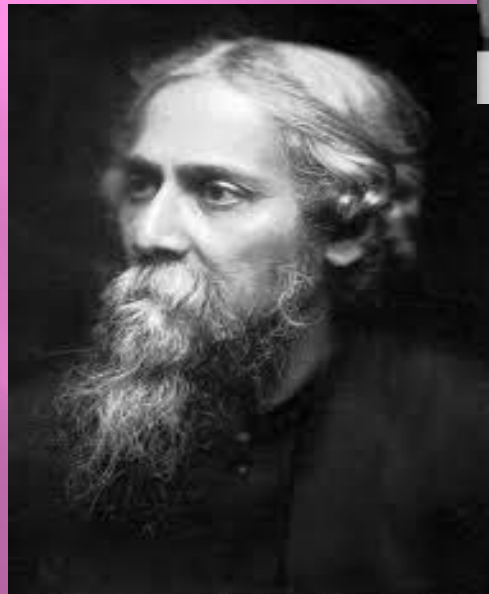




Madame Irene Curie-Joliot Opening the Institute



Rabindranath Tagor



C. V. Raman



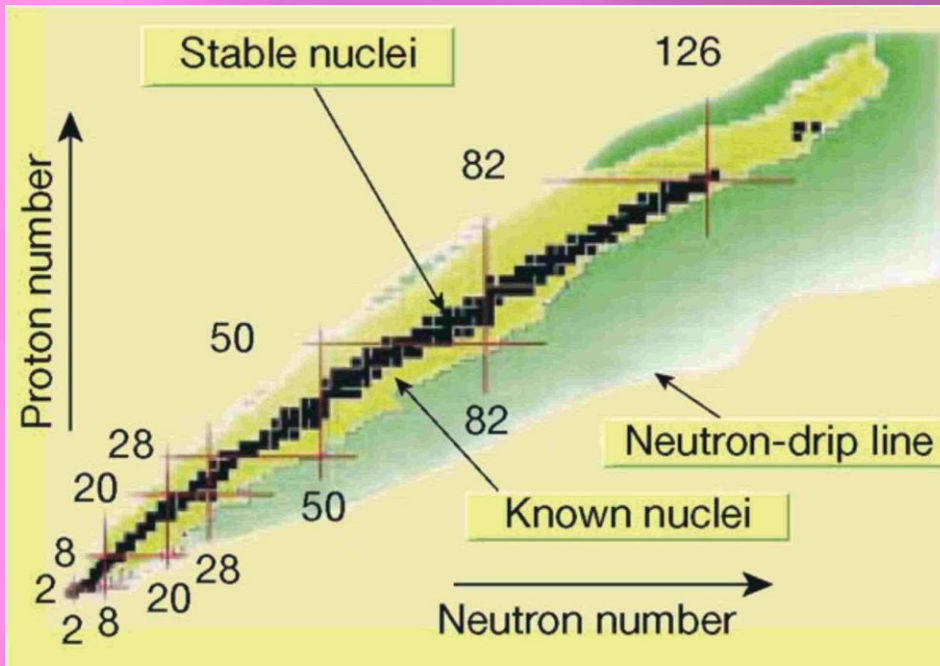
NUCLEAR SYMMETRY ENERGY FROM NUCLEAR OBSERVABLES

Bijay K. Agrawal
Saha Institute of Nuclear Physics
Kolkata, India



Outline of talk

- ✓ Introduction / Motivation
- ✓ Nuclear energy density functional
- ✓ Probes for nuclear symmetry energy
- ✓ Results
- ✓ Conclusions

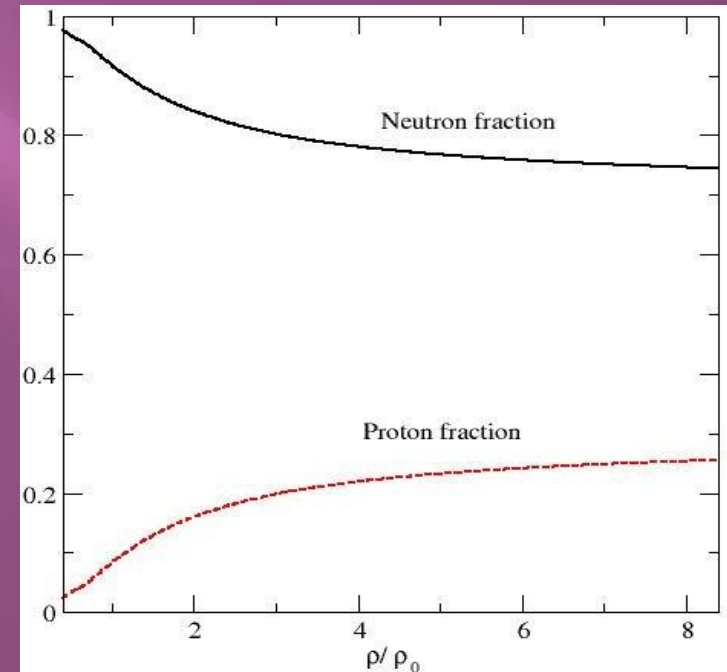
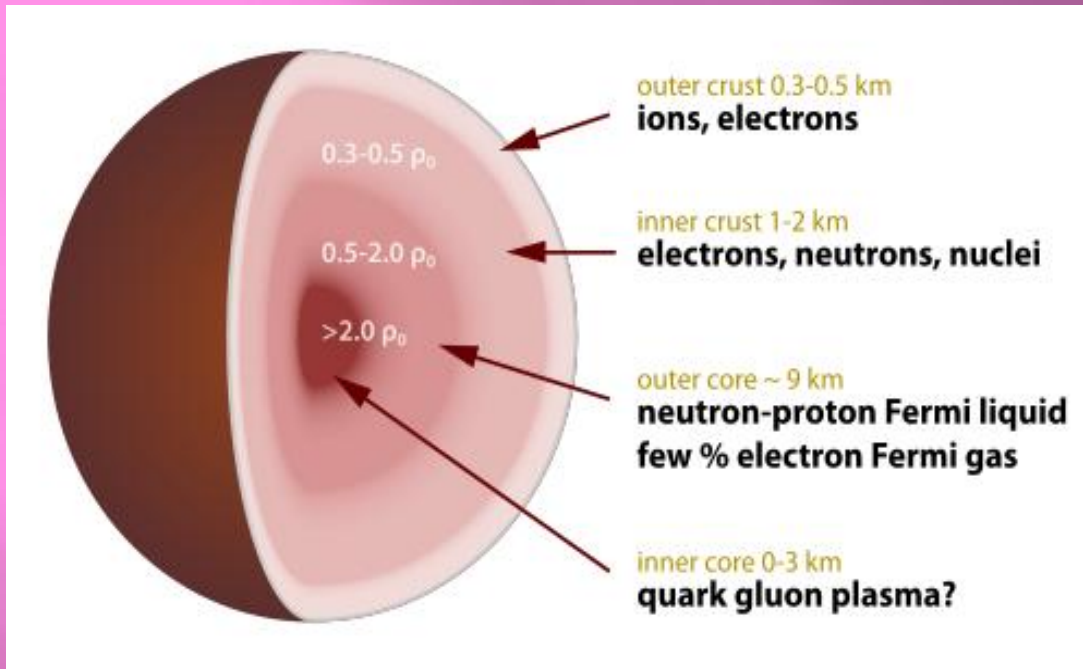


- Giant resonances
- Neutron rich nuclei
- Neutron drip line

$$B = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - (J - J_s A^{-1/3}) A I^2 + \dots$$

$$I = \frac{N - Z}{N + Z}$$

Neutron star internal structure



Structure Equations

$$\frac{dP}{dr} = -G \frac{\varepsilon \mathcal{M}}{r^2 c^2} \left[1 + \frac{P}{\varepsilon} \right] \left[1 + \frac{4\pi r^3 P}{\mathcal{M} c^2} \right] \left[1 - \frac{2G\mathcal{M}}{rc^2} \right]^{-1}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \frac{\varepsilon}{c^2}$$

Equation of State (EOS) for Asymmetric nuclear matter

$$E(\rho, I) = E(\rho, 0) + J(\rho) I^2 + \dots$$

$$E(\rho, 0) = E(\rho_0, 0) + \frac{1}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 K + \dots$$

$$J(\rho) = J(\rho_0) + \left(\frac{\rho - \rho_0}{3\rho_0} \right) L(\rho_0) + \dots$$

$$\rho_0 = 0.16 \text{ fm}^{-3}$$

$$E(\rho_0, 0) = -16 \text{ MeV}$$

$$K = 220 \text{ MeV}$$

$$J(\rho_0) \approx 32 \text{ MeV}$$

$$L(\rho_0) = 20 - 120 \text{ MeV} ?$$

Proton fraction at β equilibrium x_β

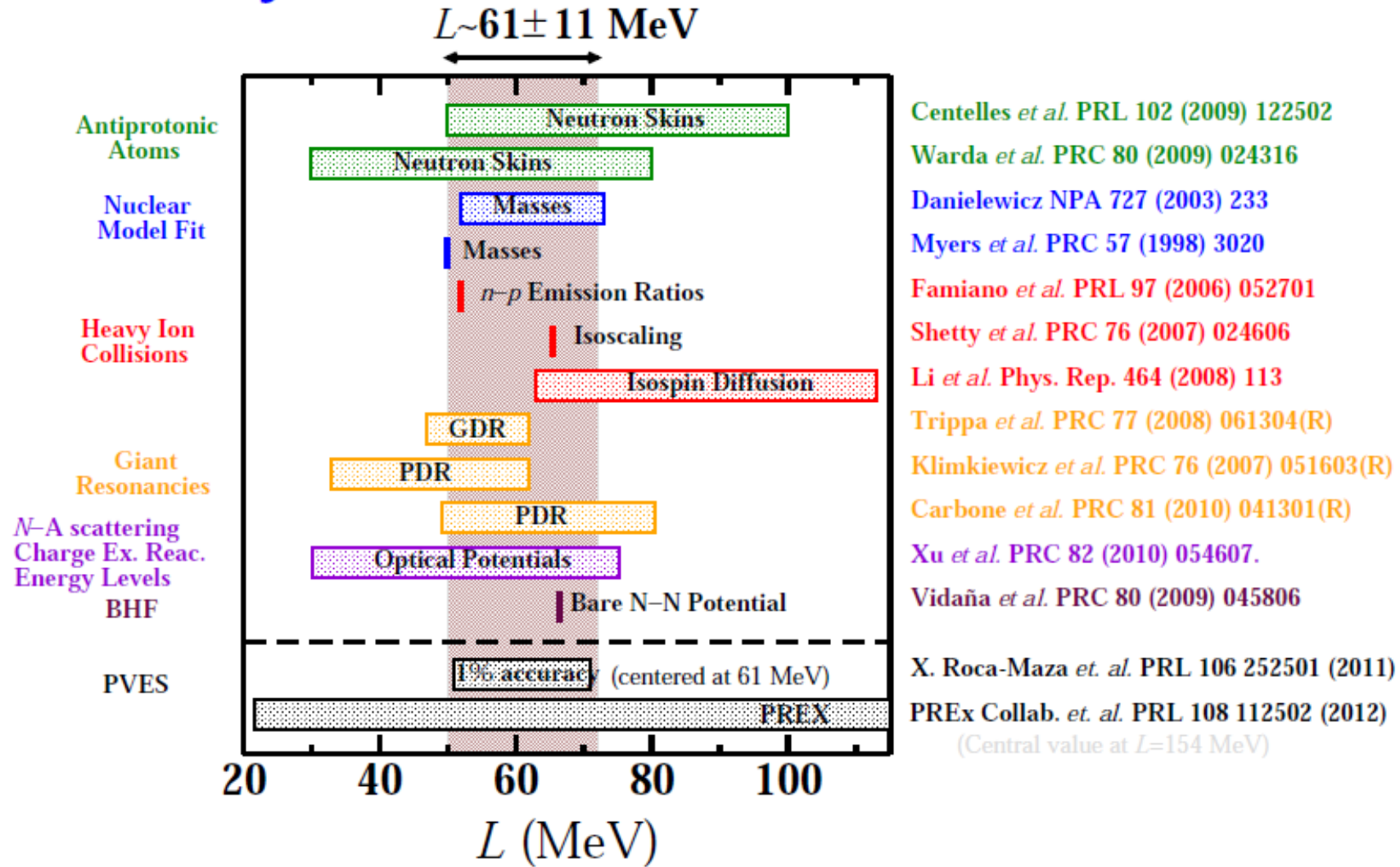
$$\hbar c (3\pi^2 \rho x_\beta)^{1/3} = 4(1 - 2x_\beta) J(\rho)$$

