Black sphere approach to proton elastic scattering and reaction cross section

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Question

Are nuclei black? ………………………… *in some sense, yes.*

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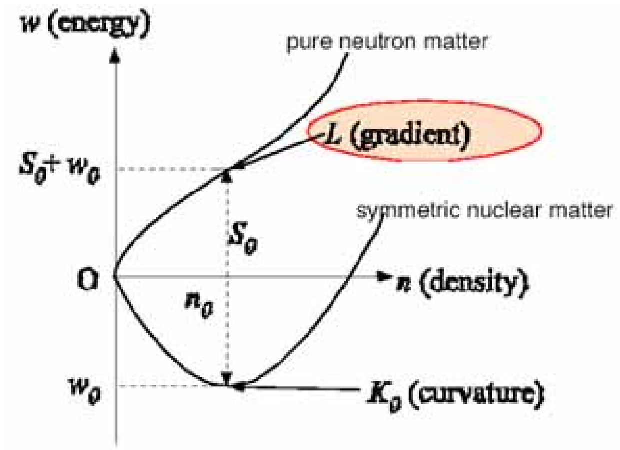
Introduction



Density distribution of neutron-rich unstable nuclei

↕

Equation of state (EOS) of asymmetric nuclear matter

↕

Neutron star structure

Stellar collapse

Heavy-ion collisions at intermediate and relativistic energies

Future systematic data for proton elastic scattering and reaction cross section for heavy unstable nuclei in RIKEN & GSI

How ???

Information about the symmetry energy at subnuclear densities

Introduction (contd.)

EOS of asymmetric nuclear matter

↕ *Systematic analysis* — **Macroscopic nuclear model**

Ref. Oyamatsu & Iida, PTP **109** (2003) 631.

Density distribution of neutron-rich nuclei

↕ *Systematic analysis* — **"Black" sphere model**

Ref. Kohama, Iida, & Oyamatsu,
PRC **69** (2004) 064316; **72** (2005) 024602.

Reaction cross section & proton elastic scattering for neutron-rich nuclei

For stable nuclei:

A "black" sphere radius, *a*, deduced from the observed diffraction peak in the proton elastic differential cross section, can be regarded as a "reaction radius," inside which the reaction with an incident proton occurs.

The "black" sphere picture allows us to construct formula for the reaction cross section without introducing any adjustable energy-dependent parameter.

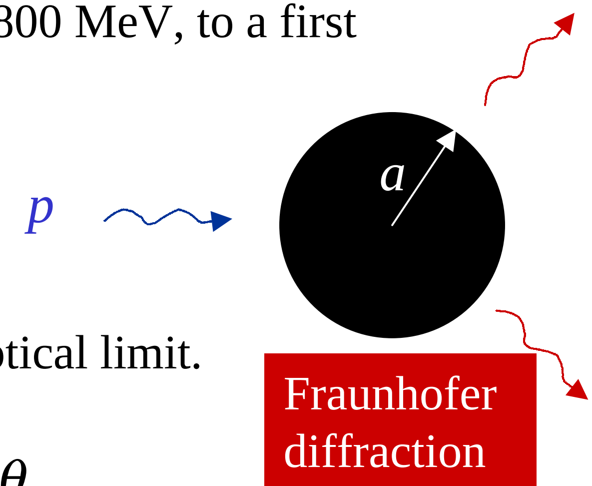
For unstable nuclei: important future work!

"Black" sphere model

"Black" sphere picture of a nucleus

Ref. Kohama, Iida, & Oyamatsu, PRC **69** (2004) 064316.

For protons of incident energies of order or greater than 800 MeV, to a first approximation,



- A nucleus is absorptive to protons.

$$\sigma_{pN}\, n_0\, a \gg 1$$

- Protons behave according to wave optics in the optical limit.

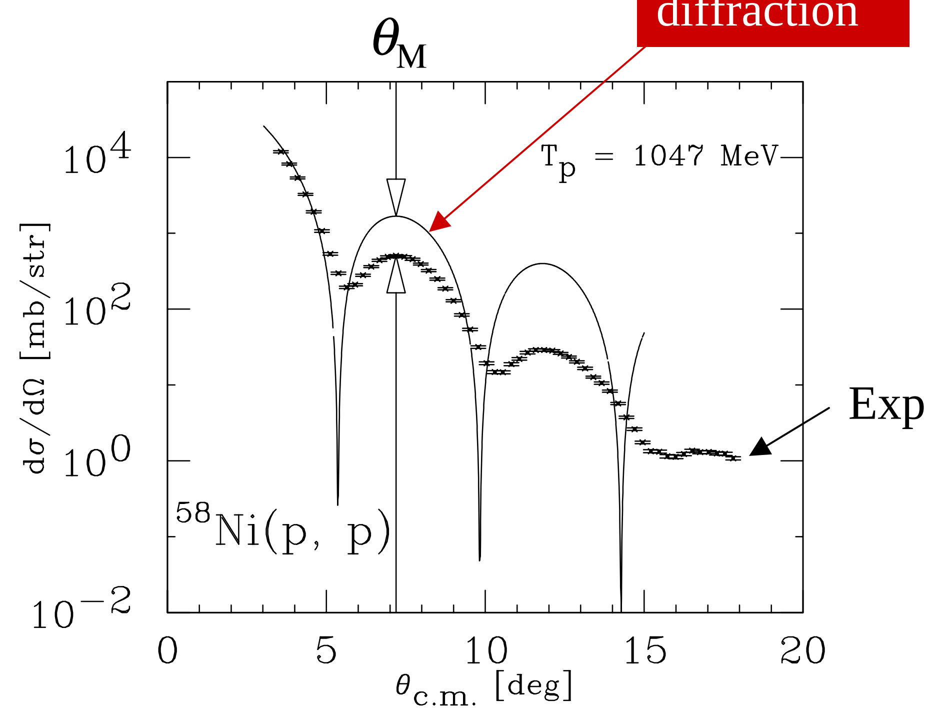
$$\lambda_p \ll a$$

Differential p-A elastic cross section

At the first peak of angle θ_M,

$$2ka\sin\left(\frac{\theta_M}{2}\right) = 5.1356\cdots$$

k: proton momentum
in the C.M. system



"Black" sphere model (contd.)

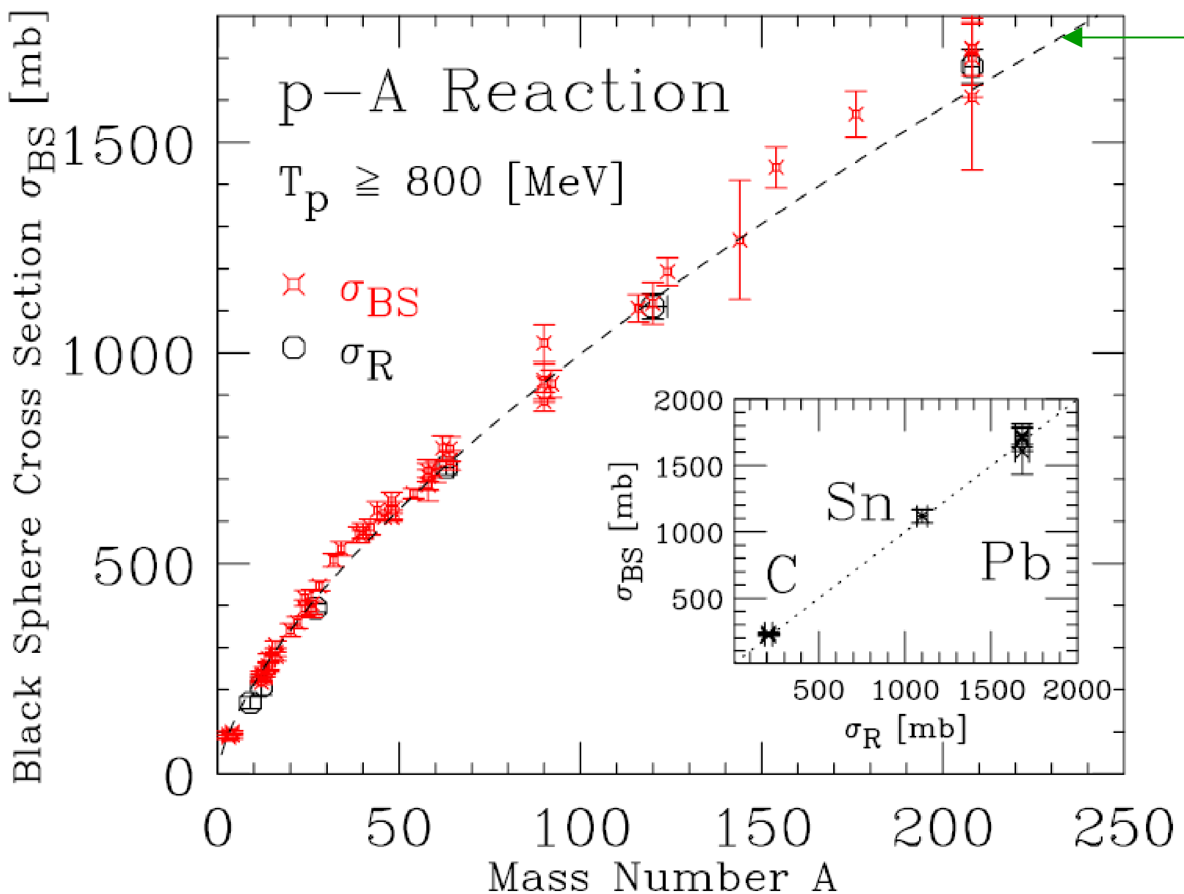
"Black" sphere cross section vs. p-A reaction cross section (stable nuclei)

Ref. Kohama, Iida, & Oyamatsu, PRC **72** (2005) 024602.

