

Cluster Structure and Isoscalar Monopole Excitation in Light Nuclei

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Place of Kanto Gakuin University (KGU)



Outline of my talk

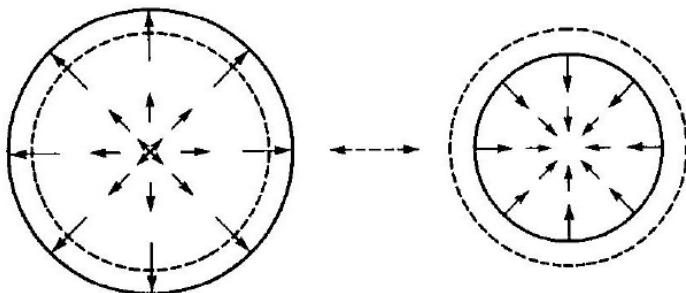
- 1. Introduction**
- 2. IS monopole strengths in light nuclei**
Typical case : ^{16}O
RPA and SRPA calculations, compared with Exp.
- 3. 4α OCM for 0^+ states in ^{16}O**
(OCM=Orthogonality Condition Model)
- 4. IS monopole strength function with 4α OCM,
compared with Exp.**
- 5. Emphasize two features of IS monopole excitations.**
- 6. Dual nature of ^{16}O ground state.**
- 7. Monopole strengths in other nuclei**
- 8. Summary**

- Isoscalar Monopole Mode
 \leftrightarrow density fluctuation

Typical example

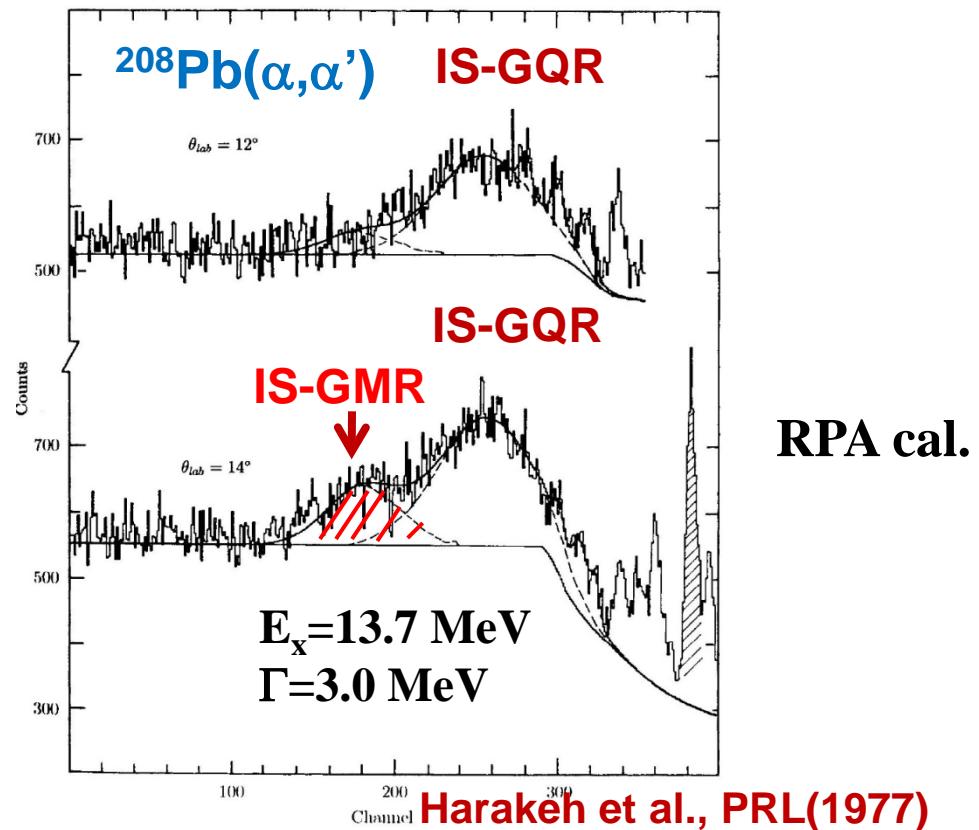
IS-GMR (heavy, medium-heavy nuclei)

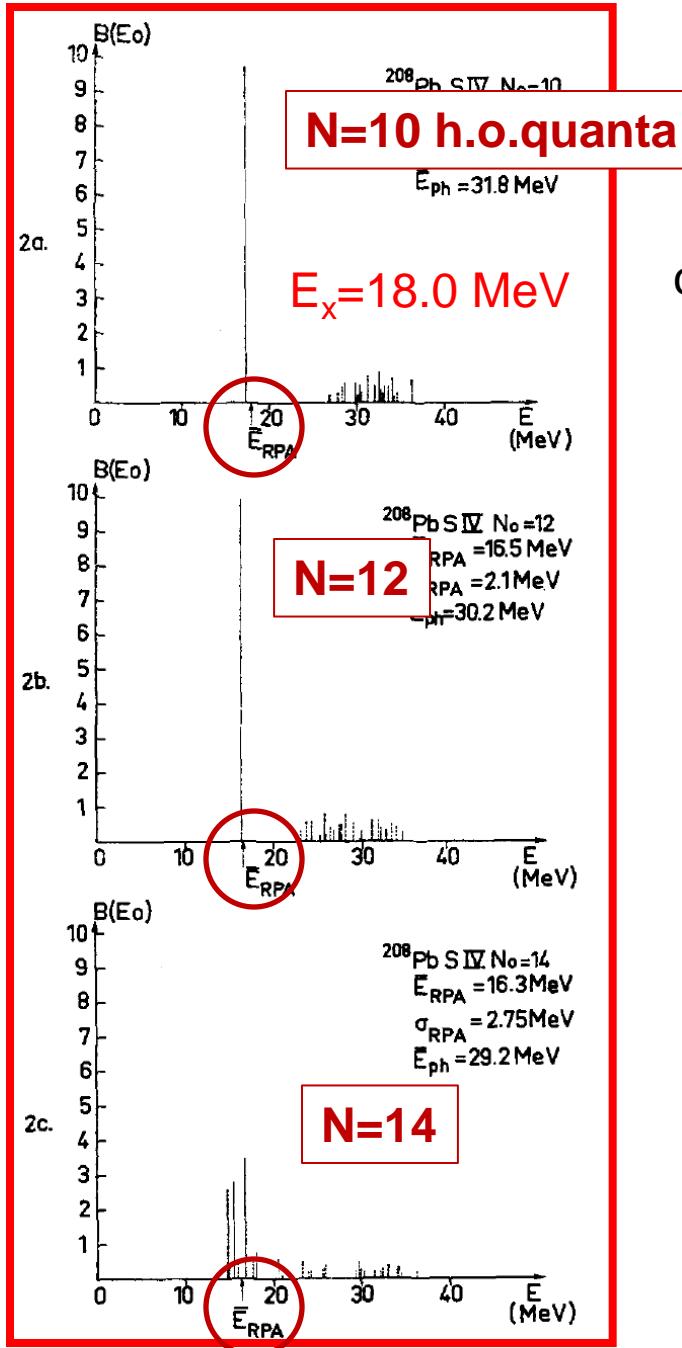
exhaust EWSR $\sim 100\%$



breathing mode

$$E_x \simeq 80/A^{-1/3} \quad [\text{MeV}]$$





collective motion with coherent 1p-1h states

IS-GMR is well reproduced
by RPA cal.

Blaizot, Gogny, Grammaticos, NPA(1976)

- Light Nuclei (p -, sd -shell,,,)
Isoscalar monopole strengths are fragmented.
- For example, ^{16}O , ($^{12}\text{C}, ^{11}\text{B}, ^{13}\text{C}, ^{24}\text{Mg}, \dots$)
IS-monopole response fun. of $^{16}\text{O}(\alpha, \alpha')$
 - (i) discrete peaks at $E_x \leq 15$ MeV
~20% of EWSR
large $M(E0)$ states \Leftrightarrow cluster states
 - (ii) three-bump structure : $E=18, 23, 30$ MeV

IS Monopole Strength Function of ^{16}O

Exp. vs Cal.

$^{16}\text{O}(\alpha, \alpha')$

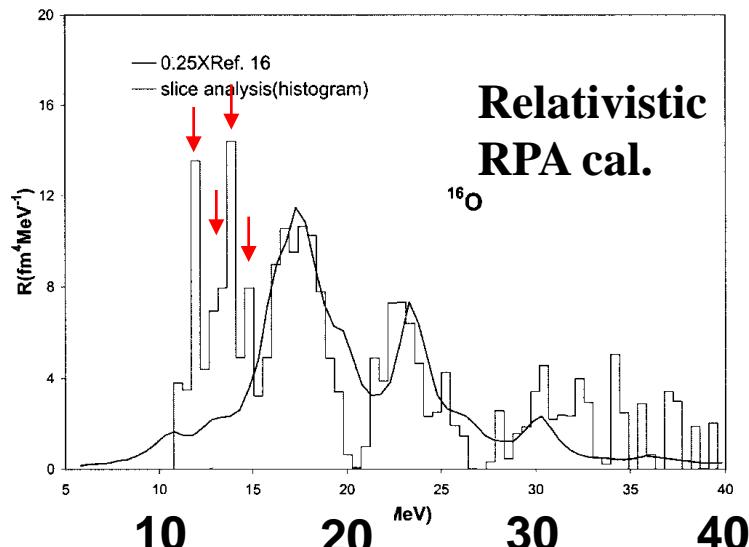


FIG. 7. The histogram is converted to monopole strength function shows the monopole response function from Ref. [16] multiplied by 0.25 and shifted by 4.2 MeV.

Exp. condition: $E_x > 10$ MeV

Exp: histogram

Lui et al., PRC 64 (2001)

discrete peaks at $Ex \leq 15$ MeV
three bumps at 18, 23, 30 MeV

Cal: real line

Relativistic RPA

Ma et al., PRC 55 (1997)

Multiplied by 0.25

Shifted by 4.2 MeV

Not well reproduced by RRPA cal.

Non-rela. Mean-field calculations of IS-monopole strengths for ^{16}O

(1) RPA calculation: 1p-1h excitations

Blaizot et al., NPA265 (1976)

(2) Second-order RPA (SRPA) calculations:
: 1p1h + 2p2h

- i) Drozdz et al., PR197(1980): D1-force
- ii) Papakonstantino et al., PLB671(2009): UCOM
- iii) Gambacurta et al., PRC81(2010) :
full SRPA calculation with Skyrme force

Non-rela. Mean-field calculations of IS-monopole strengths for ^{16}O

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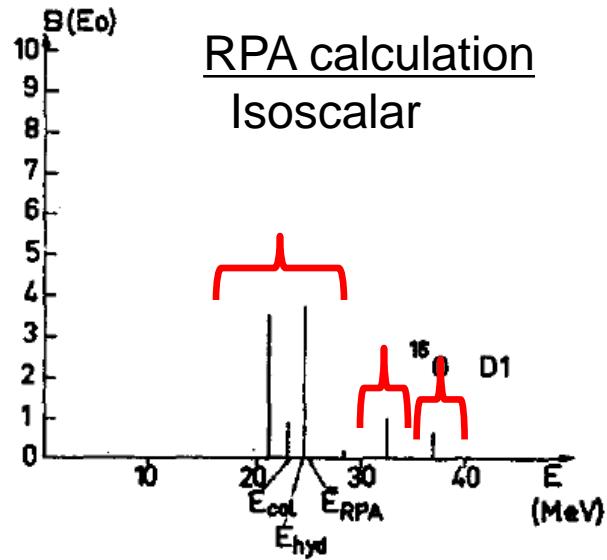
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full SRPA calculation with Skyrme force

RPA calculation



Blaizot et al., NPA(1976)

Experiment

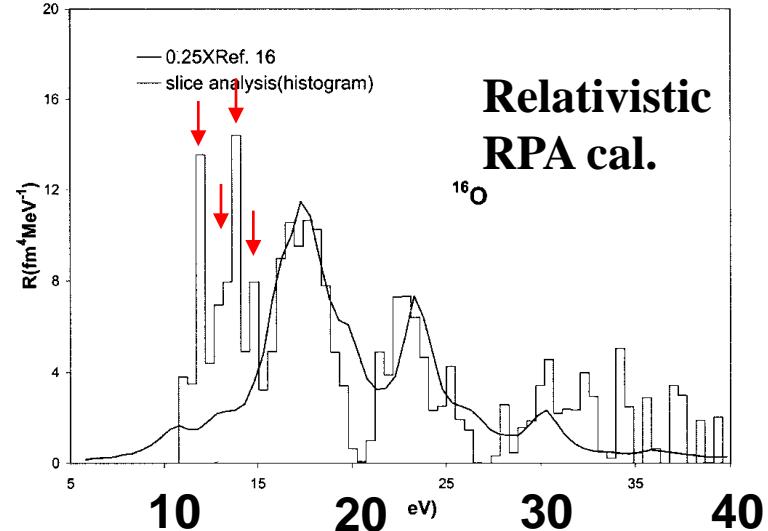


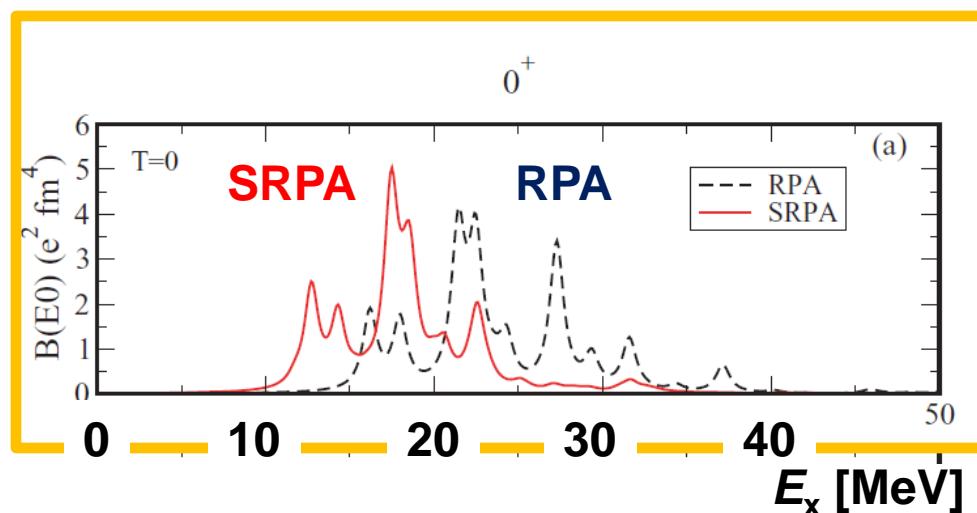
FIG. 7. The histogram is the experimental E_0 strength converted to monopole response function. The black line shows the model calculation. The red arrows indicate the experimental features.

Exp. condition: $E_x > 10 \text{ MeV}$

- (1) Reproduction of 3-bump structure,
but the energy positions are by about 3-5 MeV
higher than the data.
- (2) No reproduction of discrete peaks at $E_x \leq 15 \text{ MeV}$

Non-rel. RPA calculations

SRPA (+RPA) calculation



Gambacurta et al., PRC(2010)

Experiment

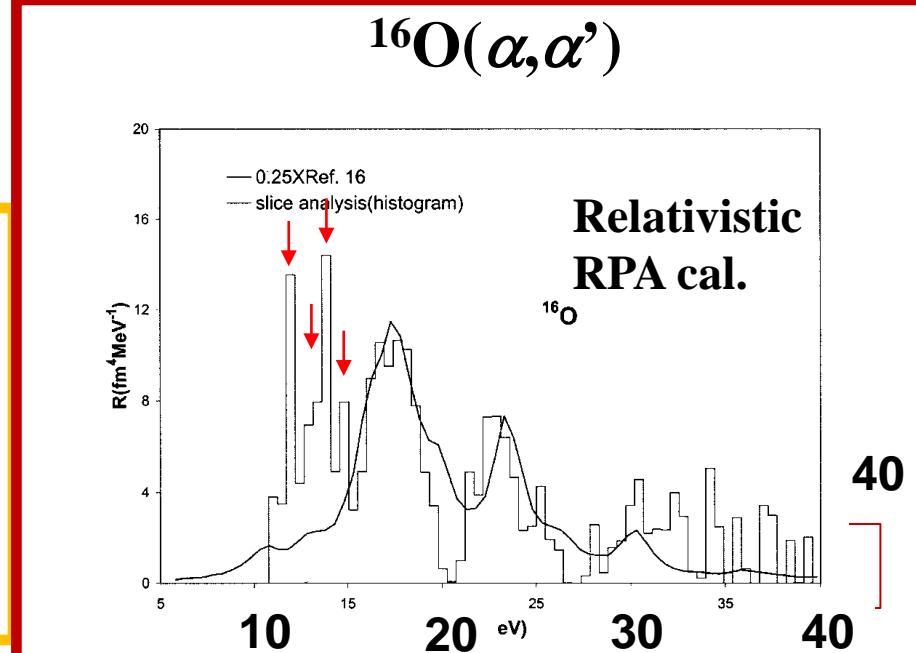


FIG. 7. The histogram is the experimental $E0$ strength converted to monopole response function. The black line shows the relativistic RPA calculation. Red arrows point to specific peaks in the histogram. The plot is labeled ^{16}O and Relativistic RPA cal.

Exp. condition: $E_x > 10$ MeV

- (1) Gross structure at higher energy region ($E_x > 18$ MeV), i.e. 3-bump structure, is reproduced by SRPA calculation.
- (2) Discrete peaks at $E_x \leq 15$ MeV are not reproduced well. In particular, the transition to 2nd 0^+ state ($E_x = 6.1$ MeV) is not seen in SRPA (+RPA) calculation.

	Experiment				4 α OCM		
	Ex [MeV]	R [fm]	M(E0) [fm 2]	Γ [MeV]	R [fm]	M(E0) [fm 2]	Γ [MeV]
0^+_1	0.00	2.71			2.7		
0^+_2	6.05		3.55		3.0	3.9	
0^+_3	12.1		4.03		3.1	2.4	
0^+_4	13.6		no data	0.6	4.0	2.4	0.60
0^+_5	14.0		3.3	0.185	3.1	2.6	0.20
0^+_6	15.1		no data	0.166	5.6	1.0	0.14

over 15%
of total EWSR

20%
of total EWSR

Experiment

$^{16}\text{O}(\alpha, \alpha')$

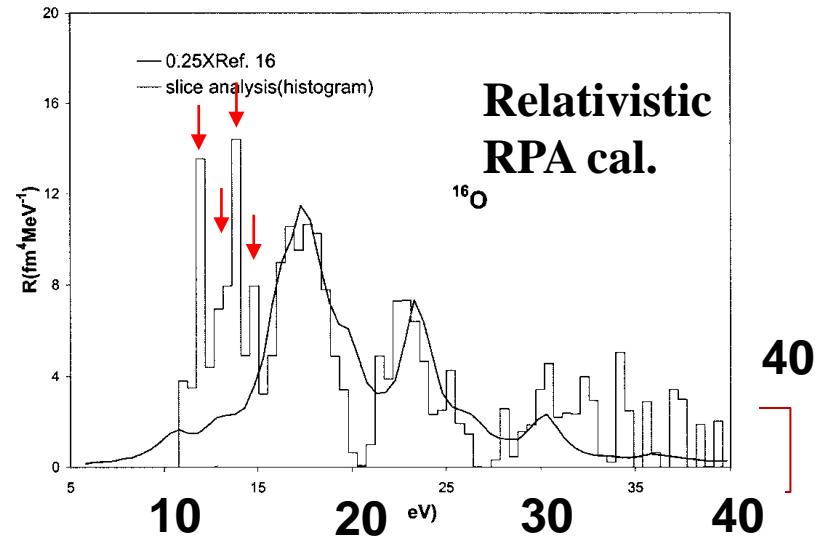
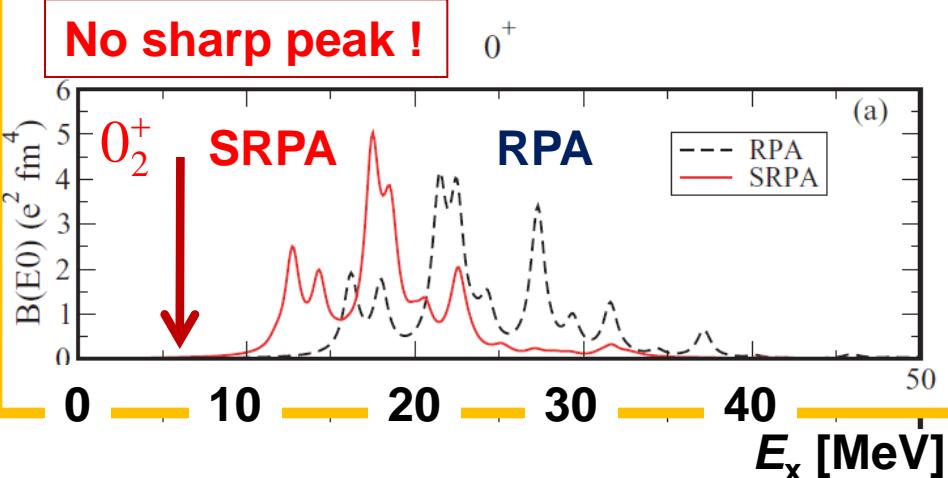


FIG. 7. The histogram is the experimental E_0 strength converted to monopole response function. The black line shows the model calculation. The experimental condition is indicated by the red bracket.

