RIKEN Dec., 2008

(expecting experimentalists as an audience)

One-particle motion in nuclear many-body problem

(The 3rd lecture, V.3)

Giant resonances (GR) and sum rules in stable and unstable nuclei

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The figures with figure-numbers but without reference, are taken from

the basic reference : A.Bohr and B.R.Mottelson, Nuclear Structure, Vol. I & II

★　“巨大”とは何と比較して巨大なのですか。

Harakeh &van der Woude, “Giant Resonances”, Oxford, 2001

When IVGDR was found in photo-neutron cross sections, it had a resonance shape, but the width was typically of the order of 5 MeV, which was an order of magnitude larger than the resonances known in nuclei at that time (~ 1960 ies).
Thus, it was called “Giant Resonance”.

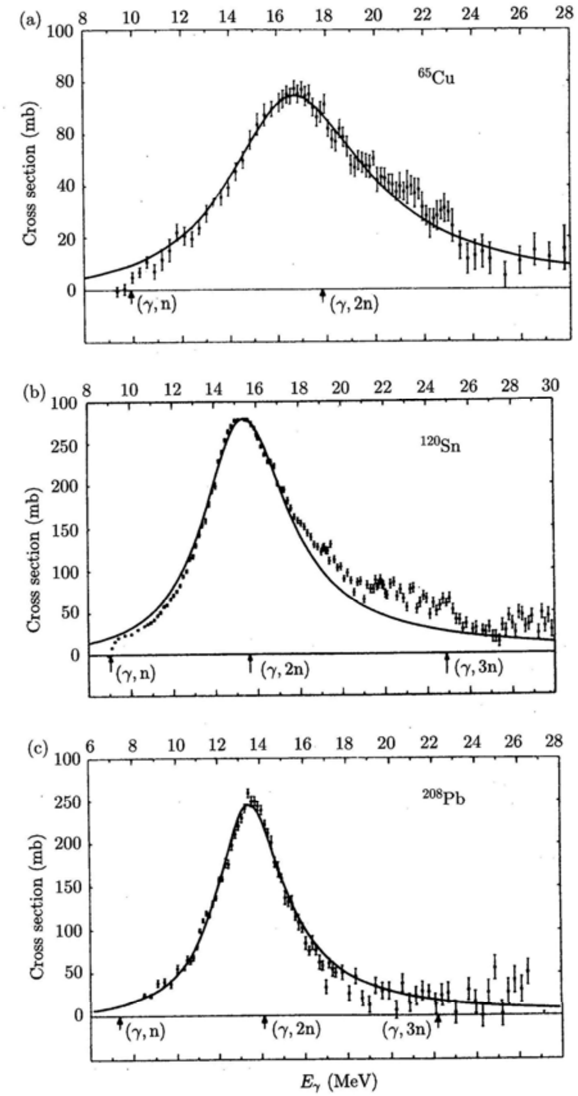


FIG. 1.2. The photo-neutron cross section $\sigma(\gamma, n)$ as a function of the photon energy for the three nuclei ^{208}Pb, ^{120}Sn and ^{65}Cu. Note that for these nuclei $\sigma(\gamma, n) \approx \sigma_{abs}(\gamma)$. From reference (BER75).

Photon energy resolution = several hundreds (< 500) keV.

★　巨大共鳴の流体モデルはわかったような気がするのですが、その適用限界を教えていただければ嬉しいです。

The hydrodynamical model consists of incompressible neutron and proton fluids.

Nuclei consist of nucleons, and the population of GR by, for example, γ-absorption is via one-particle operator.
Thus, the presence of quantum-mechanical shell-structure of one-particle levels in nuclei sets a limitation on the applicability of the hydrodynamical model.

If a collective mode consumes an appropriate sum-rule

$\longrightarrow$ Possibility of being approximately described by a macroscopic hydrodynamical model

Taking the simplest model for nuclei, namely harmonic-oscillator model, the above possibility exists if all one-particle excitations by a given operator have the same energy, one-frequency.

ex.

IVGDR is this example ; all one-particle excitations have ΔN=1, $\Delta E = 1\hbar\omega_0$

~~GQR~~ , since ΔN=0 and 2.

The hydrodynamical model is not directly applicable to spin-dependent modes.

