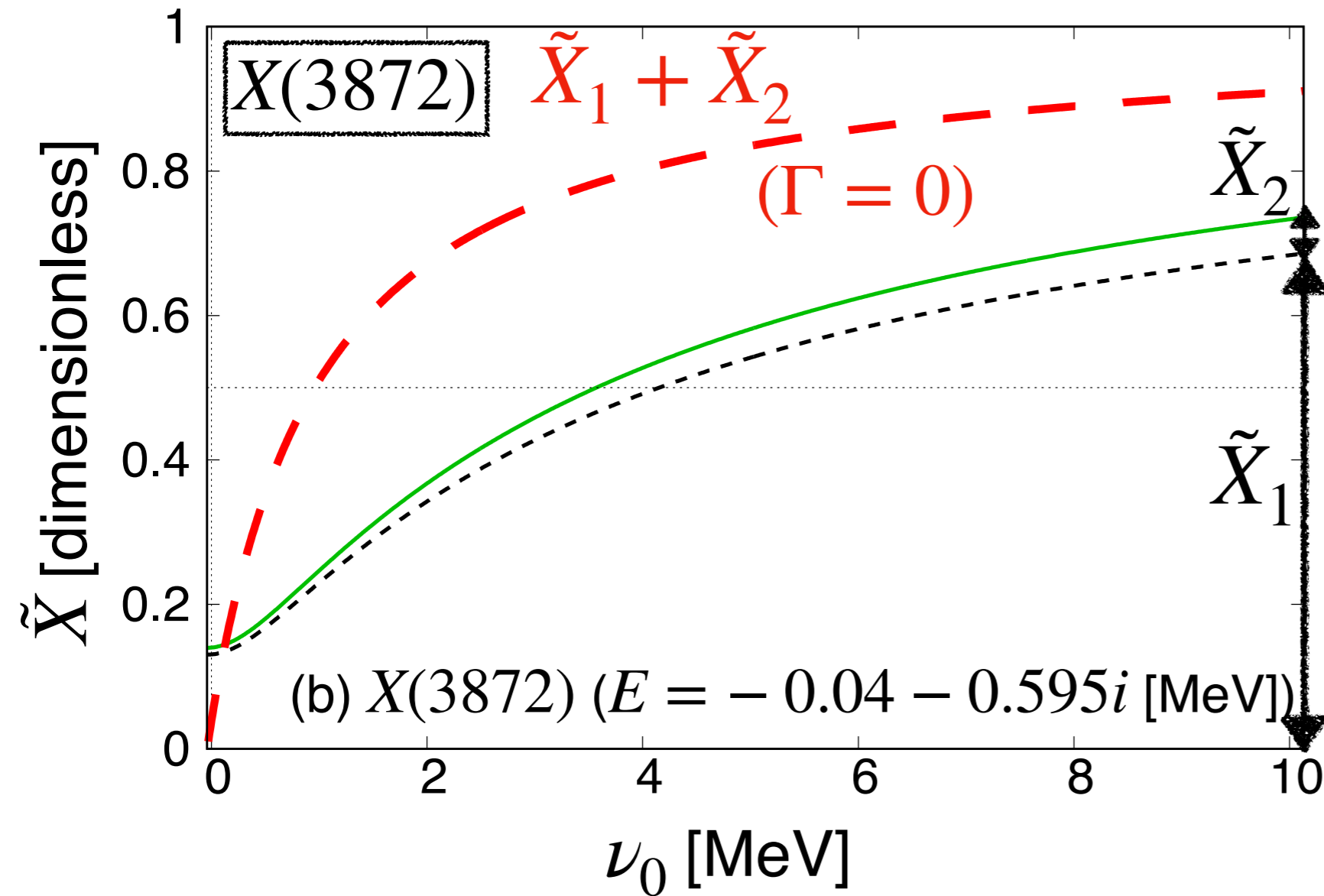
\because coupled ch. contribution (small $\Delta\omega$)

- difference of $\tilde{X}_1 + \tilde{X}_2$ ($\Gamma = 0$) and $\tilde{X}_1 + \tilde{X}_2$ is too small

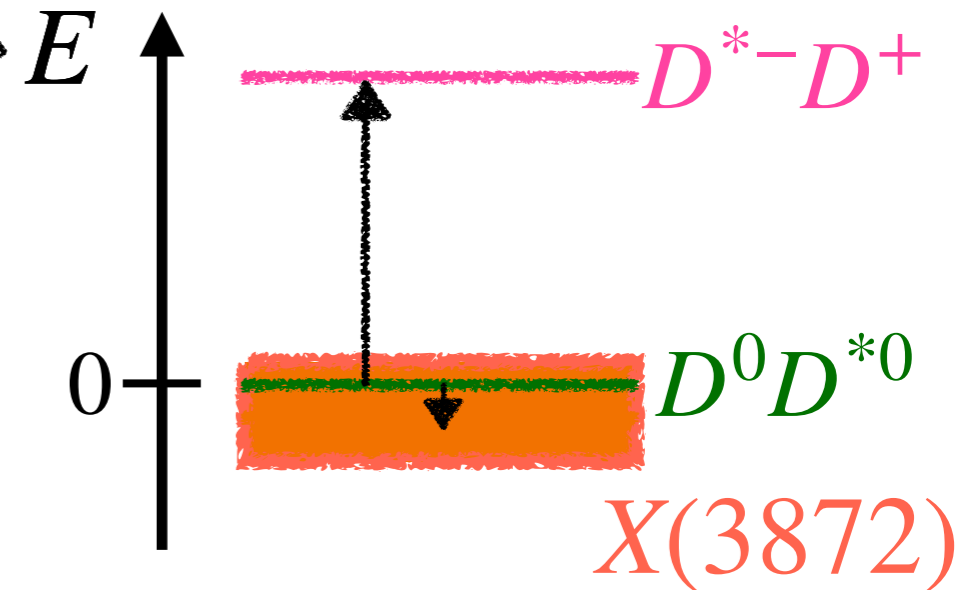
\longrightarrow We can neglect decay contribution

$\because \Gamma \ll B$

Application to $X(3872)$



$\Lambda = 140$ MeV
(π meson)



- difference of $\tilde{X}_1 + \tilde{X}_2(\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is large

∴ large decay width contribution

- \tilde{X}_2 is much smaller than \tilde{X}_1

→ coupled ch. effect is small

New interpretation scheme

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

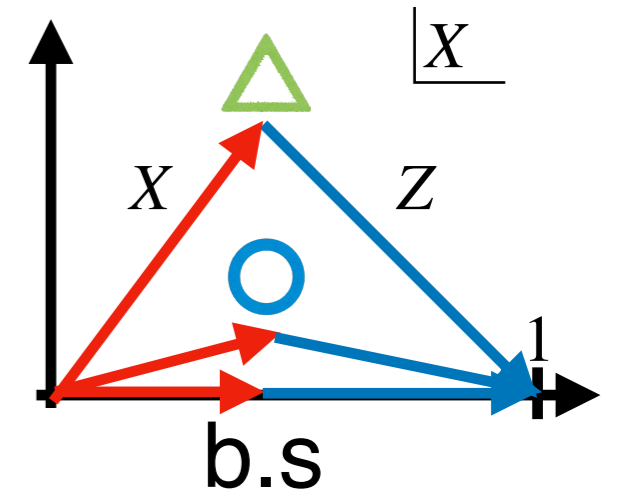
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- interpretation for complex X of resonances?

$$X \in \mathbb{C} \text{ and } X+Z = 1$$

- If $\text{Im } X$ is large, it seems that reasonable interpretation is impossible \triangle

complex X plane



● our proposal

T. Berggren, Phys. Lett. B 33, 547 (1970).

- i) \mathcal{X} : probability of certainly finding $|\text{composite}\rangle$
- ii) \mathcal{F} : probability of certainly finding $|\text{elementary}\rangle$
- iii) \mathcal{Y} : probability of uncertain identification

● conditions for sensible interpretation

- normalization : $\mathcal{X} + \mathcal{Y} + \mathcal{F} = 1$ for probabilistic interpretation

- in bound state limit : $\mathcal{X} \rightarrow X$, $\mathcal{F} \rightarrow Z$ and $\mathcal{Y} \rightarrow 0$

\mathcal{Y} characterizes uncertainty of resonance

structure of T_{cc} & $X(3872)$

● compositeness X in previous model with our scheme

- fix ν_0 from quark models

- $X_2 + Z \rightarrow Z$

T_{cc} $\nu_0 = 7 \text{ MeV}$ M. Karliner and J. L. Rosner, Phys. Rev. Lett. 119, 202001 (2017).

$$X_{D^0 D^{*+}} = 0.541 - 0.007i$$

$$\rightarrow \mathcal{X} = 0.537, \mathcal{Y} = 0.008, \mathcal{Z} = 0.456$$

higher channel contribution

$X(3872)$ $\nu_0 = 78.36 \text{ MeV}$ S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).