Neutron star properties

- Core-crust transition properties
- Cooling
- Composition
- Mass-radius relation ship

Knowledge of $J(\rho_0)$ and $L(\rho_0)$ is indispensable !!

Status of L



Centelles *et al.* PRL 102 (2009) 122502

Warda *et al.* PRC 80 (2009) 024316

Danielewicz NPA 727 (2003) 233

Myers *et al.* PRC 57 (1998) 3020

Famiano *et al.* PRL 97 (2006) 052701

Shetty *et al.* PRC 76 (2007) 024606

Li *et al.* Phys. Rep. 464 (2008) 113

Trippa *et al.* PRC 77 (2008) 061304(R)

Klimkiewicz *et al.* PRC 76 (2007) 051603(R)

Carbone *et al.* PRC 81 (2010) 041301(R)

Xu *et al.* PRC 82 (2010) 054607.

Vidaña *et al.* PRC 80 (2009) 045806

X. Roca-Maza *et al.* PRL 106 252501 (2011)

PREx Collab. *et al.* PRL 108 112502 (2012)

(Central value at $L=154$ MeV)

Definitions

Symmetryenergy

$$J(\rho) = \frac{1}{2} \left(\frac{\partial^2 E(\rho, I)}{\partial I^2} \right)_{I=0}$$
$$\approx E(\rho, 1) - E(\rho, 0)$$

Slope parameter

$$L(\rho) = 3\rho \frac{dJ}{d\rho}$$

$$J \equiv J(\rho_0)$$

$$L \equiv L(\rho_0)$$

Nuclear energy density functional

1. Non-Relativistic mean field model
2. Relativistic mean field model
3. Density dependant meson exchange model

Non-Relativistic mean field model : Skyrme type

$$\mathcal{E} = \mathcal{E}_{kin} + \mathcal{E}_0 + \mathcal{E}_3 + \mathcal{E}_{eff}$$

$$\mathcal{E}_{kin} = \left(\frac{\hbar^2}{2m} \right) \tau$$

$$\mathcal{E}_0 = \frac{1}{4} t_0 \left[(2 + x_0) \rho^2 - (2x_0 + 1) (\rho_p^2 + \rho_n^2) \right]$$

$$\mathcal{E}_3 = \frac{1}{24} t_3 \rho^\alpha \left[(2 + x_3) \rho^2 - (2x_3 + 1) (\rho_p^2 + \rho_n^2) \right]$$

$$\mathcal{E}_{eff} = \frac{1}{8} \left[t_1 (2 + x_1) + t_2 (2 + x_2) \right] \tau \rho + \frac{1}{8} \left[t_2 (2x_2 + 1) - t_1 (2x_1 + 1) \right] (\tau \rho_p + \tau \rho_n)$$

$\tau \Rightarrow$ kinetic energy density, $\rho \Rightarrow$ number density

$m \Rightarrow$ nucleon mass,

$t_i x_i$ ($i = 0, 1, 2$ and 3) determined from finite nuclei properties.

Relativistic mean field model

$$\mathcal{E} = \mathcal{E}_{kin} + \mathcal{E}_{lin} + \mathcal{E}_{\sigma} + \mathcal{E}_{\omega} + \mathcal{E}_{\rho} + \mathcal{E}_{\sigma\omega\rho}$$

$$\mathcal{E}_{lin} = \sum_{J=n,p} \bar{\Psi}_J \left[i\gamma^{\mu} \partial_{\mu} + (g_{\sigma}\sigma - M) + \left(g_{\omega}\gamma^{\mu}\omega_{\mu} + \frac{1}{2} g_{\rho}\gamma^{\mu}\tau_{\mu}\rho_{\mu} \right) \right] \Psi$$

$$\mathcal{E}_{\sigma} = -\frac{1}{2} m_{\sigma}^2 \sigma^2 - \frac{k_3}{6M} g_{\sigma} m_{\sigma}^2 \sigma^3 - \frac{k_4}{24M^2} m_{\sigma}^2 \sigma^4$$

$$\mathcal{E}_{\omega} = \frac{1}{2} m_{\omega}^2 \omega^2 + \frac{1}{24} \zeta_0 g_{\omega}^2 \omega^4$$

$$\mathcal{E}_{\rho} = \frac{1}{2} m_{\rho}^2 \tilde{\rho}^2$$

$$\begin{aligned} \mathcal{E}_{\sigma\omega\rho} = & \frac{\eta_1}{2M} g_{\sigma} m_{\omega}^2 \sigma \omega^2 + \frac{\eta_2}{4M^2} g_{\sigma}^2 m_{\omega}^2 \sigma^2 \omega^2 + \frac{\eta_{\rho}}{2M} g_{\sigma} m_{\rho}^2 \sigma \tilde{\rho}^2 \\ & + \frac{\eta_{1\rho}}{4M^2} g_{\sigma}^2 m_{\rho}^2 \sigma^2 \tilde{\rho}^2 + \frac{\eta_{2\rho}}{4M^2} g_{\omega}^2 m_{\rho}^2 \sigma^2 \omega^2 \tilde{\rho}^2 \end{aligned}$$

Density dependant meson exchange model DDME

$$g_i(\rho) = g_i(\rho_{sat}) f_i(x) \text{ for } i = \sigma, \omega$$

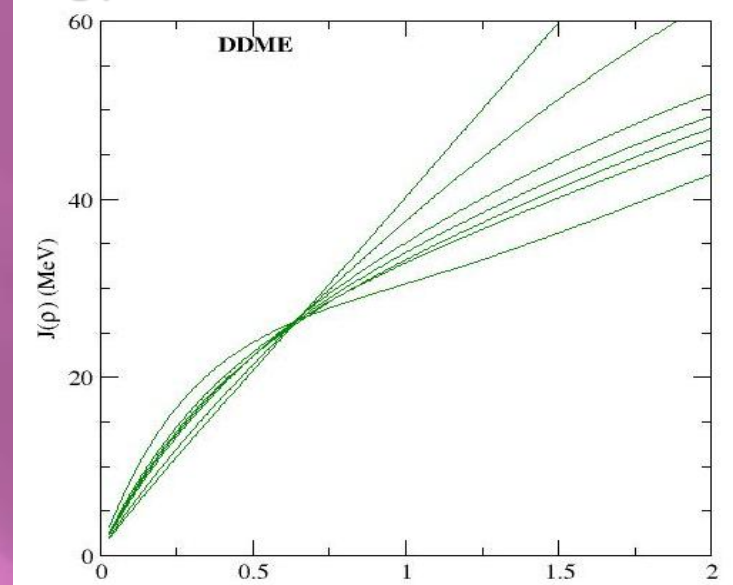
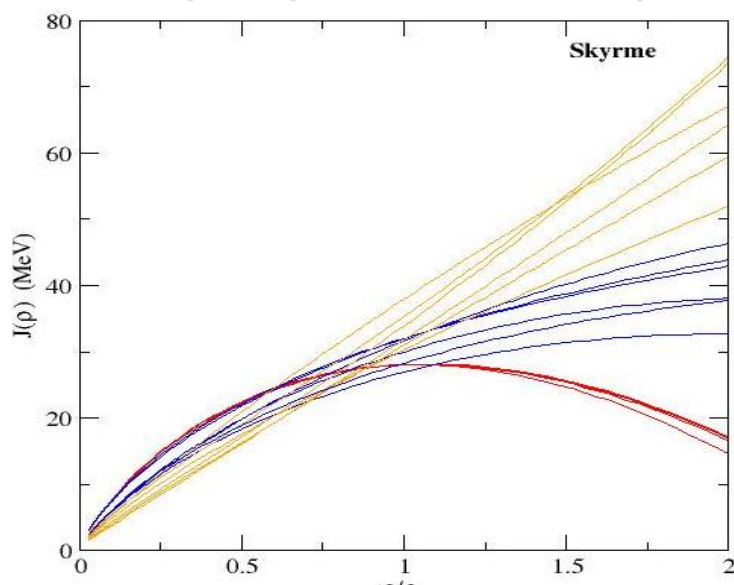
$$g_\rho(\rho) = g_\rho(\rho_{sat}) \exp[-a_\rho(x-1)]$$

