An Introduction to the Ion-Optics of Magnet Spectrometers

U. Tokyo, RIKEN
The 14th RIBF Nuclear Physics Seminar

Series of Three Lectures
CNS, University of Tokyo
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The Lecture Series

1st Lecture: 2/27/06, 10:30 am: Formalism of ion-optics and design of a complete system

2nd Lecture: 2/27/06, 1:30 pm: Ion-optical elements, design, systems

3rd Lecture: 2/27/06, 3:00 pm: Experiments with dispersion matched high resolution spectrometers

3rd Lecture

3rd Lecture: 2/27/06, 3:00 pm:
Experiments with dispersion matched,
high resolution spectrometers

- Resolving power & resolution of a spectrometer
- A fully dispersion matched beam line/spectrometer
- Experiments with dispersion matched systems
- Dispersion matching for a secondary beam spectrometer (SHARAQ)

- Review of 1st & 2nd Lecture (4 – 6)
- Resolving power of a spectrometer(7)
- Dispersion matching (7 - 11)
- Dispersion matching & experiments with Grand Raiden (12 – 18)
- Dispersion matching with K600 and K > 0 (19 - 22)
- Dispersion matching for SHARAQ, achromatic analysis (23 – 27)
- Secondary beam and limits of dispersion matching (28)

Review 1st Lecture

Lorentz Force: $$\vec{F} = \underset{\text{Electric force}}{q\vec{E}} + \underset{\text{Magnetic force}}{q\vec{v}\ x\ \vec{B}} \qquad (1)$$

TRANSPORT of Ray X_0 $$X_n = R\,X_0 \qquad (3)$$

using Matrix R $$R = R_n\,R_{n-1}\,\ldots\,R_0 \qquad (4)$$

TRANSPORT of σ Matrix (Phase space ellipsoid) $$\sigma_1 = R\sigma_0\,R^T \qquad (10)$$

Beam emittance: $$\varepsilon = \sqrt{\sigma_{11}\sigma_{22} - (\sigma_{12})^2} \qquad (5)$$

Taylor expansion, higher orders, solving the equation of motion,
phases of a separator project

Review 2nd Lecture

Ampere's Law:

$$\text{NI (Ampere turns)} = \frac{\text{B (T)} * \text{g (m)}}{4\pi * 10^{-7}\ \text{(m/A)}} \qquad (17)$$

Properties and design of magnets: Dipoles, Quadrupoles, Hexapole, Octupoles

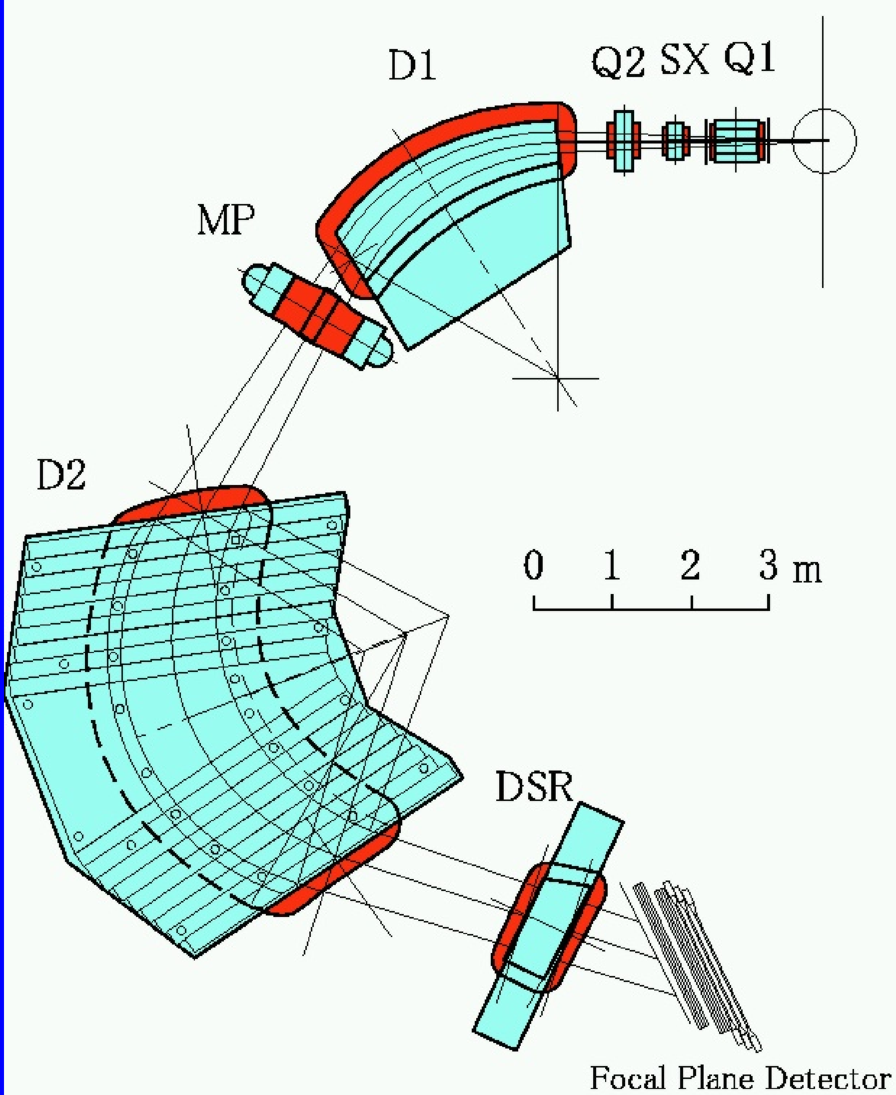
Ion-optics of magnet systems: Quadrupole triplet, Magnet spectrometers, Wien filter

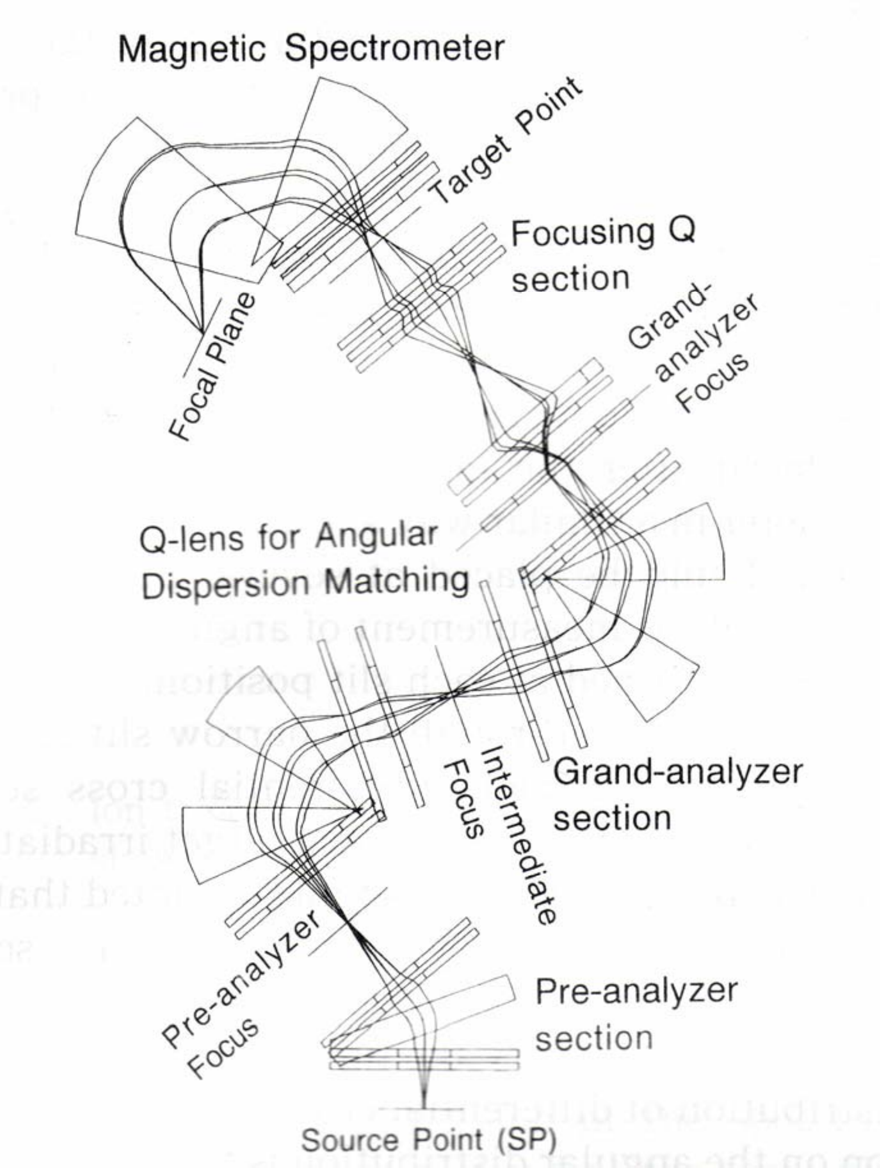
Diagnostics and field measurements:

Grand Raiden High Resolution Spectrometer

Max. Magn. Rigidity: 5.1 Tm
Bending Radius: 3.0 m
Solid Angle: 3 msr
Resolving power p/dp: 37000

Beam Line/Spectrometer fully matched





Spectrometer Design

(1^{st} Order Resolving Power)

Spectrometer Transfer Matrix S

Dispersion: $S_{16} = dx/(dp/p) = D$

Magnification: $S_{11} = dx(f.p.) / dx(tgt) = M$

Beam size: $2x_0$ (target, dispersive direction, monochromatic)

Resolving Power: $$R_p = \frac{p}{\Delta p} = \frac{D}{M * 2x_0} \qquad (22)$$

Note: R_p depends on x_0, if not given here $x_0 = 1$ mm

Note: **Resolving Power** is the "best possible 1^{st} order resolution a spectrometer can provide, disregarding higher order aberrations.

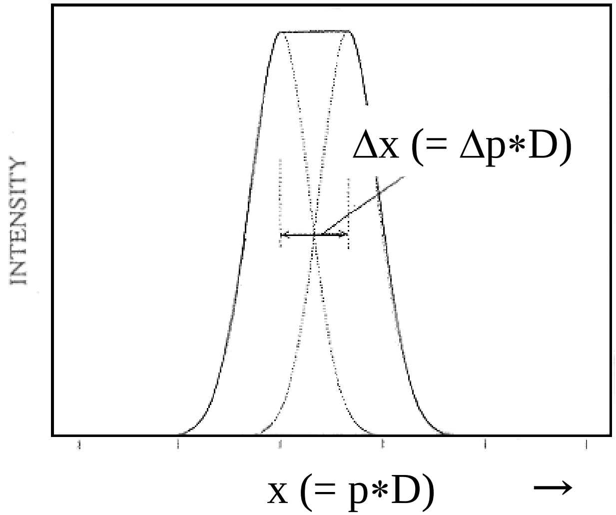
Resolution is what is measured in the Focal Plane.

