

# Energy density optimization: UNEDF2

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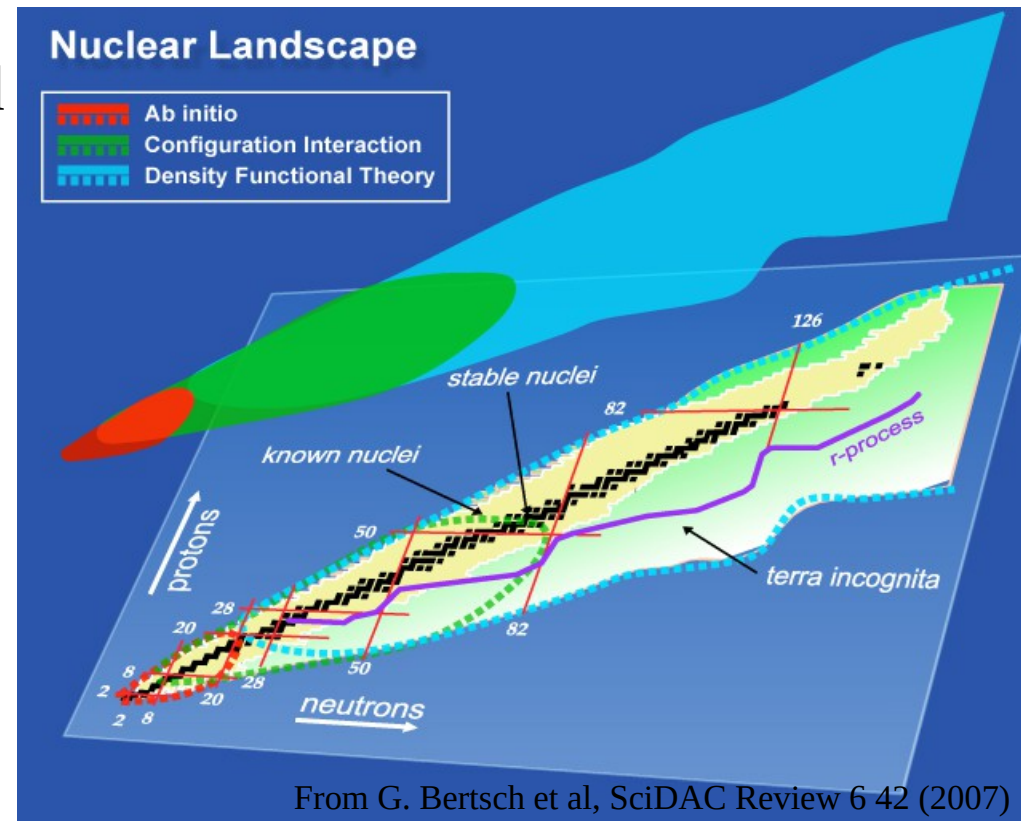


## Skyrme energy density

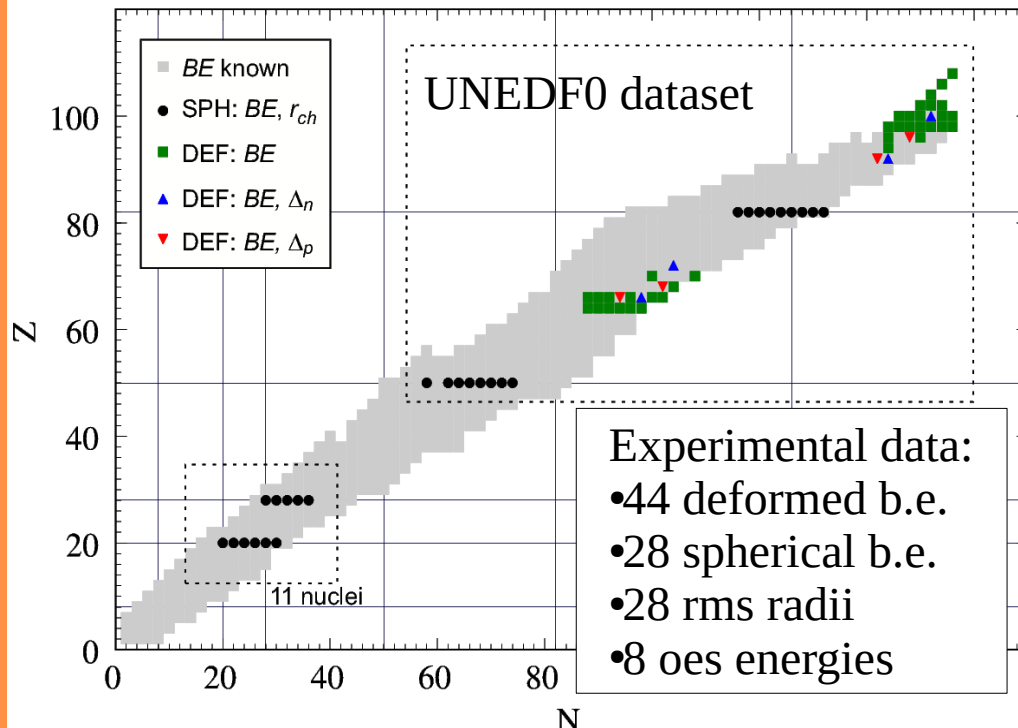
$$H_t^{even}(r) = C_t^\rho \rho_t^2 + C_t^\tau \rho_t \tau_t + C_t^{\Delta\rho} \rho_t \Delta \rho_t + C_t^{\nabla J} \rho_t \nabla \cdot \mathbf{J}_t + C_t^J \mathbf{J}_t^2$$