σ_{BS} : geometrical cross section of a "black" sphere $\sigma_{BS} = \pi a^2$

σ_R: measured reaction cross section for stable nuclei (800 MeV < T_p < 1000 MeV)

Ref. Bauhoff, At. Data Nucl. Data Tables **35** (1986) 429.



$\propto A^{2/3}$

$\sigma_{BS} \cong \sigma_R$

The black sphere radius *a* corresponds to the reaction radius.

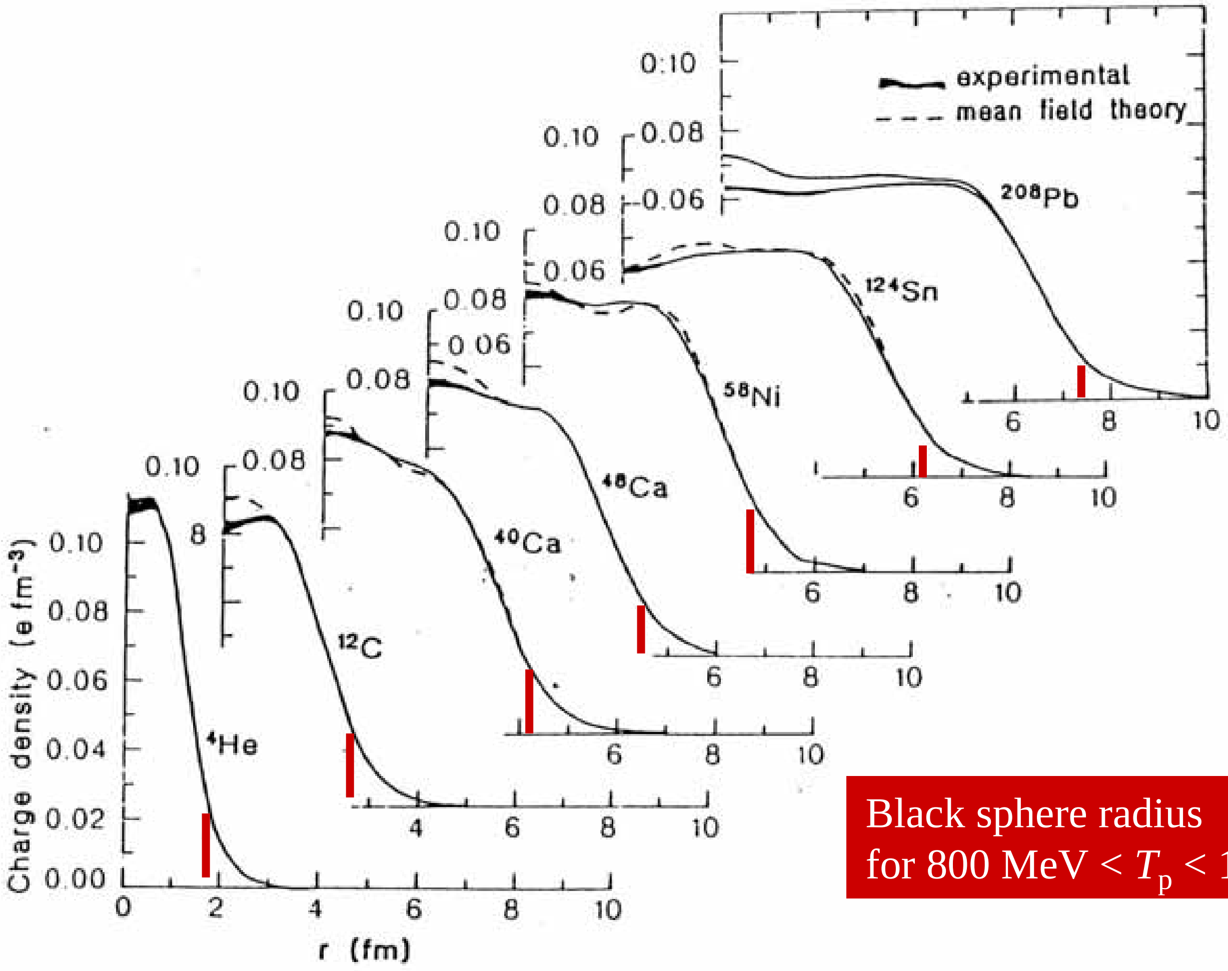


Fig. 2.4. Nuclear ground state charge distributions (Fro 83).

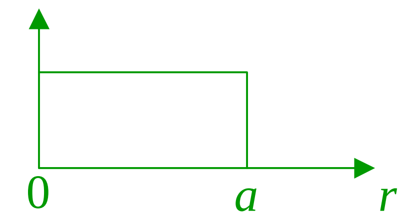
Black sphere radius
for 800 MeV < T_p < 1000 MeV)

“Black” sphere model (contd.)

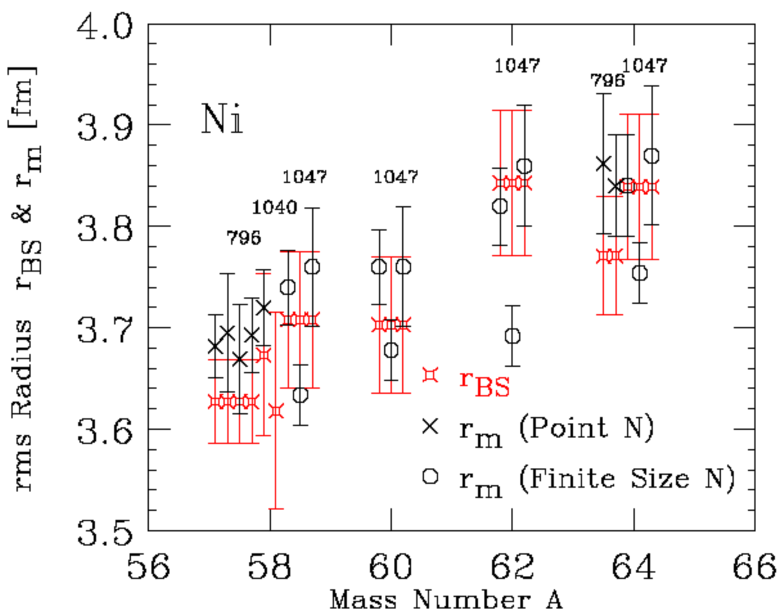
“Black” sphere radius vs. nuclear matter radius (stable nuclei , T_p > 800 MeV)

