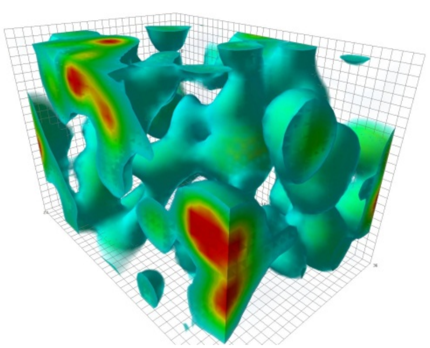
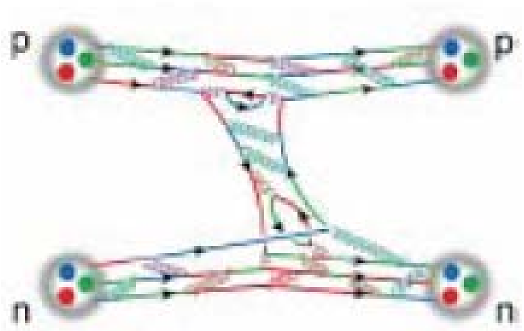
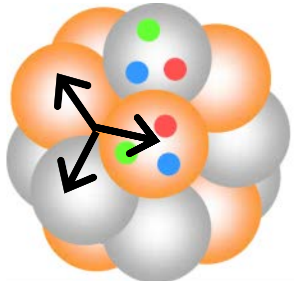
Lattice QCD studies for Two- and Three-Nucleon Forces

Takumi Doi

(Nishina Center, RIKEN)

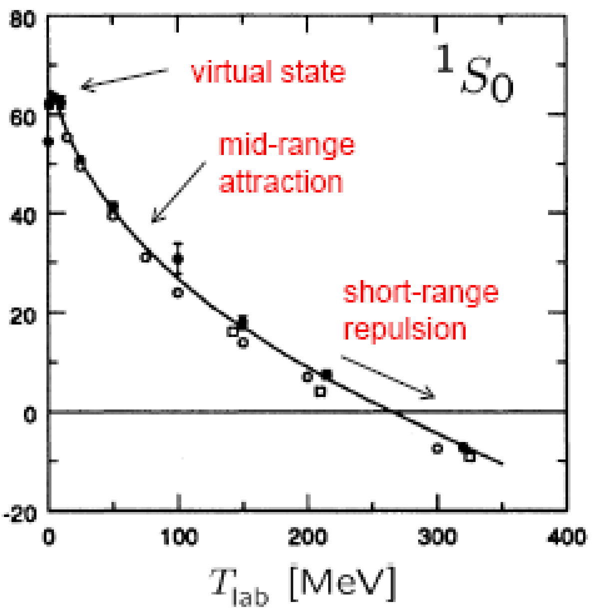




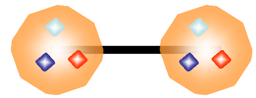


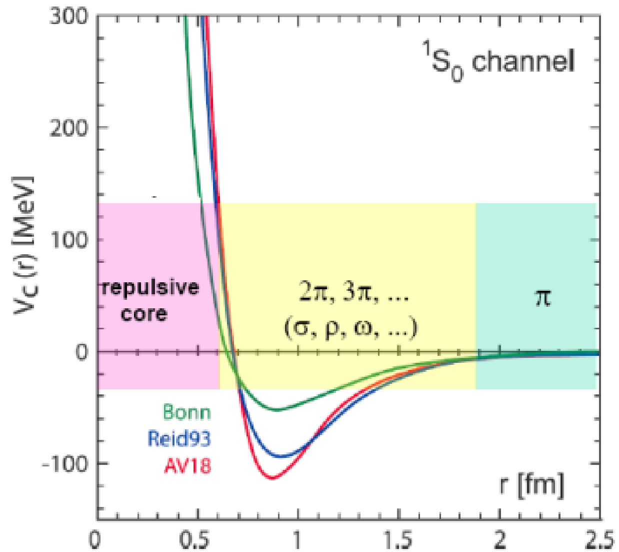
- Hadron Interactions
  - Bridging different worlds:
    Particle Physics / Nuclear Physics / Astrophysics
  - **Frontier:** 1st principle calc by Lattice simulations
- Outline
  - Introduction
  - Theoretical framework for lattice hadron forces
  - Lattice results at heavier quark masses
    - **Two-baryon interactions (NN, YN, YY)**
    - **Three-baryon interactions (NNN)**
  - Challenges toward physical quark mass point
    - ➔ **Nuclear Physics on the Lattice**
  - Summary / Prospects

(1) Build a foundation for nuclear physics

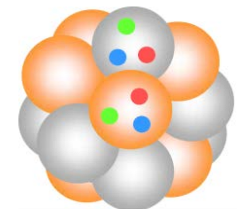


NN phase shifts
from experiments

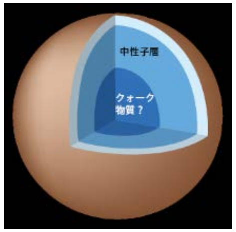




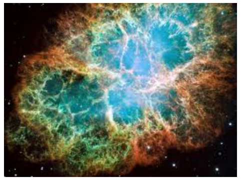
Phenomenological
Nuclear Forces



Nuclei



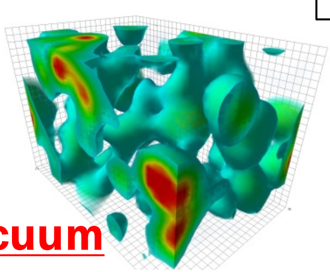
Neutron Stars



Super Novae

Various
applications

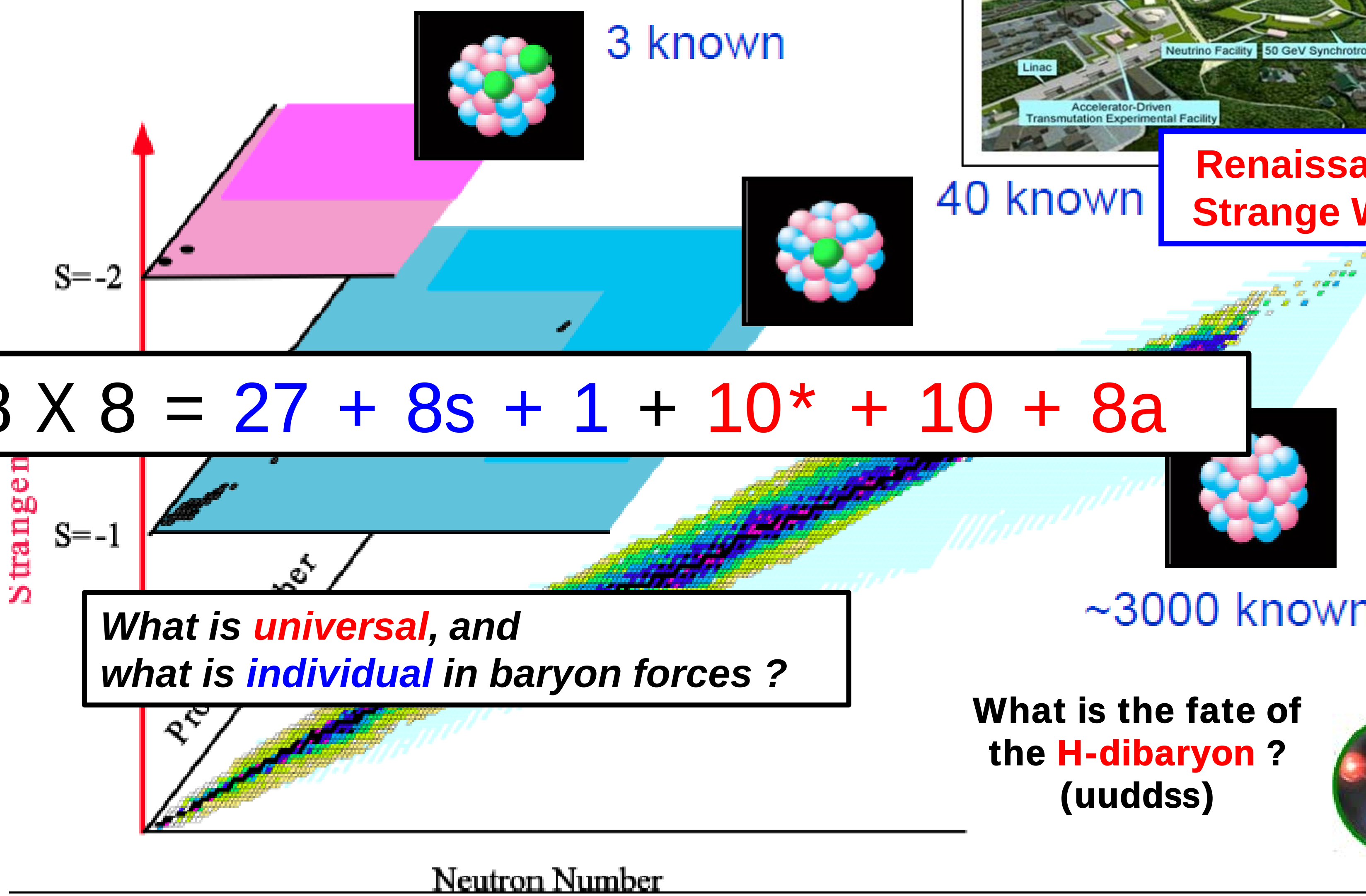
- ***Nuclear Forces*** ***play crucial roles***
  - *Yet, no clear connection to QCD so far*

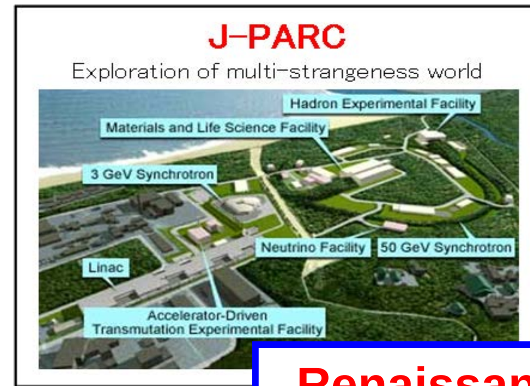


QCD Vacuum

(2) Predict Unknown Interactions (YN, YY, NNN)

3D Nuclear Chart





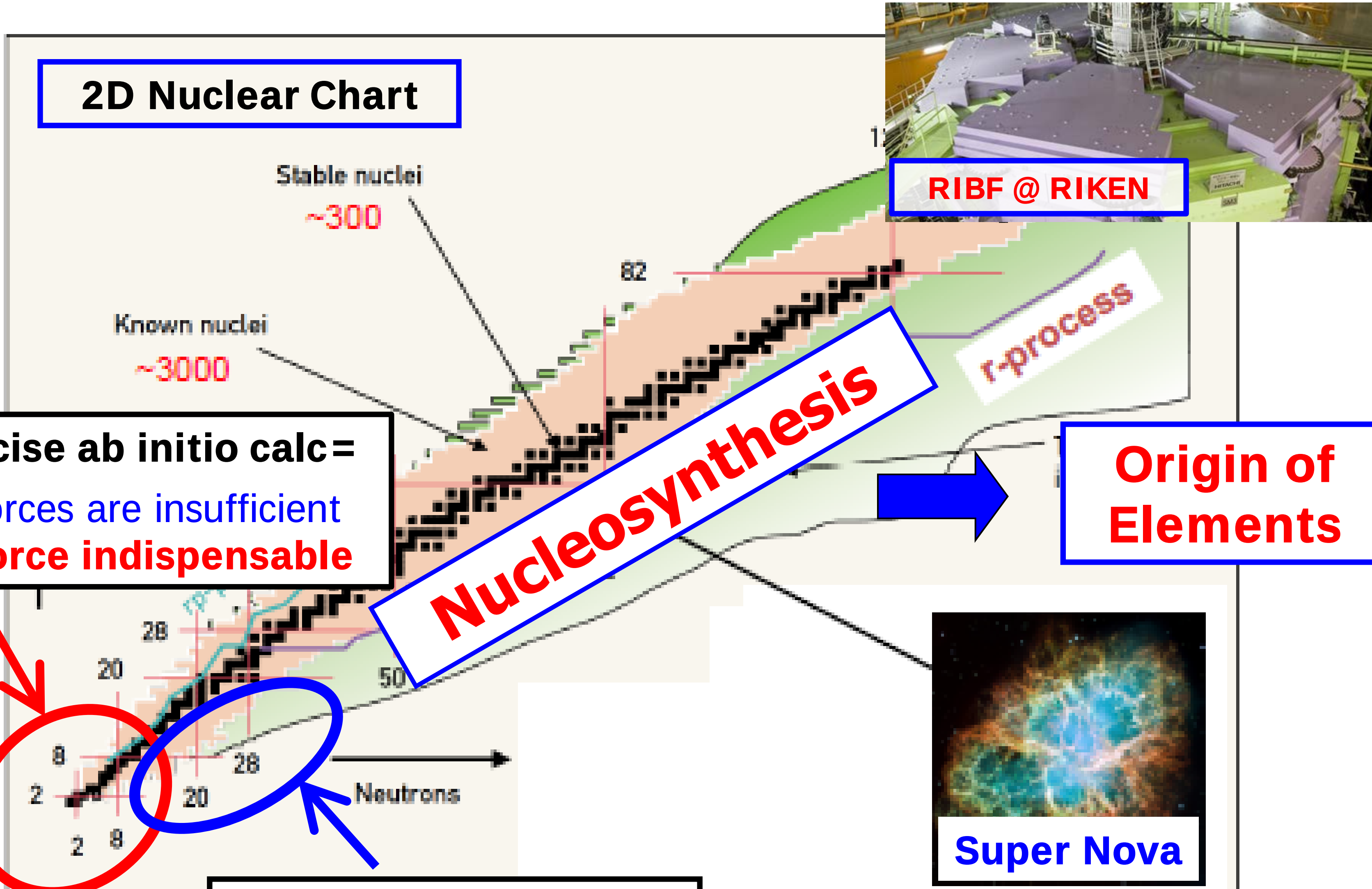
Renaissance in Strange World !

$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$

What is universal, and what is individual in baryon forces ?

What is the fate of the H-dibaryon ? (uuddss)

(2) Predict Unknown Interactions (YN, YY, NNN)



Origin of Elements

=Precise ab initio calc=

2N-forces are insufficient

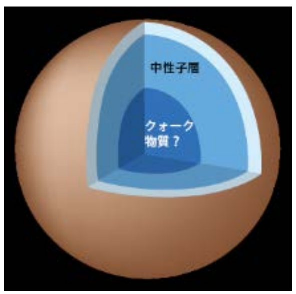
3N-force indispensable

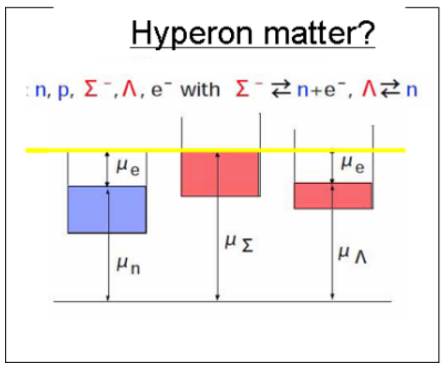
Paradigm Shift in
Unstable Nuclei
(New Magic Numbers !)

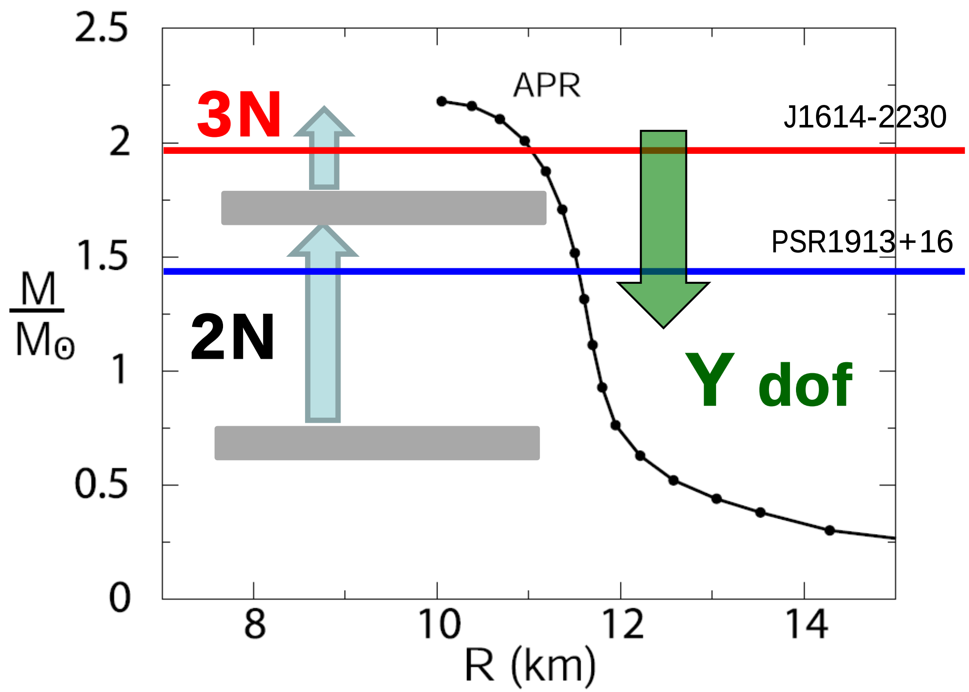
← **Important role of 3N-force**

Dense Matter ← Interactions of YN, YY, NNN,… are crucial

- Neutron Stars, Super Novae ←→ EoS







How to sustain a neutron star against gravitational collapse ?

Akmal et al.('98), Nishizaki et al.('02), Takatsuka et al.('08)

Revival of quark matter ? (Masuda et al.)

EoS of Neutron Star by Gravitational Waves ?