$$C_t^\rho = C_{t0}^\rho + C_{tD}^\rho \rho_0^\gamma, \quad t=0,1$$

- Skyrme EDF is constructed from bilinear terms of density matrices and their derivatives up to the 2nd order
- Each term multiplied by a constant **coupling constant** (except density dependent  $C^\rho$ )
- Volume part of the ED can be parameterized with infinite nuclear matter parameters
- Coupling constants adjusted to experimental input



# Energy density optimization: UNEDF0 and UNEDF1



UNEDF0: M. K., T. Lesinski, J. Moré, W. Nazarewicz, J. Sarich, N. Schunck, M. V. Stoitsov, S. Wild, PRC 82, 024313 (2010)

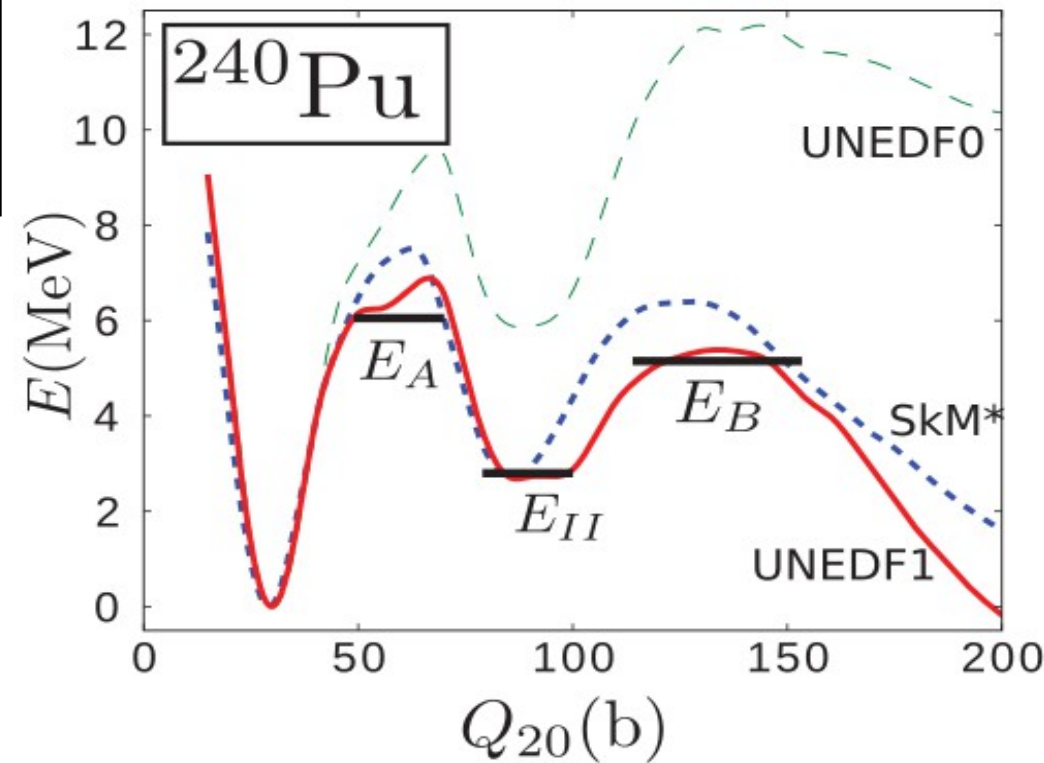
UNEDF1: M. K., J. McDonnell, W. Nazarewicz, P.-G. Reinhard, J. Sarich, N. Schunck, M. V. Stoitsov, S. Wild, PRC 85, 024304 (2012)

- Optimization of Skyrme-like ED with respect of 12 parameters at deformed HFB level