Resolution is also affected (deteriorated) by:

Spectrometer aberrations, beam properties, target effects, detector resolution

Note: **"Resolution"** in Energy $R_E = \frac{E}{\Delta E} = 0.5 * R_p$

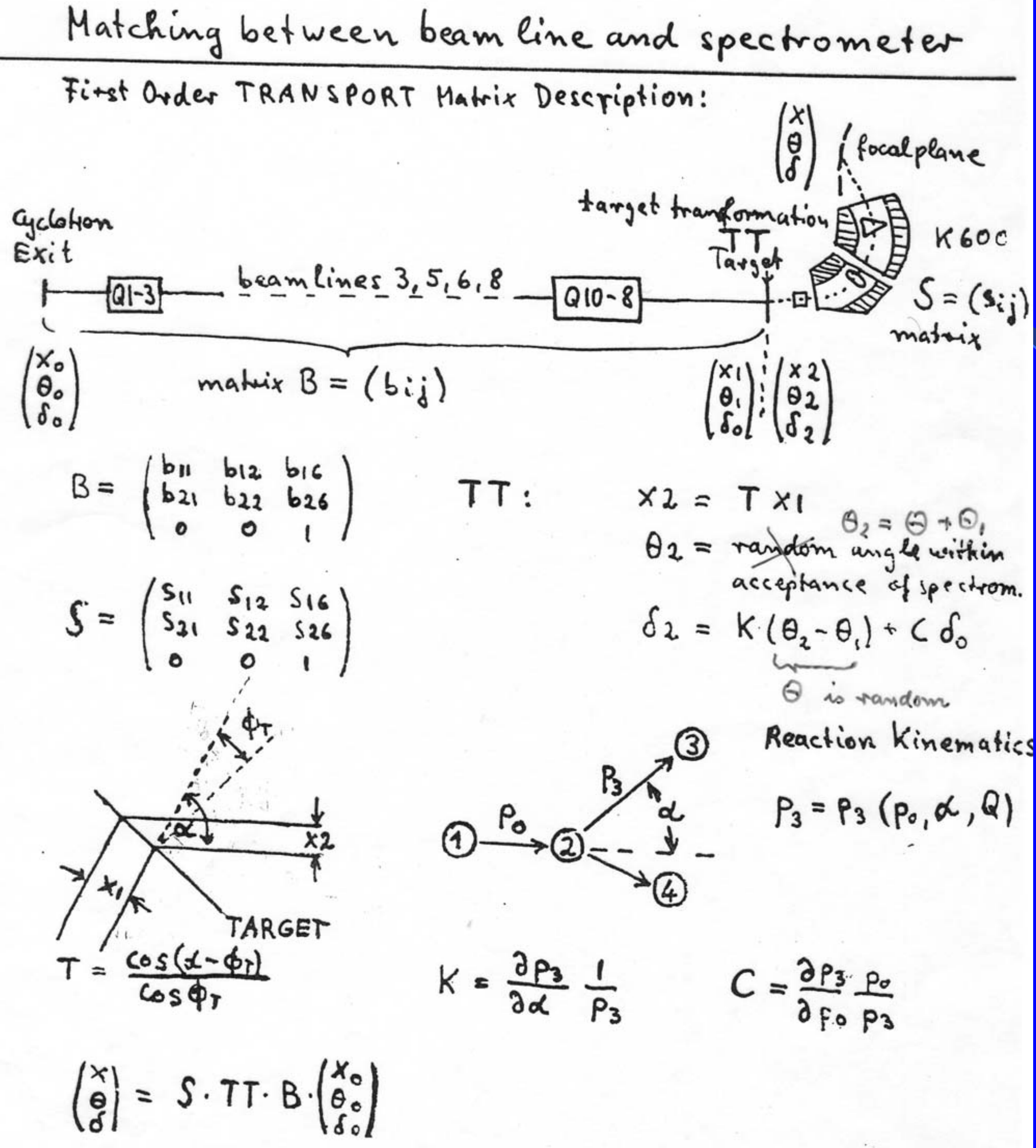
because $E = p^2/m$ (non-relativistic)



Peaks are "resolved" when Δx = FWHM

Dispersion Matching

- High resolution experiments
- Secondary beam (large dp/p)



Matching between beam line and spectrometer

First Order TRANSPORT Matrix Description:

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{16} \\ b_{21} & b_{22} & b_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{16} \\ s_{21} & s_{22} & s_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

TT: $x_2 = T\, x_1$

$\theta_2 = \Theta + \theta_1$

$\theta_2 =$ ~~random~~ angle within acceptance of spectrom.

$$\delta_2 = K \cdot \underbrace{(\theta_2 - \theta_1)}_{\Theta \text{ is random}} + C\,\delta_0$$

$$T = \frac{\cos(\alpha - \phi_T)}{\cos\phi_T}$$

Reaction Kinematics

$$P_3 = P_3(P_0, \alpha, Q)$$

$$K = \frac{\partial P_3}{\partial \alpha}\,\frac{1}{P_3} \qquad C = \frac{\partial P_3}{\partial P_0}\,\frac{P_0}{P_3}$$

$$\begin{pmatrix} x \\ \theta \\ \delta \end{pmatrix} = S \cdot TT \cdot B \cdot \begin{pmatrix} x_0 \\ \theta_0 \\ \delta_0 \end{pmatrix}$$

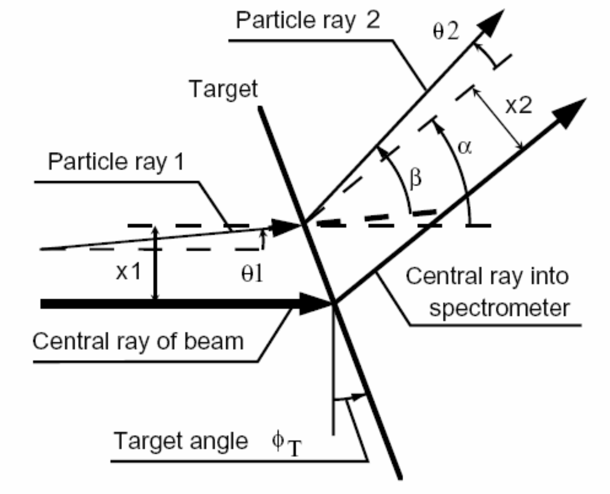
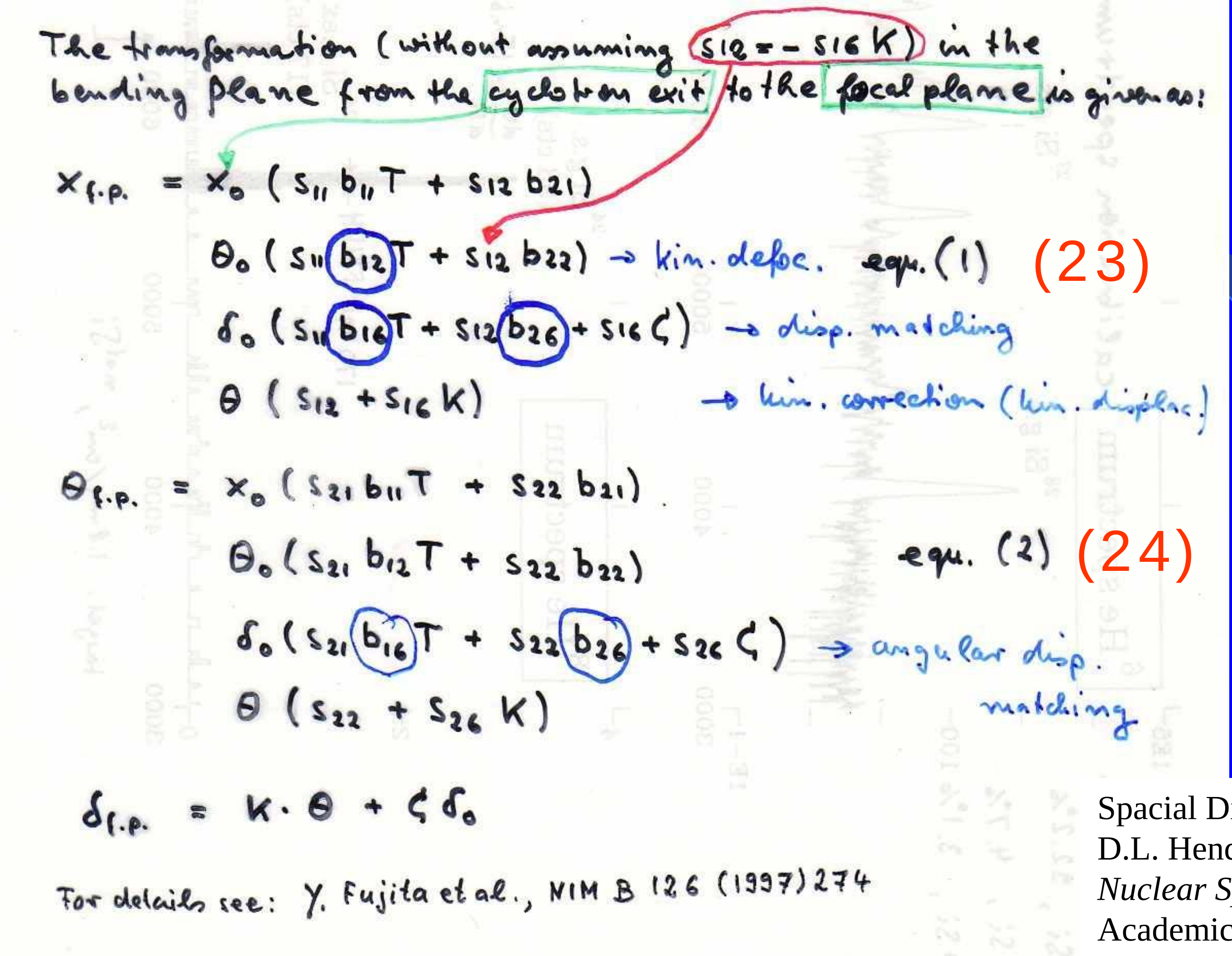


Fig. 1. Schematic layout of the incident particle 1 and the outgoing particle 2 relative to the beam and spectrometer.

Solution of first order Transport and Complete Matching



The transformation (without assuming $s_{12} = -s_{16}K$) in the bending plane from the cyclotron exit to the focal plane is given as:

$$x_{f.p.} = x_0 (s_{11} b_{11} T + s_{12} b_{21})$$

$$\theta_0 (s_{11} \boxed{b_{12}} T + s_{12} b_{22})$$ → kin. defoc. equ. (1) (23)

$$\delta_0 (s_{11} \boxed{b_{16}} T + s_{12} \boxed{b_{26}} + s_{16} C)$$ → disp. matching

$$\theta (s_{12} + s_{16} K)$$ → kin. correction (kin. displac.)

$$\theta_{f.p.} = x_0 (s_{21} b_{11} T + s_{22} b_{21})$$

$$\theta_0 (s_{21} b_{12} T + s_{22} b_{22})$$ equ. (2) (24)

$$\delta_0 (s_{21} \boxed{b_{16}} T + s_{22} \boxed{b_{26}} + s_{26} C)$$ → angular disp. matching

$$\theta (s_{22} + s_{26} K)$$

$$\delta_{f.p.} = K \cdot \theta + C \delta_0$$

For details see: Y. Fujita et al., NIM B 126 (1997) 274

Complete Matching

For best **Resolution** in the focal plane, minimize the coefficients of all terms in the expression of **x f.p.**

For best **Angle Resolution** Minimize Coefficients of δ_0 in expression of Y **f.p.**

Note: Also the beam focus **b12** on target is important (**b12** = 0 for kinem. k = 0)

Spacial Dispersion Matching:
D.L. Hendrie In: J. Cerny, Editor,
Nuclear Spectroscopy and Reactions, Part A,
Academic Press, New York (1974), p. 365.

Hendrie, Dispersion Matching $b_{16} = -\frac{D}{M} * \frac{C}{T}$ (23')

D = s_{16} = Spectrometer dispersion

M = s_{11} = Spectrometer magnification

Spacial and Angular Dispersion Matching

Solutions for b_{16} and b_{26} under conditions that both δ_0-coefficients = 0 in (23) and (24)

$$s_{11}\, b_{16}\, T + s_{12}\, b_{26} + s_{16}\, C = 0$$

$$s_{21}\, b_{16}\, T + s_{22}\, b_{26} + s_{26}\, C = 0$$

Solutions:

$$b_{16} = -\frac{s_{16}}{s_{11}}(1 + s_{11}\, s_{26}\, K - s_{21}\, s_{16} K)\,\frac{C}{T} \quad (25)$$

Spacial Dispersion Matching

$$b_{26} = (s_{21}\, s_{16} + s_{11}\, s_{26})\, C \quad (26)$$

Angular Dispersion Matching

$$b_{12} = -\frac{s_{12}\, b_{22}}{s_{11}\, T} = \frac{s_{16}\, b_{22}\, K}{s_{11}\, T} \quad (27)$$