$$X_{D^0 D^{0*}} = 0.919 - 0.079i$$

$$\rightarrow \mathcal{X} = 0.890, \mathcal{Y} = 0.028, \mathcal{Z} = 0.081$$

large bare state energy $\nu_0 \gg E_{\text{typ}}$

structure of T_{cc} & $X(3872)$

● compositeness X in previous model with our scheme

- fix ν_0 from quark models

- $X_2 + Z \rightarrow Z$

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Summary

T. Kinugawa and T. Hyodo Phys. Rev. C 109 , 045205 (2024).

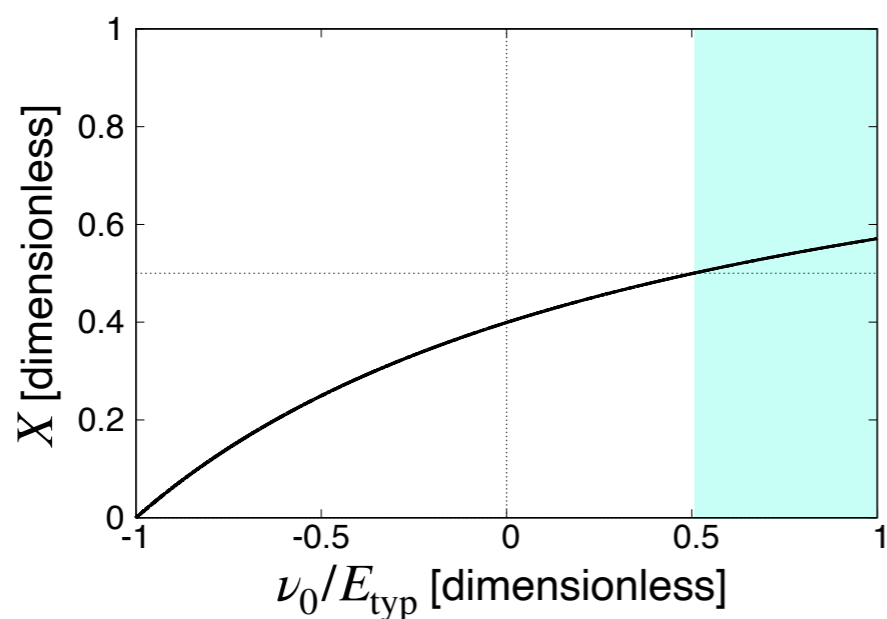
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T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

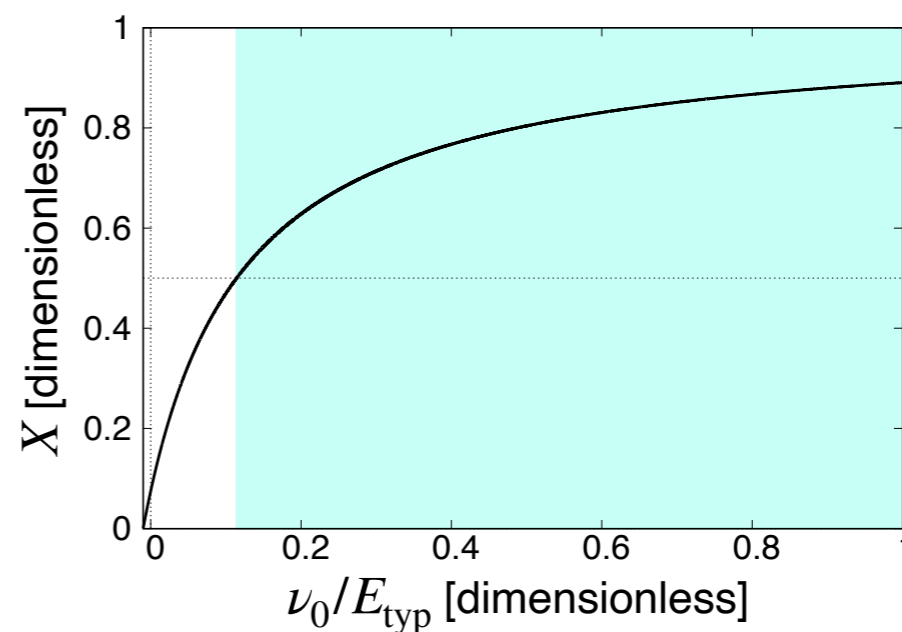
- internal structure of exotic hadrons ← compositeness

- shallow bound states

→ fine tuning is necessary to realize elementary dominant state



typical b.s.



shallow b.s.

T_{cc} : important coupled ch. effect with negligible decay effect

$X(3872)$: important decay effect with negligible coupled ch. effect



Back up

Universality for near-th. resonances ¹⁸

- near-threshold **bound** (and virtual) states

$a_0 \rightarrow \infty$ and universality holds in $B \rightarrow 0$ limit 

$\rightarrow X \rightarrow 1$ (completely composite)

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
T. Kinugawa, T. Hyodo, 2303.07038 [hep-ph].

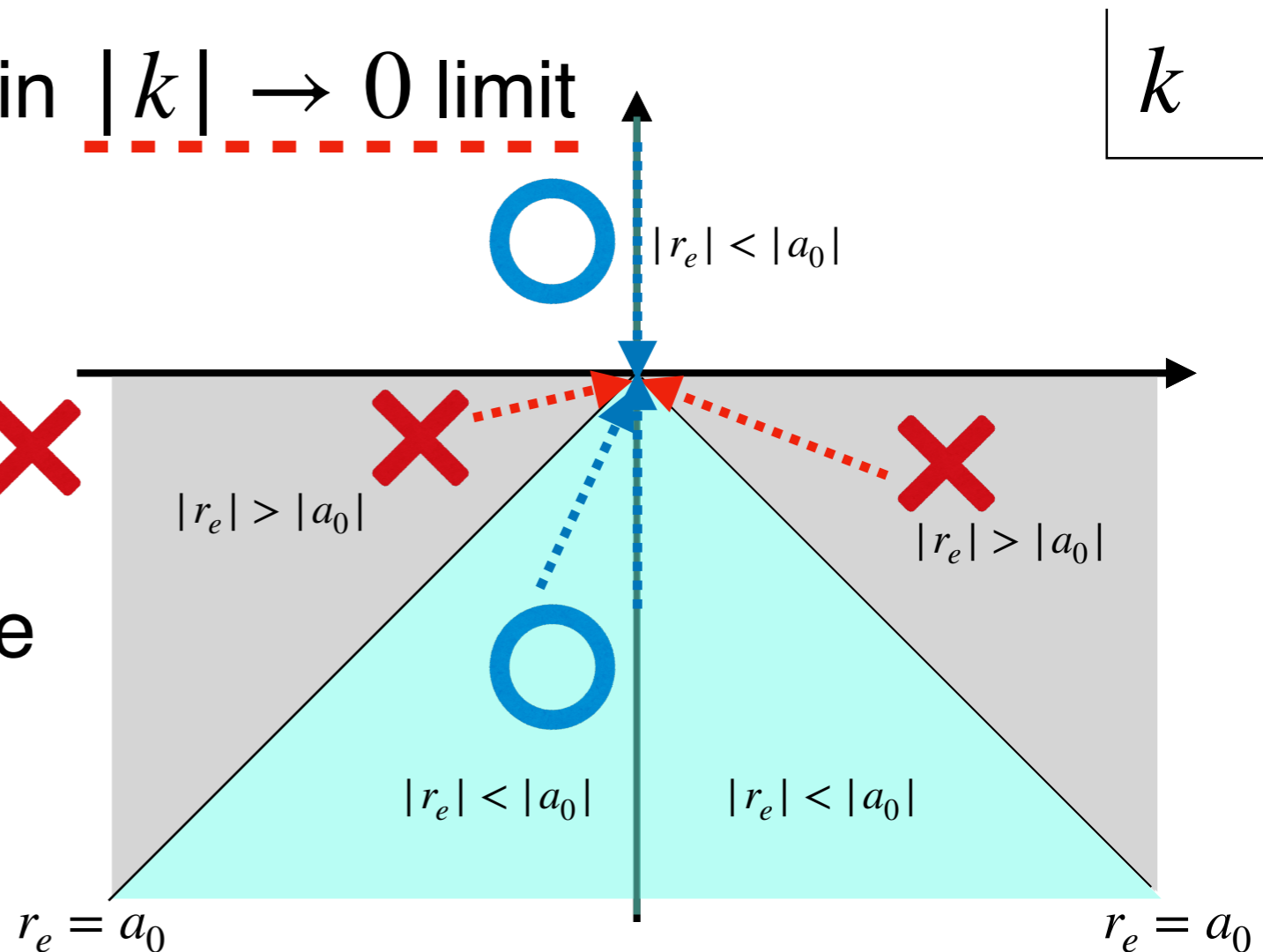
- near-threshold **resonances**

$a_0 \rightarrow \infty$ but also $|r_e| \rightarrow \infty$ in $|k| \rightarrow 0$ limit