r_{BS} : root-mean-square “black” sphere radius for a rectangular distribution

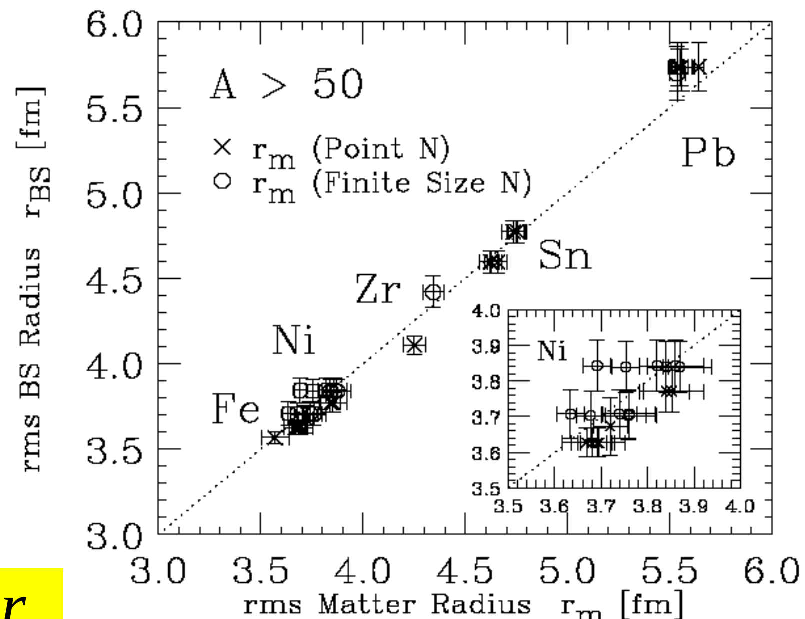
$$r_{BS} = \sqrt{\frac{3}{5}}a$$



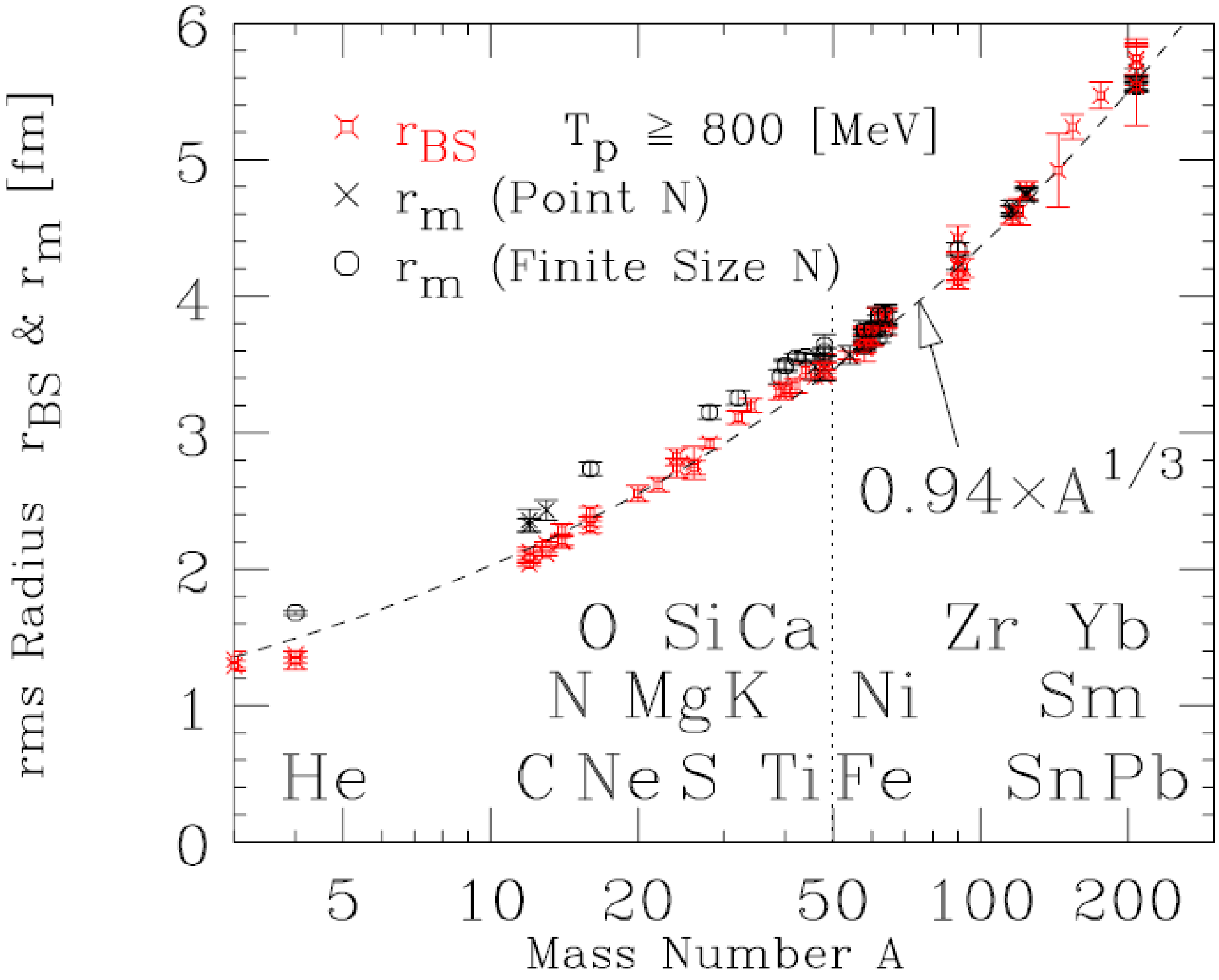
r_m: root-mean-square matter radius determined from conventional scattering theories so as to reproduce the overall diffraction pattern in proton elastic scattering



$r_{BS} \cong r_m$



"Black" sphere model (contd.)



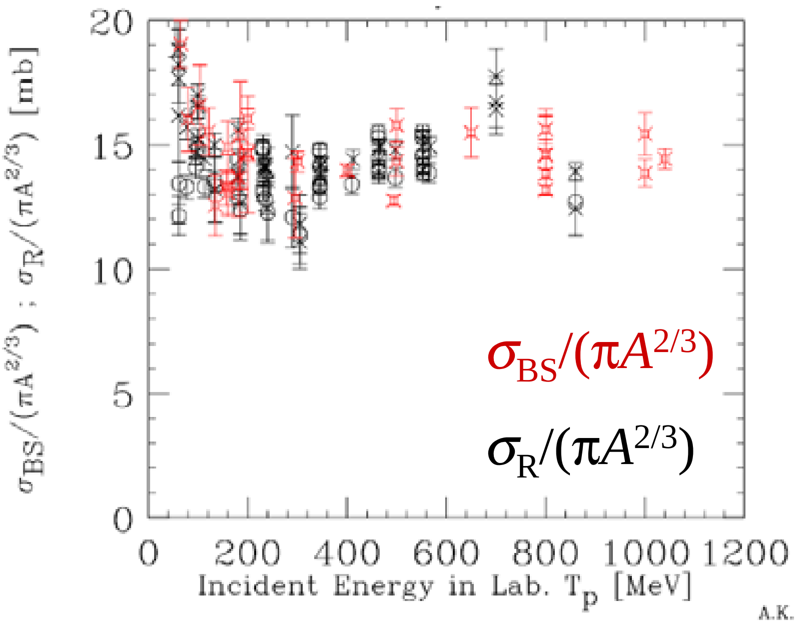
Systematic deviation between r_{BS} and r_m for A<50

It is not r_m but r_{BS} that follows the $A^{1/3}$ law!

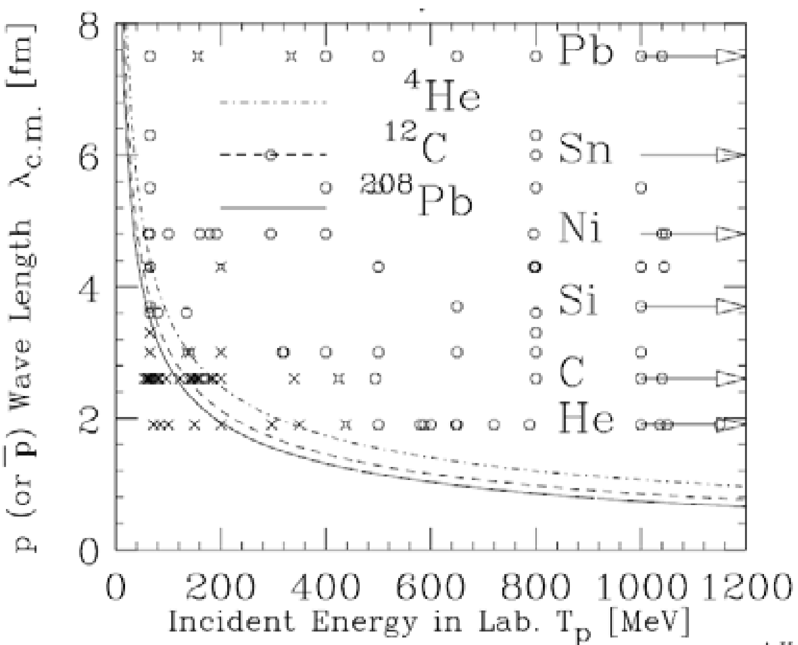
“Black” sphere model (contd.)

Extension to lower energies (stable nuclei)

The agreement between σ_{BS} vs. σ_R is still good?