Exp. condition: $E_x > 10 \text{ MeV}$

SRPA (+RPA) calculation

No sharp peak !



- (1) Gross structure at higher energy region ($E_x > 18 \text{ MeV}$), i.e. 3-bump structure, is reproduced by SRPA calculation.
- (2) Discrete peaks at $E_x \leq 15 \text{ MeV}$ are not reproduced well. In particular, the transition to 2nd 0^+ state ($E_x = 6.1 \text{ MeV}$) is not seen in SRPA (+RPA) calculation.

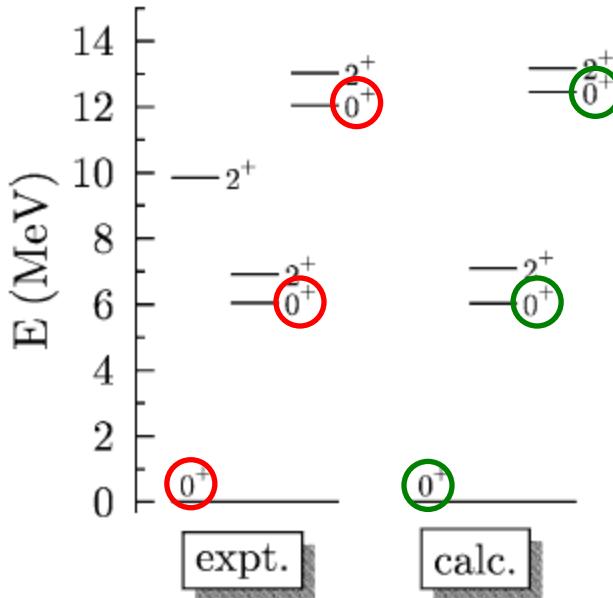
RPA and SRPA calculations for ^{16}O

- (1) Gross structure at higher energy region, i.e. 3-bump structure, looks likely to be approximately reproduced by RPA and SPRA calculations.
- (2) Discrete peaks at $E_x \leq 15$ MeV are not reproduced well. In particular, the transition to 2nd 0+ state ($E_x = 6.1$ MeV) is not seen in SRPA and RPA calculations.
This peak should exit sharply at $E_x = 6.1$ MeV because $\Gamma \leq 1$ eV.

Beyond mean-field calculation for ^{16}O

M. Bender, P.-H. Heenen, Nucl. Phys. A713, (2003)

Analysis with self-consistent $0p$ - $0h$, $2p$ - $2h$, and $4p$ - $4h$ HF states **SLy4**



Transition	$M(E0)/\text{fm}^2$	
	Expt.	Calc.
$0_2^+ \rightarrow 0_1^+$	3.55 ± 0.21	5.735
$0_3^+ \rightarrow 0_1^+$	4.03 ± 0.09	0.690

State	$ \langle Jk Jnp - nh \rangle ^2$			
	0_1^+	0_2^+	0_3^+	0_4^+
0p-0h	0.737	0.195	0.006	0.0001
2p-2h (p)	0.094	0.098	0.018	0.003
2p-2h (n)	0.099	0.098	0.019	0.002
4p-4h	0.027	0.212	0.193	0.003
8p-8h	0.000	0.000	0.000	0.034

Purposes of my talk

- What kind of states contribute to the discrete peaks?
- Recently, 4α OCM(直交条件模型) succeeded in describing the structure of the lowest six 0^+ states up to 4α threshold ($E_x \approx 15$ MeV).
Funaki et al., PRL101(2008)
- We will study the IS monopole strength function with the 4α OCM.

OCM: Orthogonality Condition Model

Cluster-model analyses of ^{16}O

- $\alpha + ^{12}\text{C}$ OCM

Y. Suzuki, PTP55 (1976), 1751

- $\alpha + ^{12}\text{C}$ GCM

M. Libert-Heinemann, D. Bay et al., NPA339 (1980)

- 4α THSR wf Not include $\alpha + ^{12}\text{C}$ configuration.

Tohsaki, Horiuchi, Schuck, Roepke, PRL87 (2001)

Funaki, Yamada et al., PRC82(2010)

- 4α OCM 4α -gas, $\alpha + ^{12}\text{C}$, shell-model-like configurations

Funaki, Yamada et al., PRL101 (2008)

Reproduction of lowest six 0^+ states up to 4α threshold (15MeV)

$^{16}\text{O} = \alpha + ^{12}\text{C}$ cluster model

Y. Suzuki, PTP55 (1976), 1751

Even-parity

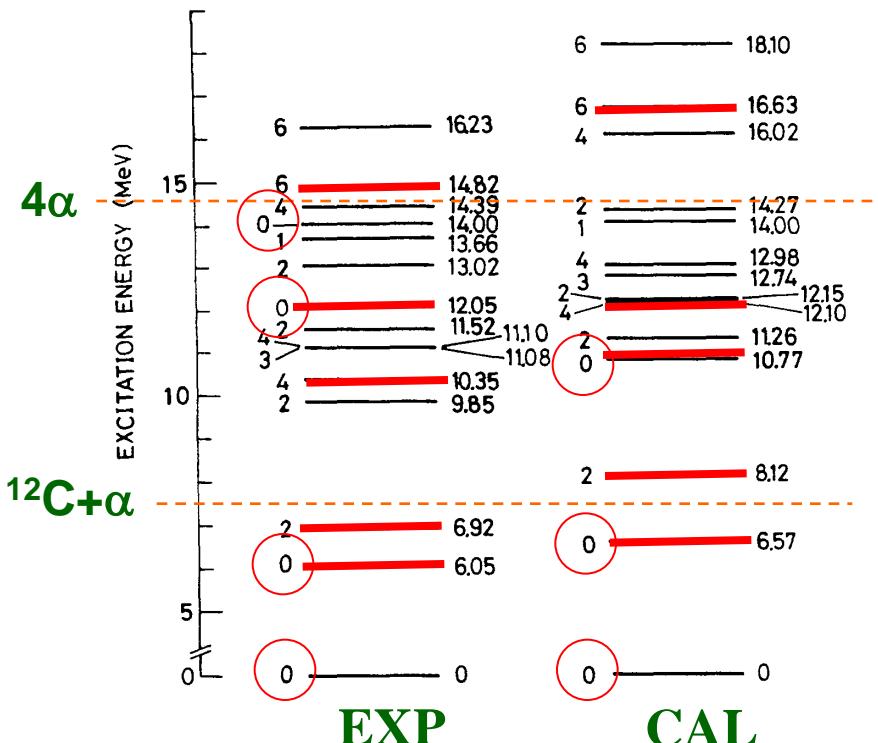


Fig. 2 (a). Energy levels of ^{16}O for the even-parity states [Ref. 30)].

Odd-parity

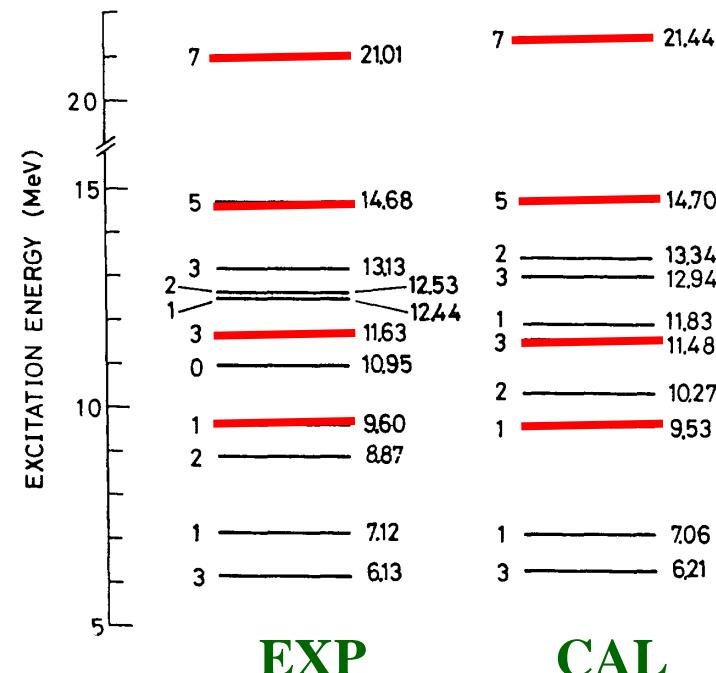
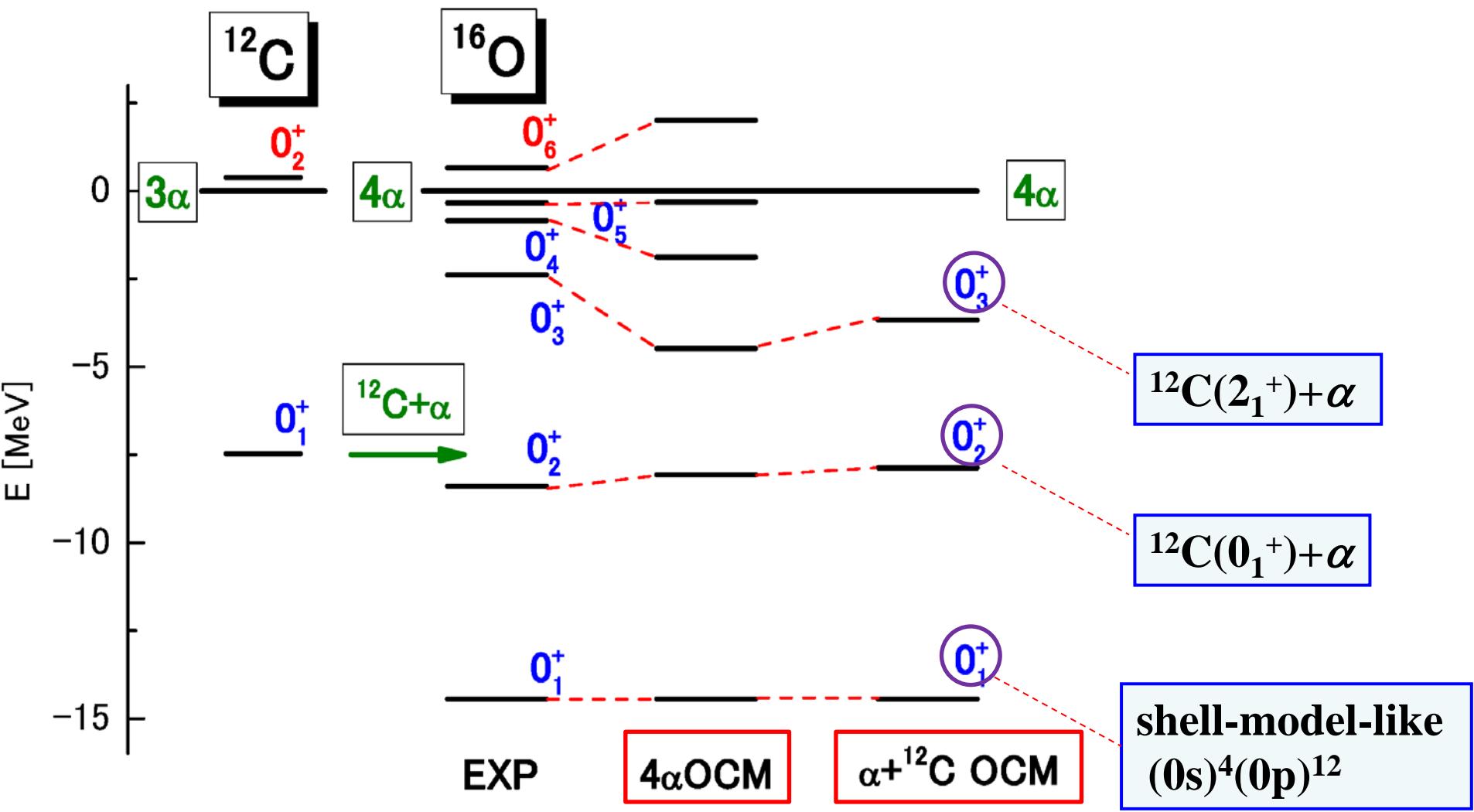


Fig. 2 (b). Energy levels of ^{16}O for the odd-parity states [Ref. 30)].

— $^{12}\text{C} + \alpha$: molecular states



Funaki et al.,
 PRL101 (2008) Suzuki,
 PTP55 (1976)

OCM (orthogonality condition model)

- An approximation of **RGM** (resonating group method)
- Relative motions among c.o.m. of clusters are exactly solved under an orthogonality condition arising from Pauli-Blocking effects

For example of **$n\alpha$** system,

S. Sato, Prog. Thor. Phys. 40 (1968)

Fermion w.f.: $\Phi^{(F)} = A \left\{ \prod_{i=1}^N \phi_{\alpha_i}^{\text{int}} \chi^{\text{rel}} \right\}, \quad (H - E) \Phi^{(F)} = 0, \quad \langle \Phi^{(F)} | \Phi^{(F)} \rangle = 1,$

RGM eq.: $(H - EN) \chi^{\text{rel}} = 0, \quad \langle \chi^{\text{rel}} | N | \chi^{\text{rel}} \rangle = 1,$

α -cluster w.f.: $\Phi^{(B)} = \sqrt{N} \chi^{\text{rel}}, \quad \left(\frac{1}{\sqrt{N}} H - \frac{1}{\sqrt{N}} - E \right) \Phi^{(B)} = 0, \quad \langle \Phi^{(B)} | \Phi^{(B)} \rangle = 1,$

Approximation: $\frac{1}{\sqrt{N}} H - \frac{1}{\sqrt{N}} \Rightarrow T + \sum_{i < j} V_{2\alpha}^{\text{eff}}(i, j) + \sum_{i < j < k} V_{3\alpha}^{\text{eff}}(i, j, k) = T + V^{\text{eff}}$ **Orthogonality condition**

OCM equation: $(T + V^{\text{eff}} - E) \Phi^{(B)} = 0 \quad \text{with} \quad \langle u_F | \Phi^{(B)} \rangle = 0, \quad \langle \Phi^{(B)} | \Phi^{(B)} \rangle = 1$

u_F : **Pauli forbidden states**, $N u_F = 0 \quad \text{or} \quad A \left\{ \prod_{i=1}^N \phi_{\alpha_i}^{\text{int}} u_F \right\} = 0$

$\Phi^{(B)}$: **Symmetrized w.f. with relative (Jacobi) coordinates**

Easy to formulate **$2\alpha+t$** OCM and **$3\alpha+n$** OCM based on GEM

Framework of 4α OCM

Total w.f. : internal w.f.s of α clusters \times relative w.f.

$$\tilde{\Psi}(J^\pi) = \boxed{\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4)} \times \boxed{\Psi(J^\pi)}.$$

$\phi(\alpha) : (0s)^4$

$\langle u_F | \Psi(J^\pi) \rangle = 0$: Orthogonality condition arising from
Paul-blocking effects:
i.e. orthogonal to \mathcal{U}_F (Pauli-forbidden states)

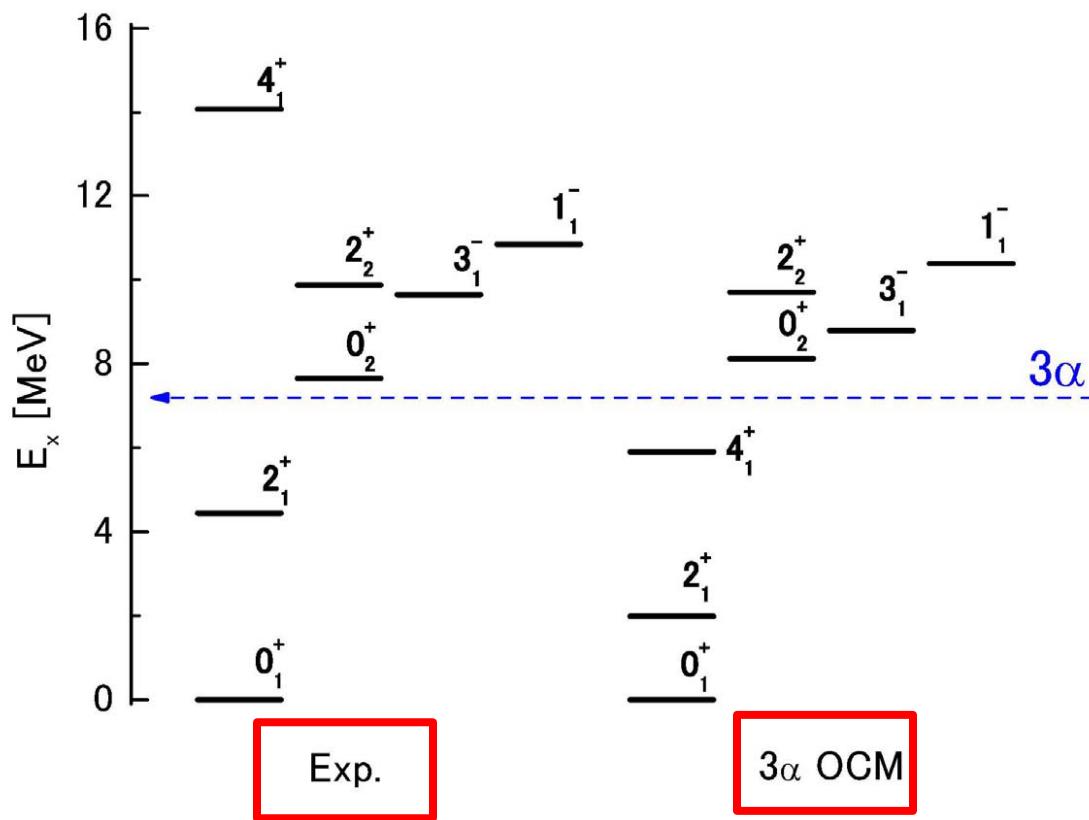
Hamiltonian for $\Psi(J^\pi)$: 4-body Hamiltonian

$$H_{\text{OCM}} = \sum_{i=1}^4 T_i - T_{cm} + \sum_{i < j}^4 \left[V_{2\alpha}^{(N)}(i, j) + V_{2\alpha}^{(\text{Coul})}(i, j) \right]$$

$$+ \sum_{i < j < k}^4 V_{3\alpha}(i, j, k) + V_{4\alpha}(1, 2, 3, 4)$$

2 α potential: phase shifts
Energy spectra of $^{12}\text{C}(0+, 2+, 4+, 3-, 1-)$
Energy of ^{16}O g.s.