★　重い核と軽い核で、励起機構にどのような違いがあるのでしょうか。

GRs in heavier nuclei have often a better resonance shape than those in lighter nuclei.

This may be due to the fact that in heavier nuclei ;

(a) GR can be more collective, since many more 1p-1h configurations are available.

(b) The spread of energies of 1p-1h excitations ($\sim A^{-1/3}$) is smaller, while the p-h interaction which couples 1p-1h excitations with different energies has no strong A-dependence.

Thus, building up a collective state out of available 1p-1h configurations is easier.

Lighter nuclei have less clear distinction between the surface and the inside.

This makes a difference, for example, when a probe used is sensitive only to the surface or GR is of a surface type.

★　中性子過剰だとどうなりそうか、ということにも少し触れてもらえると楽しいです。

Neutron-excess gives an essential difference in charge-exchange GR, $t_{\pm}$ GR, from the case of N=Z nuclei.

ex. Some t_{+}GR may disappear due to the Pauli principle.

ex. $E_x(t_{-}GR) > E_x(t_{+}GR)$ in the presence of neutron excess.

Neutron excess $\longrightarrow$ Excitations made by Isoscalar (i.e. isospin-independent) operators carry an isovector transition density ----- When neutrons and protons move in the same way in nuclei with N > Z, $\delta\rho_n - \delta\rho_p \neq 0$.

Giant resonances and sum rules

7.1. Introduction

7.2. Sum rules

7.2.1. Sum rules for (1 or t_z) excitations

Classical oscillator sum (= energy-weighted sum)
Sum-rule in axially-symmetric quadrupole-deformed nuclei

7.2.2. Sum rules for ($t_\pm$) charge-exchange excitations

Difference, $S_- - S_+$, of non-energy-weighted sums

7.3. Giant resonances of IS or t_z type (excitations within the same nuclei)

7.3.1. Isovector giant dipole resonance (IVGDR)

7.3.2. Isoscalar and isovector giant quadrupole resonance (ISGQR and IVGQR)

7.3.3. Isoscalar giant monopole resonance (ISGMR) - compression mode

7.4. Giant resonances of charge-exchange (n→p or p→n) type
(excitations to the neighboring nuclei)

7.4.1. Fermi transitions (IAS)

7.4.2. Gamow-Teller (GT) resonance (incl. magnetic giant dipole resonance)

7.4.3. Isovector spin giant monopole resonance (IVSGMR)

7.4.4. Isovector spin giant dipole resonance (IVSGDR)

7.5. Giant resonances in nuclei far away from the stability line

7.5.1. ISGQR of nuclei with weakly-bound neutrons

- an example of threshold strength

7.5.2. β-decay to GTGR in drip line nuclei

β^- decay to $GTGR_-$ in very neutron-rich light nuclei

β^+ decay to $GTGR_+$ in medium-heavy (N>Z) proton-drip-line nuclei

References:

M.N.Harakeh and A.vander Woude, “Giant Resonances”, 2001, Oxford.

鈴木敏男, 原子核の巨大共鳴状態, 物理学最前線 19 (共立出版) 1988.

7. Collective motion based on particle-hole excitations
- giant resonances and sum-rules

7.1. Introduction

Collective motion :

Many nucleons participate coherently in the motion so that a given observable (transition) is much enhanced compared with a single-particle estimate.

The best-established collective motion in nuclei is rotational motion of deformed nuclei.

The properties of very low-energy collective states are sensitive both to pair correlations and to the shell-structure around the Fermi levels.
Only those particles close to the Fermi levels contribute to the pair correlation.

In contrast, many (if not all) particles in a nucleus participate in giant resonances (GR), so that
(a) the properties of GR are almost independent of the shell-structure around the Fermi level,
(b) depend on the bulk properties, and
(c) are expressed as a smooth function of Z, N and A.

The total transition strength should be limited by a "sum rule", which depends on the ground-state properties.
Due to the collective nature, GR consumes the major part of the sum rule that is defined for respective collectivity.

$\longrightarrow$ Then, GR may correspond to a classical picture of collective motion.

Usefulness of sum-rules

If an observed peak consumes the major part of the sum-rule, the peak expresses a collective mode.
Moreover, there are almost no other collective excitations carrying the strength of the same operator *F*,
while the mode created with the operator acting on the ground state is approximately an eigenstate of the Hamiltonian.

