Study of Scattering Amplitude in the Complex Scaling Method

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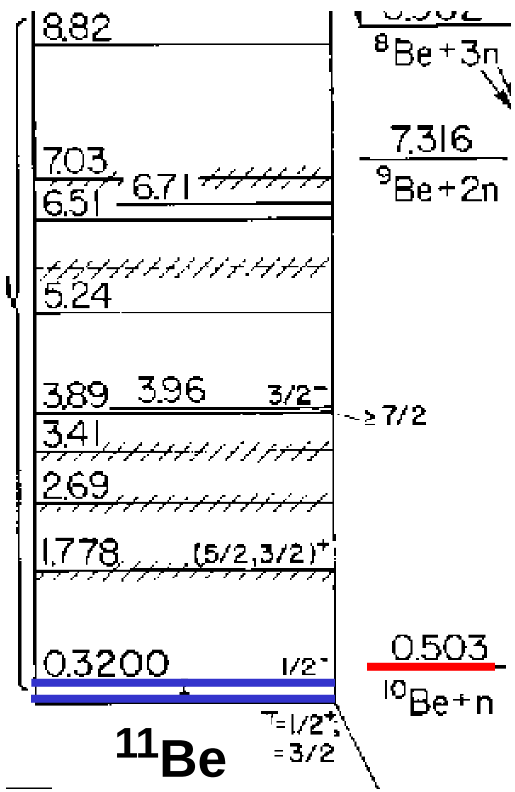
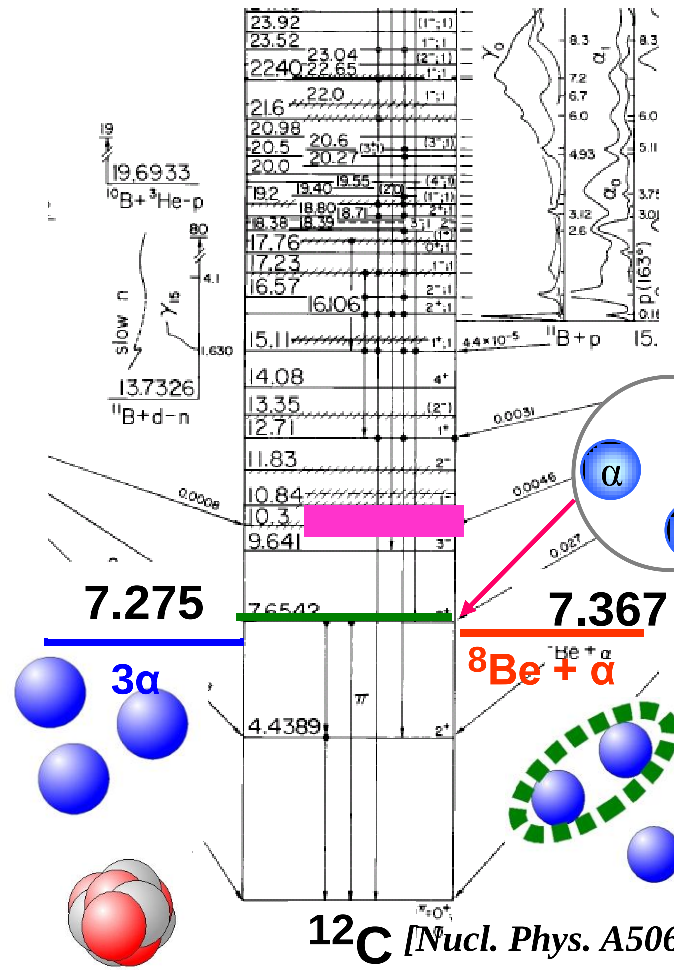
Topics:

1. ***Continuum Level Density for Coupled-channel systems***
2. ***Scattering T-Matrix Calculations in the Complex Scaling Method***

Motivation and Purposes

- For unstable nuclei, or even for stable nuclei, a unified description of bound and resonant states is necessary.
- Besides the observation of the quantized bound and resonant states, the majority of information comes from scattering states.
- We show that the complex scaling method can provides us with promising tools for such a purpose.

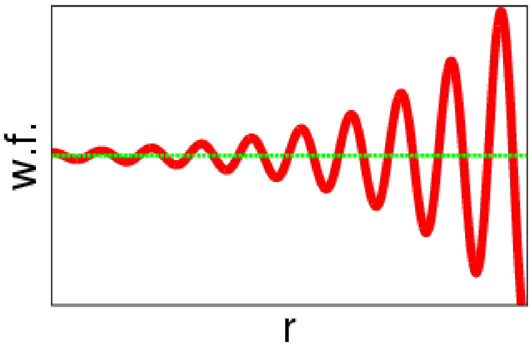




[Nucl. Phys. A506 (1990) 1]

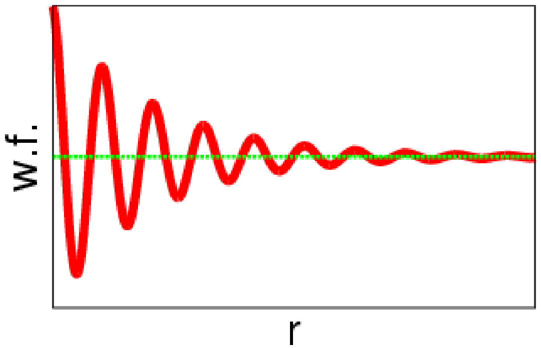
Complex Scaling Method

- The Complex Scaling Method(CSM) is well known as a tool to investigate the resonant state.
- In the CSM, the coordinate is rotated in the complex plane and the wave function of resonance is changed into a damping form.



$U(\theta): \quad r \to r\exp(i\theta)$

θ : scaling parameter



Resonance: $exp(ikr) \text{ -> } \alpha\, exp(-(k_R sin\theta - k_I cos\theta)r) \text{ -> } 0,$

$k = k_R - ik_I, \ (tan\theta > k_I/k_R)$

- Therefore, resonant states as well as bound states can be treated with squared integrable functions.
- Many calculations have been devoted to find resonant states of cluster nuclei or to calculate resonances of three-body problems.

Eigenvalues of a Complex Scaled Hamiltonian

- Carrying out the diagonalization of the complex scaled Hamiltonian, $H(\theta)$, we can gain information about the bound and resonant states.
- By contrast, the continuum spectra of the $H(\theta)$ are distributed on the 2θ-line from every threshold as shown in Fig. 1.

- ***In this talk, we show that rotated continuum states have important physical meanings in addition to bound and resonant states.***

- Keyword :
  1. Basis function method
  2. Extended completeness relation in CSM
  3. The complex-scaled Green function (CLD, T-matrix)

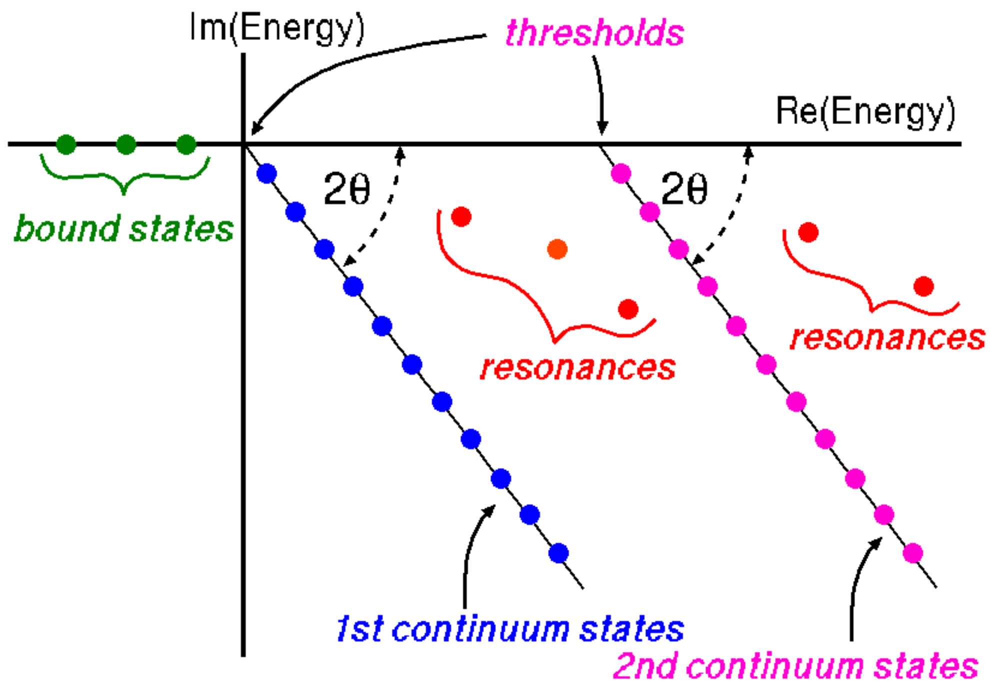


Fig.1 : Schematic energy eigenvalue distribution for a complex scaled Hamiltonian.