- Full GR simulation of binary neutron star mergers w/ Shen-EoS (stiff) and Hyperon(Λ)–EoS (soft)

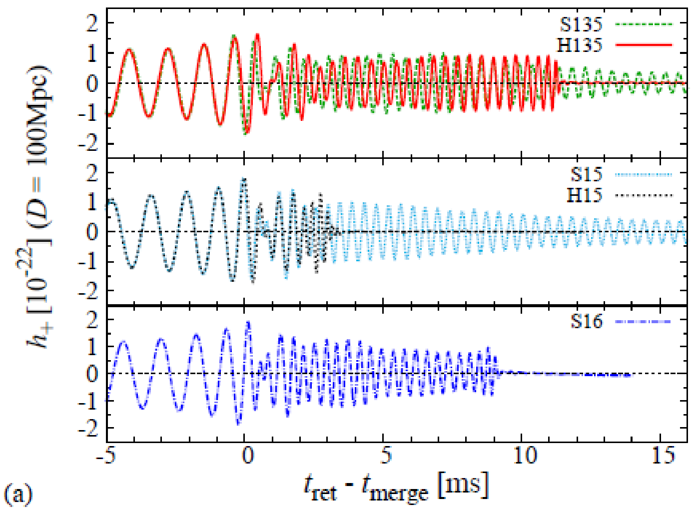


FIG. 4: (a) GWs observed along the axis perpendicular to th
D = 100 Mpc. (b) The effective amplitude of GWs defined by
noise amplitudes of a broadband configuration of Advanced Las
and Large-scale Cryogenic Gravitational wave Telescope (LCGT

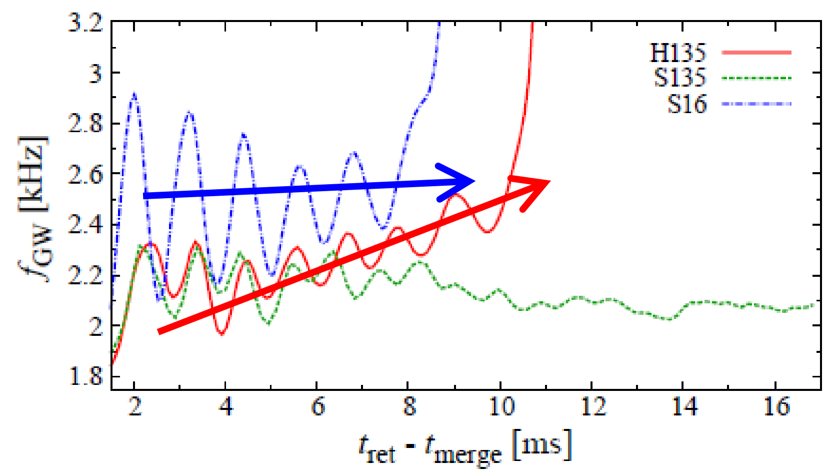
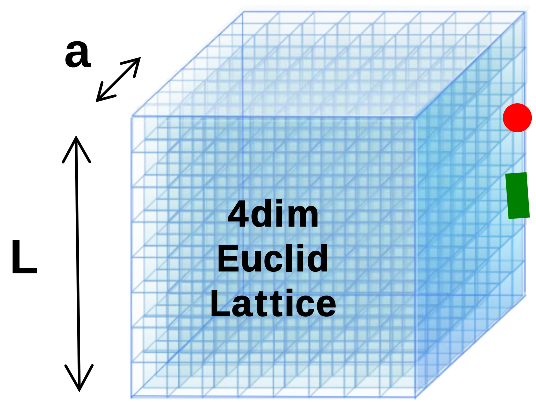


FIG. 5: $f_{\rm GW}(t)$ in the HMNS phase, smoothed by a weighted spline, for H135 (solid red), S135 (dashed green), and S16 (dashed-dotted blue).

Y.Sekiguchi et al., arXiv:1110.4442[astro-ph.HE]

Lattice QCD
First-principle calculation of QCD

$$Z = \int dU\, dq d\bar{q}\; e^{-S_E}$$



quarks on sites

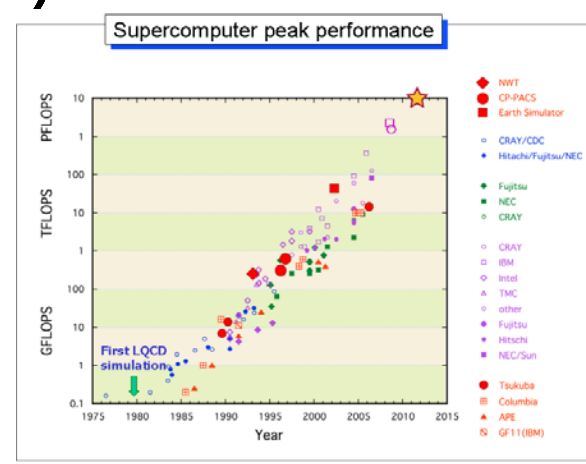
$$q(x)$$

gluons on links

$$U_\mu(x, x+\mu) = \exp[-iaA_\mu]$$

K.G. Wilson

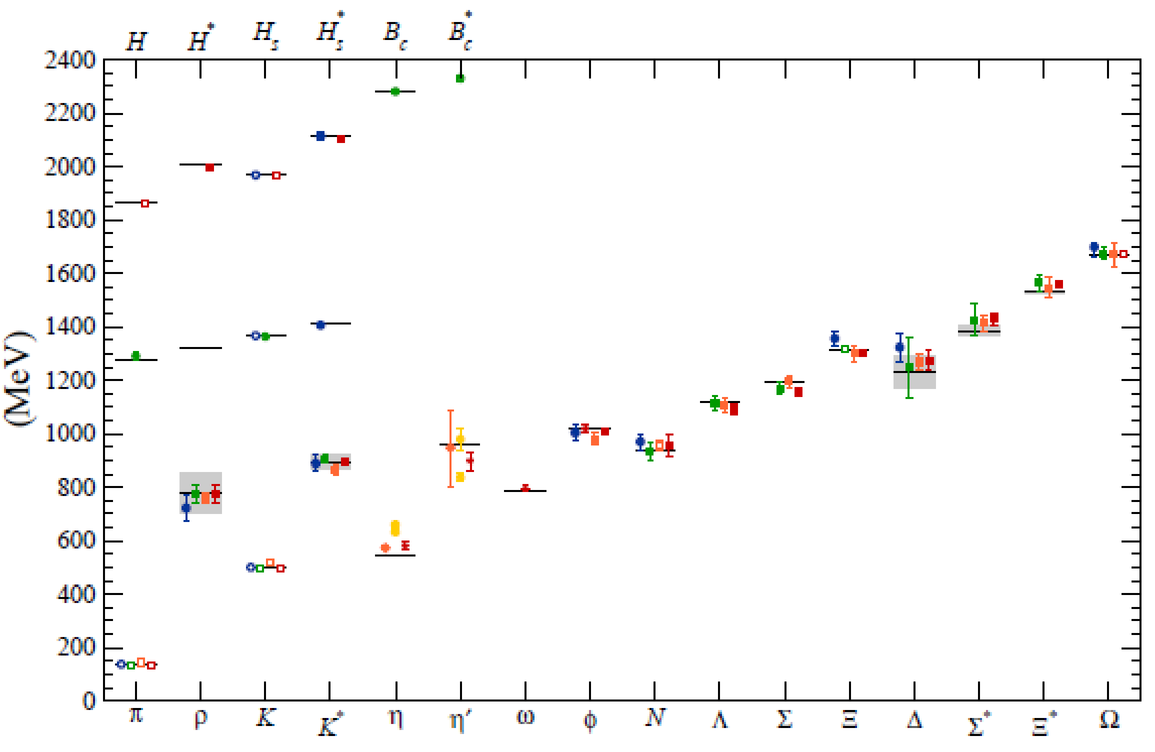
- **Well-defined reguralized system (finite a and L)**
- **Gauge-invariance manifest**
- **Fully-Nonperturbative**
- **DoF ~ 10^9 ➔ Monte-Carlo w/ Euclid time**



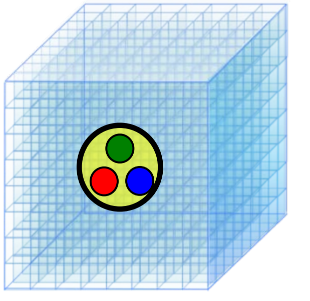
Significant theoretical and hardware advances

Status of Lattice QCD

Hadron spectrum well reproduced !



Summary by Kronfeld, arXiv:1203.1204



Inputs:

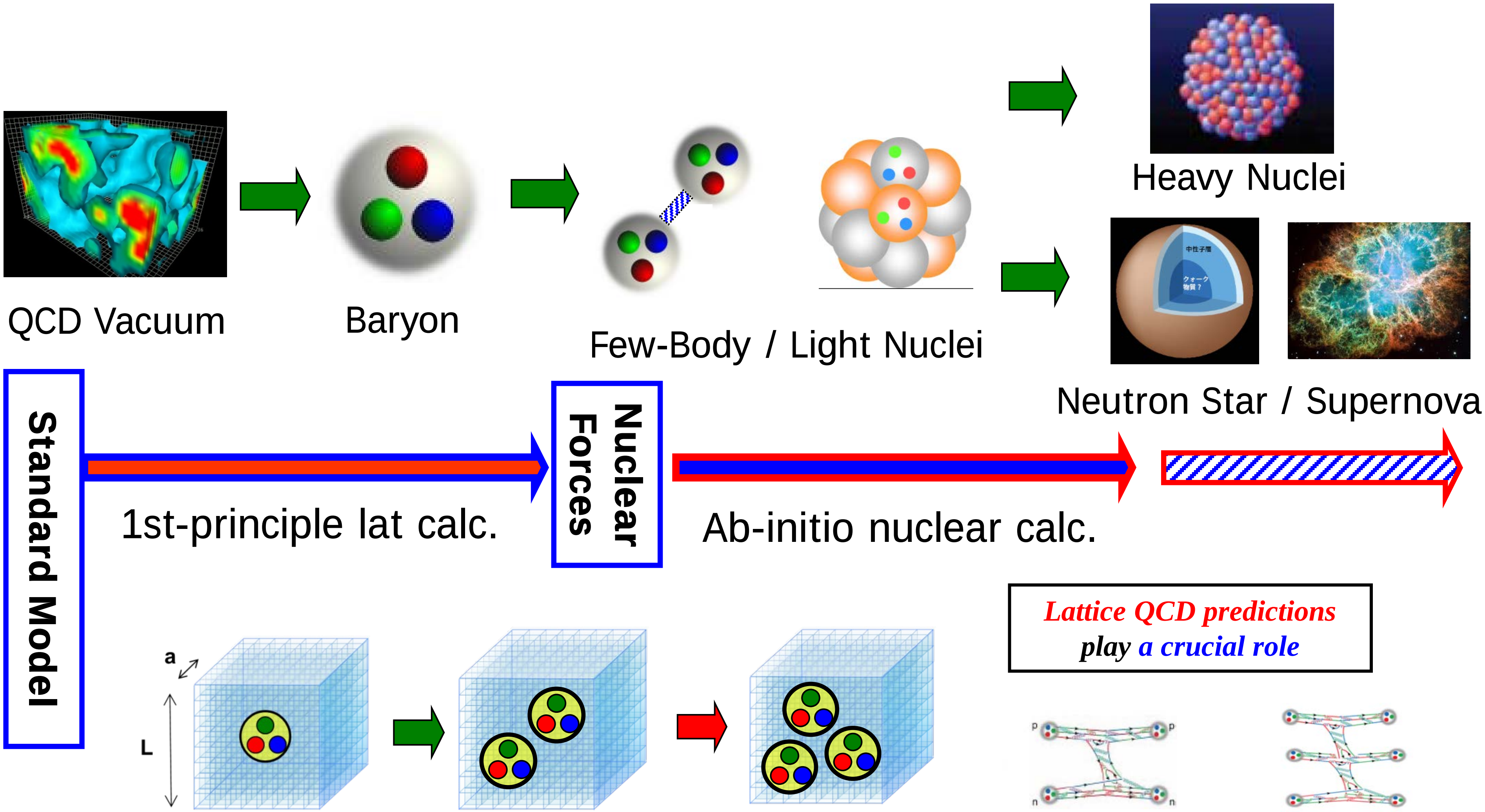
- **quark masses (m_u, m_d, m_s)**
- **coupling constant α_s**

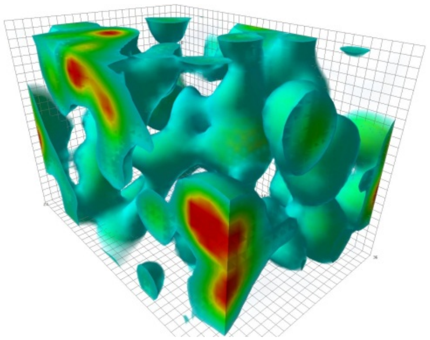
Fully dynamical (unquenched) QCD simulations
at the physical quark mass point already performed

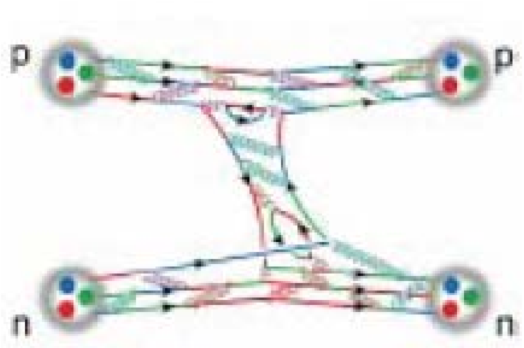
PACS-CS Coll., PRD81(2010)074503
BMW Coll., JHEP1108(2011)148

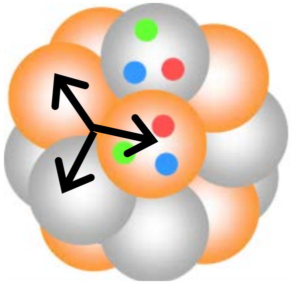
Roadmap:

Nuclear Physics and Astrophysics from Lat QCD









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Nuclear Physics on the Lattice

- NN phase shift (Luscher's formula)
  - Fukugita et al. PRL73(1994)2176
  - NPLQCD Coll, reviewed in Prog.Part.Nucl.Phys 66(2010)1

- NBS wave function ➔ NN potential
  - Ishii-Aoki-Hatsuda PRL99(2007)022001, PTP123(2010)89
  - HAL QCD Coll. (2009-), **reviewed in PTEP2012 (2012) 01A105 [arXiv:1206.5088]**

- Light nuclei on the lattice
  - Yamazaki-Kuramashi-Ukawa (PACS-CS Coll.) PRD81(2010)111504, PRD84(2011)054506

Other approaches, e.g., strong coupling limit

de Forcrand and Fromm, PRL104(2010)112005

Hadrons to Atomic nuclei from Lattice QCD
(HAL QCD Collaboration)

S. Aoki, N. Ishii, H. Nemura, K. Sasaki, M. Yamada (Univ. of Tsukuba)

B. Charron (Univ. of Tokyo)

T. Doi, T. Hatsuda , Y. Ikeda, K. Murano (RIKEN)

T. Inoue (Nihon Univ.)

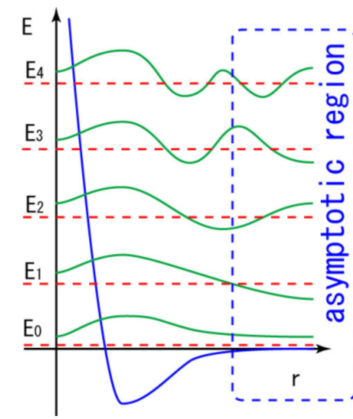
Nuclear Forces from Lattice QCD [HAL QCD method]

- Potential is constructed so as to reproduce the NN phase shifts (or, S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

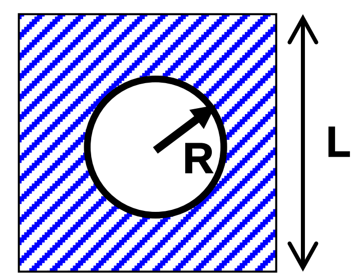
$$\boxed{\psi(\vec{r}) = \langle 0|\ N(\vec{r})N(\vec{0})\ |N(\vec{k})N(-\vec{k}); in\rangle}$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \qquad r > R$$



  – Wave function ←→ phase shifts

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

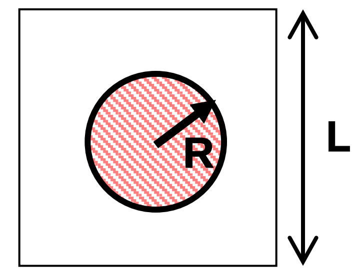


M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
CP-PACS Coll., PRD71(2005)094504
Ishizuka, PoS LAT2009 (2009) 119

“Potential” as a representation of S-matrix [HAL QCD method]

- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



  – U(r,r’): faithful to the phase shift by construction
    - U(r,r’) below inelastic threshold is

$$U(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{m} \sum_{n,n'}^{n_{\rm th}} (\nabla_{\boldsymbol{r}}^2 + k_n^2)\psi_n(\boldsymbol{r}) \mathcal{N}_{nn'}^{-1} \psi_{n'}^*(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r} \psi_n^*(\boldsymbol{r})\psi_{n'}(\boldsymbol{r})$$

    - U(r,r’): E-independent, while non-local in general
    - Non-locality ➔ derivative expansion　　Okubo-Marshak(1958)

$$U(\vec{r}, \vec{r'}) = \underbrace{V_c(r)}_{\textbf{LO}} + \underbrace{S_{12} V_T(r)}_{\textbf{LO}} + \underbrace{\vec{L}\cdot\vec{S} V_{LS}(r)}_{\textbf{NLO}} + \underbrace{\mathcal{O}(\nabla^2)}_{\textbf{NNLO}}$$

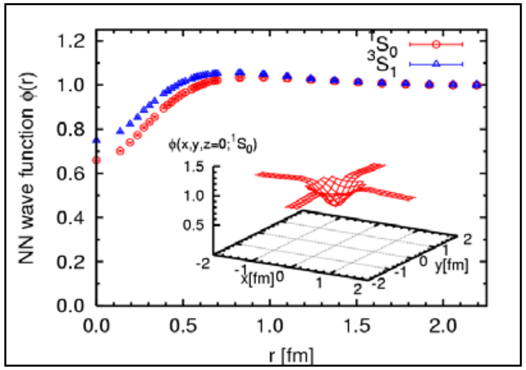
Aoki-Hatsuda-Ishii PTP123(2010)89

Check on convergence: K.Murano et al., PTP125(2011)1225

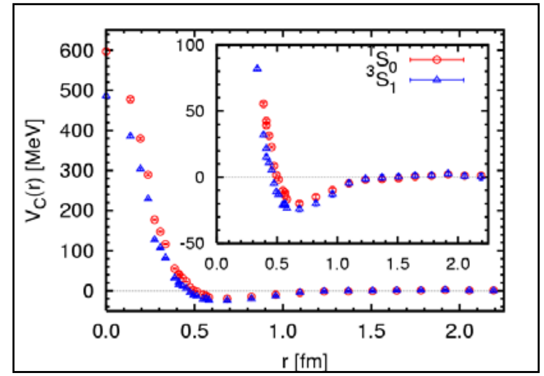
Our Approach [HAL QCD method]

Lattice QCD

NBS wave func.



