Introduction to Perturbative QCD

Lecture 3

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Outline for Lecture 3

- Cross section with one identified hadron
  - Lepton-hadron deeply inelastic scattering (DIS)

- Factorization for IR sensitive cross sections

- What is the predictive power of pQCD?

- DGLAP evolution equation

- Parton distribution functions

Excellent resource – CTEQ summer school website

http://www.phys.psu.edu/~cteq
Deep inelastic scattering

Recall:

\[ E' \frac{d \sigma^{\text{DIS}}}{d^3 k'} = \frac{1}{2s} \left( \frac{1}{Q^2} \right)^2 L^{\mu \nu}(k, k') W^{\mu \nu}(q, p) \]

Hadronic tensor:

\[ W_{\mu \nu}(q, p, S) = \frac{1}{4\pi} \int d^4 z \ e^{i q \cdot z} \left\langle p, S \left| J_\mu^+(z) J_\nu(0) \right| p, S \right\rangle \]

\[ W_{\mu \nu} = -\left( g_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1(x_B, Q^2) + \frac{1}{p \cdot q} \left( p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left( p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) F_2(x_B, Q^2) \]

\[ + i M_p \varepsilon^{\mu \nu \rho \sigma} q_\rho \left[ \frac{S_\sigma}{p \cdot q} g_1(x_B, Q^2) + \frac{(p \cdot q) S_\sigma - (S \cdot q) p_\sigma}{(p \cdot q)^2} g_2(x_B, Q^2) \right] \]

Structure functions – infrared sensitive:

\[ F_1(x_B, Q^2), F_2(x_B, Q^2), g_1(x_B, Q^2), g_2(x_B, Q^2) \]
Perturbative QCD Factorization

- Cross sections with identified hadrons are infrared sensitive and non-perturbative
  
  Typical hadronic scale: \( 1/R \sim 1 \text{ fm}^{-1} \sim \Lambda_{QCD} \)
  
  Energy exchange in hard collisions: \( Q >> \Lambda_{QCD} \)

  \[\Rightarrow \quad \text{pQCD works at } \alpha_s(Q), \text{ but not at } \alpha_s(1/R)\]

- PQCD can be useful iff quantum interference between perturbative and nonperturbative scales can be neglected

\[\sigma_{\text{phy}}(Q, 1/R) \sim \hat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)\]

Factorization \(\leftrightarrow\) needs a “long-lived” parton state
Picture of factorization in DIS

- **Time evolution:**
  - Long-lived parton state

- **Unitarity – summing over all hard jets:**
  - Interaction between the “past” and “now” are suppressed!

Interaction between the “past” and “now” are suppressed!
Factorization in DIS

\[ \sigma_{\text{DIS tot}} \sim OQM \]

Now

Past

Connection

Predictive power of pQCD

- short-distance and long-distance are separately gauge invariant
- short-distance part is Infrared-Safe, and calculable
- long-distance part can be defined to be Universal
Long-lived parton states

- Feynman diagram representation:

\[ W^{\mu\nu} \propto \ldots \]

- Perturbative pinched poles:

\[
\int d^4k \ H(Q,k) \left( \frac{1}{k^2 + i\varepsilon} \right) \left( \frac{1}{k^2 - i\varepsilon} \right) T(k, \frac{1}{r_0}) \Rightarrow \infty \ \text{perturbatively}
\]

- Perturbative factorization:

\[
k^\mu = xp^\mu + \frac{k^2 + k_T^2}{2xp \cdot n} n^\mu + k_T^\mu
\]

\[
\int \frac{dx}{x} d^2k_T \ H(Q, k^2 = 0)
\]

\[
\int dk^2 \left( \frac{1}{k^2 + i\varepsilon} \right) \left( \frac{1}{k^2 - i\varepsilon} \right) T(k, \frac{1}{r_0})
\]

\[ \text{Nonperturbative matrix element} \]

\[ \text{Short-distance} \]
Collinear factorization

- **Collinear approximation, if**
  \[ Q \sim x p \cdot n \gg k_T, \sqrt{k^2} \]