$$\because |a_0| \leq |r_e|$$

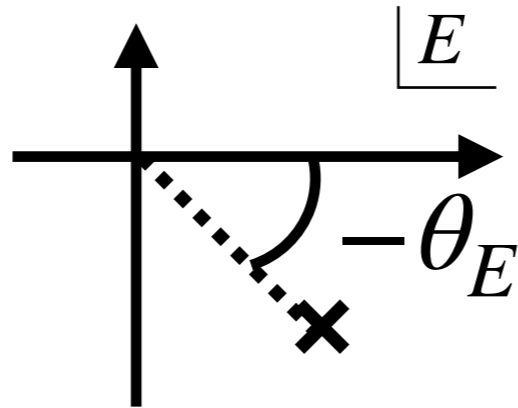
\rightarrow universality does not hold 

Near-threshold resonances are **not** necessarily composite dominant



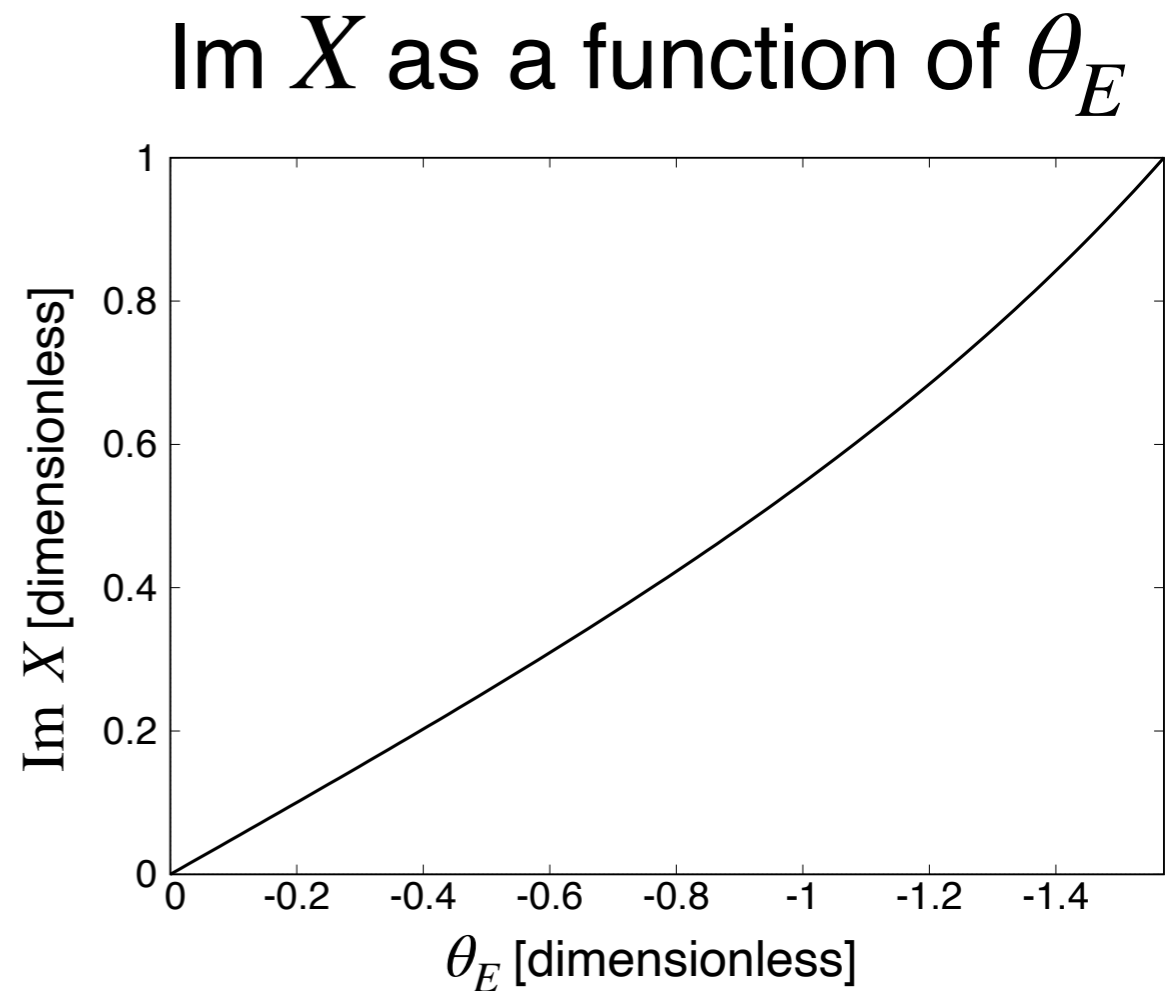
Compositeness in ERE

$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan \theta_k = -i \tan(\theta_E/2)$$



$$(k = |k| e^{i\theta_k}, E = |E| e^{i\theta_E})$$

→ X in ERE is pure imaginary



- in general, compositeness X of unstable resonances becomes **complex** by definition

- complex X **cannot** be directly interpreted as a probability



Complex compositeness

- probabilistic interpretation?

$$X \in \mathbb{C} \text{ and } X+Z = 1$$

- If $\text{Im } X$ is large, it seems that reasonable interpretation is impossible $\times \triangle$

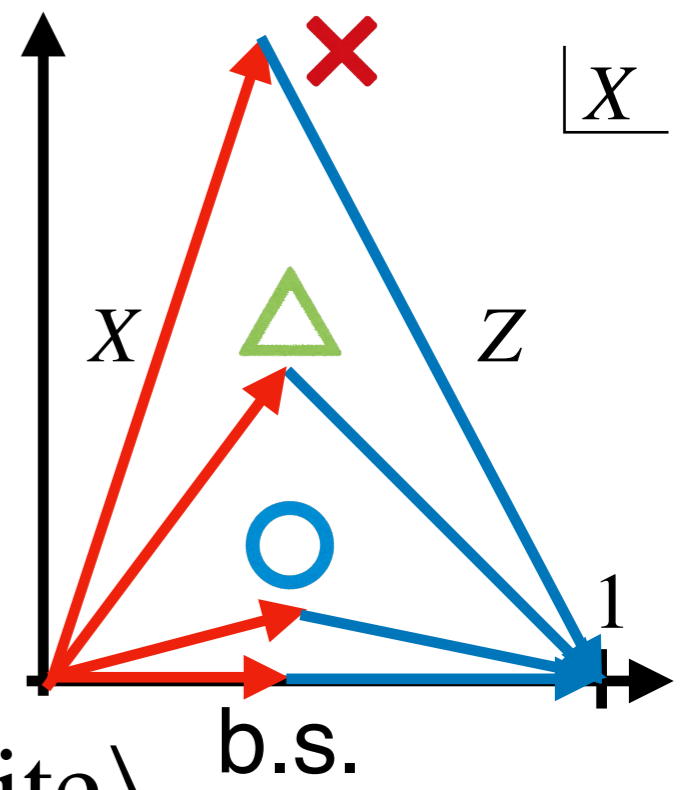
- our proposal

i) \mathcal{X} : probability of certainly finding $|\text{composite}\rangle$

ii) \mathcal{F} : probability of certainly finding $|\text{elementary}\rangle$

iii) \mathcal{Y} : probability of uncertain identification

complex X plane

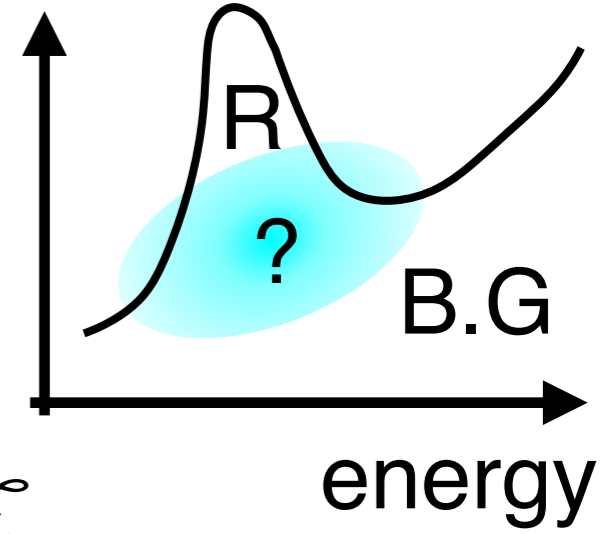


uncertain appears from T. Berggren, Phys. Lett. B 33, 547 (1970).

- finite lifetime (uncertainty in energy)

- separation from B.G.

spectrum



$$\text{complex compositeness } X \in \mathbb{C} \longrightarrow \mathcal{X}, \mathcal{Y}, \mathcal{F}$$

Definition

● conditions for sensible interpretation

- normalization : $\mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathcal{X} \rightarrow X, \mathcal{Z} \rightarrow Z$ and $\mathcal{Y} \rightarrow 0$