Lat Nuclear Force



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq e^{i\delta_l(k)}\sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

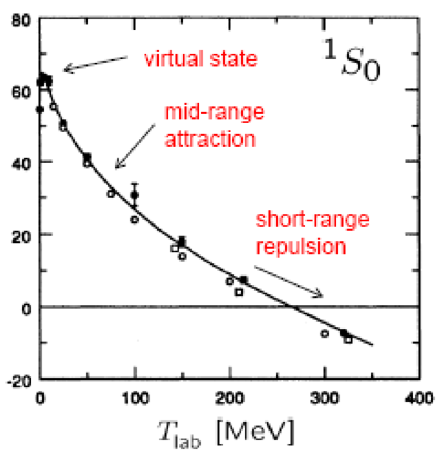
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}'\,U(\vec{r},\vec{r}')\psi(\vec{r}')$$

Lat potential is faithful to phase shift by construction

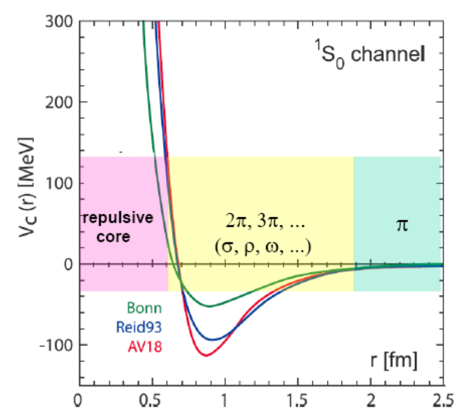
Analog to …

Scattering Exp.

Phase shifts



Phen. Potential



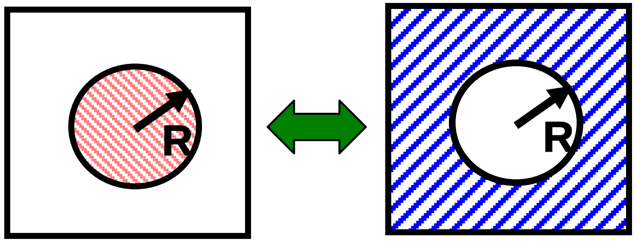
A few remarks on the Lattice Potential

- Potential is NOT an observable and is not unique:
  They are, however, phase-shift equivalent potentials.
  - Choosing the pot. (sink op.) ⇔ choosing the "scheme"

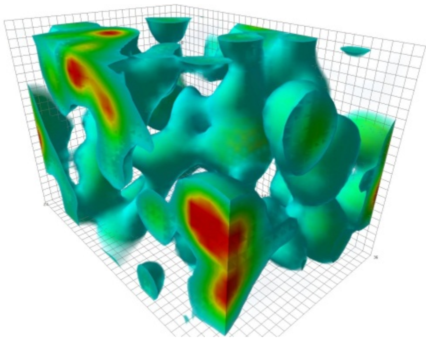
- We study potential (+ phase shifts), since:
  - Convenient to understand physics
  - Essential to study many-body

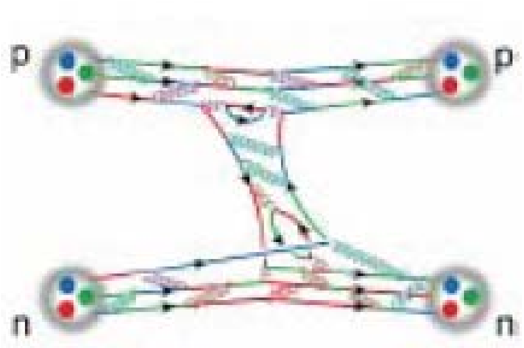
$$Lat \rightarrow \boxed{\delta_E \rightarrow U(\boldsymbol{r})} \rightarrow \text{many-body}$$

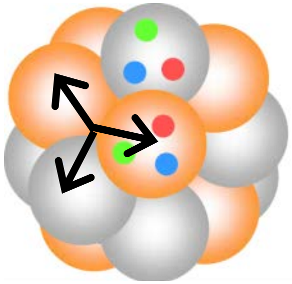
$$\boxed{Lat \rightarrow \underset{\delta_E}{\rightarrow} \underset{\leftarrow}{\rightarrow} U(\boldsymbol{r})} \rightarrow \text{many-body}$$



  - Finite V artifact better under control
  - Excited states better under control







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    - Three-baryon interactions
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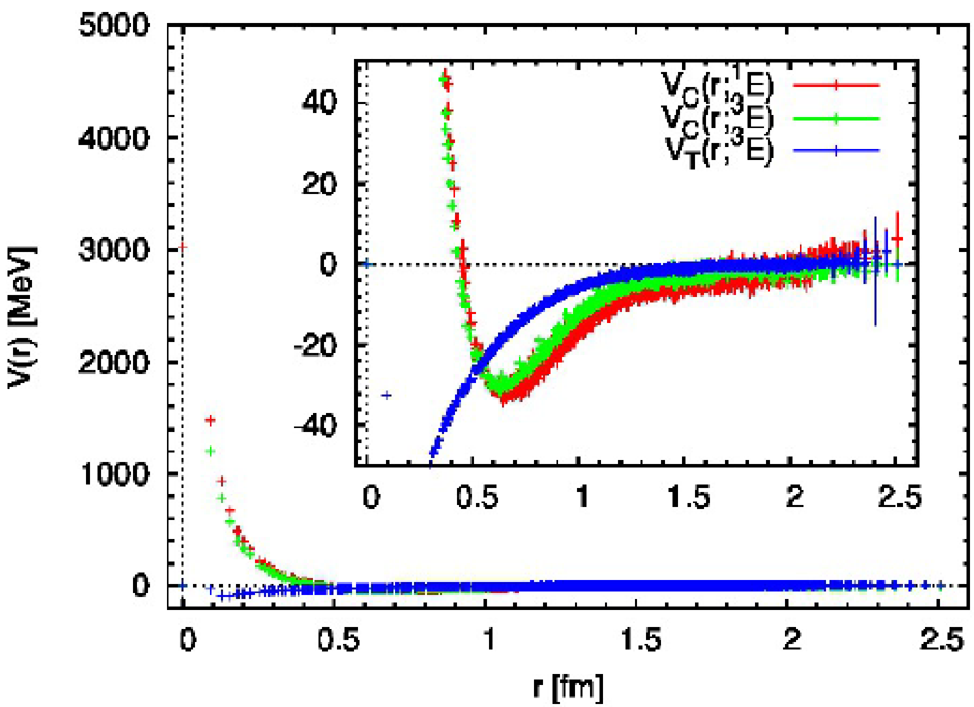
(1) NN potential on the lattice (positive parity)

$^{2S+1}L_J$

- "di-neutron" channel $^1S_0$ ➔ central force
- "deuteron" channel $^3S_1 - ^3D_1$ ➔ central & tensor force



Coupled channel study
for deuteron channel

$$(H_0 + V_C + V_T S_{12})\psi = E\psi$$

$$\psi = \psi_S + \psi_D$$

P	: projection to L=0
Q=(1-P)	: projection to L=2

⇩

$$P(H_0 + \boxed{V_C} + \boxed{V_T} S_{12})\psi = EP\psi$$

$$Q(H_0 + \boxed{V_C} + \boxed{V_T} S_{12})\psi = EQ\psi$$

Nf=2+1 clover (PACS-CS), 1/a=2.2GeV,
L=2.9fm, m_π=0.7GeV, m_N=1.6GeV

N.Ishii et al. (HAL QCD Coll.)
PLB712(2012)437

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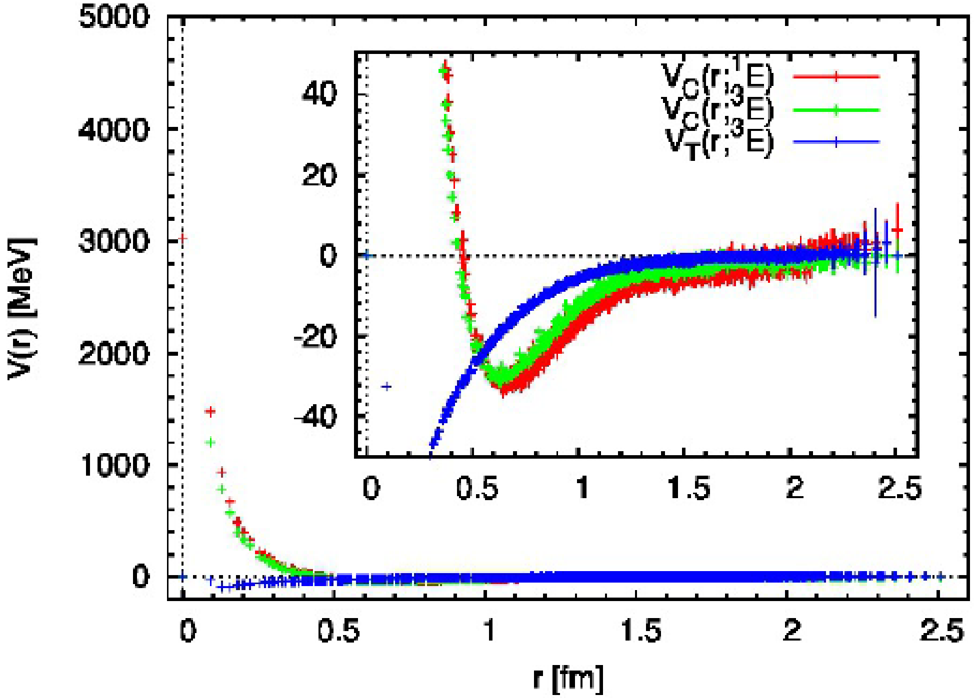

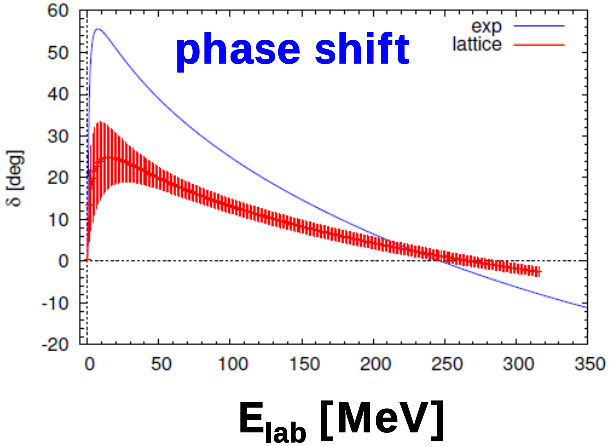

Not Bound $a(^1S_0) = 1.6(1.1)$ fm

Nf=2+1 clover (PACS-CS), 1/a=2.2GeV,
L=2.9fm, m_π=0.7GeV, m_N=1.6GeV

N.Ishii et al. (HAL QCD Coll.)
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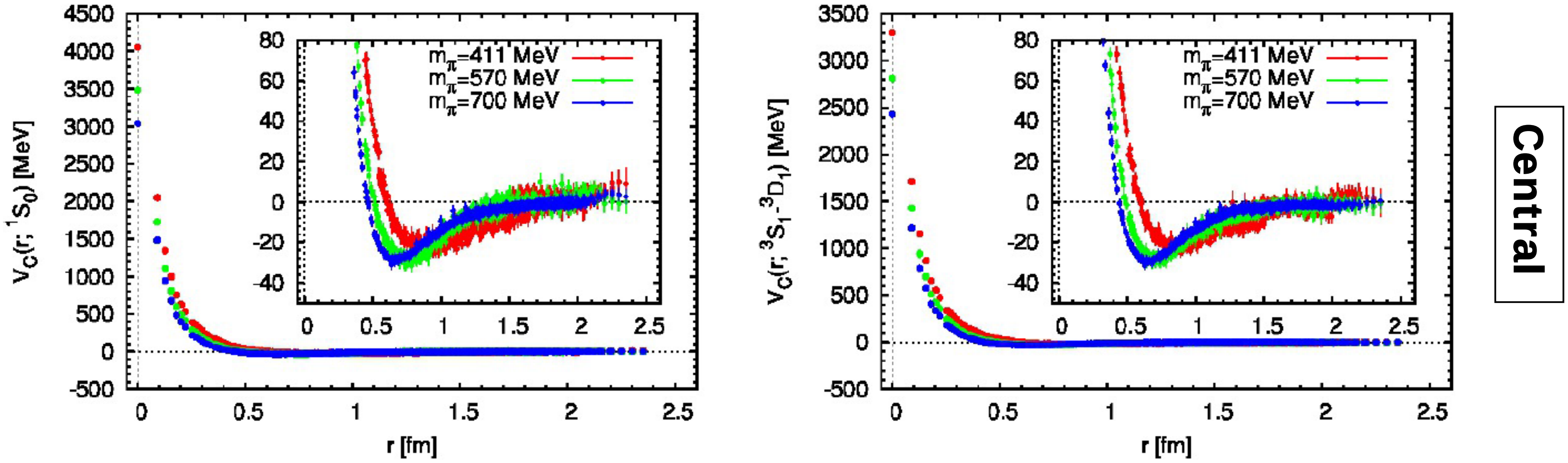
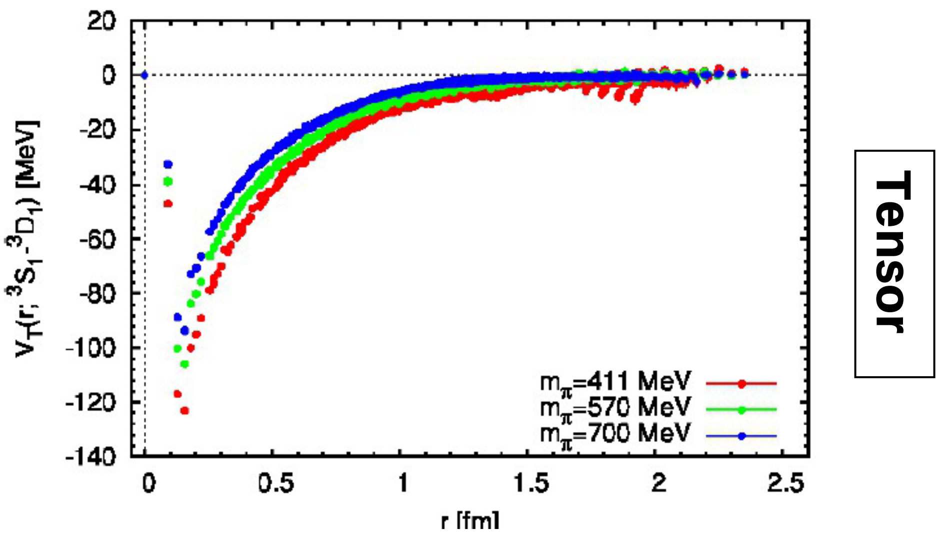
Quark mass dependence

Central in $^1S_0$

$^3S_1$-$^3D_1$ channel





<u>**Lighter mass**</u> corresponds to…

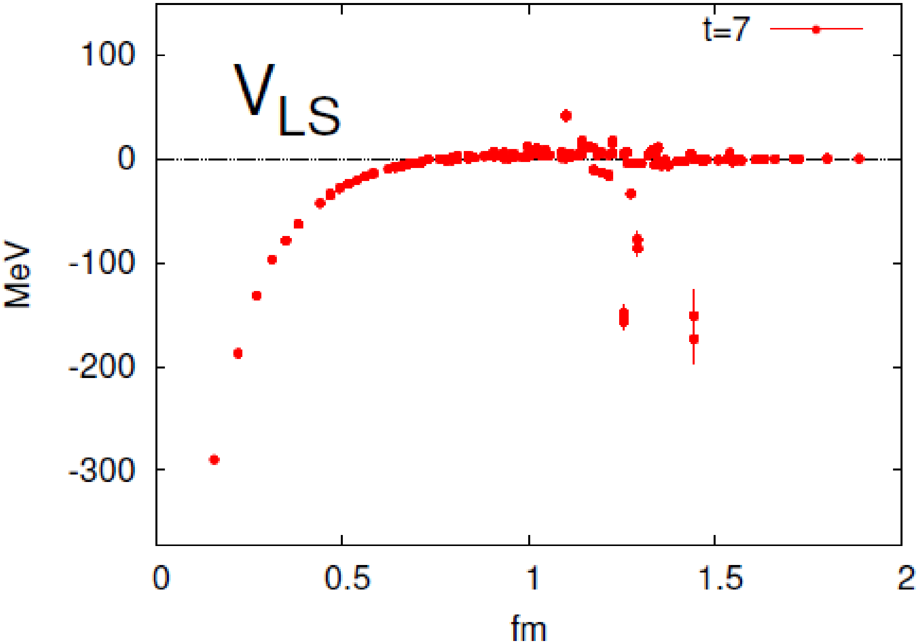
- **Longer interaction range**
- **Larger Repulsive Core**
- **Stronger Tensor Force**

N.Ishii @ Lat2012

NN potential on the lattice (negative parity)

$^{2S+1}L_J$

- S=1 channel: $^3P_0,\ ^3P_1,\ ^3P_2 - ^3F_2$
  - Central & tensor forces in LO
  - Spin-orbit force in NLO
    - Inject a momentum ➔ $J^P = A_1^-, T_1^-, T_2^-$



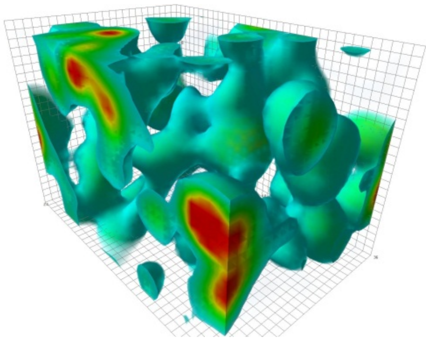
Nf=2 clover (CP-PACS), L=2.5fm, mπ=1.1GeV

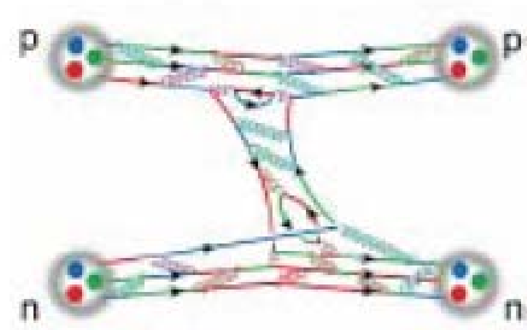
$\vec{L}\cdot\vec{S} = +1$ for $^3P_2$

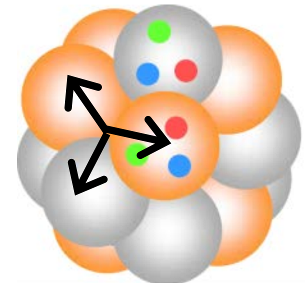
Superfluidity $^3P_2$ in neutron star ←→ neutrino cooling

←→ **Cas A NS: cooling is being measured !**

[K.Murano]







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$$8 \times 8 = \underbrace{27 + 8s + 1}_{\text{symmetric}} + \underbrace{10^* + 10 + 8a}_{\text{anti-symmetric}}$$