$$x = \frac{\rho}{\rho_{sat}},$$

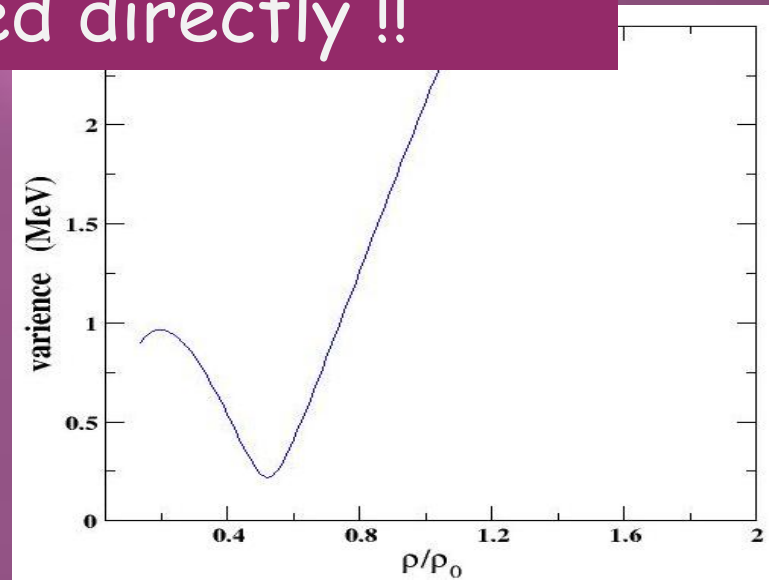
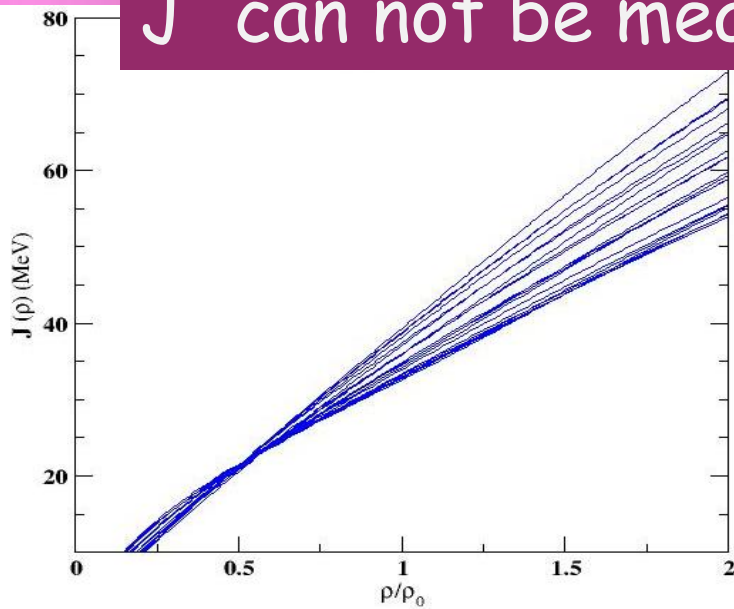
$$f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}$$

$$f_i(1) = 1$$

Density dependence of symmetry energy



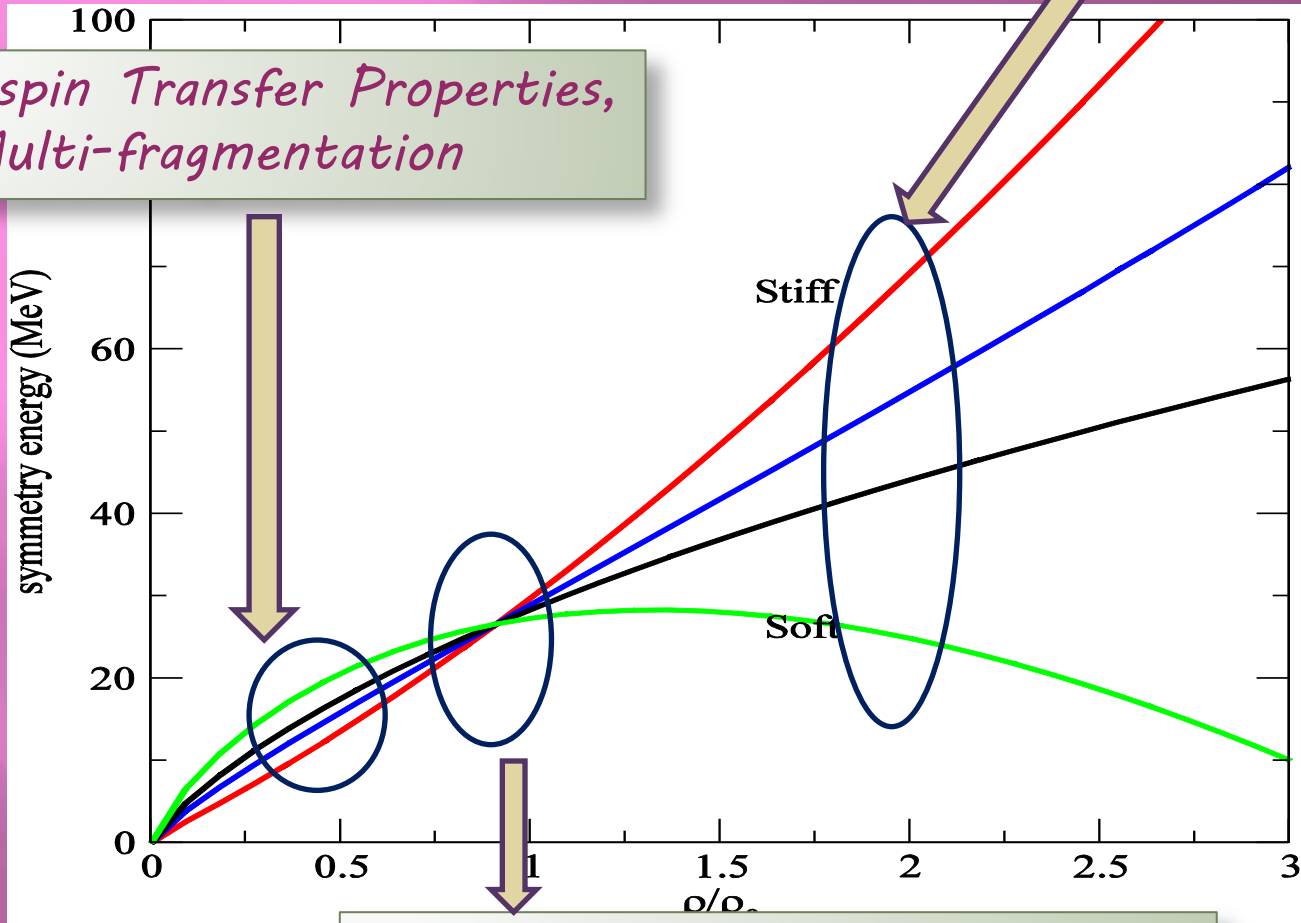
J can not be measured directly !!



Probes for the nuclear symmetry energy

Supra saturation densities Supernovae, Nucleosynthesis, Rel. HIC, Neutron Stars

Isospin Transfer Properties, Multi-fragmentation



$\rho = \rho_0$
structure and low energy excitations of asymmetric nuclei, skin thickness, Isovector giant resonances

Nuclear Structure: Neutron-skin, Giant resonance

Probes at ρ_0

- ▣ Neutron skin thickness
- ▣ Giant dipole resonance
 - i) Pigmy dipole resonance
 - ii) Dipole polarizability
- ▣ Giant quadrupole resonance
- ▣ Nuclear binding energies

Covariance Analysis

Consider a model described by coupling constants:
(P_1, P_2, \dots, P_N)

At the optimum parameter set P_0

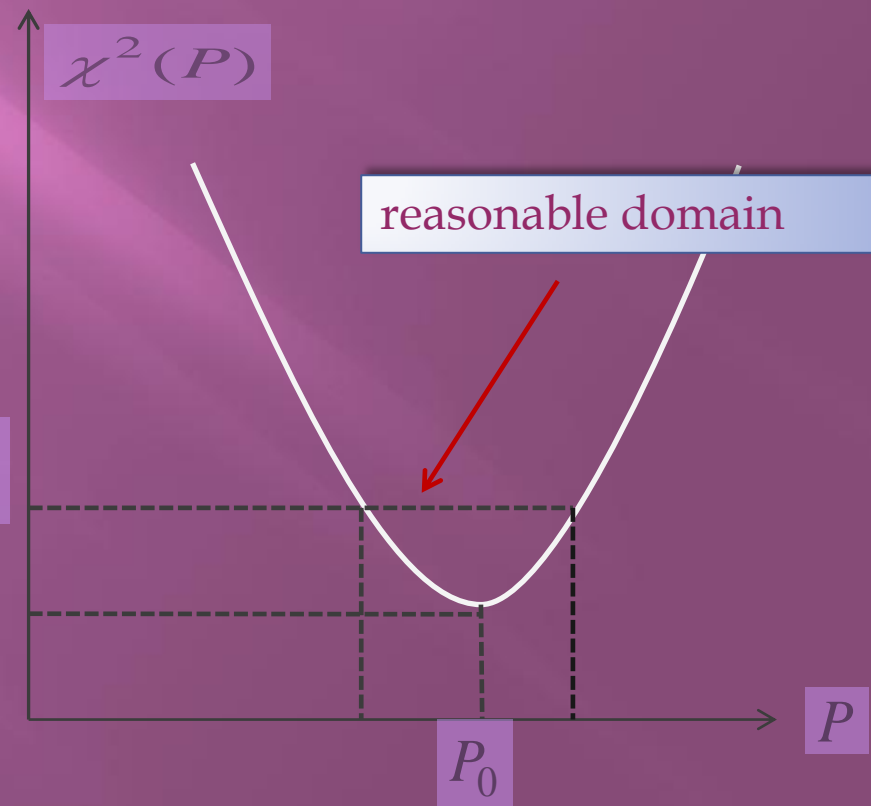
$$\chi^2(P_0) = \chi^2_{\min}$$

Reasonable domain of
parameter satisfy:

$$\chi^2(P) - \chi^2(P_0) \leq 1$$

$$\chi^2(P_0) + 1$$

$$\chi^2(P_0)$$



$|c_{AB}| = 1$: Fully correlated
 $c_{AB} = 0$: Uncorrelated or Statistically independent

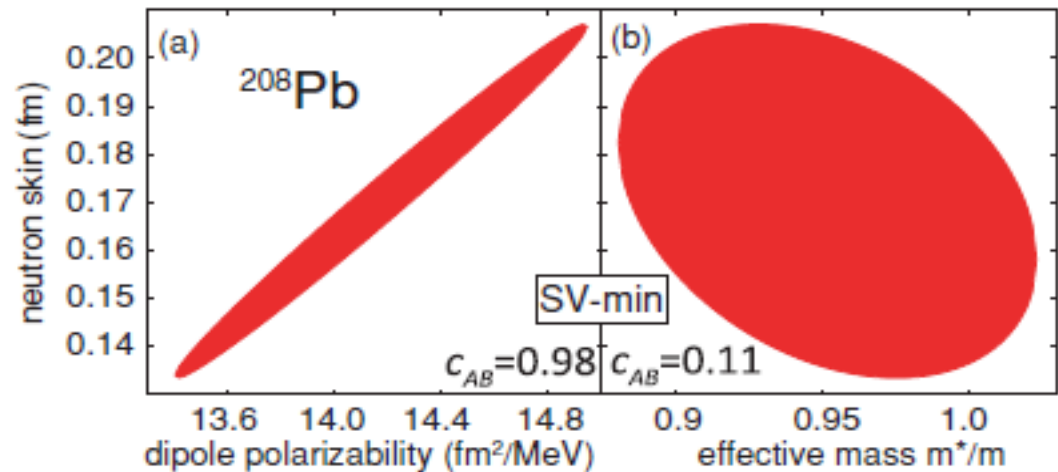
$$C_{AB} = \frac{\overline{\Delta A \Delta B}}{\sqrt{\overline{\Delta A^2} \overline{\Delta B^2}}}$$

$$\overline{\Delta A \Delta B} = \sum_{ij} \partial_{pi} A(M^{-1})_{ij} \partial_{pj} B$$

$$\overline{\Delta A^2} = \sum_{ij} \partial_{pi} A(M^{-1})_{ij} \partial_{pj} A$$

$$M_{ij} = \partial_{pi} \partial_{pj} \chi^2 \Big|_{P_0}$$

$$\partial_{pi} A = \frac{\partial A}{\partial pi} \Big|_{P_0}$$



pygmy str.