- **Scheme dependence**

- **Collinear factorization**

  Parton’s transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** \( \nu, Q^2 \rightarrow \infty, \text{ while } x_B \text{ fixed} \)

  Feynman’s parton model and Bjorken scaling

  \[
  F_2(x_B, Q^2) = x_B \sum_f e_f^2 \phi_f(x_B) + O(\alpha_s) + O\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)
  \]
Necessary condition for factorization

"Any uncanceled long-distance divergence of a partonic scattering cross section has to be process-independent"

On hadron state:

\[ \sigma_H(Q, 1/R) \sim \sum_a \hat{\sigma}_a(Q) \otimes \varphi_{a/H}(1/R) + O(1/QR) \]

On parton state:

\[ \sigma_p(Q, 1/R) \sim \sum_a \hat{\sigma}_a(Q) \otimes \varphi_{a/p}(1/R) + O(1/QR) \]

Process dependent partonic cross section (Feynman diagrams)

Process-independent Parton-level pdfs (Feynman diagrams)

Example:

\[ W_{\mu\nu}^{(1)} \propto \text{All uncanceled divergences are absorbed into PDFs} \]
Parton distribution functions (PDFs)

- Predictive power of pQCD relies on the factorization and the universality of PDFs

- PDFs as matrix elements of two parton fields:
  - quark distribution as an example,
    \[
    \phi_q(x, \mu^2) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle h(p) | \bar{\psi}_q(0) \frac{\gamma^+}{2} \mathcal{P} e^{-ig \int_0^{y^-} dw^- A^+(w^-)} \psi_q(y^-) | h(p) \rangle
    \]
  - corresponding diagram in momentum space,
    \[\frac{\gamma^+}{2p^+} \delta(x - \frac{k^+}{p^+}) \frac{d^4k}{(2\pi)^4}\]
  - \(|h(p)\rangle\) can be a hadron, or a nucleus state, as well as a parton state

+ UV CT

Gives the \(\mu\)-dependence
An instructive exercise for high orders

- **Consider a cross section:** \( \sigma(Q^2, m^2) = \sigma_0 \left[ 1 + \alpha_s I + O(\alpha_s^2) \right] \)

- **Leading quantum correction:**
  \[ I = \int_0^\infty dk^2 \frac{1}{k^2 + m^2} \frac{Q^2}{Q^2 + k^2} \]

- **Analysis of the integral:**
  \[ I = \int_{k^2 \ll Q^2} dk^2 \frac{1}{k^2 + m^2} + \int_{k^2 \sim Q^2} dk^2 \frac{1}{k^2} \frac{Q^2}{Q^2 + k^2} + O(m^2/Q^2) \]

- **Result for the cross section:**
  \[ \sigma = \left( 1 + \alpha_s \int_{k^2 \ll Q^2} dk^2 \frac{1}{k^2 + m^2} \right) \left( 1 + \alpha_s \int_{k^2 \sim Q^2} dk^2 \frac{1}{k^2} \frac{Q^2}{Q^2 + k^2} \right) \]
  \[ + O(\alpha_s^2) + O(m^2/Q^2) \]
  \[ \equiv f \times \hat{\sigma} + O(\alpha_s^2) + O(m^2/Q^2). \]
Scaling violation and factorization

- NLO partonic diagram to structure functions:

  \[ \int_{0}^{-Q^2} \frac{dk_1^2}{k_1^2} \sim 0 \]

  \[ t_{AB} \to \infty \]

  Diagram has both long- and short-distance physics

- Factorization, separation of short- from long-distance:

  \[ C^{(0)} \times \phi^{(1)} \]

  \[ = \]

  \[ C^{(1)} \times \phi^{(0)} \]

