Repulsive aspects of pairing correlation in nuclear fusion reaction

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Simulate of heavy ion collision using TDHF


FIG. 2. Contour lines of the density integrated over the coordinate normal to the scattering plane for an $^{16}\text{O}+^{16}\text{O}$
collision at $E_{\text{lab}}=105$ MeV and incident angular momentum $L=13\hbar$. The times $t$ are given in units of $10^{-23}$ sec.
### Several mean-field theories

<table>
<thead>
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<th>For static</th>
<th>For dynamics</th>
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<tbody>
<tr>
<td><strong>No Pairing</strong></td>
<td>Hartree-Fock(HF)</td>
<td>Time-Dependent HF (TDHF, RPA)</td>
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<td><strong>With BCS Pairing</strong></td>
<td><strong>HF+BCS</strong></td>
<td><strong>TDHF+BCS</strong></td>
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<tr>
<td><strong>With Pairing</strong></td>
<td>Hartree-Fock-Bogoliubov (HFB)</td>
<td>TDHFB (QRPA)</td>
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</table>

※ RPA: Random-Phase Approximation
※ QRPA: Quasi-particle RPA
What kind of pairing effect is expected in low-energy Heavy ion collision?

- **Fusion or Fission cross section**
- **Level crossing**
  - Energy Dissipation
  - Neck formation
  - Odd-even effects for spontaneous fission half-lives?
- **Pair transfer reaction**
  - Nuclear Josephson effect

Method  S.E. et al.: PRC82 (2010) 034306

Statics: HF+BCS
Dynamics: Cb-TDHFB

\( ph \)-channel Int. : Skyrme (SkM*, SLy4d)

\( pp(hh) \)-channel : \( \delta \)-type (time-reversal)

\[ \hat{\mathcal{V}}^\tau(r_1, \sigma_1; r_2, \sigma_2) = V_0^\tau \frac{1 - \hat{\sigma}_1 \cdot \hat{\sigma}_2}{4} \delta(r_1 - r_2) \]

: spin-singlet zero-range interaction

Points:
To solve Cb-TDHFB in 3D space enable us to study nuclear dynamics including deformation and pairing, self-consistently.

For the simulation of collision
Prepare the target and projectile nuclei with HF+BCS, put them with some \( b \) and distance, also add velocity to them, and their time will be evolved by Cb-TDHFB.

Cb-TDHFB Eqs.

\[ i\hbar \frac{\partial}{\partial t} \phi_l(t) = \{ h(t) - \varepsilon_l(t) \} \phi_l(t) \quad : \text{Canonical basis (for } \tilde{l} \text{ also)} \]

\[ i\hbar \frac{\partial}{\partial t} \rho_l(t) = \kappa_l(t) \Delta^*_l(t) - \Delta_l(t) \kappa^*_l(t) \quad : \text{Occupation Prob. (for } l > 0) \]

\[ i\hbar \frac{\partial}{\partial t} \kappa_l(t) = \{ \varepsilon_l(t) + \varepsilon_{\tilde{l}}(t) \} \kappa_l(t) + \Delta_l(t) (2 \rho_l(t) - 1) \quad : \text{Pair Prob. (for } l > 0) \]
Example: Photo-absorption cross section of $^{172}$Yb

$\beta = 0.32$

$\Delta_n = 0.76$ [MeV]

$\Delta_p = 0.55$ [MeV]

$\Gamma = 1.0$ [MeV]

3D Cb-TDHFB

Cb-TDHFB can reproduce the photo-absorption cross section of $^{172}$Yb.

- Heavy nucleus
- Deformed nucleus
- Including pairing

Total cal. cost: **300 CPU hours**
(with a Single processor; Intel Core i7 3.0 GHz)

Box size: $R = 15$ [fm], mesh = 1 [fm] (3D-Spherical)

Canonical-basis space (HF+BCS g.s.): 146 states for neutron, 98 states for proton

Experimental data:

Dipole mode

$$\hat{F}^N = - (Ze/A)(\hat{z} + \hat{x} + \hat{y})$$

$$\hat{F}^P = (Ne/A)(\hat{z} + \hat{x} + \hat{y})$$
Setup for collision

Incident Energy: 18 - 20 [MeV] \( (E_{\text{cm}} = 9.0 \text{ -- } 10 \text{ [MeV]}, V_{\text{Coul.}} \sim 9 \text{ MeV}) \)

Impact parameter: 2.8 - 3.1 [fm]

Effective Interaction: Skyrme force (SkM*), Contact pairing

Projectile: \(^{22}\text{O}\), Target: \(^{22}\text{O}\) (HF g.s. has also spherical shape)

\# of canonical-basis for HF+BCS g.s. ; \( (N, Z) = (32 \text{ (16+16)}, 16 \text{ (8+8)}) \)

Average of gap energy; \( \bar{\Delta}_n = 2.066 \text{ [MeV]} \) \( V_0^n = -412.5 \text{ [MeV]} \)

Calculation space (3D meshed box):

Length of box for \((x, y, z)\) is \(36, 20, 40\text{[fm]} \) meshed by \(1.0 \text{ [fm]} \)
Simulation of $^{22}$O + $^{22}$O collision with $b = 3.0$ [fm] and $E_{cm}=10$ [MeV]

Time-evolution of Neutron density distribution
Simulation of $^{22}\text{O} + ^{22}\text{O}$ collision with $b = 3.0$ [fm] and $E_{\text{cm}} = 10$ [MeV]

Time-evolution of Neutron density distribution

\[ \sigma_F = 2\pi \int_0^{b_f} db \ b \quad \sigma_F^{\text{BCS}} < \sigma_F^{\text{HF}} \]
From simulation of $^{22}\text{O} + ^{22}\text{O}$ collision with $b = 2.8 – 3.1 \text{ [fm]}$ and $E_{\text{cm}} = 10 \text{ [MeV]}$,

$$2.8 \text{ fm} < b^B_f < 2.9 \text{ fm} < b^{Bw}_f < 3.0 \text{ fm} < b^H_f < 3.1 \text{ fm}$$

Pairing correlation does not increase the fusion cross section (in this work).