$\partial_\rho(E/A)_{\text{neut}}$

r_{neut}

neutron skin

$\alpha_D(^{208}\text{Pb})$

$\partial_\rho a_{\text{sym}}$

a_{sym}

GDR

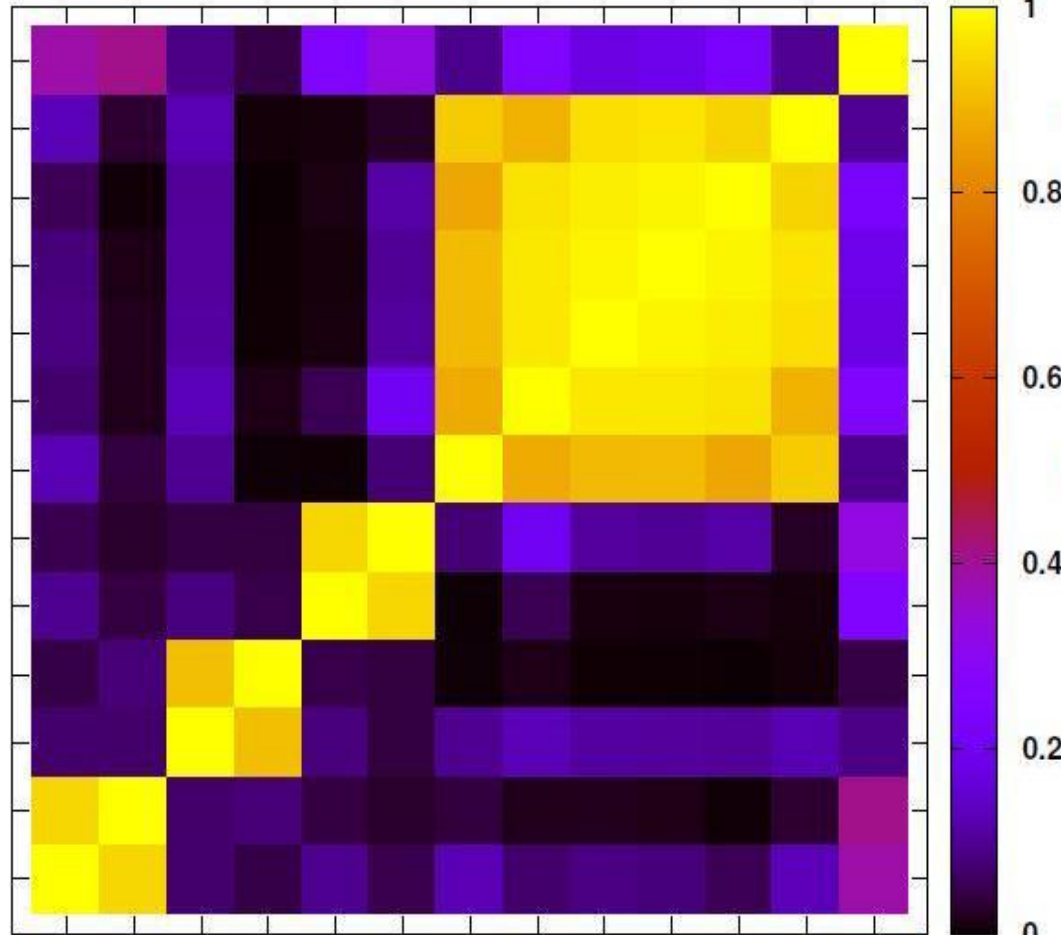
κ

GQR

m^*/m

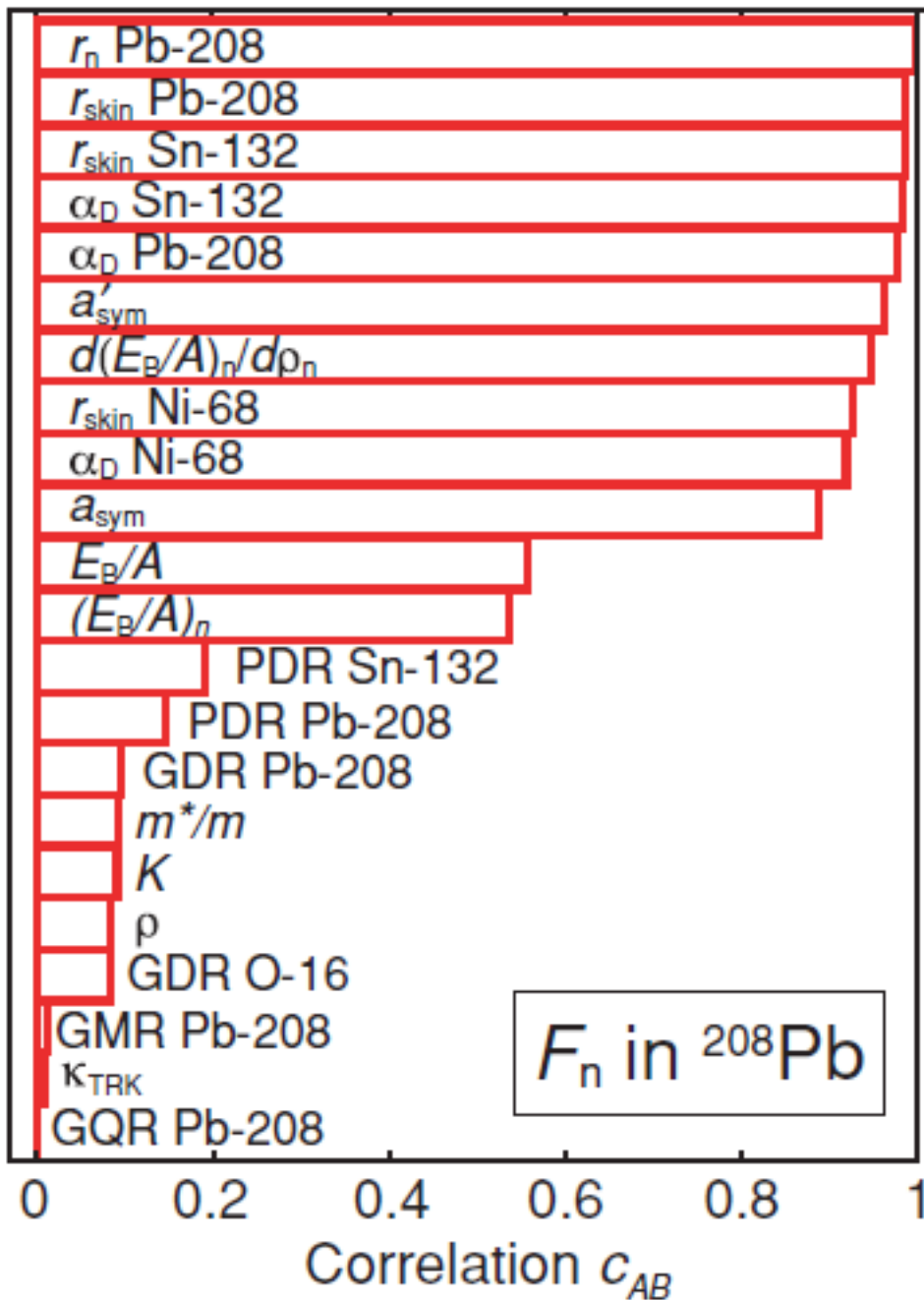
GMR

K



isovector

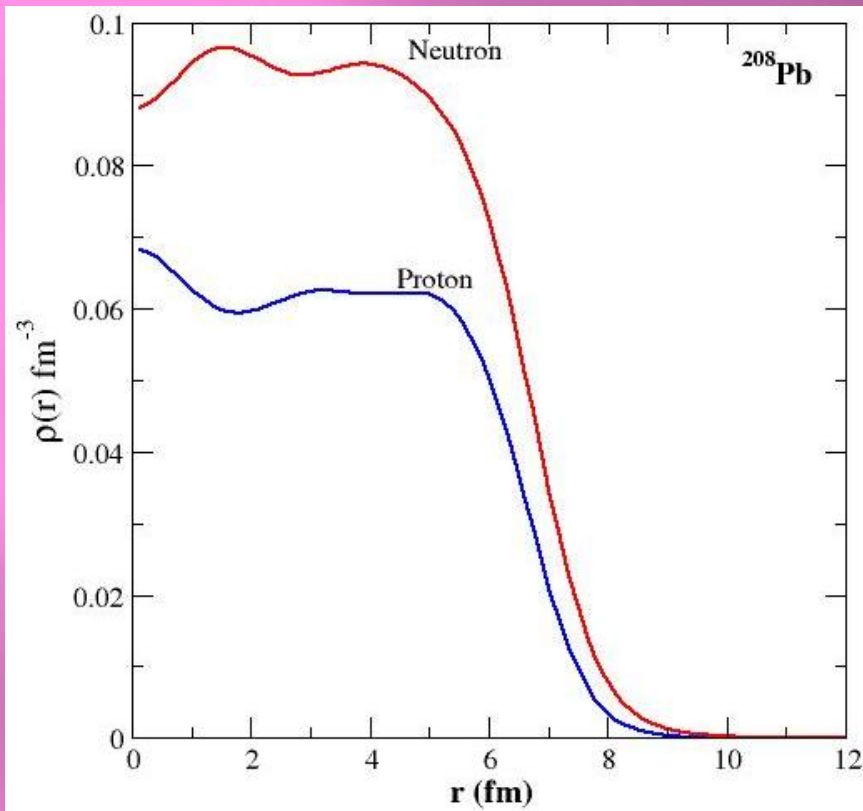
isoscalar



Good isovector indicators

Poor isovector indicators

Neutron skin thickness

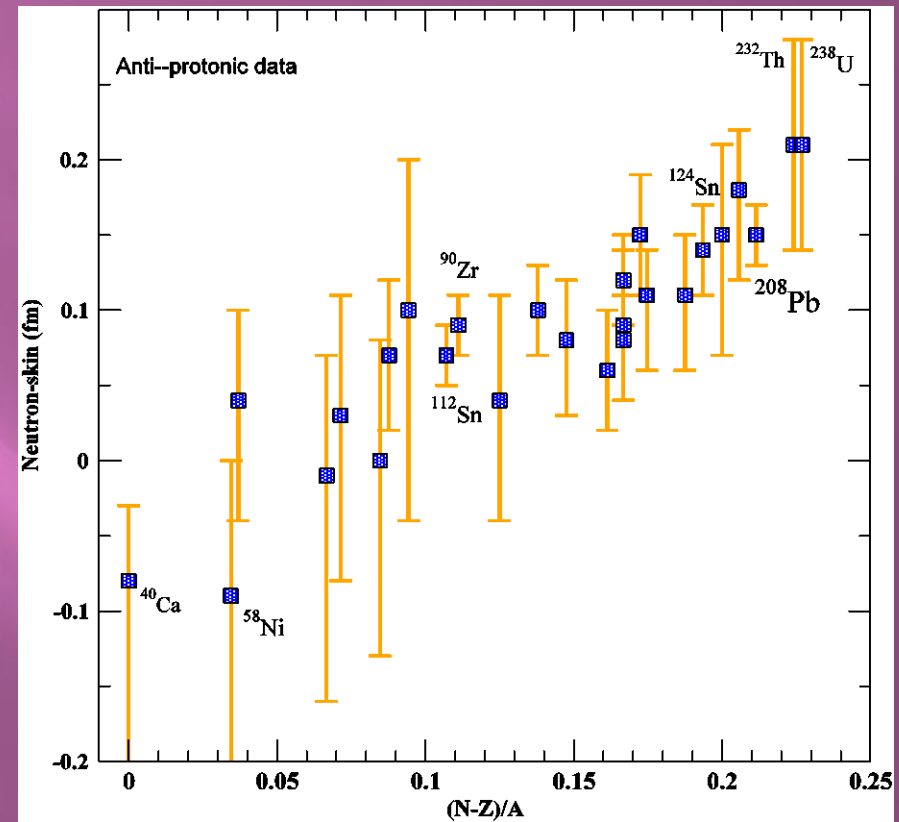
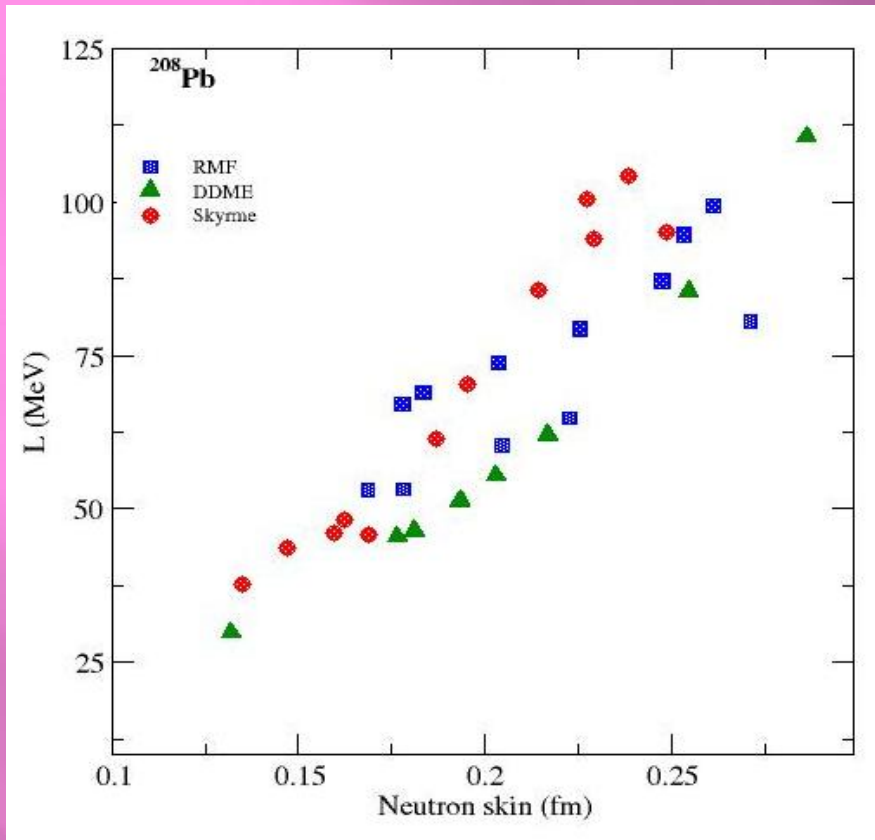