Topic 1: Continuum Level Density

- The continuum level density (CLD), $\Delta(E)$, plays an important role in the study of scattering theory since the CLD relate the scattering *S*-matrix and the Green function as

$$\Delta(E) = (2\pi)^{-1}\mathrm{Im}\frac{d}{dE}\ln\,\det S(E), \quad \Delta(E) = -\frac{1}{\pi}\mathrm{Im}\left[\mathrm{Tr}\left[G(E) - G_0(E)\right]\right]$$

[Ref: S.Shlomo, Nucl. Phys. A 539 (1992), 17.]

[Ref:A.T. Kruppa, Phys. Lett. B 431 (1998), 237.]

- CLD is constructed from two part : the full Green function part, $\rho(E)$, and the asymptotic Green function part, $\rho_0(E)$

$$\Delta(E) = \rho(E) - \rho_0(E)$$

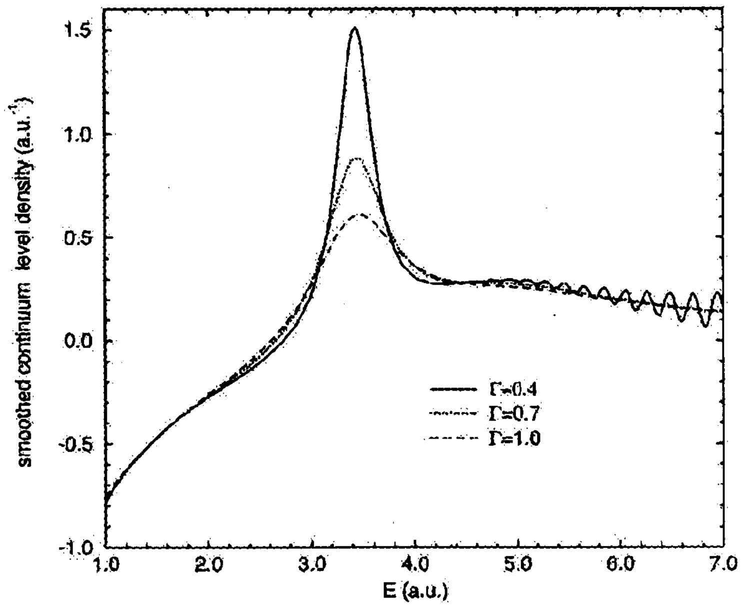
$$\rho(E) = -\frac{1}{\pi}Im\left\{Tr\left[\frac{1}{E-H}\right]\right\}, \qquad \rho_0(E) = -\frac{1}{\pi}Im\left\{Tr\left[\frac{1}{E-H_0}\right]\right\}$$

Continuum Level Density with basis function method

- A. T. Kruppa and K. Arai studied CLD in a framework of a finite number(M) of basis function and the Strutinsky smoothing procedure.

$$g_M(E) = \sum_i^M \delta(E - \epsilon_i) - \sum_j^M \delta(E - \epsilon_0^j)$$

- Their calculation succeeded well.
  - Their results depend on smoothing parameter
  - Only smoothed CLD is obtained(not “raw” CLD)



[ref: A. T. Kruppa and K. Arai, Phys. Rev. A59(1999)3556]

Continuum Level density in the complex scaling method

[Ref: R. Suzuki, T. Myo and K. Kato, Prog. Theor. Phys. 113 (2005), 1273.]

- Density of state in CSM

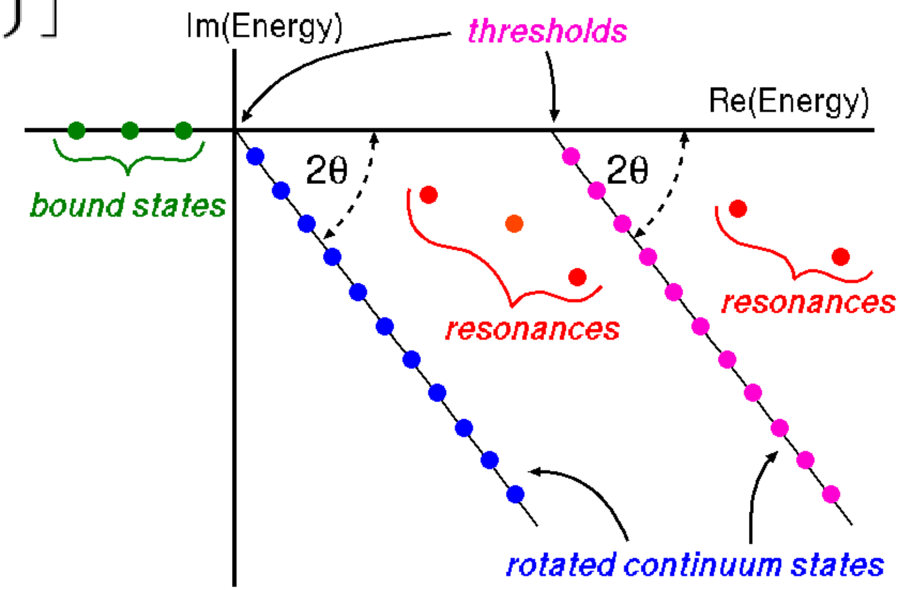
$$\rho(E) \;=\; -\frac{1}{\pi}\mathrm{Im}\left[Tr\left\{\frac{1}{E-H(\theta)}\right\}\right]$$

- CSM

$$U(\theta):\quad \boldsymbol{r}\to\boldsymbol{r}\exp(i\theta),$$

$$\boldsymbol{k}\to\boldsymbol{k}\exp(-i\theta)$$

θ :scaling parameter



- Extended Completeness relation

$$\mathbf{1} = \sum_B |\Phi_B(\boldsymbol{\xi})\rangle\langle\widetilde{\Phi}^*_B(\boldsymbol{\xi}')| + \sum_R^{N_\theta} |\Phi_R(\boldsymbol{\xi})\rangle\langle\widetilde{\phi}^*_R(\boldsymbol{\xi}')| + \int dk_\theta |\Phi_{k_\theta}(\boldsymbol{\xi})\rangle\langle\widetilde{\Phi}^*_{k_\theta}(\boldsymbol{\xi}')|$$

[ref: T. Myo, A. Ohnishi and K. Kato. Prog. Theor. Phys. 99(1998)801]

New smoothing mechanism in the complex scaling method

$$\rho(E) \approx \sum_B \delta(E-e_B) + \frac{1}{\pi}\sum_R \frac{\Gamma_R/2}{(E-E_R)^2+\Gamma_R^2/4} + \frac{1}{\pi}\sum_C \frac{E_C^I}{(E-E_C^R)^2+E_C^{I2}}$$

