The β Decay of $^{38}$Ca: Sensitive Test of Isospin Symmetry-Breaking Corrections from Mirror Superallowed $0^+ \rightarrow 0^+$ transitions

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Superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$ decay

$$ft = \frac{K}{G_V^2 \langle M_F \rangle^2}$$

- $f = \text{statistical rate function } f(Z, Q_{EC})$
- $t = \text{partial half-life } f(t_{1/2}, \text{BR})$
- $G_V = \text{vector coupling constant}$
- $M_F = \text{Fermi matrix element}$

Including corrections

$$\mathcal{A} = ft (1 + \delta'_R) [1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta^V_R)}$$

- $\delta_C = \text{isospin-symmetry breaking correction}$
- $\delta'_R, \delta_{NS} = \text{radiative correction (transition dependent)}$
- $\Delta_R = \text{radiative correction (transition independent)}$
Superallowed $0^+ \rightarrow 0^+$ nuclear $\beta$ decay

\[ ft = \frac{K}{G_V^2 \langle M_F \rangle^2} \]

- $f$ = statistical rate function $f(Z, Q_{EC})$
- $t$ = partial half-life $f(t_{1/2}, BR)$
- $G_V$ = vector coupling constant
- $M_F$ = Fermi matrix element

Including corrections

\[ \mathcal{A} = ft(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta_R^V)} \]

- $f(Z, Q_{EC}) \sim 1.5\%$
- $f($nuclear structure$) \sim 0.3 - 1.5\%$
- $f($interactions$) \sim 2.4\%$
Approximately 150 individual measurements made contributions with compatible precision.

The 13 best-known transitions: the $f_t$ values for 10 cases have been measured to 0.1% precision or better; 3 more cases to <0.3% precision.

Results:

$G_V$ constant
- verified to $\pm 0.013\%$

$|V_{ud}| = \frac{G_V}{G_\mu}$

$= 0.97425 \pm 0.00022$

CKM unitarity
- satisfied at $\pm 0.06\%$

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$

$= 0.99990 \pm 0.00060$
Error budget for $V_{ud}$ determined from $0^+ \rightarrow 0^+$ decays

\[ V_{ud} = 0.97425 \pm 0.00022 \]

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73 046301 (2010)

\[ \Delta R \]

\[ \delta_C - \delta_{NS} \]

\[ \delta_R \]

Parts in $10^4$

Z of daughter

Shell Model Saxon-Woods radial functions

Shell Model Hartree-Fock radial functions

PRC 79, 055502 (2009)
Testing $\delta_C$ calculations by experiment

Our strategy is to compare the ft values from a pair of mirror superallowed decays.

Accepting CVC is valid:
$$\mathcal{R} = ft(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \text{CONST}$$

Then, ratio of ft values for a pair of mirror superallowed transitions is
$$\frac{ft^a}{ft^b} = 1 + (\delta'^b_R - \delta'^a_R) + (\delta^b_{NS} - \delta^a_{NS}) - (\delta^b_C - \delta^a_C)$$

a: decay of the $T_Z = -1$ parent (need to be measured)
b: decay of the $T_Z = 0$ parent (well-known to better than ±0.1%)
Testing $\delta_C$ calculations by experiment

\[
\frac{ft^a}{ft^b} = 1 + (\delta'^b - \delta'^a) + (\delta^b_{NS} - \delta^a_{NS}) - (\delta^b_C - \delta^a_C)
\]

a: decay of the $T_Z = -1$ parent
b: decay of the $T_Z = 0$ parent

<table>
<thead>
<tr>
<th>$\text{A of mirror pairs}$</th>
<th>$\text{ft}^a/\text{ft}^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>SW</td>
</tr>
<tr>
<td>34</td>
<td>HF</td>
</tr>
<tr>
<td>38</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td></td>
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</table>

The figure shows the relative change in $ft$ between two decay pathways as a function of the atomic mass $A$ of mirror pairs. The equation provides a method to test $\delta_C$ calculations experimentally.
Testing $\delta_C$ calculations by experiment

\[ \frac{ft^a}{ft^b} = 1 + (\delta_R^b - \delta_R^a) + (\delta_{NS}^b - \delta_{NS}^a) - (\delta_C^b - \delta_C^a) \]

- **a:** decay of the $T_Z = -1$ parent
- **b:** decay of the $T_Z = 0$ parent

![Graph showing the ratio of $ft^a$ to $ft^b$ with data points for different nuclei.](Image)
Experimental set-up for branching-ratio measurement

**Momentum**

**Achromat**

**Recoil**

**Separator**

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$^1\text{H}(^{39}\text{K}, 2n)^{38}\text{Ca}$ at 30A MeV

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$t_{1/2}(^{38}\text{Ca}) = 443.77(35)\ text{ms}$

$T_{\text{collect}} = 1.6\ s$

$T_{\text{move}} = 0.169\ s$

$T_{\text{count}} = 1.54\ s$

---

Calibrated for absolute efficiency to ±0.2%. 
Challenges for the decay of $^{38}\text{Ca}$

Complex decays require direct branching-ratio measurements approaching ±0.1%.

Our approach: \( \text{BR}_F = 1 - \Sigma \text{BR}_{GT} \)

\[
R_i = \frac{N_{\beta\gamma_i}}{N_{\beta} \varepsilon_{\gamma_i}} \frac{\varepsilon_{\beta}}{\varepsilon_{\beta_i}} k
\]

\( \Sigma \text{BR}_{GT} \) precision ±(Y×10⁻³)/3

<table>
<thead>
<tr>
<th>Experimental corrections</th>
<th>±Y×10⁻³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real-coincidence summing</td>
<td>+2.6(3)%</td>
</tr>
<tr>
<td>Dead time + pile-up</td>
<td>+1.37(1)%</td>
</tr>
<tr>
<td>Energy dependence of (\beta)-detection efficiency</td>
<td>+0.38(4)%</td>
</tr>
<tr>
<td>(\gamma) detection in (\beta) detector</td>
<td>-0.043(4)%</td>
</tr>
</tbody>
</table>
Results for the beta-decay branching of $^{38}$Ca

$\Sigma BR_{GT} = 22.72(16)\%$

$BR_F = 77.28(16)\%$

This completes the data required for a precise $ft$-value result for $^{38}$Ca to the determination of $V_{ud}$.

$$ft^a(^{38}\text{Ca} \to ^{38m}\text{K}) = 3062.3 \pm 6.8 \text{ s}$$

$$ft^b(^{38m}\text{K} \to ^{38}\text{Ar}) = 3051.5 \pm 0.9 \text{ s}$$

$$ft^a/ft^b = 1.0036 \pm 0.0022$$

Since 2009

$Q_{EC}$
Summary

• The branching ratio for the superallowed transition of $^{38}$Ca has been measured to ±0.2% precision for the first time.

• This is the first addition to the set of well-known superallowed transitions in nearly a decade.

• Isospin-symmetry-breaking correction is experimentally tested by measurements of $^{38}$Ca.

• It can be further tested and improved by adding new transitions from $T_Z = -1$ parents with a higher experimental precision.
Collaborators at TAMU

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