Up to now

- **Same nuclei collision** with the incident energy around Coulomb barrier → Pairing correlation *can have repulsive aspects* in fusion reaction.
  - $\sigma_H \sim 283 \text{ mb} \rightarrow \sigma_B \sim 246 \text{ mb}$ be small about 15% → The repulsive effects *depends on* the strength of pairing.

From now on

- **In a little bit heavier system?** ← To increase pair number
- **Case of different nuclei collision?** ← Large difference of chemical potential will accelerate Pair transfer.
- **Is the pairing effects visible in much heavier system?**
In a little bit heavier system  

Point: increase of pair number

Projectile: $^{52}\text{Ca}$, Target: $^{52}\text{Ca}$  
In both methods, the g.s. is spherical shape.

$E_{cm} = 51.5$ MeV ($V_{\text{Coul.}} \sim 49$ MeV)  
Impact parameter: 2.2 - 2.6 [fm]

Effective Interaction: SkM*, Contact pairing

$V_0^n = -438.1$ MeV  
To reproduce $\Delta_n$ of $^{52}\text{Ca}$

\[
2.4 \text{ fm} < b_f^B < 2.45 \text{ fm} < b_f^H < 2.5 \text{ fm}
\]

$\rightarrow \sigma_H \sim 189 \text{ mb} \rightarrow \sigma_B \sim 181 \text{ mb} \text{ be small about 5\%}$
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\]
Case of different nuclei collision

Projectile: $^{22}\text{O}$, Target: $^{52}\text{Ca}$

In both methods, the g.s. is spherical shape.

$E_{cm} = 25$ MeV ($V_{\text{coul.}} \sim 20.8$ MeV)

Impact parameter: $3.0 - 4.5$ [fm]

Effective Interaction: SkM*, Contact pairing

$V_0^n = -425.3$ MeV $\leftarrow$ Average of strength in $^{22}\text{O}$ and $^{52}\text{Ca}$

Point: Difference of chemical potential

$\sigma_H \sim 528$ mb $\rightarrow \sigma_B \sim 503$ mb be small about 5%

ARIS2014, 14.6.6
S.Ebata
Case of different nuclei collision

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$E_{\text{cm}} = 25 \text{ MeV} \ (V_{\text{Coul.}} \sim 20.8 \text{ MeV})$  

Impact parameter: $3.0 - 4.5 \text{ [fm]}$

Effective Interaction: SkM*, Contact pairing

$V_0^n = -425.3 \text{ MeV}$  

Average of strength in $^{22}\text{O}$ and $^{52}\text{Ca}$

$4.0 \text{ fm} < b_f^B < 4.1 \text{ fm} < b_f^H < 4.25 \text{ fm}$

$\rightarrow \sigma_H \sim 528 \text{ mb} \rightarrow \sigma_B \sim 503 \text{ mb}$ be small about 5%

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Is the pairing effect visible in much heavier system?

**Projectile** : $^{96}\text{Zr}$, **Target** : $^{124}\text{Sn}$

$E_{cm} = 228$ MeV ($V_{\text{coul.}} = 216, 225$ MeV)

HF+BCS : Both ground states are spherical shape.

HF : $^{96}\text{Zr}$ ; spherical, $^{124}\text{Sn}$ ; oblate shape.

**Effective Interaction** : SLy4d, Contact pairing

$V_0^n = -412.5$ MeV  \(\rightarrow\) *Very strong in order to check the effect*

$V_0^p \equiv 0.0$ MeV  \(\rightarrow\) *Neglect proton pairing*

HF g.s. of $^{124}\text{Sn}$ has Oblate shape. $\beta = -0.1$

**Coulomb barrier for $^{96}\text{Zr} + ^{124}\text{Sn}$ (F.D.)**

**Diagram**

- $^{96}\text{Zr}$
- $^{124}\text{Sn}$
- $V_{FD} = 225$ MeV
- $V_{FD} = 216$ MeV

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Is the pairing effect visible in much heavier system?
Is the pairing effect visible in much heavier system?

Time up to scission

\[ T_{z||Z}^{\text{Sci.}} \sim 850 \text{ fm/c} \]
\[ T_{x,y||Z}^{\text{Sci.}} \sim 2070 \text{ fm/c} \]
\[ T_{B}^{\text{Sci.}} \sim 1000 \text{ fm/c} \]
We apply Cb-TDHFB to large amplitude collective phenomena such as collision, with a contact pairing functional.

- $^{22}\text{O}+^{22}\text{O}$: Pairing effects in fusion reaction, have a repulsive aspect.
- $^{52}\text{Ca}+^{52}\text{Ca}$: The repulsive aspect appears also, but its contribution becomes small.
- $^{22}\text{O}+^{52}\text{Ca}$: The similar aspects can be seen.
- $^{96}\text{Zr}+^{124}\text{Sn}$: The pairing effects gets the time from a contact to scission to be short.

**Perspective**

To analyze the detail of internal behavior

- More accurate calculation
- Behavior of single-particle levels in real-time cal.
- Energy distribution, Level crossing
- Particle number projection for the multi-particle transfer in real-time cal.
- Nucleon transfer (Pair transfer), Nuclear Josephson effects