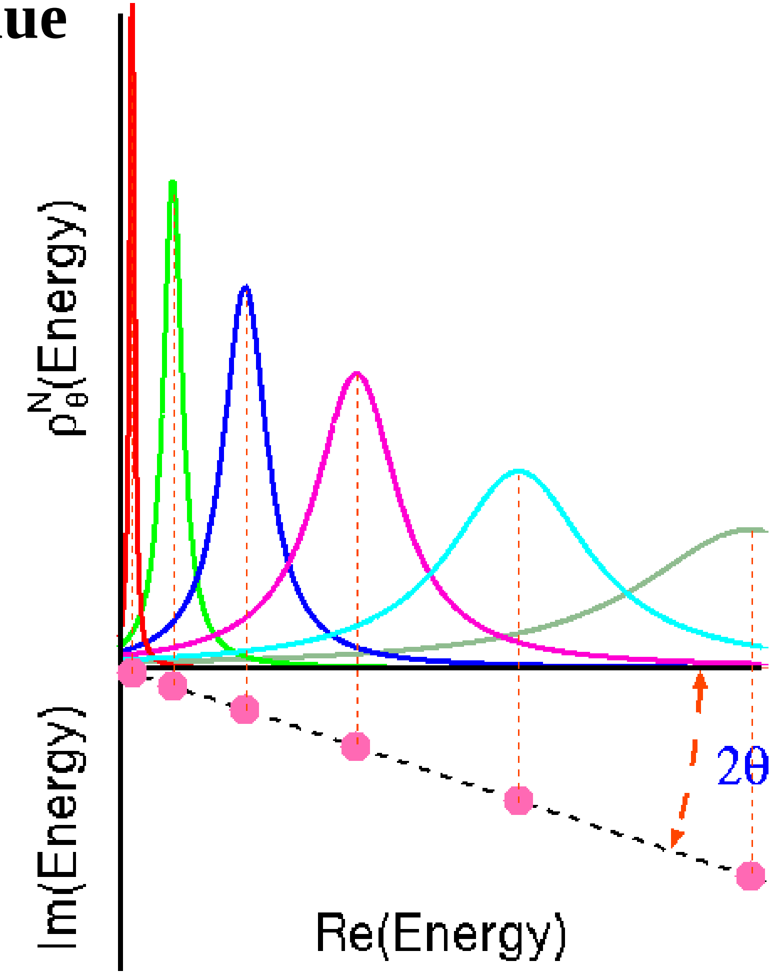
ρ(E) **is described by only the eigenvalue**

- Resonance $e_R = E_R - i\frac{\Gamma_R}{2}$
- Continuum $e_C = E_C^R - iE_C^I$

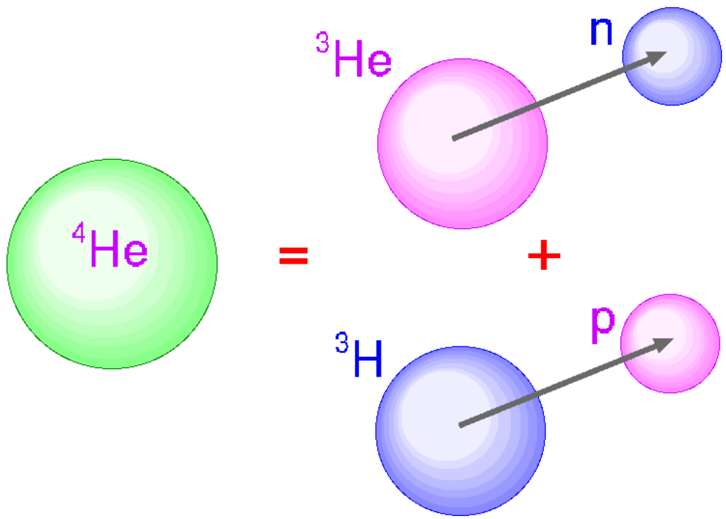
鈴 Basis function method

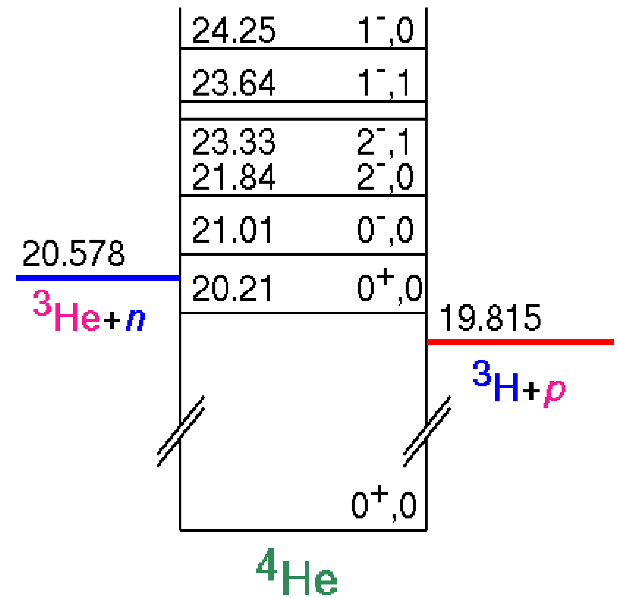
鈴Ex. Gaussian or H.O. wfs

鈴 Artificial smoothing procedure is not needed.



Numerical application of CLD:
^{4}He={^{3}H+p}+{^{3}He+n} system





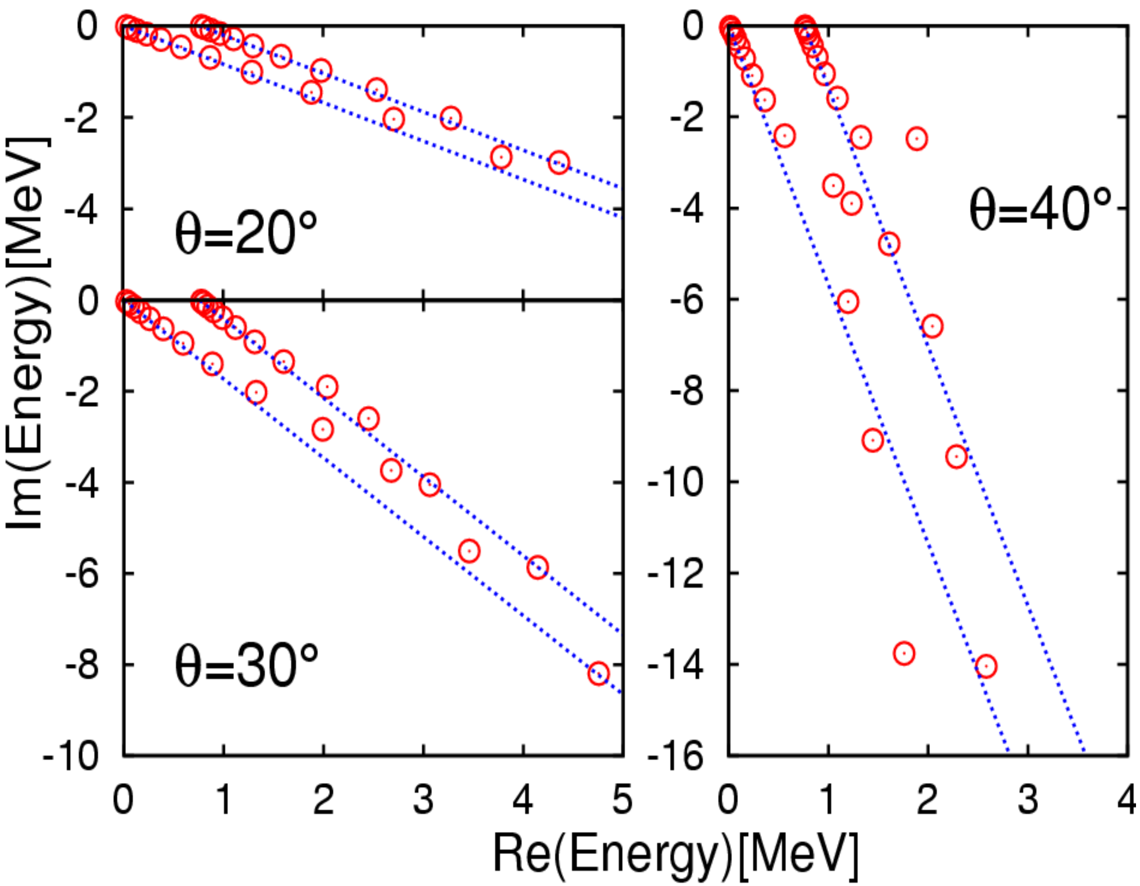
$$\left[-\frac{\hbar^2}{2\mu_{3_{H-p}}}\nabla^2 + V_d + \frac{e^2}{r}\mathrm{erf}(\sqrt{\alpha}r) - E + \epsilon_1\right]\chi_1 = V_c\chi_2$$

$$\left[-\frac{\hbar^2}{2\mu_{3_{He-n}}}\nabla^2 + V_d - E + \epsilon_2\right]\chi_2 = V_c\chi_1$$