\mathcal{Y} characterizes uncertainty of resonance

● new interpretation

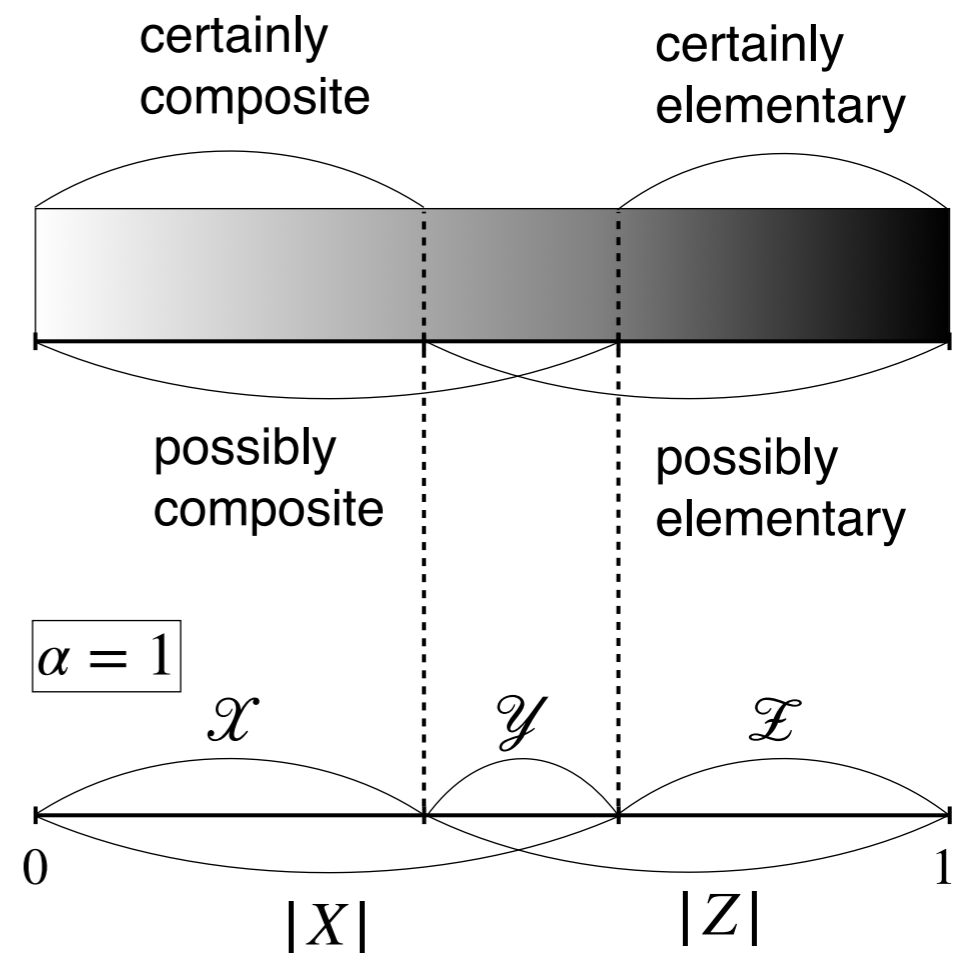
$$\mathcal{X} + \alpha \mathcal{Y} = |X|, \quad \mathcal{Z} + \alpha \mathcal{Y} = |Z|$$

$$\mathcal{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathcal{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$\mathcal{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

α reflects uncertain nature of resonances



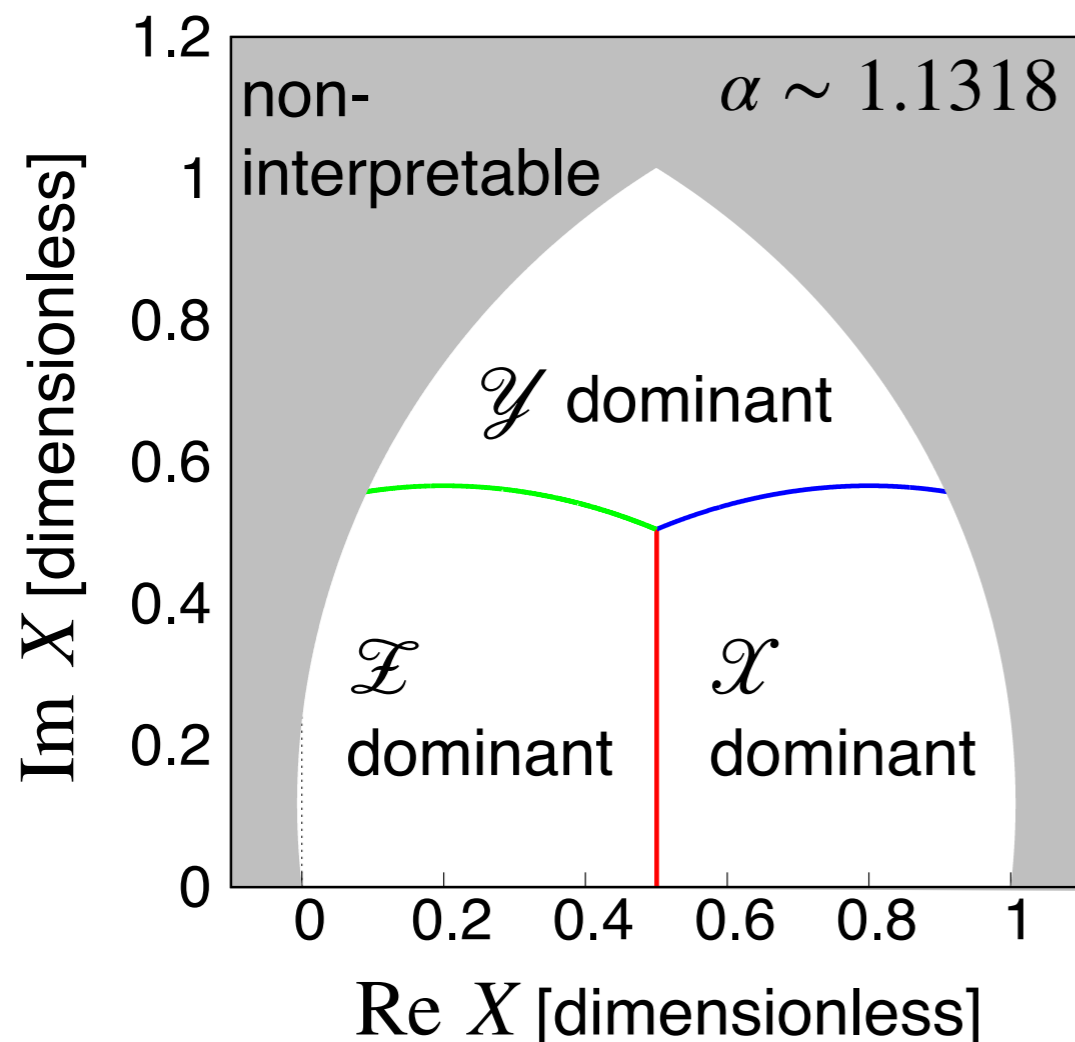
Definition

- if $\alpha > 1/2$, \mathcal{Y} is always positive but \mathcal{X}, \mathcal{Z} can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain

$\mathcal{X} < 0$ or $\mathcal{Z} < 0$

non-interpretable



our criterion for physical “state”

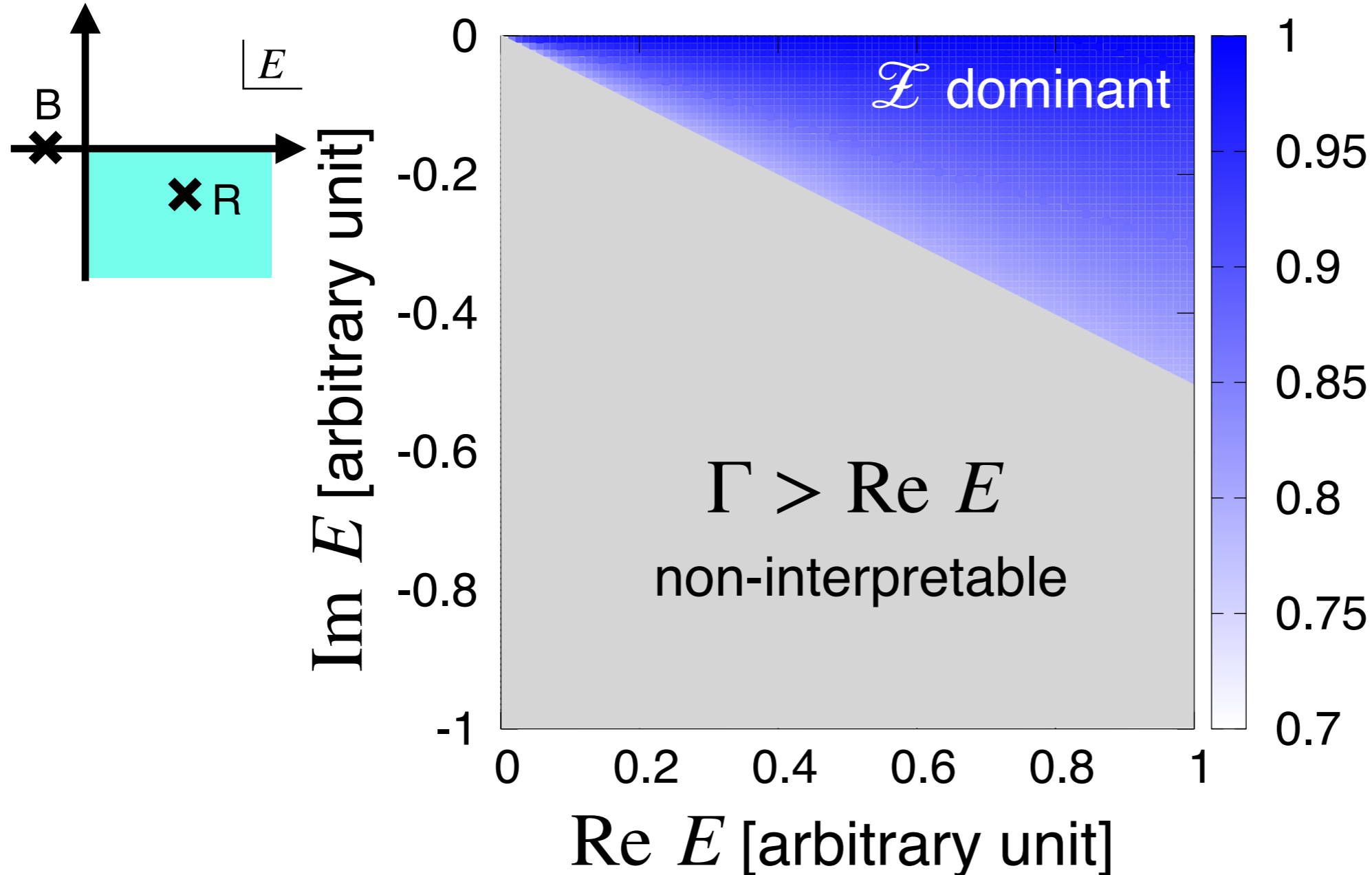
$$\Gamma \leq \text{Re } E$$

- exclude poles which we cannot regard as physical “state” from probabilistic interpretation