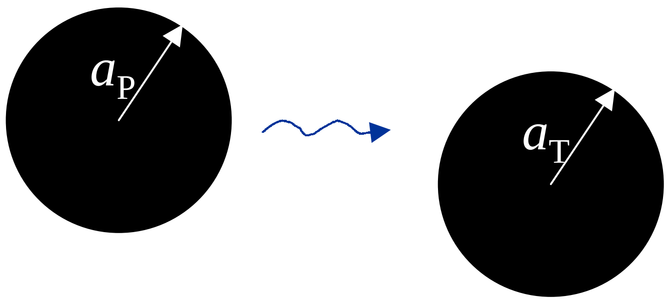


Diffraction peak is still present?

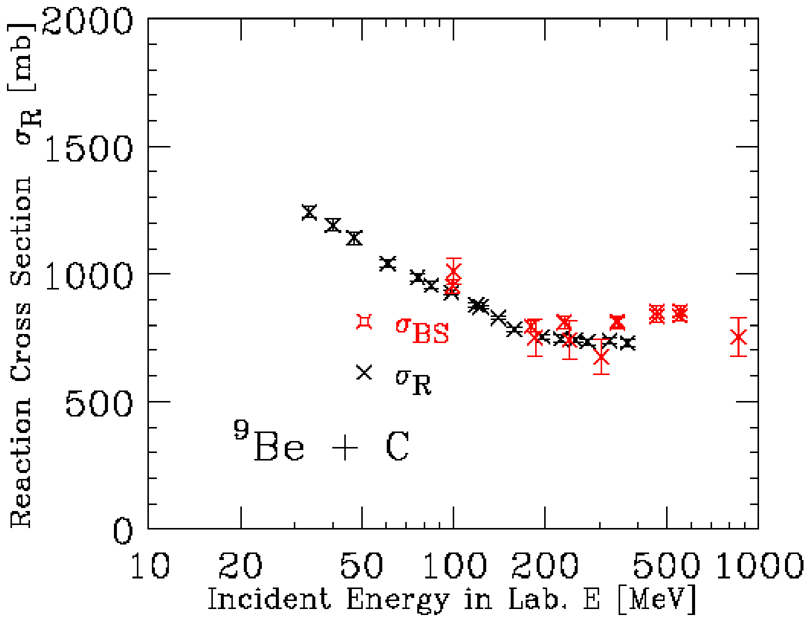


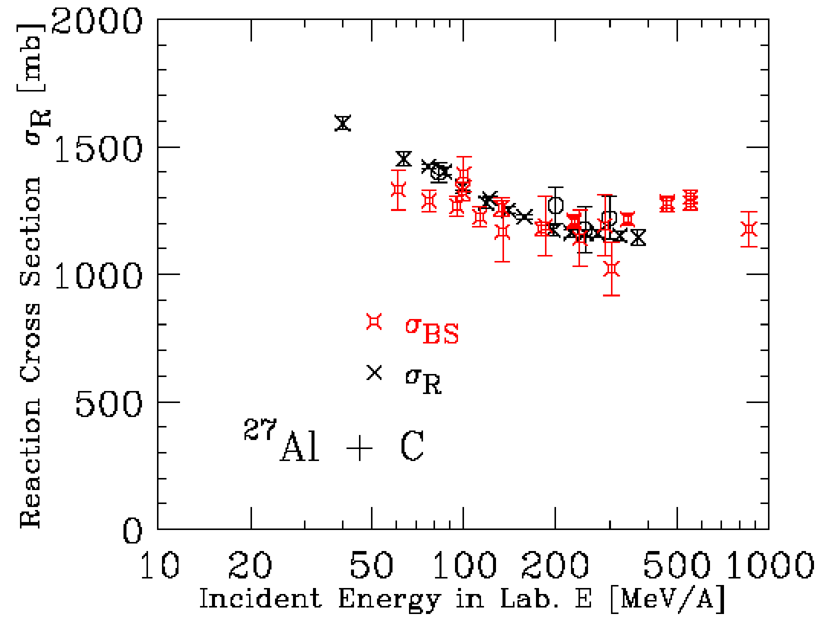
“Black” sphere model (contd.)

Extension to nucleus-nucleus reactions



$$\sigma_{BS} = \pi(a_T + a_P)^2$$





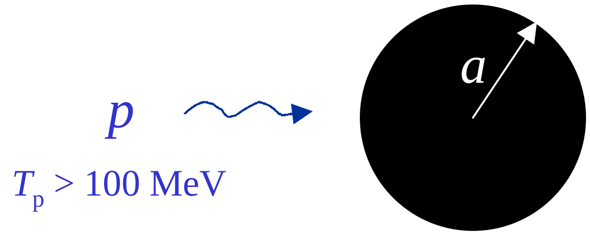
$$\sigma_{BS} \cong \sigma_R$$

Formula for reaction cross section

Ref. Iida, Kohama, & Oyamatsu, JPSJ **76** (2007) 044201.

Proton - stable nucleus

$$\sigma_R(T_p, A) = \pi a^2(T_p, A) = \pi a_0^2\left(1+\frac{\Delta a}{a_0}\right)^2$$



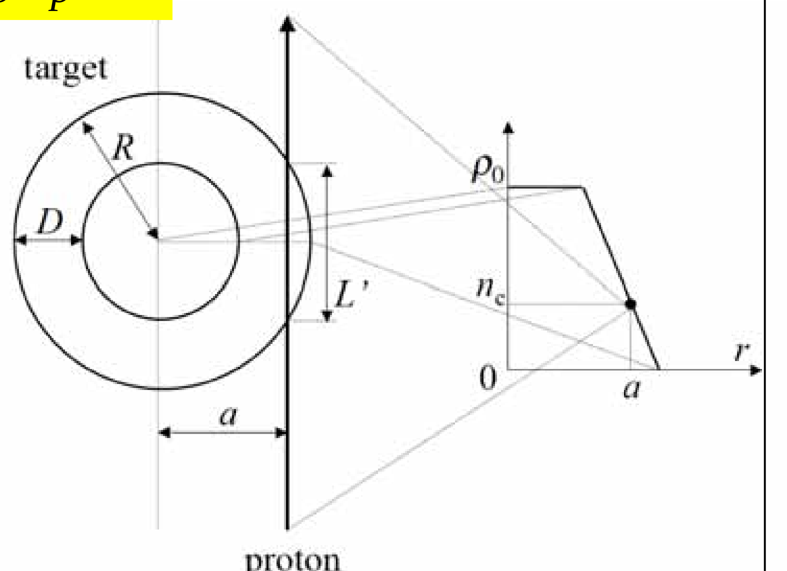
a_0: black sphere radius at T_p=800 MeV

——— determined from elastic scattering data or estimated as $a_0 \approx 1.214A^{1/3}$ fm

$\Delta a = a - a_0$ ——— determined from the condition that the proton optical depth

$\int_l dl[\sigma_{pn} n_n(\mathbf{r}) + \sigma_{pp} n_p(\mathbf{r})]$ amounts to a critical value

σ_{pN} controls the T_p dependence of σ_R .



Formula for reaction cross section (contd.)

Proton - stable nucleus

$$\sigma_{\mathrm{R}}(T_p, A) = \pi a^2 (T_p, A) = \pi a_0^2 \left(1 + \frac{\Delta a}{a_0}\right)^2, \ T_p > 100\ \mathrm{MeV}$$

$$\Delta a = \left(\frac{\rho_0 a_0}{D n_{c0}} - \frac{a_0}{L'_0} \frac{dL'}{da}\bigg|_0 \right)^{-1} \frac{\Delta \bar{\sigma}_{pN}}{\bar{\sigma}_{pN0}}$$

$$\bar{\sigma}_{pN} = (Z/A)\sigma_{pp} + (1 - Z/A)\sigma_{pn}$$

$$X_0 : X\,(= \bar{\sigma}_{pN}, a, n_c, L')\ \text{at}\ T_{\mathrm{p}} = 800\ \mathrm{MeV}$$

$$\Delta X = X - X_0$$

$$L' = 2\sqrt{R^2 - a^2}$$

$$R = R_0 + D/2 - R_0 \left(1 + 12 R_0^2 / D^2\right)^{-1}$$

$$D = 2.2\ \mathrm{fm},\ \rho_0 = 0.16\ \mathrm{fm}^{-3},\ R_0 = \left(3A/4\pi\rho_0\right)^{1/3}$$

