Memo for deformed HO wave functions

S. Shimoura
2006.11.10

1 Spherical HO wave functions

1.1 Polar coordinate respresentation (r, θ, ϕ)

$$\psi_0\left(1\mathrm{s}\right) = 2\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{00}\left(\theta,\phi\right)$$

$$\psi_m\left(1\mathrm{p}\right) = \sqrt{\frac{8}{3}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{1m}\left(\theta,\phi\right)$$

$$\psi_0\left(2\mathrm{s}\right) = \sqrt{\frac{8}{3}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha^2r^2-\frac{3}{2}\right)\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{00}\left(\theta,\phi\right)$$

$$\psi_m\left(1\mathrm{d}\right) = \sqrt{\frac{16}{15}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)^2\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{2m}\left(\theta,\phi\right)$$

$$\psi_m\left(2\mathrm{p}\right) = \frac{4}{\sqrt{15}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)\left(\alpha^2r^2-\frac{5}{2}\right)\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{1m}\left(\theta,\phi\right)$$

$$\psi_m\left(1\mathrm{f}\right) = \frac{8}{\sqrt{210}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)^3\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{3m}\left(\theta,\phi\right)$$

$$\psi_0\left(3\mathrm{s}\right) = \sqrt{\frac{8}{15}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha^4r^4-5\alpha^2r^2+\frac{15}{4}\right)\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{00}\left(\theta,\phi\right)$$

$$\psi_m\left(2\mathrm{d}\right) = \frac{8}{\sqrt{210}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)^2\left(\alpha^2r^2-\frac{7}{2}\right)\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{2m}\left(\theta,\phi\right)$$

$$\psi_m\left(1\mathrm{g}\right) = \frac{8}{\sqrt{945}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)^4\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{4m}\left(\theta,\phi\right)$$

$$\psi_m\left(3\mathrm{p}\right) = \frac{4}{\sqrt{105}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)\left(\alpha^4r^4-7\alpha^2r^2+\frac{15}{4}\right)\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{1m}\left(\theta,\phi\right)$$

$$\psi_m\left(2\mathrm{f}\right) = \frac{8}{\sqrt{945}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)^3\left(\alpha^2r^2-\frac{9}{2}\right)\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{3m}\left(\theta,\phi\right)$$

$$\psi_m\left(1\mathrm{h}\right) = \frac{16}{3\sqrt{2310}}\left(\frac{\alpha^3}{\sqrt{\pi}}\right)^{1/2}\left(\alpha r\right)^5\exp\left(-\frac{1}{2}\alpha^2r^2\right)Y_{5m}\left(\theta,\phi\right)$$

1.2 Cylindrical coordinate respresentation (ρ, z, ϕ; $r^2 = \rho^2 + z^2$)

$$
\begin{aligned}
\psi_0\,(1\mathrm{s}) &= \left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} \exp\left(-\frac{1}{2}\alpha^2 r^2\right) \\
\psi_{\pm 1}\,(1\mathrm{p}) &= \mp\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} \alpha\rho\ \mathrm{e}^{\pm i\phi}\ \exp\left(-\frac{1}{2}\alpha^2 r^2\right) \\
\psi_0\,(1\mathrm{p}) &= \sqrt{2}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} \alpha z\ \exp\left(-\frac{1}{2}\alpha^2 r^2\right) \\
\psi_0\,(2\mathrm{s}) &= \sqrt{\frac{2}{3}}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} \left(\alpha^2 r^2 - \frac{3}{2}\right) \exp\left(-\frac{1}{2}\alpha^2 r^2\right) \\
\psi_{\pm 2}\,(1\mathrm{d}) &= \sqrt{\frac{1}{2}}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} \alpha^2\rho^2\ \mathrm{e}^{\pm 2i\phi}\ \exp\left(-\frac{1}{2}\alpha^2 r^2\right) \\
\psi_{\pm 1}\,(1\mathrm{d}) &= \mp\sqrt{2}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} \alpha^2\rho z\ \mathrm{e}^{\pm i\phi}\ \exp\left(-\frac{1}{2}\alpha^2 r^2\right) \\
\psi_0\,(1\mathrm{d}) &= \sqrt{\frac{1}{3}}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2} \alpha^2\left(2z^2 - \rho^2\right) \exp\left(-\frac{1}{2}\alpha^2 r^2\right)
\end{aligned}
$$