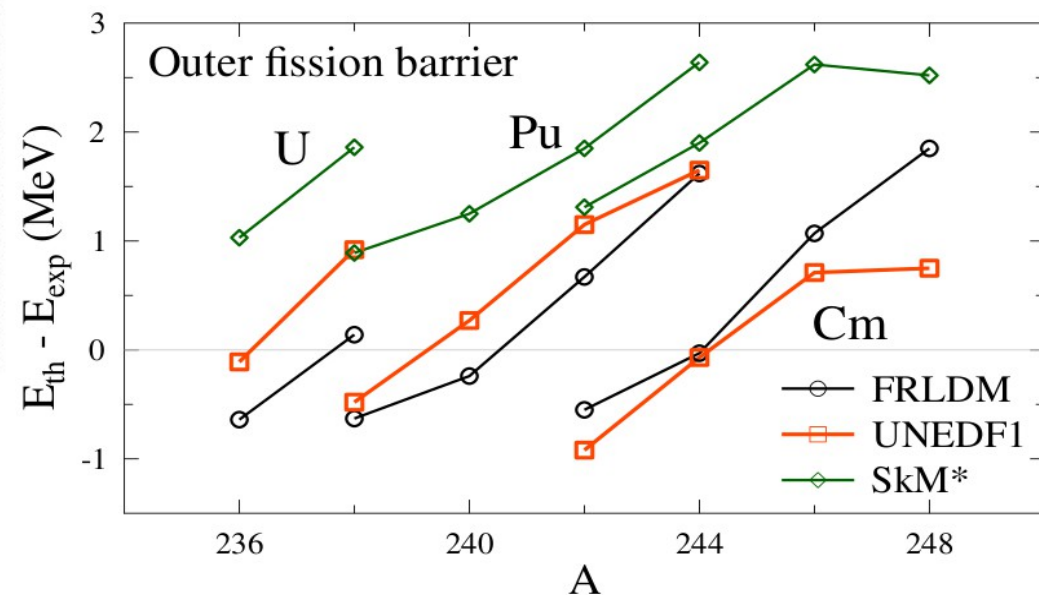
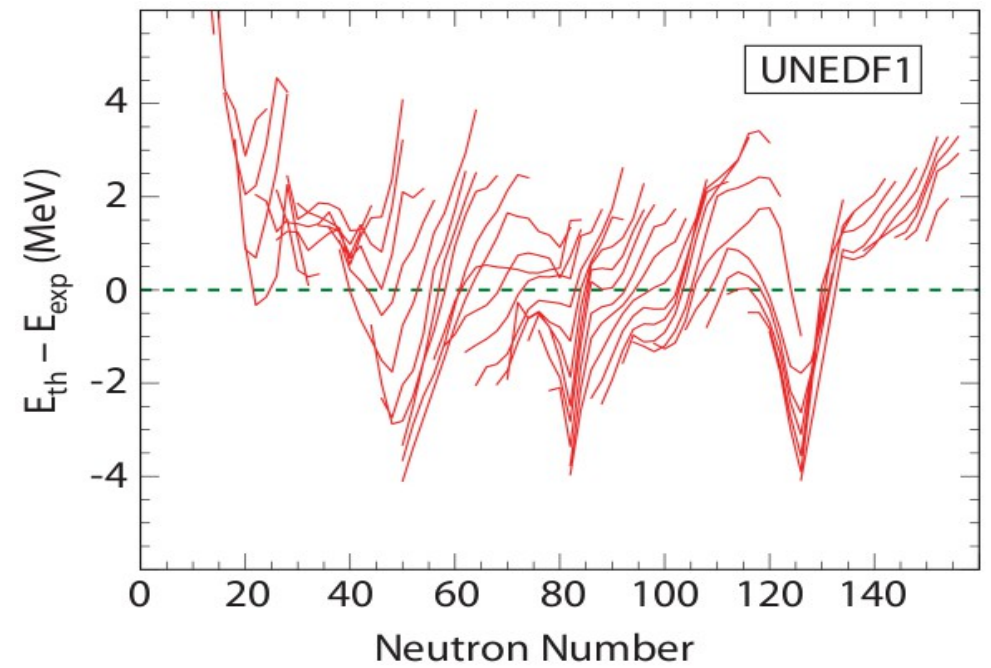