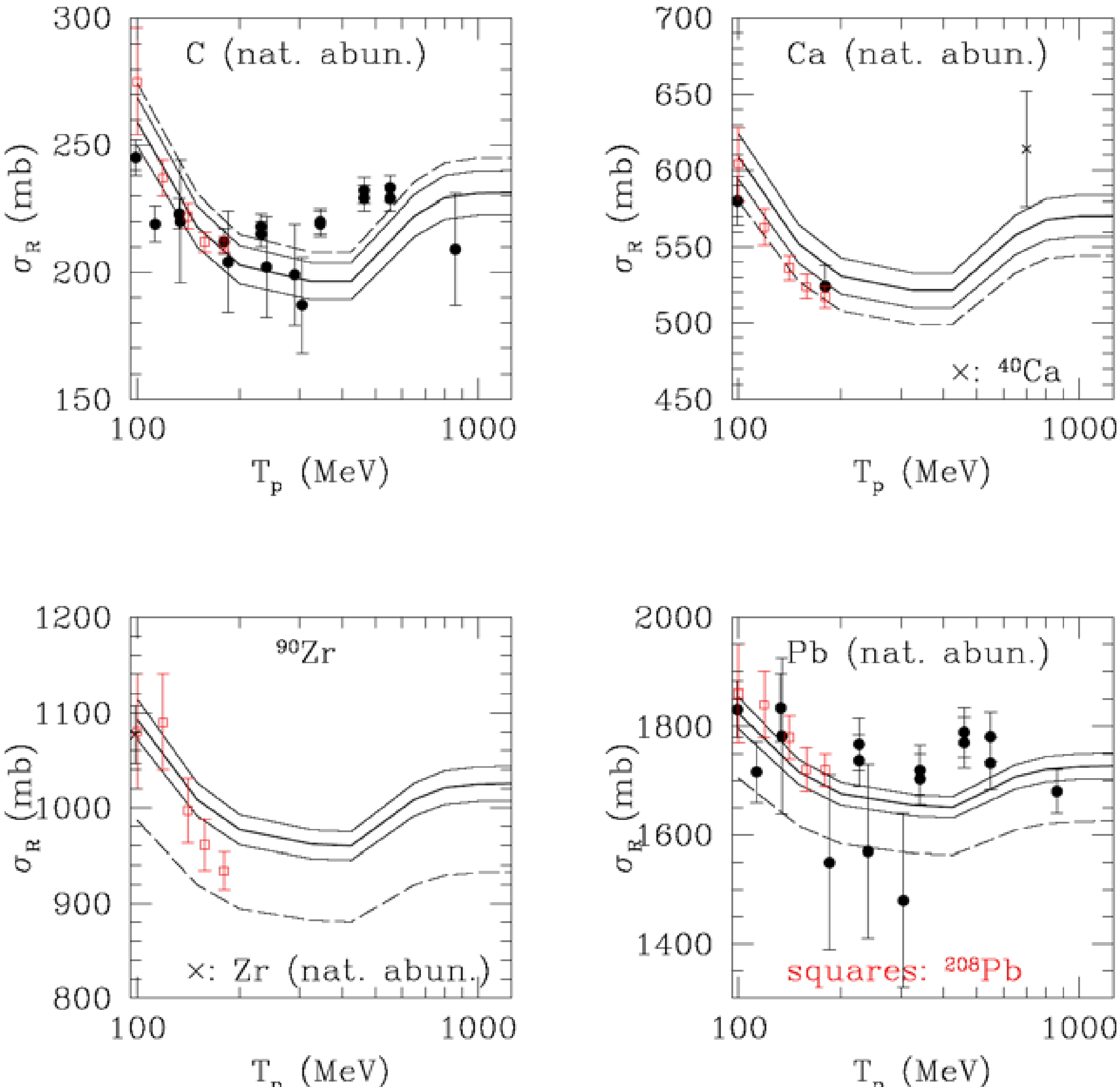
$$n_{c0} = 0.9\left(\bar{\sigma}_{pN0} L'_0\right)^{-1}$$

No energy dependent adjustable parameter

——— different from fitting formulas

Formula for reaction cross section (contd.)

Proton - stable nucleus

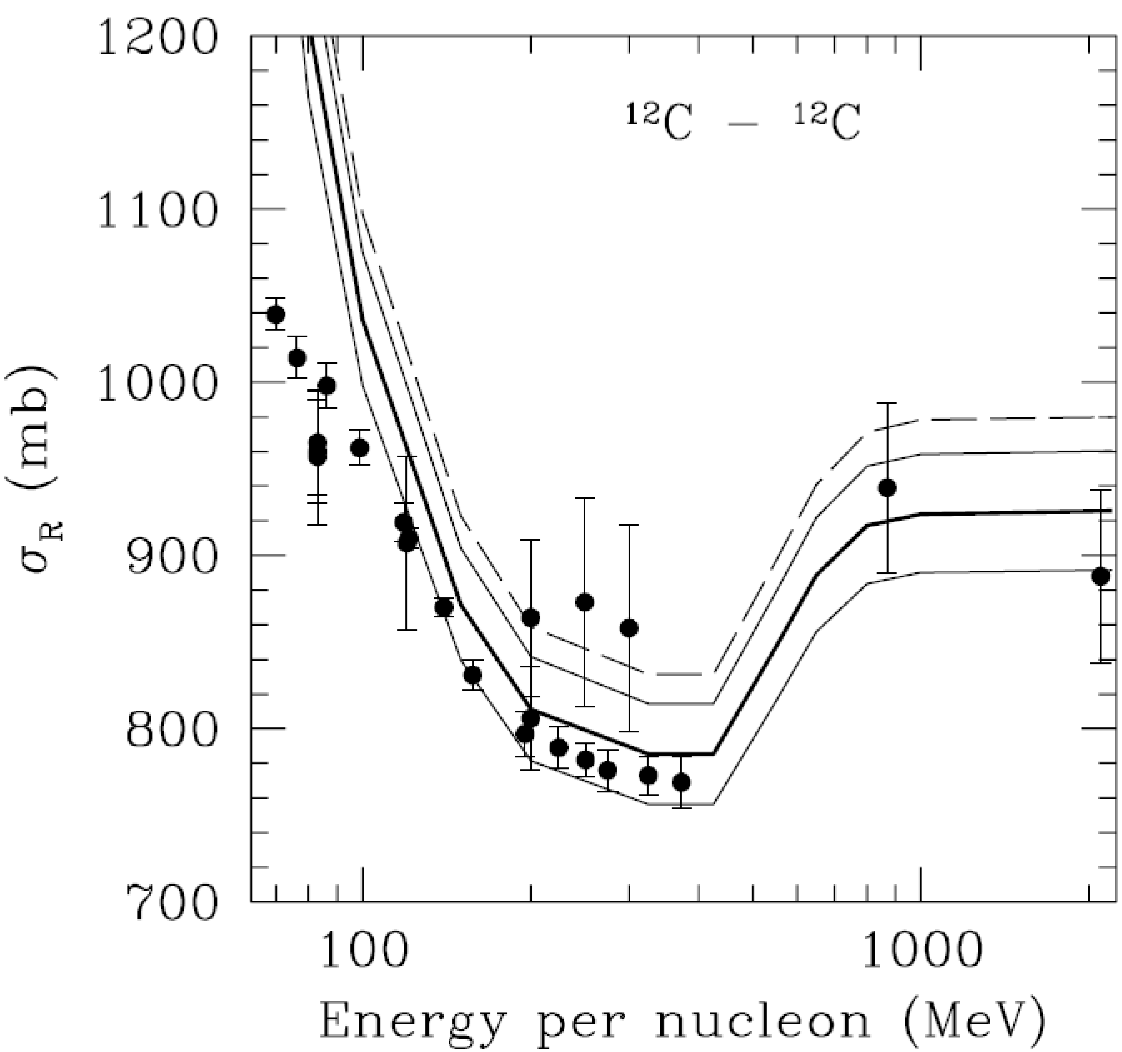


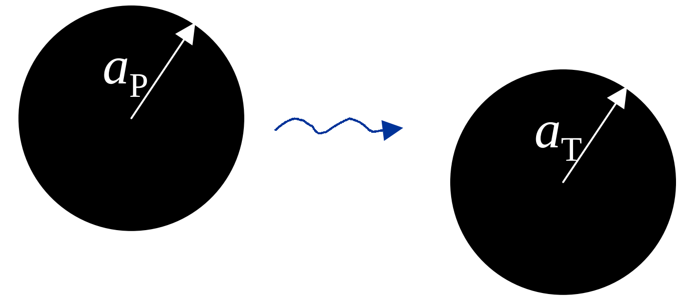
☐: data from Auce *et al.*

PRC **71** (2005) 064606.

Formula for reaction cross section (contd.)

Stable nucleus - stable nucleus





$$\sigma_R = \pi(a_T + a_P)^2$$

•: including data from Takechi *et al*., Eur. Phys. J. A **25** (2005) s1.217.

Formula for reaction cross section (contd.)

Extension to unstable nuclei

The dependence of σ_R on the neutron excess $\delta=(N-Z)/A$ has to be similar to that of the matter radius squared.