1.3 Two-dimensional HO wave functions (ρ, ϕ)

$$
\begin{aligned}
n_\perp = 0\ (\ell_z = 0) &: \left(\frac{\alpha}{\sqrt{\pi}}\right) \exp\left(-\frac{1}{2}\alpha^2\rho^2\right) \\
n_\perp = 1\ (\ell_z = \pm 1) &: \mp\left(\frac{\alpha}{\sqrt{\pi}}\right) \alpha\rho\ \mathrm{e}^{\pm i\phi} \exp\left(-\frac{1}{2}\alpha^2\rho^2\right) \\
n_\perp = 2\ (\ell_z = \pm 2) &: \sqrt{\frac{1}{2}}\left(\frac{\alpha}{\sqrt{\pi}}\right) \alpha^2\rho^2\ \mathrm{e}^{\pm 2i\phi} \exp\left(-\frac{1}{2}\alpha^2\rho^2\right) \\
n_\perp = 2\ (\ell_z = 0) &: \left(\frac{\alpha}{\sqrt{\pi}}\right)\left(\alpha^2\rho^2 - 1\right) \exp\left(-\frac{1}{2}\alpha^2\rho^2\right)
\end{aligned}
$$

1.4 *LS* coupling

$$
\begin{aligned}
\left|\frac{3}{2}\ \frac{3}{2}\right\rangle_{\mathrm{p}} &= \psi_{+1}\,(\mathrm{p})\,|\!\uparrow\rangle \\
\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{\mathrm{p}} &= \sqrt{\frac{2}{3}}\psi_0\,(\mathrm{p})\,|\!\uparrow\rangle + \sqrt{\frac{1}{3}}\psi_{+1}\,(\mathrm{p})\,|\!\downarrow\rangle \\
\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{\mathrm{p}} &= -\sqrt{\frac{1}{3}}\psi_0\,(\mathrm{p})\,|\!\uparrow\rangle + \sqrt{\frac{2}{3}}\psi_{+1}\,(\mathrm{p})\,|\!\downarrow\rangle \\
\psi_0\,(\mathrm{p})\,|\!\uparrow\rangle &= \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{\mathrm{p}} - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{\mathrm{p}} \\
\psi_{+1}\,(\mathrm{p})\,|\!\downarrow\rangle_{\mathrm{p}} &= \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{\mathrm{p}} + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{\mathrm{p}}
\end{aligned}
$$

$$\left|\frac{5}{2}\ \frac{5}{2}\right\rangle_{\mathrm{d}} = \psi_2\,(\mathrm{d})\,|\uparrow\rangle$$

$$\left|\frac{5}{2}\ \frac{3}{2}\right\rangle_{\mathrm{d}} = \sqrt{\frac{4}{5}}\psi_{+1}\,(\mathrm{d})\,|\uparrow\rangle + \sqrt{\frac{1}{5}}\psi_{+2}\,(\mathrm{d})\,|\downarrow\rangle$$

$$\left|\frac{3}{2}\ \frac{3}{2}\right\rangle_{\mathrm{d}} = -\sqrt{\frac{1}{5}}\psi_{+1}\,(\mathrm{d})\,|\uparrow\rangle + \sqrt{\frac{4}{5}}\psi_{+2}\,(\mathrm{d})\,|\downarrow\rangle$$

$$\left|\frac{5}{2}\ \frac{1}{2}\right\rangle_{\mathrm{d}} = \sqrt{\frac{3}{5}}\psi_{0}\,(\mathrm{d})\,|\uparrow\rangle + \sqrt{\frac{2}{5}}\psi_{+1}\,(\mathrm{d})\,|\downarrow\rangle$$

$$\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{\mathrm{d}} = -\sqrt{\frac{2}{5}}\psi_{0}\,(\mathrm{d})\,|\uparrow\rangle + \sqrt{\frac{3}{5}}\psi_{+1}\,(\mathrm{d})\,|\downarrow\rangle$$

$$\psi_{+1}\,(\mathrm{d})\,|\uparrow\rangle = \sqrt{\frac{4}{5}}\left|\frac{5}{2}\ \frac{3}{2}\right\rangle_{\mathrm{d}} - \sqrt{\frac{1}{5}}\left|\frac{3}{2}\ \frac{3}{2}\right\rangle_{\mathrm{d}}$$