Energy spectra of ^{12}C with 3α OCM (Complex scaling)



How to solve 4-body problem with orthogonality condition

Combining Gaussian Expansion Method (GEM) and OCM

$$\Psi(J^\pi) = \sum_\nu f_c(\nu) \times \hat{S} \left[\begin{array}{c} \text{Gaussian} \\ \left[\varphi_{\ell 1}(\xi_1, \nu_1) \varphi_{\ell 2}(\xi_2, \nu_2) \right]_{L12} \end{array} \begin{array}{c} \text{Gaussian} \\ \varphi_{\ell 3}(\xi_3, \nu_3) \end{array} \right]_J$$

$$H_{\text{OCM}} \Psi(J^\pi) = E \Psi(J^\pi)$$

$$\langle u_F | \Psi(J^\pi) \rangle = 0$$

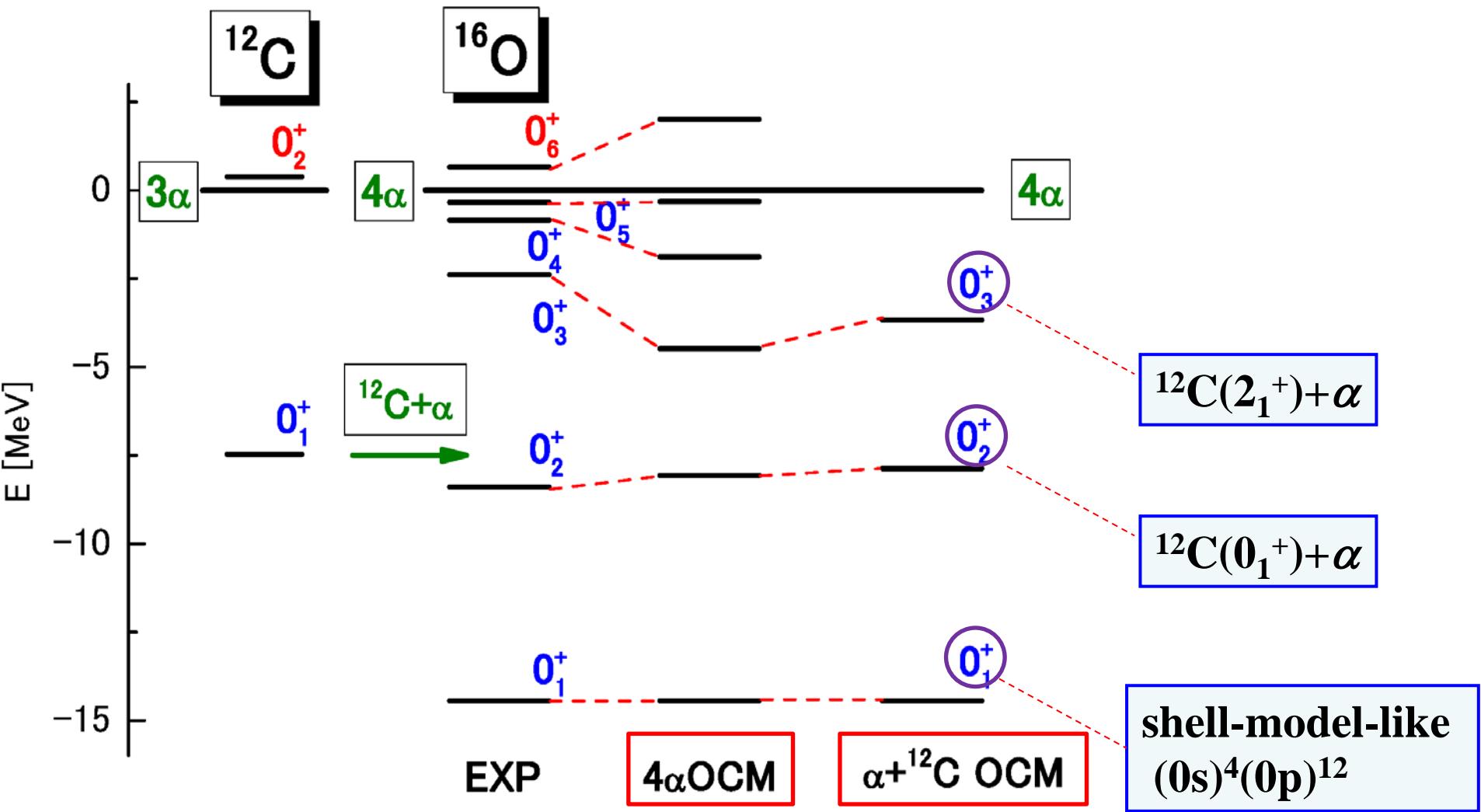
GEM (Gaussian Expansion Method):

Numerical precision is equivalent to Faddeev-Yakubousky eq.
for ${}^4\text{He}$ = 4-body problem with realistic NN forces

Kamimura: PRA38(1988),
Hiyama, Kino, Kamimura: PPNP51(2003),
Kamada et al.: PRC64(2001)

GEM+OCM: Structure study of Light Hypernuclei

Hiyama, Yamada: PPNP63(2009)



Funaki et al., Suzuki,
PRL101 (2008) PTP55 (1976)

	Experimental data				4 α OCM		
	E _x [MeV]	R [fm]	M(E0) [fm ²]	Γ [MeV]	R [fm]	M(E0) [fm ²]	Γ [MeV]
0^+_1	0.00	2.71			2.7		
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over 15%
of total EWSR

20%
of total EWSR

Single-particle IS-monopole strength

$$M(E0) \sim \langle u_f / r^2 / u_i \rangle \sim (3/5) \times R^2 = 4.4 \text{ fm}^2$$

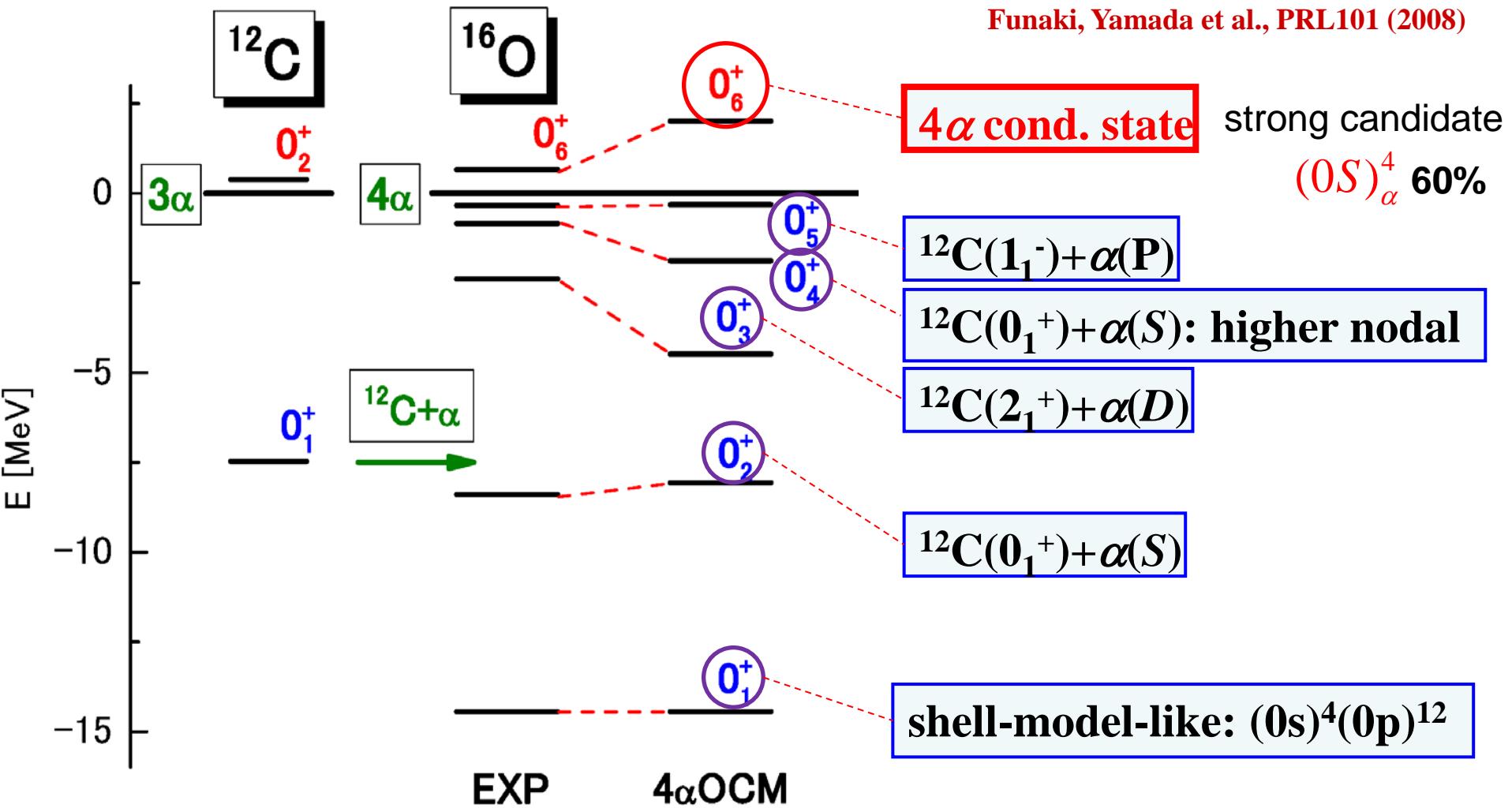
(using R = nuclear radius = 2.7 fm)

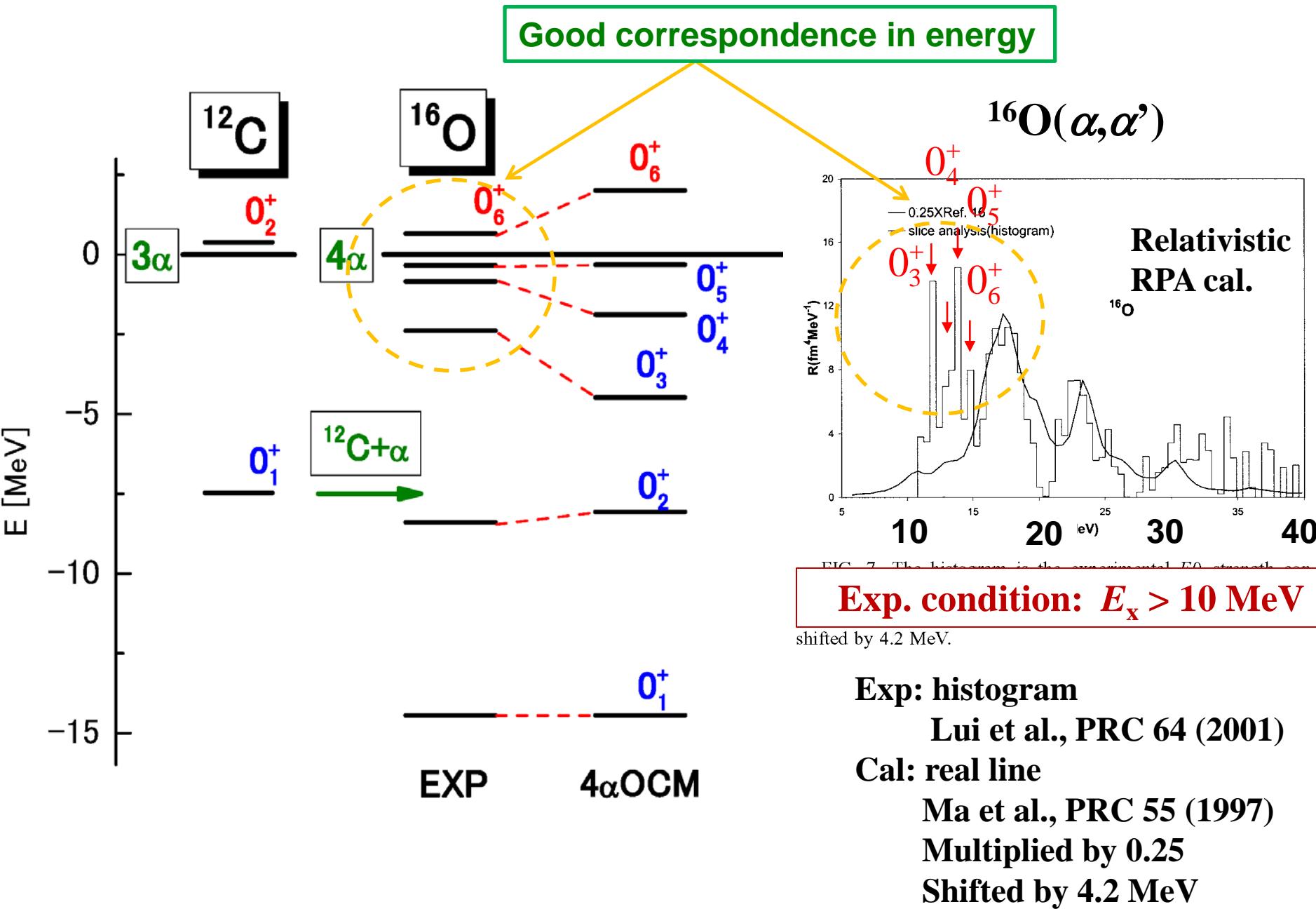
Uniform-density approximation for $u_f(r)$ and $u_i(r)$

$$\begin{aligned} u(r) &= (3/R^3)^{1/2} && \text{for } 0 \leq r \leq R \\ u(r) &= 0 && \text{for } R < r \end{aligned}$$

4α OCM calculation

Funaki, Yamada et al., PRL101 (2008)





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- Recently, 4α OCM succeeded in describing the structure of the lowest six 0^+ states up to 4α threshold ($E_x \approx 15$ MeV). Funaki et al., PRL101(2008)
- We will study the IS monopole strength function with the 4α OCM. Yamada et al., PRC85(2012)

**IS monopole strength function $S(E)$
within 4α OCM framework**

Monopole Strength Function with 4α OCM

$$S(E) = \sum_n \delta(E - E_n) |\langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle|^2, \quad \mathcal{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2$$

$$R(E) = \langle 0_1^+ | \frac{\mathcal{O}^\dagger \mathcal{O}}{E - H + i\epsilon} | 0_1^+ \rangle, \quad |0_n^+\rangle: \text{resonance state with } E_n - i\Gamma_n/2$$

$$\begin{aligned} S(E) &= -\frac{1}{\pi} \text{Im}[R(E)] \\ &= \frac{1}{\pi} \sum_n \frac{\Gamma_n / 2}{(E - E_n)^2 + (\Gamma_n / 2)^2} \left| M(0_n^+ - 0_1^+) \right|^2 \end{aligned}$$

$$M(0_n^+ - 0_1^+) = \langle 0_n^+ | \mathcal{O} | 0_1^+ \rangle : \text{calculated by } 4\alpha \text{ OCM}$$

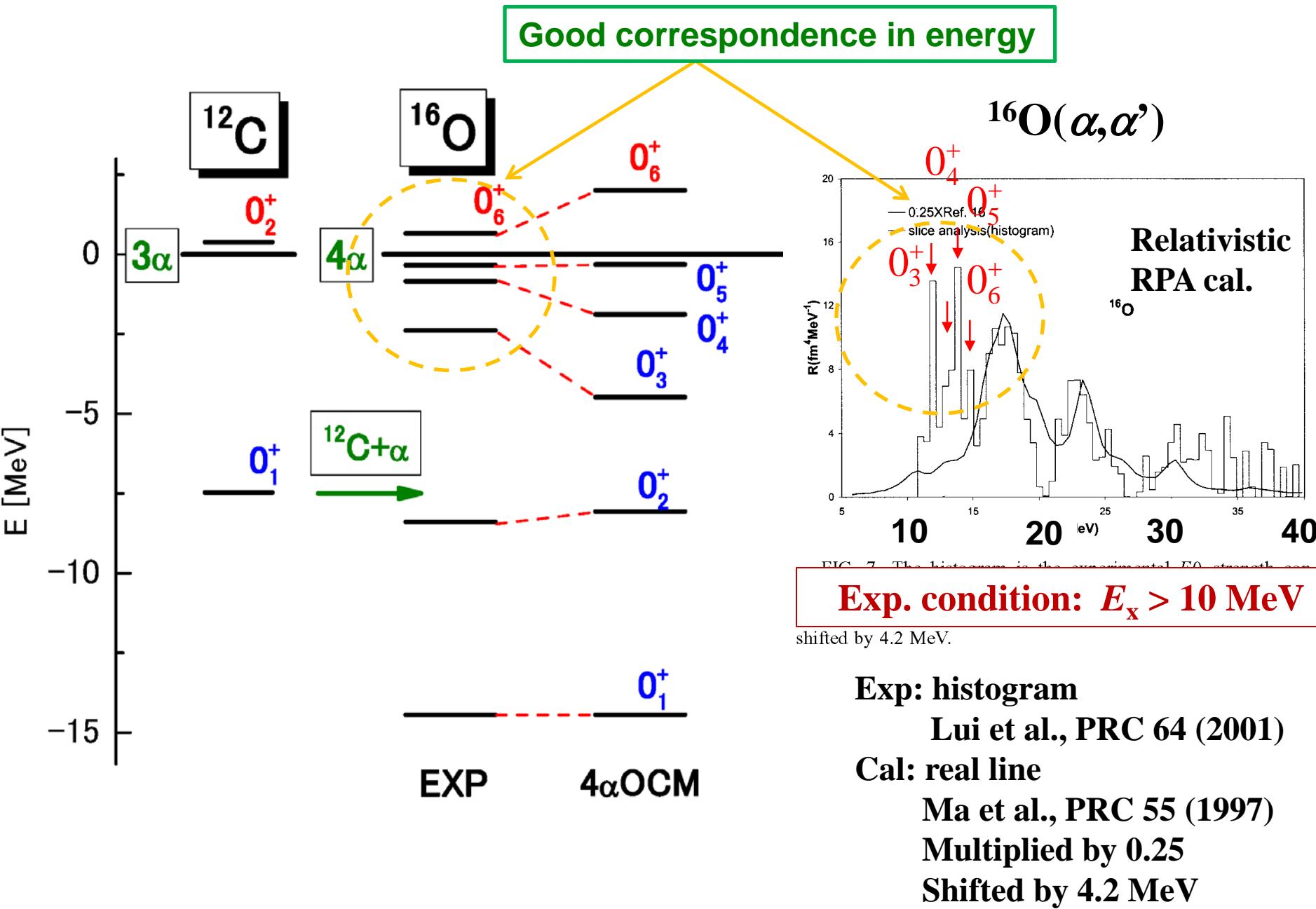
$$\Gamma_n = \sqrt{\Gamma_n(\text{OCM})^2 + (\text{exp. resolution})^2} \quad 50 \text{ keV}$$

E_n : experimental energy of n -th 0+ state

	Experiment				4 α OCM		
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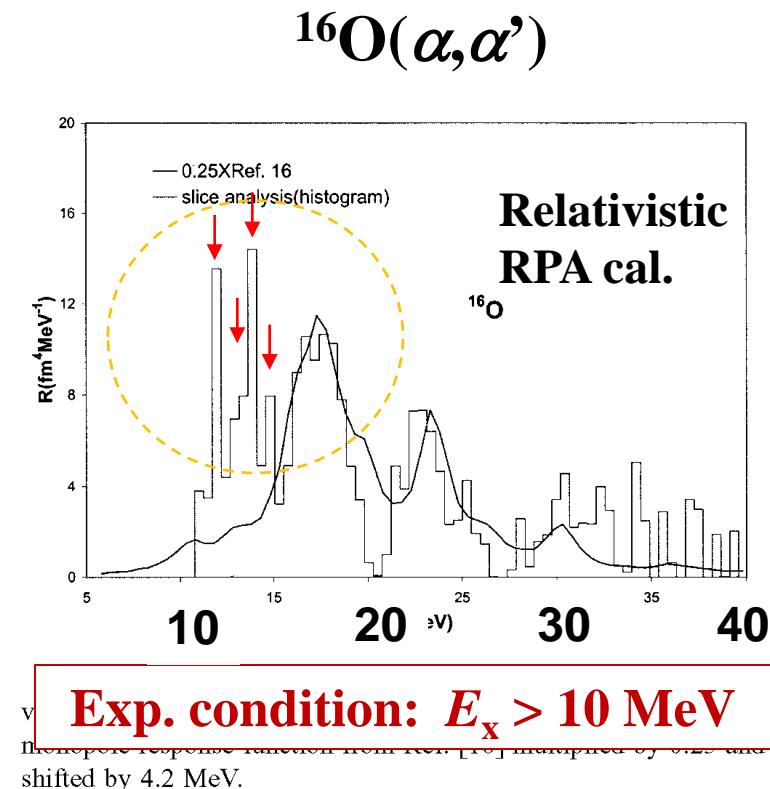
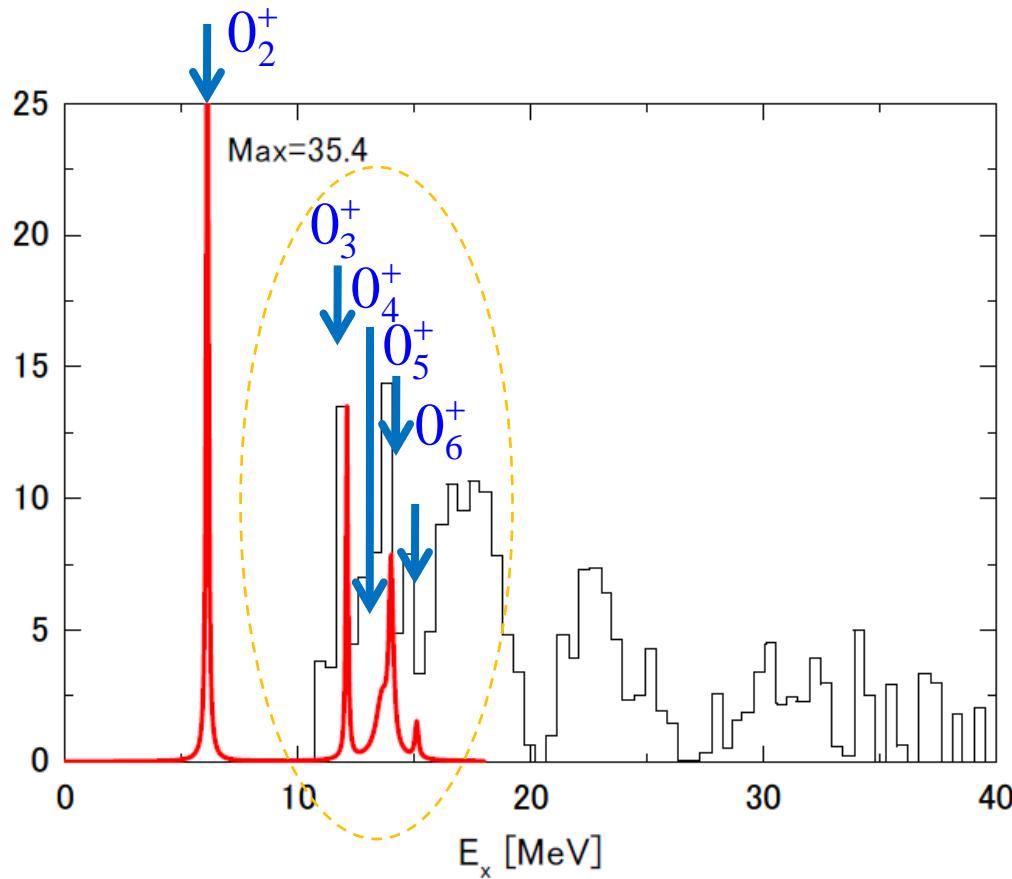
over 15%
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20%
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Exp. vs. Cal.