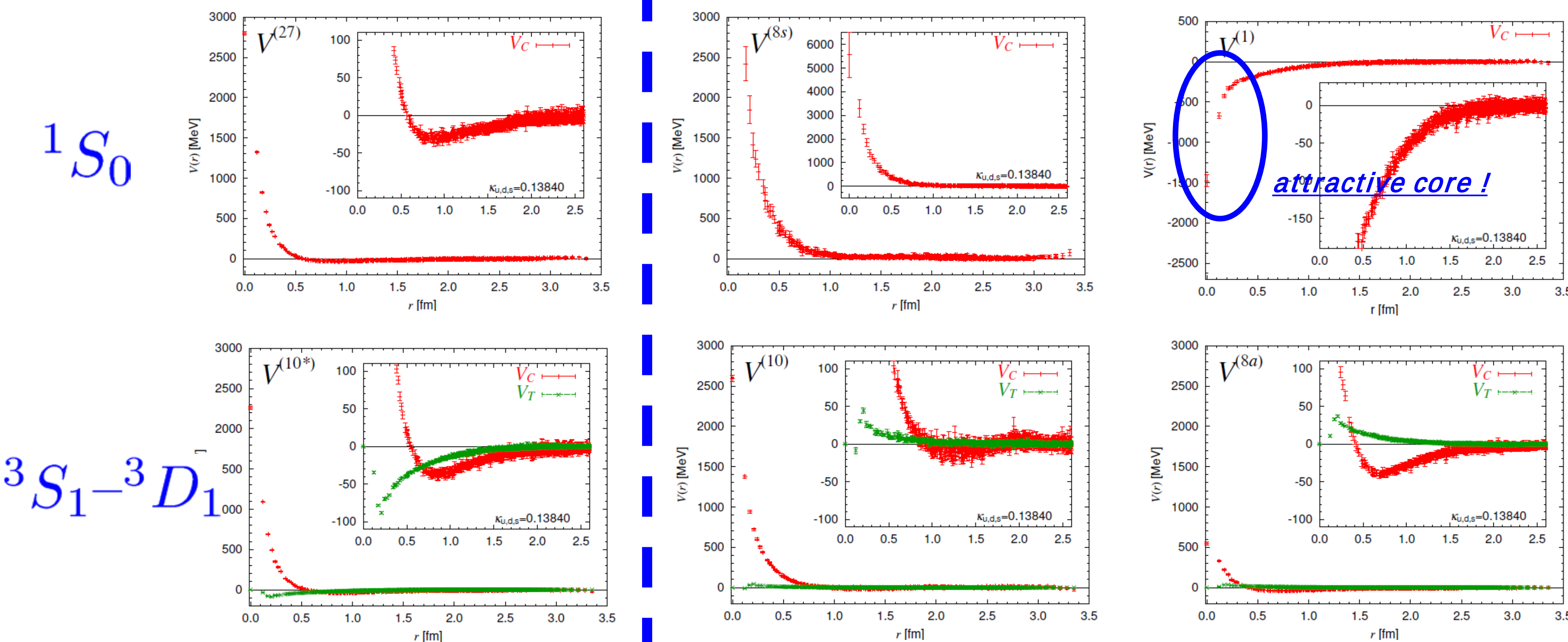
NN channel

    - Three-baryon interactions
  - Challenges toward physical quark mass point
  - Summary / Prospects

SU(3) study

(2) Hyperon forces

a=0.12fm, L=3.9fm, m(PS)= 0.47-1.2GeV



27,10*:
Same as NN

8s,10:
strong repulsive core

1s: deep attractive pocket
8a: weak repulsive core

Repulsive core
← Pauli principle !

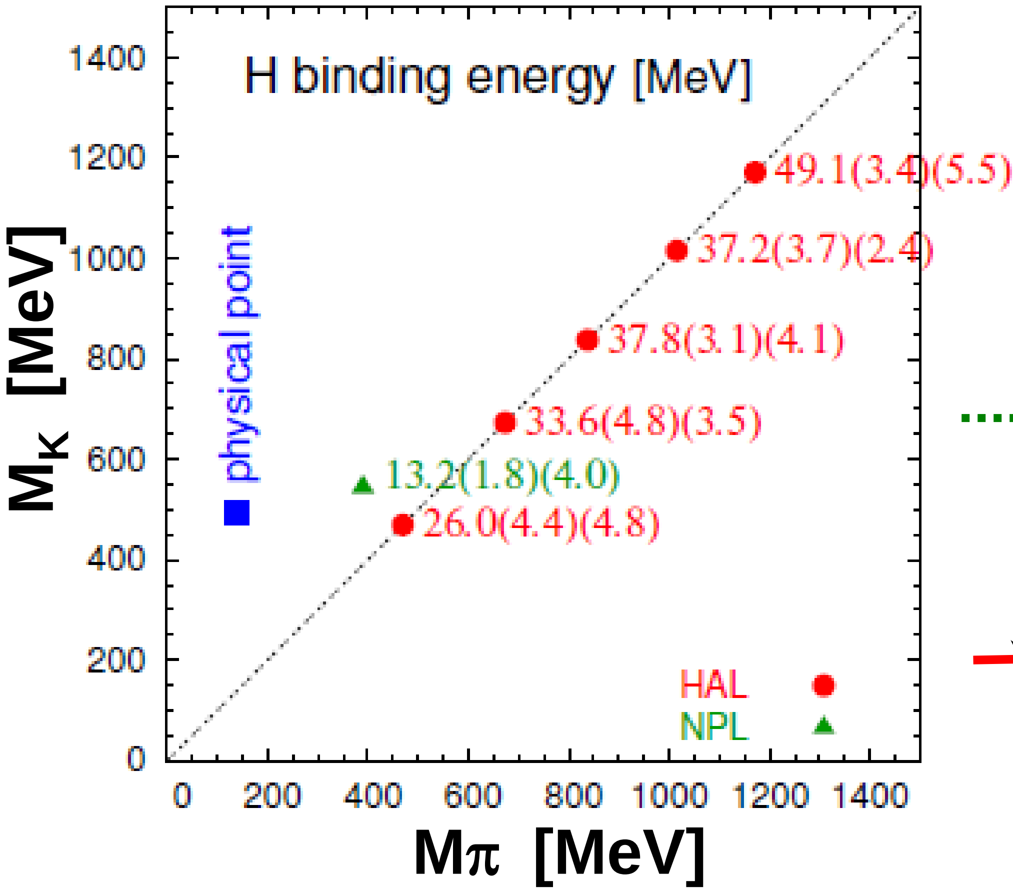
T.Inoue et al. (HAL QCD Coll.), NPA881(2012)28

Also seen in Takahashi et al. (2010), Kawanai et al. (2010)
Meson-baryon, **Y.Ikeda** et al., arXiv:1111.2663

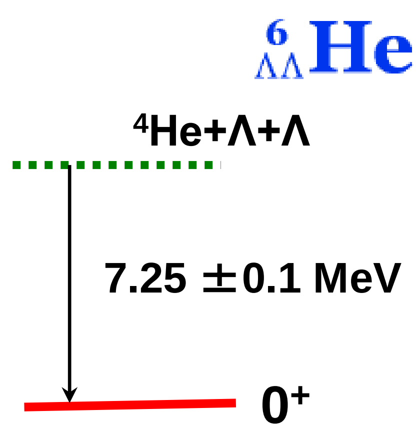
M.Oka et al., NPA464(1987)700

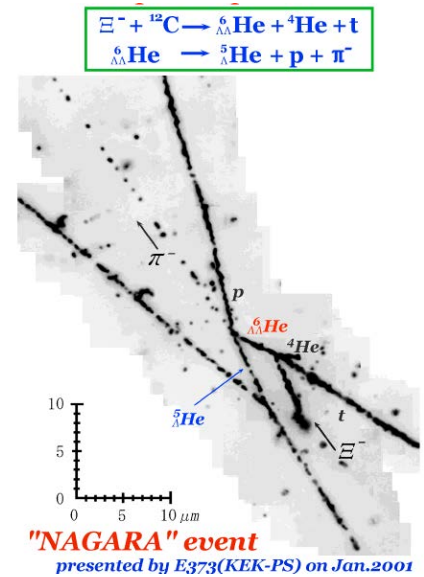
➔ Study of baryonic matter & Neutron Star [T.Inoue]

H-dibaryon (uuddss, I=0, $^1S_0$)



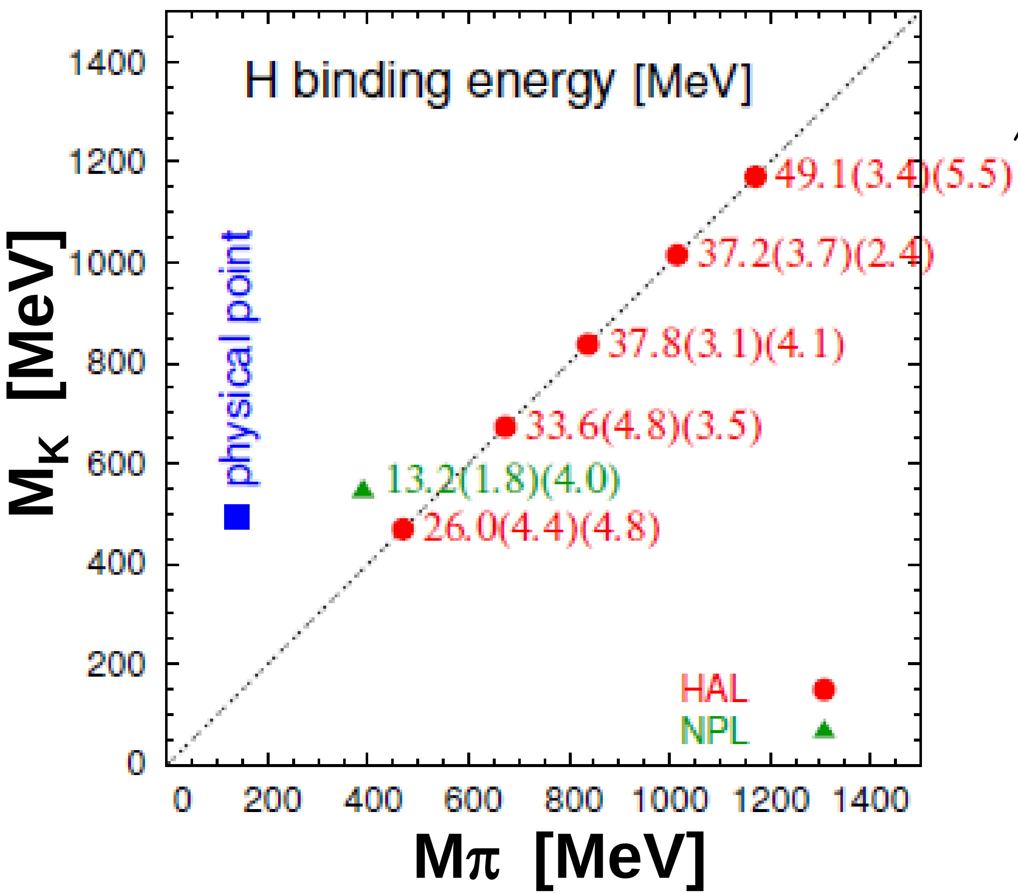
Experiments

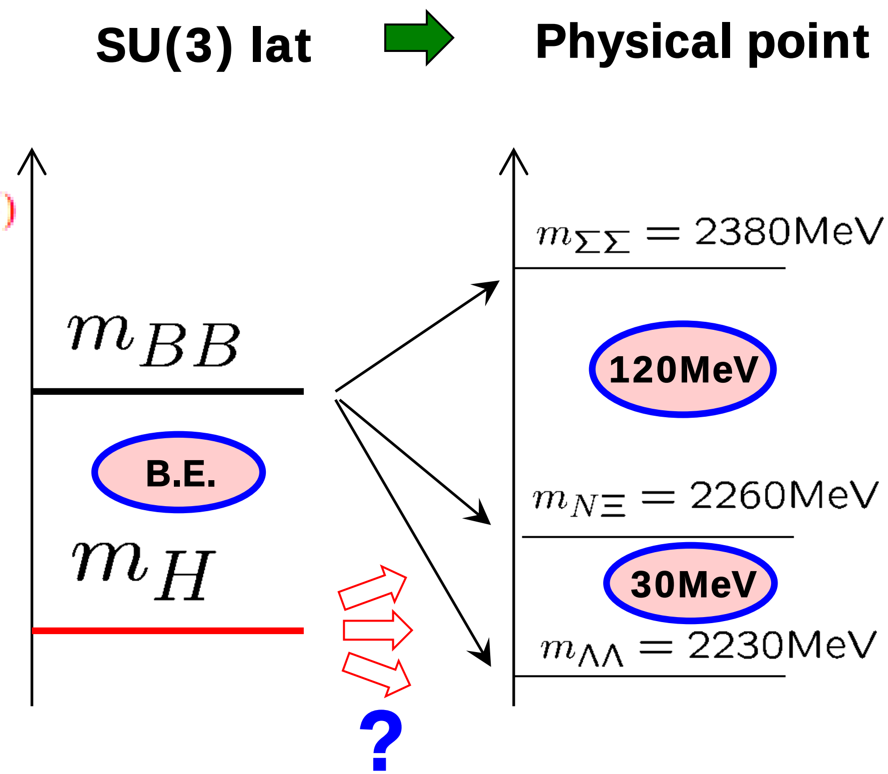




- ΛN attraction
- ΛΛ weak attraction
- No deeply bound H-dibaryon

H-dibaryon (uuddss, I=0,$^1S_0$)



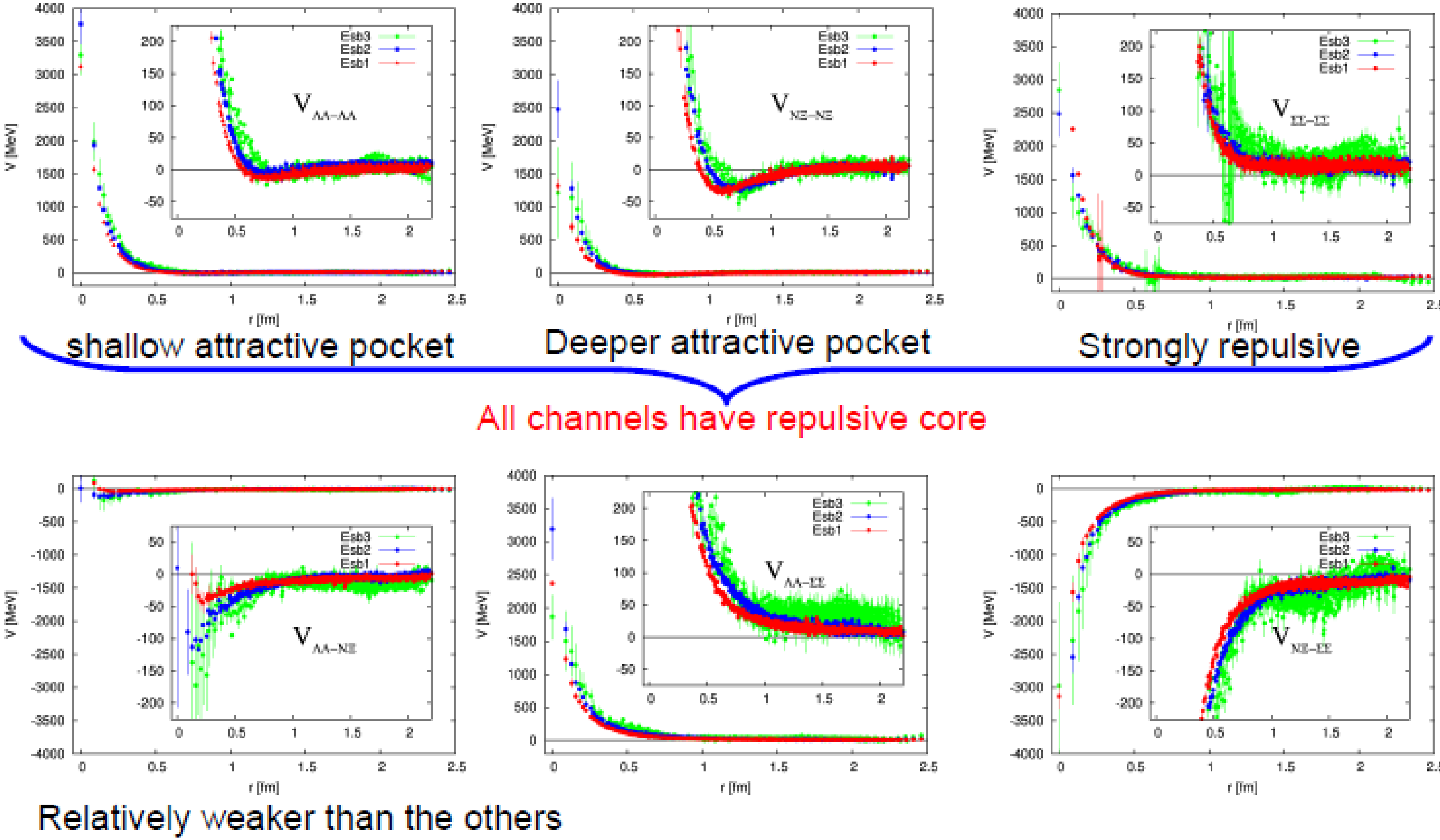


Coupled channel study is essential

$$\Lambda\Lambda\text{–}N\Xi\text{–}\Sigma\Sigma$$

→ K.Sasaki

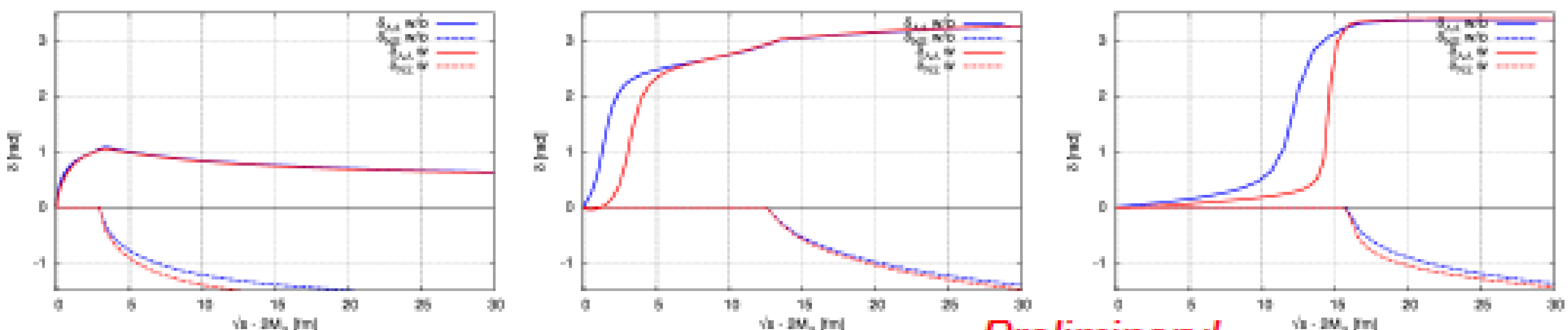
Coupled channel beyond SU(3)



[K.Sasaki]

SU(3) breaking effects on phase shift

Esb 1 | *Esb 2* | *Esb 3*



Preliminary!