  LO + evolution

  \[ NLO \]

  \[ + \]

  \[ + \]

  \[ + \]
Leading power QCD formula

- QCD corrections: pinch singularities in $\int d^4 k_i$

- Logarithmic contributions into parton distributions

- Factorization scale: $\mu_F^2$

- To separate collinear from non-collinear contribution

Recall: renormalization scale to separate local from non-local contribution
Calculation of perturbative parts

- Use DIS structure function $F_2$ as an example:

$$F_{2h}(x_B, Q^2) = \sum_{q,f} C_{q/f} \left( \frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s \right) \otimes \varphi_{f/h}(x, \mu_F^2) + O\left( \frac{\Lambda_{QCD}^2}{Q^2} \right)$$

- Apply the factorized formula to parton states: $h \rightarrow q$

Use Feynman diagrams

- Express both SFs and PDFs in terms of powers of $\alpha_s$:

**0th order:**

$$F_{2q}^{(0)}(x_B, Q^2) = C_q^{(0)}(x_B / x, Q^2 / \mu_F^2) \otimes \varphi_{q/q}^{(0)}(x, \mu_F^2)$$

$$\Rightarrow C_q^{(0)}(x) = F_{2q}^{(0)}(x) \quad \varphi_{q/q}^{(0)}(x) = \delta_{qq} \delta(1 - x)$$

**1st order:**

$$F_{2q}^{(1)}(x_B, Q^2) = C_q^{(1)}(x_B / x, Q^2 / \mu_F^2) \otimes \varphi_{q/q}^{(0)}(x, \mu_F^2) + C_q^{(0)}(x_B / x, Q^2 / \mu_F^2) \otimes \varphi_{q/q}^{(1)}(x, \mu_F^2)$$

$$\Rightarrow C_q^{(1)}(x, Q^2 / \mu_F^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}(x, \mu_F^2)$$
Leading order coefficient function

- Projection operators for SFs:

\[
W_{\mu\nu} = - \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \frac{1}{p \cdot q} \left( p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left( p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) F_2(x, Q^2)
\]

\[
F_1(x, Q^2) = \frac{1}{2} \left( -g_{\mu\nu} + \frac{4x^2}{Q^2} p_\mu p_\nu \right) W_{\mu\nu}(x, Q^2)
\]

\[
F_2(x, Q^2) = x \left( -g_{\mu\nu} + \frac{12x^2}{Q^2} p_\mu p_\nu \right) W_{\mu\nu}(x, Q^2)
\]

- 0\textsuperscript{th} order:

\[
F_{2q}^{(0)}(x) = x g^{\mu\nu} W_{\mu\nu, q}^{(0)} = x g^{\mu\nu} \left[ \frac{1}{4\pi} \right]
\]

\[
= \left( x g^{\mu\nu} \right) \frac{e_q^2}{4\pi} \text{Tr} \left[ \frac{1}{2} \gamma \cdot p \gamma_\mu \gamma \cdot (p + q) \gamma_\nu \right] 2\pi \delta \left( (p + q)^2 \right)
\]

\[
= e_q^2 x \delta(1 - x)
\]

\[
C_q^{(0)}(x) = e_q^2 x \delta(1 - x)
\]
NLO coefficient function

\[
C_q^{(1)}(x, Q^2 / \mu_F^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}(x, \mu_F^2)
\]

- **Projection operators in n-dimension:** 
  \[ g_{\mu\nu} g^{\mu\nu} = n \equiv 4 - 2\epsilon \]
  \[
  (1 - \epsilon) F_2 = x \left( -g^{\mu\nu} + (3 - 2\epsilon) \frac{4x^2}{Q^2} p^\mu p^\nu \right) W_{\mu\nu}
  \]

- **Feynman diagrams:**
  \[
  W_{\mu\nu,q}^{(1)} + \text{c.c.} + UV \text{ CT}
  \]