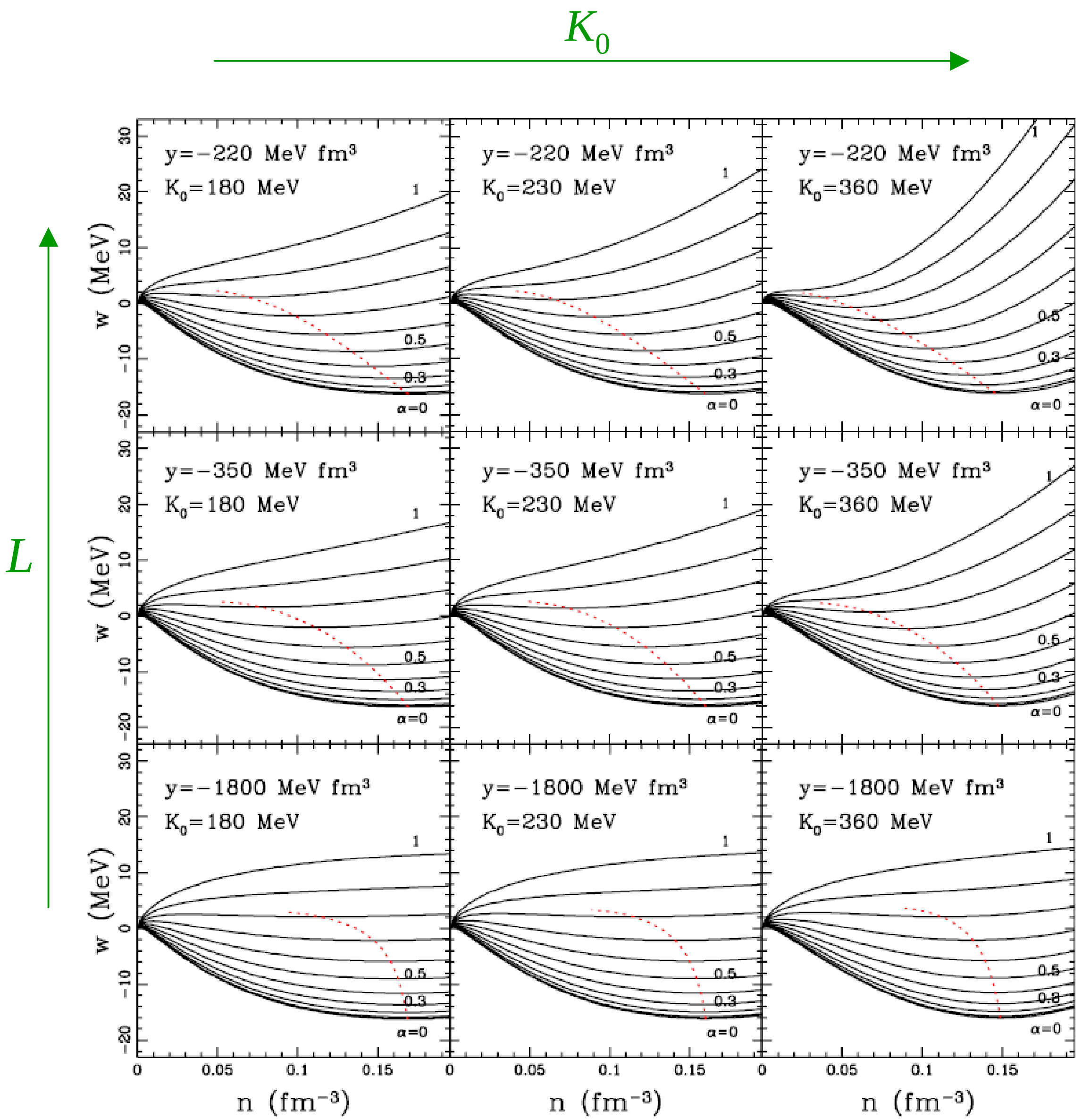
Scaling by a radius formula $R_m(A, \delta; L)$ constructed from a macroscopic nuclear model

The L dependence (R_m increases with L) is essential !

$$L = 3n_0 \left(\frac{dS}{dn} \right)_{n=n_0}$$

Ref. Oyamatsu & Iida, PTP **109** (2003) 631.

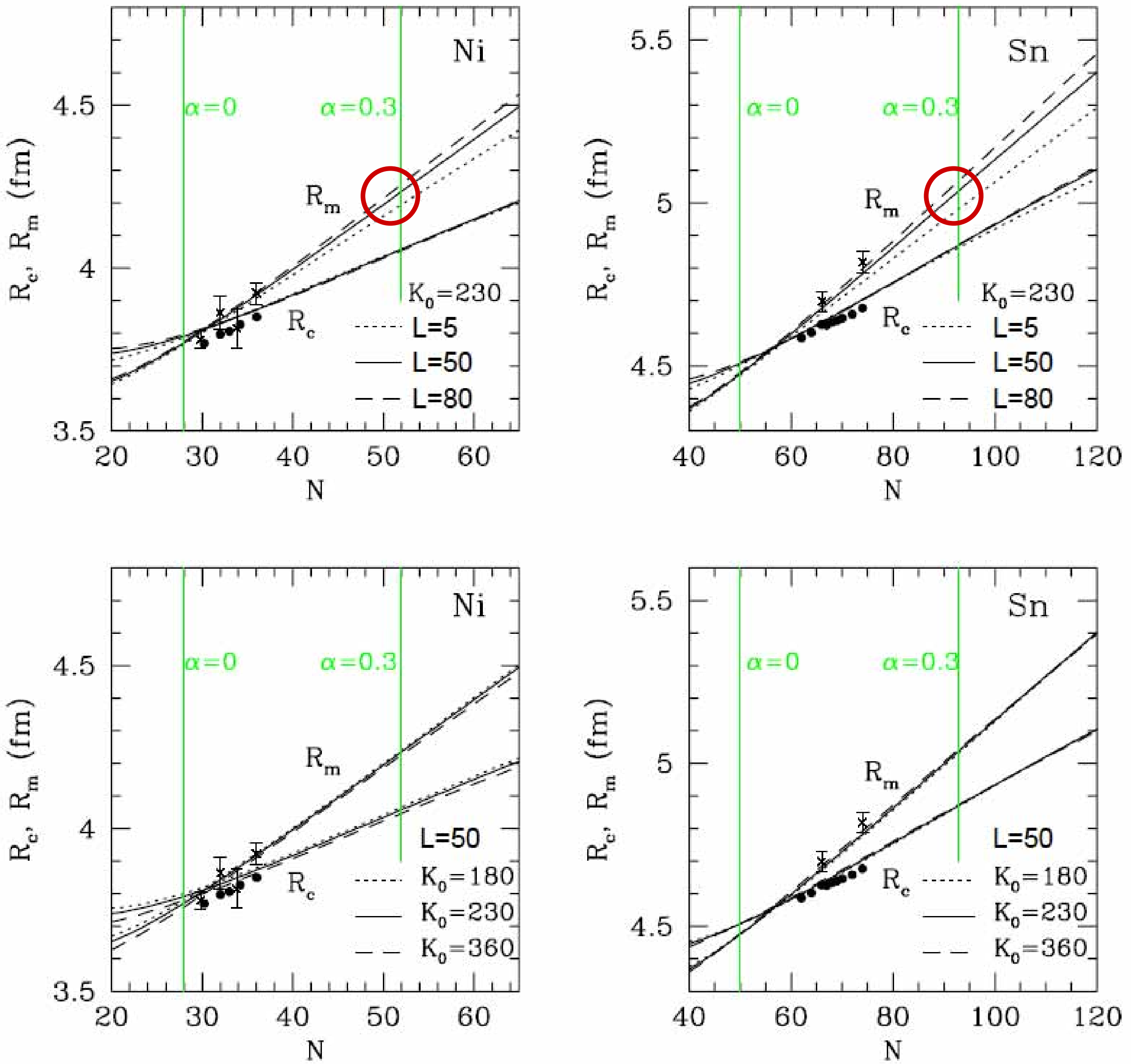
$$\sigma_R = \pi \left(a_T \frac{R_m(A_T, \delta_T; L)}{R_m(A_T, 0; L)} + a_P \frac{R_m(A_P, \delta_P; L)}{R_m(A_P, 0; L)} \right)^2$$



saturation line

(y=-$K_0S_0/3n_0L$: slope at α=0)

Ref. Oyamatsu & Iida, PTP **109**(2003)631.



