Four-body CDCC calculations applied to the scattering of Borromean nuclei

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Collaboration

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Outline

➢ Motivation
  ➢ Borromean nuclei

➢ Discretization methods based on Hyperspherical Harmonics (HH)
  ➢ Transformed Harmonic Oscillator (THO)
  ➢ Bin

➢ Four-body Continuum Discretized Coupled Channels (CDCC)

➢ Application to $^6\text{He}+$ target

➢ Summary and conclusions
Motivation: General scheme

Quantum system

→

Unbound states

⇓

Continuum spectrum

Non normalizable

⇒

Weakly bound system

Discretization method

⇒

Bound states

⇓

Discrete spectrum

Normalizable

⇒

Strongly bound system
HH method: The states of the system are expanded in Hyperspherical Harmonics

\[
\Psi_{j\mu n}(\rho, \Omega) = \sum_{\beta} R_{\beta j n}(\rho) \sum_{\nu I \nu I} \langle j_{ab} \nu I \nu | j \mu \rangle \kappa_I^I \sum_{m\sigma} \langle lm S_x \sigma | j_{ab} \nu \rangle \Upsilon_{Klm}^{l_x l_y}(\Omega) \chi_S^\sigma
\]

\[
\Omega \equiv \{ \alpha, \hat{x}, \hat{y} \}
\]

\[
\beta \equiv \{ K, l_x, l_y, l, S_x, j_{ab} \}
\]
Three-body discretization methods

- HH method: The states of the system are expanded in Hyperspherical Harmonics

\[ \Psi_{j\mu n}(\rho, \Omega) = \sum_{\beta} R_{\beta jn}(\rho) \sum_{\nu} \langle j_{ab} \nu I_{I} | j_{\mu} \rangle \kappa_{I}^{\nu} \sum_{m\sigma} \langle lm S_{x} \sigma | j_{ab} \nu \rangle \Upsilon_{Klm}(\Omega) \chi_{Sx}^{\sigma} \]

\[ \Omega \equiv \{ \alpha, \hat{x}, \hat{y} \} \]
\[ \beta \equiv \{ K, l_x, l_y, l, S_x, j_{ab} \} \]

- The hyperradial functions \( \{ R_{\beta jn} \} \) can be constructed by different discretization methods.
THO method: 2-body system


Central potential

\[ \varphi_B(r') \quad \sim \quad \phi_{0l_B}^{HO}(s) \]

\[ \int_0^r |\varphi_B(r')|^2 dr' = \int_0^s |\phi_{0l_B}^{HO}(s')|^2 ds' \]

THO basis

\[ \psi_{nl}^{THO}(r) = \varphi_B(r) s(r)^{l-l_B} L_n^{l+1/2} \left( s(r)^2 \right) \]
THO method: 3-body system

⇒ \( s(\rho) \) is calculated for each channel \( \beta \) included in the bound ground state

\[
\int_0^\rho d\rho' \rho'^{5/2} |R_{B\beta}(\rho')|^2 = \int_0^s ds' s'^{5/2} |R^{HO}_{0K}(s')|^2
\]

\[
R^{THO}_{i\beta}(\rho) = R_{B\beta}(\rho) L_i^{K+2} (s_\beta(\rho)^2)
\]
THO method: 3-body system

- $s(\rho)$ is calculated for each channel $\beta$ included in the bound ground state

$$\int_0^\rho d\rho' \rho'^{5/2} |R_{B\beta}(\rho')|^2 = \int_0^s ds' s'^{5/2} |R_{0K}^{HO}(s')|^2$$

$$R_{i\beta}^{THO}(\rho) = R_{B\beta}(\rho)L_i^{K+2}(s_\beta(\rho)^2)$$

- The Hamiltonian of the system is diagonalized in a finite THO basis with $i = 0, \ldots, n_b$
THO method: 3-body system

- $s(\rho)$ is calculated for each channel $\beta$ included in the bound ground state

\[ \int_0^\rho d\rho' \rho'^{5/2} |R_{B\beta}(\rho')|^2 = \int_0^s ds' s'^{5/2} |R_{0K}^{HO}(s')|^2 \]

\[ R_{i\beta}^{THO}(\rho) = R_{B\beta}(\rho) L_i^{K+2} (s_\beta(\rho)^2) \]

- The Hamiltonian of the system is diagonalized in a finite THO basis with $i = 0, \ldots, n_b$

- Finally the hyperradial functions are obtained as

\[ R_{i\beta jn}^{THO}(\rho) = \sum_i C_{n}^{i\beta j} R_{i\beta}^{THO}(\rho) \]
Bin method (I)