Examples of Giant Resonances experimentally studied in β-stable nuclei are

(a) Excitations in the same nuclei (IS = Isoscalar, IV = Isovector)

	spin-parity	operator	observed peak energy
IS GMR*	0+	$\sum_k r_k^2$	80 $A^{-1/3}$ MeV (for A > 90)
IS GDR*	1–	$\sum_k r_k^3 Y_{1\mu}(\hat{r}_k)$	
IV GDR	1–	$\sum_k \tau_z(k)\vec{r}_k$	79 $A^{-1/3}$ MeV (for A > 50)
IS GQR	2+	$\sum_k r_k^2 Y_{2\mu}(\hat{r}_k)$	63 $A^{-1/3}$ MeV (for A > 60)
IV GQR	2+	$\sum_k \tau_z(k) r_k^2 Y_{2\mu}(\hat{r}_k)$	
IV spin GR	1+	$\sum_k \tau_z(k)\vec{\sigma}_k$	

GRs have width of several MeV (except IAS) and exhaust the major part of respective sum-rule.

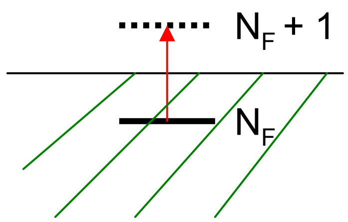
* compression mode

(b) Excitations to neighboring nuclei

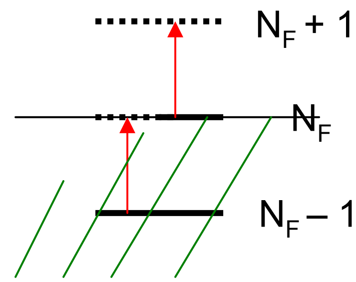
	spin-parity	operator
IAS	0+	$\sum_k t_\pm(k)$
GT GR	1+	$\sum_k t_\pm(k)\vec{\sigma}_k$
IV GQR	2+	$\sum_k t_\pm(k) r_k^2 Y_{2\mu}(\hat{r}_k)$
IV spin GMR*	1+	$\sum_k t_\pm(k)\vec{\sigma}_k (r_k^2 - \langle r^2 \rangle_{excess})$
IV spin GDR	0–, 1–, 2–	$\sum_k t_\pm(k) r_k (Y_1(\hat{r}_k)\vec{\sigma}_k)_{J\pi}$

Examples of selection rules in spherically-symmetric harmonic-oscillator potential

1) Operator $rY_{1\mu}$ (or x, y, z) $\Longrightarrow$ $\Delta N=1$ ($E_x = \hbar\omega_0$) excitations

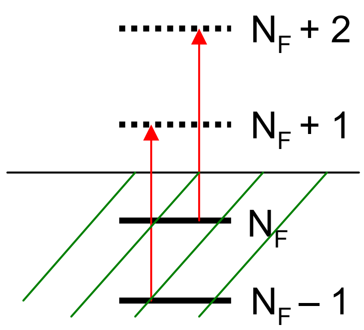


Closed-shell configuration

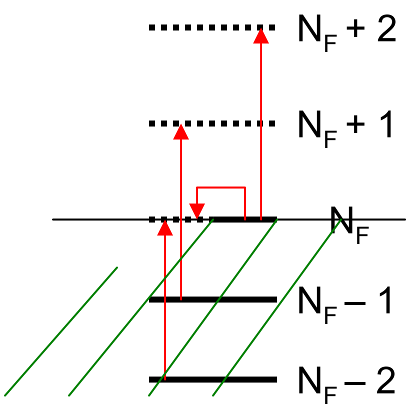


Partially-occupied N_F shell

2) Operator $r^2 Y_{2\mu}$ (or x^2, y^2, z^2) $\Longrightarrow$ $\Delta N=0$ or 2 ($E_x = 0\hbar\omega_0$ or $2\hbar\omega_0$) excitations



Closed-shell configuration



Partially-occupied N_F shell

In realistic potentials the above selection rules do not exactly work, but work approximately.

Observed one-particle energies are not well reproduced

by Hartree-Fock calculations using Skyrme interactions with m^* (= (0.6-0.8) m).