$$\psi_{+2}\,(\mathrm{d})\,|\downarrow\rangle = \sqrt{\frac{1}{5}}\left|\frac{5}{2}\ \frac{3}{2}\right\rangle_{\mathrm{d}} + \sqrt{\frac{4}{5}}\left|\frac{3}{2}\ \frac{3}{2}\right\rangle_{\mathrm{d}}$$

$$\psi_{0}\,(\mathrm{d})\,|\uparrow\rangle = \sqrt{\frac{3}{5}}\left|\frac{5}{2}\ \frac{1}{2}\right\rangle_{\mathrm{d}} - \sqrt{\frac{2}{5}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{\mathrm{d}}$$

$$\psi_{+1}\,(\mathrm{d})\,|\downarrow\rangle = \sqrt{\frac{2}{5}}\left|\frac{5}{2}\ \frac{1}{2}\right\rangle_{\mathrm{d}} + \sqrt{\frac{3}{5}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{\mathrm{d}}$$

1.5 Representation with asymptotic quantum numbers $|Nn_3\Lambda\Omega\rangle$

$$|110\ 1/2\rangle = \sqrt{2}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\alpha z\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|\uparrow\rangle = \psi_0\,(1\mathrm{p})\,|\uparrow\rangle$$

$$= \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{p}} - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{p}}$$

$$|101\ 1/2\rangle = -\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\alpha\rho\ \mathrm{e}^{i\phi}\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|\downarrow\rangle = \psi_1\,(1\mathrm{p})\,|\downarrow\rangle$$

$$= \sqrt{\frac{1}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{p}} + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{p}}$$

$$|101\ 3/2\rangle = -\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\alpha\rho\ \mathrm{e}^{i\phi}\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|\uparrow\rangle = \psi_1\,(1\mathrm{p})\,|\uparrow\rangle = \left|\frac{3}{2}\ \frac{3}{2}\right\rangle_{1\mathrm{p}}$$

$$|220\ 1/2\rangle = \sqrt{\frac{1}{2}}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\left(2\alpha^2 z^2 - 1\right)\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|\uparrow\rangle = \left(\sqrt{\frac{1}{3}}\psi_0\,(2\mathrm{s}) + \sqrt{\frac{2}{3}}\psi_0\,(1\mathrm{d})\right)|\uparrow\rangle$$

$$= \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{2\mathrm{s}} - \sqrt{\frac{4}{15}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{d}} + \sqrt{\frac{2}{5}}\left|\frac{5}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{d}}$$

$$|200\ 1/2\rangle = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\left(\alpha^2\rho^2 - 1\right)\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|\uparrow\rangle = \left(\sqrt{\frac{2}{3}}\psi_0\,(2\mathrm{s}) - \sqrt{\frac{1}{3}}\psi_0\,(1\mathrm{d})\right)|\uparrow\rangle$$

$$= \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle_{2\mathrm{s}} + \sqrt{\frac{2}{15}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{d}} - \sqrt{\frac{1}{5}}\left|\frac{5}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{d}}$$