Vc(r), Vd(r), : obtained to reproduce the experimental phase shift

[Ref: M. Teshigawara, doctor thesis, Hokkaido University, 1993]

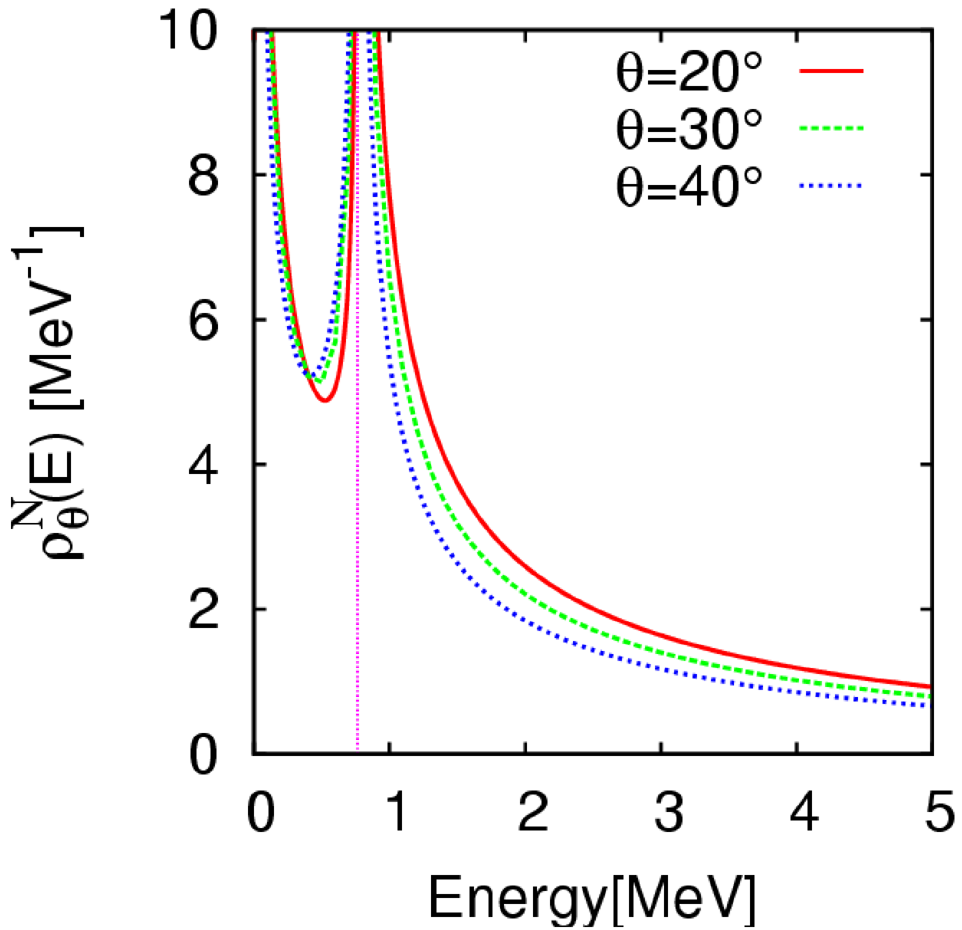
Eigenvalue distributions of $^3P_1$ state



We expand the relative wave function using the Gaussian basis functions. We use 30 basis functions for one channel. Therefore total basis number is N=60.

[Ref: M. Kamimura, Phys. Rev. A38 (1988) 621.]

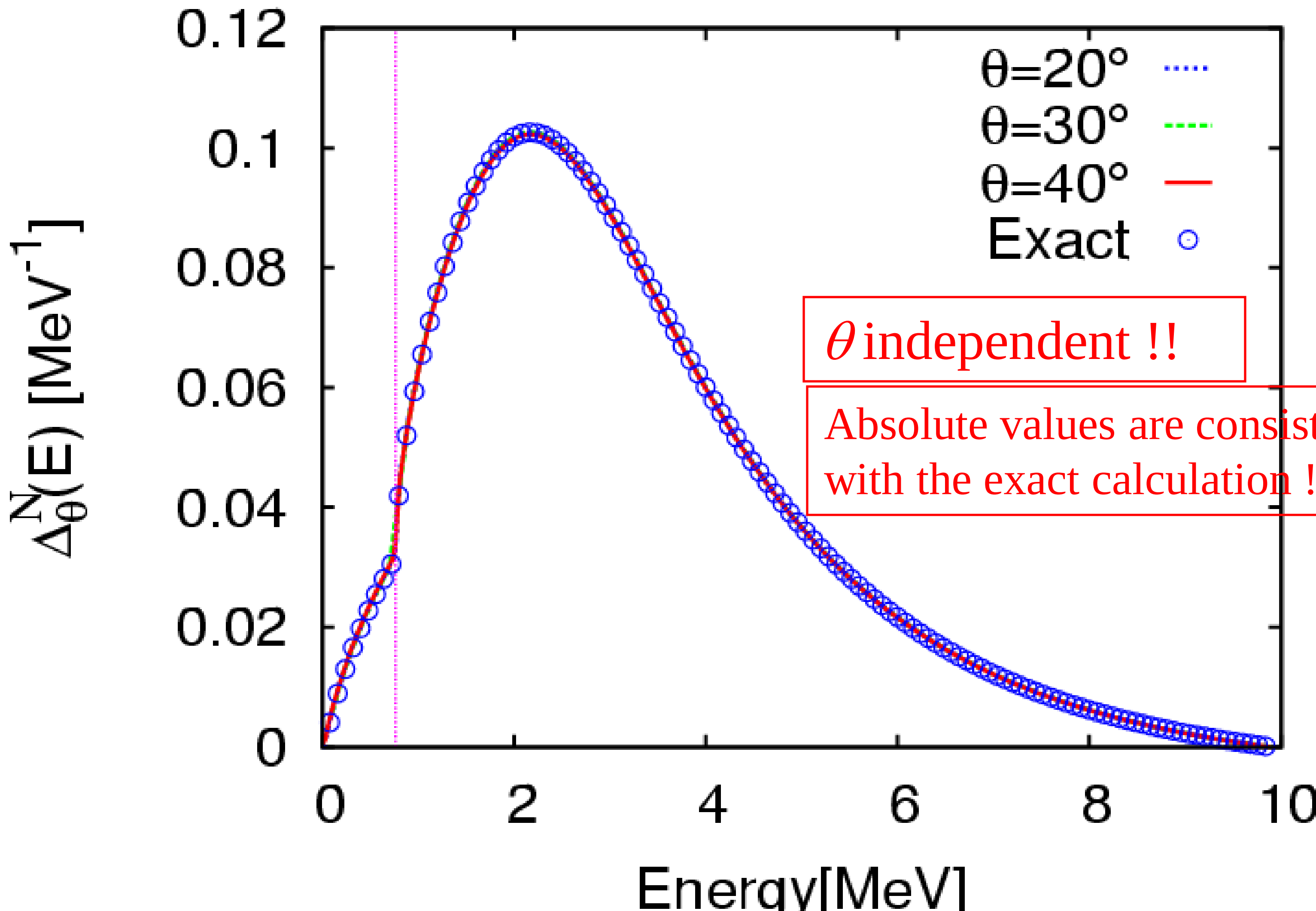
Calculated $\rho(E)$



- Description of smoothing behaviour is very successful in CSM with basis function method and slightly depends on the scaling parameter θ.
- To remove θ dependence, we subtract the free density term $\rho_0(E)$.

$$\Delta(E) = \rho(E) - \rho_0(E)$$

Calculated continuum level density : Δ(E)



θ independent !!

Absolute values are consistent with the exact calculation !!

