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Verification of Significance for Solar Cycles in ^{14}C-concentration Variation by Consideration of Measurement Uncertainty

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We report a significance test of the peaks in periodogram by taking into account of measurement uncertainties, which are relatively large compared with the data variation. As an illustration, we verified significance of solar cycles observed in the variation of ^{14}C-concentration in the period of 880-965AD by Miyahara *et al.*[1] with two ways.

1. Test of significance under consideration of only measurement uncertainties

Tested power spectrum

Tested power spectrum was estimated by the discrete fourier transformation.

- The power p_j and the frequency f_j :

$$p_j = \frac{|X_j|^2}{N\Delta t}, \quad f_j = \frac{j}{N\Delta t}$$

X_j : Fourier component

- The even time-interval data were prepared from the original data [1].

Δt = 1 [yr]: Sampling interval
N = 85 : Number of data points

- Miyahara *et al.* [1] reported 9-year periodicity (±1 year for the 68% confidence level against high frequency (< 3 years) noise) and 18-year period by use of wavelet analysis.
- The authors did not consider the (relatively large) measurement uncertainty.

⇒ We deal with the measurement uncertainty.

We found power peaks including 17-year, 9.4-year and 8.5-year periods.

Test of significance

Null hypothesis

Variation of data is due to measurement uncertainties

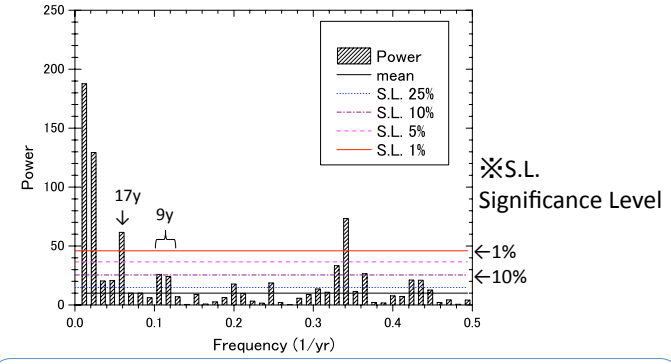
Under this, appearance-probability distributions were simulated for each p_j using Monte Carlo method.

The obtained appearance-probability distributions for p_j was found to be approximated by

$$f(p_j) = \frac{1}{10} e^{-\frac{p_j}{10}}. \quad \text{... Eq. (*)}$$

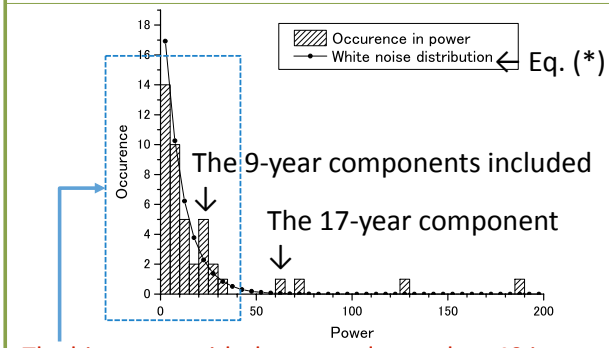
⇒ The fluctuation due to measurement uncertainties is represented by a white noise with $\sigma_0^2 = 10.0$.

σ_0^2 : A variance supposed when all the measurement error bars might to be the same.



- Only four peak components have S.L. lower than 1%.
- The 17-year component is significant at the S.L. of 1%.
- The 9-year components are not significant enough.

Histogram for chance occurrence in power spectrum



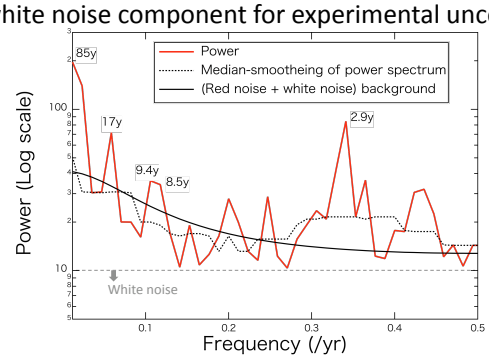
The histogram with the power lower than 40 is reproduced by the white noise. The components there may be caused by the fluctuation due to measurement uncertainties.

2. Red noise background estimation and significance of the power peaks

Assumption

- The measurement uncertainties correspond to 1σ of a normal distribution.
- The measured values in repeating experiments should fluctuate according to the normal distribution.

We simulated the power spectra by Monte Carlo method 100,000 times and obtained a mean value of power for each frequency. We determined a red noise background (usual for climatic phenomena) by fitting median-smoothed values with the 1st-order auto-regressive model [2], combined with the white noise component for experimental uncertainties.



The power values of the 17-year and 9-year period exceed the (red + white) noise background with the probability of 87% and 61%, respectively.

3. Conclusion

- Test of significance for the peaks was performed by taking into account of measurement uncertainties.
- The 17-year peak was significant enough.
- The significance of the 9-year peaks were much less than that of the 17-year peak.

[1] H. Miyahara et al., EPSL **272**, 290 (2008).
[2] Mann and Lees, Climatic Change **33**, 409 (1996).