We report a significance test of the peaks in periodogram by taking into account of measurement uncertainties, which are relatively large compared with the data variation. As an illustration, we verified significance of solar cycles observed in the variation of $^{14}$C-concentration in the period of 880-965AD by Miyahara et al.\cite{1} with two ways.

1. Test of significance under consideration of only measurement uncertainties

Tested power spectrum

Tested power spectrum was estimated by the discrete fourier transformation.

- The power $p_j$ and the frequency $f_j$:
  \[ p_j = \frac{\left| X_j \right|^2}{N \Delta \tau}, \quad f_j = \frac{j}{N \Delta \tau} \]

$X_j$: Fourier component  
- The even time-interval data were prepared from the original data \cite{1}.
  
  $\Delta \tau = 1$ [yr]: Sampling interval  
  $N = 85$: Number of data points

We found power peaks including 17-year, 9.4-year and 8.5-year periods.

Test of significance

Null hypothesis

Variation of data is due to measurement uncertainties. Under this, appearance-probability distributions were simulated for each $p_j$ using Monte Carlo method.

The obtained appearance-probability distributions for $p_j$ was found to be approximated by

\[ f(p_j) = \frac{1}{10} 10^{-\frac{p_j}{10}}, \quad \ldots \text{Eq. (*)} \]

The fluctuation due to measurement uncertainties is represented by a white noise with $\sigma_0^2 = 10.0$.

$\sigma_0^2$: A variance supposed when all the measurement error bars might to be the same.

- Only four peak components have S.L. lower than 1%.
- The 17-year component is significant at the S.L. of 1%.
- The 9-year components are not significant enough.

2. Red noise background estimation and significance of the power peaks

Assumption

- The measurement uncertainties correspond to $1\sigma$ of a normal distribution.
- The measured values in repeating experiments should fluctuate according to the normal distribution.

We simulated the power spectra by Monte Carlo method 100,000 times and obtained a mean value of power for each frequency. We determined a red noise background (usual for climatic phenomena) by fitting median-smoothed values with the 1st-order auto-regressive model \cite{2}, combined with the white noise component for experimental uncertainties.

The power values of the 17-year and 9-year period exceed the (red + white) noise background with the probability of 87% and 61%, respectively.

3. Conclusion

- Test of significance for the peaks was performed by taking into account of measurement uncertainties.
- The 17-year peak was significant enough.
- The significance of the 9-year peaks were much less than that of the 17-year peak.

\[1\] H. Miyahara et al., EPSL 272, 290 (2008).
\[2\] Mann and Lees, Climatic Change 33, 409 (1996).