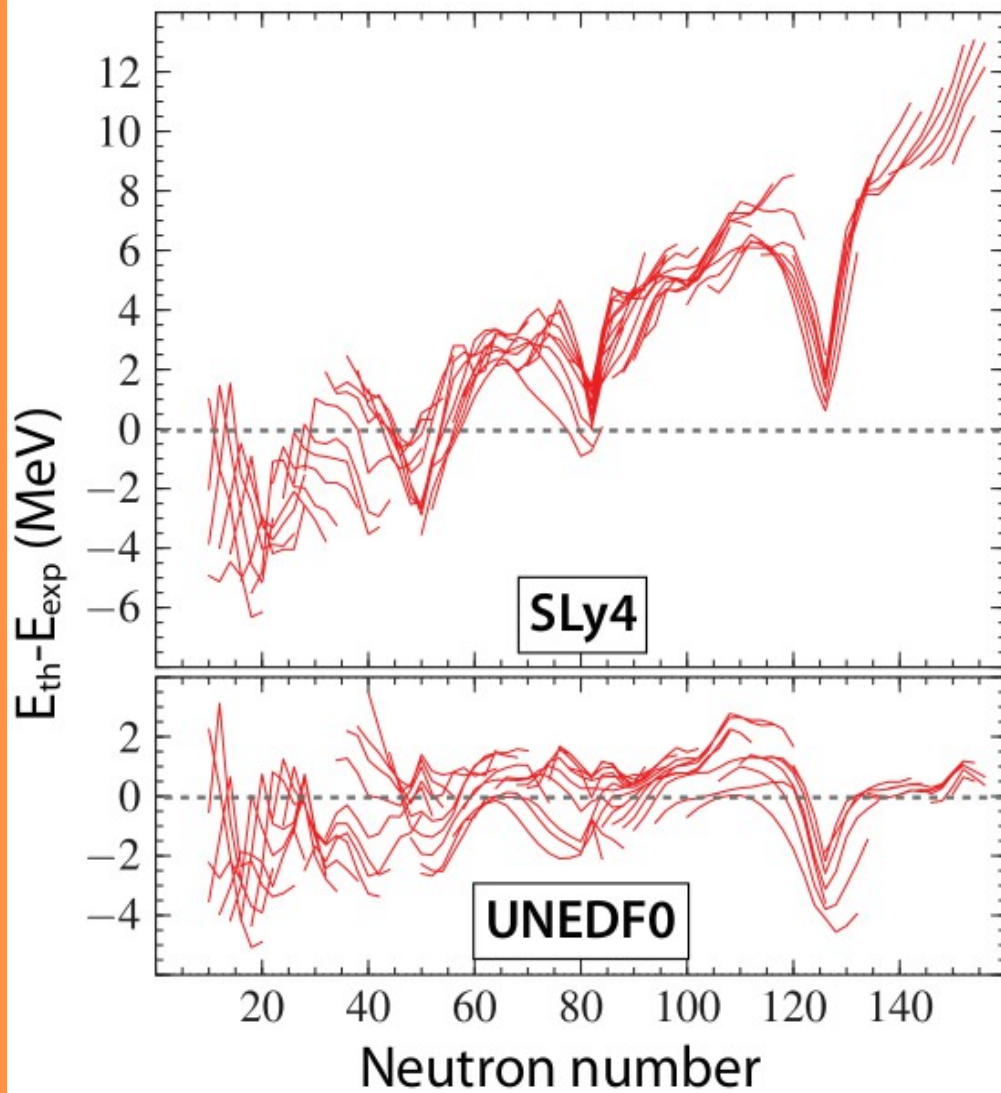
$$\rho_c, E^{NM}/A, K^{NM}, a_{sym}^{NM}, L_{sym}^{NM}, M_s^{-1}$$