$\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ dominant regions and non-interpretable region

Structure of near-th. resonances

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- resonances are **not composite dominant state** ($\mathcal{Z} \gtrsim 0.8$)
- different from near-threshold bound states (composite dominant $X \sim 1$ and $Z \sim 0$)

resonances with previous works

$$\bar{Z} = 1 - \sqrt{\left| \frac{1}{1 - 2r_e/a_0} \right|}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

$$\tilde{Z}_{KH} = \frac{1 + |Z| - |X|}{2}$$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

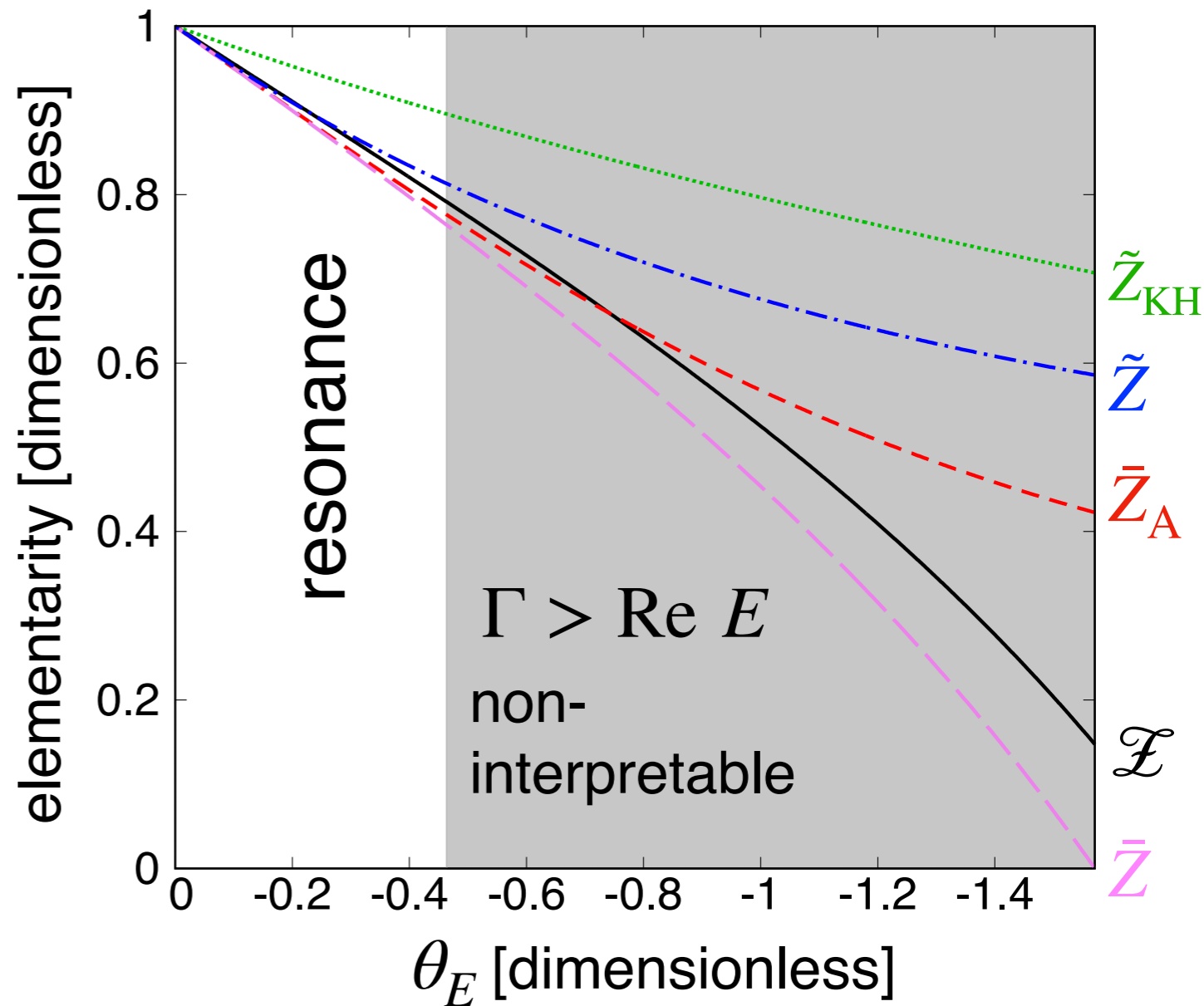
$$\tilde{Z} = \frac{|Z|}{|X| + |Z|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC **93**, 035204 (2016).

$$\bar{Z}_A = 1 - \sqrt{\frac{1}{1 + |2r_e/a_0|}}$$

I. Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A **57**, 101 (2021).

interpretations as a function of θ_E



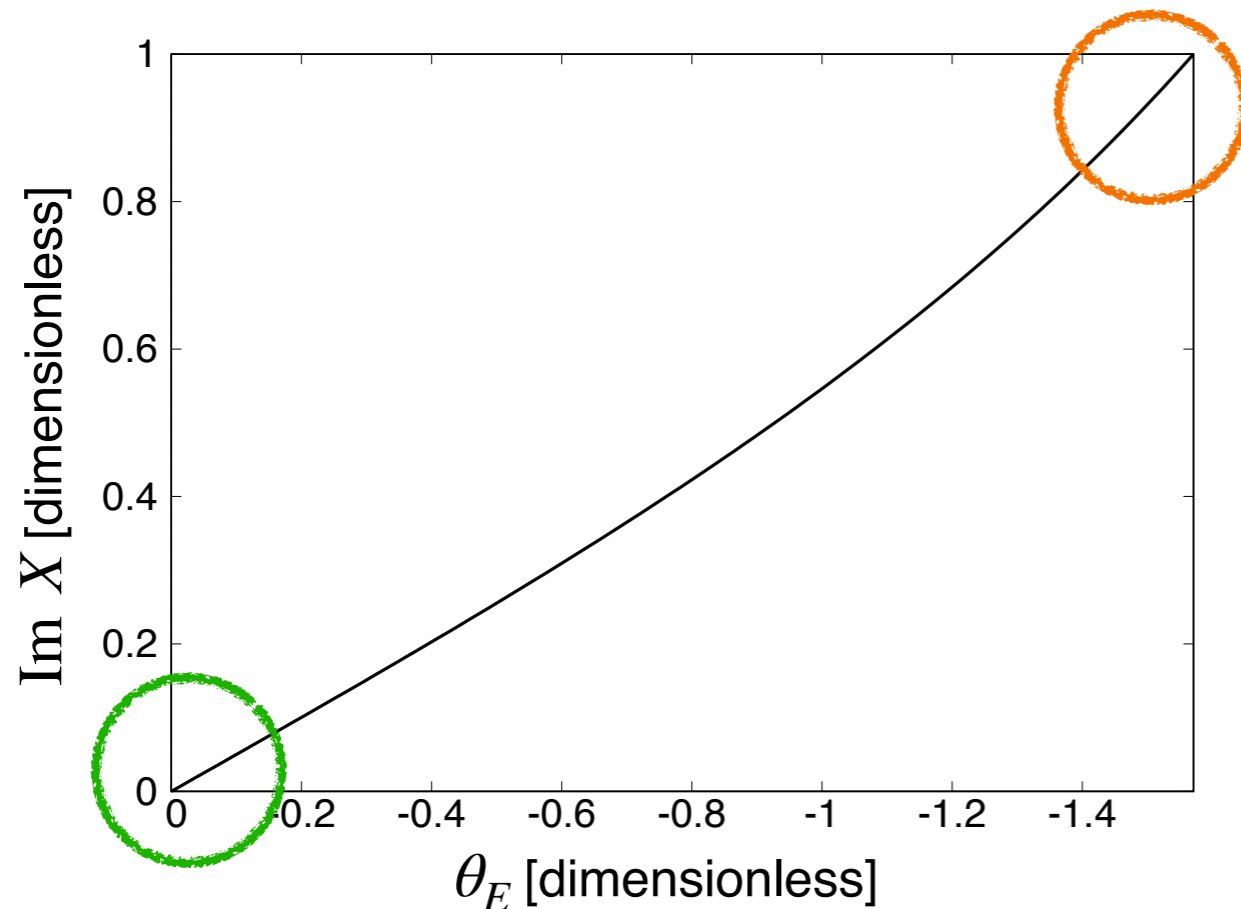
- all interpretations show resonances are elementary dominant

Compositeness in ERE

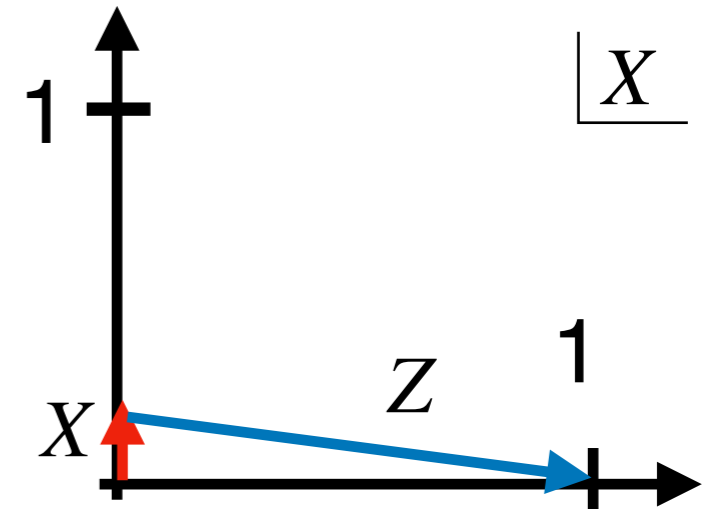
$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan \theta_k \quad (k = |k| e^{i\theta_k})$$