- **Virtual**
  \[
  + \quad + \text{c.c.} \quad + \text{c.c.} \quad + UV \text{ CT}
  \]

- **Calculation:**
  \[ -g^{\mu\nu} W_{\mu\nu,q}^{(1)} \quad \text{and} \quad p^\mu p^\nu W_{\mu\nu,q}^{(1)} \]
Contribution from the trace of $W_{\mu\nu}$

- **Lowest order in n-dimension:**
  \[-g^\mu\nu W_{\mu\nu,q}^{(0)} = e_q^2 (1 - \varepsilon) \delta(1 - x)\]

- **NLO virtual contribution:**
  \[-g^\mu\nu W_{\mu\nu,q}^{(1)V} = e_q^2 (1 - \varepsilon) \delta(1 - x)\]
  \[\ast \left(-\frac{\alpha_s}{\pi}\right) C_F \left[\frac{4\pi\mu_F^2}{Q^2}\right]^{\varepsilon} \frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left[\frac{1}{\varepsilon^2} + \frac{3}{2}\frac{1}{\varepsilon} + 4\right]\]

- **NLO real contribution:**
  \[-g^\mu\nu W_{\mu\nu,q}^{(1)R} = e_q^2 (1 - \varepsilon) C_F \left(-\frac{\alpha_s}{2\pi}\right) \left[\frac{4\pi\mu_F^2}{Q^2}\right]^{\varepsilon} \frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\]
  \[\ast \left\{-\frac{1-\varepsilon}{\varepsilon} \left[1 - x + \left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right] + \frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)} + \frac{2\varepsilon}{1-2\varepsilon}\right\}\]
The “+” distribution:

\[
\left( \frac{1}{1-x} \right)^{1+\varepsilon} = -\frac{1}{\varepsilon} \delta(1-x) + \frac{1}{(1-x)^{+}} + \varepsilon \left( \frac{\ln(1-x)}{1-x} \right)^{+} + O(\varepsilon^2)
\]

\[
\int_{z}^{1} dx \frac{f(x)}{(1-x)^{+}} \equiv \int_{z}^{1} dx \frac{f(x) - f(1)}{1-x} + \ln(1-z)f(1)
\]

One loop contribution to the trace of \( W_{\mu\nu} \):

\[
-g^{\mu\nu} W_{\mu\nu,q}^{(1)} = e_{q}^{2} (1 - \varepsilon) \left( \frac{\alpha_{s}}{2\pi} \right) \left\{ -\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x) \ln \left( \frac{Q^2}{\mu_{F}^2 (4\pi e^{-\gamma_E})} \right) \right. \\
+ C_{F} \left[ \left( 1 + x^2 \right) \left( \frac{\ln(1-x)}{1-x} \right)^{+} - \frac{3}{2} \left( \frac{1}{1-x} \right)^{+} - \frac{1 + x^2}{1-x} \ln(x) \right. \\
+ 3 - x - \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-x) \left. \right\}
\]

Splitting function:

\[
P_{qq}(x) = C_{F} \left[ \frac{1 + x^2}{(1-x)^{+}} + \frac{3}{2} \delta(1-x) \right]
\]
One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)\nu}_{\mu\nu, q} = 0 \quad p^\mu p^\nu W^{(1)R}_{\mu\nu, q} = e_q^2 C_F \frac{\alpha_s}{2\pi} \frac{Q^2}{4x}$$