Focusing Condition

Spacial and Angular Dispersion Matching

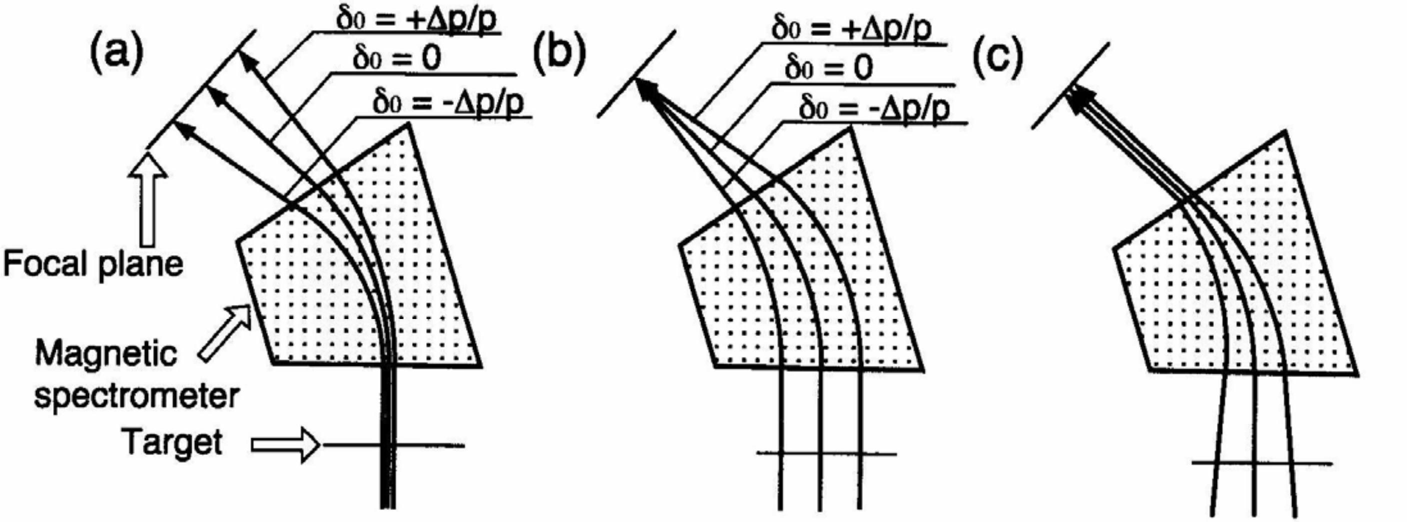
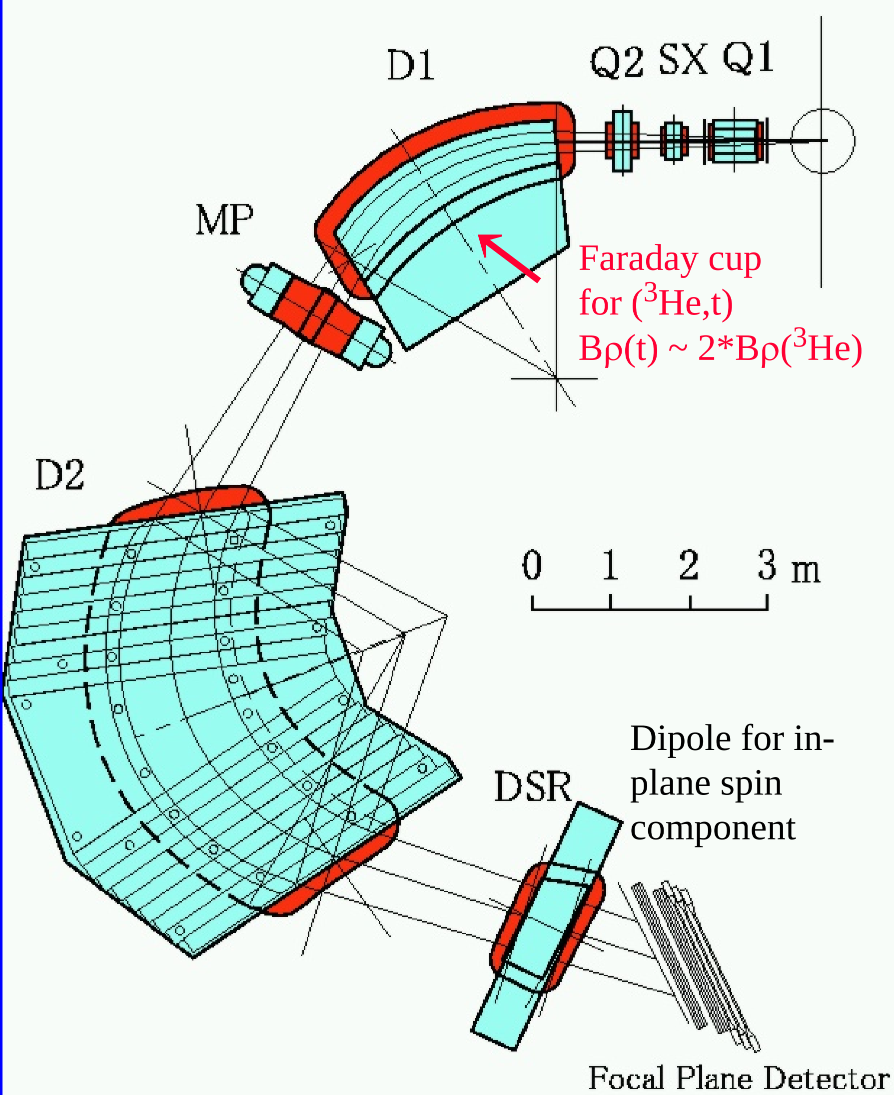


Figure 2.2: Schematic ion trajectories under different matching conditions of a beam line

Grand Raiden High Resolution Spectrometer

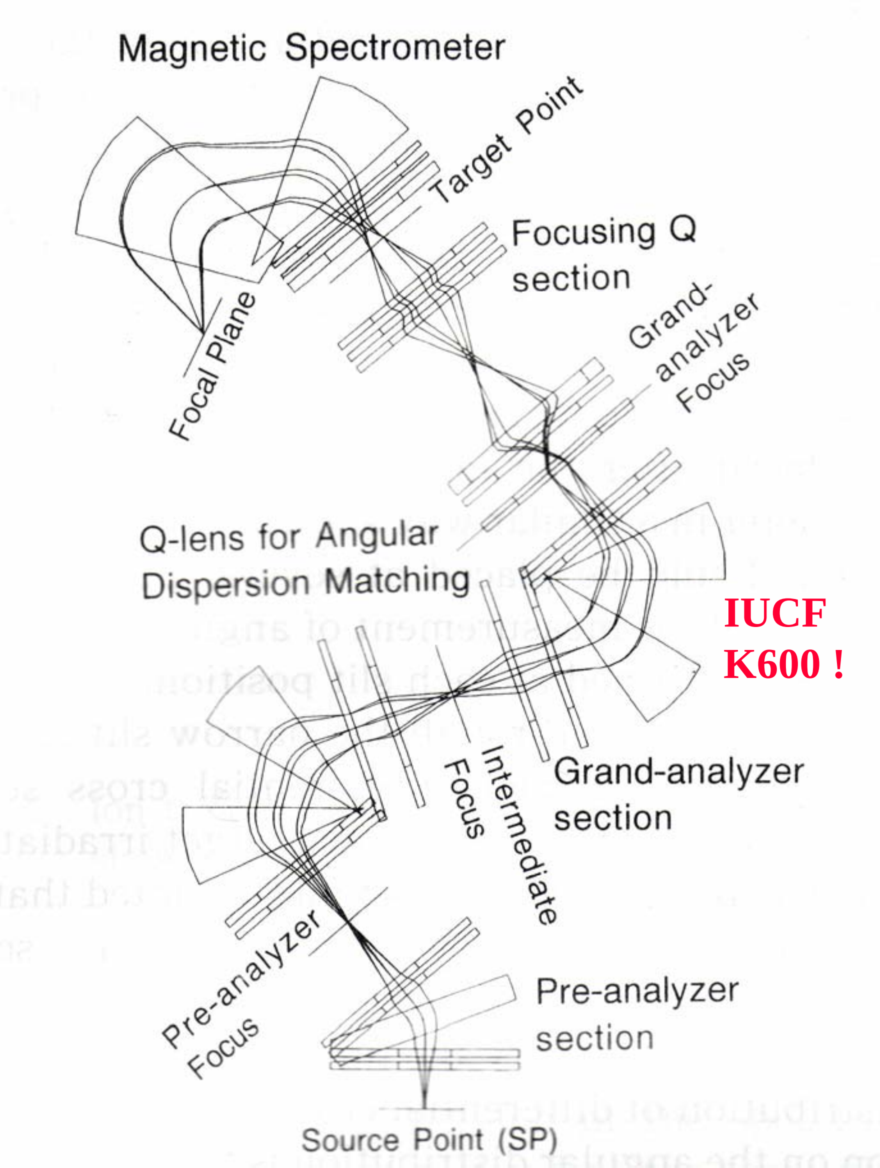
Max. Magn. Rigidity:	5.1 Tm
Bending Radius ρ_0:	3.0 m
Solid Angle:	3 msr
Resolv. Power p/dp	37000



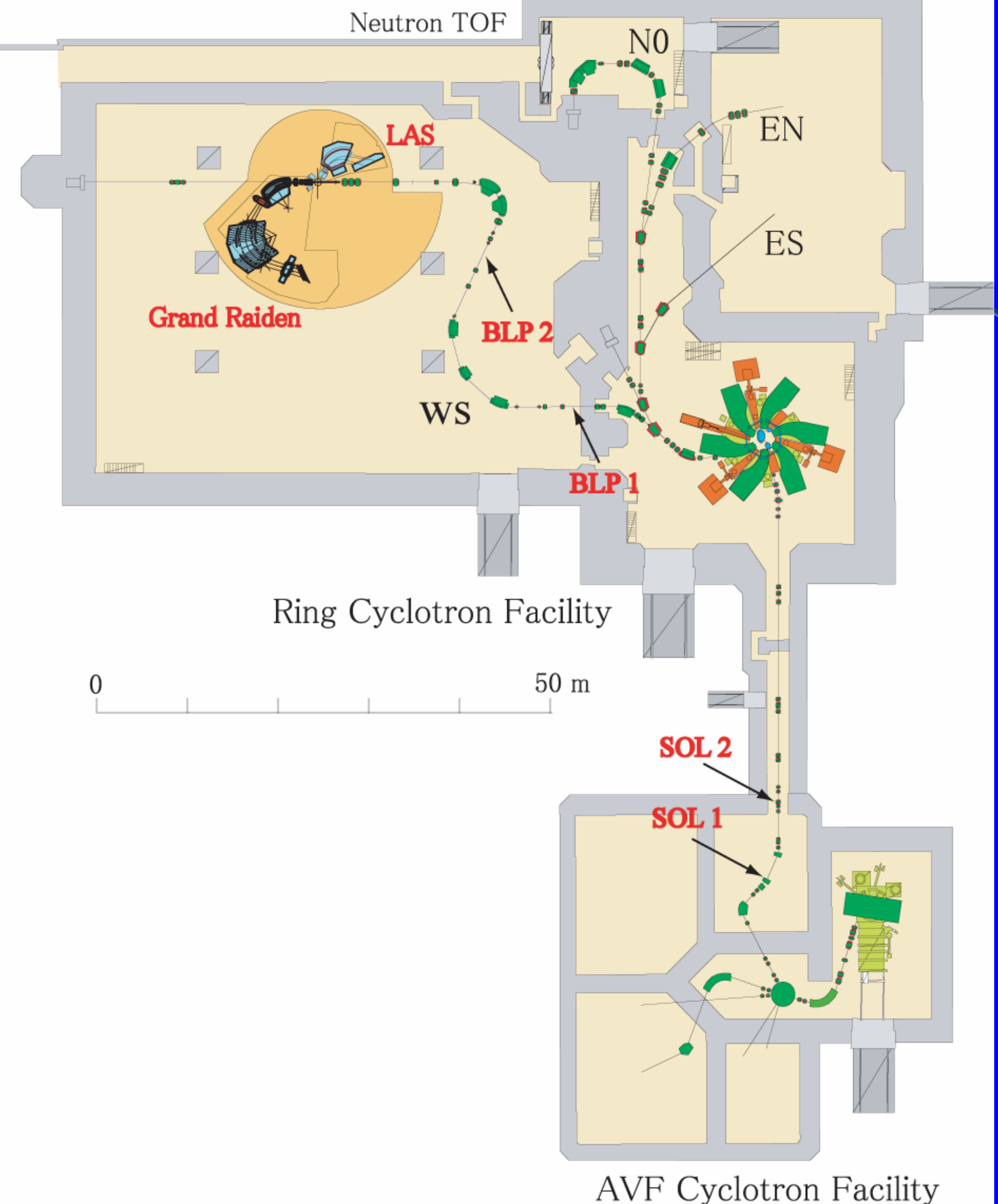
Faraday cup for (^{3}He,t) $B\rho(t) \sim 2{*}B\rho(^3He)$

Dipole for in-plane spin component

Beam Line/Spectrometer fully matched



IUCF K600 !