$$
\begin{aligned}
|211\ 1/2\rangle &= -\sqrt{2}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\alpha^2 z\rho\ \mathrm{e}^{i\phi}\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|{\downarrow}\rangle = \psi_1\,(1\mathrm{d})\,|{\downarrow}\rangle \\
&= \sqrt{\frac{3}{5}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{d}} + \sqrt{\frac{2}{5}}\left|\frac{5}{2}\ \frac{1}{2}\right\rangle_{1\mathrm{d}} \\
|211\ 3/2\rangle &= -\sqrt{2}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\alpha^2 z\rho\ \mathrm{e}^{i\phi}\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|{\uparrow}\rangle = \psi_1\,(1\mathrm{d})\,|{\uparrow}\rangle \\
&= -\sqrt{\frac{1}{5}}\left|\frac{3}{2}\ \frac{3}{2}\right\rangle_{1\mathrm{d}} + \sqrt{\frac{4}{5}}\left|\frac{5}{2}\ \frac{3}{2}\right\rangle_{1\mathrm{d}} \\
|202\ 3/2\rangle &= \frac{1}{\sqrt{2}}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\alpha^2\rho^2\ \mathrm{e}^{2i\phi}\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|{\downarrow}\rangle = \psi_2\,(1\mathrm{d})\,|{\downarrow}\rangle \\
&= \sqrt{\frac{4}{5}}\left|\frac{3}{2}\ \frac{3}{2}\right\rangle_{1\mathrm{d}} + \sqrt{\frac{1}{5}}\left|\frac{5}{2}\ \frac{3}{2}\right\rangle_{1\mathrm{d}} \\
|202\ 5/2\rangle &= \frac{1}{\sqrt{2}}\left(\frac{\alpha}{\sqrt{\pi}}\right)^{3/2}\alpha^2\rho^2\ \mathrm{e}^{2i\phi}\exp\left(-\frac{1}{2}\alpha^2 r^2\right)|{\uparrow}\rangle = \psi_2\,(1\mathrm{d})\,|{\uparrow}\rangle = \left|\frac{5}{2}\ \frac{5}{2}\right\rangle_{1\mathrm{d}}
\end{aligned}
$$

2 Deformed HO wave functions

$$\begin{aligned}
\omega_1 &= \omega_2 = \omega_\perp \ ; \ \frac{\omega_\perp}{\omega_3} = \frac{1+\frac{1}{3}\delta}{1-\frac{2}{3}\delta} = \frac{\alpha_\perp^2}{\alpha_3^2} \ ; \ \alpha_3^3 = \frac{1-\frac{2}{3}\delta}{1+\frac{1}{3}\delta}\alpha_0^3 \\
\alpha_3^2 &= \left(\frac{1-\frac{2}{3}\delta}{1+\frac{1}{3}\delta}\right)^{2/3}\alpha_0^2 \ ; \ \alpha_\perp^2 = \left(\frac{1+\frac{1}{3}\delta}{1-\frac{2}{3}\delta}\right)^{1/3}\alpha_0^2 \\
\omega_3 &= \left(\frac{1-\frac{2}{3}\delta}{1+\frac{1}{3}\delta}\right)^{2/3}\omega_0 \ ; \ \omega_\perp = \left(\frac{1+\frac{1}{3}\delta}{1-\frac{2}{3}\delta}\right)^{1/3}\omega_0 \\
E\left([Nn_3\Lambda]\right) &= \left(n_3+\frac{1}{2}\right)\hbar\omega_3 + \left(N-n_3+1\right)\hbar\omega_\perp \\
&= \hbar\omega_0 \frac{N+\frac{3}{2}+\left(\frac{1}{3}N-n_3\right)\delta}{\left(1-\frac{2}{3}\delta\right)^{1/3}\left(1+\frac{1}{3}\delta\right)^{2/3}} \\
&= \hbar\omega_0\left\{\left(N+\frac{3}{2}\right)+\left(\frac{N}{3}-n_3\right)\delta+\left(\frac{N}{9}+\frac{1}{6}\right)\delta^2+\left(\frac{5N}{81}-\frac{n_3}{9}+\frac{1}{27}\right)\delta^3+\cdots\right\} \\
\alpha_3^2 z^2+\alpha_\perp^2\rho^2 &= (\alpha_0 r)^2\left(\frac{1+\frac{1}{3}\delta}{1-\frac{2}{3}\delta}\right)^{1/3}\left(1-\frac{\delta}{1+\frac{1}{3}\delta}\cos^2\theta\right) \\
r^2 Y_{20} &= \sqrt{\frac{5}{16\pi}}\left(2z^2-\rho^2\right)
\end{aligned}$$

