Analysis of polarized proton-$^6$He elastic scattering based on an improved di-neutron model

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Contents:

1. Conventional “di-neutron (2n)” model for $^6$He
   • Something wrong!
   • What’s wrong?

2. “Improved di-neturon model” for $^6$He
   • A constraint upon “2n”-α wave function

3. Adiabatic-Recoil approximation (ARA)
   • “2n” size-effects on the elastic scattering
     → via break-up effect / folding potential

4. Effect of “2n”-p interaction ignored in ARA
Problems of previous analyses

• **Folding model** : *no break-up effect* is included:
  – how important the breakup effect is?
  → estimate it within the framework of *di-neutron* model
  – BUT!

• *“standard” di-neutron model* on the market, which many authors have been using, *may not be realistic*
  – Size of the whole nucleus (\(^6\)He) inevitably exceed the measured size.
  → Need modification/improvement to the “di-neutron” model
\( \alpha - 2n \) mean distance given by the S.E.M. so as to reproduce observed B.E.(\(^6\text{He}) = 0.976 \text{ MeV} \\

\[
\overline{R}_{6\text{He}}^2 = \frac{2}{3} \overline{R}_\alpha^2 + \frac{1}{3} \overline{R}_{2n}^2 + \frac{2}{9} \overline{r}_{\alpha-2n}^2
\]

- known from exp.data \\
\( \overline{R}_{6\text{He}} \approx 2.44 \text{ fm}, \quad \overline{R}_\alpha \approx 1.36 \text{ fm} \)

- should be! \\
\( \overline{r}_{\alpha-2n} \leq 4.6 \text{ fm} \)

even when \( \overline{R}_{2n} = 0 \)
Conventional “di-neutron” model for $^6\text{He}$

$\alpha+(2n)$ relative wave function is assumed to be 2s state and is given by the separation energy method (SEM) with the use of a single attractive Woods-Saxon potential with a standard geometry (e.g. $R=1.2 \times A^{1/3}$ fm, $a=0.5$ fm)

- very small B.E. (0.976 MeV)
  - $\rightarrow$ w.f. has a very long-range tail
  - $\rightarrow$ $R_{\alpha-2n} \geq 4.7$ fm $\sim 5.1$ fm
- leads to an unrealistic size of the whole nucleus $^6\text{He}$

$R_{r.m.s.}(^6\text{He})$ and B.E. $\alpha-2n$ cannot be reproduced simultaneously by the standard di-neutron model, as long as one uses a single attractive potential. $\rightarrow$ need $\alpha+n+n$ three-body treatment?
$^6\text{He}$

\[ \alpha \rightarrow r_{\alpha-2n} \rightarrow \text{"2n"} \]

\[ \sim 50\% \]

\[ \sim 50\% \]

\[ \text{"di-neutron" type} \]

\[ \text{"cigar" type} \]

\[ r_{\alpha-2n} \]
Surface-barrier simulates three-body effect (e.g. effects of non-zero angular momentum components):

\[
\bar{R}_\text{He}^2 = \frac{2}{3} \bar{R}_\alpha^2 + \frac{1}{3} \bar{R}_{2n}^2 + \frac{2}{9} \bar{r}_{\alpha-2n}^2
\]

Proposed by Sakuragi, Hirabayashi, Funada in di-neutron model for \(^{11}\text{Li}\)

(Niigata Symposium on Unstable nuclei, 1991)
\[ \overline{R}_{\text{He}}^2 = \frac{2}{3} \overline{R}_\alpha^2 + \frac{1}{3} \overline{R}_{2n}^2 + \frac{2}{9} r_{\alpha-2n}^2 \]

**Exp.data**

**free-parameter** ← “2n” size

Total size of \(^6\text{He}\) should be fixed to exp. value!

\[ \langle R_{2n} \rangle = 2.5 \text{ fm} \]
\[ \langle R_{\alpha-2n} \rangle = 3.42 \text{ fm} \]
“2n” - $\alpha$ relative wave function