RCNP Facility Layout Osaka, Japan

$D = S_{16} = 17$ cm/% = 17 m

$M = S_{11} \sim -0.45$

Dispersion on target:

$B_{16} = D/M = -37$ m

Resolving power:

$2x_0 = 1$ mm

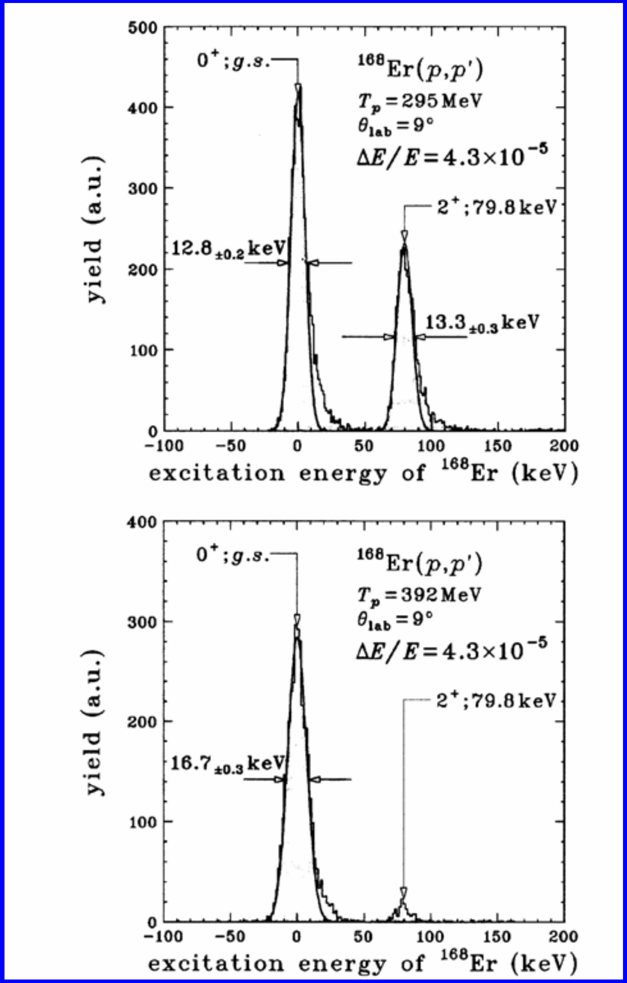
$R = p/\Delta p = 37000$

Dispersion matched beam line WS to the high resolution spectrometer Grand Raiden

Momentum and Angular Resolution

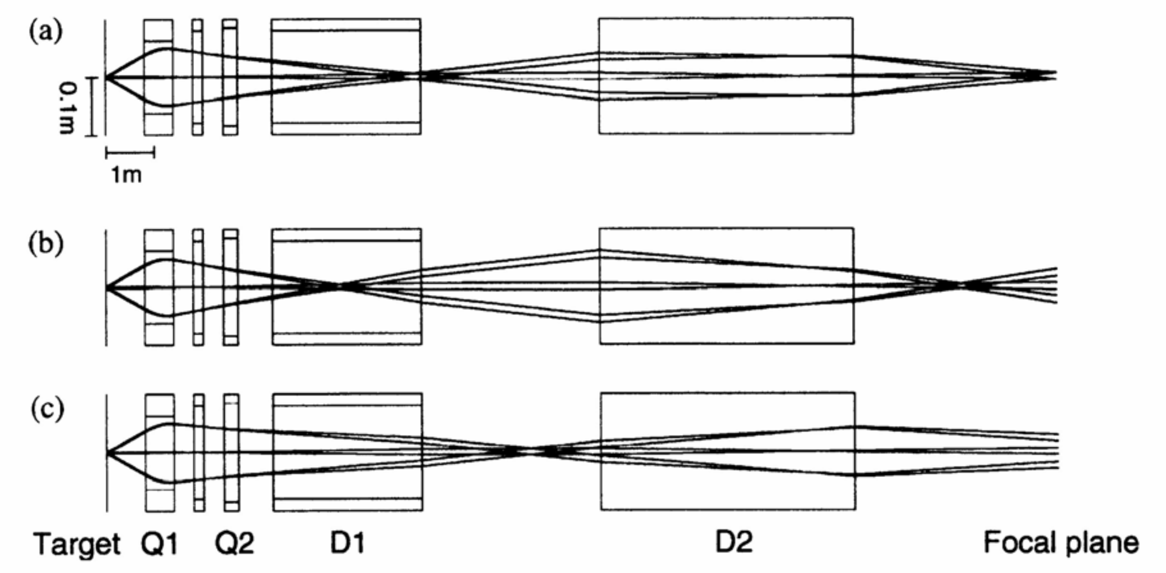
Spacial & Angular Dispersion Matching & Focus Condition allows

Energy Resolution: $E/\Delta E=23000$, $\Delta p/p = 40000$, despite beam spread: $E/\Delta E = 1700 - 2500$



Angular resolution: ΔY_{scatt} = SQRT($\Delta Y^2_{hor}+\Delta\Phi^2$) = 4 - 8 msr

At angles close to beam (e.g. 0 deg) vert. angle component is needed → Overfocus mode, small target dimension, because (y|y) is large,
Limitation: multiple scattering in detector

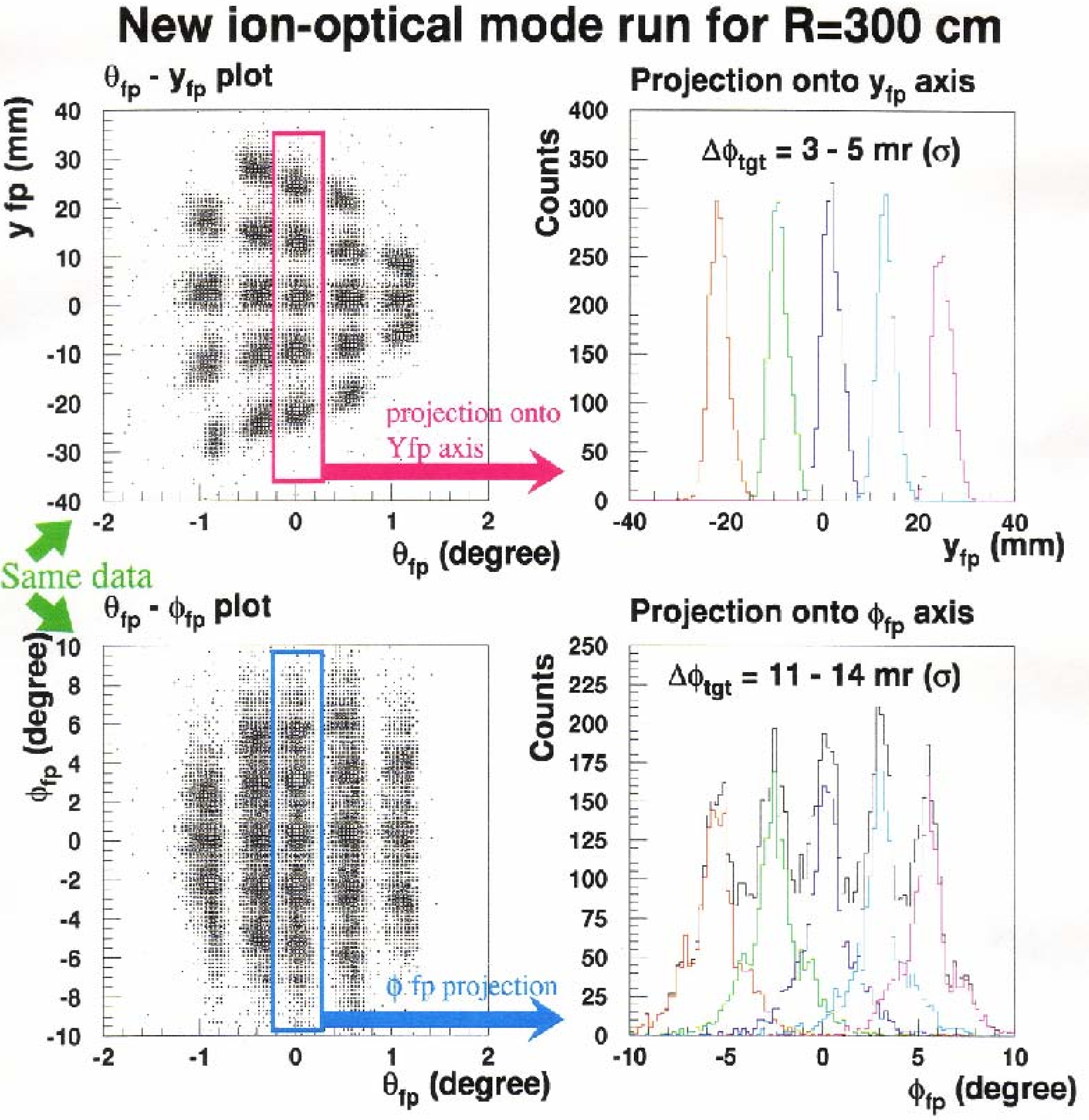


Refs.: Y.Fujita et al, NIM B126(1997)274, H.Fujita et al. NIM A 469(2001)55, T.Wakasa et al, NIM A482(2002)79

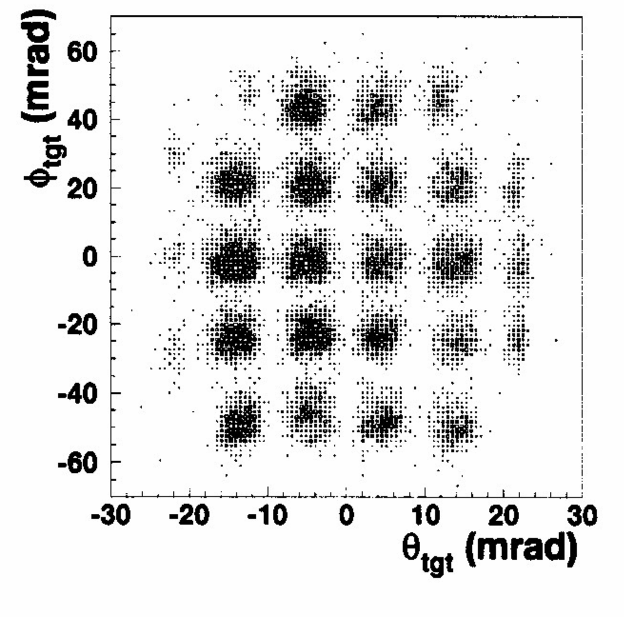
Grand Raiden Angle Calibration

Data suggest: Use y_{fp} not Φ_{fp} to calibrate angle!

Over-focus mode (b)



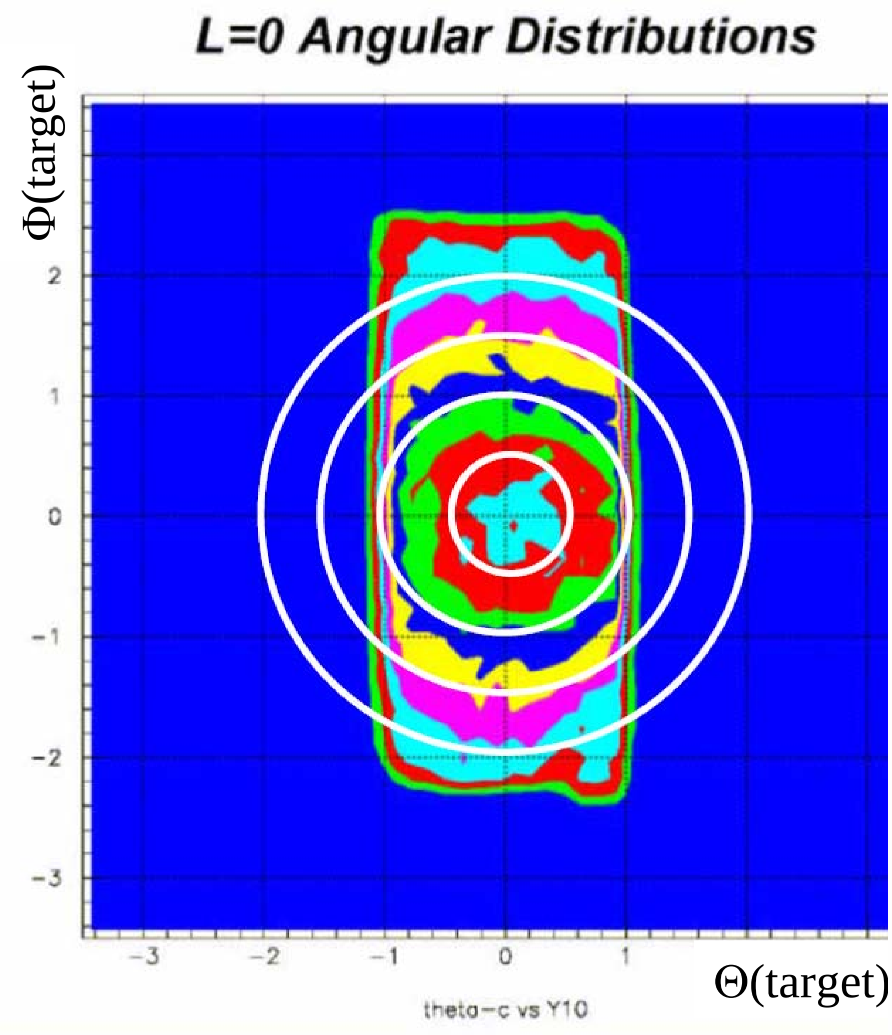
Calibrated!