IS monopole S(E) with 4α OCM



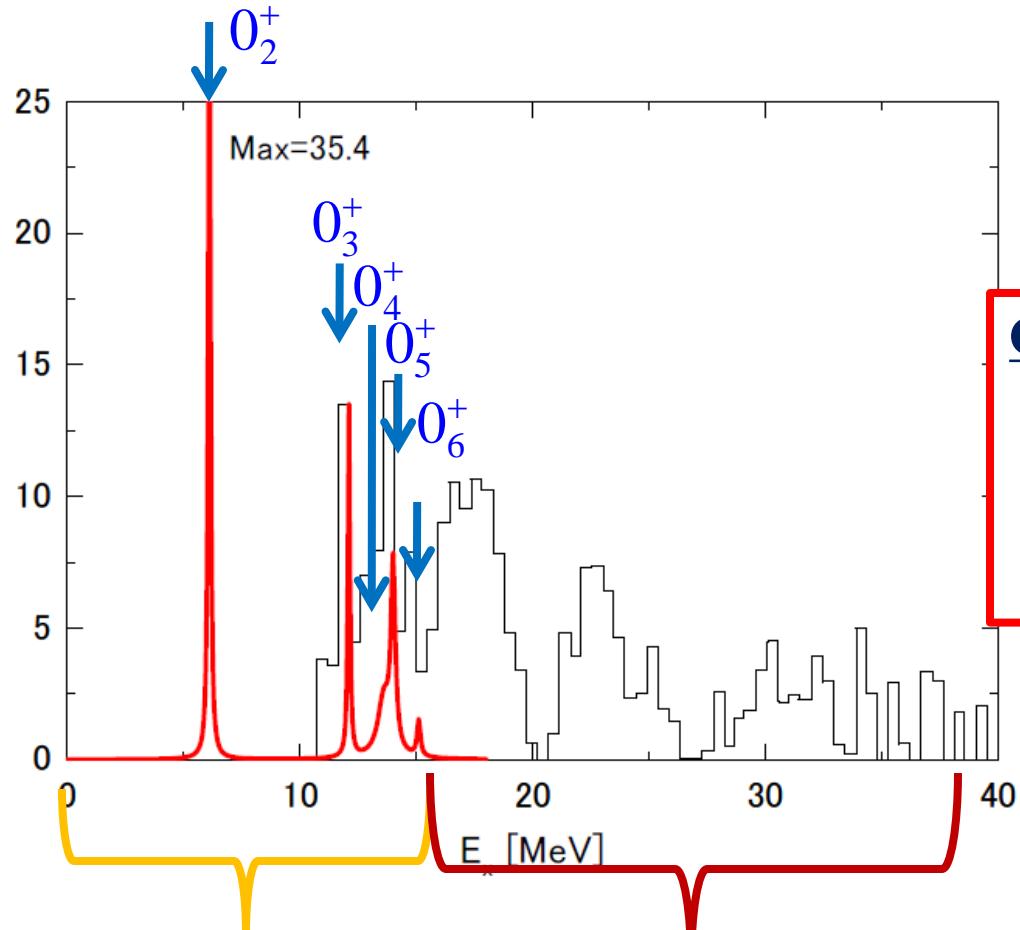
Exp. condition: $E_x > 10 \text{ MeV}$

It is likely to exist discrete peaks on a small bump at $E_x < 15 \text{ MeV}$

This small bump may come from the contribution from continuum states of $\alpha + ^{12}\text{C}$

Exp. vs. Cal.

IS monopole S(E) with 4α OCM



Two features
in IS monopole excitations

Origin: dual nature of G.S. of ^{16}O
(1) α -clustering degree of freedom
(2) mean-field-type one
 $(0s)^4(0p)^{12} : \text{SU}(3)(00) = ^{12}\text{C} + \alpha :$
Bayman-Bohr theorem

Excitation to cluster states
(α -cluster type)

Monopole excitation
of mean-field type (RPA)

Dual nature of ground state of ^{16}O

mean-field character and α -clustering character

Ground state of ^{16}O

(λ, μ)

Dominance of doubly-closed-shell structure: $(0s)^4(0p)^{12} = \text{SU}(3)(0,0)$

Cluster-model calculations: 4α OCM, 4α THSR, $\alpha + ^{12}\text{C}$ OCM, ...

Mean-field calculations : RPA, QRPA, RRPA,.....

Supported by no-core shell model calculations:

Dytrych et al., PRL98 (2007)

Bayman & Bohr, NPA9 (1958/59)

Bayman-Bohr theorem : $\text{SU}(3)[f](\lambda\mu)$ is equivalent to “a cluster-model wf”

Doubly-closed-shell w.f., $(0s)^4(0p)^{12}$, is mathematically equivalent to a single α -cluster w.f.

This means that the ground state w.f. of ^{16}O originally has an α -clustering degree of freedom together with mean-filed-type degree of free dom.

We call dual nature of g.s.

Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

$$\frac{1}{\sqrt{16!}} \det |(0s)^4(0p)^{12}| \times [\phi_{cm}(\mathbf{R}_{cm})]^{-1} : \text{closed shell}$$

$$= N_0 \sqrt{\frac{12!4!}{16!}} A \left\{ \left[u_{40}(\xi_3, 3\nu) \phi_{L=0}({}^{12}\text{C}) \right]_{J=0} \phi(\alpha) \right\}$$

relative wf (S-wave)

$$= N_2 \sqrt{\frac{12!4!}{16!}} A \left\{ \left[u_{42}(\xi_3, 3\nu) \phi_{L=2}({}^{12}\text{C}) \right]_{J=0} \phi(\alpha) \right\}$$

relative wf (D-wave)

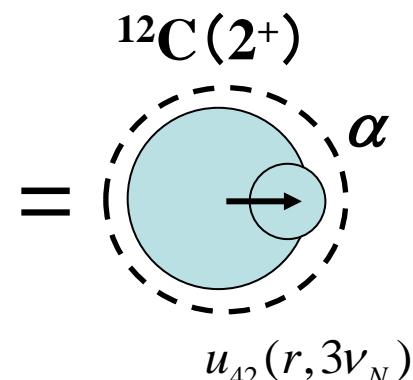
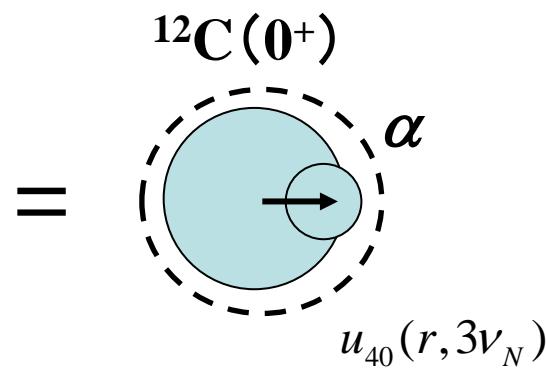
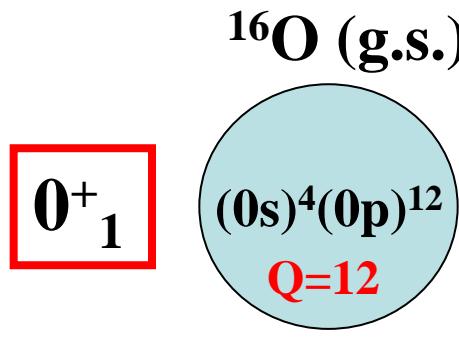
c.o.m. w.f. of ${}^{16}\text{O}$

$$\phi_{cm}(\mathbf{R}_{cm}) = \left(\frac{32\nu}{\pi} \right)^{3/4} \exp(-16\nu \mathbf{R}_{cm}^2)$$

α -degree of freedom

→ G.S. has mean-field-type and α -cluster degrees of freedom.

We call dual nature of g.s.



Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

$$\frac{1}{\sqrt{16!}} \det \left| (0s)^4 (0p)^{12} \right| \times [\phi_{cm}(\mathbf{R}_{cm})]^{-1} : \text{closed shell}$$

$$= N_0 \sqrt{\frac{12!4!}{16!}} A \left\{ \left[u_{40}(\xi_3, 3\nu) \phi_{L=0}({}^{12}\text{C}) \right]_{J=0} \phi(\alpha) \right\}$$

relative wf (S-wave)

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c.o.m. w.f. of ${}^{16}\text{O}$

$$\phi_{cm}(\mathbf{R}_{cm}) = \left(\frac{32\nu}{\pi} \right)^{3/4} \exp(-16\nu \mathbf{R}_{cm}^2)$$

} **α -degree of freedom**

→ G.S. has mean-field-type and α -cluster degrees of freedom.

Excitation of mean-field-type degree of freedom in g.s
 → 1p1h states (3-bump structure)

Excitation of α -cluster degree of freedom in g.s
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Next we will see the excitation mechanism of monopole operator in detail.

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Excitation of mean-field-type degree of freedom in g.s

→ 1p1h states are produced (3-bump structure)

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Interesting characters of IS monopole operator

$$\mathcal{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2$$

$$= \sum_{i=1}^4 (\mathbf{r}_i - \mathbf{R}_\alpha)^2 + \sum_{i=5}^{16} (\mathbf{r}_i - \mathbf{R}_{^{12}\text{C}})^2 + 3(\mathbf{R}_\alpha - \mathbf{R}_{^{12}\text{C}})^2$$

internal part of α

internal part of ^{12}C

relative part acting
on relative motion
of α and ^{12}C

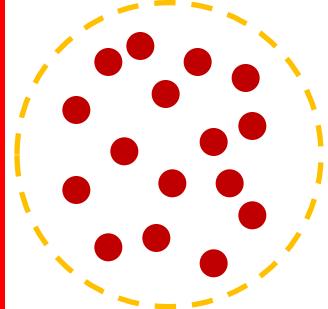
$$= \sum_{k=1}^4 \sum_{i=1}^4 (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2 + \sum_{k=1}^4 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\text{cm}})^2$$

internal part of each α -cluster

relative parts acting
on relative motions of 4 α 's
with respect to c.o.m. of ^{16}O

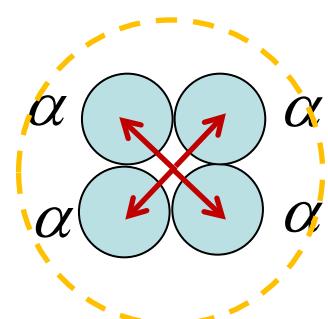
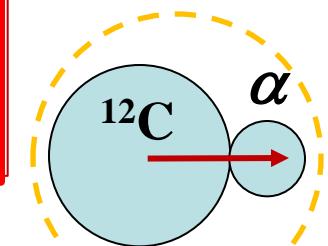
Decompositions into Internal part and relative part play an important role in understanding monopole excitations of ^{16}O .

16 nucleons



^{12}C

α



IS monopole operator

Yamada et al., PTP120(2008)

$$\mathcal{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2 = \underbrace{\sum_{i=1}^4 (\mathbf{r}_i - \mathbf{R}_\alpha)^2 + \sum_{i=5}^{16} (\mathbf{r}_i - \mathbf{R}_{12C})^2}_{\text{internal parts}} + \underbrace{3(\mathbf{R}_\alpha - \mathbf{R}_{12C})^2}_{\text{relative part}}$$
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$$\begin{aligned} \mathcal{O}|(0s)^4(0p)^{12}\rangle &\propto \mathcal{O}\mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \quad [\mathcal{O}, \mathcal{A}] = \mathcal{O}\mathcal{A} - \mathcal{A}\mathcal{O} = 0 \\ &= \mathcal{A} \left\{ \mathcal{O} u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \end{aligned}$$

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\$\phi(\alpha; 2\hbar\omega - \text{ex.})\$
\$\phi(^{12}\text{C}; 2\hbar\omega - \text{ex.})\$
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IS monopole operator

Yamada et al., PTP120(2008)

$$\mathcal{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2 = \underbrace{\sum_{i=1}^4 (\mathbf{r}_i - \mathbf{R}_\alpha)^2}_{\text{internal parts}} + \underbrace{\sum_{i=5}^{16} (\mathbf{r}_i - \mathbf{R}_{12C})^2}_{\text{relative part}} + 3(\mathbf{R}_\alpha - \mathbf{R}_{12C})^2$$

$$= \mathcal{O}(\alpha) + \mathcal{O}(^{12}\text{C}) + 3\xi_3^2$$

$$\begin{aligned} \boxed{\mathcal{O}|(0s)^4(0p)^{12}\rangle} &\propto \mathcal{O}\mathcal{A}\{u_{40}(\xi_3)\phi_{0+}(^{12}\text{C})\phi(\alpha)\} \\ &= \mathcal{A}\{\mathcal{O}u_{40}(\xi_3)\phi_{0+}(^{12}\text{C})\phi(\alpha)\} \\ &= \mathcal{A}\{u_{40}(\xi_3)\phi_{0+}(^{12}\text{C})[\mathcal{O}(\alpha)\phi(\alpha)]\} \\ &\quad + \mathcal{A}\{u_{40}(\xi_3)[\mathcal{O}(^{12}\text{C})\phi_{0+}(^{12}\text{C})]\phi(\alpha)\} \\ &\quad + \mathcal{A}\{[3\xi_3^2 u_{40}(\xi_3)]\phi_{0+}(^{12}\text{C})\phi(\alpha)\} \end{aligned}$$

excitation of cluster state

$\phi(\alpha; 2\hbar\omega - \text{ex.})$

$\phi(^{12}\text{C}; 2\hbar\omega - \text{ex.})$

$2\hbar\omega$ excitation of relative motion

When final state is a $^{12}\text{C}(0+)+\alpha$ cluster state with relative wf χ having $2\hbar\omega$ ex.,

$$\begin{aligned} M(0_2^+ - 0_1^+) &\propto \langle \chi(\xi_3)\phi_{0+}(^{12}\text{C})\phi(\alpha)|\mathcal{O}|(0s)^4(0p)^{12}\rangle \\ &\propto \langle \chi(\xi_3)|3\xi_3^2|u_{40}(\xi_3)\rangle \sim \langle u_{60}|\mathbf{r}^2|u_{40}\rangle \quad \text{s.p. strength} \end{aligned}$$

$6\hbar\omega \quad 4\hbar\omega$

More elegant proof

Yamada et al., PTP120(2008)

$$|0_1^+\rangle \propto \mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\}$$

$$|0_2^+\rangle \propto \mathcal{A} \left\{ \chi_0(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\}$$

$$\chi_0(\xi_3) = \sum_{Q \geq 6} c_Q u_{Q0}(\xi_3)$$

$$\begin{aligned} M(0_1^+ - 0_2^+) &= \langle 0_1^+ | \mathcal{O} | 0_2^+ \rangle \\ &\propto \langle \mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} | \mathcal{O} | \mathcal{A} \left\{ \chi_0(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \\ &\propto c_6 \langle \mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} | \mathcal{O} | \mathcal{A} \left\{ u_{60}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \\ &\propto c_6 \langle u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) | \mathcal{A} \left\{ \mathcal{O} u_{60}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \\ &= c_6 \langle u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) | \mathcal{A} \left\{ (3\xi_3^2 u_{60}(\xi_3)) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \end{aligned}$$

$$\therefore \langle u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) | \mathcal{A} \left\{ u_{60}(\xi_3) (\mathcal{O}(^{12}\text{C}) \phi_{0+}(^{12}\text{C})) \phi(\alpha) \right\} \rangle = 0$$

$$\langle u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) | \mathcal{A} \left\{ u_{60}(\xi_3) \phi_{0+}(^{12}\text{C}) (\mathcal{O}(\alpha) \phi(\alpha)) \right\} \rangle = 0$$

conservation law of harmonic oscillator quanta in bra an ket

$$\begin{aligned} \mathcal{O} &= \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2 \\ &= \mathcal{O}(\alpha) + \mathcal{O}(^{12}\text{C}) + 3\xi_3^2 \end{aligned}$$

More elegant proof

Yamada et al., PTP120(2008)

$$|0_1^+\rangle \propto \mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\}$$

$$|0_2^+\rangle \propto \mathcal{A} \left\{ \chi_0(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\}$$

$$\chi_0(\xi_3) = \sum_{Q \geq 6} c_Q u_{Q0}(\xi_3)$$

$$\begin{aligned} M(0_1^+ - 0_2^+) &= \langle 0_1^+ | \mathcal{O} | 0_2^+ \rangle \\ &\propto \langle \mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} | \mathcal{O} | \mathcal{A} \left\{ \chi_0(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \\ &\propto c_6 \langle \mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} | \mathcal{O} | \mathcal{A} \left\{ u_{60}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \\ &\propto c_6 \langle u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) | \mathcal{A} \left\{ \mathcal{O} u_{60}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \\ &= c_6 \langle u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) | \mathcal{A} \left\{ (3\xi_3^2 u_{60}(\xi_3)) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \end{aligned}$$

Using expansion of $3\xi_3^2 u_{60}(\xi_3)$

$$3\xi_3^2 u_{60}(\xi_3) = \sum_{Q \geq 4} u_{Q0}(\xi_3) \langle u_{Q0} | 3\xi_3^2 | u_{60} \rangle$$

$$\begin{aligned} M(0_1^+ - 0_2^+) &\propto c_6 \langle u_{40} | 3\xi_3^2 | u_{60} \rangle \\ &\quad \times \langle u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) | \mathcal{A} \left\{ u_{40}(\xi_3) \phi_{0+}(^{12}\text{C}) \phi(\alpha) \right\} \rangle \\ &\propto c_6 \langle u_{40} | 3\xi_3^2 | u_{60} \rangle \end{aligned}$$

$$\begin{aligned} \mathcal{O} &= \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2 \\ &= \mathcal{O}(\alpha) + \mathcal{O}(^{12}\text{C}) + 3\xi_3^2 \end{aligned}$$

Bayman-Bohr theorem

Nucl. Phys. 9, 596 (1958/1959)

$$\frac{1}{\sqrt{16!}} \det |(0s)^4(0p)^{12}| \times [\phi_{cm}(\mathbf{R}_{cm})]^{-1} : \text{closed shell}$$

$$= N_0 \sqrt{\frac{12!4!}{16!}} A \left\{ \left[u_{40}(\xi_3, 3\nu) \phi_{L=0}({}^{12}\text{C}) \right]_{J=0} \phi(\alpha) \right\}$$

relative wf (S-wave)

$$= N_2 \sqrt{\frac{12!4!}{16!}} A \left\{ \left[u_{42}(\xi_3, 3\nu) \phi_{L=2}({}^{12}\text{C}) \right]_{J=0} \phi(\alpha) \right\}$$

relative wf (D-wave)

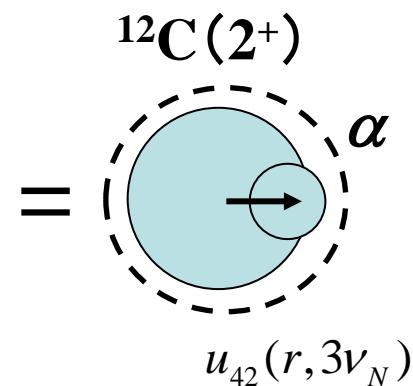
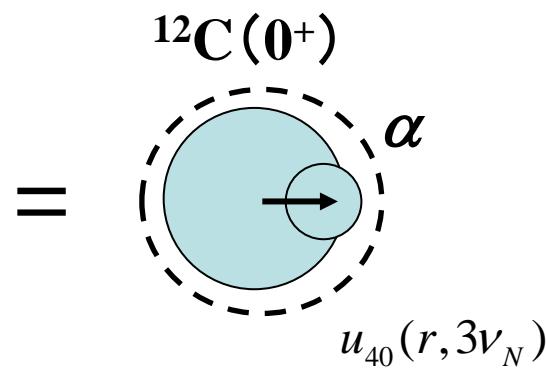
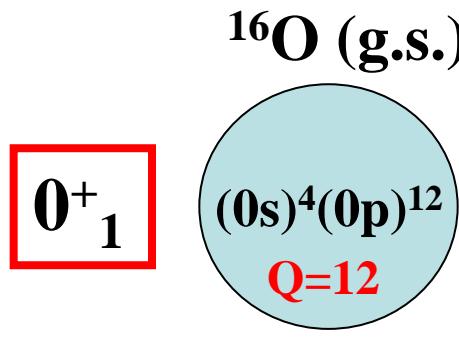
c.o.m. w.f. of ${}^{16}\text{O}$

$$\phi_{cm}(\mathbf{R}_{cm}) = \left(\frac{32\nu}{\pi} \right)^{3/4} \exp(-16\nu \mathbf{R}_{cm}^2)$$

α -degree of freedom

→ G.S. has mean-field-type and α -cluster degrees of freedom.