Continuum states can be expanded in HH as

$$\Psi_{\kappa j \mu}(\rho, \Omega, \Omega_\kappa) = \sum_{\beta \beta'} R_{\beta \beta' j}(\kappa \rho) \mathcal{Y}_{\beta j \mu}(\Omega) \times \sum_{m' \sigma'} \langle l' m' S' x \sigma' | j \mu \rangle \Upsilon_{K' l' y m'}(\Omega_\kappa)$$

\{\beta'\} incoming; \{\beta\} outgoing; \kappa = \sqrt{2m\varepsilon}/\hbar
Bin method (I)

- Continuum states can be expanded in HH as
  \[ \Psi_{\kappa j \mu}(\rho, \Omega, \Omega_\kappa) = \sum_{\beta \beta'} R_{\beta \beta' j}(\kappa \rho) \mathcal{Y}_{j \mu}(\Omega) \times \sum_{m' \sigma'} \langle l' m' S'_x \sigma' | j \mu \rangle \Upsilon_{K' l' m'}^{l' l'}(\Omega_\kappa) \]
  \{\beta'\} incoming; \{\beta\} outgoing; \kappa = \sqrt{2m\varepsilon/\hbar}

- Bins for each incoming channel are calculated as
  \[ R_{j \beta \{\beta' \varepsilon_{av}\}}^{\text{bin}}(\rho) = \frac{2}{\sqrt{\pi N_{\beta' j}}} \int_{\kappa}^{\kappa + \Delta \kappa} d\kappa f_{\beta' j}(\kappa) R_{\beta \beta' j}(\kappa \rho) \]
  \[ f_{\beta' j}^{n-r}(\kappa) = e^{-i\delta_{\beta' j}(\kappa)} \quad f_{\beta' j}^{r}(\kappa) = \sin \delta_{\beta' j}(\kappa) e^{-i\delta_{\beta' j}(\kappa)} \]
  \[ N_{\beta' j} = \int_{\kappa}^{\kappa + \Delta \kappa} d\kappa |f_{\beta' j}(\kappa)|^2 \]
Bin method (II)

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- So $S$-matrix is diagonalized for every $\varepsilon$ obtaining the eigenchannels and eigenphases.
- Then bins are calculated for each eigenchannel such as explained before for incoming channels.
- Now we include only up to $n_{ec}$ eigenchannels that corresponds to the biggest phase-shifts.
4-body CDCC formalism

Coupled channels system

\[
\left[ -\frac{\hbar^2}{2m_r} \left( \frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + \varepsilon_{n_j} - E \right] f^J_{Ln_j}(R) \\
+ \sum_{L'n_j'} i^{L'\text{L}} V^J_{Ln_j,L'n_j'}(R) f^J_{L'n_j'}(R) = 0
\]
Coupling potentials

\[ V_{Ln_j,L'n'_j'}^J(R) = \langle Ln_j JM | \sum_{k=1}^{3} \hat{V}_{kt}(\vec{r}_k) | L'n'_j' JM \rangle \]

where

\[ \Phi_{Ln_j}^{JM}(\hat{R}, \vec{x}, \vec{y}) = \sum_{\mu M L} \psi_{j \mu n}(\vec{x}, \vec{y}) \langle LM L j \mu | JM \rangle Y_{L M L} (\hat{R}) \]

multipolar expansion

\[ V_{Ln_j,L'n'_j'}^J(R) = \sum_Q (-1)^{J-j} \hat{L} \hat{L}' \left( \begin{array}{ccc} L & Q & L' \\ 0 & 0 & 0 \end{array} \right) \times W(LL'_j j' j, Q J) F_{n_j,n'_j'}^Q(R) \]
Form factors

\[
F_{n_j, n'_j}(R) = (-1)^{Q + 2j - j' \hat{j} \hat{j}'} (2Q + 1) \\
\times \sum_{\beta \beta'} \sum_{k=1}^{3} \sum_{k_k} N_{\beta \beta_k} N_{\beta' \beta_k'} \\
\times (-1)^{l_{x_k} + S_{x_k} + j'_{abk} - j_{abk} - I_k} \delta_{l_{x_k} l'_{x_k}} \delta_{S_{x_k} S'_{x_k}} \\
\times \hat{l}_{y_k} \hat{l}'_{y_k} \hat{l}_{k} \hat{l}'_{k} \hat{j}_{abk} \hat{j}'_{abk} \left( \begin{array}{ccc} l_{y_k} & Q & l'_{y_k} \\ 0 & 0 & 0 \end{array} \right) \\
\times W(l_k l'_k l_{y_k} l'_{y_k}; Q l_{x_k}) W(j_{abk} j'_{abk} l_k l'_k; Q S_{x_k}) \\
\times W(j j' j_{abk} j'_{abk}; Q I_k) \int \int (\sin \alpha_k)^2 (\cos \alpha_k)^2 \rho^5 d\alpha_k d\rho \\
\times R_{\beta j n}(\rho) \varphi_{K_k}^{l_{x_k} l_{y_k}} (\alpha_k) \nu_{Q}^{l_{x_k} l'_{y_k}} (\alpha_k) R_{\beta' j' n'}(\rho)
\]
$^6$He Hamiltonian