Scattering Angle

reconstructed from focal plane measurements
using complete dispersion matching techniques

E(^{3}He) = 420 MeV



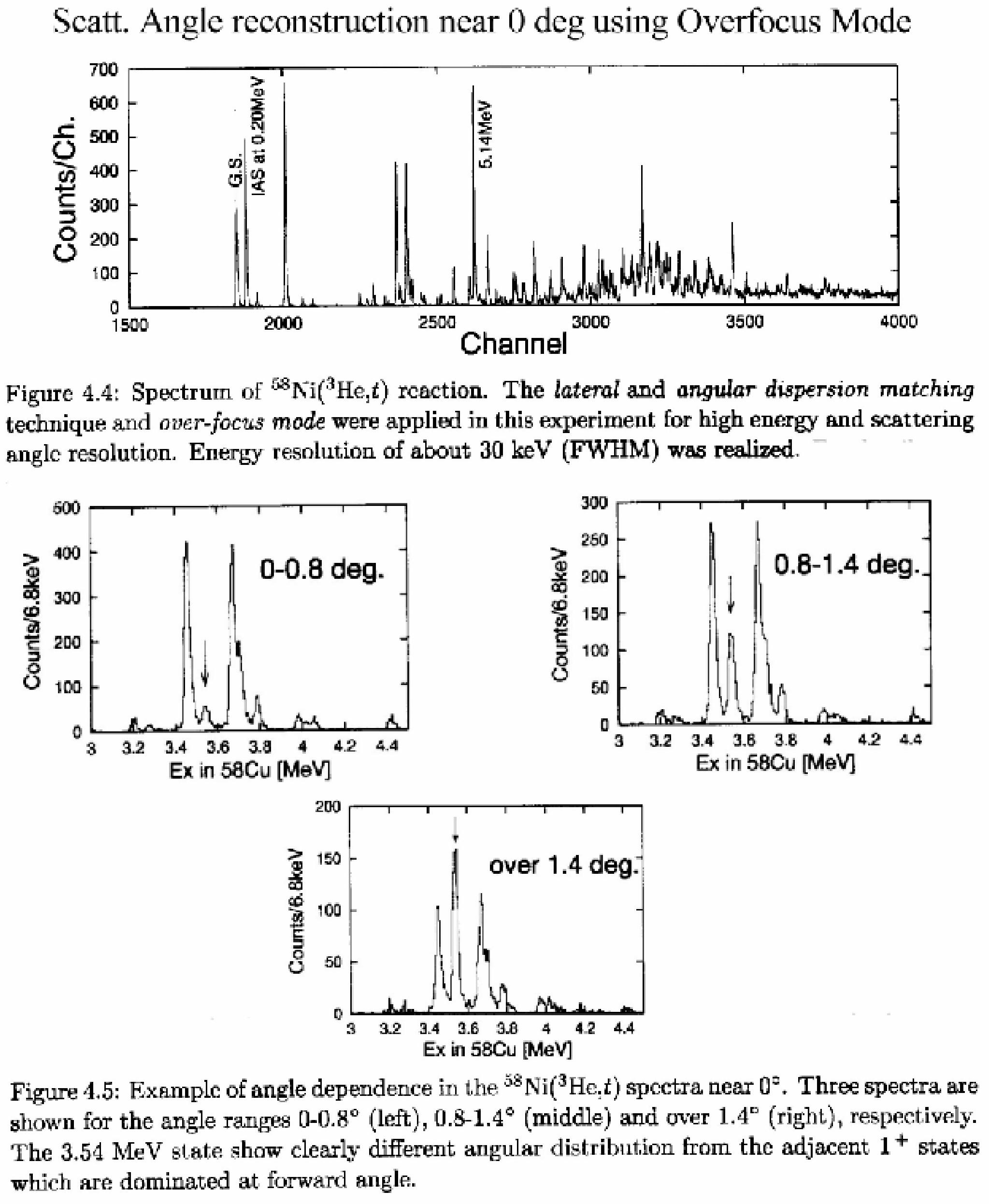

Scatt. Angle reconstruction near 0 deg using Overfocus Mode

Figure 4.4: Spectrum of ^{58}Ni(^{3}He,t) reaction. The *lateral* and *angular dispersion matching* technique and *over-focus mode* were applied in this experiment for high energy and scattering angle resolution. Energy resolution of about 30 keV (FWHM) was realized.

Figure 4.5: Example of angle dependence in the ^{58}Ni(^{3}He,t) spectra near 0°. Three spectra are shown for the angle ranges 0-0.8° (left), 0.8-1.4° (middle) and over 1.4° (right), respectively. The 3.54 MeV state show clearly different angular distribution from the adjacent 1^+ states which are dominated at forward angle.

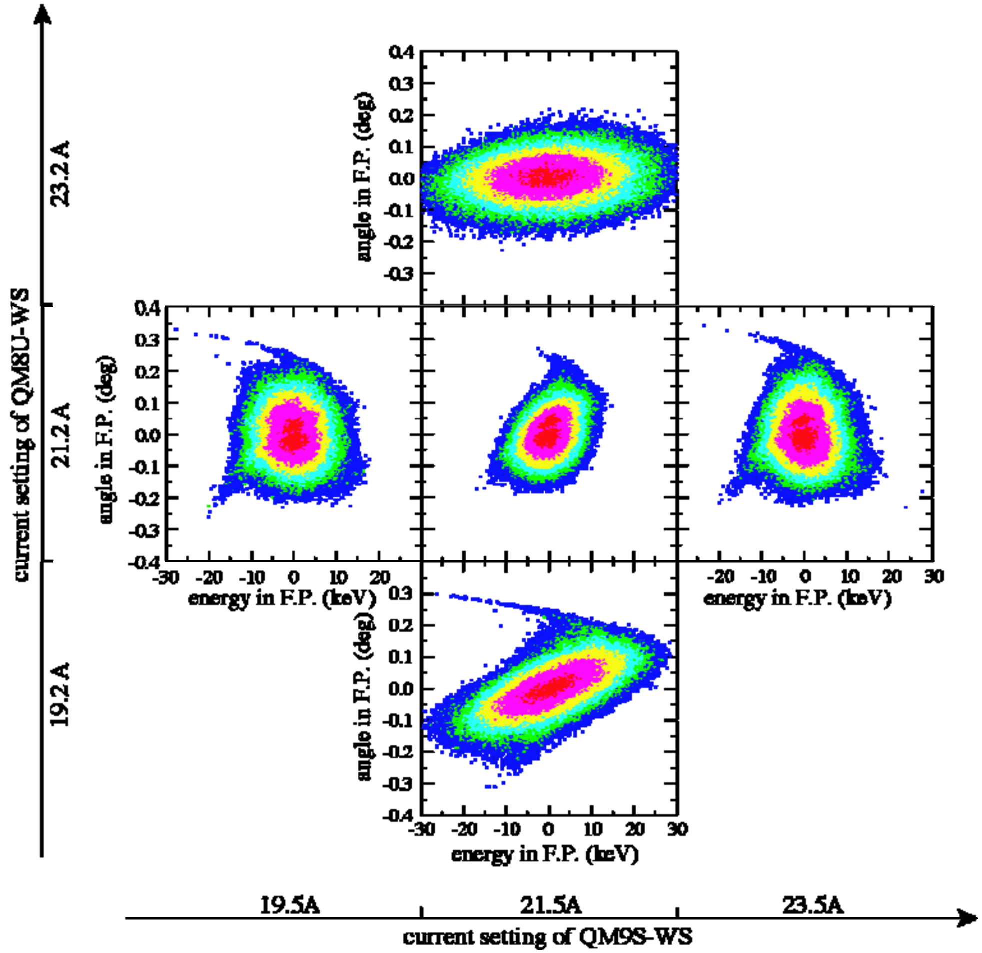
Horizontal Beam Profiles in the Focal Plane of Grand Raiden

- QM8U
  →Control lateral
  dispersion
- QM9S
  →Control angular
  dispersion
- Lateral and angular dispersions can be controlled independently
- References

  H. Fujita et al., NIMA

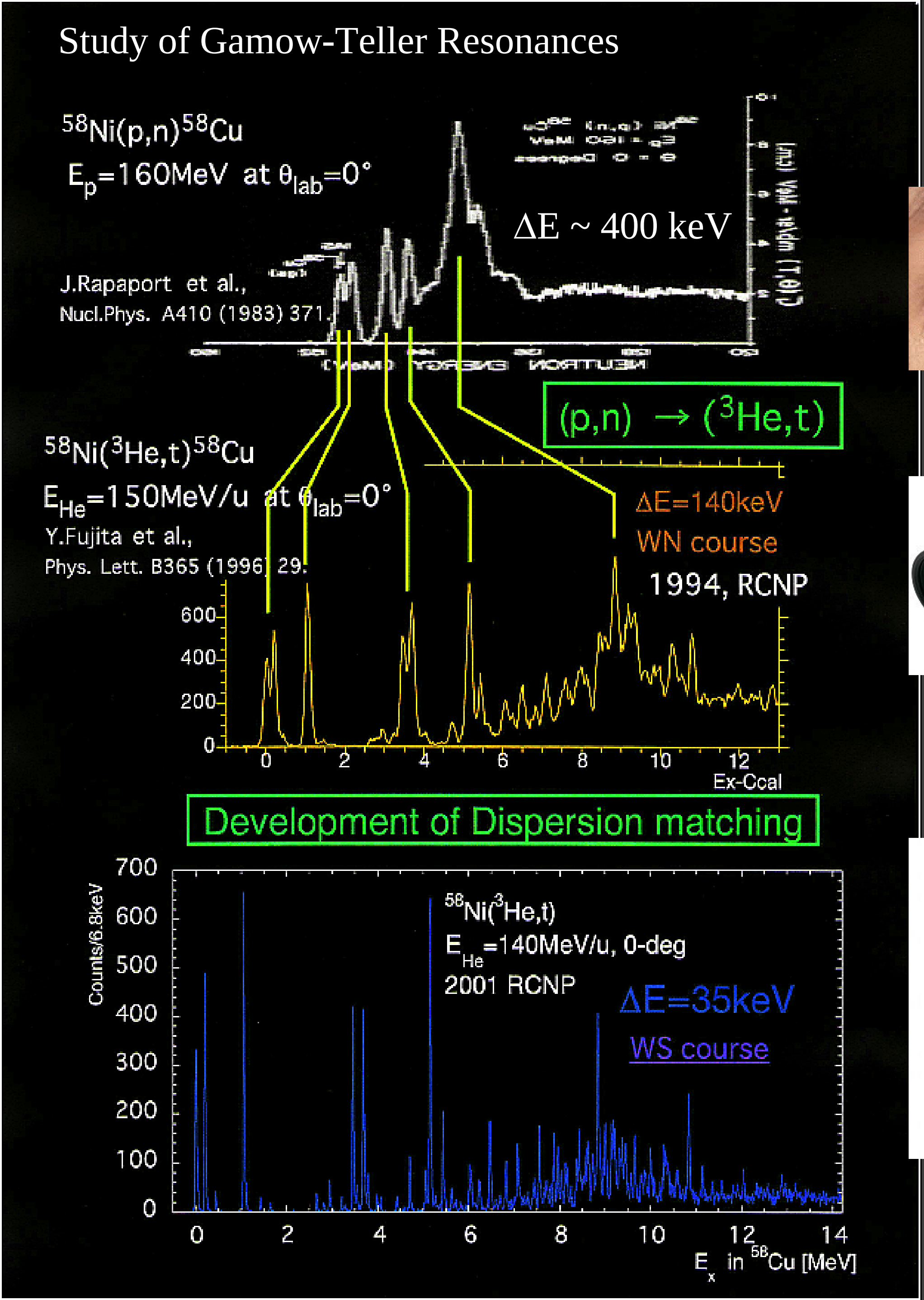
  T. Wakasa et al., NIMA

Dispersion matching for K = 0 with faint beam



Effect of Dispersion Matching

(Optical Resolution compared)



Study of Gamow-Teller Resonances

^{58}Ni(p,n)^{58}Cu

E_p=160MeV at θ_{lab}=0°

J.Rapaport et al.,
Nucl.Phys. A410 (1983) 371.