$$R_{skin} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$$

$$\langle r_q^2 \rangle = \frac{\int r^2 \rho_q(r) d^3r}{\int \rho_q(r) d^3r}$$

$$q = p, n$$

L from neutron skin thickness (R_{skin})



$$L = 75 \pm 25 \text{ MeV}$$

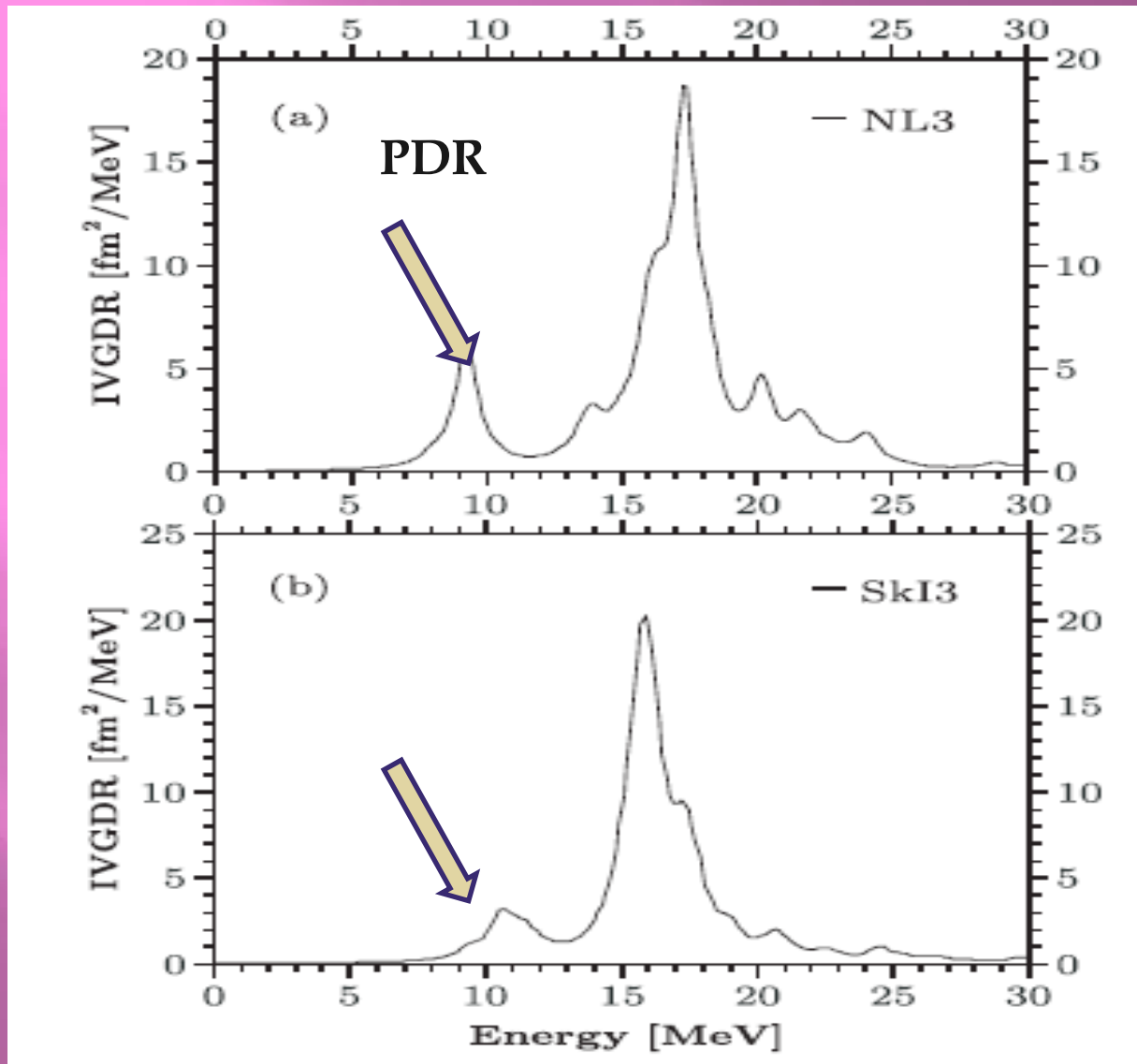
M. Centelles et al PRL 102,122502 (2009)

First PREX data PRL 108, 112502 (2012)

$$R_{skin} = 0.33^{+0.16}_{-0.18} \text{ fm}$$

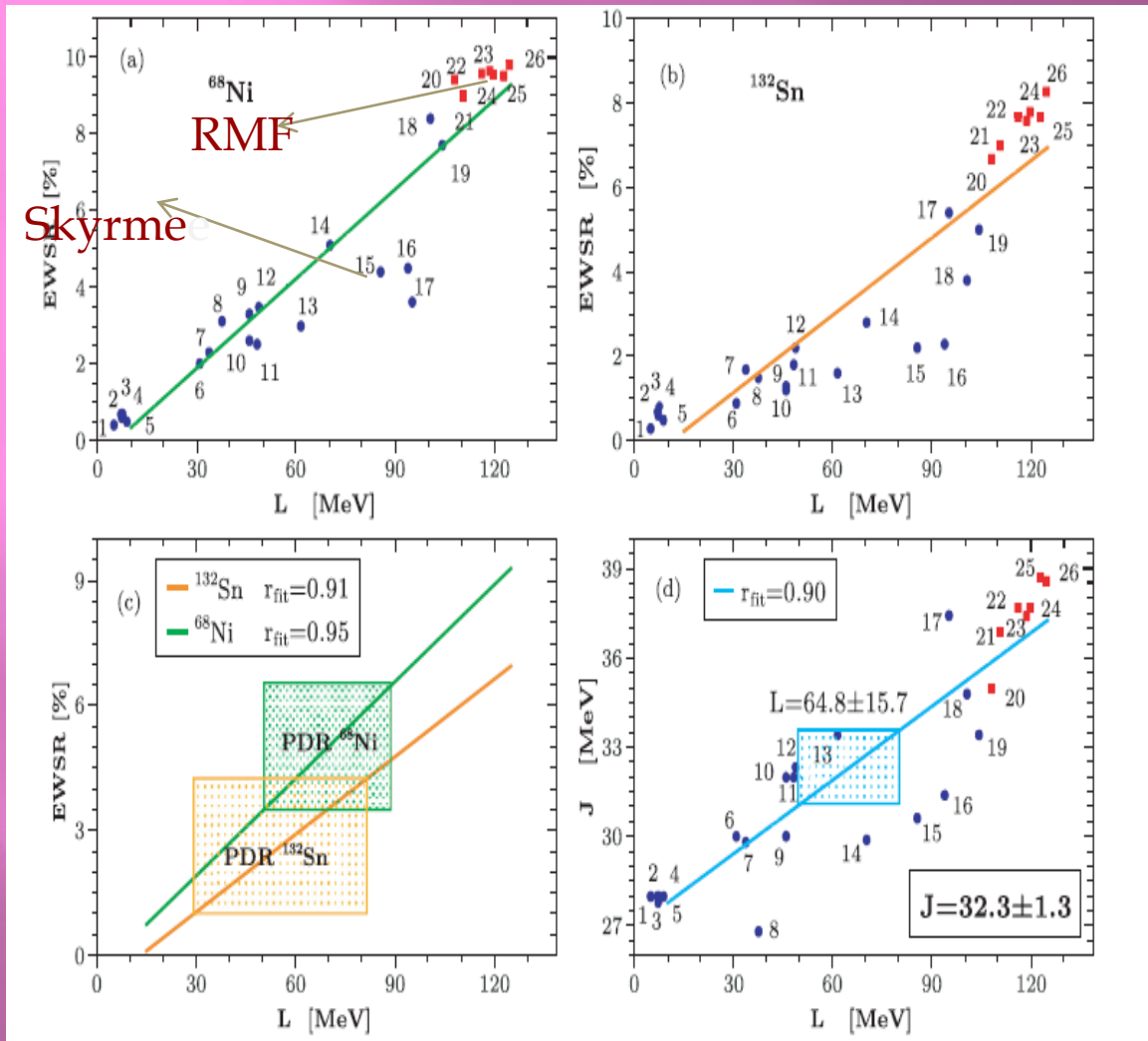
PREX II DATA awaited!!