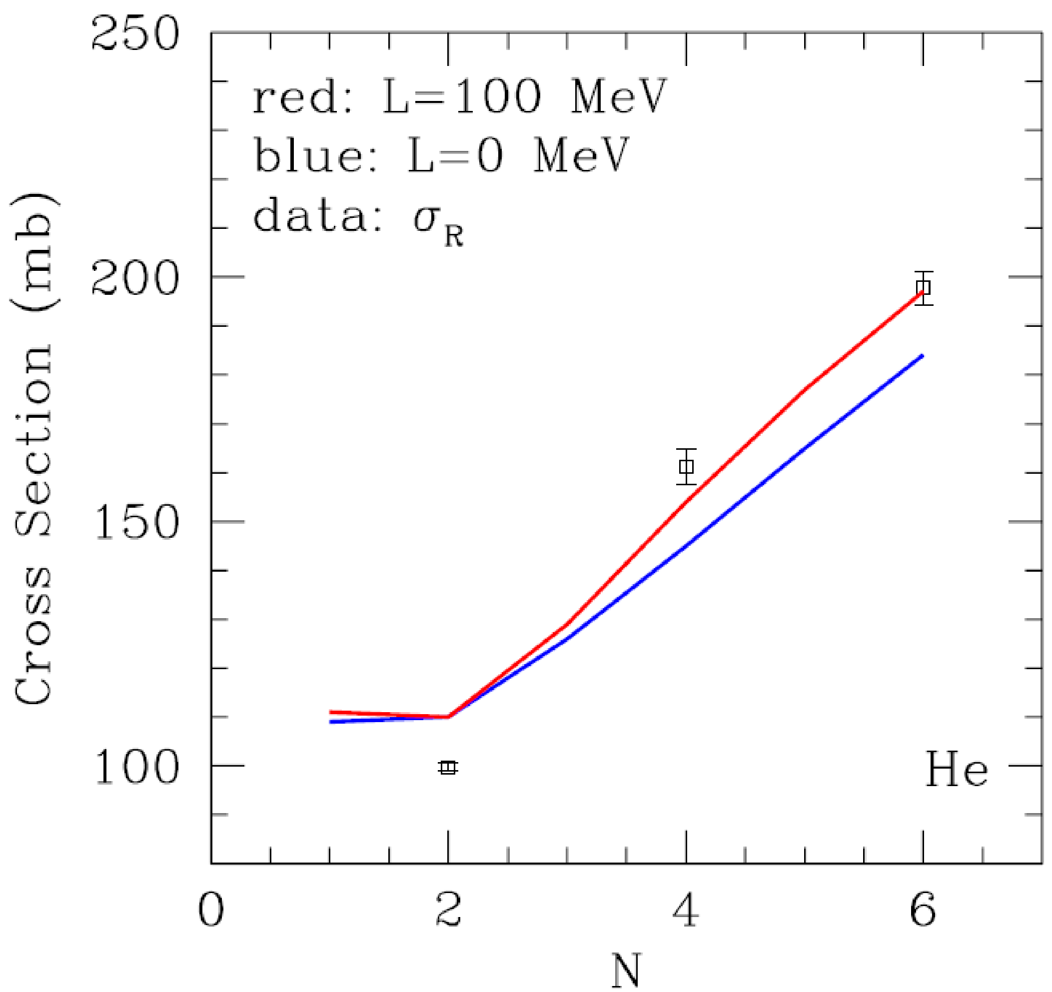
Ref. Oyamatsu & Iida,
PTP **109**(2003)631.

Comparison with empirical data

vs. reaction cross section data

———deduced from elastic scattering (p target, 678 MeV < E/A < 702 MeV)

Ref. Neumaier *et al.*, NPA **712** (2002) 247.

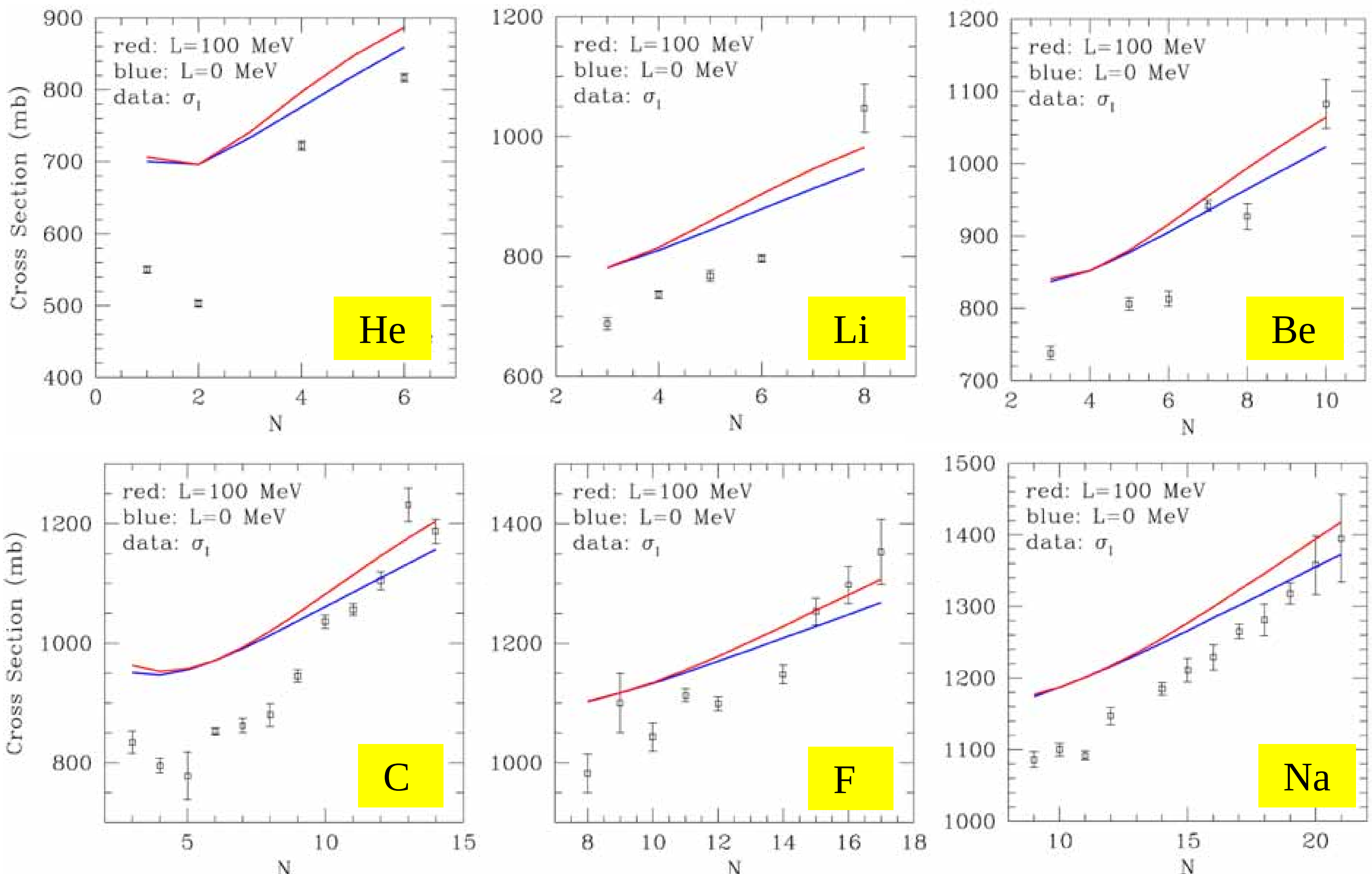


Comparison with empirical data (contd.)

vs. interaction cross section data

σ_I: measured interaction cross section (C target, 620 MeV < *E/A* < 1050 MeV)

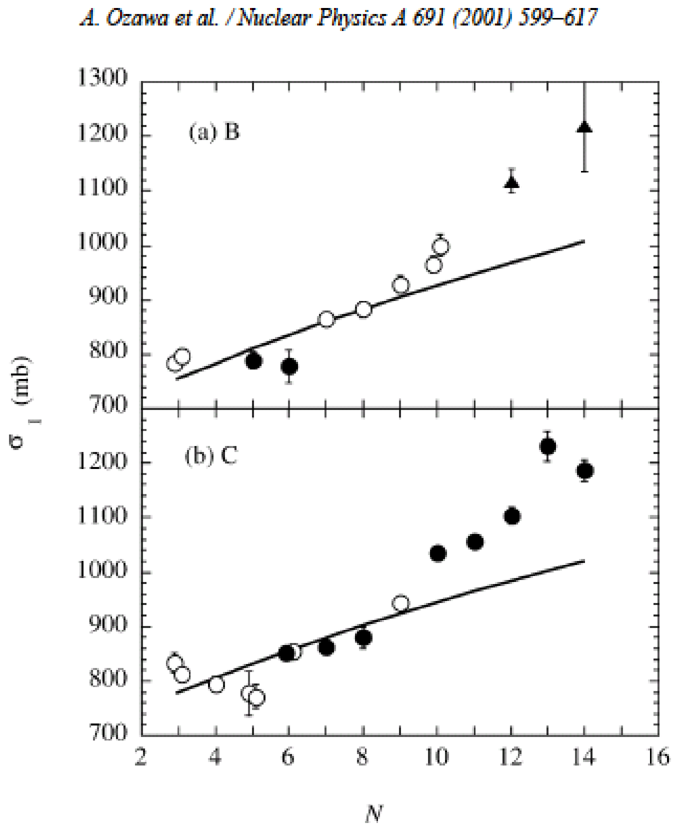
Ref. Ozawa, Suzuki, & Tanihata, NPA **693** (2001) 32.