In contrast,

observed energy of ISGQR are often reproduced

by RPA based on the Hartree-Fock calculation with the same Skyrme interaction (so-called self-consistent RPA).

Note that the parameters related to ISGQR are well taken care of, when Skyrme parameters are determined.

In this lecture we do not further go into detail of [Skyrme H.F. + RPA] calculation.

Instead, we try to understand GRs, sometimes using the result of [Skyrme H.F. + RPA] calculation, but mostly using the models which are as simple as possible.

Shape oscillations - typical vibrational excitations when nuclear matter is incompressible.

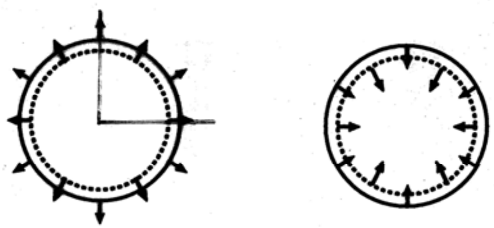
Compression modes → information on nuclear compressibility

$$\rho_{n0}^{tr}(\vec{r}) \equiv \langle n | \sum_{k=1}^{A} \delta(\vec{r} - \vec{r}_k) | 0 \rangle \equiv \underline{\rho_{\lambda}^{tr}(r)} Y_{\lambda\mu}(\hat{r})$$

radial transition density

proton neutron

IS Monopole - compression mode



G.F.Bertsch and S.F.Tsai
Physics Reports **18**, (1975) 125.

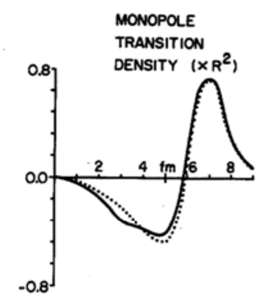


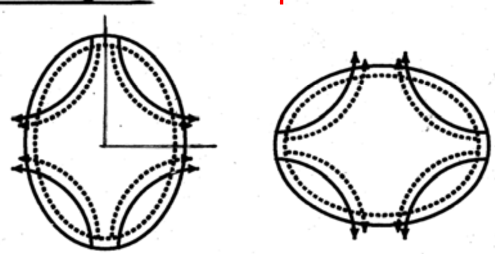
Fig. 10. Monopole transition density in ^{208}Pb. The solid curve is the result for the transition density operator $R^2\Sigma_i\rho(r_i - R)$, for the SkI interaction between the ground and the 20 MeV state. Units are $\pi\,\text{fm}^{-1}$. The dotted curve is the prediction of the Tassie model, eq. (43b). Here, the overall magnitude of the curve has no significance.

Tassie model

(---------)

$$3\rho_0(r) + r\frac{d\rho_0(r)}{dr}$$

IS Quadrupole - shape oscillation



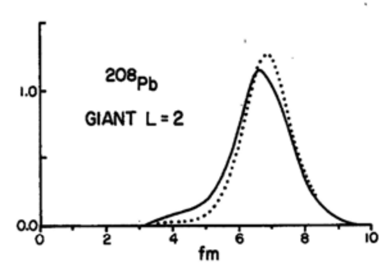


Fig. 9. Isoscalar quadrupole transition density (× r^2) in ^{208}Pb with the SkI interaction to the giant state predicted at 11.4 MeV. The solid curve is the calculation; the dashed curve is the collective model, eq. (43a).

$$r\frac{d\rho_0(r)}{dr}$$

In heavier nuclei GR may show a resonance (Lorentzian ?) shape and the properties can be systematic, while those of GR in medium weight and light nuclei are more individual.
In very light nuclei GR strength distribution is split into several fragments.

∵) In lighter nuclei the collectivity is weaker, or a number of p-h configurations to contribute to GR is smaller.

In lighter nuclei the difference of the relevant p-h excitation energies may be large compared with the interaction between them,

---

Transition densities of GR with good accuracy is not experimentally available.

Example of transition density of
IS shape oscillation ;

3^- state of ^{208}Pb at Ex = 2.61 MeV

Experimental data are taken from
(e , e') in J.Heisenberg and I.Sick,
P.L. **32B** (1970) 249.