→ X in ERE is pure imaginary

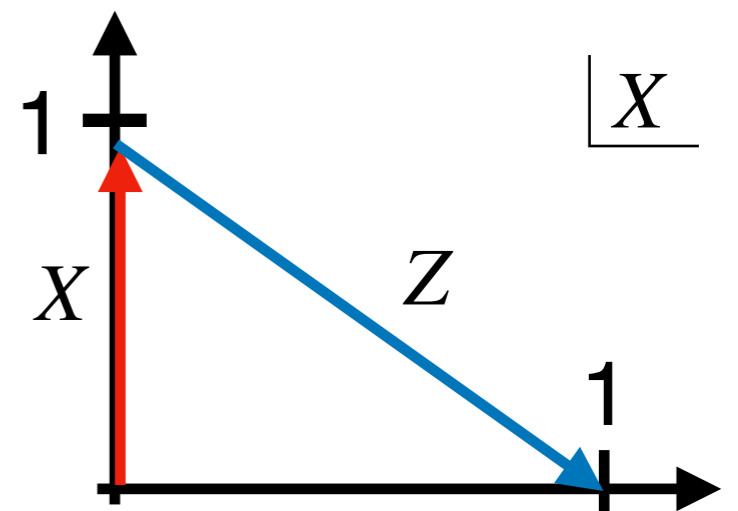
X as a function of θ_E ($E = |E| e^{i\theta_E}$)



small width ($\theta_E \sim 0$)

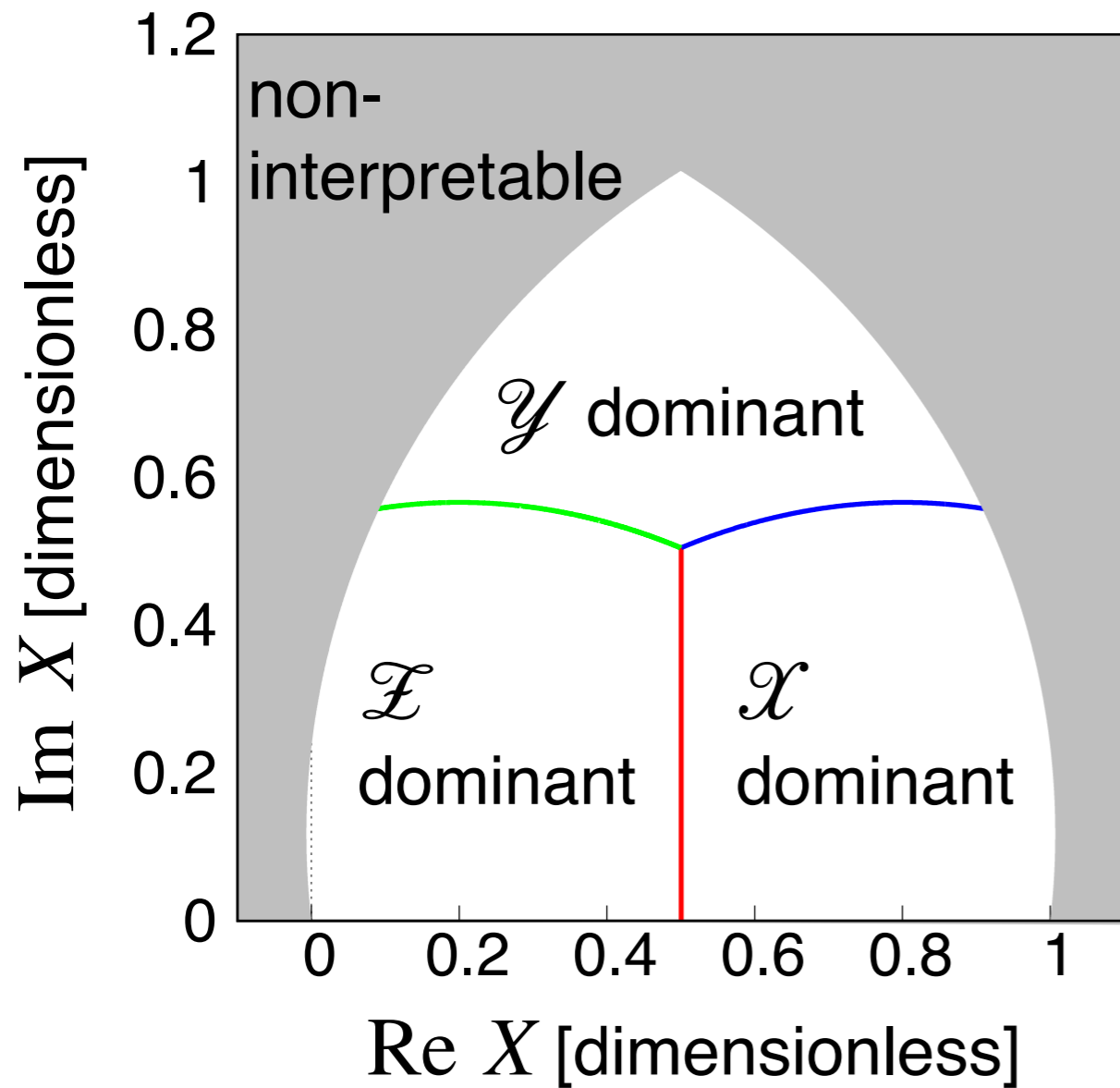


large width ($\theta_E \sim -\pi/2$)



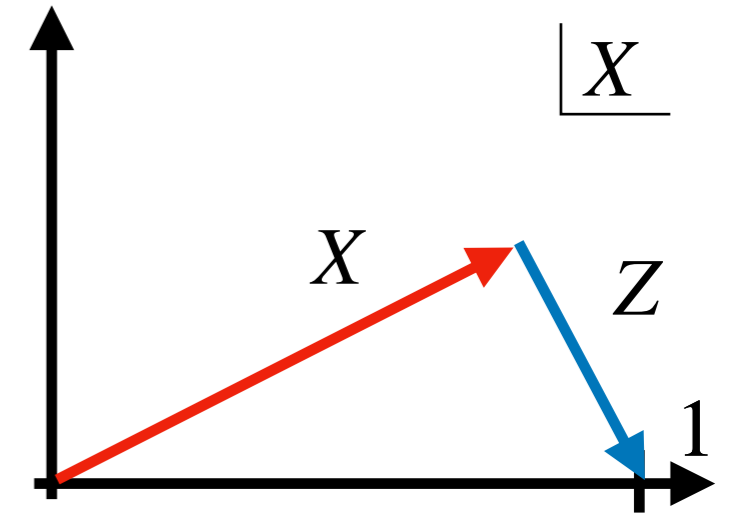
$\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ dominant region

- $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ dominant region in complex X plane