Exact solution is calculated by solving the scattering problem and obtained by eigenphase shift :

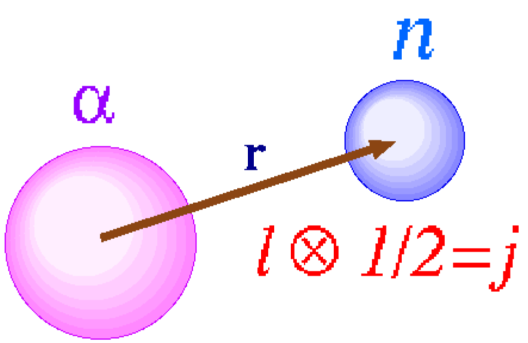
$$\Delta(E) = \frac{1}{\pi} \sum_j \frac{d\delta_j}{dE}$$

Single channel calculation

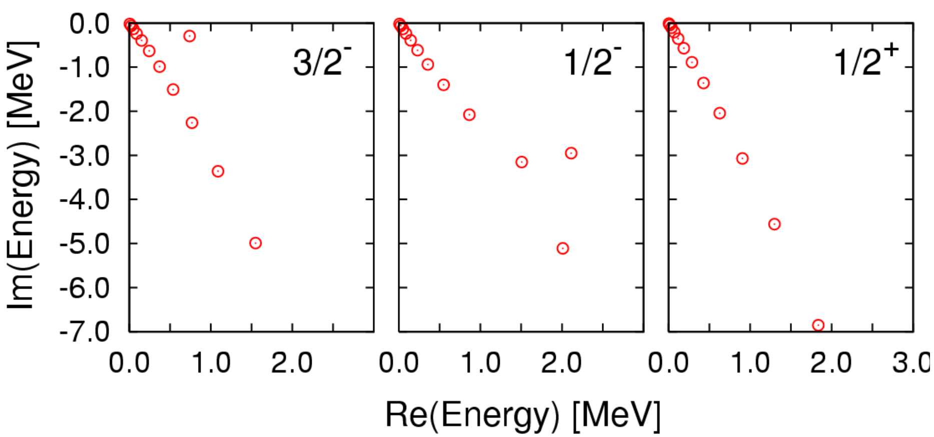
[Ref: R. Suzuki, T. Myo and K. Kato, Prog. Theor. Phys. 113 (2005), 1273.]

$$\Delta(E) = -\frac{1}{\pi} Im\left[Tr\left[G(E) - G_0(E)\right]\right] \quad and \quad \Delta(E) = \frac{1}{\pi}\frac{d\delta_l}{dE}$$

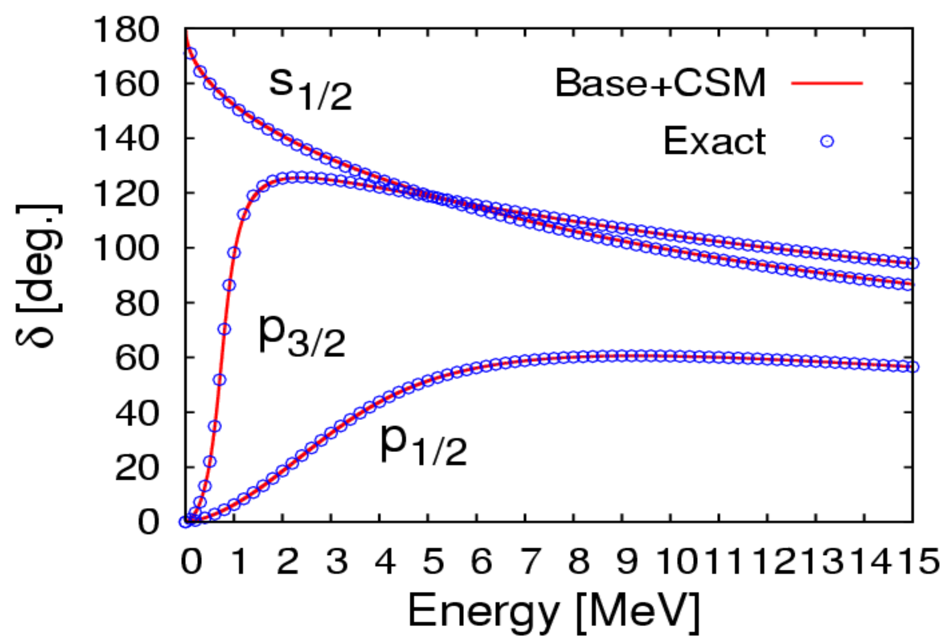
$$\delta(E) = \pi \int_0^E dE' \Delta(E') dE'$$



Energy eigenvalue distributions



Phase shift of ^{5}He



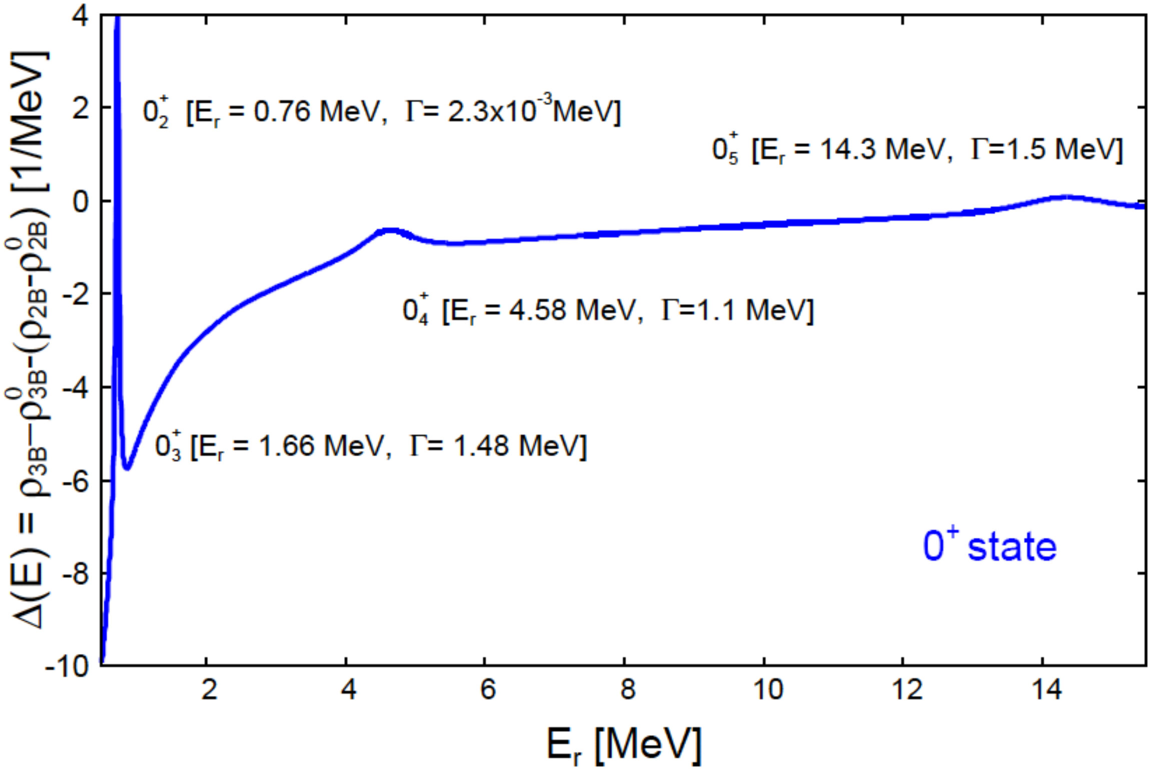
Three-body CLD (3α calculation)

[Ref: C. Kurokawa et al.,]

• *Formalism*

$$\Delta^{'}(E) = \rho_{3B}(E) - \rho_{3B}^{0}(E) - \left(\rho_{2B}(E) - \rho_{2B}^{0}(E)\right)$$
$$= -\frac{1}{\pi}\,\mathrm{Im}\left\{\mathrm{Tr}\left[\frac{1}{E-H_{3B}} - \frac{1}{E-H_{3B}^{0}} - \left(\frac{1}{E-H_{2B}} - \frac{1}{E-H_{2B}^{0}}\right)\right]\right\}$$

• *Numerical application : CLD is calculated using the eigenvalues*



Topic 2: Scattering T-Matrix Calculations in the Complex Scaling Method

[Ref: A.T. Kruppa, R. Suzuki and K. Kato, PRC 75, 044602(2007).]

