Lectures on the Longitudinal Spin Structure of the Nucleon

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Outline of the lectures

**Lecture 1**: some fundamentals about spin; helicity “sum rule”; helicity parton distributions; evolution and angular momentum

**Lecture 2**: inward bound: “femto-spectroscopy”

**Lecture 3**: longitudinal spin physics at RHIC - part I

**Lecture 4**: longitudinal spin physics at RHIC - part II

Disclaimer: 4 hours are barely enough to discuss the main principles, recent experimental findings, and theoretical foundations & developments
Lecture 2

inward bound:
“femto-spectroscopy“
main theme of this lecture:

What can we learn about the nucleon spin structure from polarized lepton-nucleon collisions?

we have to be prepared that ...

“In an increasingly complex world, sometimes old questions require new answers.”
to tackle such questions we can use the World’s most powerful “microscopes”

e.g. the DESY-HERA accelerator complex

here we can even do “sub-femto-science”
Deep-inelastic scattering: kinematics

take a snapshot of the nucleon with a DIS microscope:

relevant kinematics for \( l(k,s) N(P,S) \rightarrow l'(k') X \)

- momentum transfer: \( q^\mu = k^\mu - k'^\mu \)
- "Bjorken"-x: \( x = Q^2/(2p \cdot q) \) and \( Q^2 = -q^2 \)
- energy-transfer: \( y = \frac{p \cdot q}{p \cdot k} \frac{E - E'}{E} \)
- spin vectors: \( s \cdot k = 0 \) and \( S \cdot P = 0 \)

"deep-inelastic": \( Q^2 \gg 1 \text{ GeV}^2 \)  
"scaling limit": \( Q^2 \rightarrow \infty \), \( x \) fixed

resolution: \( \frac{\hbar}{Q} \approx 2 \times 10^{-16} \text{ m} \)

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Longitudinal Spin Structure of the Nucleon
Deep-inelastic scattering: cross section

first analysis of DIS does not require a lot of knowledge about QCD

electroweak theory tells us how
the virtual vector boson couples:
(let's assume only photon exchange)

cross section:
\[
\frac{4\alpha^2}{s} \frac{d^3\vec{k}'}{2|\vec{k}'| Q^4} L^{\mu\nu}(k, q, s) W_{\mu\nu}(p, q, S)
\]

phase space
scat. lepton

photon propagator

leptonic tensor
from QED

hadronic tensor
contains information
about hadronic structure

(can be easily generalized to W/Z-boson exchange)
Deep-inelastic scattering: cross section

space-time symmetries, hermiticity, current conservation, ...
dictate the possible Lorentz-structures of the hadronic tensor:

\[ \mathcal{W}^{\mu\nu}(P, q, S) = \frac{1}{4\pi} \int d^4z \, e^{i q \cdot z} \langle P, S \mid J_\mu(z) J_\nu(0) \mid P, S \rangle \]

\[ = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x, Q^2) + \left(P^\mu - \frac{P \cdot q}{q^2} q^\mu\right) \left(P^\nu - \frac{P \cdot q}{q^2} q^\nu\right) F_2(x, Q^2) \]

\[ + i M \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[ \frac{S_\sigma}{P \cdot q} g_1(x, Q^2) + \frac{S_\sigma (P \cdot q) - P_\sigma (S \cdot q)}{(P \cdot q)^2} g_2(x, Q^2) \right] \]

two spin-dependent structure functions

how to access \( g_1 \) and \( g_2 \) experimentally?
Deep-inelastic scattering: $g_1$ and $g_2$

idea: look at $W_{\mu\nu}(P,q,S) - W_{\mu\nu}(P,q,-S)$ → unpol. part ($F_{1,2}$) drops out

specialize to leptons with helicity $\lambda$ and $\angle(k,S) \equiv \alpha$

find: $\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\,e^4}{4\pi^2Q^2} \times$

\[ \times \left\{ \cos\alpha \left[ 1 - \frac{y}{2} - \frac{m^2x^2y^2}{Q^2} \right] g_1(x, Q^2) - \frac{2m^2x^2y}{Q^2} g_2(x, Q^2) \right\} \]

\[ - \sin\alpha \cos\phi \frac{2mx}{Q} \sqrt{\left( 1 - y - \frac{m^2x^2y^2}{Q^2} \right)} \left( \frac{y}{2} g_1(x, Q^2) + g_2(x, Q^2) \right) \}\]

- **longitudinal polarization** ($\alpha = 0$): probes mainly $g_1$ (up to terms $\propto m^2/Q^2$)

- **transverse polarization** ($\alpha = \pi$): suppressed as $m/Q$

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Deep-inelastic scattering: notation, $A_{LL}$

Here we will only consider long. polarization!