ΔE ~ 400 keV

(p,n) → (^{3}He,t)

^{58}Ni(^{3}He,t)^{58}Cu

E_{He}=150MeV/u at θ_{lab}=0°

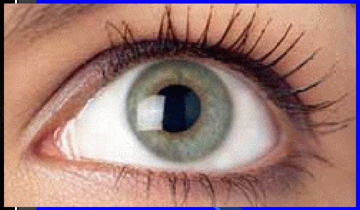
Y.Fujita et al.,
Phys. Lett. B365 (1996) 29.

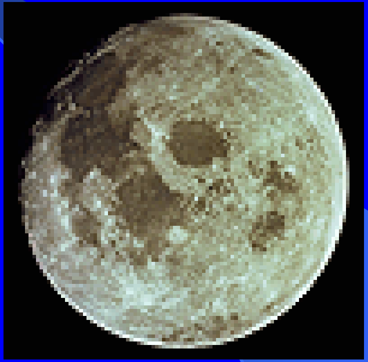
ΔE=140keV
WN course
1994, RCNP

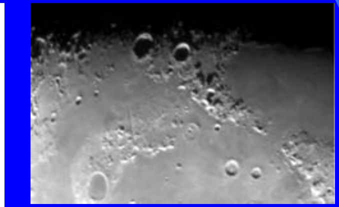
Development of Dispersion matching

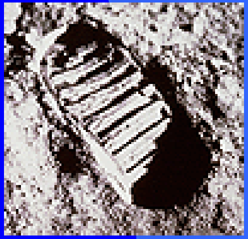
^{58}Ni(^{3}He,t)
E_{He}=140MeV/u, 0-deg
2001 RCNP

ΔE=35keV
WS course







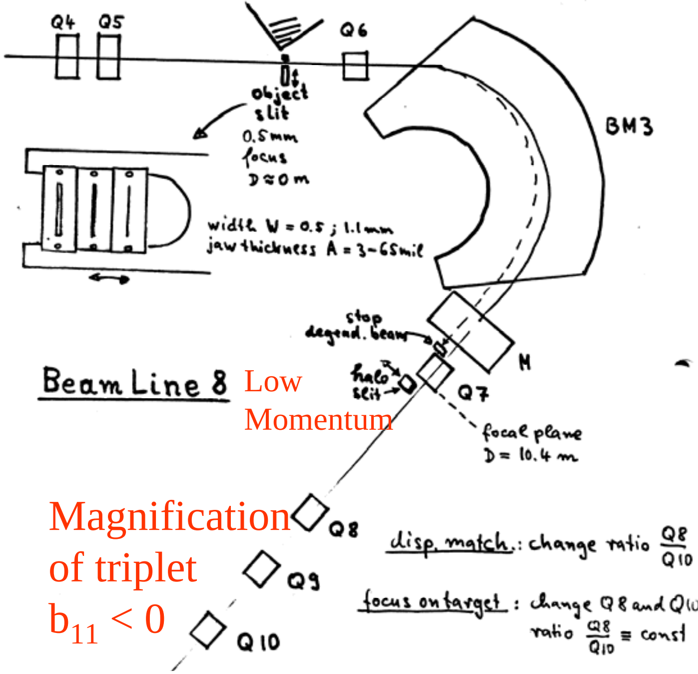


Where is the limit?

IUCF K600, dispersion matched beam line

- Thin-slit method (object 0.5 mm)
- Q7 for angular dispersion matching
- Triplet Q8-Q9-Q10 for disp. matching and focus

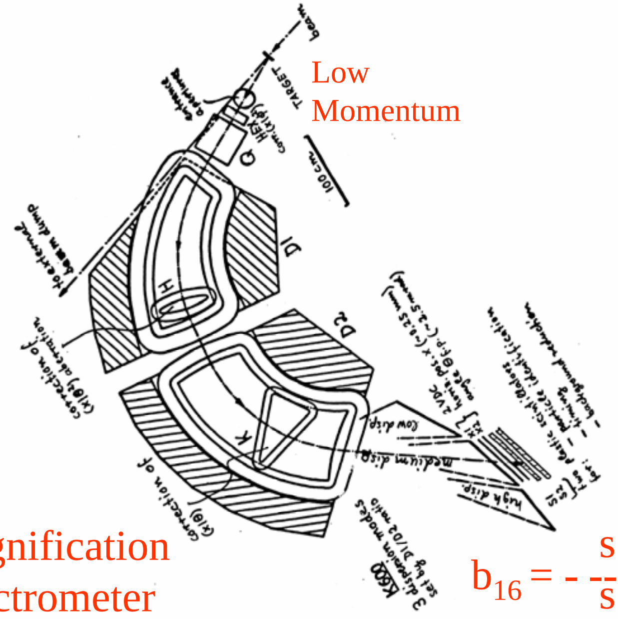
Resolving Power: $R_p = \frac{p}{\Delta p} = \frac{D}{M * 2x_0}$ (22)



Low Momentum

Magnification of triplet

$b_{11} < 0$



Low Momentum

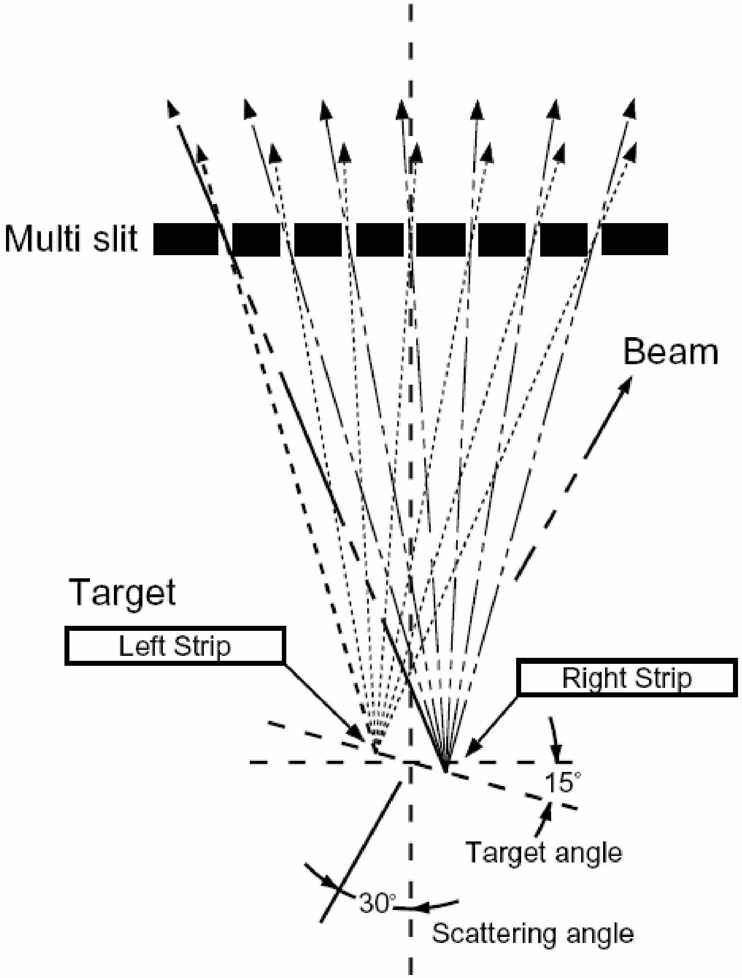
Magnification Spectrometer

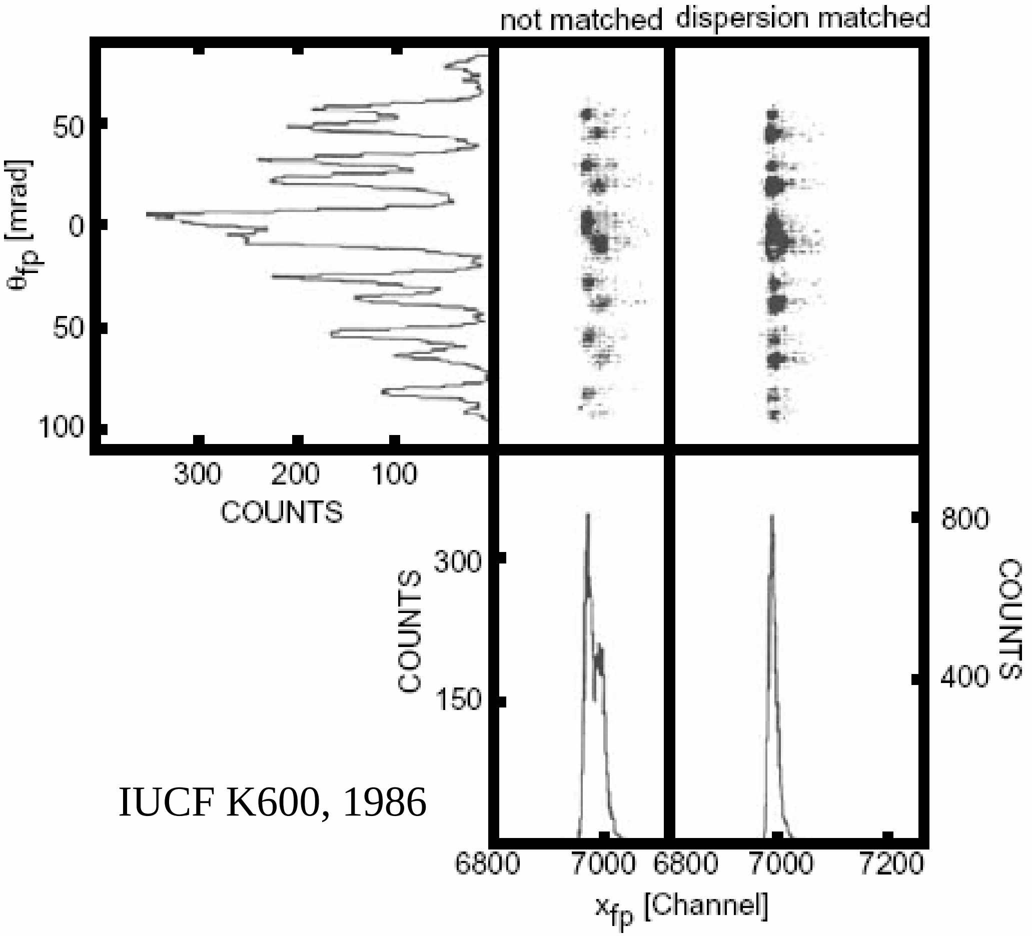
$s_{11} < 0$

$b_{16} = -\frac{s_{16}}{s_{11}}$

Diagnostic of Dispersion Matching (K > 0)