- By using Green-operator and complex scaling method

$$t_{\alpha,\beta}(E) = \langle\Phi_\alpha| \int dr F_l(k_\alpha r)V(r)F_l(k_\beta r)|\Phi_\beta\rangle$$
$$+e^{i\theta}\langle\Phi_\alpha| \int dr dr' F_l(k_\alpha r e^{i\theta})\hat{V}(re^{i\theta})G^\theta(E;r,r')\hat{V}(r'e^{i\theta})F_l(k_\beta r' e^{i\theta})|\Phi_\beta\rangle$$

- Green's function

$$G^\theta(E;r',r) = \langle r'|\frac{\mathbf{1}}{E-H(\theta)}|r\rangle$$

- Using the Extended Completeness relation in the CSM

$$\mathbf{1} = \sum_B |\Phi_B(\boldsymbol{\xi})\rangle\langle\widetilde{\Phi}_B(\boldsymbol{\xi}')| + \sum_R^{N_\theta} |\Phi_R(\boldsymbol{\xi})\rangle\langle\widetilde{\phi}_R(\boldsymbol{\xi}')| + \int dk_\theta |\Phi_{k_\theta}(\boldsymbol{\xi})\rangle\langle\widetilde{\Phi}_{k_\theta}(\boldsymbol{\xi}')|$$

Green term

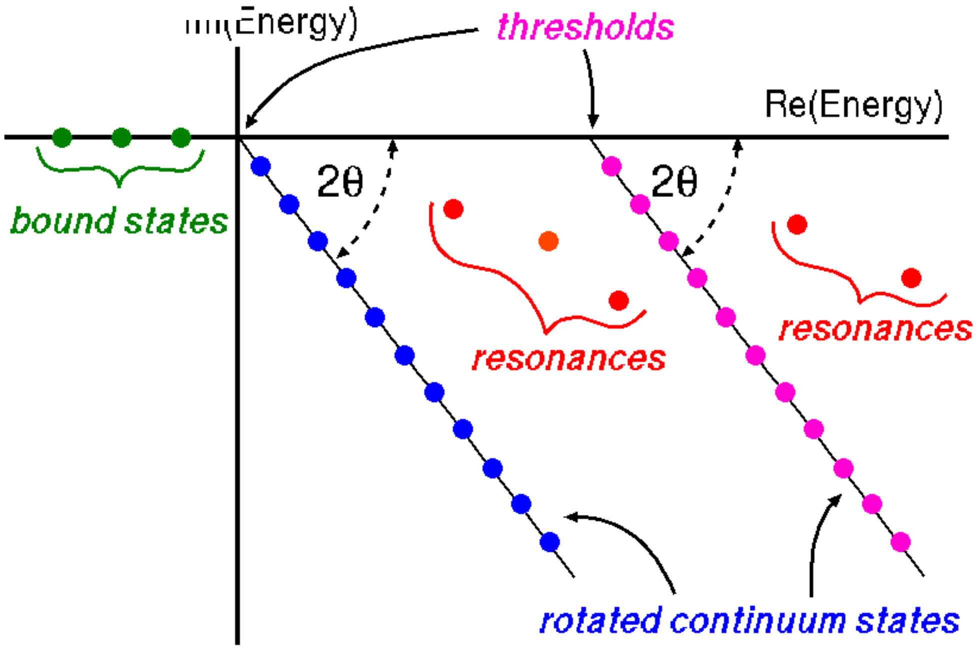
- 2nd term is approximated as

$$e^{i\theta}\left[\sum_B \frac{d_B(\theta)d_B(\theta)}{E-E_B^\theta}+\sum_R \frac{d_R(\theta)d_R(\theta)}{E-E_R^\theta}+\sum_C \frac{d_C(\theta)d_C(\theta)}{E-E_C^\theta}\right]$$

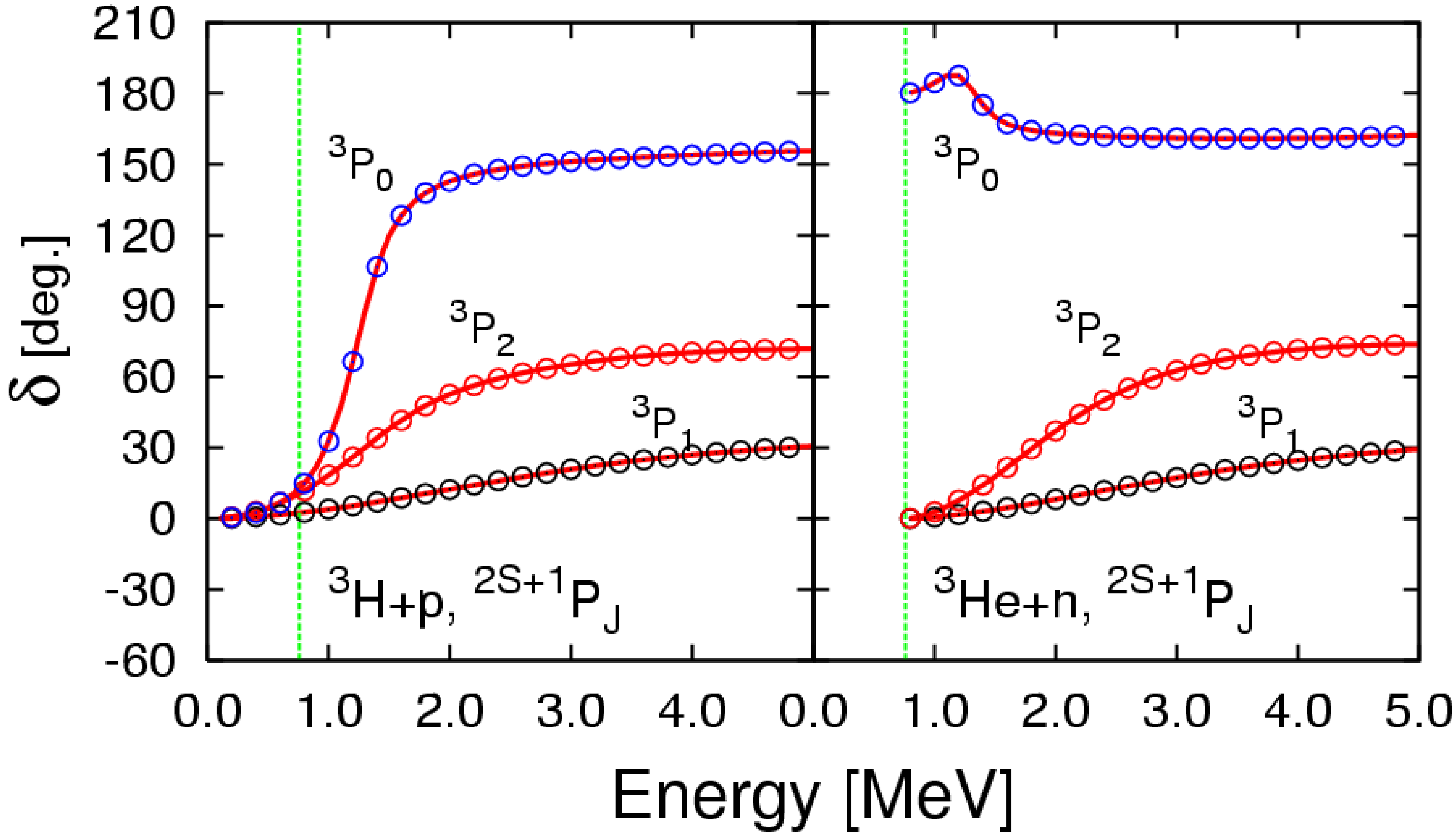
where

$$d_i(\theta) = \int dr \psi_i^\theta V(re^{i\theta}) F_l(kre^{i\theta})$$

- Basis function method



Numerical application : $^4He=\{^3H+p\}+\{^3He+n\}$ system



The *P*-wave phase shifts of the ^{3}H+*p* and ^{3}He+*n*. The solid lines are determined by CS calculation and the circles are calculated by integrating the differential equations with the Runge-Kutta method.