We call dual nature of g.s.

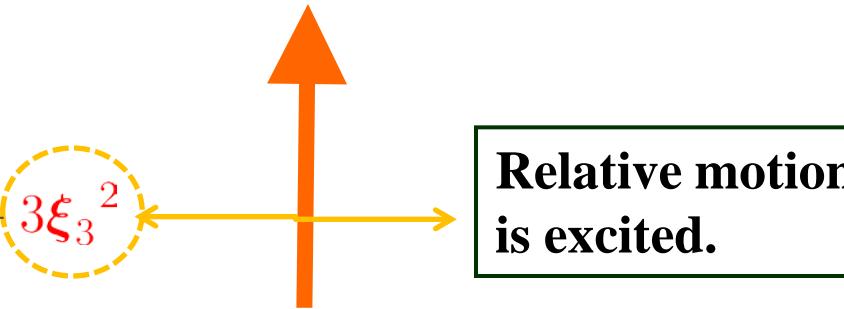


Monopole transitions: $0^+_1 - 0^+_2$, $0^+_1 - 0^+_3$

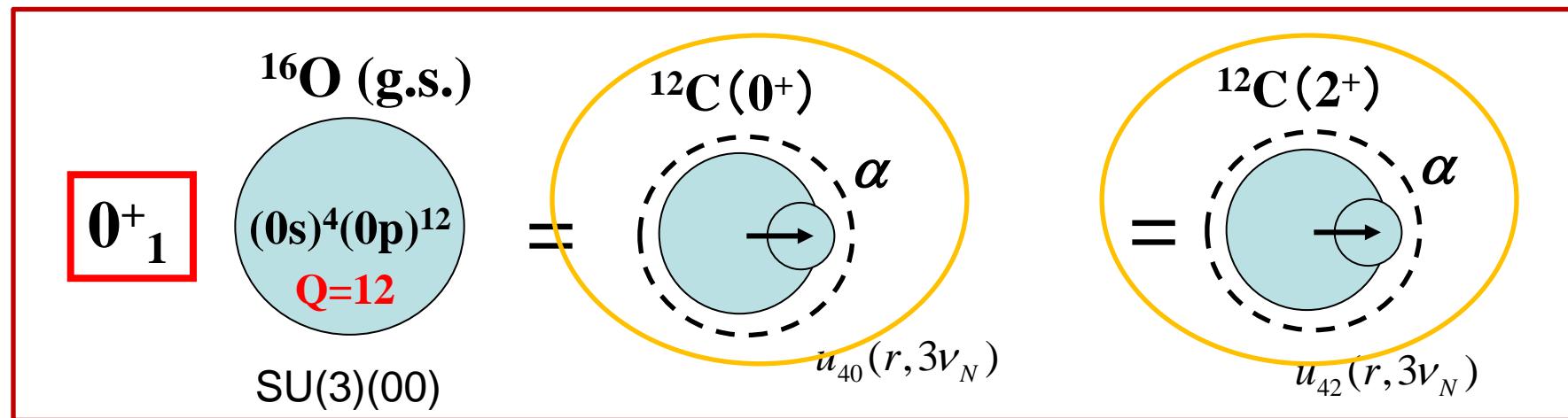
Monopole operator

$$\mathcal{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2$$

$$= \mathcal{O}(\alpha) + \mathcal{O}(^{12}\text{C}) + 3\xi_3^2$$



Yamada et al.,
PTP120 (2008)

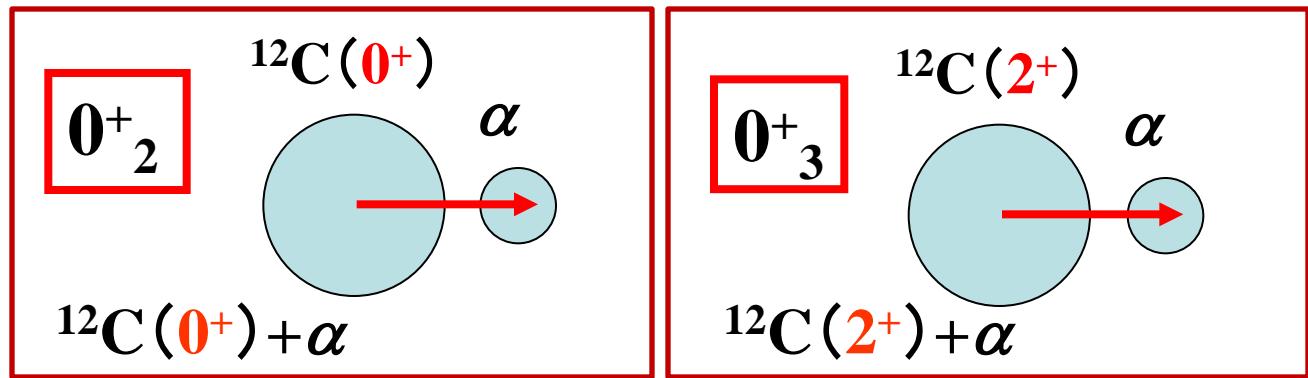


Monopole transitions: $0^+_1 - 0^+_2$, $0^+_1 - 0^+_3$

Monopole operator

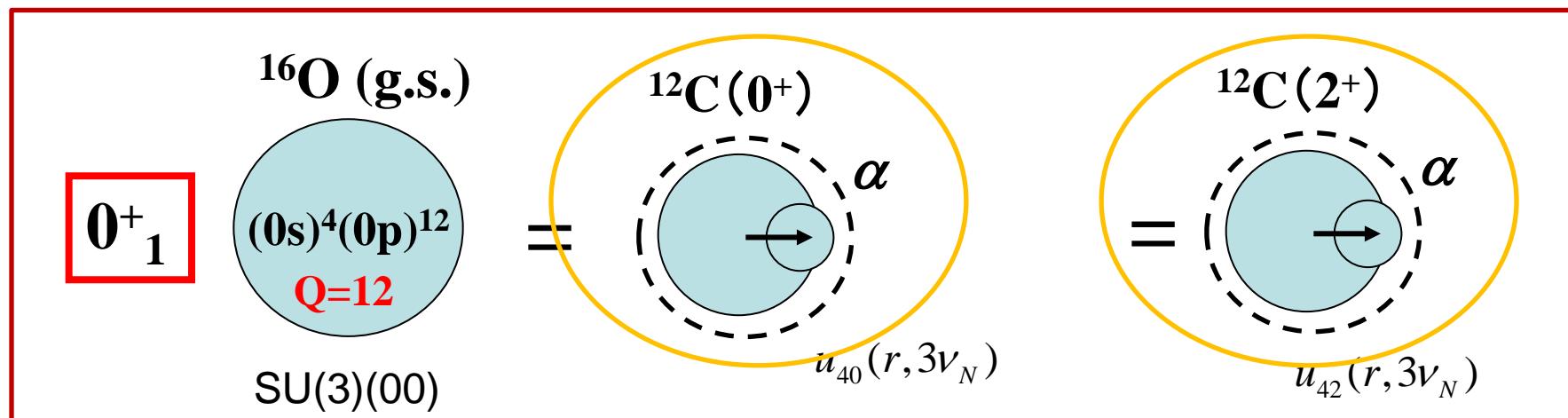
$$\mathcal{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2$$

$$= \mathcal{O}(\alpha) + \mathcal{O}(^{12}\text{C}) + 3\xi_3^2$$



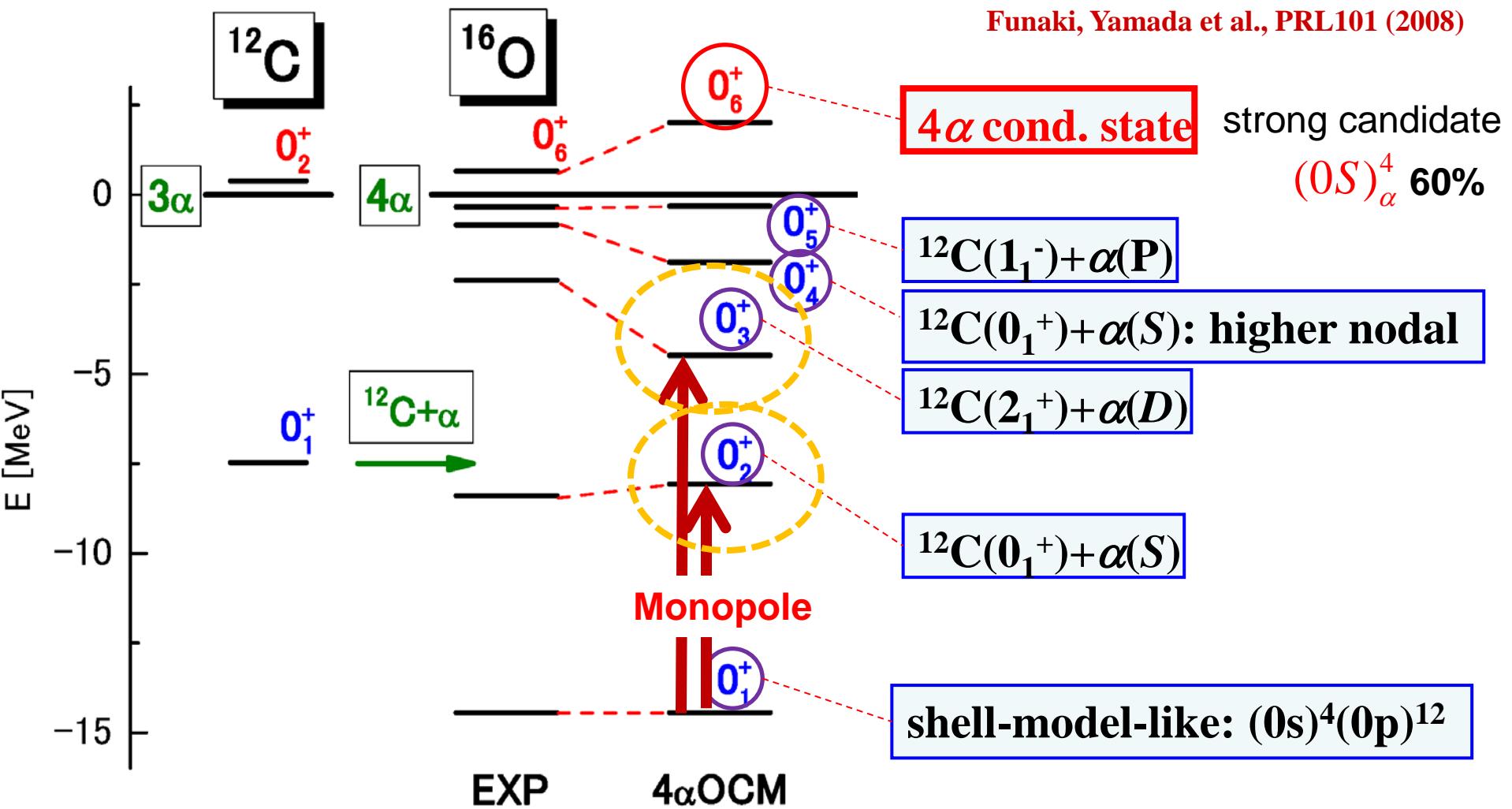
Relative motion
is excited.

Yamada et al.,
PTP120 (2008)

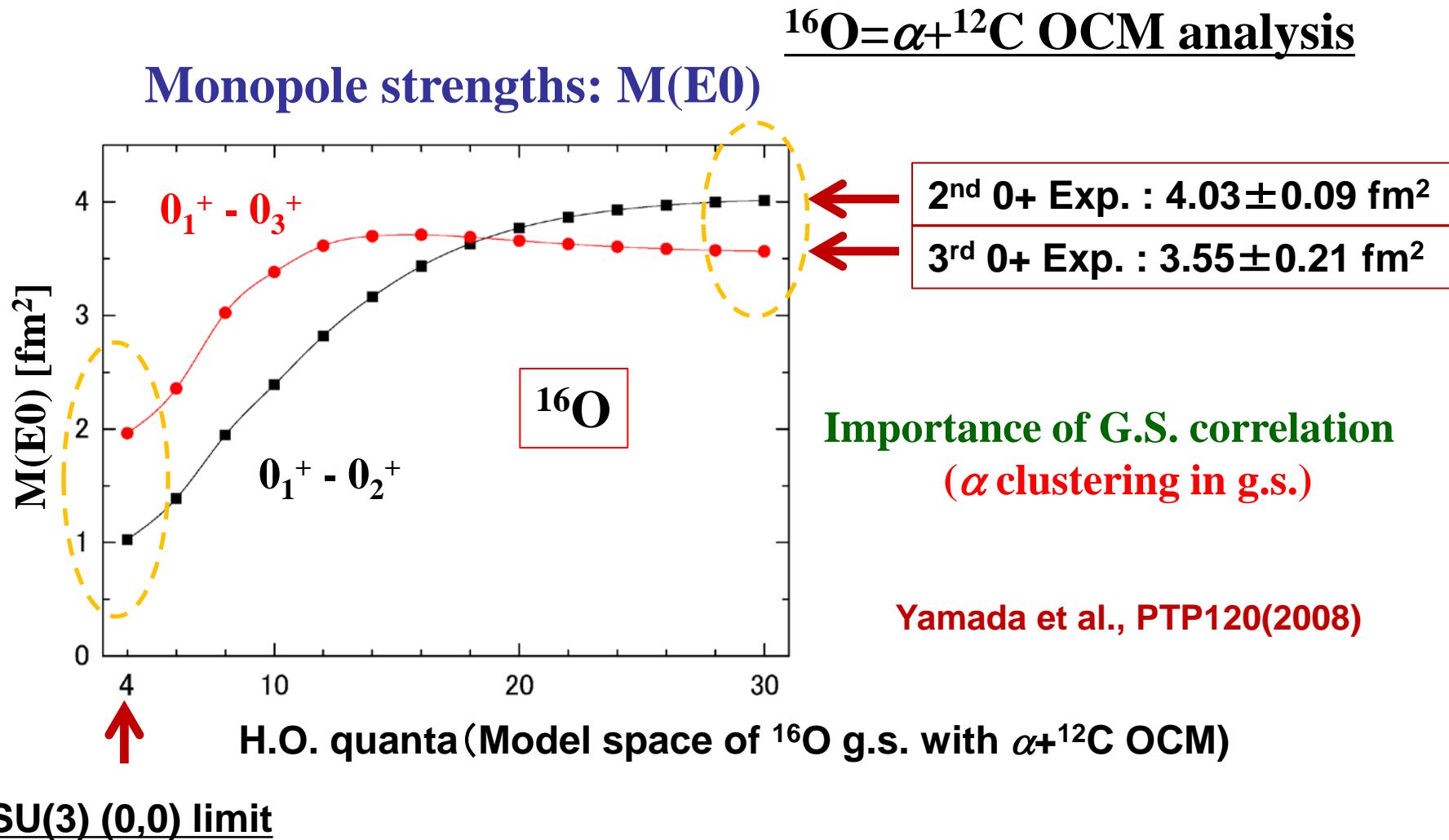


4 α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



Importance of α -type ground-state correlation in monopole strengths



Monopole excitation to $0^+_{2,3}$ and 0^+_6 state

$0^+_{2,3}$ states:

$^{12}\text{C}(0_1^+, 2_1^+) + \alpha$

main configuration

Bayman-Bohr theorem:

$(0s)^4(0p)^{12}$ has an $\alpha + ^{12}\text{C}(0_1^+, 2_1^+)$ degree of freedom

relative motion ($\alpha - ^{12}\text{C}$) is excited by IS monopole operator

0^+_6 state:

4α -gas like structure

main configuration

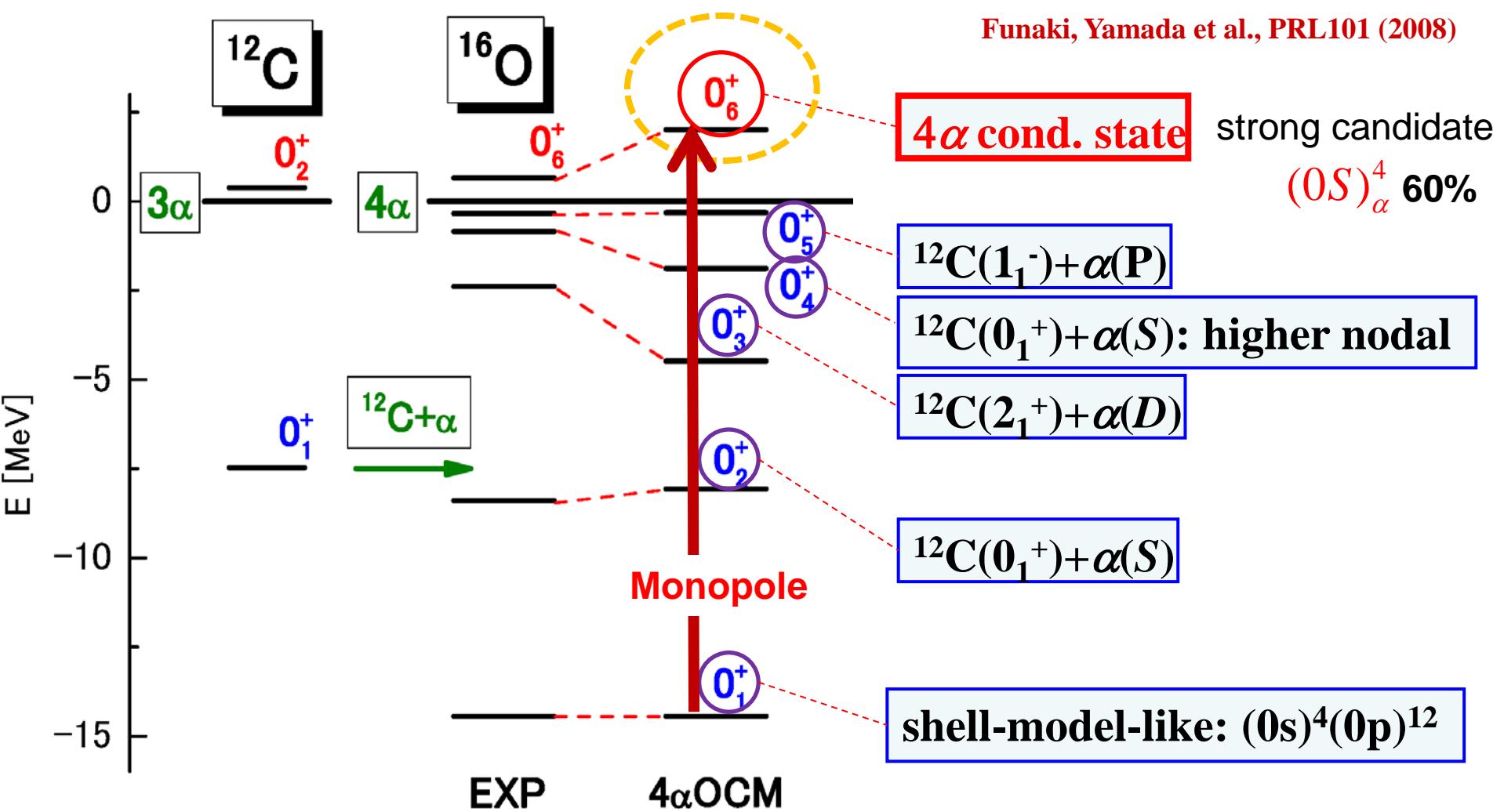
Bayman-Bohr theorem:

$(0s)^4(0p)^{12}$ has a 4α degree of freedom

relative motion among 4α is excited by IS monopole operator

4 α OCM calculation

Funaki, Yamada et al., PRL101 (2008)

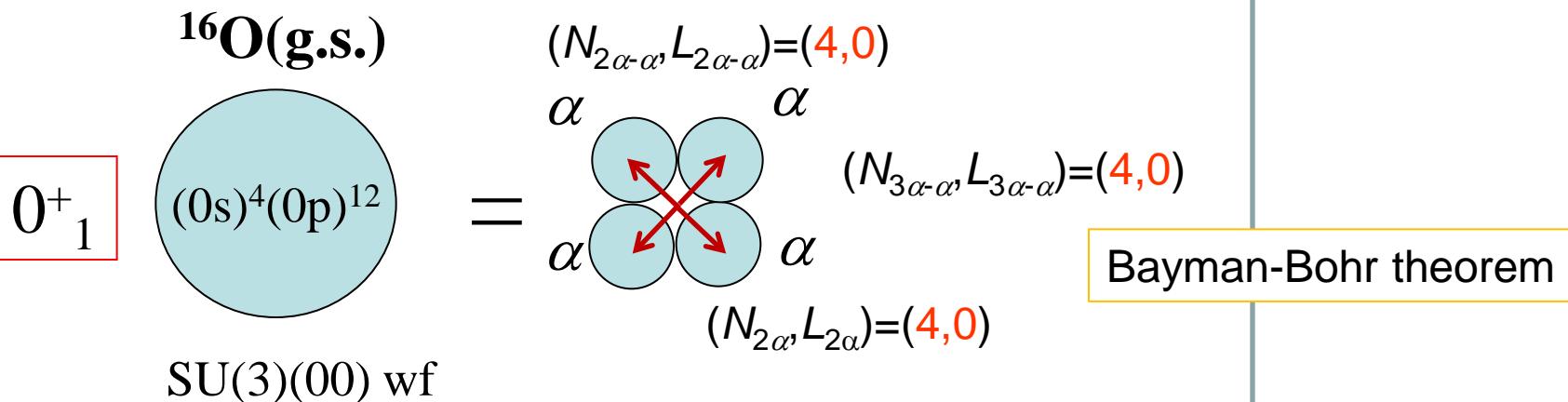


Bayman-Bohr theorem

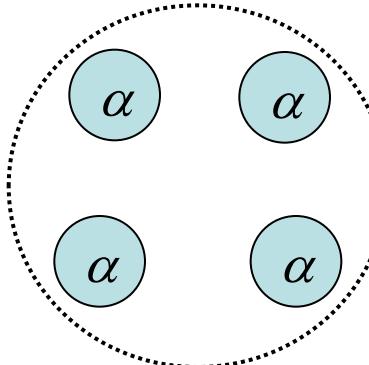
$$\frac{1}{\sqrt{16!}} \det |(0s)^4(0p)^{12}| \times [\phi_{\text{cm}}(\mathbf{R}_{\text{cm}})]^{-1} : \text{closed shell}$$

$$= \hat{N}_0 \sqrt{\frac{4!4!4!4!}{16!}} \mathcal{A} \left\{ \left[u_{40}(\xi_3, 3\nu) \left[u_{40}(\xi_2, \frac{8}{3}\nu) u_{40}(\xi_1, 2\nu) \right]_{L=0} \right]_{J=0} \right. \\ \left. \times \phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4) \right\} \quad \text{4}\alpha\text{-cluster wf}$$

→ G.S. has a 4α -cluster degree of freedom.