2.1 Representation with asymptotic quantum numbers $|Nn_3\Lambda\rangle$

Wave functions

$$\begin{aligned}
|000\rangle &= \left(\frac{\alpha_0}{\sqrt{\pi}}\right)^{3/2}\exp\left\{-\frac{1}{2}\left(\alpha_3^2z^2+\alpha_\perp^2\rho^2\right)\right\} \\
|110\rangle &= \sqrt{2}\left(\frac{\alpha_0}{\sqrt{\pi}}\right)^{3/2}\alpha_3 z\exp\left\{-\frac{1}{2}\left(\alpha_3^2z^2+\alpha_\perp^2\rho^2\right)\right\} \\
|101\rangle &= -\left(\frac{\alpha_0}{\sqrt{\pi}}\right)^{3/2}\alpha_\perp\rho\ \mathrm{e}^{i\phi}\exp\left\{-\frac{1}{2}\left(\alpha_3^2z^2+\alpha_\perp^2\rho^2\right)\right\} \\
|220\rangle &= \sqrt{\frac{1}{2}}\left(\frac{\alpha_0}{\sqrt{\pi}}\right)^{3/2}\left(2\alpha_3^2z^2-1\right)\exp\left\{-\frac{1}{2}\left(\alpha_3^2z^2+\alpha_\perp^2\rho^2\right)\right\} \\
|200\rangle &= \left(\frac{\alpha_0}{\sqrt{\pi}}\right)^{3/2}\left(\alpha_\perp^2\rho^2-1\right)\exp\left\{-\frac{1}{2}\left(\alpha_3^2z^2+\alpha_\perp^2\rho^2\right)\right\} \\
|211\rangle &= -\sqrt{2}\left(\frac{\alpha_0}{\sqrt{\pi}}\right)^{3/2}\alpha_3\alpha_\perp z\rho\ \mathrm{e}^{i\phi}\exp\left\{-\frac{1}{2}\left(\alpha_3^2z^2+\alpha_\perp^2\rho^2\right)\right\} \\
|202\rangle &= \frac{1}{\sqrt{2}}\left(\frac{\alpha_0}{\sqrt{\pi}}\right)^{3/2}\alpha_\perp^2\rho^2\ \mathrm{e}^{2i\phi}\exp\left\{-\frac{1}{2}\left(\alpha_3^2z^2+\alpha_\perp^2\rho^2\right)\right\}
\end{aligned}$$

$$\langle Nn_3\Lambda|\, z^2\, |Nn_3\Lambda\rangle = \frac{n_3+\frac{1}{2}}{\alpha_3^2} = \frac{\left(n_3+\frac{1}{2}\right)\left(1+\frac{1}{3}\delta\right)}{\alpha_0^2\left(1-\frac{2}{3}\delta\right)^{2/3}\left(1+\frac{1}{3}\delta\right)^{1/3}}$$

$$\langle Nn_3\Lambda|\, \rho^2\, |Nn_3\Lambda\rangle = \frac{N-n_3+1}{\alpha_\perp^2} = \frac{\left(N-n_3+1\right)\left(1-\frac{2}{3}\delta\right)}{\alpha_0^2\left(1-\frac{2}{3}\delta\right)^{2/3}\left(1+\frac{1}{3}\delta\right)^{1/3}}$$

$$\langle Nn_3\Lambda|\, r^2\, |Nn_3\Lambda\rangle = \frac{\left(N+\frac{3}{2}\right)+\left(n_3-\frac{2}{3}N-\frac{1}{2}\right)\delta}{\alpha_0^2\left(1-\frac{2}{3}\delta\right)^{2/3}\left(1+\frac{1}{3}\delta\right)^{1/3}}$$

$$\langle Nn_3\Lambda|\, 2z^2-\rho^2\, |Nn_3\Lambda\rangle = \frac{\left(3n_3-N\right)+\frac{2}{3}\left(N+\frac{3}{2}\right)\delta}{\alpha_0^2\left(1-\frac{2}{3}\delta\right)^{2/3}\left(1+\frac{1}{3}\delta\right)^{1/3}} = Q_0$$

2.2 Spectroscopic factors as a function of deformation

$$\begin{aligned}\Psi_\ell^{[Nn_30]}(r) &= 2\pi\int_{-1}^{1} d\cos\theta\ \Psi^{[Nn_30]}(\vec{r})\, Y_{\ell 0}^*(\theta) \\ &= \sqrt{(2\ell+1)\pi}\int_{-1}^{1} d\cos\theta\ \Psi^{[Nn_30]}(\vec{r})\, P_\ell(\cos\theta)\end{aligned}$$

$$\left\langle \psi_0\,(1\mathrm{s})\,|\Psi_{\ell=0}^{[000]}\right\rangle = 1-\frac{1}{24}\delta^2-\frac{1}{72}\delta^3-\frac{31}{3456}\delta^4-\frac{23}{5184}\delta^5-\frac{5797}{2239488}\delta^6-\cdots$$