$$C_0^{\rho\Delta\rho}, C_1^{\rho\Delta\rho}, V_0^n, V_0^p, C_0^{\rho\nabla J}, C_1^{\rho\nabla J}$$

- UNEDF1 was the first parameterization which was systematically optimized at the deformed HFB level for fission studies
- UNEDF1 included data on 4 fission isomers states ( $^{226}\text{U}$ ,  $^{238}\text{U}$ ,  $^{240}\text{Pu}$ ,  $^{242}\text{Cm}$ )



# UNEDF0 and UNEDF1: Performance



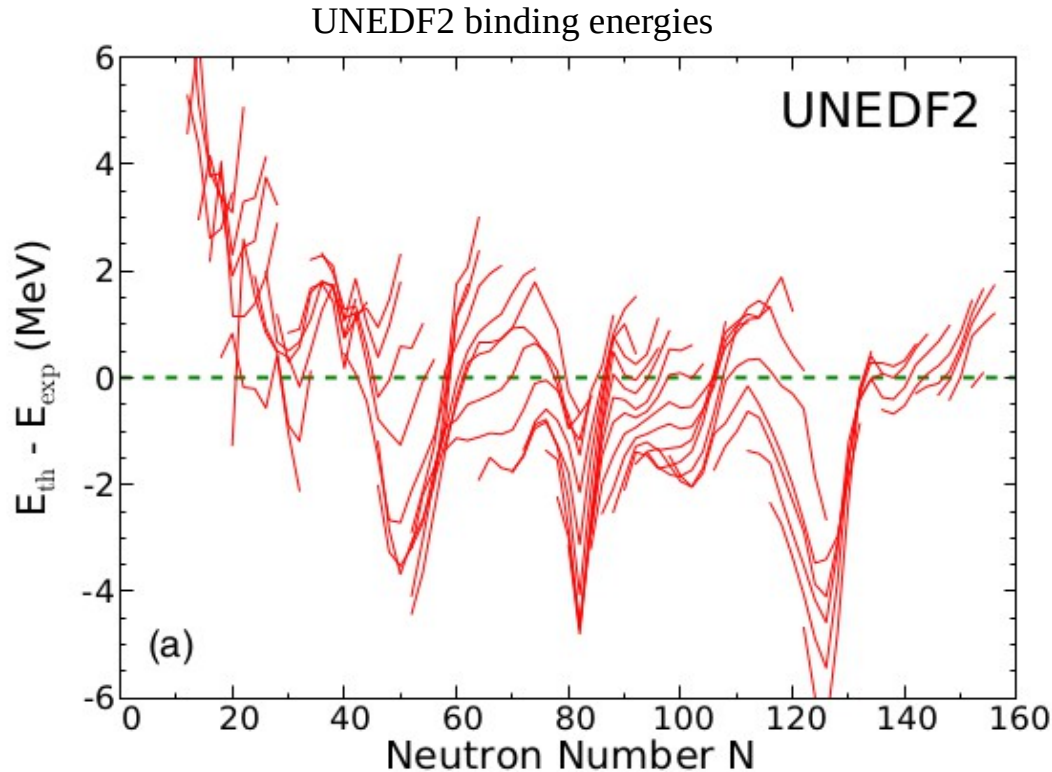
# Energy density optimization: UNEDF2

- Optimization of Skyrme-like ED with respect of 14 parameters at deformed HFB level: Tensor terms now included

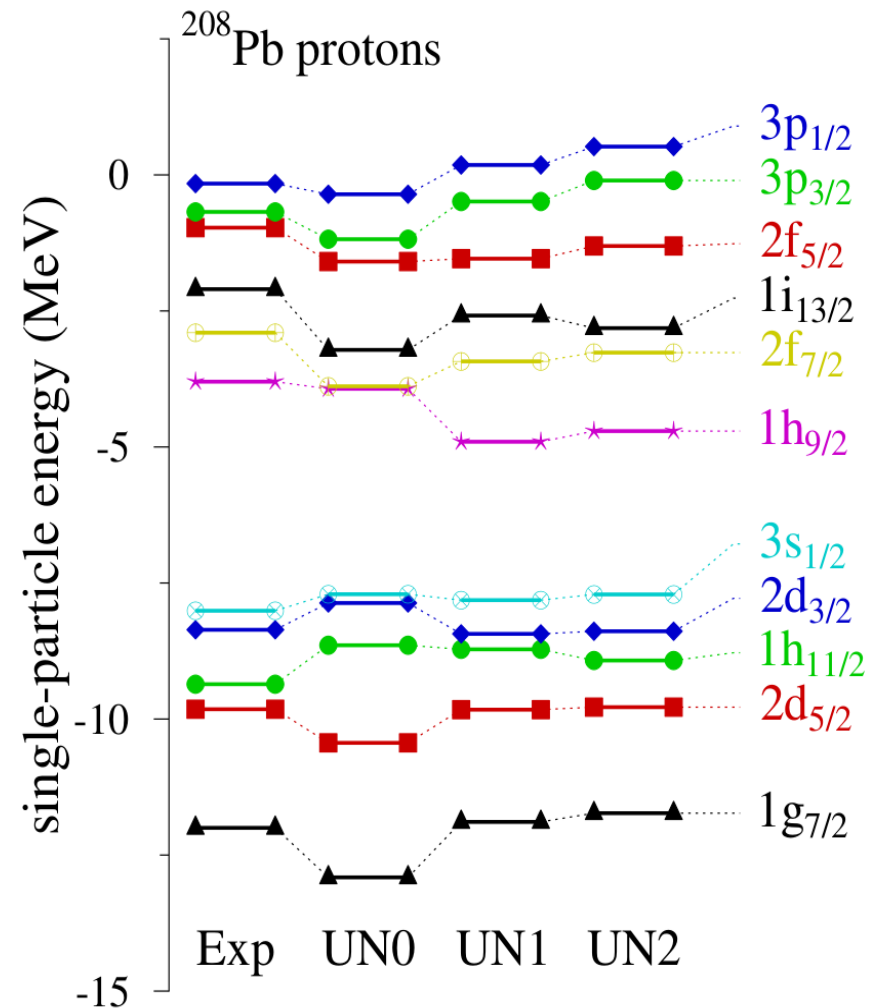
$$\rho_c, E^{NM}/A, K^{NM}, a_{sym}^{NM}, L_{sym}^{NM}, M_s^{-1}$$

$$C_0^{\rho\Delta\rho}, C_1^{\rho\Delta\rho}, V_0^n, V_0^p, C_0^{\rho\nabla J}, C_1^{\rho\nabla J}, \boxed{C_0^J, C_1^J}$$