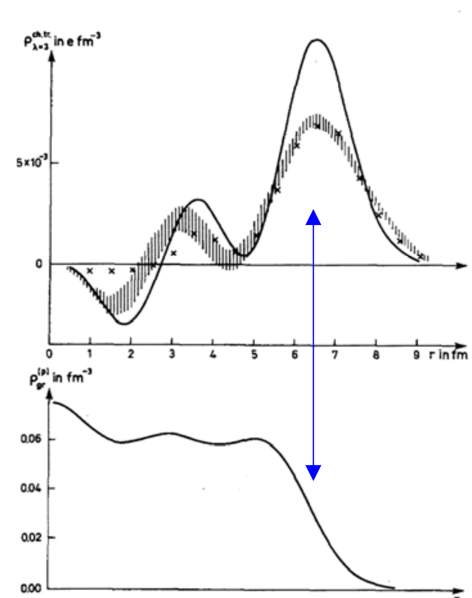


Fig. 1. The upper part represents the radial transition charge density of the 3^- state at 2.61 MeV in ^{208}Pb. The solid line shows the calculated value, while for the experimental values the dashed band was taken from ref. [4] and the crosspoints were from ref. [3]. The lower part of the figure expresses the calculated charge density of the ground state of ^{208}Pb.

I.H., P.L. **66B** (1977) 410

7.2. Sum rules

In this lecture we treat nucleons as elementary particles, neglecting possible contributions from internal degree of freedom of nucleons.

$\Rightarrow$ Valid in the energy interval, well below internal excitations of nucleons .

7.2.1. Sum rules for (1 or t_z) excitations

Classical oscillator sum - sum of energy-weighted transition strength

$$S(F_\lambda) \equiv \sum_a (E_a - E_0) B(F_\lambda; 0 \to aI_a) = \sum_a (E_a - E_0) \left| \langle a | F_\lambda | 0 \rangle \right|^2$$

$$= \frac{1}{2} \langle 0 | \left[F_\lambda, [H, F_\lambda] \right] | 0 \rangle$$ where $H = \sum_i t_i + \sum_{i<j} \mathrm{v}_{ij}$, $H|0\rangle = E_0|0\rangle$, $H|a\rangle = E_a|a\rangle$

If v_{ij} does not explicitly depend on the momentum of particles, $\mathrm{v}_{ij}(\cancel{\vec{p}_k})$, and if one-particle operator F_λ depends only on $\vec{r}_k$ $\quad F_\lambda = \sum_k F_\lambda(\vec{r}_k)$ $\Rightarrow$

$$\left[\sum_{i<j} \mathrm{v}_{ij}, F_\lambda \right] = 0$$ Thus, $[H, F_\lambda] \Rightarrow \left[\sum_i t_i, F_\lambda \right]$

Then,

$$\boxed{S(F_\lambda) = \langle 0 | \sum_k \frac{\hbar^2}{2m_k} \left(\nabla_k F_\lambda(\vec{r}_k) \right)^2 | 0 \rangle}$$

Note that the sum is expressed as a ground-state expectation value of one-body operator - insensitive to the many-body correlation in the ground state – "model independent". Sums with other energy weightings involve two- or many-body operators.

In particular, if $F_{\lambda\mu} = f(r) Y_{\lambda\mu}(\hat{r})$, we obtain

$$\boxed{S(F_\lambda)_{class} = \frac{2\lambda+1}{4\pi}\frac{\hbar^2}{2m} A \left\langle \left(\frac{df}{dr}\right)^2 + \lambda(\lambda+1)\left(\frac{f}{r}\right)^2 \right\rangle} \qquad (7.1)$$

where $\langle \ \rangle$ expresses the average per particle in the ground state of *A* particles.

For *Eλ* transitions with $\lambda \geq 2$, neglecting the correction due to the center of mass motion,

$$F_{\lambda\mu} \Rightarrow e r^\lambda Y_{\lambda\mu}(\hat{r})$$ only for protons, then,

$$\boxed{S(E\lambda)_{class} = \frac{\lambda(2\lambda+1)^2}{4\pi}\frac{\hbar^2}{2m} Z e^2 \underline{\left\langle r^{2\lambda-2} \right\rangle_{proton}}}$$

radial average for protons in the ground state

For *E0* transitions,

$$F_{\lambda\mu} \Rightarrow e r^2$$ only for protons, then,

$$\boxed{S(E0)_{class} = \frac{2\hbar^2}{m} Z e^2 \left\langle r^2 \right\rangle_{proton}}$$

For *E1* transitions ;

Since isoscalar dipole operator corresponds to the center of mass motion that must not create an excitation, the dipole operator which creates excitations is necessarily of isovector character.