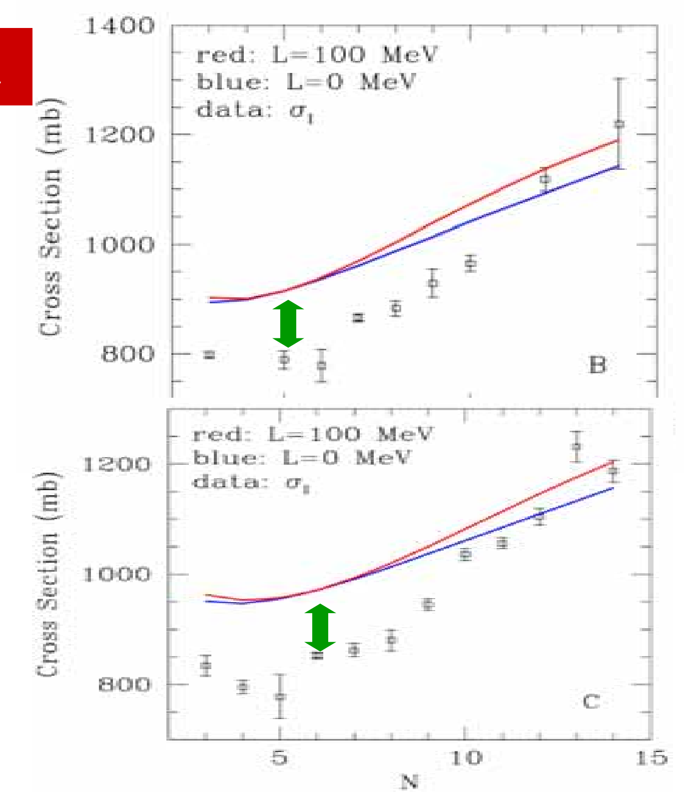
Comparison with empirical data (contd.)

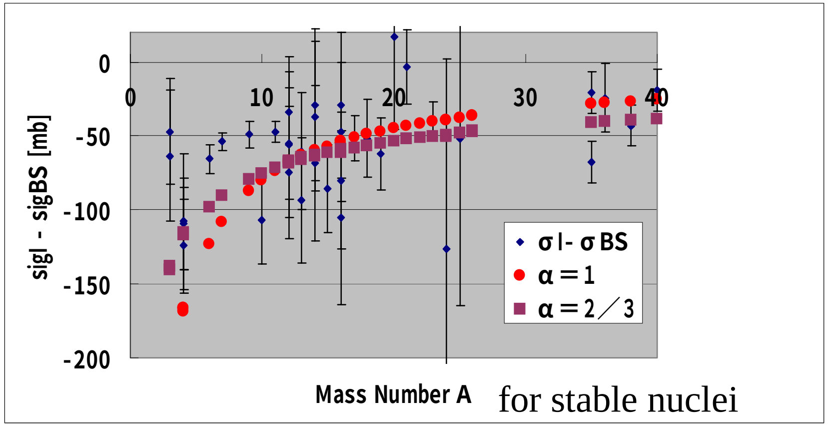
What is the standard value of the reaction cross section?

Past



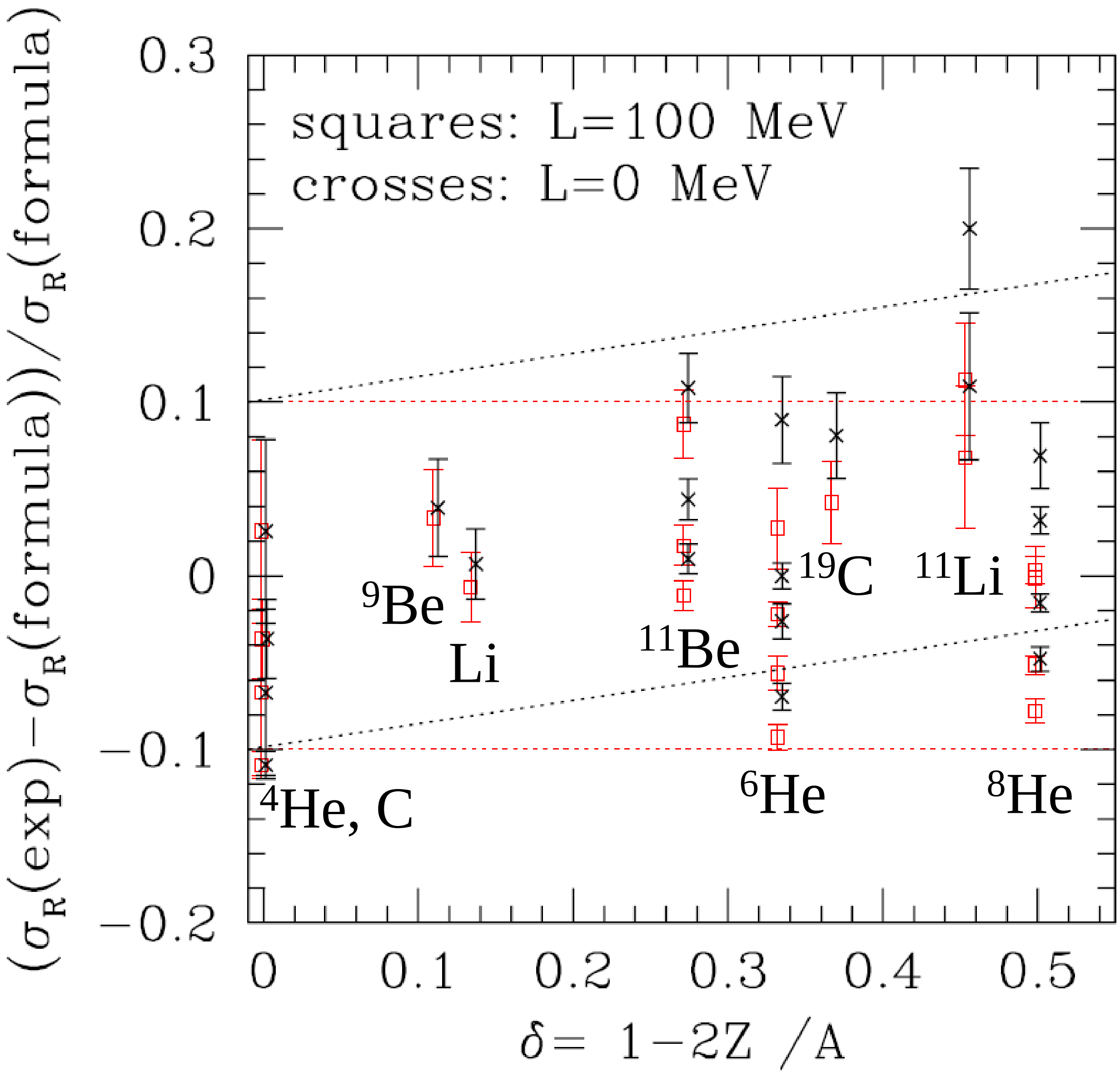
Present





Comparison with empirical data (contd.)

Summary



Empirical data:

σ_I for ^{6,8}He, ^{11}Li, ^{11}Be, ^{19}C

σ_R for ^{4,6,8}He, nat Li, ^{9}Be, nat C

Conclusion

1. The “black” sphere geometrical cross section obtained from *a*

   — well reproduces the systematic data on the proton-nucleus reaction cross section measured for stable nuclei.

3. Comparison of the reaction cross section formula with empirical data

   - good agreement with the existing reaction cross section data
   - interesting relation with the interaction cross section data

     —— a significant difference for stable nuclei

     —— no significant difference for very neutron-rich nuclei to within uncertainties in *L*

2. Awaiting simultaneous measurements of proton elastic scattering and reaction/interaction cross section for unstable neutron-rich nuclei

   — σ_{BS} vs. $\sigma_{R(I)}$: important check of the widely accepted speculation that one can deduce the neutron halo structure from measurements of the reaction/interaction cross section