For a longitudinally polarized beam and target define:

$$d\Delta_{LL}\sigma \equiv d\sigma(\leftrightarrow) - d\sigma(\rightarrow\rightarrow)$$

$$\propto g_1(x, Q^2) + 2y\frac{M_N^2 x^2}{Q^2} g_2(x, Q^2)$$

$$\simeq g_1(x, Q^2) \quad \text{will lead us to } \Delta q(x, Q^2) \quad \bullet \rightarrow - \rightarrow \bullet$$

And for the experimentally relevant double-spin asymmetry:

$$A_{LL} = \frac{d\sigma(\leftrightarrow) - d\sigma(\rightarrow\rightarrow)}{d\sigma(\leftrightarrow) + d\sigma(\rightarrow\rightarrow)}$$

$$\simeq D(y) \frac{g_1(x, Q^2)}{F_1(x, Q^2)}$$

$D(y)$: degree of polarization of the virtual photon
DIS in the QCD improved parton model

to establish the connection between $g_1$ and $\Delta q$ we have
to invoke the QCD improved parton model

foundation:

time scale of hard interaction $\ll$ time scale for parton-parton int.

lepton scatters off “free” partons, incoherently

this allows for a factorization of short-distance and long-distance physics:

long-distance (non-pert.): probability to find a parton in the nucleon with
light-cone momentum $\xi \text{p}^+$: $\rightarrow$ pdfs!

short-distance (pert.): cross section for electron-parton scattering
now it is easy to see why a measurement of $g_1$ provides us with information on the helicity parton densities $\Delta q$:

$$g_1(x, Q^2) \propto e_q^2 [q^+ - q^- + \bar{q}^+ - \bar{q}^-](x, Q^2) = e_q^2 [\Delta q + \Delta \bar{q}](x, Q^2)$$
DIS: space-time “picture”

But why does the QCD parton model work?

this can be best understood in a reference frame where the proton moves very fast and \( Q \gg m_h \) is big

(recall light-cone kinematics from Lecture 1)

<table>
<thead>
<tr>
<th>4-vector</th>
<th>hadron rest frame</th>
<th>Breit frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>((p^+, p^-, \vec{p}_T))</td>
<td>(\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0}))</td>
<td>(\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0}))</td>
</tr>
<tr>
<td>((q^+, q^-, \vec{q}_T))</td>
<td>(\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0}))</td>
<td>(\frac{1}{\sqrt{2}}(-Q, Q, \vec{0}))</td>
</tr>
</tbody>
</table>

in general \((a^+, a^-, \vec{a}_T) \rightarrow (e^\omega a^+, e^{-\omega} a^-, \vec{a}_T) = (a'^+, a'^-, \vec{a}')\)

here: \(e^\omega = Q/(x m_h)\)
simple estimate for typical time-scale of interactions among the partons inside a fast-moving hadron:

**rest frame:** \( \Delta x^+ \sim \Delta x^- \sim \frac{1}{m} \)

**Breit frame:** \( \Delta x^+ \sim \frac{1}{m} \frac{Q}{m} = \frac{Q}{m^2} \) large

\( \Delta x^- \sim \frac{1}{m} \frac{m}{Q} = \frac{1}{Q} \) small

interactions between partons are spread out inside a fast moving hadron

How does this compare with the time-scale of the hard scattering?
now let the virtual photon meet our fast moving hadron ...

struck quark kicked into $x^-$ direction

interaction localized to within $\Delta x^+ \sim 1/Q$

upshot:

- partons are free during the hard interaction
- hadron effectively consists of partons that have momenta $(p_i^+, p_i^-, \bar{p}_i)$
- convenient to introduce momentum fractions $0 < \xi_i \equiv p_i^+/p^+ < 1$
Spin-dependent DIS @ NLO of QCD

The structure function $g_1$ is known up to NLO accuracy:

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \left\{ \left[ \Delta q + \Delta \bar{q} \right](x, Q^2) + \ldots \right\}$$

QCD corrections ("coefficient functions" $\Delta C_{q,g}$)

Kodaira; Ratcliffe; Bodwin, Qiu; van Neerven, Zijlstra; ...
Lessons from polarized DIS

more than 25 years of beautiful data on polarized DIS

... but what have we learned??
Lessons from polarized DIS: the early days, a "sum rule", and a surprise

- an early measurement of $g_1^p$
- data extend to smaller $x$
- precision not spectacular

however, if plotted differently

- there is a discrepancy with a theoretical expectation
- it becomes top-cite 1000+
- it caused a flurry; we still benefit from the afterglow
Lessons from polarized DIS: the early days, a “sum rule”, and a surprise

let’s analyze the EMC result for the x-integral of $g_1$

- it will be very useful to rewrite $g_1$ as: (only for simplicity we stick to LO)

$$g_{1,p,n}(x, Q^2) = \pm \frac{1}{12} \Delta a_3(x, Q^2) + \frac{1}{36} \Delta a_8(x, Q^2) + \frac{1}{9} \Delta a_0(x, Q^2)$$

isospin-symmetry

$\Delta u^n = \Delta d^p$, etc.