$$\left\langle \psi_0\,(2\mathrm{s})\,|\Psi_{\ell=0}^{[000]}\right\rangle = -\frac{1}{108\sqrt{6}}\delta^3+\cdots$$

$$\left\langle \psi_0\,(3\mathrm{s})\,|\Psi_{\ell=0}^{[000]}\right\rangle = \frac{1}{6\sqrt{30}}\left(\delta^2+\frac{1}{3}\delta^3+\frac{11}{72}\delta^4+\cdots\right)$$

$$\left\langle \psi_0\,(1\mathrm{d})\,|\Psi_{\ell=2}^{[000]}\right\rangle = \frac{1}{2\sqrt{3}}\left(\delta+\frac{1}{6}\delta^2+\frac{1}{24}\delta^3+\frac{5}{432}\delta^4+\frac{61}{31104}\delta^5-\cdots\right)$$

$$\left\langle \psi_0\,(1\mathrm{g})\,|\Psi_{\ell=4}^{[000]}\right\rangle = \frac{3}{4\sqrt{105}}\left(\delta^2+\frac{1}{3}\delta^3+\frac{11}{72}\delta^4+\cdots\right)$$

$$\left\langle \psi_0\,(1\mathrm{s})\,|\Psi_{\ell=0}^{[220]}\right\rangle = -\frac{1}{3\sqrt{2}}\left(\delta+\frac{1}{6}\delta^2+\frac{7}{216}\delta^3+\frac{1}{144}\delta^4-\frac{31}{31104}\delta^5-\cdots\right)$$

$$\left\langle \psi_0\,(2\mathrm{s})\,|\Psi_{\ell=0}^{[220]}\right\rangle = \frac{1}{\sqrt{3}}\left(1-\frac{11}{72}\delta^2-\frac{11}{216}\delta^3-\frac{695}{31104}\delta^4-\frac{431}{46656}\delta^5-\frac{27799}{6718464}\delta^6-\cdots\right)$$

$$\left\langle \psi_0\,(3\mathrm{s})\,|\Psi_{\ell=0}^{[220]}\right\rangle = \frac{2}{3\sqrt{15}}\left(\delta+\frac{1}{6}\delta^2-\frac{29}{216}\delta^3-\frac{11}{144}\delta^4-\frac{1399}{31104}\delta^5-\cdots\right)$$

$$\left\langle \psi_0\,(1\mathrm{d})\,|\Psi_{\ell=2}^{[220]}\right\rangle = \sqrt{\frac{2}{3}}\left(1-\frac{17}{72}\delta^2-\frac{17}{216}\delta^3-\frac{1115}{31104}\delta^4-\frac{707}{46656}\delta^5-\frac{46729}{6718464}\delta^6-\cdots\right)$$

$$\left\langle \psi_0\,(1\mathrm{g})\,|\Psi_{\ell=4}^{[220]}\right\rangle = \sqrt{\frac{6}{35}}\left(\delta+\frac{1}{6}\delta^2-\frac{1}{9}\delta^3-\frac{7}{108}\delta^4-\frac{1249}{31104}\delta^5+\cdots\right)$$

$$\left\langle \psi_0\left(1\mathrm{s}\right)|\Psi^{[200]}_{\ell=0}\right\rangle = \frac{1}{6}\left(\delta+\frac{1}{6}\delta^2+\frac{13}{216}\delta^3+\frac{1}{48}\delta^4+\frac{31}{31104}\delta^5+\cdots\right)$$
$$\left\langle \psi_0\left(2\mathrm{s}\right)|\Psi^{[200]}_{\ell=0}\right\rangle = \sqrt{\frac{2}{3}}\left(1-\frac{5}{72}\delta^2-\frac{5}{216}\delta^3-\frac{467}{31104}\delta^4-\frac{347}{46656}\delta^5-\frac{28933}{6718464}\delta^6-\cdots\right)$$
$$\left\langle \psi_0\left(3\mathrm{s}\right)|\Psi^{[200]}_{\ell=0}\right\rangle = -\frac{\sqrt{2}}{3\sqrt{15}}\left(\delta+\frac{1}{6}\delta^2+\frac{1}{54}\delta^3-\frac{205}{31104}\delta^5-\cdots\right)$$
$$\left\langle \psi_0\left(1\mathrm{d}\right)|\Psi^{[200]}_{\ell=2}\right\rangle = -\sqrt{\frac{1}{3}}\left(1-\frac{11}{72}\delta^2-\frac{11}{216}\delta^3-\frac{887}{31104}\delta^4-\frac{623}{46656}\delta^5-\frac{47863}{6718464}\delta^6-\cdots\right)$$
$$\left\langle \psi_0\left(1\mathrm{g}\right)|\Psi^{[200]}_{\ell=4}\right\rangle = -\sqrt{\frac{3}{35}}\left(\delta+\frac{1}{6}\delta^2-\frac{1}{35}\delta^3-\frac{5}{216}\delta^4-\frac{625}{31104}\delta^5-\cdots\right)$$