- Single particle energies included in the optimization. These are handled with blocked HFB calculations



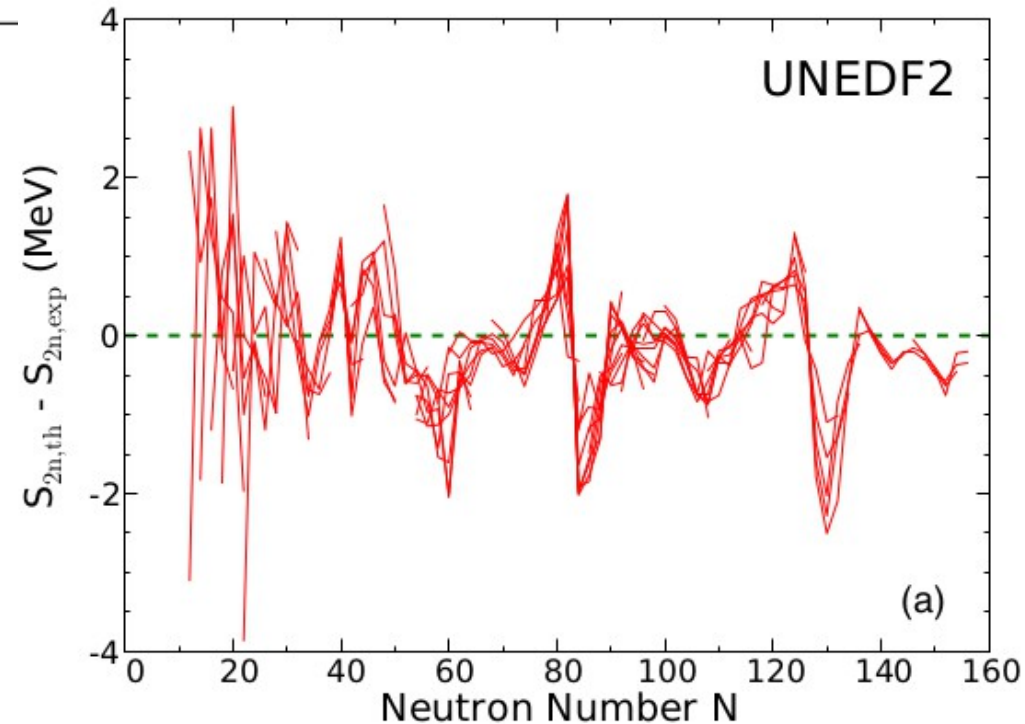
UNEDF2: M.K., J. McDonnell, W. Nazarewicz, E. Olsen, P.-G. Reinhard, J. Sarich, N. Schunck, S.M. Wild, D. Davesne, J. Erler, A. Pastore, Phys. Rev. C 89 054314 (2014)