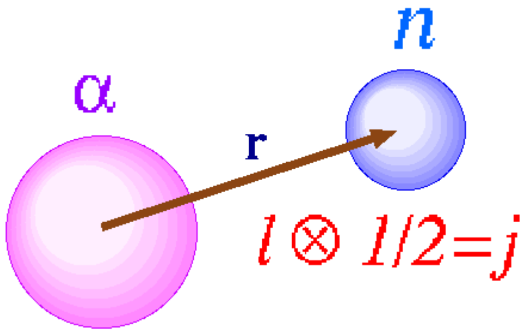
Conclusions and summary

- We showed that the complex scaling method provides us with very useful tools to describe bound and unbound states in a unified way.
  - resonant states are described with L^2 basis functions in the same manner as bound states.
  - rotated continuum states have also a important role.
  - It is easily to apply to three-body systems.

- As applications, we showed the continuum level density (CLD) and scatteing matrix calculations using only square integrable functions.

Application to ^{5}He=α+n system

$$\left[T_{\rm rel} + V_{\alpha n}(r) + \lambda\,|\phi_{\rm PF}\rangle\langle\phi_{\rm PF}| - E\right]\psi^{J}_{\rm rel}(\boldsymbol{r}) = 0$$



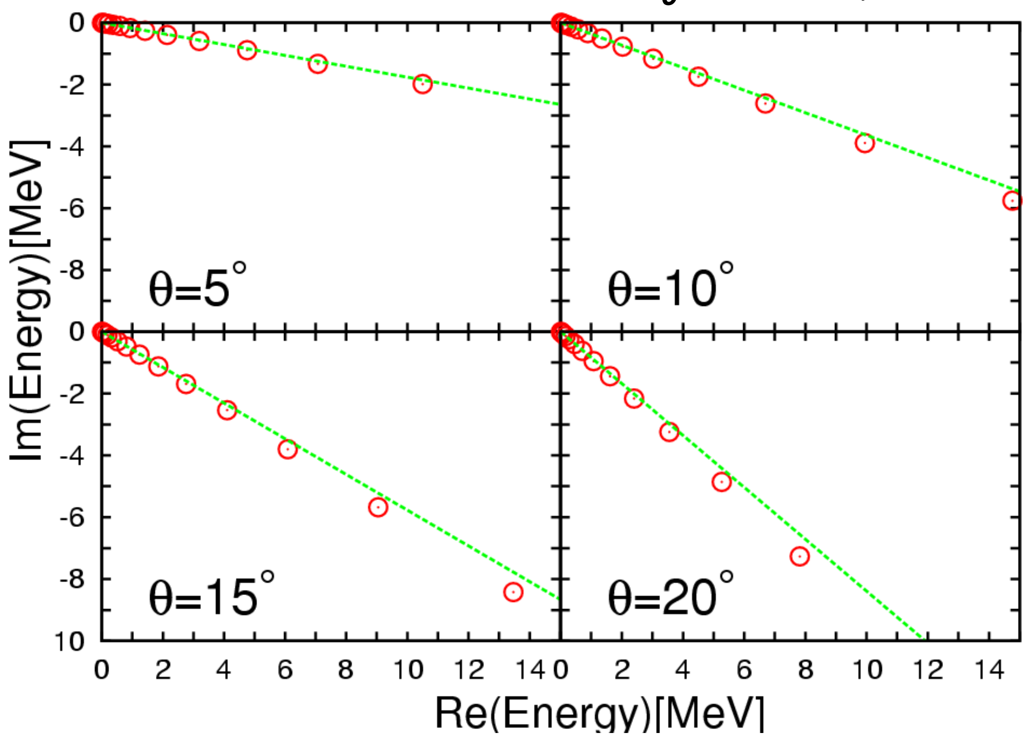
- $V_{\alpha n}(r)$: KKNN potential [3]
- Pauli principle is treated by OCM [4]

λ : 10^6 MeV, PF : $0s_{1/2}$

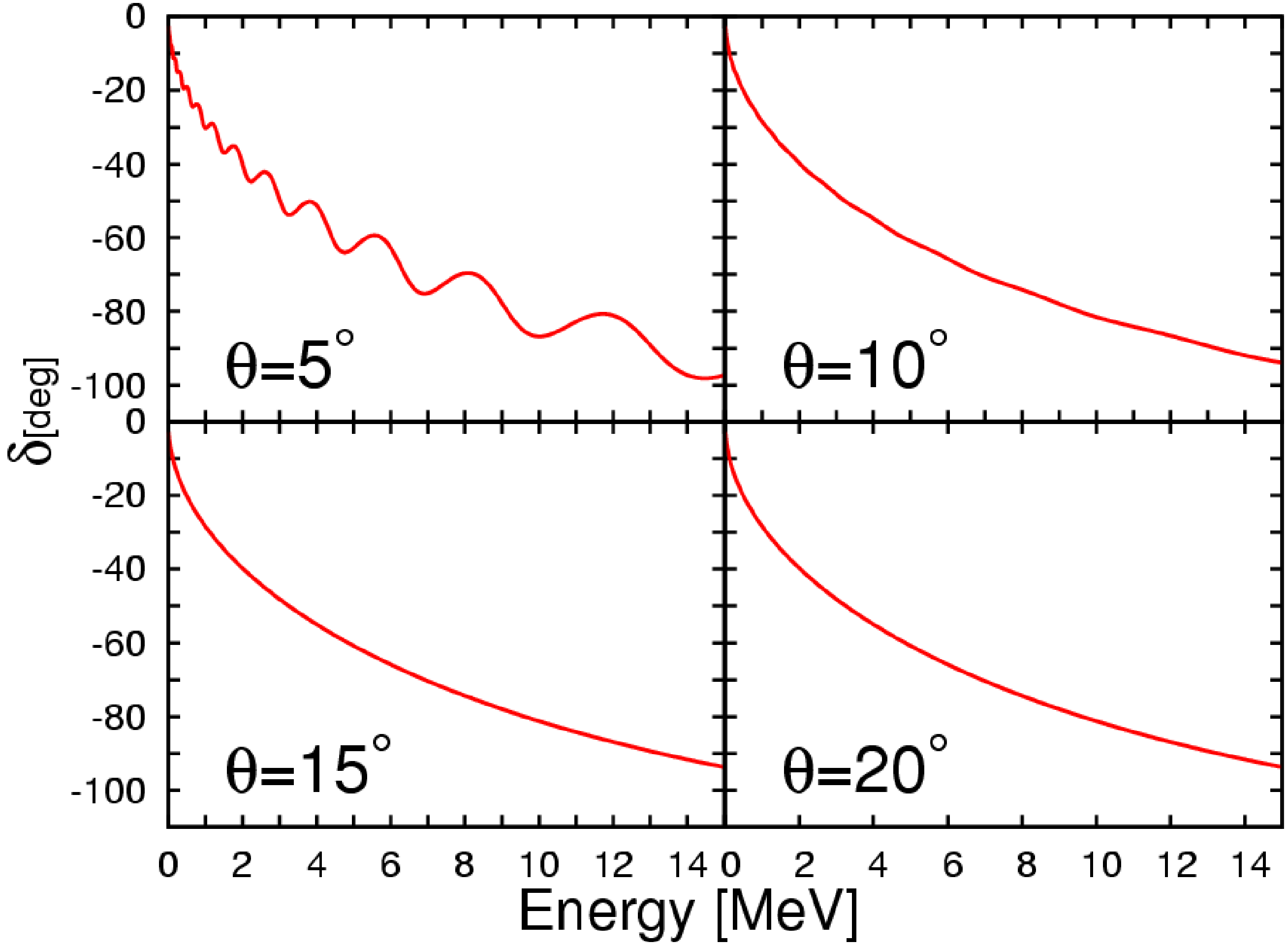
[3] H. Kanada, et al, Prog. Theor. Phys. 61(1979)1327.