- Esb1:
  - No difference between two types of potentials.
  - Bound H-dibaryon
- Esb2:
  - ΛΛ phase shift is visibly changed
  - H-dibaryon is near the ΛΛ threshold
- Esb3:
  - ΛΛ phase shifts are visibly changed.
  - The H-dibaryon resonance energy is close to NΞ threshold..

In unit of MeV	*Esb 1*	*Esb 2*	*Esb 3*
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter

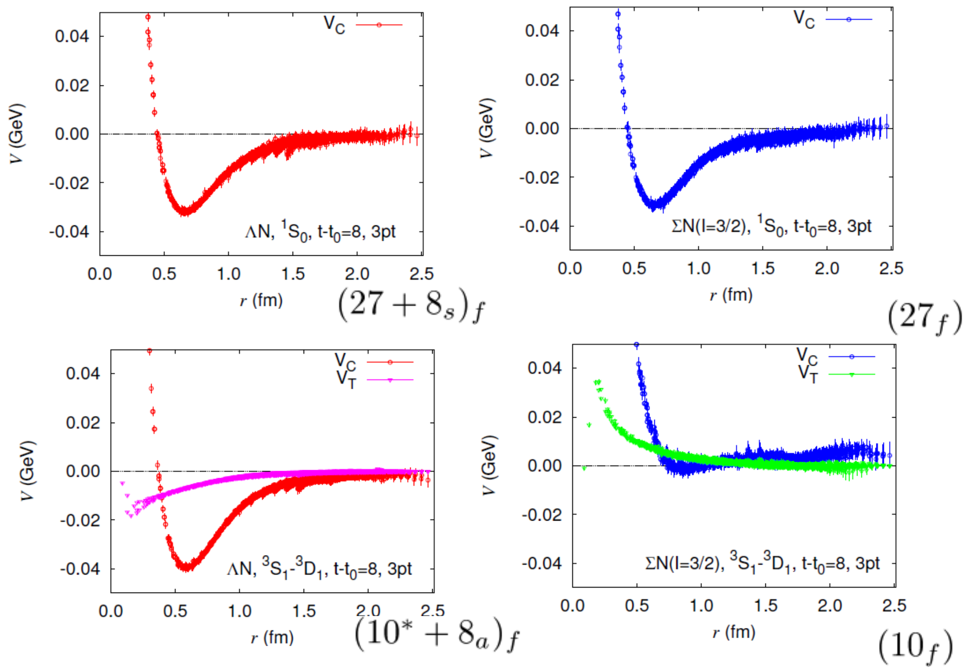
Hyperon Interactions in S= -1

ΛN $\qquad$ $\Sigma N(I=3/2)$

(single channel study)

$^1S_0$

$^3S_1\text{–}^3D_1$



$\Lambda N(^1S_0, ^3S_1)$

$\Sigma N(I=3/2, ^1S_0)$

➔Attractive

Not bound

$\Sigma N(I=3/2, ^3S_1)$

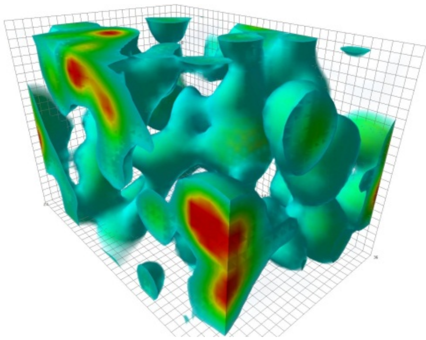
➔Repulsive

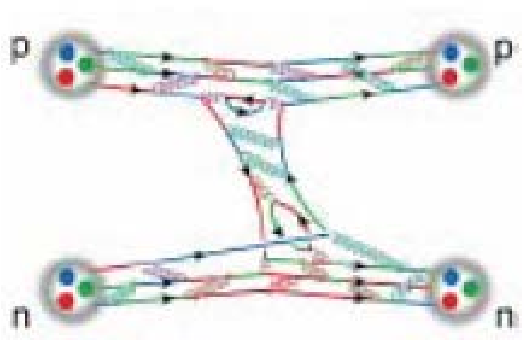
Nemura et al.
Nf=2+1, L=2.9fm,
mπ= 0.70GeV
arXiv:1203.3320

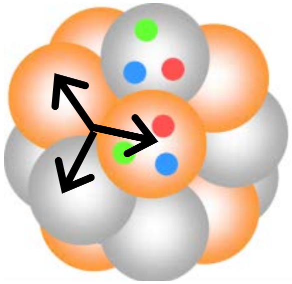
Crucial input for the core of neutron star and hyper-nuclei

[H. Nemura]

ΛN-ΣN coupled channel study is also in progress





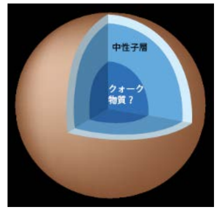


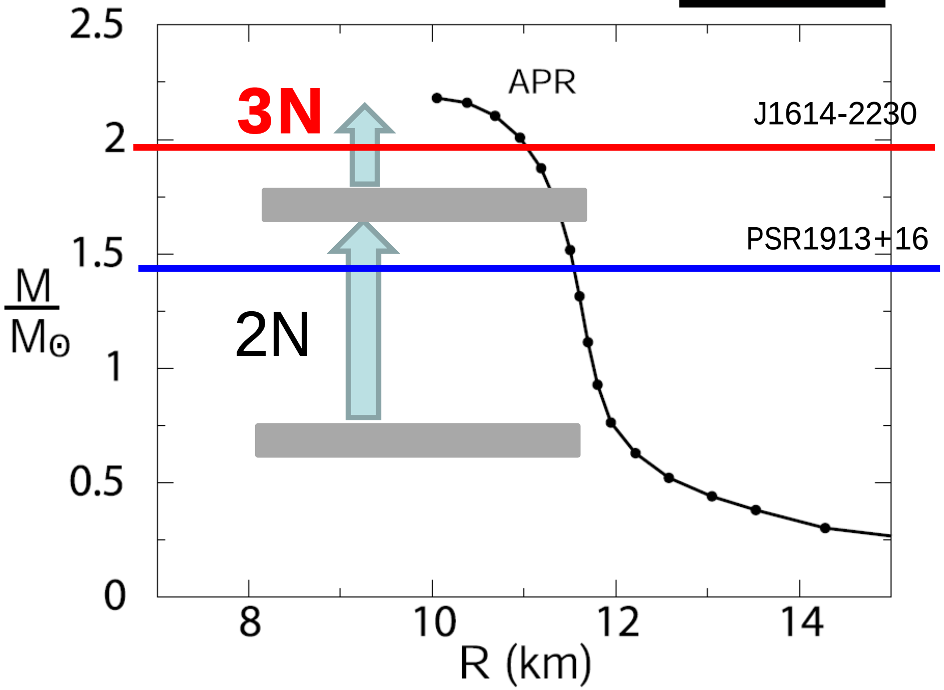
- <u>Outline</u>
  - Introduction
  - Theoretical framework for lattice hadron forces
  - Lattice results at heavier quark masses
    - Two-baryon interactions (1) nuclear- (2) hyperon- forces
    - **Three-baryon interactions (NNN)**
  - Challenges toward physical quark mass point
  - Summary / Prospects

Crucial role of 3NF at short range

Neutron Star

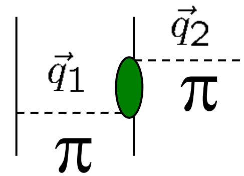
（Densest system in the Universe）





Short-range repulsive 3NF is required

In phenomenological models:



2πE-3NF:

➔ too strong attraction

Tucson-Melbourne

$$\sim \frac{F_\pi^2(\vec{q}_1^2)(\vec{\sigma}\cdot\vec{q}_1)}{\vec{q}_1^2+m_\pi^2} F(\vec{q}_1,\vec{q}_2) \frac{F_\pi^2(\vec{q}_2^2)(\vec{\sigma}\cdot\vec{q}_2)}{\vec{q}_2^2+m_\pi^2}$$

$F_\pi(\vec{q}^2) = \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2+\vec{q}^2}\right)$ $\quad \Lambda_\pi \sim 700\text{MeV} \ll 1.3\text{GeV}$

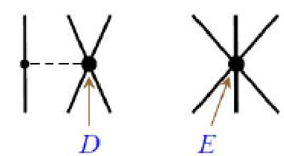
➔ (effective) short-range repulsion by cut-off

Urbana/Illinois

2πE-3NF + explicit short-range repulsive term

Chiral EFT

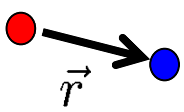
Short-range LECs fitted



Can we understand it directly from QCD ?

3NF calculation in Lat QCD

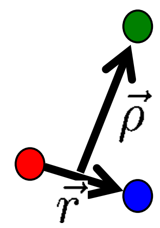
- **In the case of 2N system…**
  - Calc 4pt func ➔NBS amp.



$$\psi(\vec r) = \langle 0|N(\vec x+\vec r;t)N(\vec x;t)|2N\rangle$$

- **Extention to 3N system**
  - Calc 6pt func ➔NBS amp. of 3N

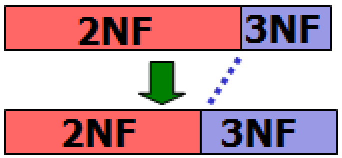
$$\psi(\vec r,\vec\rho) = \langle 0\boxed{N(\vec x+\vec r)}\boxed{N(\vec x)}\boxed{N(\vec x+\vec r/2+\vec\rho)}|3N\rangle$$



  - Obtain 3NF through

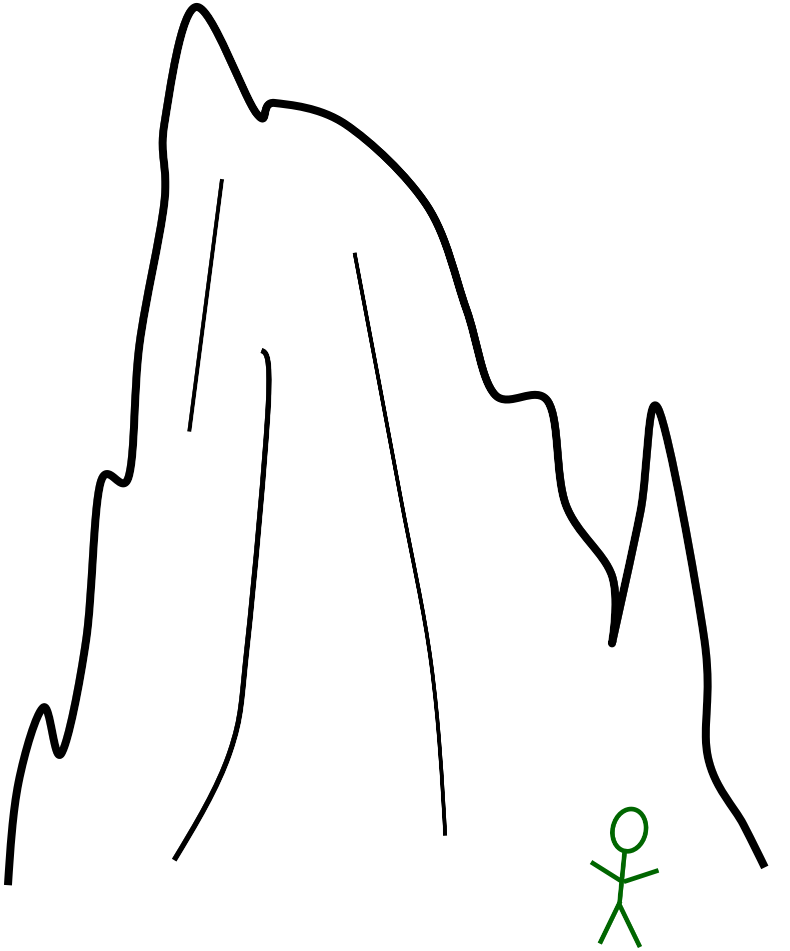
$$(E-H_0^r-H_0^\rho)\psi(\vec r,\vec\rho) = \left[\underbrace{\textstyle\sum_{i<j} V_{ij}(\vec r_{ij})}_{\text{by 2N calc}} + V_{3NF}(\vec r,\vec\rho)\right]\psi(\vec r,\vec\rho)$$

– The combination of (2NF, 3NF) ➔ observables
  - ➔ systematic determination by Lat QCD



The Challenges

- Enormous computational cost for correlators



The Challenges

- Enormous computational cost for correlators
  - # of Wick contraction (permutation)

$$N_{\rm perm} = N_u! \times N_d! \sim [\,(\tfrac{3}{2}A)!\,]^2$$ for mass number A

(← **a factor of 2^A speedup** by inner-baryon exchange)

  - # of color / spinor contractions

$$N_{\rm loop} = 6^A \cdot 4^A$$

(color) (spinor)

(← **a factor of 2^A speedup** by "half-spin" method)

$$N = \epsilon_{abc}(q^T C\gamma_5 q)q$$

  - Total cost: $N_{\rm perm} \times N_{\rm loop}$
    - ^{2}H : 9 x 144 = 1 x 10^3
    - ^{3}H : 360 x 1728 = 6 x 10^5
    - ^{4}He : 32400 x 20736 = 7 x 10^8

c.f. T.Yamazaki et al., PRD81(2010)111504
$N_{\rm perm} = 1107$ for ^{4}He in the isospin limit

See also TD (HAL QCD Coll.) PoS LAT2010, 136

Breaking News !

Unified contraction algorithm

TD, M.Endres, arXiv:1205.0585, CPC184(2013)117

- Traditional algorithm

color/spinor contractions (ξ')

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Permutations

color ϵ_{abc}, spinor $(C\gamma_5)$, etc.

- New algorithm

[impose the same spacial label at source]

  – Permutation applies to color/spinor indices at "Coeff"

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Sum over color/spinor unified list

  – **Permutation DONE beforehand**
    - (Wick contraction and color/spinor contractions are unified)
  – Significant improvement

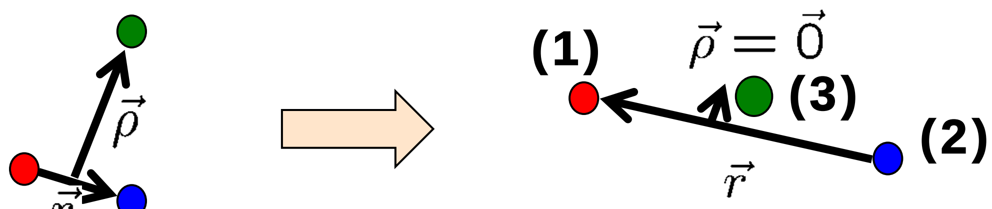
$\boxed{\times 192 \text{ for } {}^3\mathrm{H}/{}^3\mathrm{He},\ \times 20736 \text{ for } {}^4\mathrm{He},\ \times 10^{11} \text{ for } {}^8\mathrm{Be}}$

(x add'l. speedup)

3NF calculation in Lat QCD

- We fix the geometry of 3N (← this is not an approximation)
- We study **linear setup**

We consider Triton channel

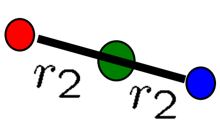


$(\vec{r}_2 \equiv \vec{r}/2)$

  - → $L^{(1,2)\text{-pair}} = L^{\text{total}} = 0$ or 2 only
  - → **Bases are only three**, labeled by **$^1S_0$, $^3S_1$, $^3D_1$ for (1,2)-pair**

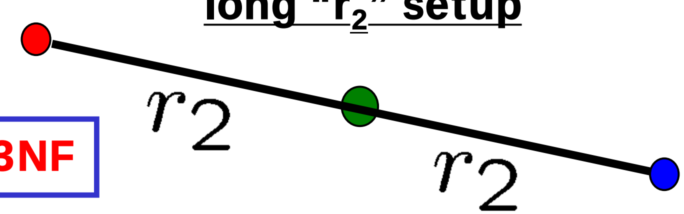
- **Linear setup** with various distance **"r_2"**

short "r_2" setup



Study r_2-dependence of 3NF

long "r_2" setup



Extraction of Genuine 3NF

- **Genuine 3NF** can be extracted from **3x3 coupled channel**
  - Both of parity-even 2NF and parity-odd potential required

$$\hat{H}_0 \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} + \left(\underset{(V_{2N}+V_{3NF})}{V} \right) \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} = E \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix}$$

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

$\boxed{V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1} \text{ : (P = odd)}}$

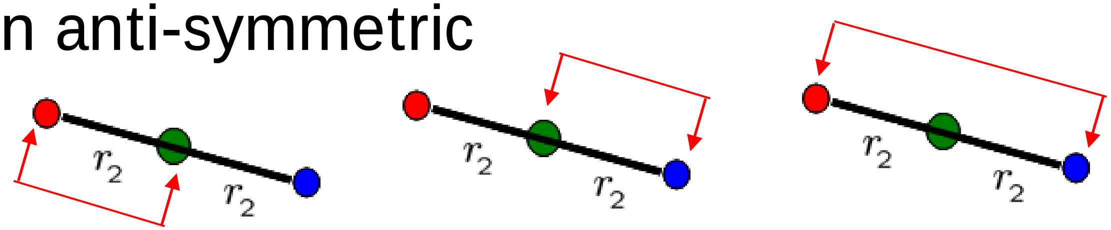
Target to be determined

- S/N : parity-even 2NF > parity-odd 2NF in Lat QCD
  - → **Desirable to extract 3NF w/ parity-even 2NF only**

Solution using "symmetric" wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric

$\psi_S =$



- ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- Bases are rotated as $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$\boxed{|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)}$$

$$|\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

All pair P=even

$$\hat{H}_0 \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \hat{V}_{3NF} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} = E \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix}$$

No V(P=odd)

Solution using "symmetric" wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric
  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- 3x3 coupled channel is reduced to
  - **<u>one channel with only 3NF unknown</u>**
  - two channels with $V_C^{I,S=0,0}$, $V_C^{I,S=1,1}$, $V_T^{I,S=1,1}$,(3NF) unknown