One loop contribution to $F_2$ of a quark:

$$F^{(1)}_{2q} (x, Q^2) = e_q^2 x \frac{\alpha_s}{2\pi} \left\{ -\frac{1}{\varepsilon} \right\}_C P_{qq} (x) \left( 1 + \varepsilon \ln(4\pi e^{-\gamma_E}) \right) + P_{qq} (x) \ln \left( \frac{Q^2}{\mu_F^2} \right)$$

$$+ C_F \left[ (1 + x^2) \left( \frac{\ln(1-x)}{1-x} \right)_+ - 3 \left( \frac{1}{1-x} \right)_+ - \frac{1+x^2}{1-x} \ln(x) + 3 + 2x - \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-x) \right]$$

$$\Rightarrow \infty \quad \text{as} \quad \varepsilon \to 0$$

One loop contribution to quark PDF of a quark:

$$\phi^{(1)}_{q/\bar{q}} (x, \mu_F^2) = \left( \frac{\alpha_s}{2\pi} \right) P_{qq} (x) \left\{ \left( \frac{1}{\varepsilon} \right)_C + \left( -\frac{1}{\varepsilon} \right)_U \right\} + \text{UV-CT}$$

Different UV-CT = different factorization scheme!
Common UV-CT terms:

- **MS scheme:** \[ \text{UV-CT}_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x) \left( \frac{1}{\mathcal{E}_{\text{UV}}} \right) \]

- **DIS scheme:** choose a UV-CT, such that \[ C_q^{(1)}(x, Q^2 / \mu_F^2) \bigg|_{\text{DIS}} = 0 \]

One loop coefficient function:

\[
C_q^{(1)}(x, Q^2 / \mu_F^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_q^{(1)}(x, \mu_F^2)
\]

\[
C_q^{(1)}(x, Q^2 / \mu^2) = e_q^2 x \frac{\alpha_s}{2\pi} \left\{ P_{qq}(x) \ln \left( \frac{Q^2}{\mu^2} \right) \right\}
\]

\[
+ C_F \left[ (1 + x^2) \left( \frac{\ln(1-x)}{1-x} \right)_+ - \frac{1}{2} \left( -\frac{1}{1-x} \right)_+ - \frac{1 + x^2}{1-x} \ln(x) + 3 + 2x - \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-x) \right]
\]
Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale:

\[ \mu_F^2 \frac{d}{d \mu_F^2} F_2(x_B, Q^2) = 0 \]

Evolution (differential-integral) equation for PDFs:

\[ \sum_f \left[ \mu_F^2 \frac{d}{d \mu_F^2} C_f \left( \frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s \right) \right] \otimes \varphi_f(x, \mu_F^2) + \sum_f C_f \left( \frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s \right) \otimes \mu_F^2 \frac{d}{d \mu_F^2} \varphi_f(x, \mu_F^2) = 0 \]

- PDFs and coefficient functions share the same logarithms:

  PDFs:
  
  \[ \log \left( \frac{\mu_F^2}{\mu_0^2} \right) \quad \text{or} \quad \log \left( \frac{\mu_F^2}{\Lambda_{QCD}^2} \right) \]

  Coefficient functions:
  
  \[ \log \left( \frac{Q^2}{\mu_F^2} \right) \quad \text{or} \quad \log \left( \frac{Q^2}{\mu^2} \right) \]

DGLAP evolution equation:

\[ \mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j} \left( \frac{x}{x'}, \alpha_s \right) \otimes \varphi_j(x', \mu_F^2) \]
DGLAP evolution of PDFs

- **DGLAP equations:**
  \[
  \mu_F^2 \frac{\partial}{\partial \mu_F^2} \phi_i(x, \mu_F^2) = \sum_j P_{i/j} \left( \frac{x}{x'}, \alpha_s \right) \otimes \phi_j(x', \mu_F^2)
  \]

- **Splitting functions:**
  - Splitting functions have to be process independent
  - Can be then derived in many different ways
    - from the log part of the C's
    - from the anomalous dimension of the nonlocal operators defining the PDFs

- **Predictive power of pQCD:**
  Once the boundary condition is fixed by the data, the scale dependence of PDFs is a prediction of pQCD
Global QCD analysis of PDFs