Pigmy Dipole Resonance (PDR)



A. Carbon et.al
PRC 81,041301 (2010)

Pigmy Dipole Resonance (PDR)



$$EWSR = \int_{E_1}^{E_2} ES(E) dE$$

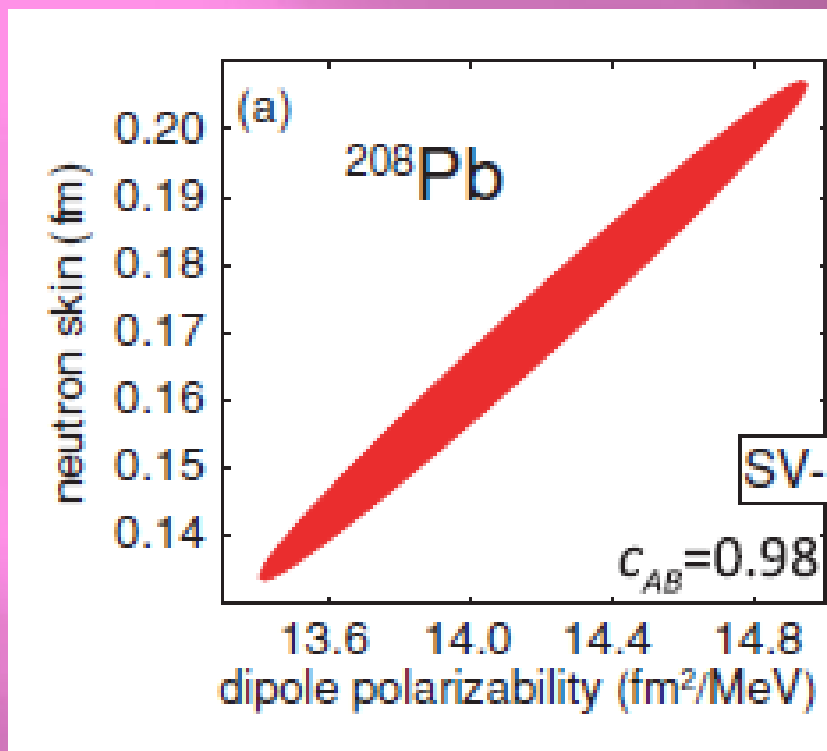
$$L(\rho_0) = 64.8 \pm 15.7$$

$$J(\rho_0) = 32.3 \pm 1.3$$

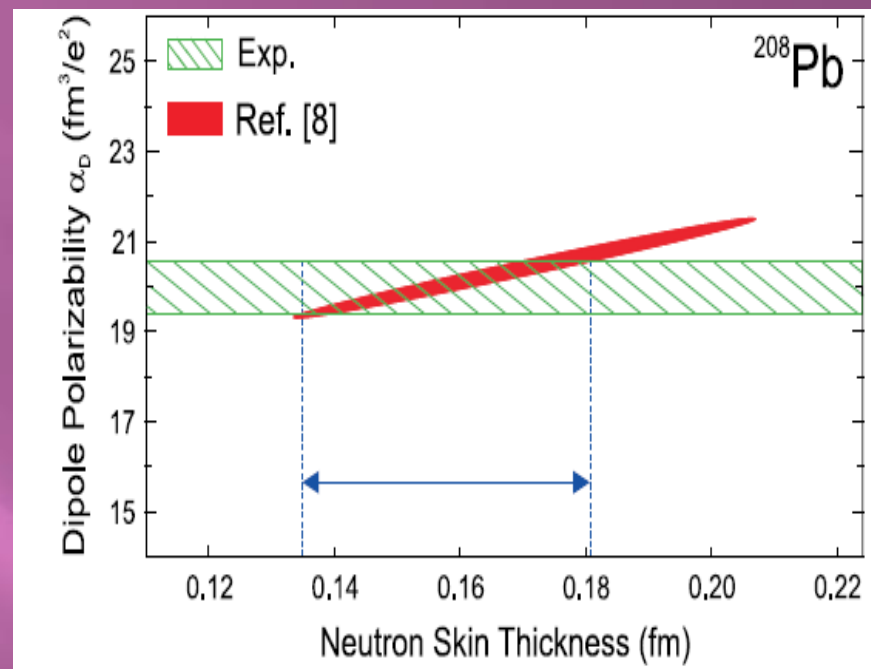
$$L = 64.8 \pm 15.7 \text{ MeV}$$

A. Carbon et al
 PRC 81,041301 (2010)

L and R_{skin} from Diapole polarizability



Reinhard *et.al*, PRC 81, 051303 (R) 2010

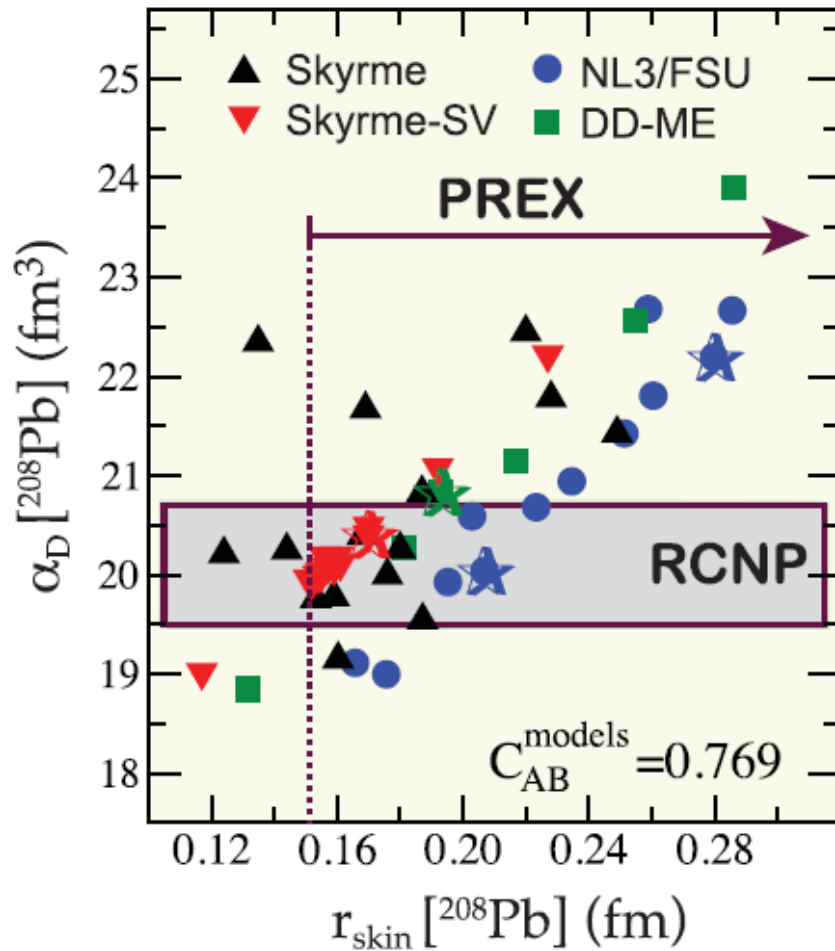


High resolution (p,p')
measurement of α_D .
Tamii *et. al* PRL 107 (2011)

$$R_{skin} = 0.156^{+0.025}_{-0.021} \text{fm}$$

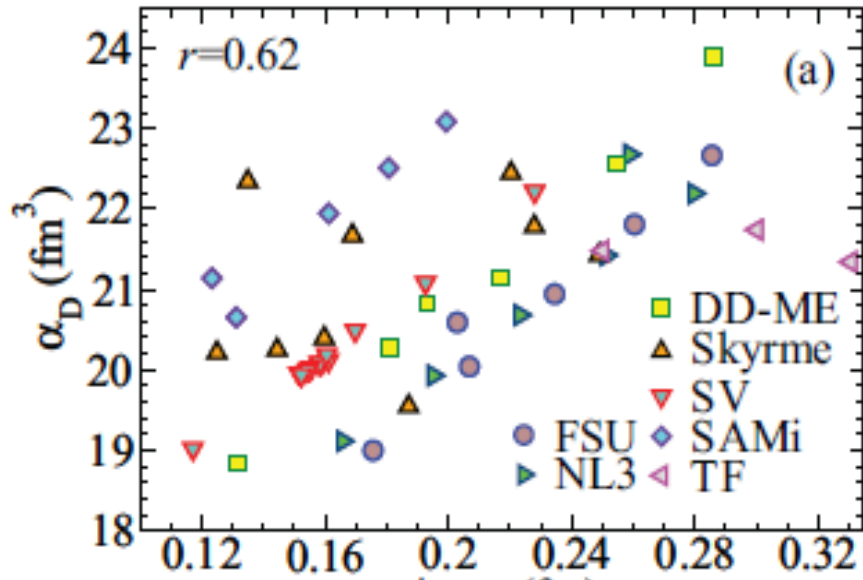
$$L = 41 \pm 14 \text{MeV}$$

L and R_{skin} from Diapole polarizability

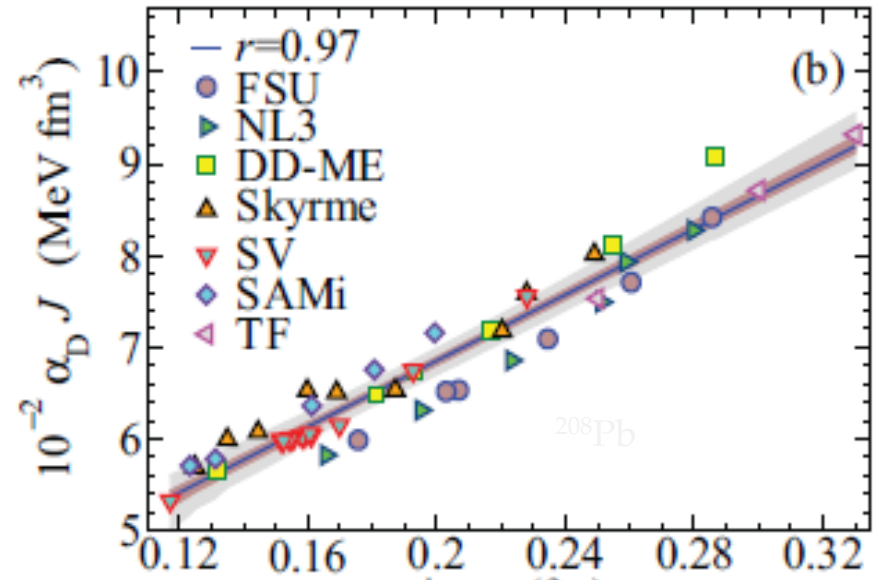


$R_{\text{skin}} = 0.168 \pm 0.022 \text{ fm}$
 $L = 47 \pm 12 \text{ MeV}$

Piekarewicz, B.K. Agrawal, et.al PRC (R) 2012



neutron skin (fm)



