Phase-space representation for nuclear potentials

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Outline

Nucleon-nucleon interaction
- Realistic NN potentials
- Unitary transformations and effective interactions

Phase-space representation
- Kirkwood representation
- Momentum dependence in phase-space representation
- Results for realistic NN potentials
NN interaction and correlations

Realistic NN potentials

- describing two-nucleon properties (scattering, deuteron) with high accuracy
- different potentials available, e.g.
  - Argonne V18 \( \text{Wiringa, Stoks, Schiavilla, PRC 51, 38 (1995)} \)
  - \( N^3 \text{LO from Chiral effective field theory} \) \( \text{Entem, Machleidt, PRC 68, 041001 (2003)} \)

\( S = 1, \ T = 0 \)

- repulsive core: nucleons can not get closer than \( \approx 0.5 \text{ fm} \) → central correlations
- strong dependence on the orientation of the spins due to the tensor force → tensor correlations
- the nuclear force will induce strong short-range correlations in the nuclear wave function
NN interaction and correlations

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Use unitary transformations to obtain “soft” effective realistic interaction
Unitary Correlation Operator Method

- NN interaction induces strong **central** and **tensor correlations**
  → Many-body methods working with (superpositions of) Slater determinants require huge model space sizes

**Unitary Correlation Operator Method (UCOM):**
Unitary transformation $C$ imprints correlation on simple model state $|\Psi\rangle$

$$|\hat{\Psi}\rangle = C |\Psi\rangle = C_\Omega C_r |\Psi\rangle$$

- Work with transformed operators and simple model states, e.g.

$$\langle \hat{\Psi} | H | \hat{\Psi}' \rangle = \langle \Psi | C^\dagger HC | \Psi' \rangle =: \langle \Psi | H_{\text{eff}} | \Psi' \rangle$$

- $H_{\text{eff}}$ for local NN potential (e.g. Argonne V18) contains **quadratic momentum dependence** replacing short-range repulsion and short-range tensor:

$$H_{\text{eff}} = C_r^\dagger \left( \frac{\vec{p}^2}{2\mu} + V(r) \right) C_r = \frac{\vec{p}^2}{2\mu} + V_{\text{eff}}(r, p)$$

**Similarity Renormalization Group**

- **Similarity Renormalization Group (SRG):**


  Evolve Hamiltonian and unitary transformation matrix

  \[
  \frac{dH_\alpha}{d\alpha} = (2\mu)^2 \left[ [T_{\text{int}}, H_\alpha], H_\alpha \right] \rightarrow H_\alpha = U_\alpha^\dagger H U_\alpha
  \]

  - Unitary transformation \( U_\alpha \) to obtain “soft” effective realistic interaction
  - SRG drives the Hamiltonian towards a band-diagonal structure
  - Performed in matrix element representation \( \rightarrow \text{momentum dependence} \)?

**AV18:** \( \langle k(L0)J; T | V_\alpha | k'(L0)J; T \rangle \) in MeVfm\(^3\)

\[
\alpha = 0.00\text{fm}^4 \text{ (bare)}
\]
Similarity Renormalization Group

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AV18: \( \langle k(L0)J; T | V_\alpha | k'(L0)J; T \rangle \) in MeVfm\(^3\)

\[ \alpha = 0.01\text{fm}^4 \]
Similarity Renormalization Group

- **Similarity Renormalization Group (SRG):**

  Evolve Hamiltonian and unitary transformation matrix

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- Unitary transformation $U_\alpha$ to obtain “soft” effective realistic interaction
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- Performed in matrix element representation → momentum dependence?

AV18: $\langle k(L0)j; T | V_\alpha | k'(L0)j; T \rangle$ in MeVfm$^3$

$\alpha = 0.04$fm$^4$
Representation of NN interactions

- effective realistic NN potentials given usually as momentum matrix elements
- can be used in many-body calculations with shell model or plane wave basis
- but not for Fermionic Molecular Dynamics and cluster models
- matrix element representation not intuitive, not transparent

Other representation to study and visualize NN potentials

- How does the potential look in position space?
- What happens to the repulsive core?
- What is the range of the interaction?
- What is the momentum dependence?

⇒ Phase-space representation
Phase-space representation

- **Kirkwood representation** [Kirkwood, Phys. Rev., 44, 31 (1933)]

  - **phase-space distribution** for a given state $|\phi\rangle$:
    \[
    f_{ps}(\vec{r}, \vec{p}) = (2\pi)^{3/2} \langle \vec{r}|\phi\rangle \langle \phi|\vec{p}\rangle \langle \vec{p}|\vec{r}\rangle
    \]

  - **phase-space representation** of an operator $O$:
    \[
    O_{ps}(\vec{r}, \vec{p}) = (2\pi)^{3/2} \langle \vec{r}|O|\vec{p}\rangle \langle \vec{p}|\vec{r}\rangle
    \]