$$\int_0^\infty r^2dr\left|\Psi^{[000]}_{\ell=0}\left(r\right)\right|^2 = 1-\frac{1}{12}\delta^2-\frac{1}{36}\delta^3-\frac{11}{720}\delta^4-\frac{23}{3240}\delta^5-\frac{2083}{544320}\delta^6-\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[000]}_{\ell=2}\left(r\right)\right|^2 = \frac{1}{12}\delta^2+\frac{1}{36}\delta^3+\frac{5}{504}\delta^4+\frac{2}{567}\delta^5+\frac{124}{108864}\delta^6+\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[000]}_{\ell=4}\left(r\right)\right|^2 = \frac{3}{560}\delta^4+\frac{1}{280}\delta^5+\frac{173}{73920}\delta^6+\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[000]}_{\ell=6}\left(r\right)\right|^2 = \frac{5}{14784}\delta^6+\cdots$$

$$\int_0^\infty r^2dr\left|\Psi^{[220]}_{\ell=0}\left(r\right)\right|^2 = \frac{1}{3}-\frac{1}{60}\delta^2-\frac{1}{180}\delta^3-\frac{19}{5040}\delta^4-\frac{43}{22680}\delta^5-\frac{611}{544320}\delta^6-\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[220]}_{\ell=2}\left(r\right)\right|^2 = \frac{2}{3}-\frac{13}{84}\delta^2-\frac{13}{252}\delta^3-\frac{1}{63}\delta^4-\frac{11}{2268}\delta^5-\frac{1489}{1197504}\delta^6-\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[220]}_{\ell=4}\left(r\right)\right|^2 = \frac{6}{35}\delta^2+\frac{2}{35}\delta^3-\frac{29}{6160}\delta^4-\frac{263}{27720}\delta^5-\frac{5933}{786240}\delta^6-\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[220]}_{\ell=6}\left(r\right)\right|^2 = \frac{15}{616}\delta^4+\frac{5}{308}\delta^5+\frac{29}{4032}\delta^6+\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[220]}_{\ell=8}\left(r\right)\right|^2 = \frac{7}{2574}\delta^6+\cdots$$

$$\int_0^\infty r^2dr\left|\Psi^{[200]}_{\ell=0}\left(r\right)\right|^2 = \frac{2}{3}-\frac{1}{20}\delta^2-\frac{1}{160}\delta^3-\frac{1}{105}\delta^4-\frac{17}{3780}\delta^5-\frac{449}{181440}\delta^6-\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[200]}_{\ell=2}\left(r\right)\right|^2 = \frac{1}{3}-\frac{1}{28}\delta^2-\frac{1}{84}\delta^3-\frac{1}{336}\delta^4-\frac{1}{1512}\delta^5-\frac{19}{399168}\delta^6-\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[200]}_{\ell=4}\left(r\right)\right|^2 = \frac{3}{35}\delta^2+\frac{1}{35}\delta^3+\frac{1}{3080}\delta^4-\frac{41}{13860}\delta^5-\frac{22511}{8648640}\delta^6-\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[200]}_{\ell=6}\left(r\right)\right|^2 = \frac{15}{1232}\delta^4+\frac{5}{616}\delta^5+\frac{167}{44352}\delta^6+\cdots$$
$$\int_0^\infty r^2dr\left|\Psi^{[200]}_{\ell=8}\left(r\right)\right|^2 = \frac{7}{5148}\delta^6+\cdots$$