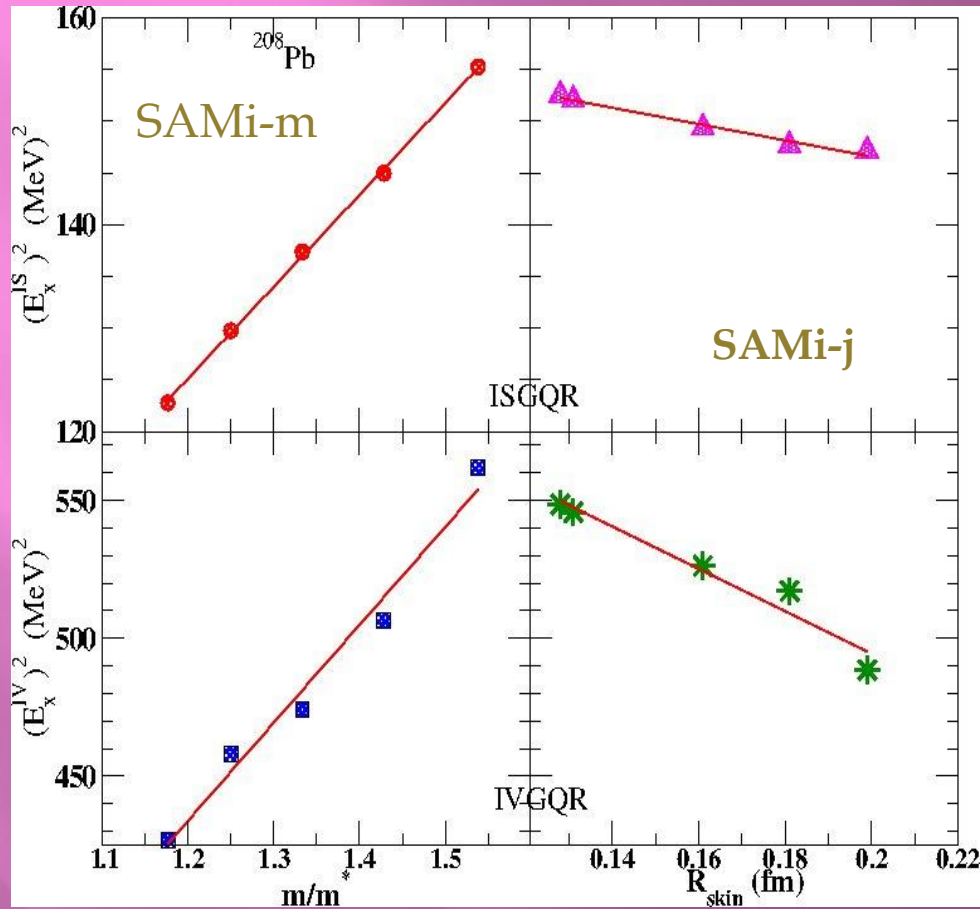
neutron skin (fm)

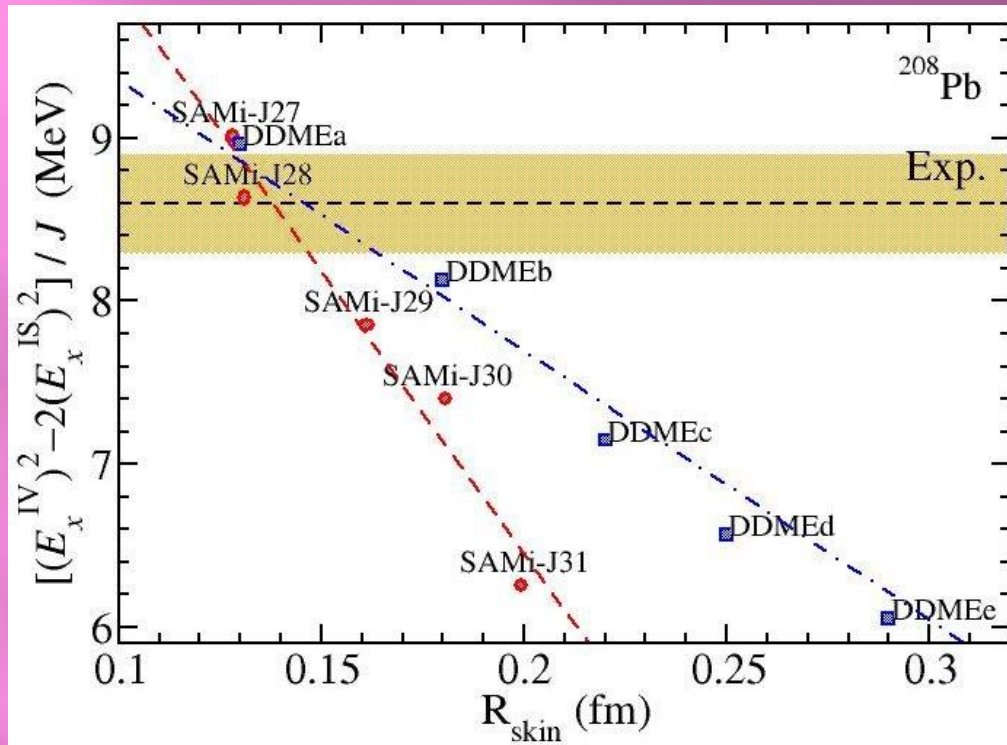
Using $\alpha_D = 20.1 \pm 0.6 \text{ fm}^3$ † in ^{208}Pb and $J = 31 \pm 2 \text{ MeV}$ ††
 Neutron skin = $0.165 \pm 0.043 \text{ fm}$
 $L = 43 \pm 26 \text{ MeV}$

Agrawal, *et. al* PRC 88, 024316 (2013)

† A. Tamii *et. al*, PRL 107, 062502 (2011) †† Lattimer *et. al*, Annu.Rev. Nucl.Part.Sci., 62,485(2012)

L and R_{skin} from Giant Quadrupole Resonances





$$R_{skin} = 0.14 \pm 0.03 \text{ fm}$$

$$L = 37 \pm 18 \text{ MeV}$$

Xavi, Marco, Agrawal et.al PRC 87, 034301 (2013)

L & R_{skin} from nuclear binding energy

$$B = a_v A - a_s A^{2/3} - \left(J - J_s A^{-1/3} \right) A I^2 - a_c \frac{Z^2}{A^{1/3}} + \dots$$

$$J = 32.10 \pm 0.31$$

Arima et, al. PRC 85,2012

$$J_s = 58.91 \pm 1.08$$

$$J(\rho) = J(\rho_0) \left(\frac{\rho}{\rho_0} \right)^\gamma, \quad L = 3\gamma J(\rho_0)$$

$$a_{sym}(A) = J(\rho_A) = J(\rho_0) \left(\frac{\rho_A}{\rho_0} \right)^\gamma$$

$$J(\rho_0) - J_s A^{-1/3} = J(\rho_A) = J(\rho_0) \left[1 - 3\gamma \varepsilon_A + \frac{9}{2} \gamma(\gamma - 1) \varepsilon_A^2 \right]$$

ρ_A = effective nucleon density

L & R_{skin} from Nuclear binding energy

$$\rho_A^\gamma = \frac{1}{X_0^2} \int d^3 r \rho(r) \rho(r)^\gamma X(r)^2$$

$$J_s = 3J(\rho_0) A^{1/3} \left[\gamma \varepsilon_A - \frac{3}{2} \gamma(\gamma - 1) \varepsilon_A^2 \right]$$

$$X_0 = \frac{N-Z}{N+Z},$$

$$X(r) = \frac{\rho_n(r) - \rho_p(r)}{\rho_n(r) + \rho_p(r)},$$

$$\varepsilon_A = \frac{\rho_0 - \rho_A}{3\rho_0}$$

$$\rho_0 = 0.155 \pm 0.008 \text{ fm}^{-3}$$

$\rho(r) \rightarrow$ mean field models

L & R_{skin} from nuclear masses

$$J(\rho) = J(\rho_0) \left(\frac{\rho}{\rho_0}\right)^\gamma \quad \Rightarrow \text{I}$$

$$J(\rho) = C_k \left(\frac{\rho}{\rho_0}\right)^{2/3}$$

$$+ (J(\rho_0) - C_k) \left(\frac{\rho}{\rho_0}\right)^\gamma \quad \Rightarrow \text{II}$$

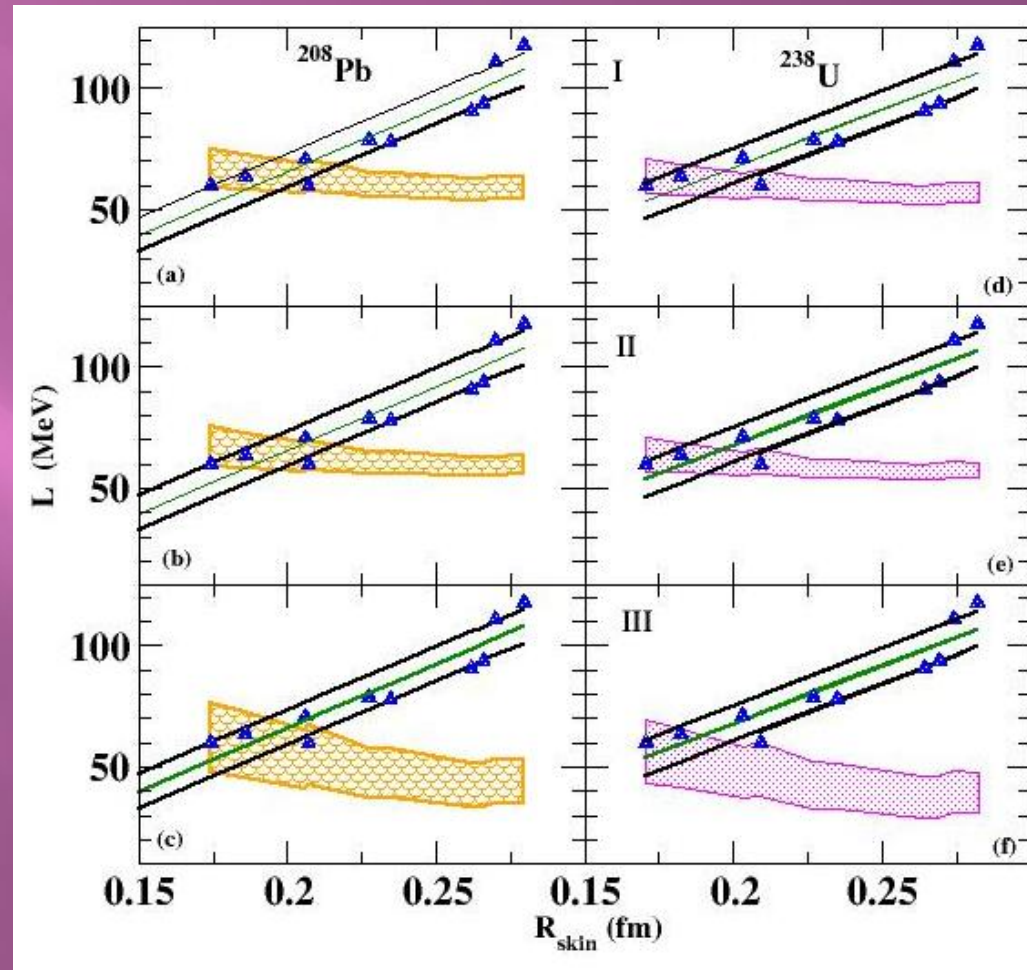
$$J(\rho) = C_k \left(\frac{\rho}{\rho_0}\right)^{2/3} + C_1 \left(\frac{\rho}{\rho_0}\right)$$

$$+ (J(\rho_0) - C_k - C_1) \left(\frac{\rho}{\rho_0}\right)^{5/3} \quad \Rightarrow \text{III}$$