# UNEDF2: Global properties

RMS deviations of various observables

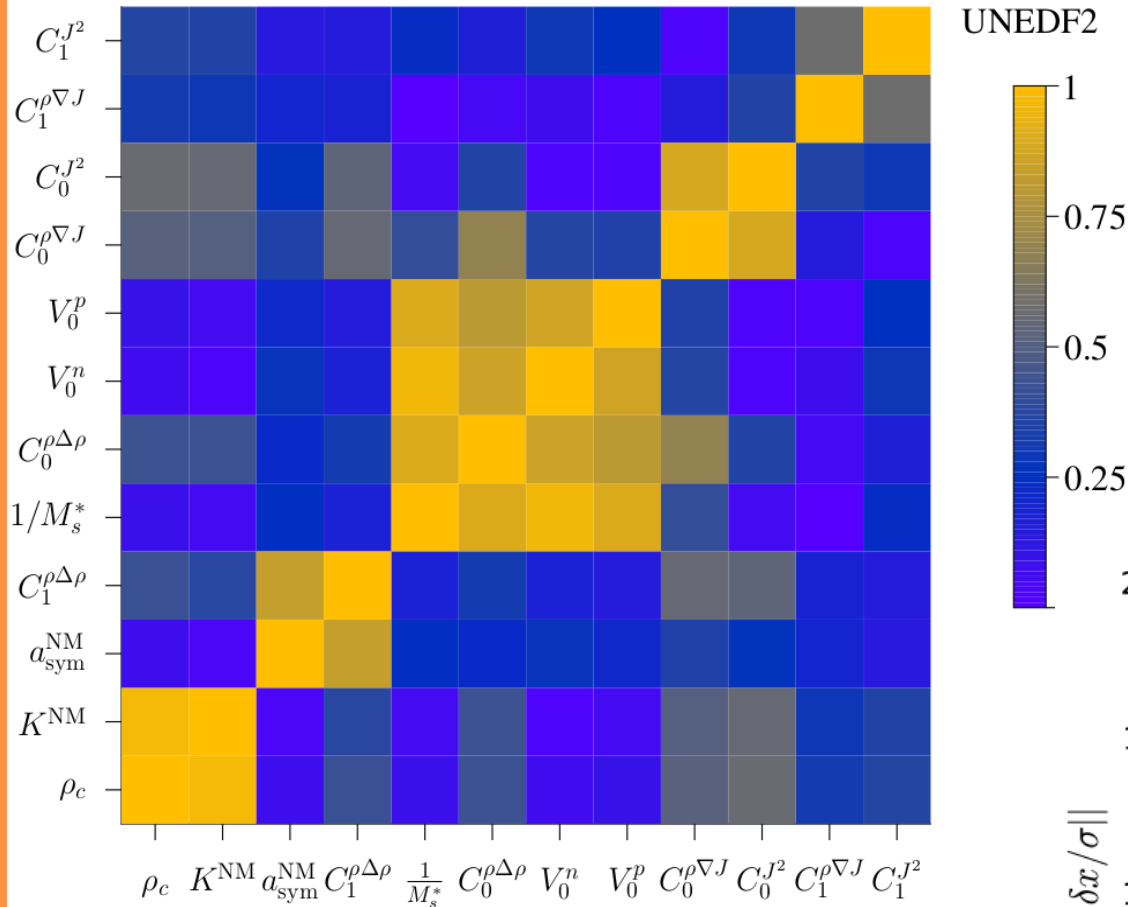
Observable	UNEDF0	UNEDF1	UNEDF2
$E$	1.428	1.912	1.950
$E (A < 80)$	2.092	2.566	2.475
$E (A \geq 80)$	1.200	1.705	1.792
$S_{2n}$	0.758	0.752	0.843
$S_{2n} (A < 80)$	1.447	1.161	1.243
$S_{2n} (A \geq 80)$	0.446	0.609	0.711
$S_{2p}$	0.862	0.791	0.778
$S_{2p} (A < 80)$	1.496	1.264	1.309
$S_{2p} (A \geq 80)$	0.605	0.618	0.572
$\tilde{\Delta}_n^{(3)}$	0.355	0.358	0.285
$\tilde{\Delta}_n^{(3)} (A < 80)$	0.401	0.388	0.327
$\tilde{\Delta}_n^{(3)} (A \geq 80)$	0.342	0.350	0.273
$\tilde{\Delta}_p^{(3)}$	0.258	0.261	0.276
$\tilde{\Delta}_p^{(3)} (A < 80)$	0.346	0.304	0.472
$\tilde{\Delta}_p^{(3)} (A \geq 80)$	0.229	0.248	0.194
$R_p$	0.017	0.017	0.018
$R_p (A < 80)$	0.022	0.019	0.020
$R_p (A \geq 80)$	0.013	0.015	0.017



- Generally, UNEDF2 gives no or only marginal improvement over to UNEDF1  
 $\Rightarrow$  Novel EDF developments required to improve precision (see Jacek's talk)