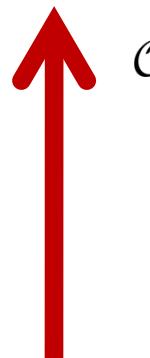
4α gas-like or $\alpha+^{12}\text{C}$ (Hoyle) $\approx 0^+_6$ state



4α -gas-like

$M(E0)=1.0 \text{ fm}^2$ by 4α OCM

Monopole transition



$$\mathcal{O} = \sum_{i=1}^{16} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2$$

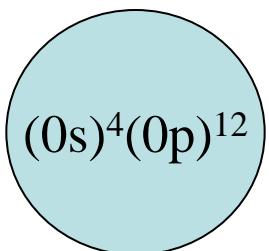
$$= \boxed{\sum_{k=1}^4 \sum_{i=1}^4 (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2} + \boxed{\sum_{k=1}^4 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\text{cm}})^2}$$

internal part

relative part

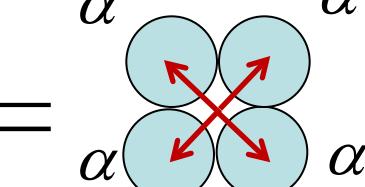
coherent
excitation

$^{16}\text{O}(\text{g.s.})$



0^+_1

$(N_{2\alpha-\alpha}, L_{2\alpha-\alpha})=(4,0)$



$(N_{3\alpha-\alpha}, L_{3\alpha-\alpha})=(4,0)$

$(N_{2\alpha}, L_{2\alpha})=(4,0)$

Bayman-Bohr theorem

SU(3)(00) wf

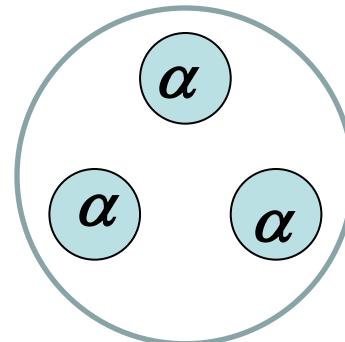
12C

3α gas-like = 0^+_2 (Hoyle) state

$$\mathcal{O} = \sum_{i=1}^{12} (\mathbf{r}_i - \mathbf{R}_{\text{cm}})^2$$

$$= \sum_{k=1}^3 \sum_{i=1}^4 (\mathbf{r}_{i+4(k-1)} - \mathbf{R}_{\alpha_k})^2 \quad \text{internal part}$$

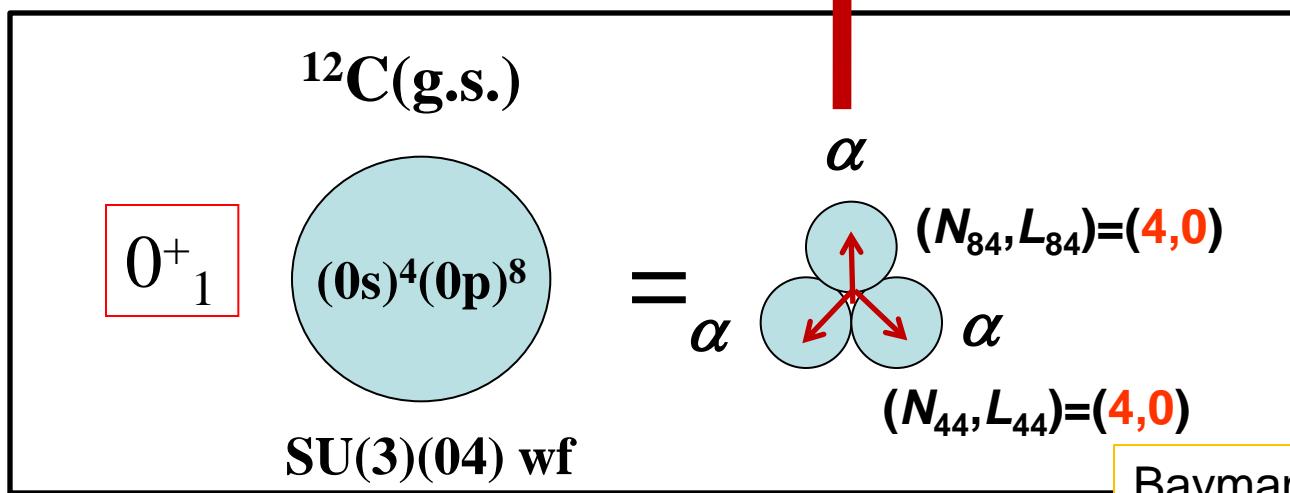
$$+ \sum_{k=1}^3 4(\mathbf{R}_{\alpha_k} - \mathbf{R}_{\text{cm}})^2 \quad \text{relative part}$$



portion of EWSR

$$M(E0) = 5.4 \pm 0.2 \text{ fm}^2 \text{ (16%)}$$

**Monopole transition
coherent excitation**



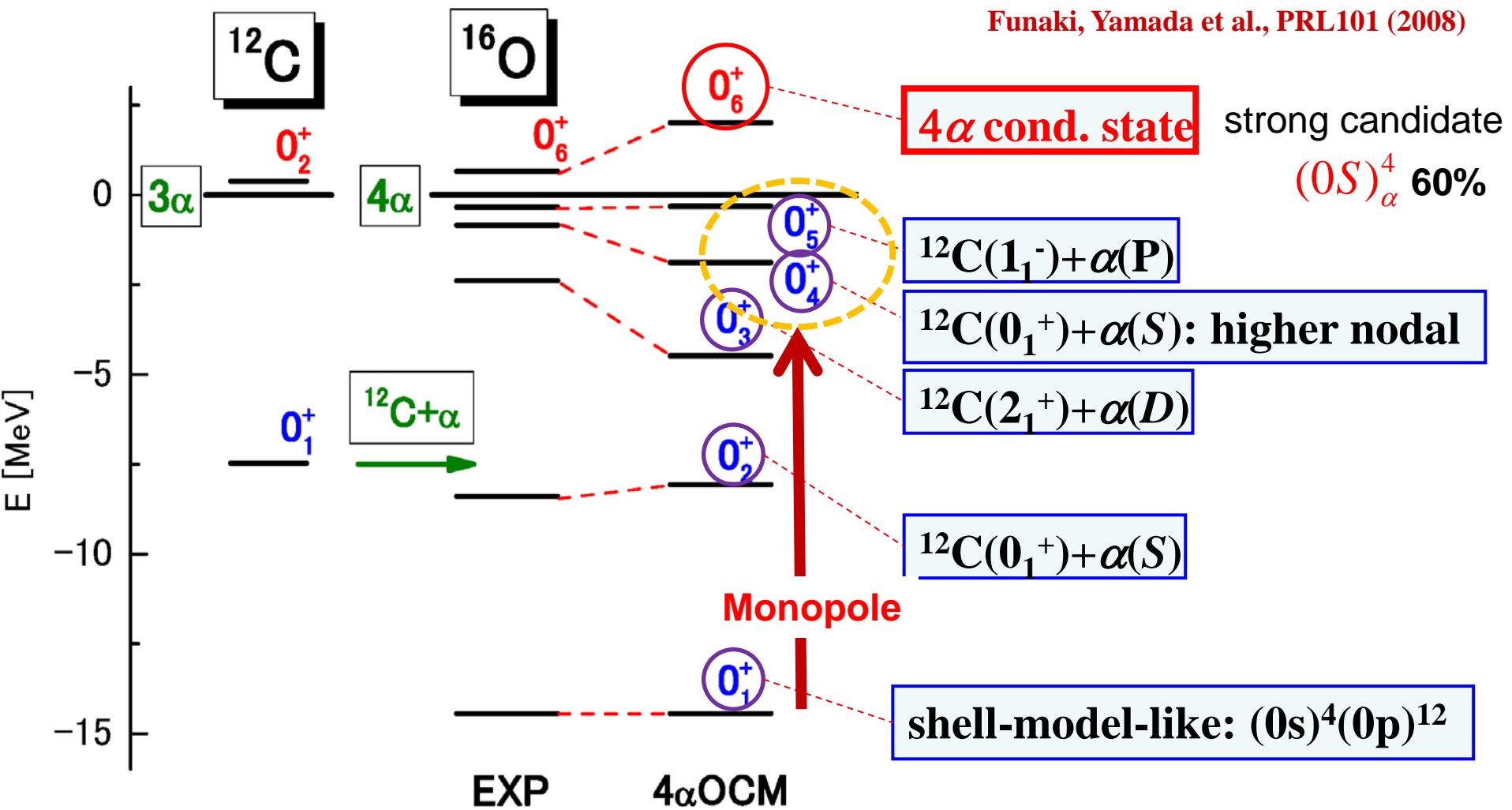
Yamada et al.,
PTP120 (2008)

Bayman-Bohr theorem

Dominance of $\text{SU}(3)(04)$: no-core shell model by Dytrych et al., PRL98 (2007)

4 α OCM calculation

Funaki, Yamada et al., PRL101 (2008)



Monopole excitation to 0^+_5 and 0^+_4 state

0^+_5 state:

$^{12}\text{C}(1_1^-) + \alpha(\text{P})$

main configuration

Bayman-Bohr theorem:

$(0\text{s})^4(0\text{p})^{12}$ has no configuration of $^{12}\text{C}(1^-) + \alpha$

Why this state is excited?

Coupling with $^{12}\text{C}(0+, 2+) + \alpha$ and $^{12}\text{C}(\text{Hoyle}) + \alpha$ configuration

Coherent contribution from these configurations

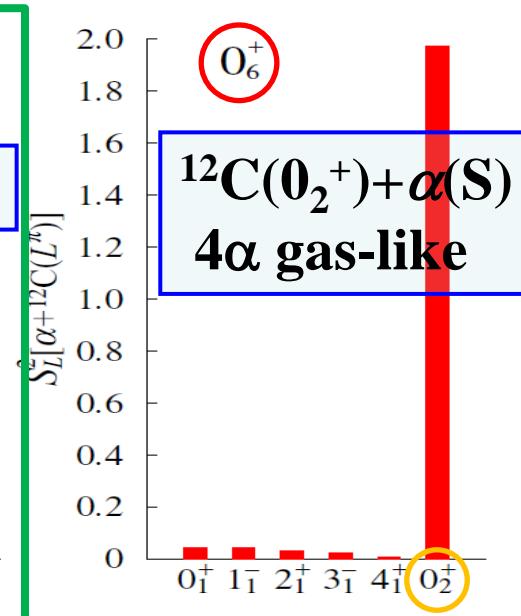
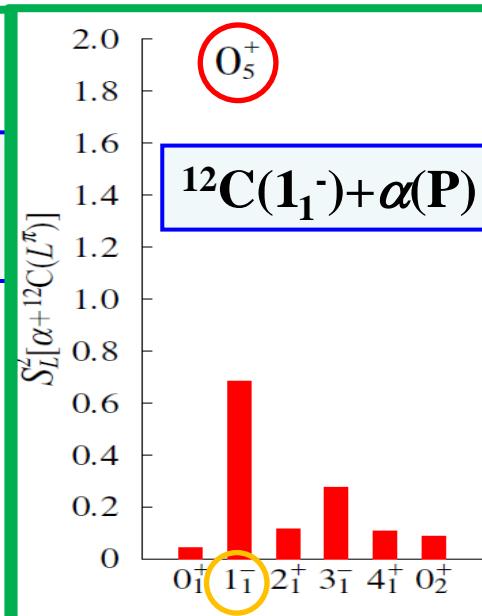
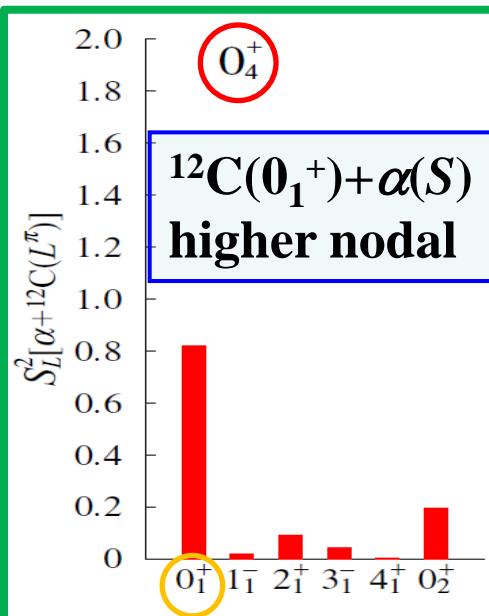
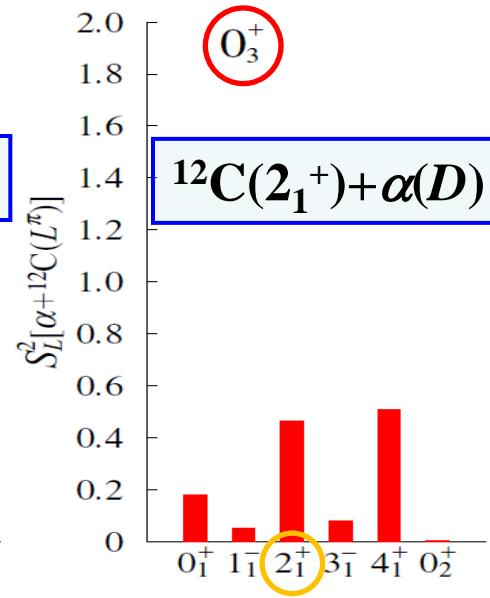
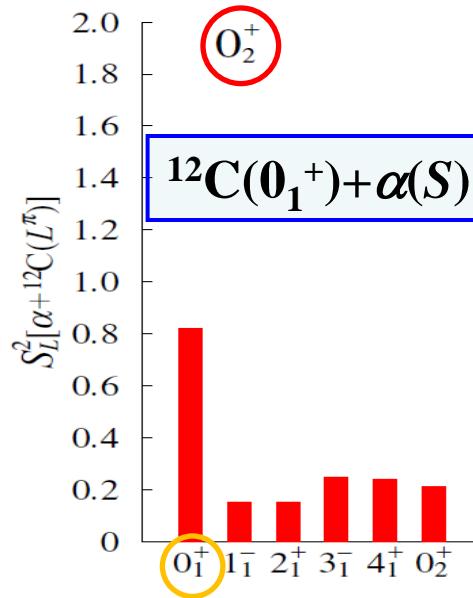
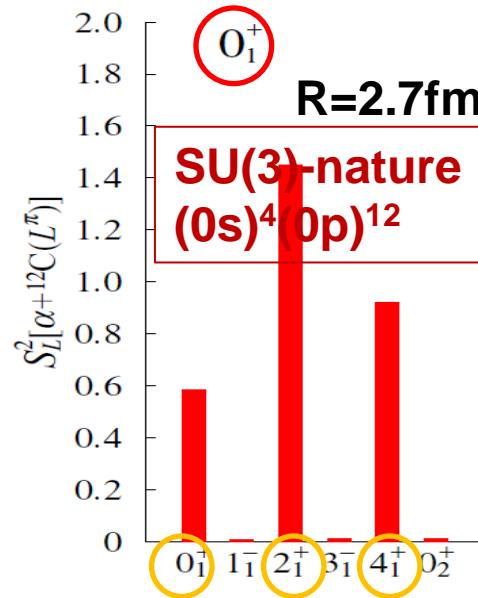
0^+_4 state:

$^{12}\text{C}(0_1^+) + \alpha(\text{S})$: higher nodal

Coherent contributions from

$^{12}\text{C}(0+, 2+) + \alpha$ and $^{12}\text{C}(\text{Hoyle}) + \alpha$ configurations

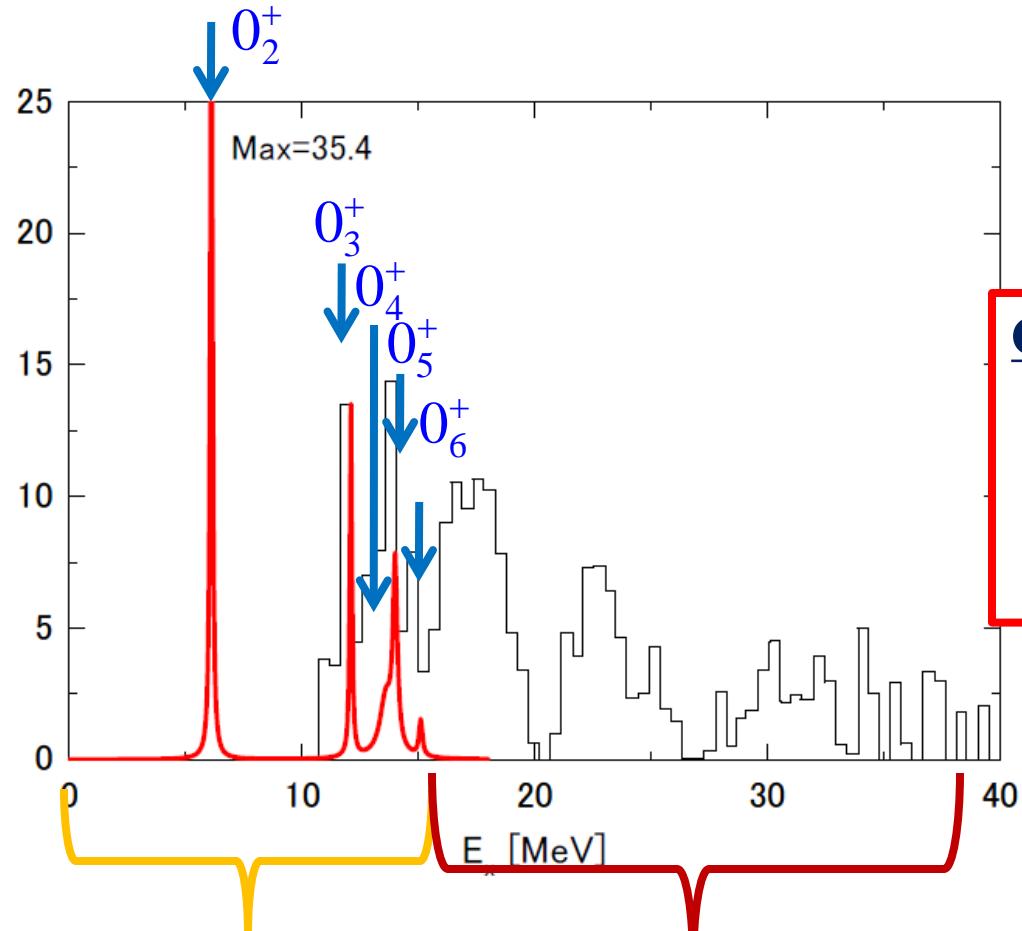
S^2 -factors of $\alpha + {}^{12}\text{C}(L^\pi)$ channels in 0^+ states of ${}^{16}\text{O}$



Funaki et al.