[4] S. Saito, Prog. Theor. Phys. 41(1969)705.

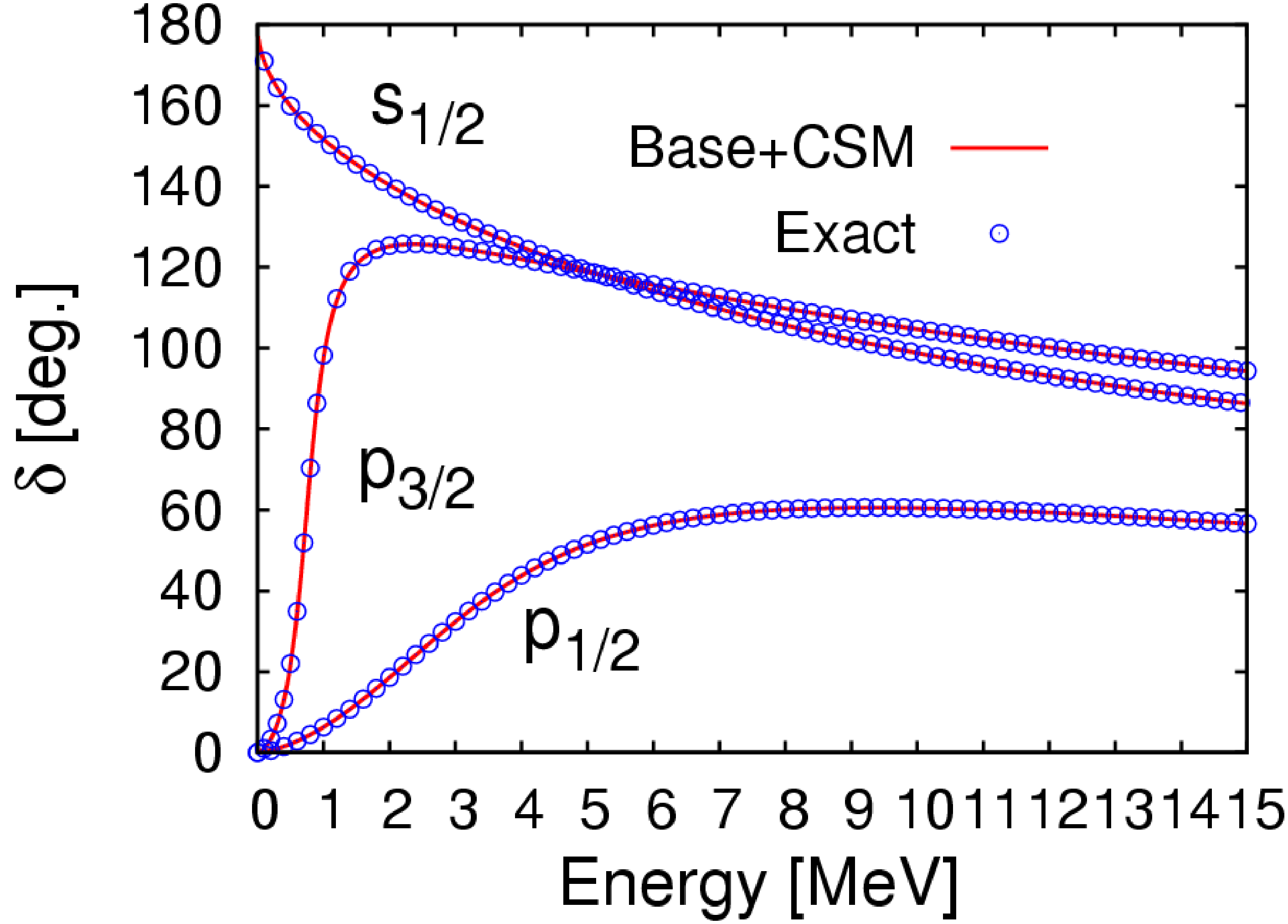
Energy eigenvalue distribution of $s_{1/2}$ state
Gaussian basis; N=30, b_0=0.2fm, γ=1.2



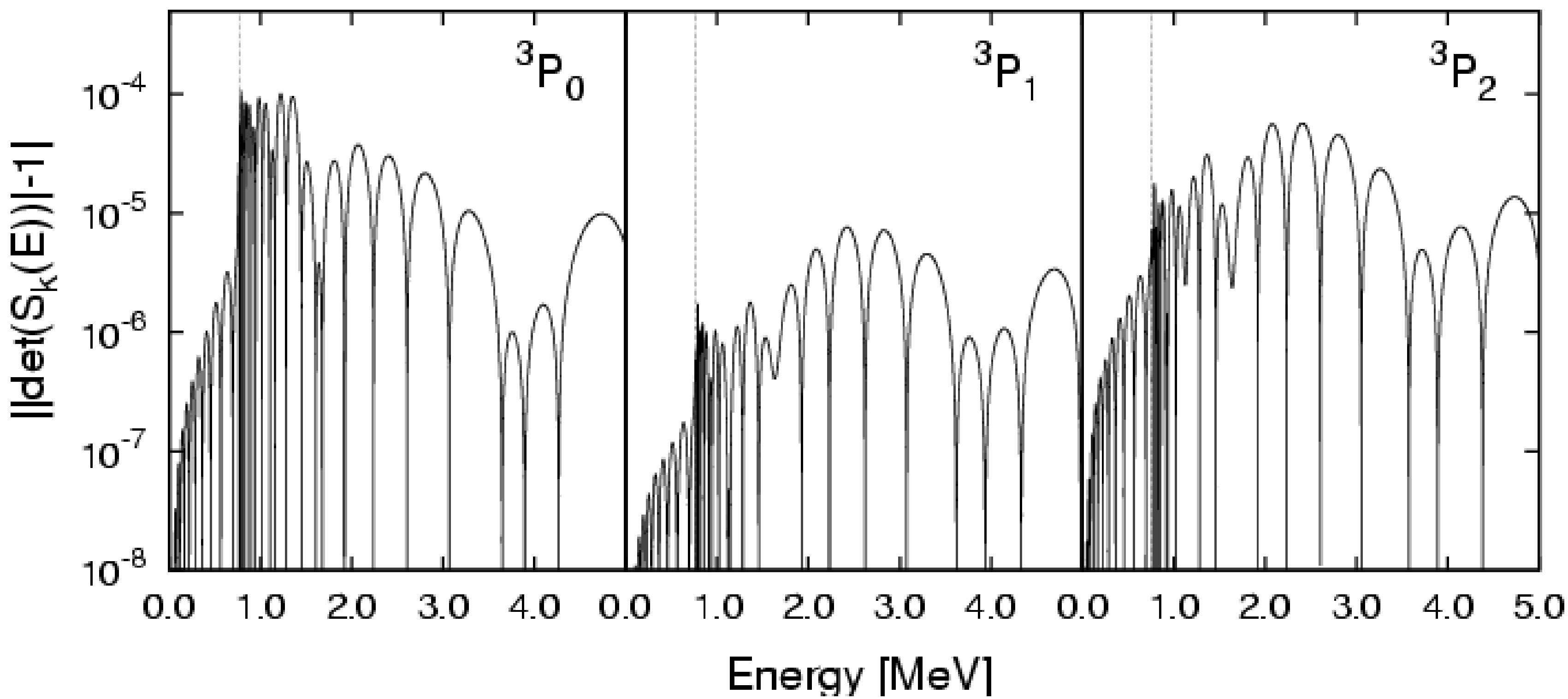
θ-dependency of phase shift of $s_{1/2}$ state



Calculated phase shift of $s_{1/2}$, $p_{1/2}$ and $p_{3/2}$ state



Check : Unitarity of calculated S-matrix



[Ref: A.T. Kruppa, R. Suzuki and K. Kato, PRC 75, 044602(2007).]

The quantity $||\det(S_k(E))|-1|$ for different partial waves as the function of the energy. The CS calculation is carried out using 30 basis functions. $S_k(E)$ is the 2x2 S-matrix in the partial wave $k={}^{2S+1}L_J$. The complex scaling parameter is 20 degree.