  - **quantum expectation value** (analogue to classical expression)
    \[
    \langle O \rangle = \langle \phi|O|\phi\rangle = \int d^3r \int d^3p f^*_{ps}(\vec{r}, \vec{p}) O_{ps}(\vec{r}, \vec{p})
    \]

- Study phase-space representation of effective NN interactions

- Multipole expansion with Legendre polynomials $P_\Lambda$:
  \[
  V_{ps}(\vec{r}, \vec{p}) = \sum_\Lambda i^\Lambda V_\Lambda(r, p) P_\Lambda(\vec{r} \cdot \vec{p}).
  \]
Phase-space representation of local $V = V(r)$

$V_{\Lambda}(r, p)$ (arb. units)

$V(r) = \exp \left\{ -\frac{r^2}{2f^2} \right\}$

local: $V = V(r)$

$\rightarrow V_{ps}(\vec{r}, \vec{p}) = V(r) = V(r)P_0(\vec{r} \cdot \vec{p})$
Phase-space representation of \( V = \frac{1}{2} (\mathbf{p}^2 V(r) + V(r)\mathbf{p}^2) \)

\( V_\Lambda(r, \mathbf{p}) \) (arb. units)

\[
\begin{align*}
\Lambda = 0 & \quad \Lambda = 1 & \quad \Lambda = 2 \\
\end{align*}
\]

\[
V(r) = \exp \left\{ -\frac{r^2}{2m^2} \right\}
\]

**quadratic momentum dependence:**

\[
\rightarrow V_{ps}(\mathbf{r}, \mathbf{p}) = \left( V(r) p^2 - \frac{1}{2} V''(r) - \frac{V'(r)}{r} \right) - i \frac{V'(r)}{r} \mathbf{r} \cdot \mathbf{p}
\]

\[
= \left( V(r) p^2 - \frac{1}{2} V''(r) - \frac{V'(r)}{r} \right) P_0(\mathbf{r} \cdot \mathbf{p}) - i \frac{V'(r)}{r} r p P_1(\mathbf{r} \cdot \mathbf{p})
\]
**Phase-space representation of** $V = V(r) \hat{L}^2$

$V_\Lambda(r, p)$ (arb. units)

\[ V(r) = \exp \left\{ -\frac{r^2}{2a^2} \right\} \]

**quadratic angular momentum dependence:** $V = V(r) \hat{L}^2$

\[
\rightarrow V_{ps}(\vec{r}, \vec{p}) = V(r)(\vec{r} \times \vec{p})^2 + 2iV(r)\vec{r} \cdot \vec{p} \\
= \frac{2}{3} V(r)(rp)^2 P_0(\hat{\vec{r}} \cdot \hat{\vec{p}}) + 2iV(r)rp P_1(\hat{\vec{r}} \cdot \hat{\vec{p}}) - \frac{2}{3} V(r)(rp)^2 P_2(\hat{\vec{r}} \cdot \hat{\vec{p}})
\]
Bare potentials in phase-space representation

$S=0$, $T=1$: $V_\Lambda(r, p)$ in MeV

**AV18**

- $\Lambda=0$
- $\Lambda=1$
- $\Lambda=2$
- $\Lambda=3$

**N$^3$LO**

- $\Lambda=0$
- $\Lambda=1$
- $\Lambda=2$
- $\Lambda=3$
UCOM Argonne potential in phase-space representation

$S = 0, T = 1: V_{\Lambda}(r, p)$ in MeV

AV18

$\Lambda = 0$  $\Lambda = 1$  $\Lambda = 2$  $\Lambda = 3$
SRG Argonne potential in phase-space representation

\( S = 0, \ T = 1: \ V_\Lambda(r, p) \) in MeV

AV18

\( \Lambda = 0 \) \hspace{1cm} \( \Lambda = 1 \) \hspace{1cm} \( \Lambda = 2 \) \hspace{1cm} \( \Lambda = 3 \)
SRG potentials in phase-space representation

\( S = 0, \, T = 1: \, V_\Lambda(r, p) \) in MeV

AV18 SRG

N\(^3\)LO SRG
Local projection

Wendt et al. introduced the **local projection** of a potential


This is the same as the phase space representation for $\Lambda = 0$ and $p = 0$:

$$V_{loc}(r) = V_{\Lambda=0}(r, p = 0)$$

![Graph](image-url)
Summary

Realistic effective interactions
- UCOM and SRG soften the interaction by unitary transformation
- transformed interactions are non-local
- no “intuitive” picture in partial wave matrix element representation

Phase-space Representation
- investigate momentum-dependence
- non-locality reflected in $p$- and $\Lambda$-dependence
- form of regulators reflected in $N^3$LO phase-space representation
- AV18 UCOM has quadratic momentum-dependence
- AV18 and $N^3$LO SRG have more complicated momentum-dependence

Outlook
- extend phase-space representation to $S = 1$ channels
- what can we learn for the operator representation for effective interactions