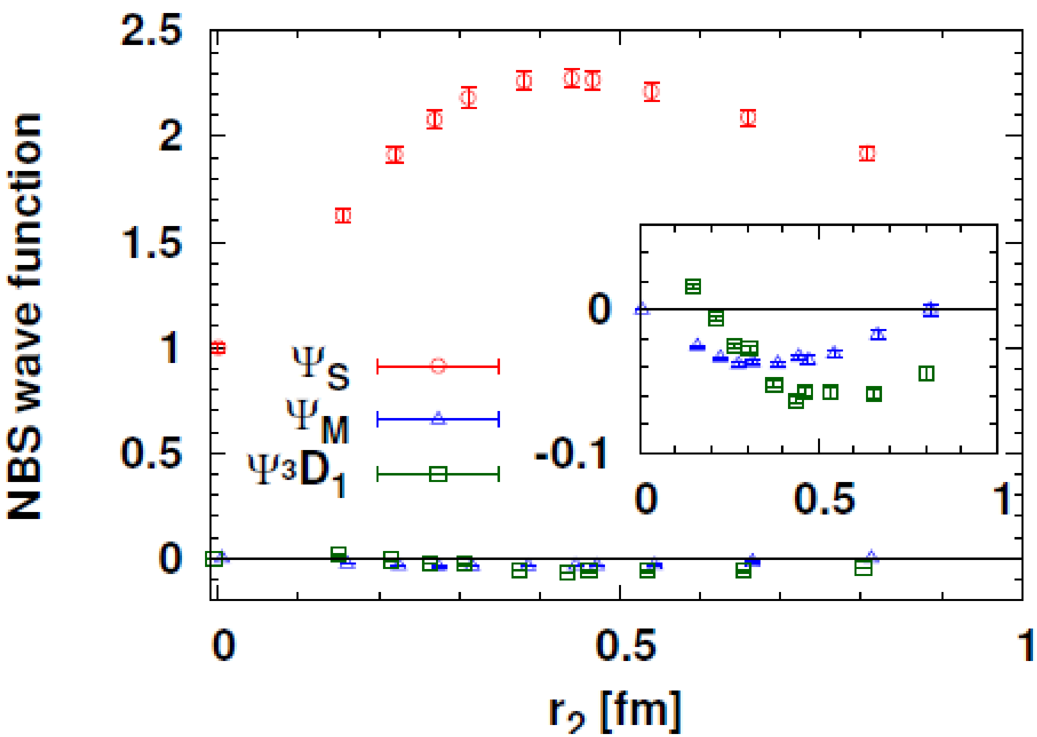
$$\begin{pmatrix} & & \\ & H_0 & \\ & & \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{3NF} & \\ & & \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} = E\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix}$$

No V(P=odd)

Target to be determined

- ➔ Even without parity-odd V, we can determine one 3NF
  - This method works for any fixed 3D-geometry other than linear

Results for wave functions

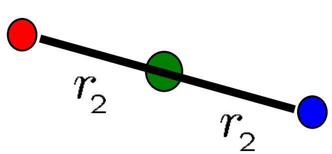


Red: Ψ_S
Blue: Ψ_M
Green: Ψ_{3D1}

Ψ_S overwhelms the wave function:

➔ Indication of the dominance of all S-wave component, higher waves suppressed

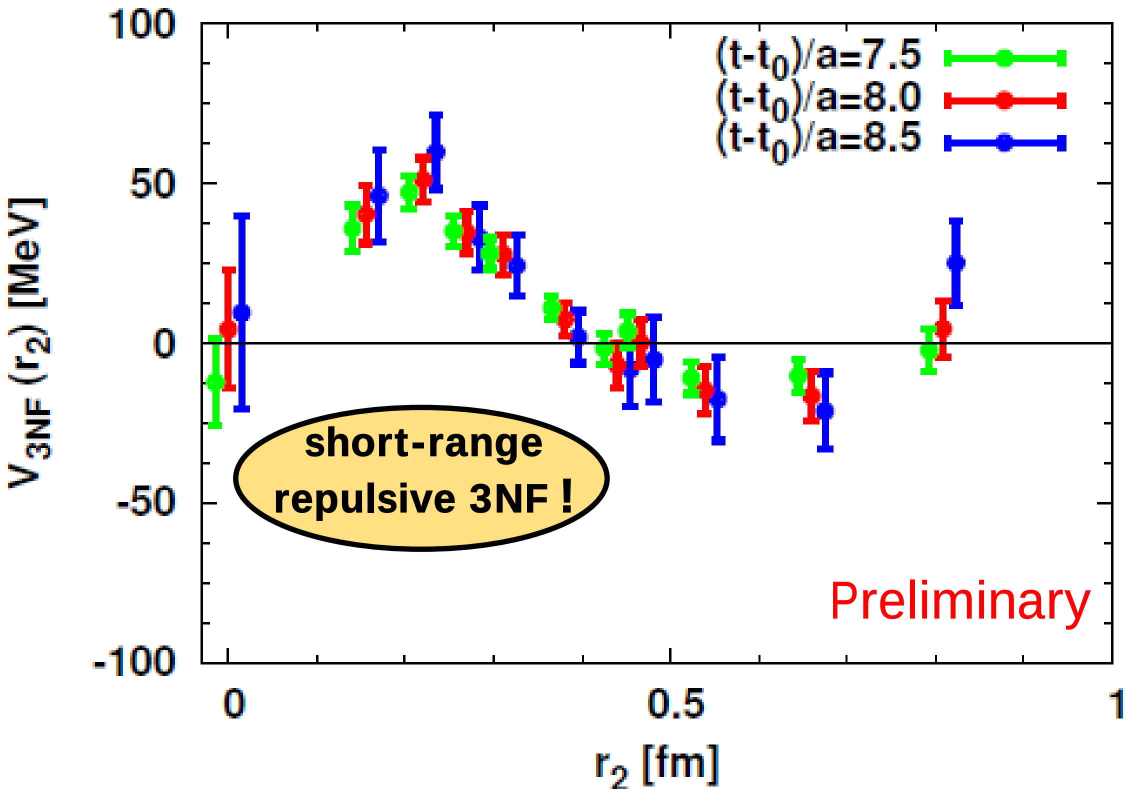
T.D. et al. (HAL QCD)
PTP127(2012)723

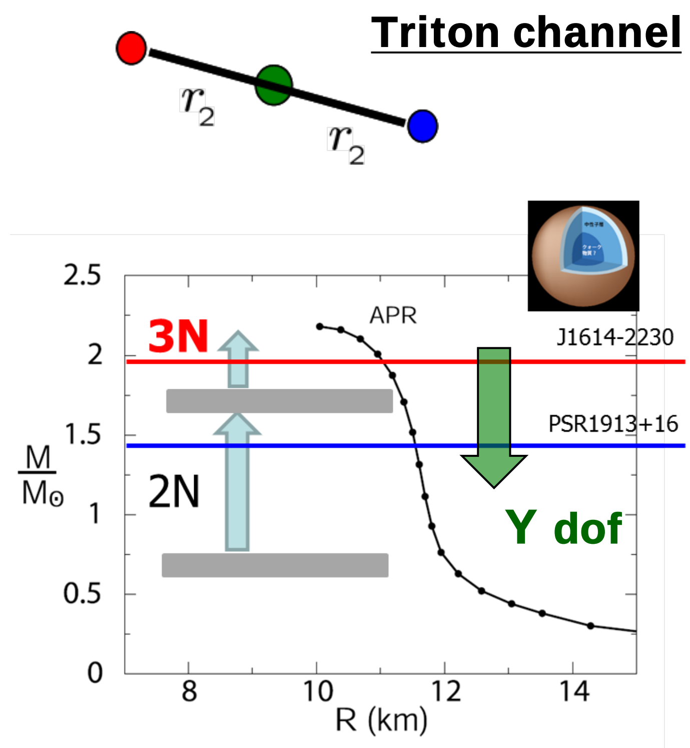


(3) 3N-forces (3NF) on the lattice

T.D. et al. (HAL QCD Coll.) PTP127(2012)723

+ t-dep method updates





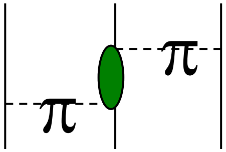
Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=1.1GeV, m_N=2.1GeV

How about other geometries ?

How about YNN, YYN, YYY ?

What is the origin of Lat 3NF ?

- 2πE-type 3NF (Fujita-Miyazawa) is unlikely
  - Strongly suppressed by m_π = 1.13GeV



- It may be attributed to quark/gluon dynamics directly
  - Recall generalized 2BF in SU(3)f …

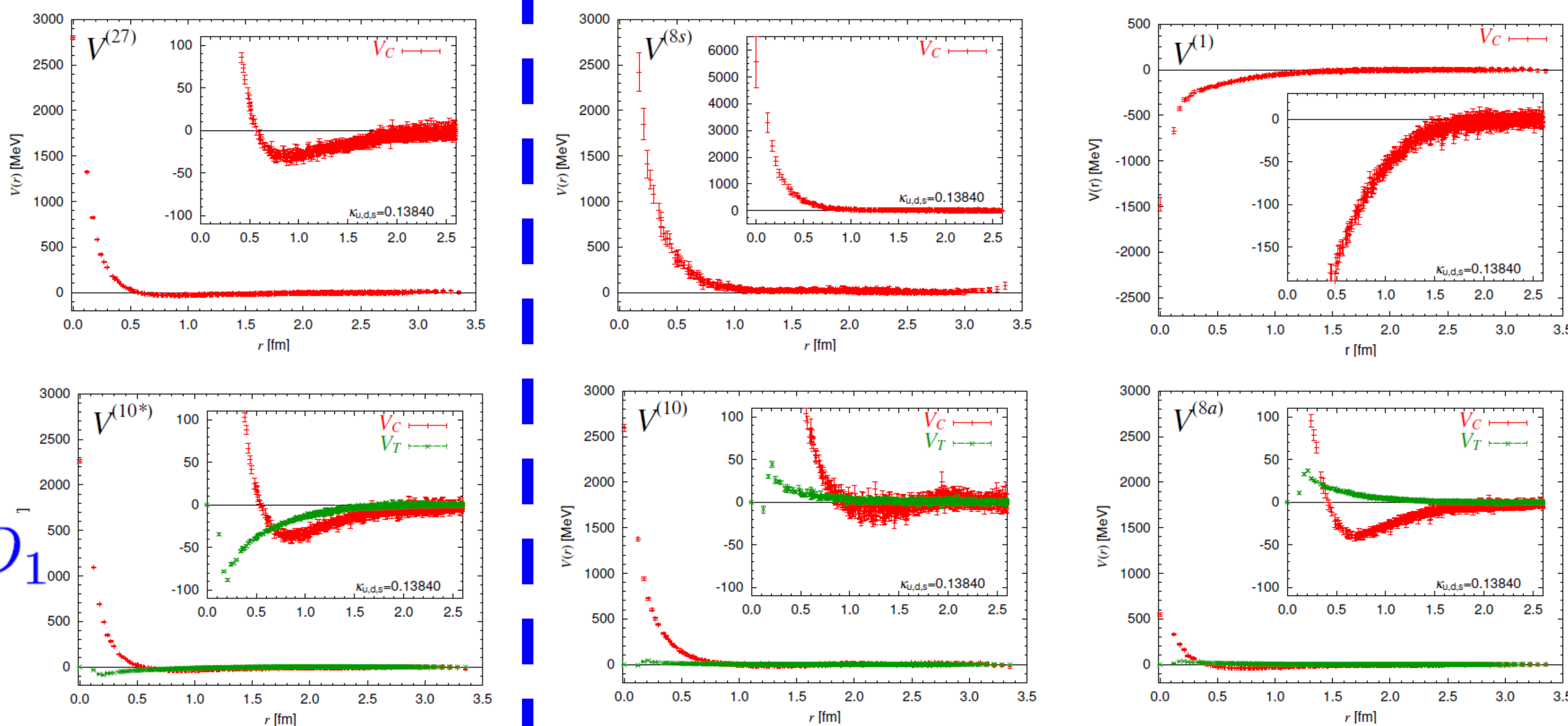
SU(3) study

BB potentials

a=0.12fm, L=3.9fm, m(PS)= 0.47-1.2GeV

$^1S_0$

$^3S_1{-}^3D_1$



27,10*:
Same as NN

8s,10:
strong repulsive core

1s: deep attractive pocket
8a: weak repulsive core

Repulsive core
← Pauli principle !

M.Oka et al., NPA464(1987)700

T.Inoue et al. (HAL QCD Coll.), NPA881(2012)28

Also seen in SU(2)c , Takahashi et al.,, PRD82(2010)094506
Charmonium-N, Kawanai-Sasaki, PRD82(2010)091501
Meson-baryon, Y.Ikeda et al., arXiv:1111.2663

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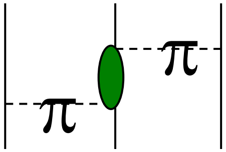
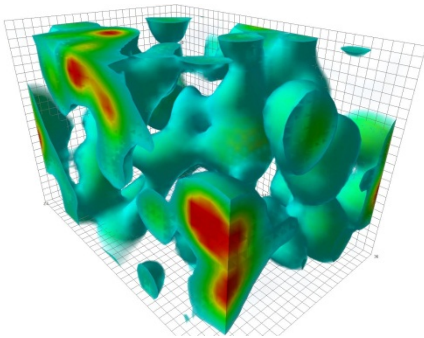
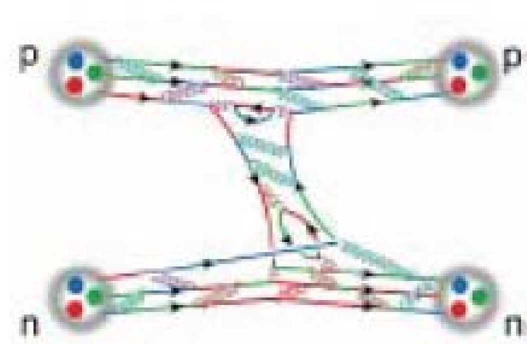
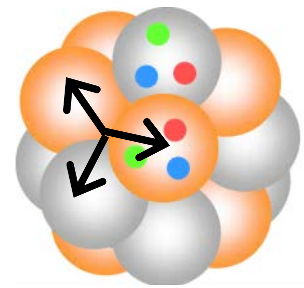


- It may be attributed to quark/gluon dynamics directly
  - Recall generalized 2BF in SU(3)f …
    - ➔ Pauli principle works well
  - What will be the Pauli-principle effect in 3NF from a viewpoint of the Quark Model ?
  - c.f. OPE (pert. QCD) predicts repulsive 3NF at short distance

S.Aoki et al., arXiv:1112.2053







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  - Challenges toward realistic potentials
    - Computational cost ➔ unified contraction everywhere
    - S/N issue
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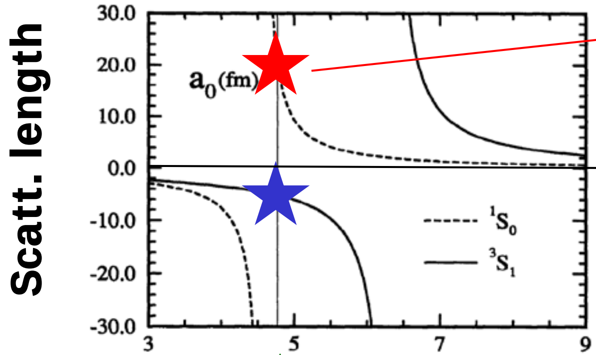
K computer

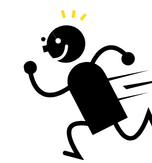
Towards realistic potential by the K computer

- **Physical mass point, Infinite V limit, continuum limit**
  - Physical $m\pi$ crucial for OPEP, chiral extrapolation won't work



$a_0(^1S_0) \sim 20\text{fm}$

"Unitary Region"



We are here

➔ m_q

m_q

Y.Kuramashi, PTPS122(1996)153

Phys. point

  - Gauge confs generation at $m\pi$ = 140MeV, L=~10fm @ K

➔ Challenge in the "measurement": S/N issue

Challenges toward the physical point

- S/N issue at light mass

Parisi, Lepage (1989)

  – To achieve ground state saturation, take $t \to \infty$

Single nucleon

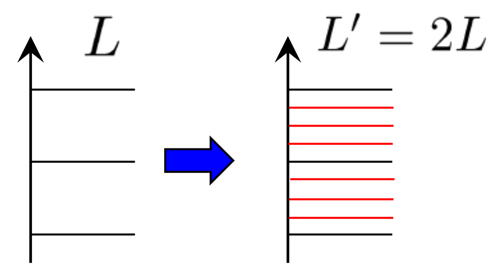
$$\frac{\mathrm{Signal}}{\mathrm{Noise}} \sim \frac{\langle N(t)\bar{N}(0)\rangle}{\sqrt{\langle N\bar{N}(t)N\bar{N}(0)\rangle}} \sim \frac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \mathbf{exp[-(m_N - 3/2m_\pi) \times t]}$$

Nucleons w/ mass number = A

$$\frac{\mathrm{Signal}}{\mathrm{Noise}} \sim \mathbf{exp[-A \times (m_N - 3/2m_\pi) \times t]}$$

  – Situation gets worse for larger volume
  🡐 large spectral density by scatt. states

$$\Delta E \simeq \frac{\vec{p}^2}{m_N} \simeq \frac{1}{m_N}\left(\frac{2\pi}{L}\right)^2 \simeq 15\mathrm{MeV} \quad \text{for } L = 10\mathrm{fm}$$



➔ Very large t >~ 100 would be required !

Solution: Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

*They are **not contaminations**, **but signals***

➔ **Schrodinger Eq. : time-independent ➔ time-dependent**

$$\left(-\frac{\partial}{\partial t}+\frac{1}{4m}\frac{\partial^2}{\partial t^2}-H_0\right)R(\boldsymbol{r},t)=\int d\boldsymbol{r}'U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t)$$

$$2\sqrt{m^2+k_n^2}=E_n=-\frac{\partial}{\partial_t}$$

Grand State (G.S.) saturation is NOT necessary !