Exp. vs. Cal.

IS monopole S(E) with 4α OCM



Two features
in IS monopole excitations

Origin: dual nature of G.S. of ^{16}O
(1) α -clustering degree of freedom
(2) mean-field-type one
 $(0s)^4(0p)^{12} : \text{SU}(3)(00) = ^{12}\text{C} + \alpha :$
Bayman-Bohr theorem

Excitation to cluster states
(α -cluster type)

Monopole excitation
of mean-field type (RPA)

Isoscalar E0 strength in ^{16}O

- Isoscalar monopole excitations

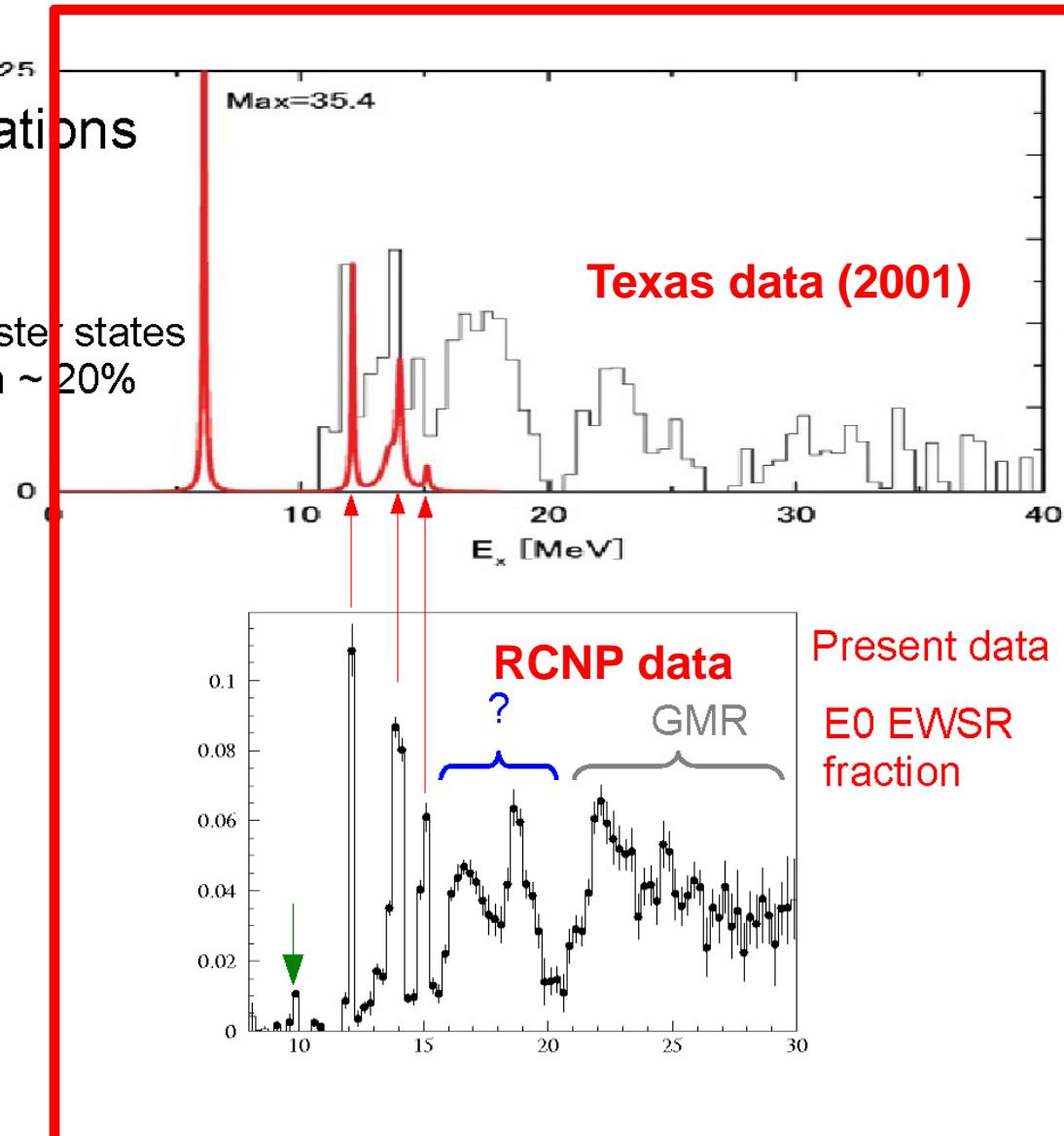
T. Yamada et al,
Phys. Rev.C 85 (2012) 034315

- Monopole excitations to α -cluster states
 $E_x \leq 16 \text{ MeV}$, EWSR fraction $\sim 20\%$

- $E_\alpha = 386 \text{ MeV} @ \text{RCNP}$

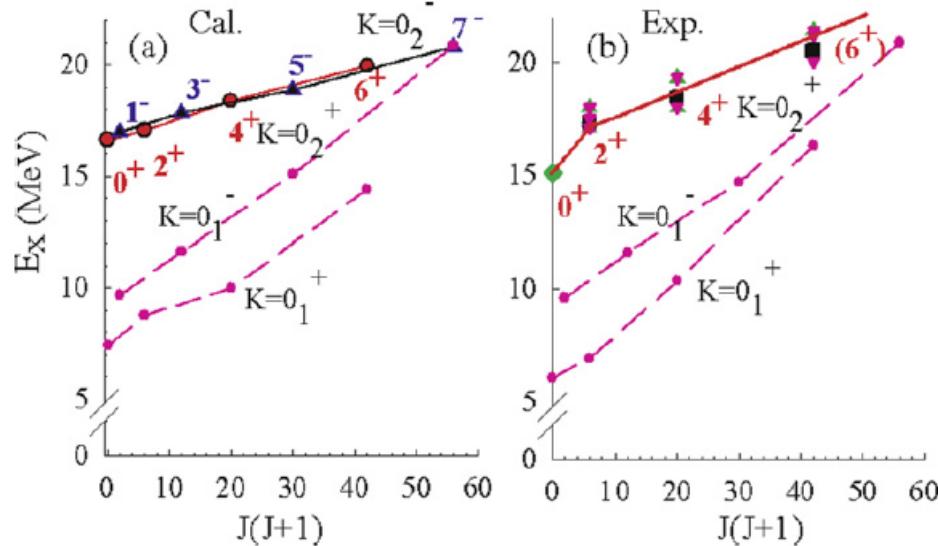
new analysis of $^{16}\text{O}(\alpha, \alpha')$

Itoh (Tohoku)'s talk,
RCNP workshop, 19 July 2012



Hoyle+ α states and Linear Chain States

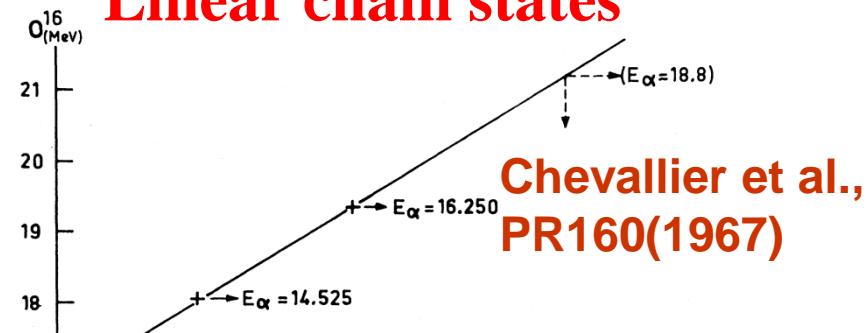
Hoyle+ α cluster



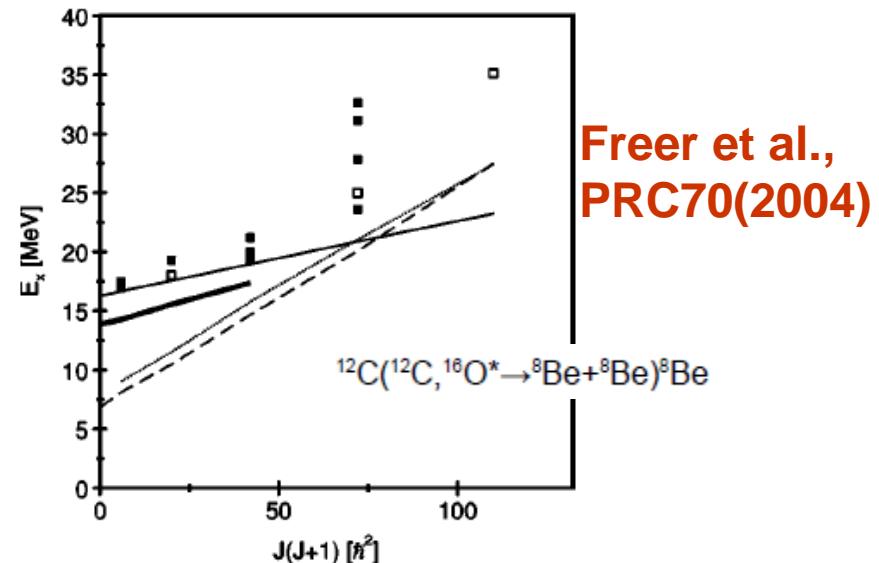
Ohkubo et al., PLB684(2010)

Funaki et al., 4 α OCM

Linear chain states



Chevallier et al., PR160(1967)



Freer et al., PRC70(2004)

Dual nature of the ground states in ^{16}O common to all $N=Z=\text{even}$ light nuclei

^{20}Ne , ^{24}Mg , ^{28}Si , ^{32}S ,

For example, ^{20}Ne g.s.

Dual nature of the ground states in ^{12}C and ^{16}O : common to all $N=Z=\text{even}$ light nuclei

^{20}Ne

$^{20}\text{Ne}, ^{24}\text{Mg}, ^{32}\text{S}, \dots, ^{44}\text{Ti}, \dots$

$$\begin{aligned}\Phi_J(^{20}\text{Ne}) &= |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\text{internal}} : \text{SU(3) wf} \\ &= N_J \sqrt{\frac{4!16!}{20!}} \mathcal{A} \left\{ u_{8J}(\underline{\mathbf{r}_{\alpha-^{16}\text{O}}}) \phi(\alpha) \phi(^{16}\text{O}) \right\} : \text{cluster wf} \\ &\quad \text{relative wf (J-wave)}\end{aligned}$$

Excitation of mean-field degree of freedom

→ $K^\pi = 2^-$ band : $5p1h$ state

Excitation of α -cluster degree of freedom

→ $\alpha + ^{16}\text{O}$ cluster states of $K^\pi = 0^+_4, 0^-$ bands

$K^\pi = 0^+_4$ band : higher nodal states

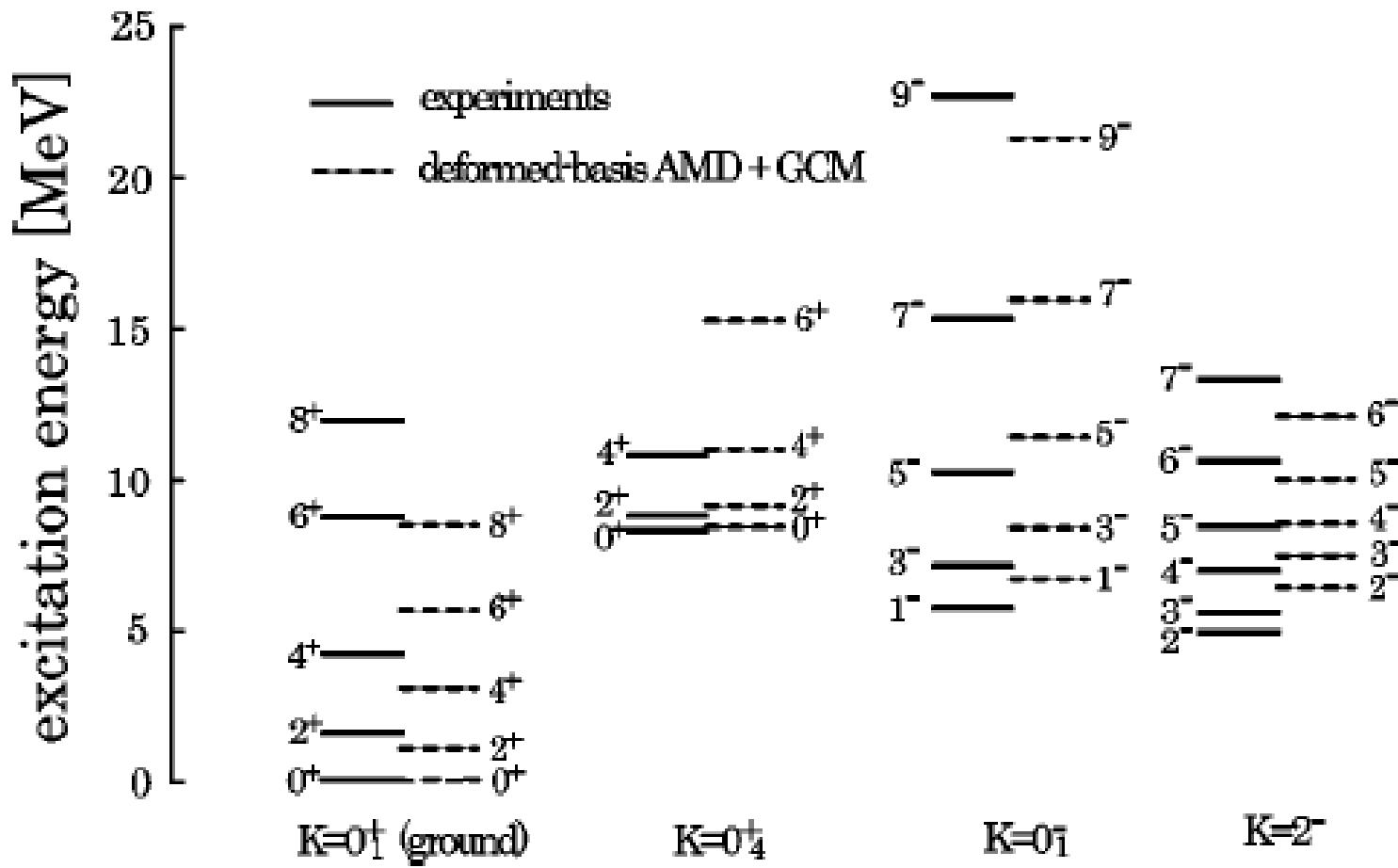
$\alpha + ^{16}\text{O}$ comp. = 80% for low spins: Q=10 quanta

$K^\pi = 0^-$ band : parity – doublet states

Almost pure $\alpha + ^{16}\text{O}$ structures for low spins: Q=9 quanta

^{20}Ne

AMD+GCM



Monopole strengths of cluster states in other light nuclei

^{16}O : 20% of total EWSR, as mentioned

^{12}C , ^{11}B , ^{20}Ne , ^{24}Mg , ...

^{12}Be , ...

^{16}O

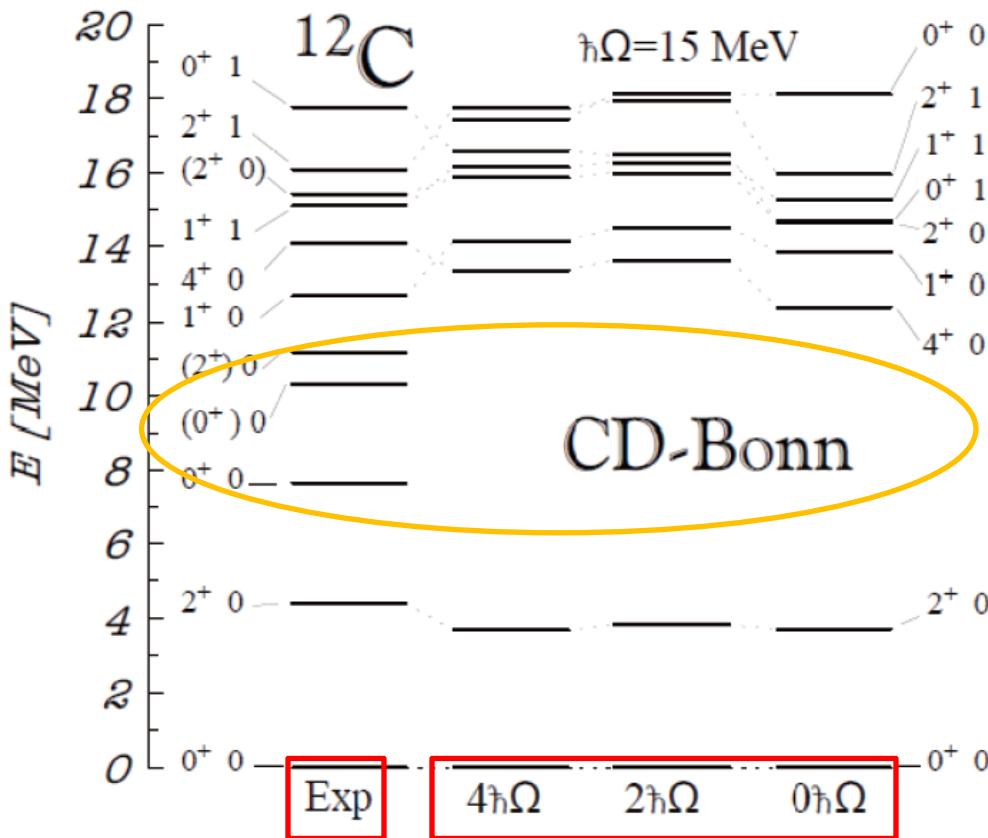
	Experiment				4 α OCM		
	Ex [MeV]	R [fm]	M(E0) [fm 2]	Γ [MeV]	R [fm]	M(E0) [fm 2]	Γ [MeV]
0^+_1	0.00	2.71			2.7		
0^+_2	6.05		3.55		3.0	3.9	
0^+_3	12.1		4.03		3.1	2.4	
0^+_4	13.6		no data	0.6	4.0	2.4	0.60
0^+_5	14.0		3.3	0.185	3.1	2.6	0.20
0^+_6	15.1		no data	0.166	5.6	1.0	0.14