\[ \hat{H}(\rho, \Omega) = \hat{T}(\rho, \Omega) + \hat{V}(\rho, \Omega) \]

\[ V = V_{n\alpha} + V_{n\alpha} + V_{nn} + V_{nn\alpha} \]

\[ n + \alpha \quad V_{n\alpha} = V_c + V_{SO} \]

$V_c, V_{SO}$: Woods-Saxon

\[ \implies \text{GPT} \quad n + n \quad V_{nn} = V_c + V_{SO} + V_t \]

$V_c, V_t, V_{SO}$: Gaussian

\[ n + n + \alpha: \text{power} \quad V_{pow} = \frac{a}{[1+(r/b)^c]} \]

Pauli forbidden states: repulsive $V_c$ for s-waves
THO basis

\[ \beta = (2,0,0,0,0,0) \]

\[ K_{\text{max}} = 8 \]
$K_{max} = 8$

$n_b = 4$
Bin: Energy spectrum

\[ K_{max} = 8 \]

1ec
$^6\text{He} + ^{64}\text{Zn} @ 13.6\text{MeV}$: elastic

$\varepsilon_{\text{max}} = 6\text{ MeV}$


One channel

THO $n_b=4$ $N=60$

bin $n_{ec}=3$ $N=46$

dineutron model
$^6\text{He} + ^{64}\text{Zn} @ 13.6\text{MeV}: \text{breakup}$

THO

$^0_+$

$1^-$

$2^+$

Bin

$^0_+$

$1^-$

$2^+$

$\sigma_{bu} (\text{mb})$

$E_x (\text{MeV})$

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2\(^+\) resonance

\[
\rho^{5/2} R_{\beta}^{\text{res}} (\rho)
\]

- **1st channel**
- **2nd channel**
- **3rd channel**

Four-body CDCC calculations applied to the scattering of Borromean nuclei – p. 19/4
$^6$He+$^{64}$Zn@10MeV: elastic


one channel

THO $n_b=3$ $N=27$

bin $n_{ec}=3$ $N=41$

dineutron model

$\varepsilon_{max} = 5$ MeV
$^{6}\text{He}+^{208}\text{Pb}@22\text{MeV}: \text{elastic}$

$\sigma/\sigma_{\text{Ruth}}$ vs $\theta_{\text{c.m.}}$ (deg)

- Louvain-la-Neuve data
- One channel
- THO $n_b=4 \ N=86$
- Bin $n_{ec}=3 \ N=45$
- Dineutron model

$\varepsilon_{max} = 8 \text{ MeV}$
\( ^6\text{He} + ^{208}\text{Pb} @ 22\text{MeV}: \text{convergence} \)
$^6$He+$^{12}$C@229.8MeV: elastic

![Graph showing elastic scattering data for $^6$He+$^{12}$C at 229.8 MeV. The graph plots the ratio of the cross section to the Ruth cross section ($\sigma/\sigma_{Ruth}$) against the angle in degrees ($\theta_{c.m.}$). The data points are compared with theoretical calculations for one channel and for two channels with different parameters. The maximum energy considered is $\varepsilon_{max} = 25$ MeV.](image-url)

- V. Lapoux et al., PRC 66, 034608 (2002)
- One channel
- THO $n_b=2$ $N=105$
- Bin $n_{ec}=2$ $N=33$

Four-body CDCC calculations applied to the scattering of Borromean nuclei – p. 23/40
Summary and conclusions

- We have presented two different discretization methods for a three-body system, THO and bin, based on expansion in HH.
- We have generalized the CDCC formalism for the application to four-body reactions.
- The formalism has been applied to the Borromean nucleus $^6\text{He}$.
- We have seen as CDCC calculations with THO or bin as discretization methods is an efficient procedure for the study of four-body reactions.