Significant advantage of potential method:

$\Delta E \simeq E_{\rm th}-E\simeq m_\pi \simeq 140{\rm MeV}$ ➔ Moderate t >~ 10 would be fine

Explicit Lat calc for I=2 pipi phase shift

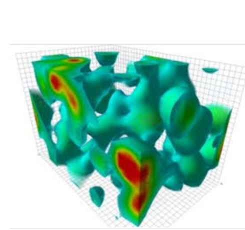
Beautiful agreement between

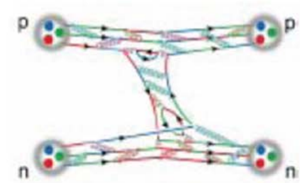
(1) Luscher's formula w/ g.s. saturation

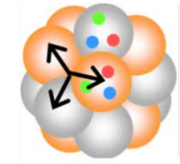
(2) the HAL QCD method w/ & w/o g.s. saturation

T.Kurth et al. (HAL-BMW Coll.) @ Lat2012

Summary and Prospects







- Hadron Interactions by 1st principle Lat calc
  - Bridging different worlds:
    Particle Physics / Nuclear Physics / Astrophysics

- Lattice QCD results for NN, **YN/YY**, NNN
  - Intriguing physics even at heavy quark masses

- **On the K computer:** <u>physical quark mass point !</u>
  - Breakthroughs in S/N issue & Comput. cost issue

Thermodynamic limit & continuum limit

➔ **Realistic hadron interactions**

➔ **Nuclear Physics on the Lattice !**

Backup Slides

Asymptotic form of BS wave function

[C.-J.D.Lin et al., NPB619,467(2001)]

For simplicity, we consider BS wave function of two pions

$$\psi_{\vec{q}}(\vec{x}) \equiv \left\langle 0 \left| N(\vec{x})\ N(\vec{0}) \right| N(\vec{q})N(-\vec{q}), in \right\rangle$$

complete set

$$1 = \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec{p})} \left| N(\vec{p}) \right\rangle \left\langle N(\vec{p}) \right| + \cdots$$

$$= \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec{p})} \left\langle 0 \left| N(\vec{x}) \right| N(\vec{p}) \right\rangle \left\langle N(\vec{p}) \left| N(\vec{0}) \right| N(\vec{q})N(-\vec{q}), in \right\rangle + I(\vec{x})$$

$$Z^{1/2} e^{i\vec{p}\cdot\vec{x}} \qquad disc. + Z^{1/2} \frac{T(\vec{p};\vec{q})}{m_N^2 - \left(2E_N(\vec{q}) - E_N(\vec{p})\right)^2 + \vec{p}^2 - i\varepsilon}$$

$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2E_N(\vec{p})} \frac{T(\vec{p};\vec{q})}{4E_N(\vec{q}) \cdot \left(E_N(\vec{p}) - E_N(\vec{q}) - i\varepsilon\right)} e^{i\vec{p}\cdot\vec{x}} \right)$$

Integral is dominated by the on-shell contribution $E_N(\vec{p}) \approx E_N(\vec{q})$

➔ T-matrix becomes the on-shell T-matrix

$$T^{(s-wave)}(s) = \frac{E(\vec{q})}{2|\vec{q}|}(-i)\left(e^{2i\delta_0(s)} - 1\right)$$

$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{2i}\left(e^{2i\delta_0(s)} - 1\right)\frac{e^{iqr}}{qr} \right) + \cdots$$

The asymptotic form

$$\boxed{\psi_{\vec{q}}(\vec{x}) = Z e^{i\delta_0(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \quad \text{(s-wave)}}$$

This is analogous
to a non-rela. wave function

Scatterings on the lattice

- Luscher's formula

M.Luscher, CMP105(1986)156
NPB354(1991)531

  – Extract the phase shifts from spectrum in finite V

$$E = 2\sqrt{m^2 + k^2}$$

$$k \cot \delta(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; q^2), \quad q = \frac{kL}{2\pi}$$

$$Z_{00}(s; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\boldsymbol{n} \in \boldsymbol{Z}^3} \frac{1}{(\boldsymbol{n}^2 - q^2)^s}$$

$$\text{low energy:} \quad k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2} \mathbf{r} k^2 + \cdots$$

Large V expansion

$$\Delta E = E - 2m = -\frac{4\pi \mathbf{a}}{m L^3} \left[1 + c_1 \frac{a}{L} + c_2 \left(\frac{a}{L} \right)^2 + \mathcal{O}(\frac{1}{L^3}) \right]$$

c_1, c_2: geometric constants

Bound state

Infinite V extrapolation has to be examined

Effective Schrodinger equation with E-independent potential

$$K(\vec{x};E) \equiv \left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E)$$

[START] local but E-dep pot. ($L^3 x L^3$ dof)

(1) We assume ψ(x; E) for different E is linearly independent with each other.

(2) ψ(x; E) has a **"left inverse"** as an integration operator as

$$E \equiv 2\sqrt{m_N^2 + k^2}$$

$$\int d^3x \tilde{\psi}(\vec{x};E')\psi(\vec{x};E) = 2\pi\delta(E-E')$$

(3) K(x; E) can be factorized as

$$\equiv m_N U(\vec{x},\vec{y})$$

$$\begin{aligned} K(\vec{x};E) &= \int \frac{dE'}{2\pi} K(\vec{x};E') \times \int d^3y \tilde{\psi}(\vec{y};E')\psi(\vec{y};E) \\ &= \int d^3y \left\{ \sum_\alpha \int \frac{dE'}{2\pi} K(\vec{x};E')\tilde{\psi}(\vec{y};E') \right\} \psi(\vec{y};E) \end{aligned}$$

(4) We are left with **an effective Schrodinger equation with an $\boxed{\text{E−independent}}$ potential U.**

$$\left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E) = m_N \int d^3y \boxed{U(\vec{x},\vec{y})}\psi(\vec{y};E)$$

Intuitive understanding

[GOAL] non-local but E-indep pot. ($L^3 x L^3$ dof)

Solution: Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

*They are **not contaminations**, **but signals***

$$\left(k_n^2/m - H_0\right)\psi_n(\boldsymbol{r}) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_n(\boldsymbol{r}')$$

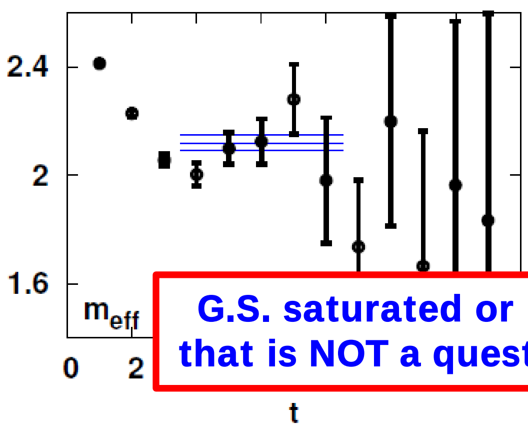
$$\Downarrow$$

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t)$$

$$2\sqrt{m^2 + k_n^2} = E_n = -\frac{\partial}{\partial t}$$

$$R(\boldsymbol{r},t) \equiv C_{NN}(\boldsymbol{r},t)/C_N(t)^2$$

Grand State (G.S.) saturation is NOT necessary !



G.S. saturated or not, that is NOT a question !

Explicit Lat calc for I=2 pipi phase shift

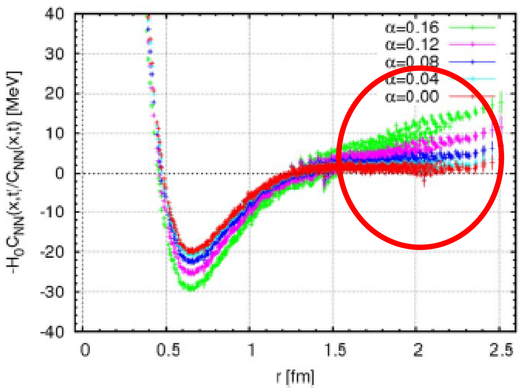
Beautiful agreement between

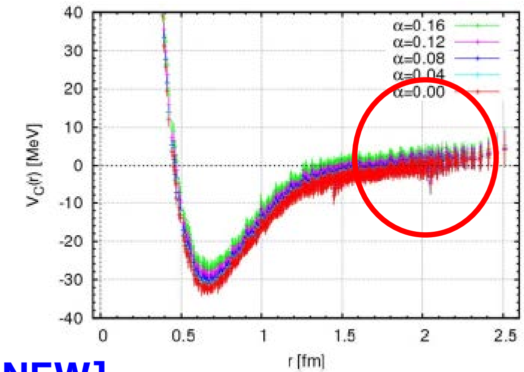
(1) Luscher's formula w/ g.s. saturation
(2) the HAL QCD method w/ & w/o g.s. saturation

Explicit check on the new t-dep HAL method

NN system

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437





[OLD]

Different sources (creation op.) ➔ different results

"contaminations" from excited states

[NEW]

Results converged

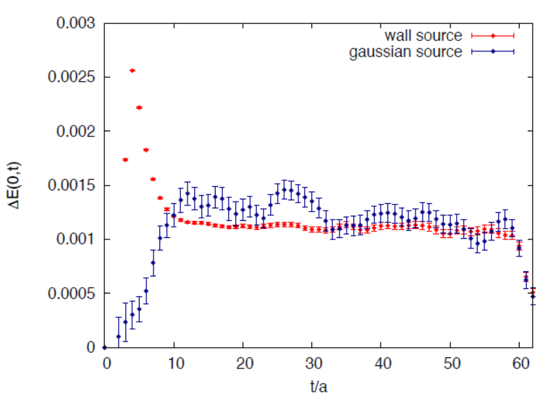
"signals" from excited states

pipi (I=2) system

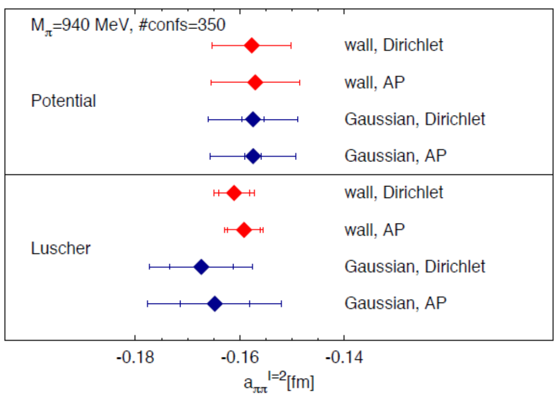
T.Kurth et al. (HAL-BMW Coll.) @ Lat2012

(Preliminary)

Effective mass



Scatt. length



Beautiful agreement between

(1) Luscher's formula

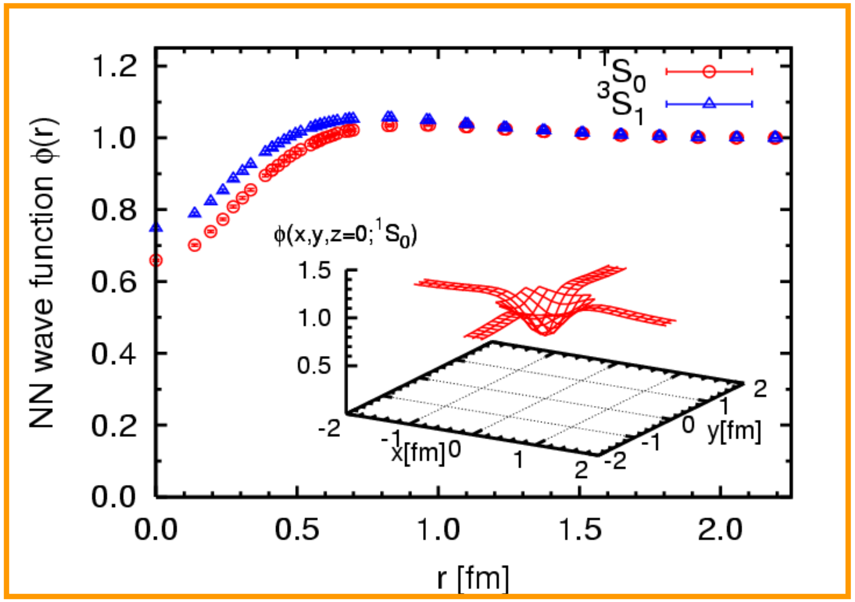
w/ g.s. saturation

(2) the HAL QCD method

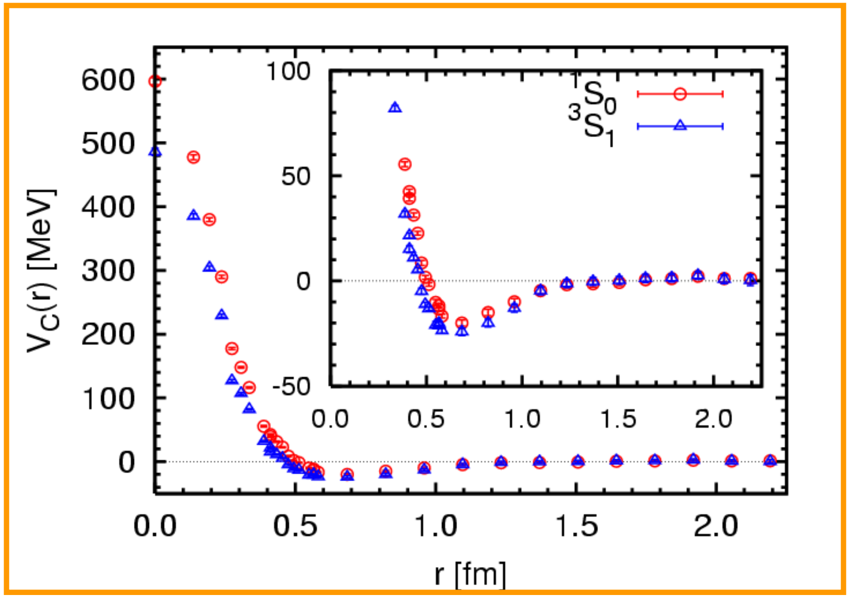
w/ & w/o g.s. saturation

Nuclear Potential (from Lat QCD)

NBS wave function



Nuclear Force

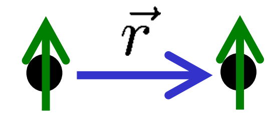


<u>**Quenched**</u> QCD
m_π = 530MeV, L=4.4fm

Ishii-Aoki-Hatsuda,
PRL99(2007)022001

Tensor Potential from Lat QCD

- Coupled channel study in $^3S_1$-$^3D_1$ channel

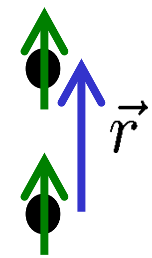


$$(H_0 + V_C + V_T S_{12})\psi = E\psi$$

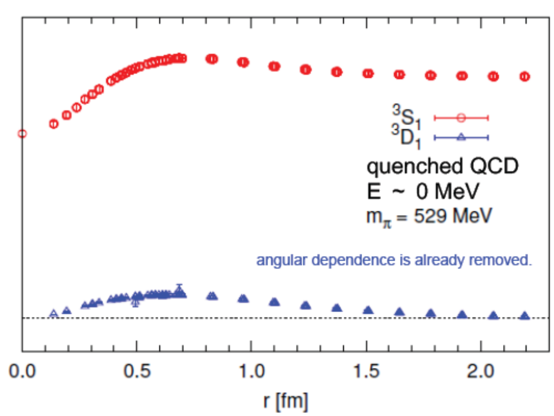
$$\psi = \psi_S + \psi_D$$

P	: projection to L=0
Q=(1-P)	: projection to L=2

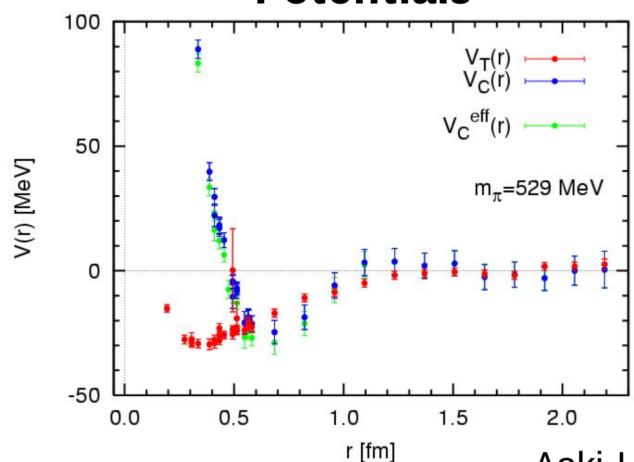
$$P(H_0 + \boxed{V_C} + \boxed{V_T} S_{12})\psi = EP\psi$$
$$Q(H_0 + \boxed{V_C} + \boxed{V_T} S_{12})\psi = EQ\psi$$



Wave function



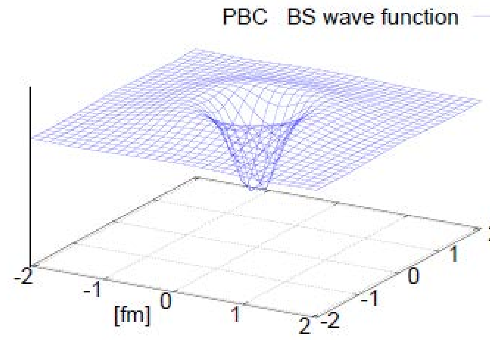
Potentials



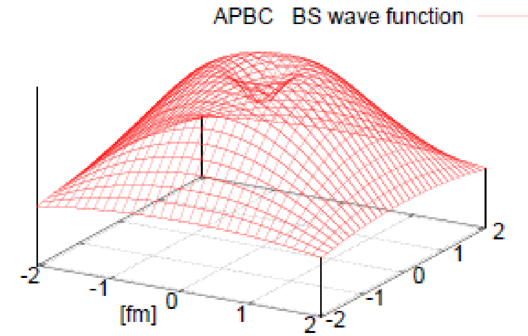
Aoki-Hatsuda-Ishii, PTP 123 (2010) 89

“Energy dependence” of LO $V_c(r)$ in velocity expansion

E ~ 0 MeV

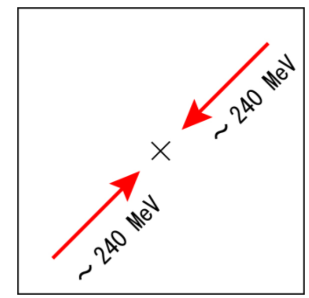


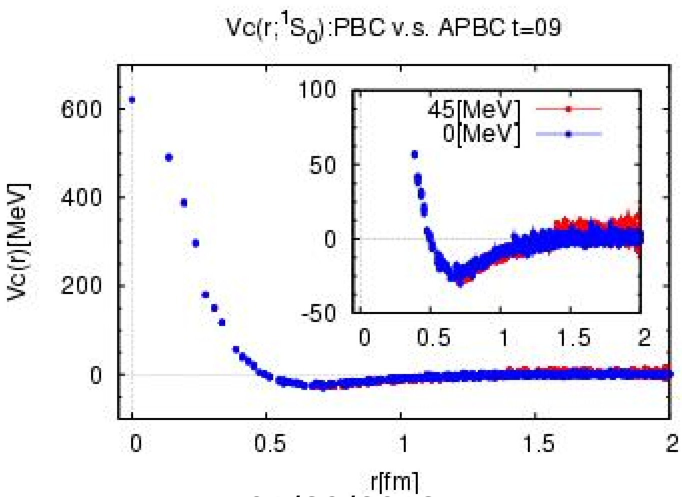
E ~ 45 MeV



Anti-periodic BC to achieve E ≠ 0

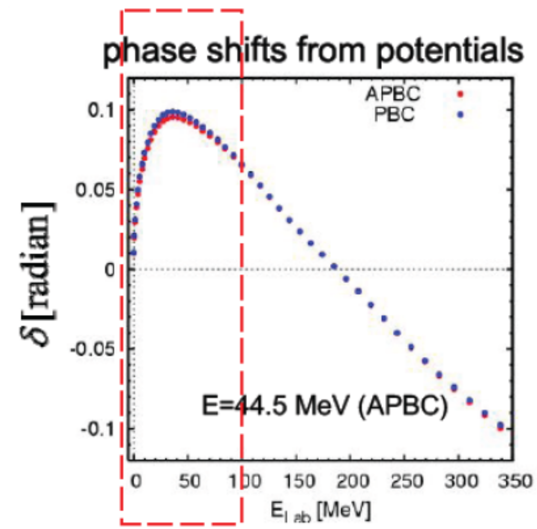
Quenched QCD
m_π =0.53GeV
a=0.137fm





In our choice of wave function,
E-dependence of the local potential
turns out to be very small at low E.