# Sensitivity analysis

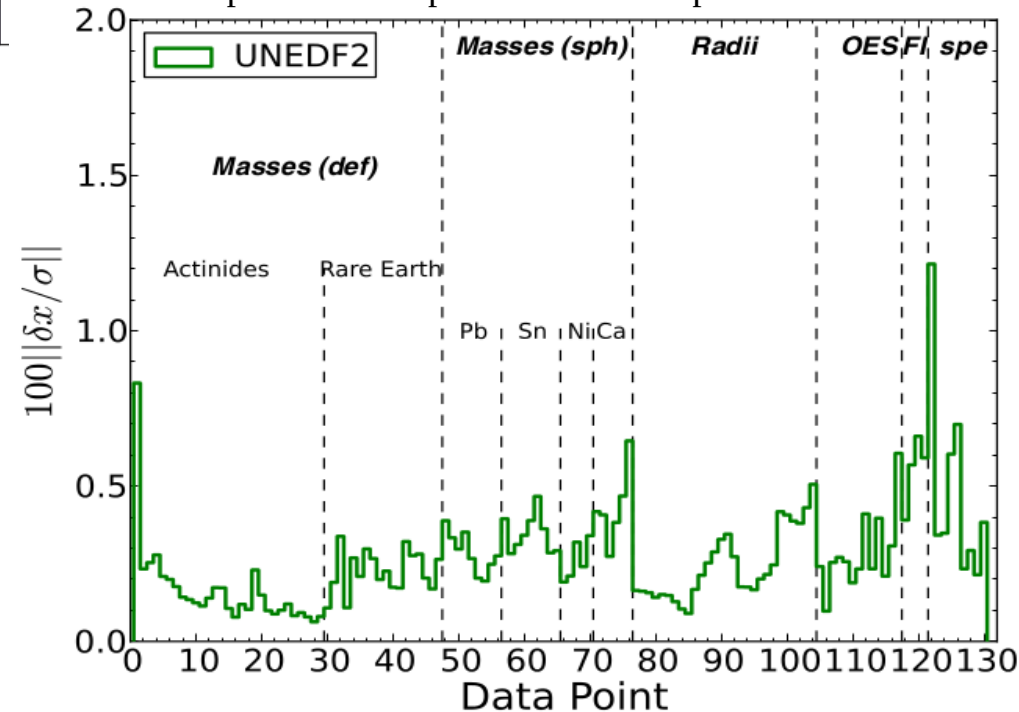
UNEDF2 correlation matrix



- Sensitivity analysis shows that UNEDF2 fully constrained with the data  
 $\Rightarrow$  Major improvements not possible in the Skyrme-EDF framework

- With UNEDF0,1 and 2, a complete sensitivity analysis was done for the obtained minimum
- Standard deviations and correlations of optimized EDF parameters
- Crucial information when addressing predictive power. E.g propagation of the error.

Impact of a datapoint to obtained parameterization



## Conclusions and outlook

- UNEDF0, 1 and 2 presents a optimization scheme of Skyrme-like EDF, which includes progressively more experimental data.
  - Many UNEDF2 properties slightly worse than with more specialized UNEDF0 or UNEDF1. UNEDF2 is the best all-around Skyrme EDF
  - Sensitivity analysis shows that further major improvements for UNEDF2 are unlikely
  - Generally, limits of the Skyrme-like EDF models have been reached: Novel EDFs required to improve precision. This conclusion is also supported by several other studies.
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- Possible future directions: EDFs with higher order derivatives, DME-based functionals, regularized higher-order functionals.
  - Future EDF optimizations schemes should include sensitivity analysis to assess the predictive power of the model



Backup slides

## UNEDF parameters

$x$	SLy4	UNEDF0	UNEDF1	UNEDF2
$\rho_c$	0.16000	0.16053	0.15871	0.15631
$E/A$	-15.972	-16.056	-15.8	-15.8
$K$	229.901	230.0	220.0	239.930
$a_{\text{sym}}$	32.004	30.543	28.987	29.131
$L$	45.962	45.080	40.005	40.0
$1/M_s^*$	1.439	0.9	0.992	1.074
$C_0^{\rho\Delta\rho}$	-76.996	-55.261	-45.135	-46.831
$C_1^{\rho\Delta\rho}$	+15.657	-55.623	-145.382	-113.164
$V_0^n$	-258.200	-170.374	-186.065	-208.889
$V_0^p$	-258.200	-199.202	-206.580	-230.330
$C_0^{\rho\nabla J}$	-92.250	-79.531	-74.026	-64.309
$C_1^{\rho\nabla J}$	-30.750	45.630	-35.658	-38.650
$C_0^{JJ}$	0.000	0.000	0.000	-54.433
$C_1^{JJ}$	0.000	0.000	0.000	-65.903

## UNEDF2 parameters

$\mathbf{x}$	$\hat{\mathbf{x}}^{(\text{fin.})}$	$\sigma$	95% CI
$\rho_c$	0.15631	0.00112	[0.154, 0.158]
$E/A$	-15.8		
$K$	239.930	10.119	[223.196, 256.663]
$a_{\text{sym}}$	29.131	0.321	[28.600, 29.662]
$L$	40.0		
$1/M_s^*$	1.074	0.052	[0.988, 1.159]
$C_0^{\rho\Delta\rho}$	-46.831	2.689	[-51.277, -42.385]
$C_1^{\rho\Delta\rho}$	-113.164	24.322	[-153.383, -72.944]
$V_0^n$	-208.889	8.353	[-222.701, -195.077]
$V_0^p$	-230.330	6.792	[-241.561, -219.099]
$C_0^{\rho\nabla J}$	-64.309	5.841	[-73.968, -54.649]
$C_1^{\rho\nabla J}$	-38.650	15.479	[-64.246, -13.054]
$C_0^{JJ}$	-54.433	16.481	[-81.687, -27.180]
$C_1^{JJ}$	-65.903	17.798	[-95.334, -36.472]

# UNEDF properties