of beam line & spectrometer using a double strip target & multi slit

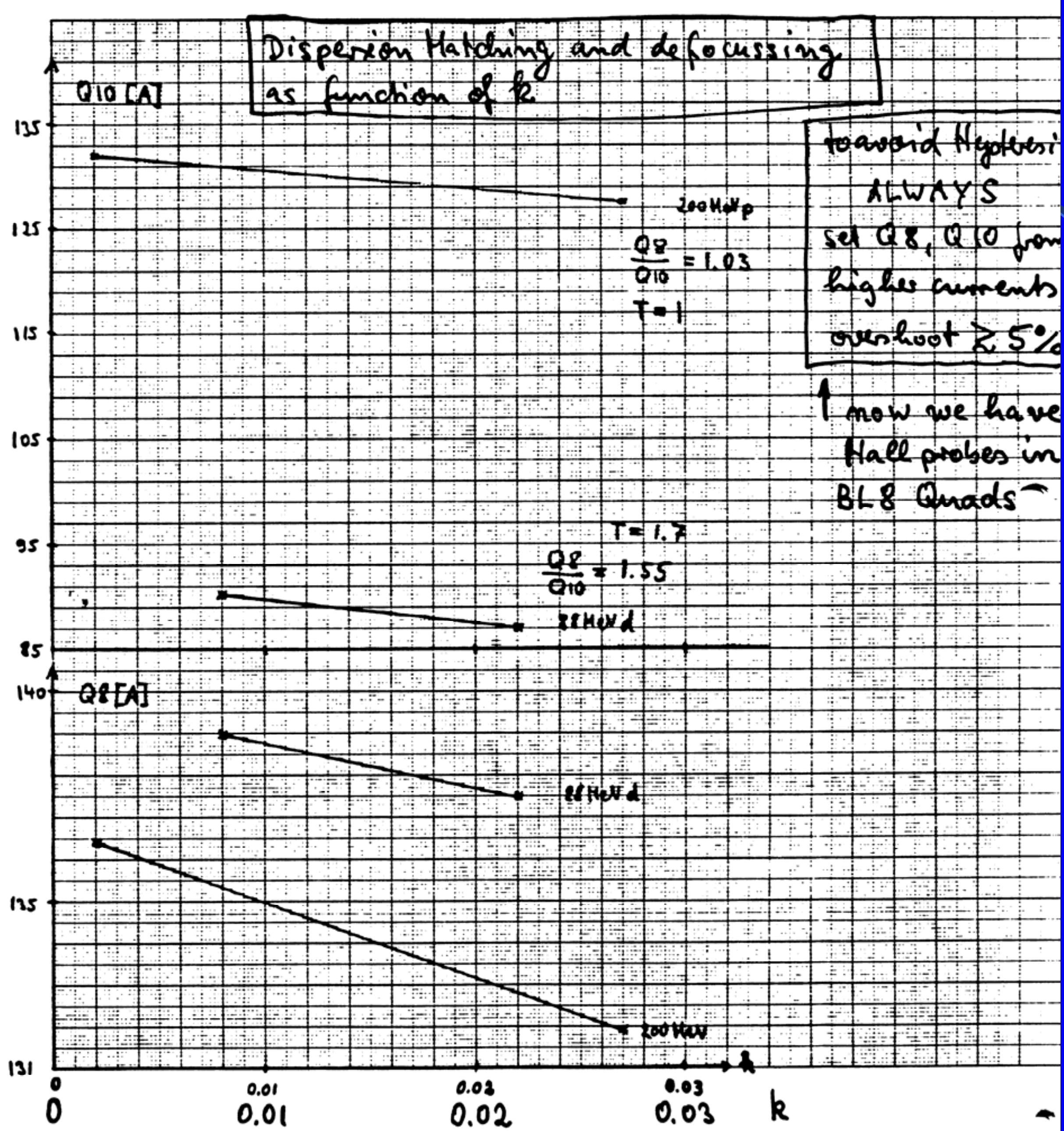




IUCF K600, 1986

Fig. 4. Scatterplots of horizontal position x_{fp} versus angle θ_{fp} and projections measured in the focal plane of the K600 using the "multi-slit system". For details, see text.

Dispersion Matching for K > 0



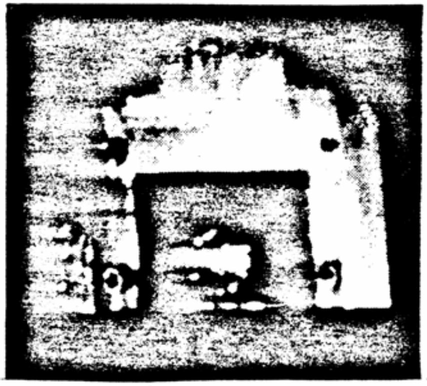
Dispersion Matching and defocussing as function of k

to avoid Hysteresi
ALWAYS
set Q8, Q10 from
higher currents
overshoot ≳ 5%

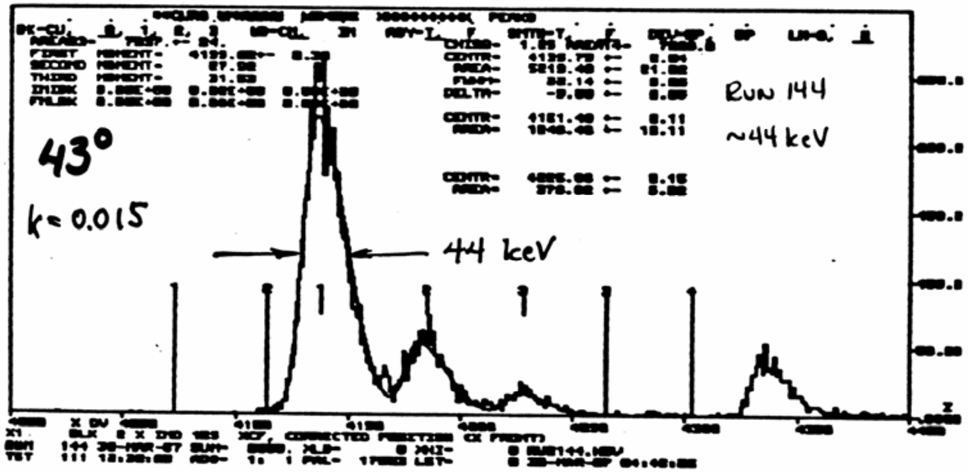
↑ now we have
Hall probes in
BL8 Quads

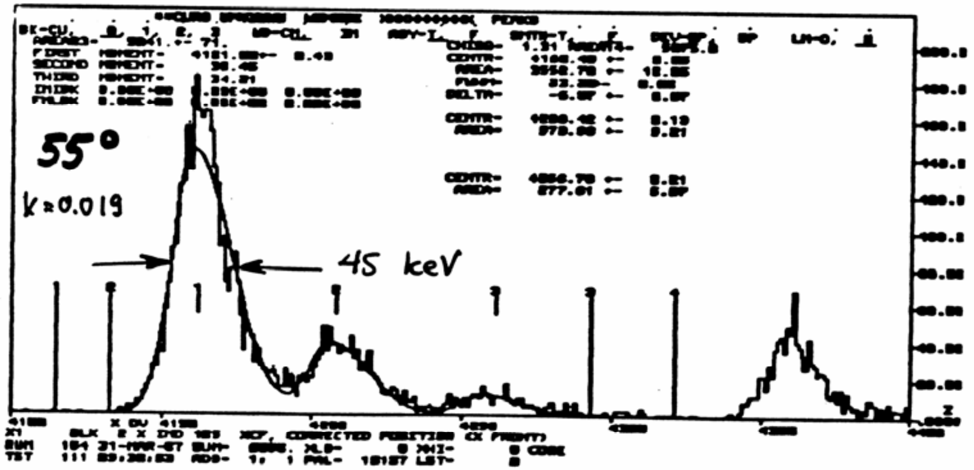
Matched spectra
K600 IUCF

$^{66}\mathrm{Zn}(d,p)^{67}\mathrm{Zn}$
88 MeV



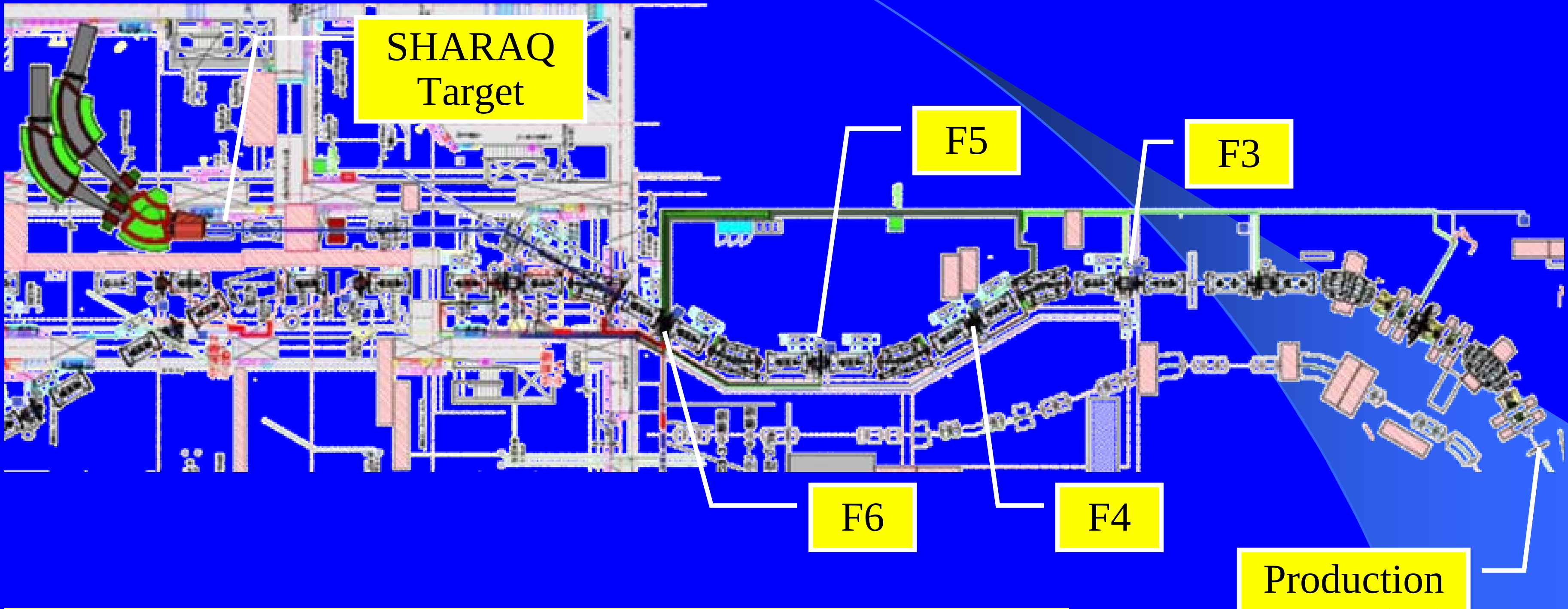
CORRECT DISPERSION
+
FOCUS DOWNSTREAM





Beam Line Layout (under revision)

SHARAQ



SHARAQ Target

F5

F3

F6

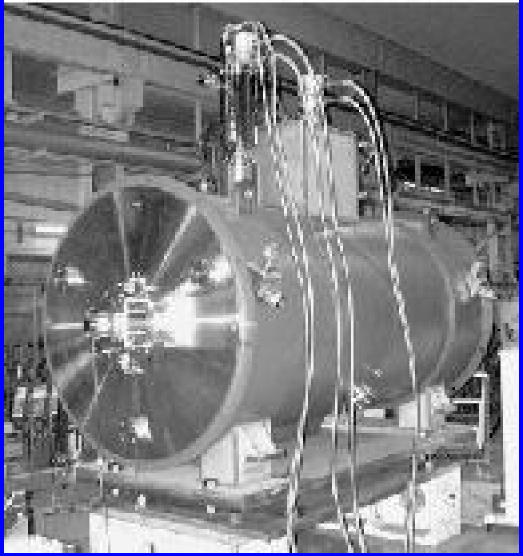
F4

Production Target (F0)

- Beam Line is shared with BigRIPS up to F6.
  - No layout freedom from F0 to F6.
- Geometrical Limitation is very tight.
  - No layout freedom for the target position.

Beam Line Elements

Superconducting Triplet Quadrupole Magnet (STQ)



Pole length (mm)	500-800-500 500-1000-500
Pole tip radius (mm)	170
Warm bore radius (mm)	140
Max. field gradient (T/m)	14.1

One hexapole coil is implemented per a STQ.