➔ *Velocity expansion is good !*

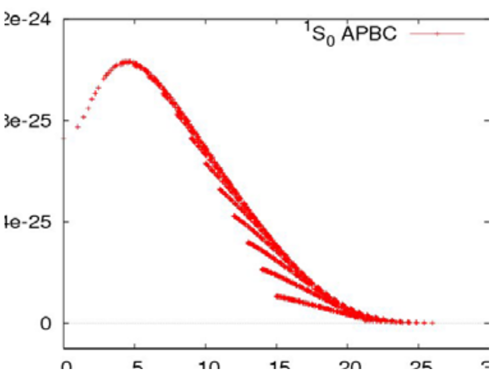


O.K. !

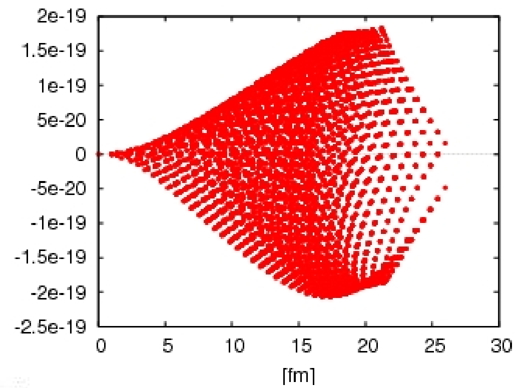
K.Murano et al., (HAL QCD Collab.)
PoS Lattice 2009 (2009) 126

L^2 dependence of $V_c(r)$ in S=0

A_1^+ NBS wave ($^1S_0$)



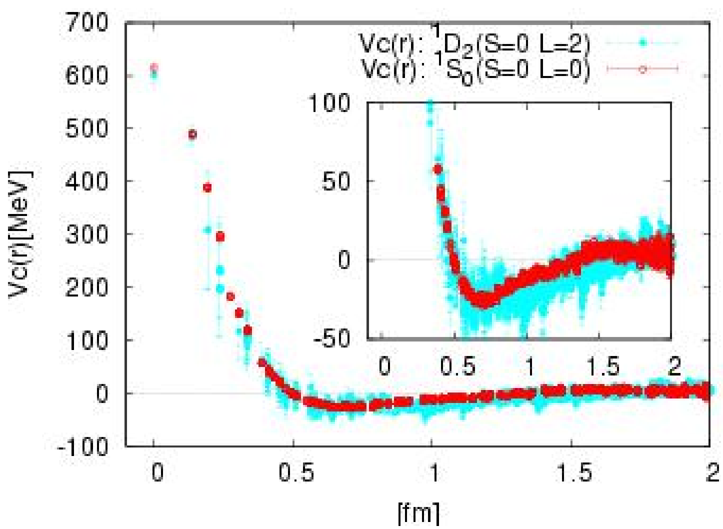
T_2^+ NBS wave ($^1D_2$)



Anti-periodic BC

$$\sim \sum Y_{2m}(\hat{r})$$

Quenched QCD
$m\pi$ =0.53GeV
a=0.137fm
E ~= 45MeV



In our choice of wave function,
L^2-dependence of the local potential
turns out to be small at low E.

(consistent within stat. error)

➔ *Velocity expansion is good !*

K.Murano (HAL QCD Collab.) @ Lattice 2010

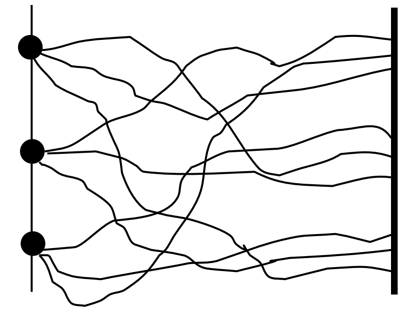
Lattice calculation setup

- Nf=2 clover fermion + RG improved gauge action (CP-PACS)
  - 598 configs x 32 measurements
  - beta=1.95, (a^{-1}=1.27GeV, a=0.156fm)
  - 16^3 x 32 lattice, L=2.5fm
    - M(π) = 1.13GeV
    - M(N) = 2.15GeV
    - M(Δ) = 2.31GeV

(κ(ud) =0.13750)

($M\pi$ L=14)

CP-PACS Coll. S. Aoki et al.,
Phys. Rev. D65 (2002) 054505



t=t(sink) t=t(src)

- Correlators
  - **Standard nucleon op** to define the wave function / potential **at sink**

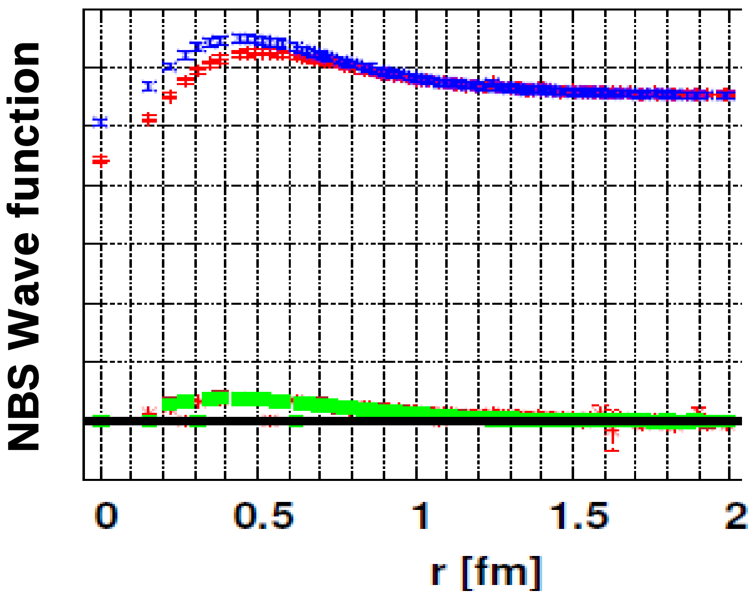
$$N = \epsilon_{abc}(q_a^T C\gamma_5 q_b) q_c$$

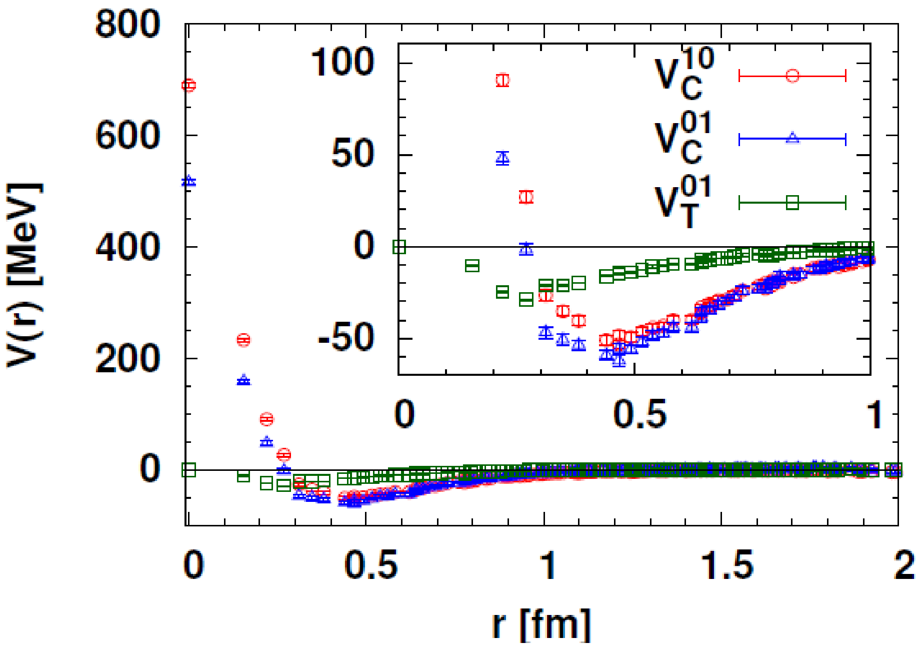
  - **Non-rela limit op** is used to create 3N state **at source**

$$G(\vec{r}_2, t-t_0) = \sum_{\vec{x}} \langle 0| \underbrace{N(\vec{x}+\vec{r}_2,t) N(\vec{x}-\vec{r}_2,t) N(\vec{x},t)}_{\text{sink}} \underbrace{\overline{NNN}(t_0)}_{\text{source}} |0\rangle$$

See also T.Yamazaki et al.,
PRD81(2010)111504

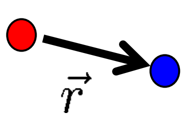
2NF (parity-even) from Lat QCD





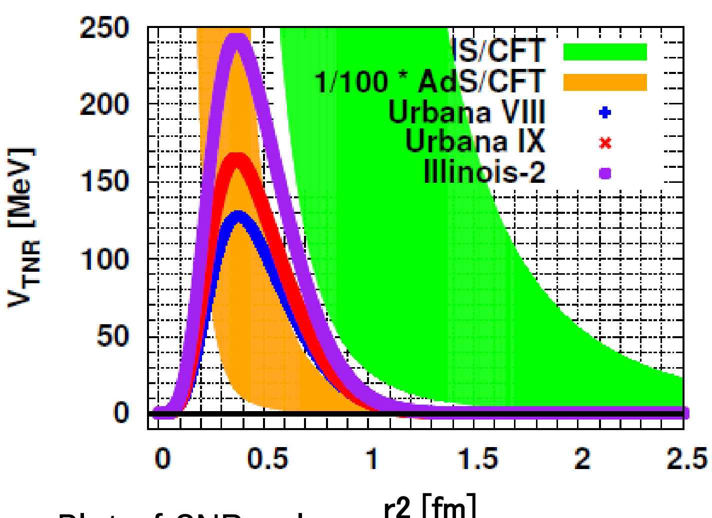
Red: $^1S_0$
Blue: $^3S_1$
Green: $^3D_1$

Nf=2, CP-PACS confs, M(π)=1.13GeV



Short-Range 3NF

- We determine 3NF effectively represented by a scalar/isoscalar functional form
  - c.f. phenomenological 3NF to reproduce saturation point of nuclear matter, etc.



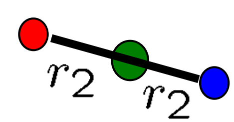
Plot of 3NR only:
there is cancellation from 3NA

$$V_{3NF} = V_{2\pi E} + (V_{3\pi R}) + V_{3NR}$$

Urbana/Illinois

$$V_{3NR} = U_0 \sum_{cyc} T^2(r_{12}) T^2(r_{13})$$

$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} T_{cut}(r)$$

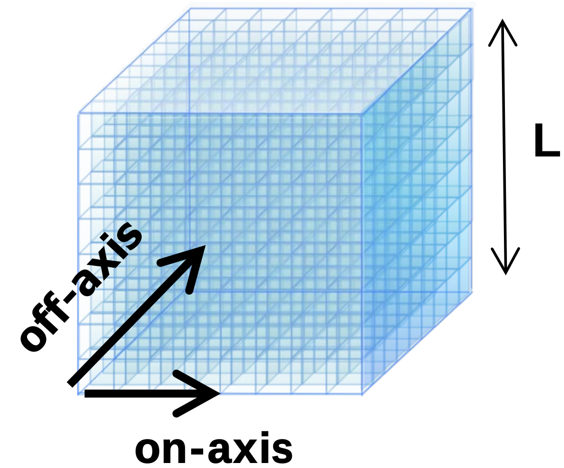


AdS/CFT: $V_{3NF} = +\text{const.} \cdot \frac{1}{r^4}$

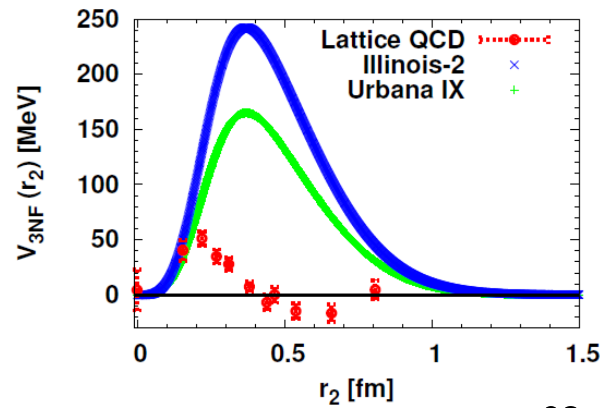
K.Hashimoto, N.Iizuka
JHEP 1011 (2010) 058

Other Systematics ?

- Finite V artifact ?
  - L=2.5fm ➔ (mπ x L) = ~14
    - $<r^2_B> <\sim (0.4\text{fm})^2$
  - Larger r_2 points $(0.5 \lesssim r_2 \lesssim 0.8 \text{ fm})$
    - ➔ carefully taken in off-axis direction

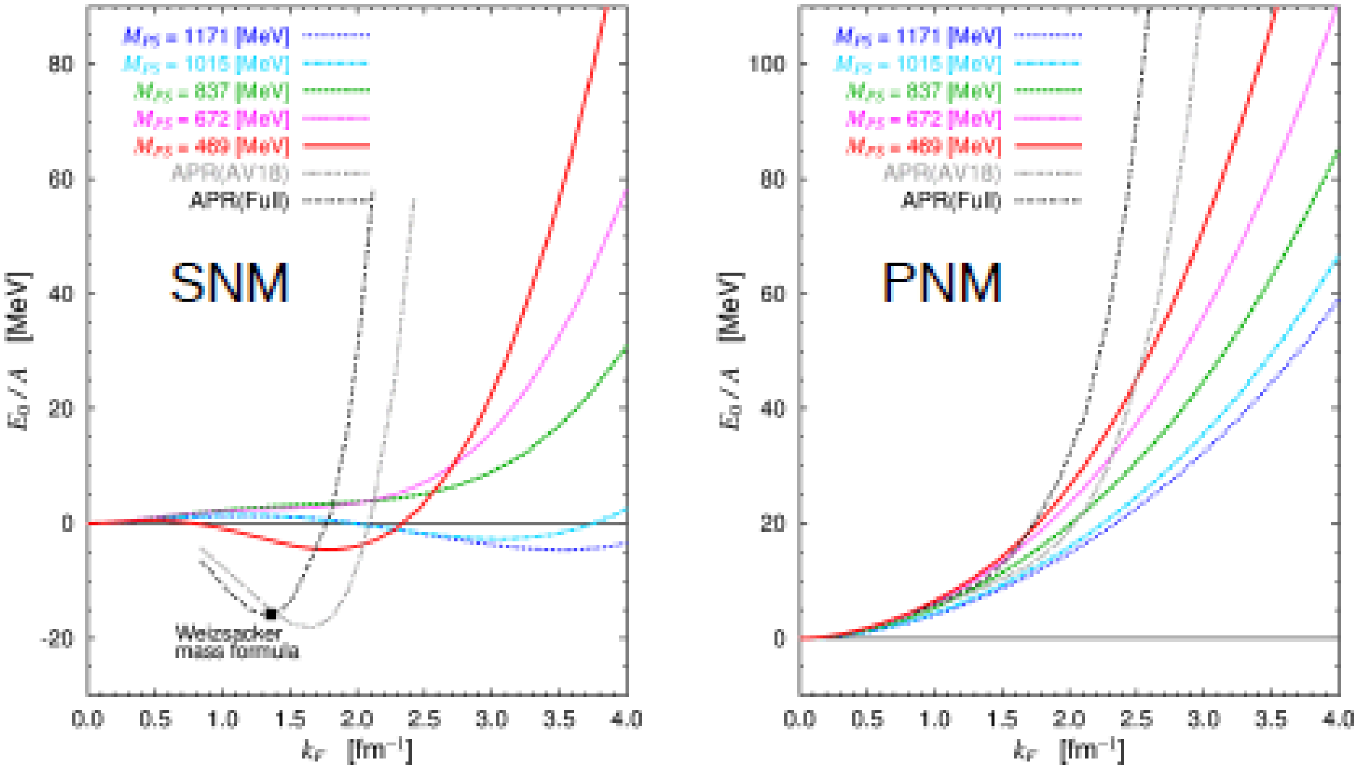


- Quark mass dependence ?
  - -- certainly.
  - calc w/ lighter mass in progress



N.B. It is not necessary that
phen 3NF and Lat 3NF exactly match.

EoS of Nuclear Matter from Lattice Forces

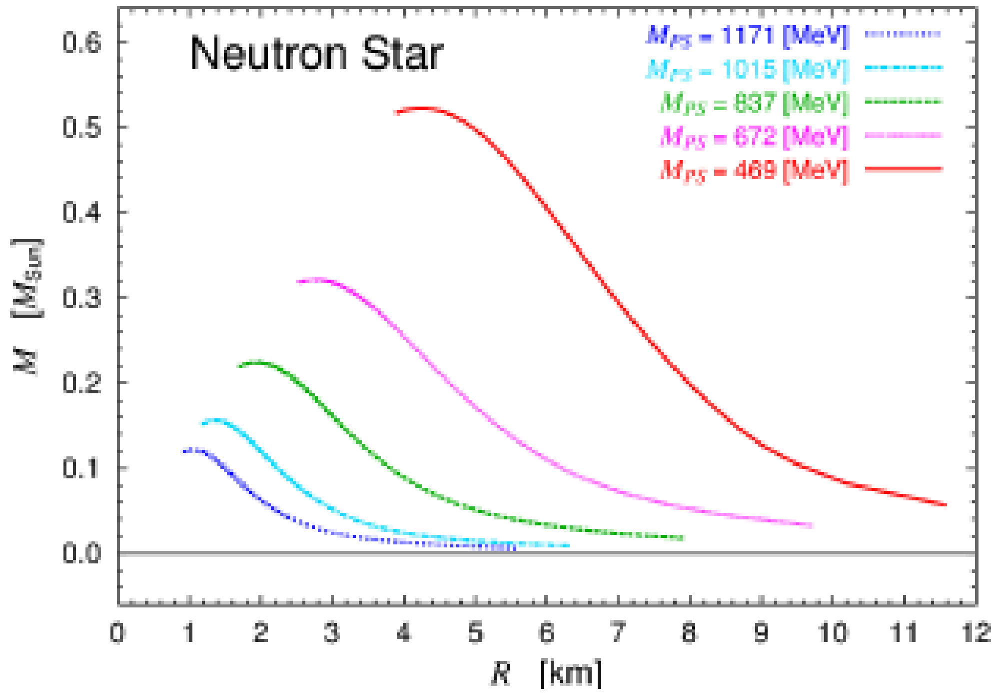


- Deviation from empirical ones due to heavy u,d quark.
- LQCD curve approaches to empirical one as mq decrease.

optimistically?

T.Inoue @ QUCS2012

Neutron Star M-R relation



M_N = 2274 MeV
M_N = 2031 MeV
M_N = 1749 MeV
M_N = 1484 MeV
M_N = 1161 MeV

Crust part is ignored.
Uniform matter only.

- Mass-radius curve of neutron stars at five value of m_q.
  - M^{max} = 0.12 – 0.52 [M_{Sun}] for M_{ps} = 1171 – 469 [MeV].
  - due to heavy nucleon and weaker repulsive core.
  - M^{max} will be much bigger with lighter u,d quark.

T.Inoue @ QUCS2012