- PDFs are extracted by using:
  - DGLAP: 
    \[ \frac{\mu_F^2}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j} \left( \frac{x}{x'}, \alpha_s \right) \otimes \varphi_j(x', \mu_F^2) \]
  - Factorized hard cross sections, e.g.:
    \[ F_{2h}(x_B, Q^2) = \sum_q C_{q/f} \left( \frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s \right) \otimes \varphi_{f/h} \left( x, \mu_F^2 \right) + O \left( \frac{\Lambda_{QCD}^2}{Q^2} \right) \]

- Data: to fix the boundary condition of DGLAP

- The order and scheme dependence of PDFs:
  - Leading order (tree-level) \( C_q \)
  - Next-to-Leading order \( C_q \)

  Calculation of \( C_q \) at NLO and beyond depends on the UVCT, the scheme dependence of \( C_q \), and the scheme dependence of PDFs
PDFs of a spin-averaged proton

- Modern sets of PDFs with uncertainties:

\[ x_f(x, Q^2) \]

Consistently fit almost all data with \( Q > 2 \text{GeV} \)
Kinematic Regions of DIS

- SLAC
- BCDMS
- NMC
- CCFR
- E665
- ZEUS
- H1

$Q^2 (GeV^2)$ | $x_{HERA} = 1$

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Comparison with DIS data

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Charm quark distributions

Large gluon at small-x \rightarrow large charm quark distribution
Uncertainties of gluon distribution

gluon at $Q = 3.16$ GeV

MRST2001

CTEQ5M

Ratio to CTEQ6

$10^{-5}$ $10^{-4}$ $10^{-3}$ $0.01$ $0.02$ $0.05$ $0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$ $0.7$ $0.8$ $0.9$ $1$

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Recover the effect of non-vanishing $k_T$

- **Sources of power corrections:**
  - Parton transverse momentum: $\langle k_T^2 \rangle/Q^2 \sim \langle k^2 \rangle/Q^2$
  - Target and parton masses: $m^2/Q^2$
  - Coherent multiple scattering: $\left(1/Q^2\right)/R^2 \left\langle F_+^+ F_-^\perp \right\rangle \langle \text{Medium length} \rangle$

- **Systematics of power corrections:**

$$\sigma_{phys}^h = \hat{\sigma}_{2}^i \otimes \left[ 1 + \alpha_s + \alpha_s^2 + \ldots \right] \otimes T_{2}^{i/h}(x)$$

$$+ \frac{\hat{\sigma}_{4}^i}{Q^2} \otimes \left[ 1 + \alpha_s + \alpha_s^2 + \ldots \right] \otimes T_{4}^{i/h}(x)$$

$$+ \frac{\hat{\sigma}_{6}^i}{Q^4} \otimes \left[ 1 + \alpha_s + \alpha_s^2 + \ldots \right] \otimes T_{6}^{i/h}(x)$$

$$+ \ldots$$

**Leading Twist**

**Factorization may not be true for power corrections!**

Need to be proved for any given process

Qiu and Vitev, PRL 2004
Improvement from the fixed order

- Beyond the Born term (lowest order), partonic hard-parts are NOT unique, due to renormalization of parton distributions

- Once \( \phi(x, \mu^2) \) is fixed in one scheme, same \( \phi(x, \mu^2) \) should be used for all calculations of partonic parts

- Coefficient has the \( P_{qq}(x) \ln \left( \frac{Q^2}{\mu_F^2} \right) \)

  Suggests to choose the scale: \( \mu_F^2 \sim Q^2 \)

- Coefficient has potentially large logarithms:

  \[ \ln(x), \quad \frac{1}{(1-x)_+}, \quad \left( \frac{\ln(1-x)}{1-x} \right)_+ \]

  Resummation of the large logarithms
Summary

- We can actually “see” and “count” the quarks and gluons – quark and gluon distributions
- PQCD factorization works for DIS to all orders as well as all powers due to OPE
- PDFs evolves – number of partons is sensitive to the probing scale
- PQCD global analysis for spin averaged cross sections results into the reasonably well-determined universal PDFs

What happen if there are more than one identified hadrons?