and similarly for the first moment/x-integral:

$$g_{1,p,n}(Q^2) = \int_0^1 dx \ g_{1,p,n}(x, Q^2)$$
Lessons from polarized DIS: the early days, a "sum rule", and a surprise

- next use that non-singlet axial currents are conserved

\[ \partial_\mu \left( \overline{\Psi} \gamma^\mu \gamma_5 \lambda_{3,8} \Psi \right) = 0 \]

\[ \Delta a_3(Q^2) \equiv \int_0^1 \Delta a_3(x, Q^2) dx \equiv \Delta a_3 \]

\[ \Delta a_8(Q^2) \equiv \int_0^1 \Delta a_8(x, Q^2) dx \equiv \Delta a_8 \]

consistent with DGLAP scale evolution to all orders in pQCD

- important: moments of \( \Delta a_{3,8} \) can be related to baryonic \( \beta \)-decays

find

\[ \Delta a_3 = g_A = 1.267 \pm 0.0035 \text{ (from neutron decay)} \]

\[ \Delta a_8 = 0.58 \pm 0.03 \text{ (assuming SU(3) symmetry)} \]
Lessons from polarized DIS: the early days, a “sum rule”, and a surprise

now we are able to fully grasp the relevance of the EMC result ...

combining everything we arrive at

\[ g_1^{p,n}(Q^2) = \left[ 0.185 \pm 0.004 - 0.024 \pm 0.004 + \frac{1}{3}(\Delta s + \Delta \bar{s})(Q^2) \right] \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) \]

from \( \Delta a_{3,8} \)

Gourdin; Ellis, Jaffe: reasonable to assume that strange quarks do not contribute to the spin of the nucleon prediction for \( x \)-integral of \( g_1 \)

EMC proved that prediction to be wrong

more recently E155: \( g_1^p = 0.118 \pm 0.004 \pm 0.007 \)
Lessons from polarized DIS: the spin puzzle/surprise

What are the lessons from EMC and its successors?

- never trust a theorist
- the quarks contribute only very little to the proton spin:
  \[ \Delta \Sigma = 0.15 \div 0.30 \]
- strange quarks polarized opposite to the nucleon spin
  \[ \Delta s + \Delta \bar{s} \simeq -0.1 \div -0.15 \]

how significant is this?

“This could be the discovery of the century. Depending, of course, on how far down it goes.”
Lessons from polarized DIS: the spin puzzle/surprise

first it was deemed to be a crisis ...

- naive quark model yields: $\Delta u = 4/3$ and $\Delta d = -1/3 \rightarrow \Delta \Sigma = 1$
- more sophisticated relativistic models: $\Delta \Sigma \simeq 0.6$

... many, sometimes confusing, twists and turns ...

- strongly advocated idea that "true quark spin" is screened by a large $\Delta g$ through the "anomaly"

  $\Delta \Sigma(Q^2) = \Delta \Sigma' - N_f \frac{\alpha_s(Q^2)}{2\pi} \Delta g(Q^2)$

  small \hspace{1cm} large

  - scenario at best controversial (gauge-inv; operators; ...)
  - perfectly O.K. as diff. factorization scheme (recall: pdfs not absolute!)

... but there are other "players" (gluon spin, orbital ang. momenta) ...

... so now it is more like a yet unsolved puzzle
Lessons from polarized DIS: yet another sum rule

In 1966 J.D. Bjorken proposed this sum rule as a strong test of QCD:

$$\text{Bj-sum: } \int_0^1 dx \left[ g_1^p(x, Q^2) - g_1^n(x, Q^2) \right] = \frac{1}{6} C_{\text{Bj}} [\alpha_s(Q^2)] g_A$$

$$C_{\text{Bj}}(\alpha_s) = 1 - \frac{\alpha_s}{\pi} - \left[ \frac{55}{12} - \frac{n_f}{3} \right] \left( \frac{\alpha_s}{\pi} \right)^2$$

Kodaira; Gorishny, Larin; Larin, Vermaseren

$$= \left[ \frac{41.4 - 7.6 n_f + \frac{115}{648} n_f^2}{\pi} \right] \left( \frac{\alpha_s}{\pi} \right)^3 \pm \ldots$$

Now it is known up to next-to-next-to-next-to-leading order

- In fact this paper triggered first exp. efforts (E80 led by V.W. Hughes)
- Non-singlet $\propto \Delta a_3$, based only on isospin symmetry
- Nicely confirmed: theory: $0.181 \pm 0.003$
  
  Experiment: $0.174 \pm 0.005 \pm 0.010$
Lessons from polarized DIS: NLO pQCD analysis

not only interested in the spin sum rule and \(x\)-integrated moments!!