Normal conducting Dipole Magnet

Pole gap (cm)	12
Bending angle (degree)	30
Mean orbit radius (m)	6
Magnetic rigidity (Tm)	9

Matching Condition for SHARAQ Beam line

by T. Kawabata

B_{ij}: Transport Matrix for Beam Line, s_{ij}: Transport Matrix for the Spectrometer

$$\begin{pmatrix} x_{\mathrm{fp}} \\ \theta_{\mathrm{fp}} \\ \delta_{\mathrm{fp}} \end{pmatrix} = \begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ \theta_0 \\ \delta_0 \end{pmatrix}$$

$$\begin{aligned} x_{\mathrm{fp}} &= \left(s_{11}b_{11} + s_{12}b_{21}\right) x_0 \\ &\quad + \left(s_{11}b_{12} + s_{12}b_{22}\right) \theta_0 \\ &\quad + \left(s_{11}b_{13} + s_{12}b_{23} + s_{13}\right) \delta_0 \end{aligned}$$

$$\begin{aligned} \theta_{\mathrm{fp}} &= \left(s_{21}b_{11} + s_{22}b_{21}\right) x_0 \\ &\quad + \left(s_{21}b_{12} + s_{22}b_{22}\right) \theta_0 \\ &\quad + \left(s_{21}b_{13} + s_{22}b_{23} + s_{23}\right) \delta_0 \end{aligned}$$

Dispersion Matching Condition

$$s_{11}b_{13} + s_{12}b_{23} + s_{13} = 0$$

Angular Matching Condition

$$s_{21}b_{13} + s_{22}b_{23} + s_{23} = 0$$

SHARAQ Spectrometer

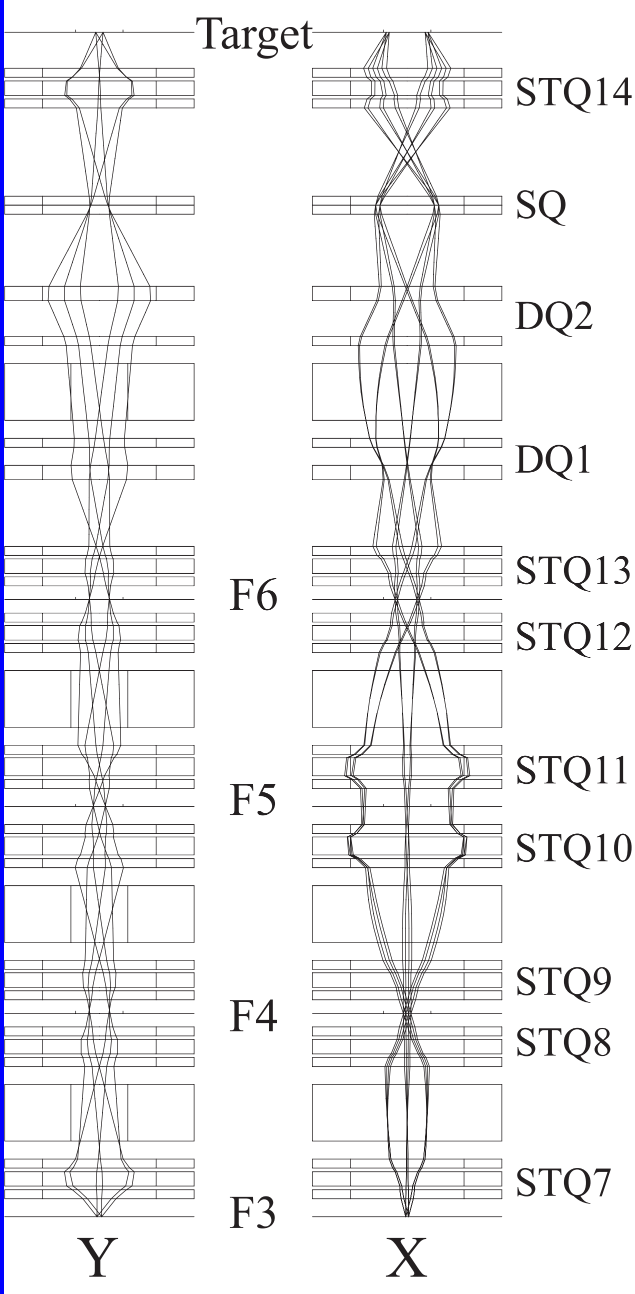
$$\begin{array}{lll} s_{11} = 0.423 & s_{12} = 0.000 & s_{13} = -5.781 \\ s_{21} = 0.744 & s_{22} = 2.364 & s_{23} = 0.557 \end{array}$$

→

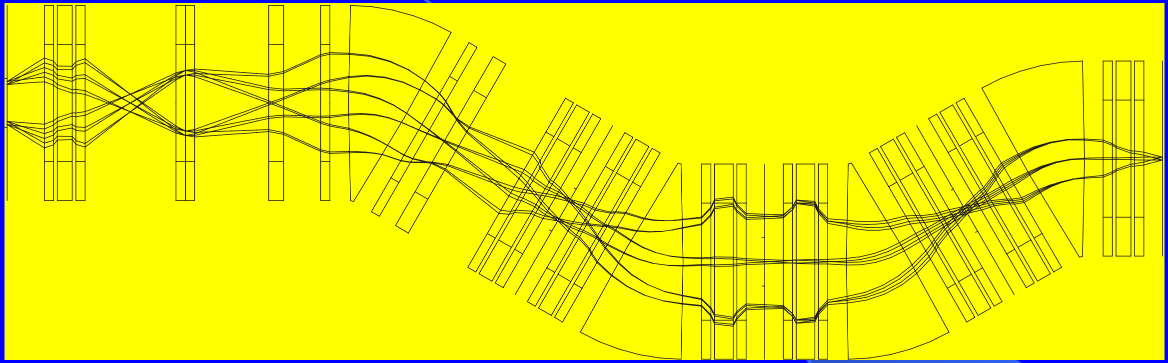
Matching Condition

$$b_{13} = 13.663$$
$$b_{23} = -4.534$$

From F3 to SHARAQ Target , GIOS calculations by T. Kawabata



$\Delta\theta_x$= +/– 10 mr, $\Delta\theta_y$= +/– 30 mr,
Δx = +/– 3 mm, Δy = +/– 5 mm, ΔP= +/– 0.3 %



$$\langle x'|x\rangle = -1.03 \quad \langle x'|\theta\rangle = 0.00 \quad \langle x'|\delta\rangle = -13.66$$

$$\langle \theta'|x\rangle = 0.32 \quad \langle \theta'|\theta\rangle = -0.97 \quad \langle \theta'|\delta\rangle = 4.53$$

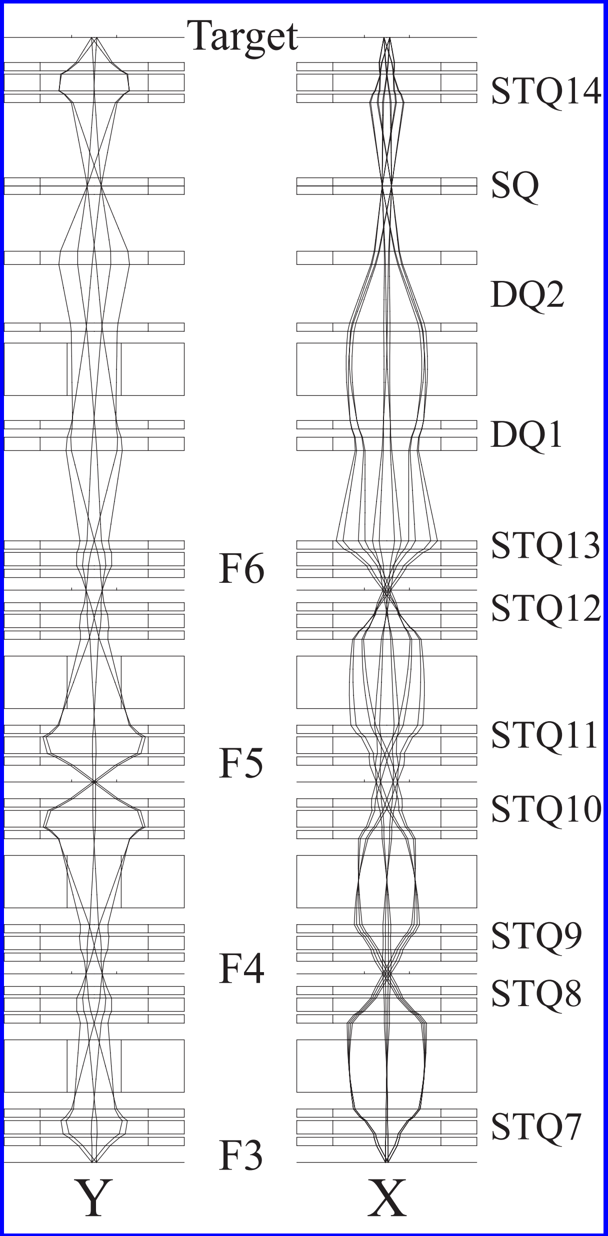
$$\langle y'|y\rangle = -1.54 \quad \langle y'|\phi\rangle = 0.00$$

$$\langle \phi'|y\rangle = 0.65 \quad \langle \phi'|\phi\rangle = -0.65$$

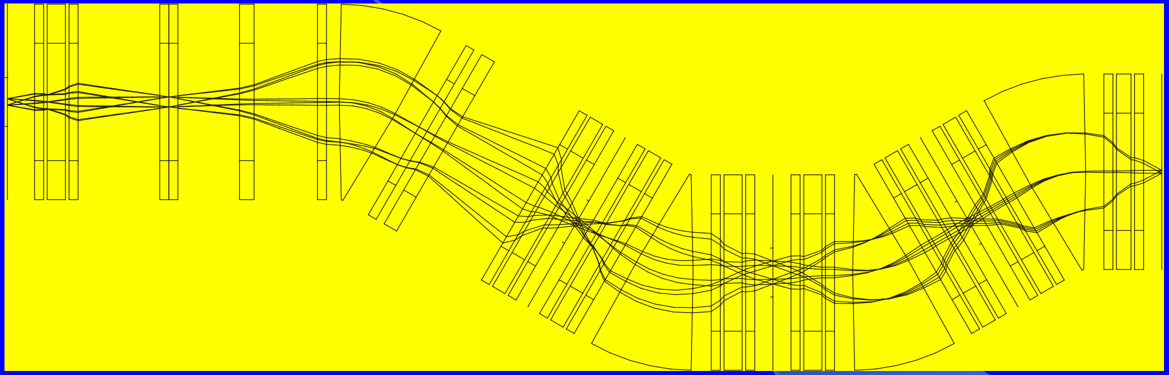
Dispersive Transport

- Double Focus at SQ.
- SQ, DQ1, and DQ2 are Normal Conducting.
- Symmetric STQ: STQ10-11, STQ9-12, STQ8-13
- Symmetric DQ: DQ1-2

Achromatic Transport (33)



$\Delta\theta_x$= +/– 20 mr, $\Delta\theta_y$= +/– 30 mr,
Δx = +/– 3 mm, Δy = +/– 5 mm, ΔP= +/– 0.3 %



$\langle x' | x \rangle = 2.16$ $\quad\langle x' | \theta \rangle = 0.00$ $\quad\langle x' | \delta \rangle = 0.00$
$\langle \theta' | x \rangle = 0.81$ $\quad\langle \theta' | \theta \rangle = 0.46$ $\quad\langle \theta' | \delta \rangle = -0.12$
$\langle y' | y \rangle = -1.19$ $\quad\langle y' | \phi \rangle = 0.00$
$\langle \phi' | y \rangle = 0.51$ $\quad\langle \phi' | \phi \rangle = -0.84$

Achromatic Transport

- Same layout with the solution #31.
- STQ7, STQ8, STQ9, and STQ14 are same setting with the solution #31.
- Symmetric: STQ10-11, STQ9-STQ12, DQ1-2.
- Large horizontal magnification.

SHARAQ is a Spectrometer for Secondary Beams (RA = radioactive)

Implications of Secondary Beam:

Comparison with spectrometers with primary beams (10^9 –10^{12}) beams (e.g. Grand Raiden)

1) Secondary beam means: low intensity 10^3 – 10^8 particles/sec, large emittance and dp/p (beam)
2) Lateral dispersion matching ensures momentum resolution is better (up to 10 times) than dp/p
3) Angular dispersion matching ensures that angle can be reconstructed ($d\Theta_{tgt}$ = 2-7 mrad)
4) Dispersion matching depends on kinematic K = (dp/dΘ)/p
5) Dispersion matching more difficult, because of large dp/p, K, large beam spot (10 cm?)
6) Dispersion matching not possible if kinematical K= (dp/dΘ)/p is too large
7) Consequence of 6) is, no dispersion matching in inverse kinematics

8) Diagnostics and measurements of secondary beam (as opposed to reaction particles) event-by-event becomes necessary for high momentum and angle resolution
9) Up to 10^6 beam part./sec use of detectors, resolution may be limited by multiple scattering.
10) > 10^6 beam part./sec when use of detectors impossible, consider momentum cutting slits

11) Beam diagnostics in beam line is very important

End Lecture 3