$V_r = 3.0 \text{ fm}$

$\phi_0 \ R_{2n} = 3.0 \text{ fm}$

$V_{R_{2n} = 3.0 \text{ fm}}$

$R_{2n} = 3.0 \text{ fm} \rightarrow R_{\alpha - 2n} = 2.76 \text{ fm}$
“2n”- $\alpha$ relative wave function

- $V_r$ [MeV]
- $r$ [fm]

$V_{R_{2n}=2.5 \text{ fm}}$

$\phi_0$ $R_{2n}=2.5 \text{ fm}$

$R_{2n}=2.5 \text{ fm} \rightarrow R_{\alpha-2n}=3.42 \text{ fm}$

0.976 MeV
“2n”- $\alpha$ relative wave function

$V(r)$ [Mev]

$V_{R2n=2.0 \text{ fm}}$

$\phi_0 \ R2n=2.0 \text{ fm}$

$R_{2n}=2.0 \ \text{fm} \rightarrow R_{\alpha-2n}=3.91 \ \text{fm}$
"2n"- $\alpha$ relative wave function

$R_{2n}=1.5\text{ fm} \rightarrow R_{\alpha-2n}=4.22\text{ fm}$
"2n" - $\alpha$ relative wave function

\[ V(r) \text{ [MeV]} \]

\[ \phi_0 \text{ R}_{2n=1.0 \text{ fm}} \]

\[ V_{R2n=1.0 \text{ fm}} \]

\[ R_{2n}=1.0 \text{ fm} \rightarrow R_{\alpha-2n}=4.43 \text{ fm} \]
“2n” - $\alpha$ relative wave function

$R_{2n}=0.0$ fm $\rightarrow R_{\alpha-2n}=4.59$ fm
How “2n” size in $^6$He can be probed by scattering with proton?

Size of “2n” correlates with $\alpha$-2n relative wave function

$$\overline{R}_{^6\text{He}}^2 = \frac{2}{3} \overline{R}_\alpha^2 + \frac{1}{3} \overline{R}_{2n}^2 + \frac{2}{9} \overline{r}_{\alpha-2n}^2$$

Exp. data

free-parameter “2n” size
\[ T_R + T_r + V_{Ab}(r) + V_{Cb}(R_{Cb}) + V_{CA}(R_{CA}) - E ] \Psi(r, R) = 0 \]

\[ (E_0 = E - \varepsilon_0) \]

\[ [T_R + V_{CA}(R_{CA}) - E_0] \Psi^{(AD)}(r, R) = 0, \]

1. b-A relative motion is slow enough compared with C-(b+A) one (Adiabatic approx.)
   \[ [T_r + V_{Ab}(r)] \Psi(r, R) \cong \varepsilon_0 \Psi(r, R) \]

2. Ignore b-C interaction if it is weak enough compared with A-C interaction:
   \[ V_{CA}(R_{CA}) \gg V_{Cb}(R_{Cb}) \approx 0 \]

Adiabatic-Recoil approx. (R.C.Johnson et al., PRL 79, 2771(1997))
Apply this approximation to \((\alpha + \text{"2n")}+p\) system

\[
\left[ T_R + V_{p\alpha}(R') - E_0 \right] \Psi_K^{(AD)}(r, R) = 0
\]

\[
\Psi_K^{(AD)}(r, R) = \varphi_0(r) \cdot e^{i\mu k \cdot r} \cdot X_K(R')
\]

\[
\left[ T_{R'} + V_{p\alpha}(R') - E_0 \right] X_K(R') = 0
\]

\[
\left( \frac{d\sigma}{d\Omega} \right)_{el} = \left| \int dr \int dR \varphi_0(r) e^{ik'r} \left\{ V_{p\alpha}(R') + V_{p2n}(R_{p2n}) \right\} \Psi_K(r, R) \right|^2
\]

\[
\approx \left| \int dr \left| \varphi_0(r) \right|^2 e^{i\mu qr} \right|^2 \times \left| \int dR' e^{-ik'R'} V_{p\alpha}(R') X_K(R') \right|^2
\]

\[
= \left| F(q) \right|^2 \times \left( \frac{d\sigma}{d\Omega} \right)_{pt}
\]