DIS data can be also used to extract the fully \(x\)-dependent helicity parton densities in a NLO pQCD analysis/fit:

in principle, we can extract:

- **sum** of all quark and antiquark densities
- the **gluon density** from (small) NLO corrections and **scaling violations**, i.e., from the \(Q^2\)-dependence of the data
Lessons from polarized DIS: NLO pQCD analysis

**detour:** In the unpol. case this works out nicely thanks to the HERA data:

- **fixed-target experiments**

- **unpol. gluon from scaling violations**

- **important ingredient for LHC's quest for the Higgs & SUSY !!**
however, in the polarized case only a much smaller portion of the kinematic plane is explored:

Lessons from polarized DIS: NLO pQCD analysis

expect a much less constrained gluon!
Lessons from polarized DIS: NLO pQCD analysis

NLO pQCD does a very good job in describing all data

from Blümlein, Böttcher

acronyms:
AAC: Asymmetry Analysis Collaboration (Hirai, Kumano, Saito)
BB: Blümlein & Böttcher
GRSV: Glück, Reya, MS, Vogelsang
Lessons from polarized DIS: NLO pQCD analysis

in terms of the individual parton densities: from Blümlein, Böttcher

\[ \Delta u_{\text{val}} \equiv \Delta u - \Delta \bar{u} \]

nicely constrained

\[ \Delta d_{\text{val}} \equiv \Delta d - \Delta \bar{d} \]

about O.K.

\[ \Delta g \]

largely unconstrained

(only stat. errors shown)

\[ \Delta u, \Delta d, \Delta s \]

no flavor separation

(SU(3) sym. assumed)
Lessons from polarized DIS: open questions

even after more than 25 years of pol. DIS there are still a lot of things we have to explore in more detail:

- the polarized gluon density and its contribution to $S_z^N = \frac{1}{2}$
  - we go after $\Delta g$ in the 3rd & 4th lecture
- quark/anti-quark and flavor decomposition
  - e.g. from semi-inclusive DIS; W’s
- the role of strangeness in the nucleon and its contribution to $S_z^N = \frac{1}{2}$
  - e.g. from semi-inclusive DIS; $\Lambda$’s
- orbital angular momentum

... and I haven't even touched transverse polarization phenomena which also have a distinguished record of surprises and puzzles
Semi-inclusive DIS: basics

Semi-Inclusive Deep-Inelastic Scattering (SIDIS): $\text{IN} \rightarrow l'\text{HX}$

$\text{SIDIS} = \text{DIS}$ with one identified hadron $H$

- scattered lepton $l'$:
  $$Q^2 = -q^2 = -(p_l - p_{l'})^2$$
  $$x = Q^2/(2p_N \cdot q)$$
  $$y = p_N \cdot q/(p_N \cdot p_l)$$

- not observed stuff $X$

- identified hadron $H$:
  $$z = p_N \cdot p_H/(p_N \cdot q)$$

Cartoon by HERMES
Semi-inclusive DIS: what can we learn?

production of the observed final-state hadron $H$ described by non-perturbative fragmentation functions $D_f^{H}(z, Q^2)$:

- probability to find a hadron $H$ with fraction $z$ of parton's $f$ momentum
- pQCD predicts scale dependence as for pdfs
- can be extracted from $e^+e^- \to H \ X$ data (e.g. from LEP)

why do we benefit from the extra hadron?

at LO:

$\gamma^* \to f \ H$

$d\Delta\sigma \sim \sum_{q=u,\bar{u},...,\bar{s}} \Delta q(x, Q^2) D_q^{H}(z, Q^2)$

extra weight for each quark

breaks the $\Delta q + \Delta \bar{q}$ deadlock of DIS plus better flavor separation
Semi-inclusive DIS in NLO pQCD

NLO pQCD corrections are available:

main feature: non-trivial x/z dependence

\[ \frac{d^3 \Delta \sigma}{dx dy dz} \simeq \sum_{q=u,\bar{u},...,\bar{s}} e_q^2 \left[ \Delta q(x, \mu_f) \left( \sum_{H} D_H^{(1)}(z, \mu_f') \right) + \int_{x}^{1} \frac{d\hat{x}}{x} \int_{z}^{1} \frac{d\hat{z}}{z} \right] \]

Furmanski, Petronzio; Altarelli et al.; de Florian, MS, Vogelsang; ...
Semi-inclusive DIS: results

- only source for $\Delta q/\Delta\bar{q}$ separation so far
- results for sea densities not conclusive (consistent with SU(3) sym. sea)
- certain models make specific predictions for breaking of SU(2) symmetry
enough for today