For example, electric-dipole excitation operator (in the direction of z-axis) should be

$$e\sum_i^{(p)} z_i \longrightarrow e\sum_i^{(p)}\left(z_i - \frac{1}{A}\left(\sum_j^{(p)} z_j + \sum_k^{(n)} z_k\right)\right) = e\sum_i^{(p)} z_i - \frac{e}{A}Z\left(\sum_j^{(p)} z_j + \sum_k^{(n)} z_k\right)$$

$$= \frac{N}{A}e\sum_i^{(p)} z_i - \frac{Z}{A}e\sum_k^{(n)} z_k$$

spin-parity = 1^-

where (p) and (n) express the sum over protons and neutrons.

Then, using (7.1), the classical oscillator sum, which should be the sum rule for IVGDR, when the interaction has $\cancel{\vec{p}_k}$, $\cancel{(\vec{\tau}\cdot\vec{\tau})}$, and $\cancel{(\vec{\sigma}\cdot\vec{\sigma})}$

$$S(E1)_{class} = \sum_{a\mu}(E_a - E_0)\left|\langle a|\frac{N}{A}e\sum_i^{(p)}(rY_{1\mu})_i - \frac{Z}{A}e\sum_k^{(n)}(rY_{1\mu})_k|0\rangle\right|^2 = \frac{9}{4\pi}\frac{\hbar^2}{2m}e^2\left(Z\left(\frac{N}{A}\right)^2 + N\left(\frac{Z}{A}\right)^2\right)$$

$$= \frac{9}{4\pi}\frac{\hbar^2}{2m}e^2\frac{NZ}{A}$$

$$S(F_\lambda = (erY_1)_p) - S(E1)_{class} = \frac{9}{4\pi}\frac{\hbar^2}{2m}Ze^2 - \frac{9}{4\pi}\frac{\hbar^2}{2m}e^2\frac{NZ}{A}$$

$$= \frac{9}{4\pi}\frac{\hbar^2}{2m}\frac{Z^2}{A}e^2$$

; oscillator strength associated with the center of mass motion ex. center of mass motion for *E2* operator

Total *E2* moment measured with respect to the center of mass

$$\sum_{k=1}^{A} e_k\left[2(z_k - Z_c)^2 - (x_k - X_c)^2 - (y_k - Y_c)^2\right]$$

where $Z_c \equiv \frac{1}{A}\sum_{k=1}^{A} z_k$, etc. and $e_k = \begin{cases} e & \text{for proton} \\ 0 & \text{for neutron} \end{cases}$

$$= \sum_{k=1}^{A} e_k\left[2z_k^2 - x_k^2 - y_k^2\right] - 4Z_c\sum_{k=1}^{A} e_k z_k + 2X_c\sum_{k=1}^{A} e_k x_k + 2Y_c\sum_{k=1}^{A} e_k y_k + \underbrace{\sum_{k=1}^{A} e_k}_{= Ze}(2Z_c^2 - X_c^2 - Y_c^2)$$

$$= \sum_{k=1}^{A}\left[e_k\left(1-\frac{2}{A}\right) + e\frac{Z}{A^2}\right](2z_k^2 - x_k^2 - y_k^2) + \frac{1}{A}\sum_{j<k}\left[-2e_j - 2e_k + 2\frac{Ze}{A}\right](2z_j z_k - x_j x_k - y_j y_k)$$

(a)

For a single-particle configuration

$$e_k \rightarrow \begin{cases} e\left(1-\frac{2}{A}+\frac{Z}{A^2}\right) & \text{for proton} \\ e\frac{Z}{A^2} & \text{for neutron} \end{cases}$$

(b)

For harmonic oscillator wave-functions and low-energy transitions ;
matrix elements of (a) receive no contributions from the recoil term, namely,
the correction term in (b) is exactly cancelled by the 2-body term in (a).