R.C. Johnson et al., PRL 79, 2771 (1997)
Adiabatic-Recoil approximation
(R.C. Johnson et al., PRL 79, 2771 (1997))

\[
\left( \frac{d\sigma}{d\Omega} \right)_{p-^6\text{He}} = |F(q)|^2 \times \left( \frac{d\sigma}{d\Omega} \right)_{\text{point}}
\]

**Form factor**

\[
F(q) = \int e^{i\mu qr} \left| \phi_{2n-\alpha}(r) \right|^2 \, dr^3,
\]

\( (\mu = m_{2n}/m_{^6\text{He}} = 1/3) \)
$p^+{}^6\text{He}$ $E_p=71$ MeV

Adiabatic–Recoil approx.

- $R_{2n}=0.0$ fm
- $R_{2n}=1.0$ fm
- $R_{2n}=1.5$ fm
- $R_{2n}=2.0$ fm
- $R_{2n}=2.5$ fm
- $R_{2n}=3.0$ fm
- $R_{2n}=3.5$ fm

$(d\sigma/d\Omega)_{pt}$

- Sakaguchi (2005)
- Korsheninnikov (1997)
$^6\text{He}\rightarrow \alpha + \text{"2n"} \text{ break-up effect}$

We compare:

a) Adiabatic-Recoil approx. (ARA) cal.
   $\rightarrow$ with break-up effect

b) Folding-model calculation
   (without $V_{p-2n}$)
   $\rightarrow$ no break-up effect
$^6\text{He} \rightarrow 2n - \alpha$

**Break-up effect**

$R_{2n}=2.5 \text{ fm}$

$R_{R.M.S}=3.42420491$

(Korsheninnikov 1997)

(Hatano 2003)

(Sakaguchi 2005)
Effect of $p$-"2n" interaction

→ folding-model (no break-up effect)
$p-^6\text{He}$ folding-model potential

\[ V(R) \text{ [MeV]} \]

\[ R \text{ [fm]} \]

**$R_{2n}=2.5 \text{ fm case}$**

- Real
- Imag.

- $p-\alpha$
- $p-"2n"
- total

\[ V_{p-\alpha} \]
\[ V_{p-"2n"} \]
Effect of p-“2n” interaction: $V_{p-2n}$

$p+^6\text{He} \quad E_p=71 \text{ MeV}$

Folding Model cal.  
($R_{2n}=2.5 \text{ fm}$)

- $V_{p\alpha}$
- $V_{p\alpha}+V_{p2n}$

- Sakaguchi (2005)
- Korsheninnikov (1997)
breakup effect

effect of p-2n int.

p+6He  $E_p=71$ MeV

Folding Model cal.

$R_{2n}=2.5$ fm  $R_{R.M.S}=3.42420491$

$V_{p\alpha}$

$V_{p\alpha}+V_{p2n}$

Summary

• We have proposed a realistic “di-neutron” model for $^6$He,
  – which well simulates the three-body nature of $^6$He and
  – reproduces both $^6$He size and binding energy simultaneously by introducing a surface barrier to 2n-$\alpha$ potential.

• $^6$He→$\alpha$+“2n” break-up effect is estimated by the use of adiabatic-recoil approximation (ARA)
  – the break-up effect is important at middle and backward angles of the elastic scattering by proton
  – ARA calculation gives vector analyzing power consistent to the observed ones within error bars.

• The “2n”-proton interaction, $V_{p-2n}$, neglected in the ARA calculation, has non-negligible effect
  – mainly due to small mass of the $\alpha$-particle core