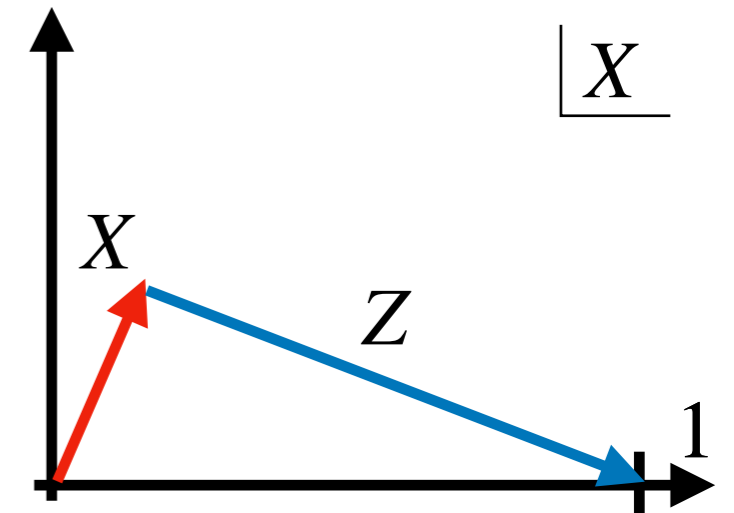


large $\text{Im } X$ is assigned to non-interpretable cases

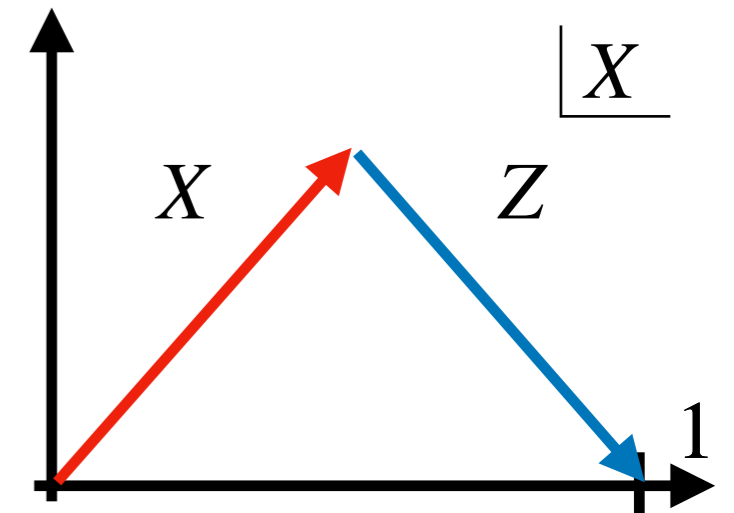
composite dominant



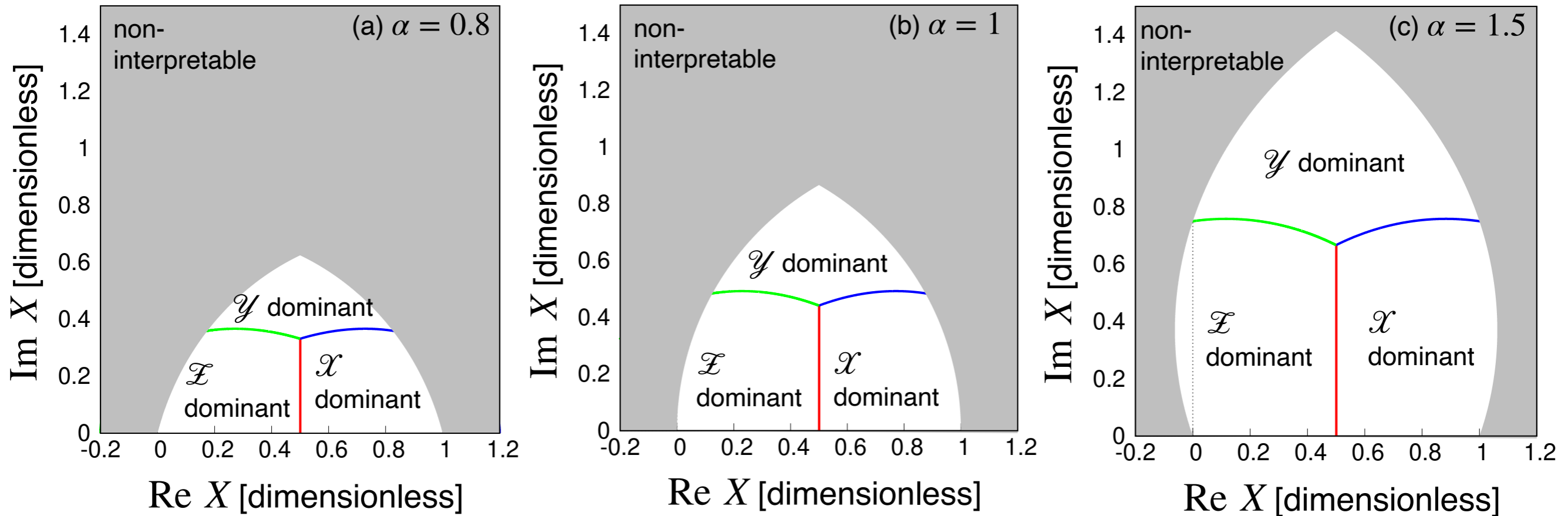
elementary dominant



uncertain



$\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ dominant region



- Interpretable regions become large with increase of α

$\alpha \rightarrow \infty \longrightarrow \mathcal{X}, \mathcal{Y}, \mathcal{Z}$ reduce to interpretation in previous work

$$\mathcal{X} \rightarrow \tilde{\mathcal{X}}, \mathcal{Z} \rightarrow \tilde{\mathcal{Z}}, \mathcal{Y} \rightarrow 0$$

Y. Kamiya and T. Hyodo,
Phys. Rev. C **93**, 035203 (2016).

Definition

● new interpretation of complex compositeness & elementarity

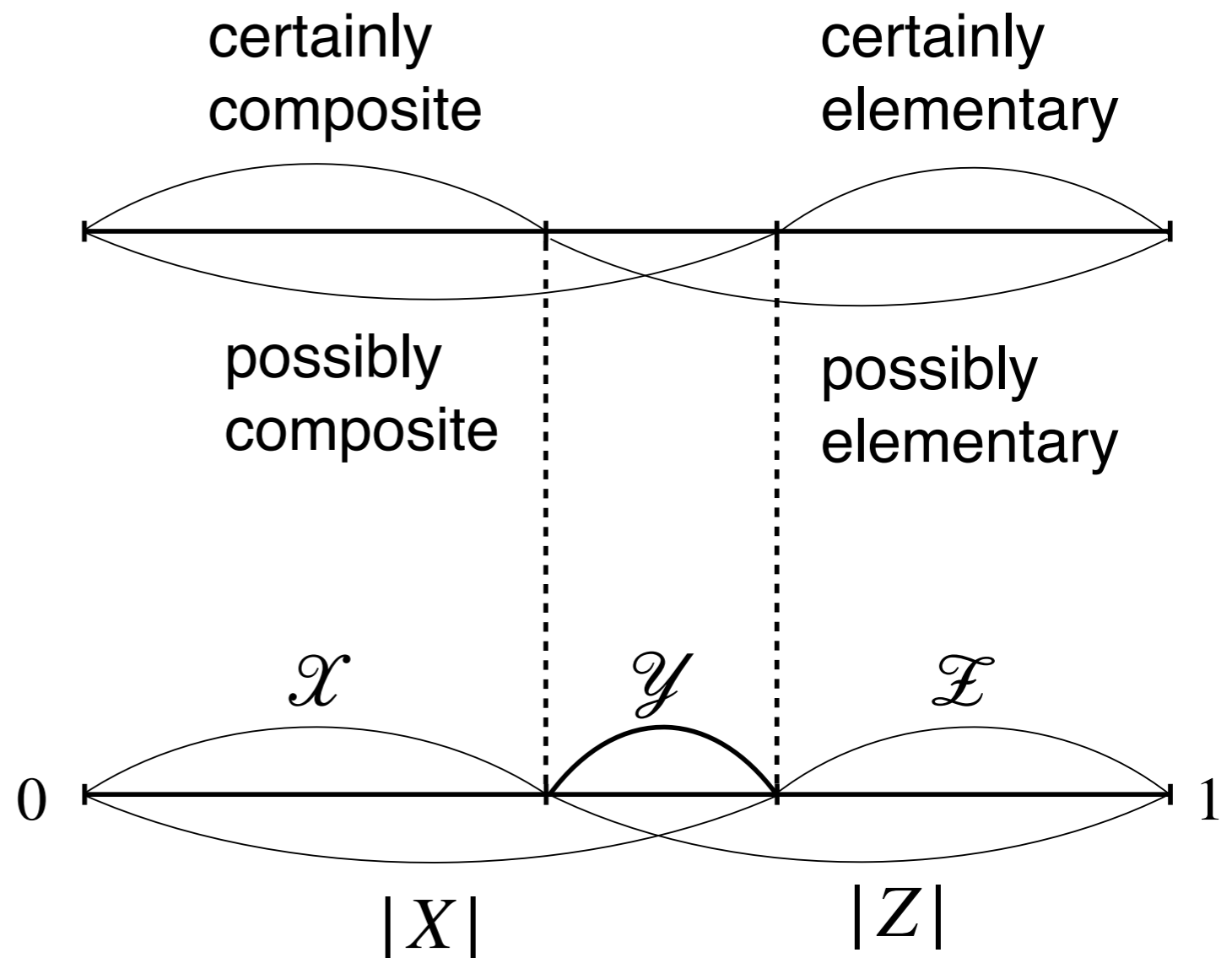
→ from Berggren's idea T. Berggren, Phys. Lett. B 33, 547 (1970).

$$\mathcal{X} + \mathcal{Y} = |X| \quad \& \quad \mathcal{L} + \mathcal{Y} = |Z|$$