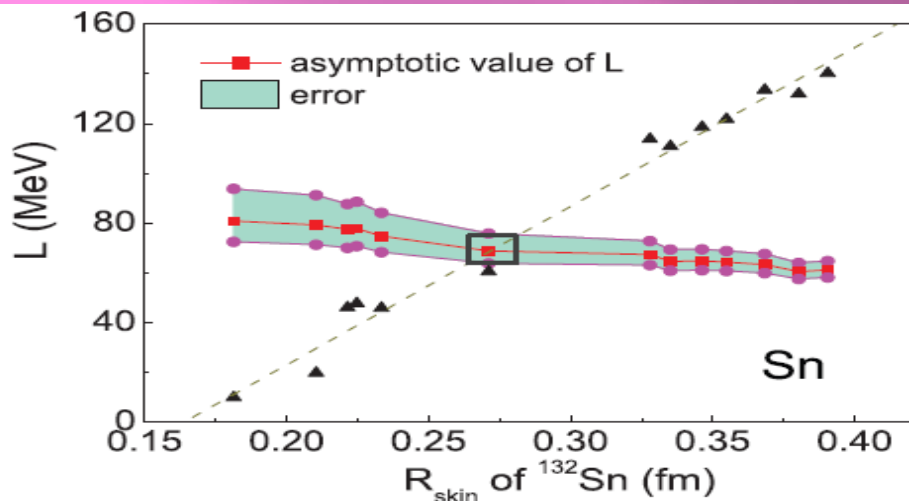
$$C_k = 13 \frac{m}{m^*}$$

$$R_{skin} = 0.190 \pm 0.016 \text{ fm}$$

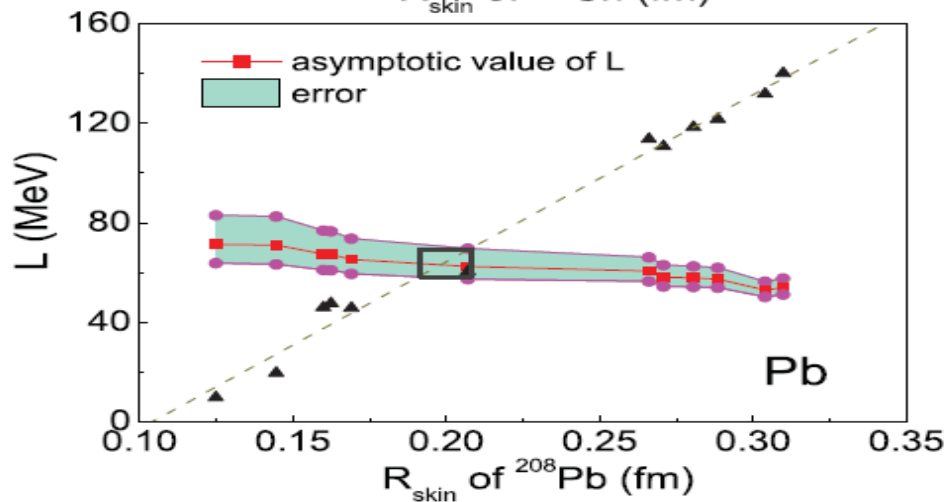
$$L = 56 \pm 12 \text{ MeV}$$



B. K. Agrawal, *et. al* PRC 87,
051306 (R) (2013)



$$L = 66 \pm 7 \text{ MeV}$$

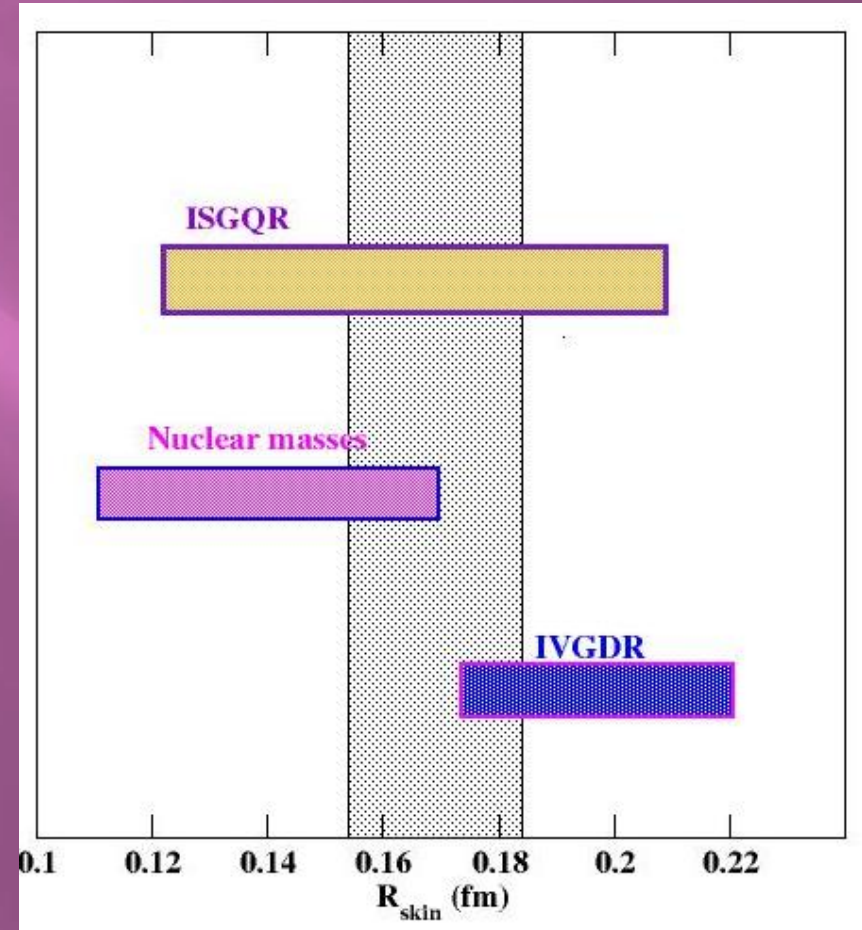
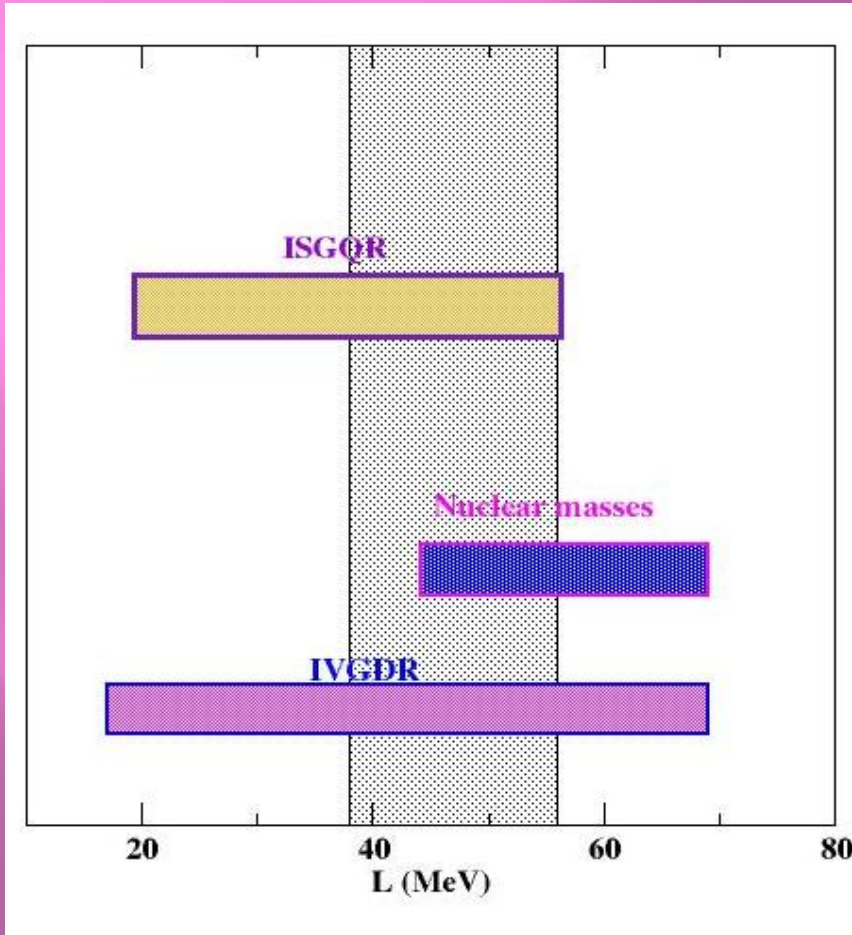


Jian Liu, et. Al Phys. Rev. C 88, 024324 (2013)

Our results

$$L = 47 \pm 9 \text{ MeV}$$

$$R_{\text{skin}} = 0.17 \pm 0.02 \text{ fm}$$



Conclusions

- Large no of the nuclear energy density functional are employed to understand the density dependence of the nuclear symmetry energy around the saturation density.
- Neutron-skin in ^{208}Pb nucleus and the symmetry energy slope parameter $L(\rho_0)$ are extracted using experimental data on giant resonances and the nuclear masses.
- Our results: $R_{\text{skin}} = 0.17 \pm 0.02 \text{ fm}$ $L(\rho_0) = 47 \pm 9 \text{ MeV}$
- More accurate data for the dipole polarizability and the neutron-skin thickness in ^{208}Pb needed to constraint in a narrow window the value of the symmetry energy slope parameter.

Collaborators

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- N. Paar (Phys. Dept. Croatia)
- M. Brenna (INFN, Italy)
- Li-Gang Cao (IMP China)



Thank you !

RIKEN Nishina Center for Accelerator-based Science