over 15%
of total EWSR

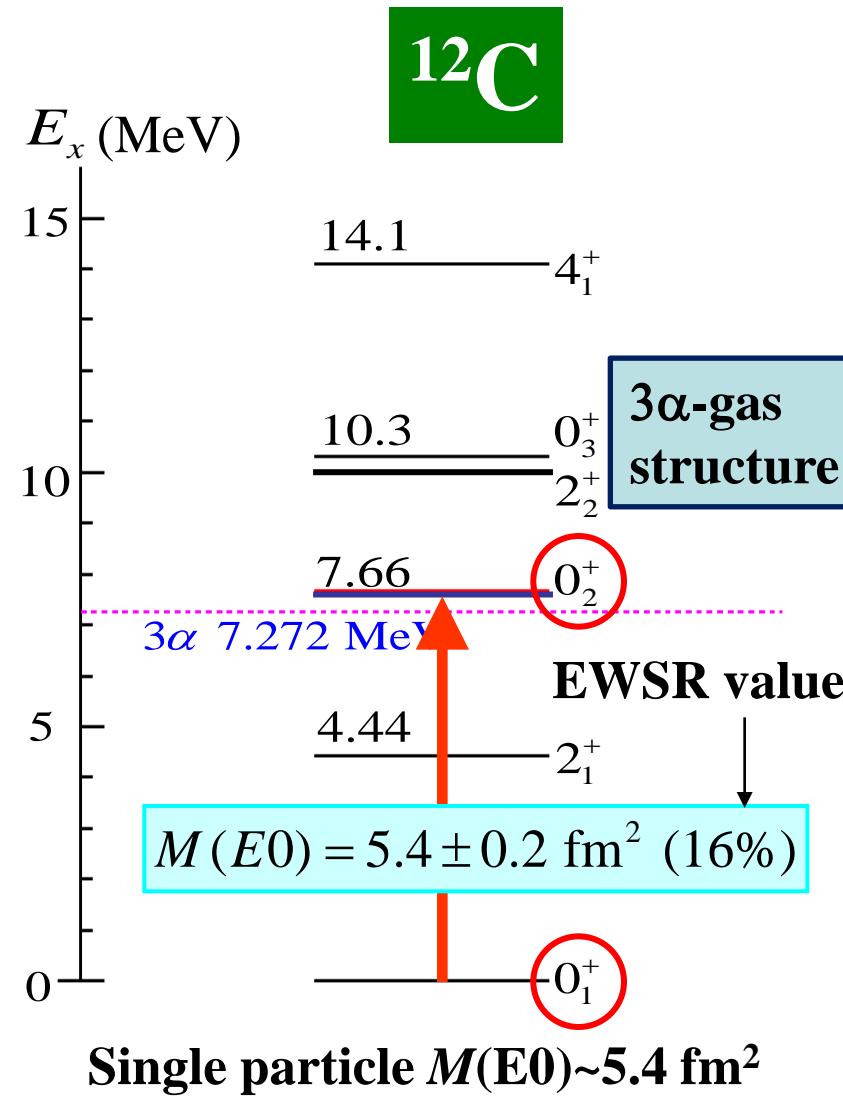
20%
of total EWSR

Monopole strengths $M(E0)$ in ^{12}C

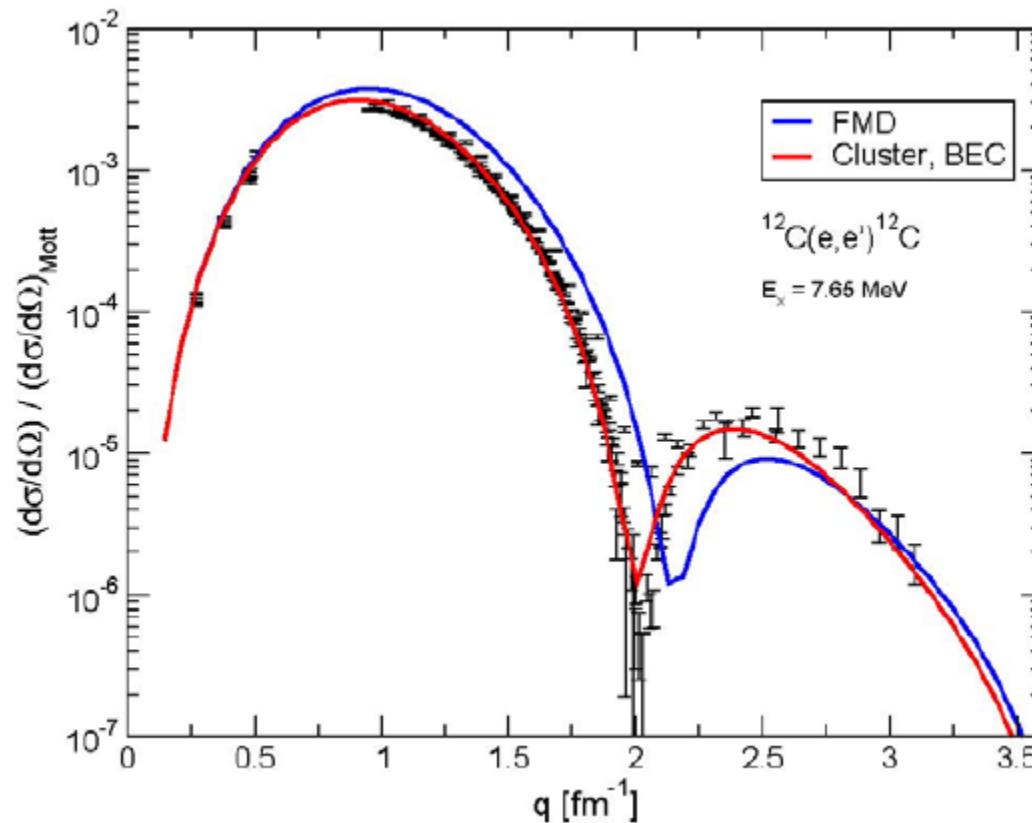
No-core shell model



P. Navr'atil et al., PRC62 (2000)



$^{12}\text{C}(\text{e},\text{e}')^{12}\text{C}$ (Hoyle)



FMD

: Chernykh et al., PRL98(2007)

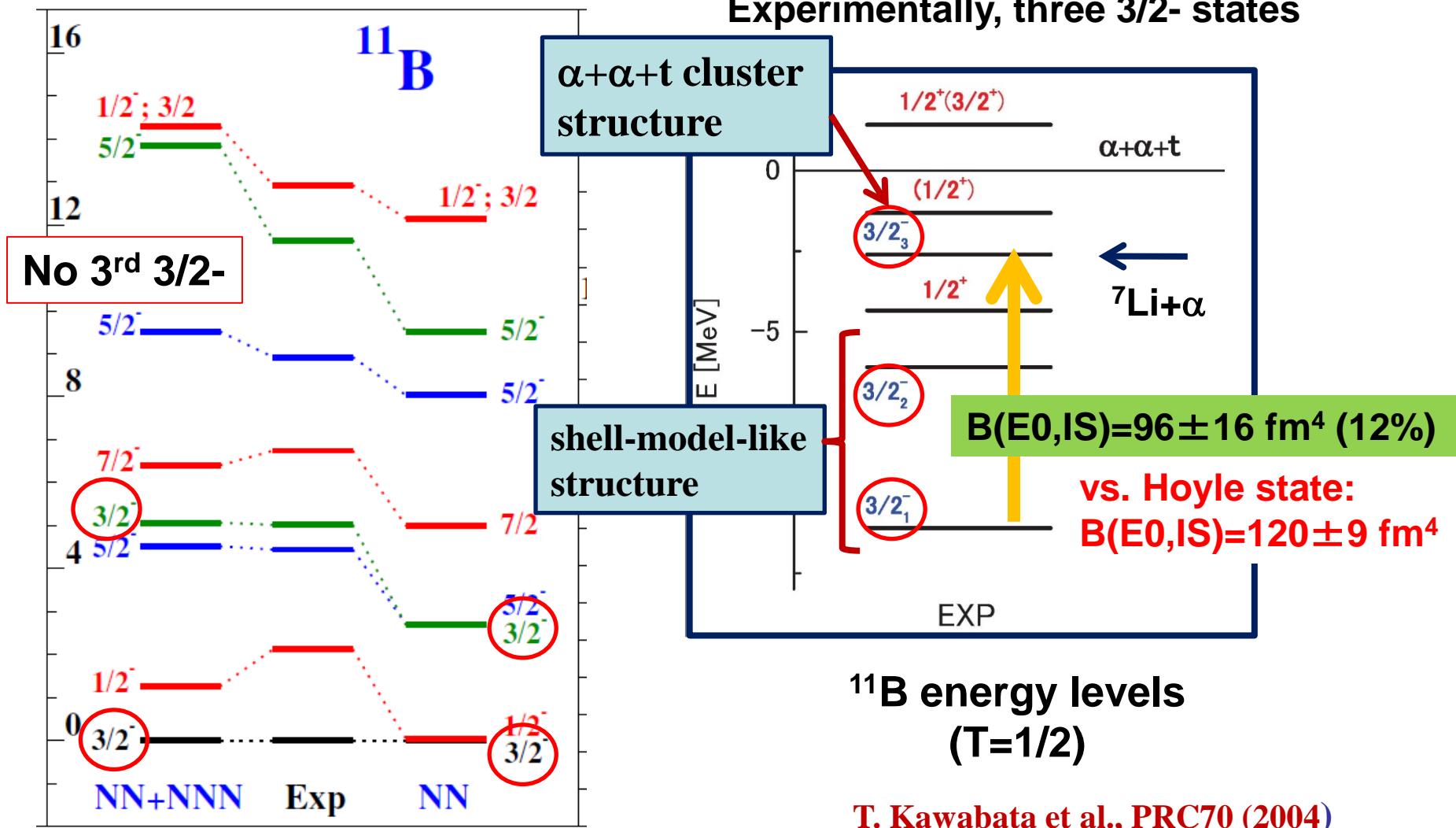
Cluster (3 α RGM): Kamimura, NPA351(1981)

BEC (3 α condensate: THSR): Funaki et al., PRC67(2003)

No-core shell model

11B

Navratil et al., JPG36(2009)

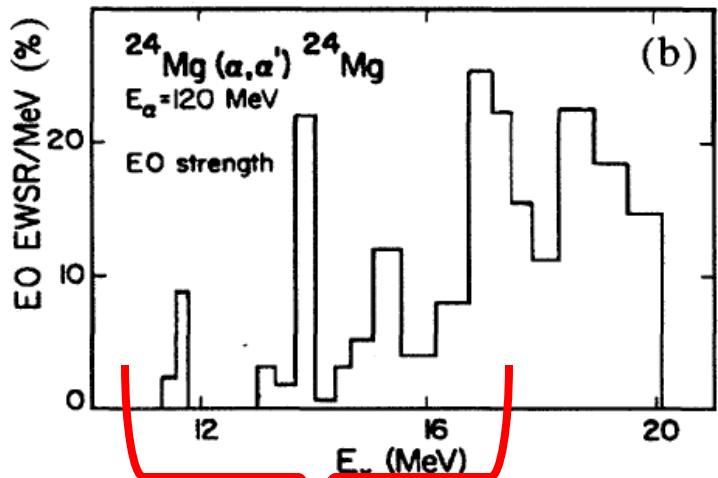
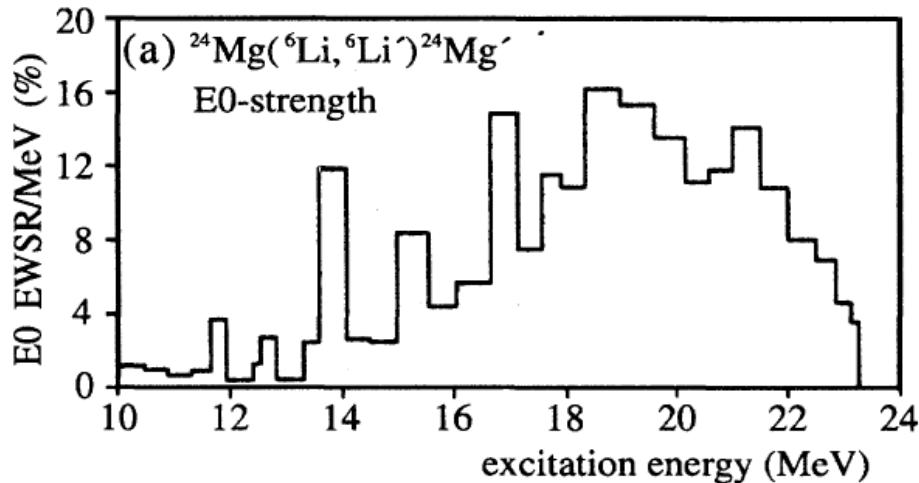


Observed levels of ^{20}Ne

	$^{12}\text{C}+\alpha+\alpha$	$^{16}\text{O}+\alpha$	
^{20}Ne	SU3(4,2)		
$^{12}\text{C}+^8\text{Be}$ thresh.	(0.0009 ± 0.0003)	(0.09 ± 0.01) (0.0002 ± 0.0001)	(<0.050) 14.31 6 ⁺
11.98	11.95 8 ⁺	12.59 6 ⁺	13.90 6 ⁺ (~0.058)
11.89 $^{12}\text{C}+2\alpha$ thresh.		12.13 6 ⁺	12.44 0 ⁺
10.78			(0.23) 10.79 4 ⁺
$^{16}\text{O}^*(6.05\text{MeV} \text{O}^+) + \alpha$ thresh.	8.78 6 ₁ (0.010 ± 0.002)	9.99 4 ₃ ⁺ (0.17 ± 0.03)	10.97 0 ₅ ⁺ (0.14)
	(0.04)	9.03 4 ₂ ⁺ (0.005)	(0.012) 10.55 4 ₄ ⁺
	7.42 2 ₁ ⁺ (0.15 ± 0.07)	7.83 2 ₃ ⁺ (0.007)	10.61 6 ⁻
	6.72 0 ₂ ⁺	7.19 0 ₃ ⁺ (>0.50)	10.26 5 ₂ (0.30)
4.73 $^{16}\text{O}+\alpha$ thresh.	4.25 4 ₁ ⁺		9.51 2 ₅ ⁺
G.S.	0. 0 ₁ ⁺		8.45 5 ⁻
			7.00 4 ⁻ 7.17 3 ₂ (0.27)
			5.62 3 ₁ ⁻ 5.78 1 ⁻ (>0.13)
			4.97 2 ⁻
			vs. Hoyle state: $M(E0, IS) = 5.4 \pm 0.2 \text{ fm}^2$
			Kimura et al.
			$K^\pi = 0_1^+$ $K^\pi = 0_2^+$ $K^\pi = 0_3^+$ $K^\pi = 0_4^+$ $(K^\pi = 2^+)$ $K^\pi = 2^-$ $K^\pi = 0^-$

$$\begin{aligned}
 \Phi_J(^{20}\text{Ne}) &= |(0s)^4(0p)^{12}(1s0d)^4 : SU(3)(80), J\rangle_{\text{internal}} \\
 &= N_J \mathcal{A} \left\{ u_{8J}(\mathbf{r}_{\alpha-(\alpha+^{12}\text{C})}) \underline{\phi(\alpha)} [u_{4L}(\mathbf{r}_{\alpha-^{12}\text{C}}) \underline{\phi(\alpha)} \phi_L(^{12}\text{C})]_0 \right\}
 \end{aligned}$$

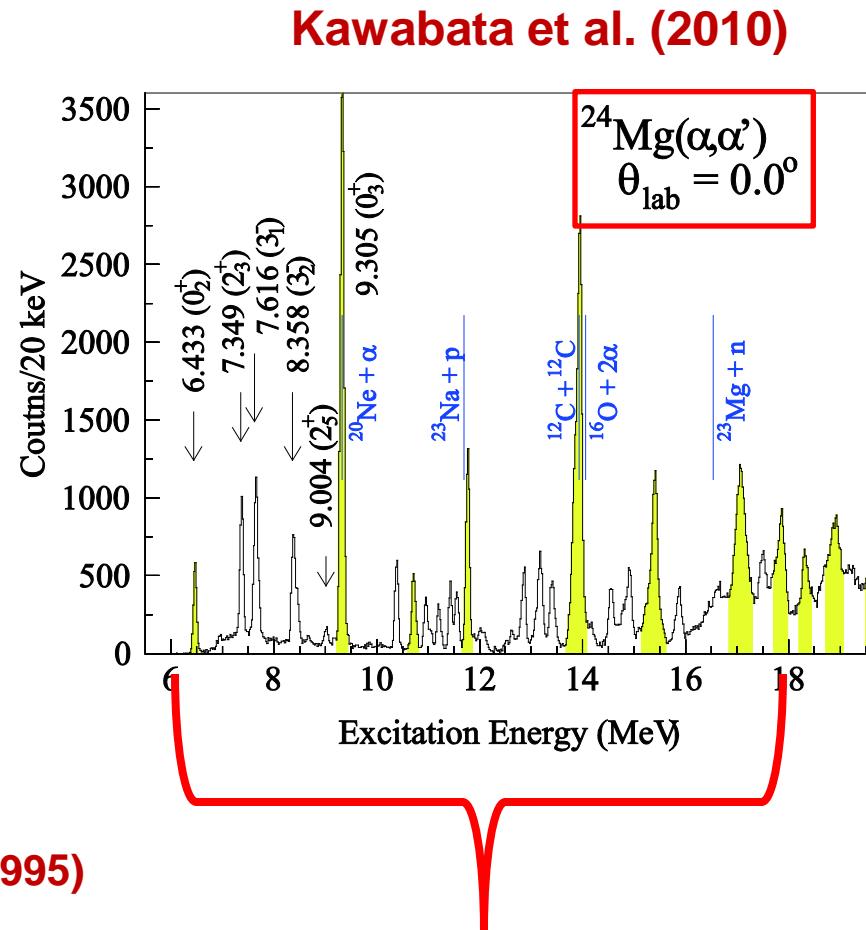
E0 strength in ^{24}Mg



Dennert et al., PRC(1995)

Excitation to cluster states? Should be studied !

AMD, OCM,....



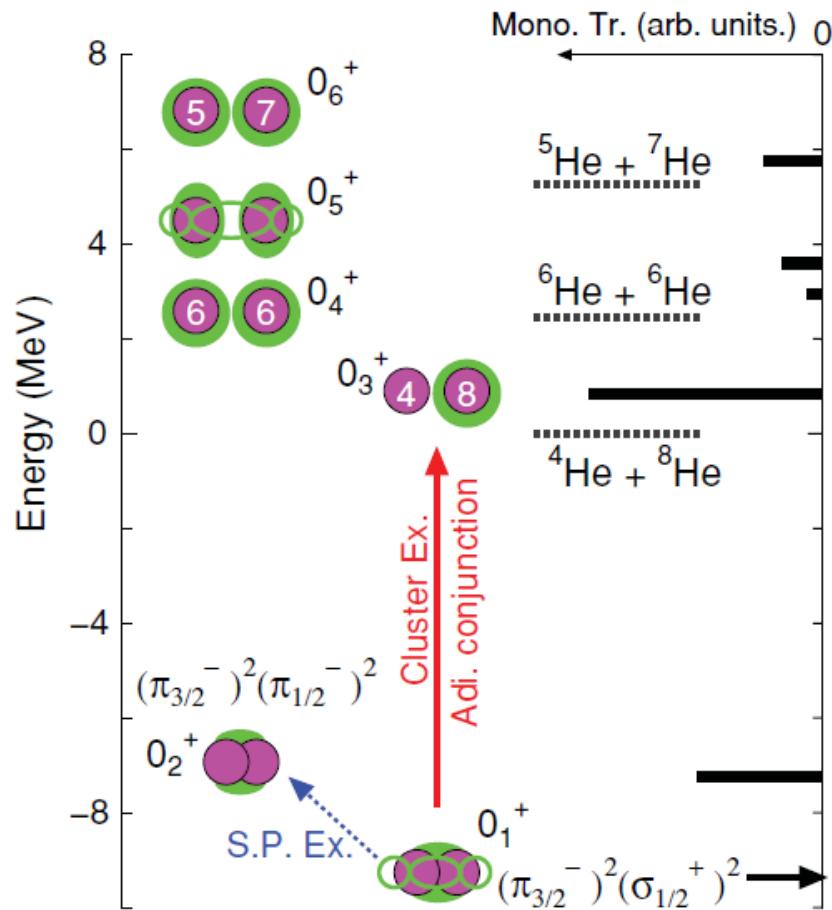
Isoscalar monopole transition in neutron-rich nuclei

12Be

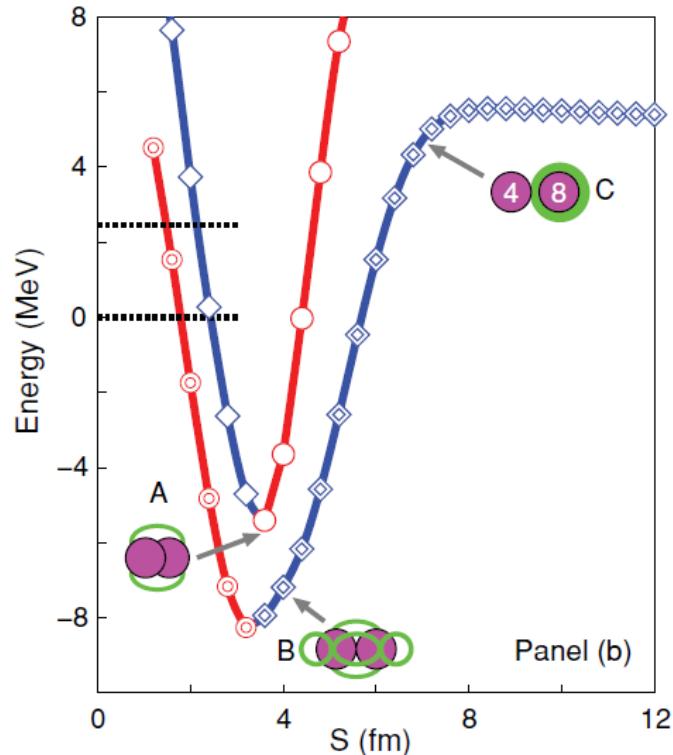
$\alpha + \alpha + 4n$ description

based on Generalized two-center cluster model

M. Ito, Phys. Rev. C 83, (2011)



Adiabatic potential



Panel (b)

Summary

- IS M(E0) trans. : useful to search for cluster states
[\leftrightarrow B(E2): nuclear deformation (Rainwater)]
They seem to have about 20% of EWSR in low energy region.
- IS monopole excitations have two features: ^{16}O (typical)
(i) α -cluster type: discrete peaks at $E_x \leq 15$ MeV
(ii) mean-field type: 3-bump structure (18,23,30 MeV)
- The origin: Dual nature of the ground state of ^{16}O .
G.S. has mean-field and α -cluster degrees of freedom
+ α -type g.s. correlation
- Dual nature is common in light nuclei.
- Two features of IS monopole excitations seems to be in general in light nuclei.

To search for cluster states in $4n$ nuclei, neutron-rich nuclei

\Rightarrow Importance of Systematic Analyses with IS M(E0) in Light Nuclei