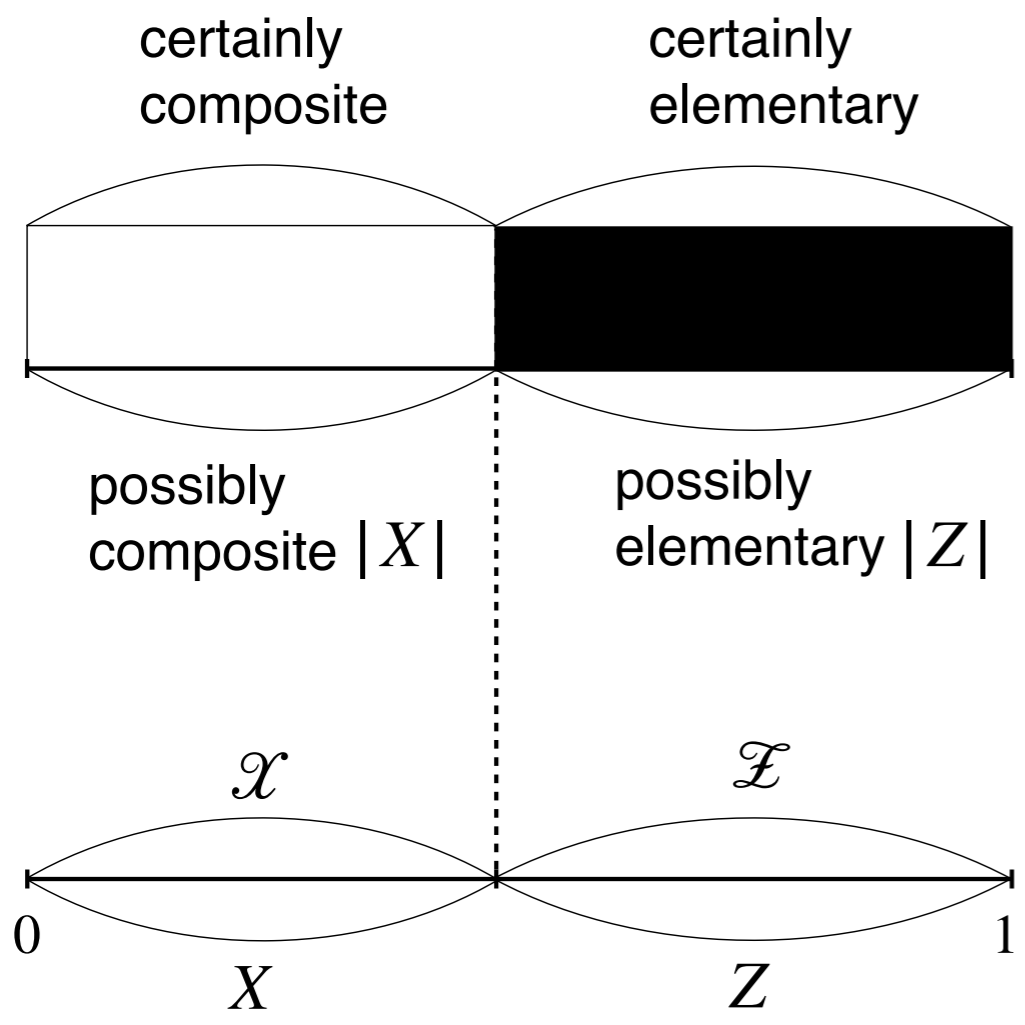
$$\mathcal{X} = 1 - |Z|$$

$$\mathcal{L} = 1 - |X|$$

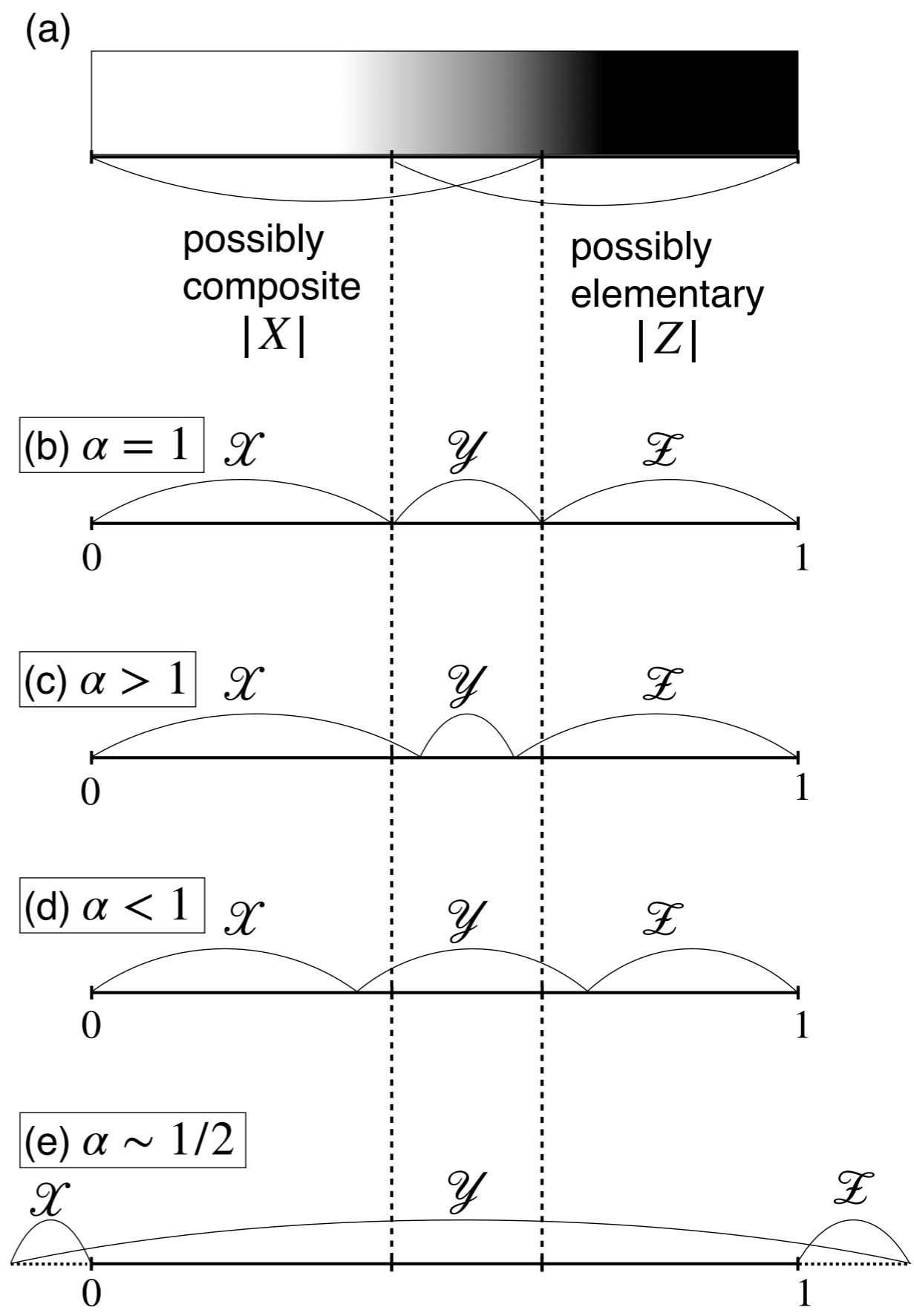
$$\mathcal{Y} = |X| + |Z| - 1$$



interpretation



bound state



resonance

uncertainty in resonances

a single
measurement



sum of measurements of a bound states / resonances

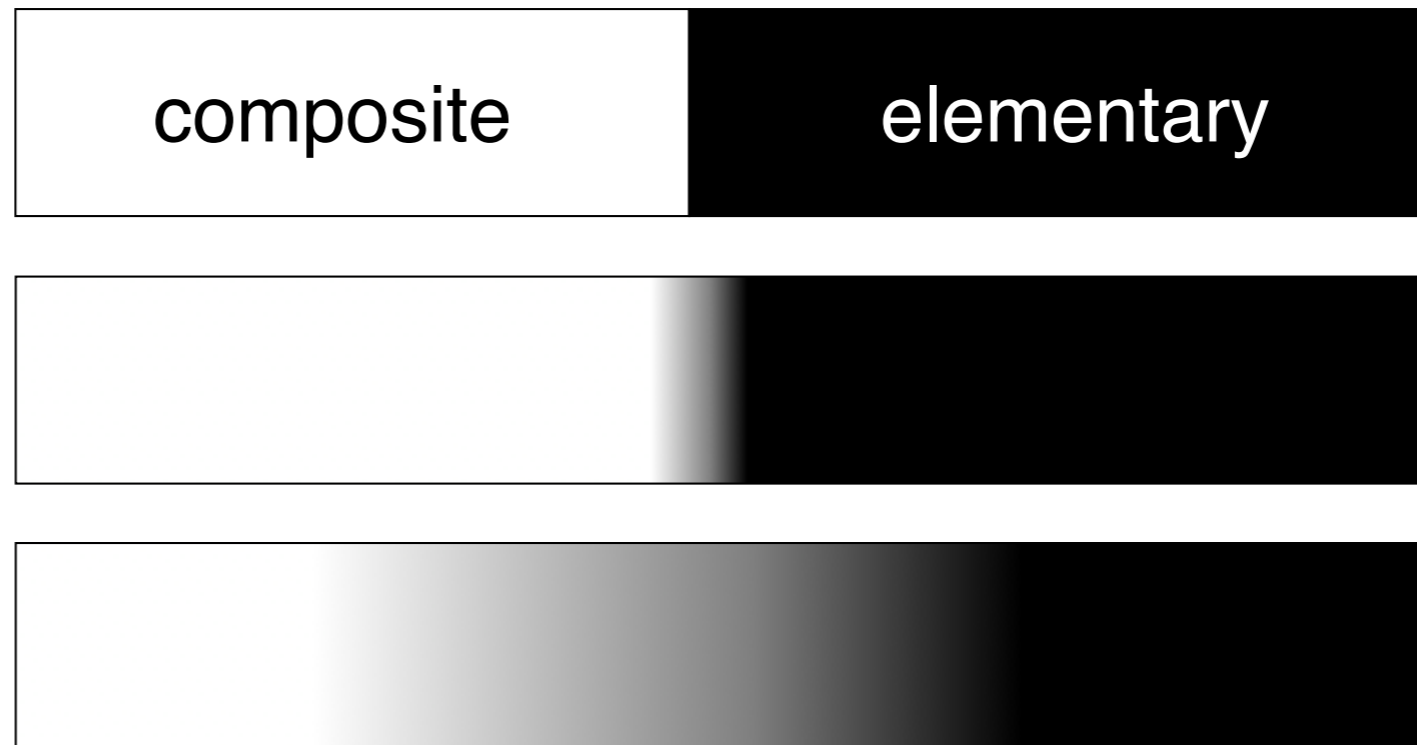
bound state

composite

elementary

narrow
resonance

broad
resonance



measurements

Structure of near-th. resonances

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