Distribution of S(*E2*) in axially-symmetric deformed nuclei

(1) Low-energy IS ($r^2 Y_{2\nu}$) excitations ($\Delta N \approx 0$)

- rotational excitation ($\nu = \pm 1$)
- gamma-vibration ($\nu = \pm 2$)
- beta-vibration ($\nu = 0$)
- (one-particle excitations)

a) rotation excitation – main *E2* oscillator strength in the low-energy region

For even-even nuclei $$(E_2 - E_0) B(E2; K=0, I=0 \to K=0, I=2) = \frac{3\hbar^2}{\Im} \frac{5}{16\pi} e^2 Q_0^2$$

For odd-A nuclei $$\sum_I \left[E(K_0, I) - E(K_0, I_0)\right] B(E2; K_0 I_0 \to K_0 I) = \frac{3\hbar^2}{\Im} \frac{5}{16\pi} e^2 Q_0^2$$

Observed moments of inertia $\Im_{obs} \approx 5\Im_{irrot} \quad \longrightarrow \quad S(E2)_{rot} \approx (0.05) S(E2)_{class}$

comparable to low-energy 2+ mode in spherical nuclei

b) gamma vibration ($r^2 Y_{2\pm 2}$ type surface vibration) takes less than a few % of $S(E2)_{class}$

beta vibration ($r^2 Y_{20}$ Type surface vibration) takes less than $(0.01) S(E2)_{class}$

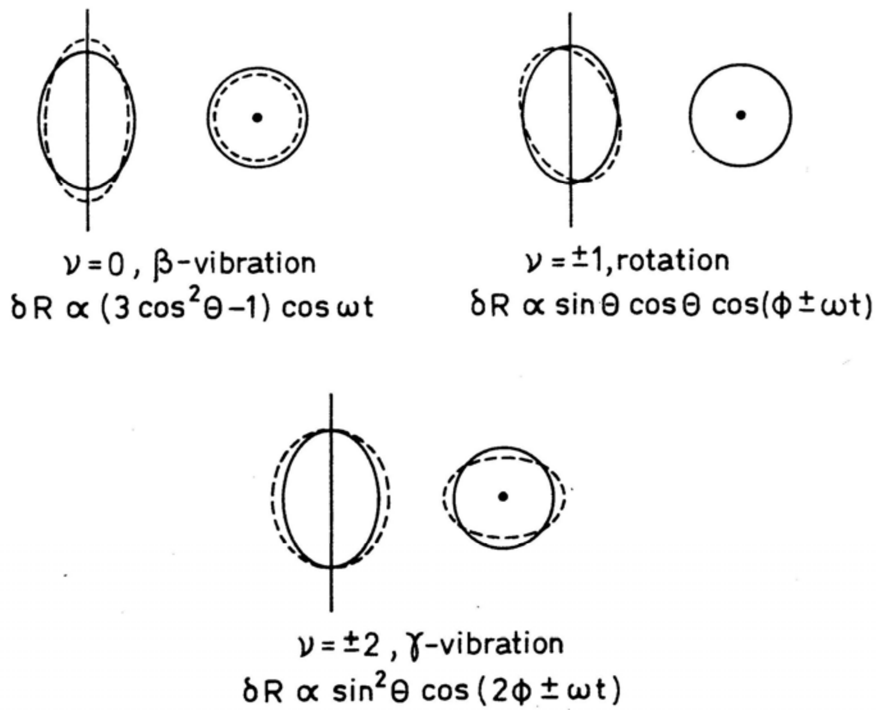


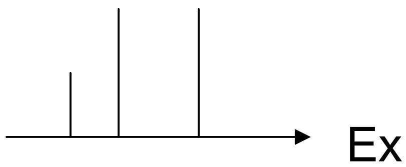
Figure 6-3 Quadrupole shape oscillations in a spheroidal nucleus. The figure shows projections of the nuclear shape in directions perpendicular and parallel to the symmetry axis.

(2) High-energy ($\Delta N \approx 2$) excitations

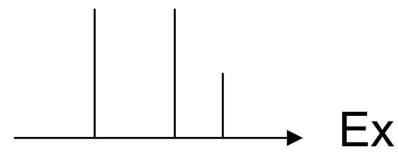
E2 strength will split depending on the quantum number $|\nu|$ of $Y_{2\nu}$.

The E2 strength of GR with $\nu = 0, \pm1, \pm2$ is expected to be approximately

1 : 2 : 2



prolate shape



oblate shape

OBS. L-S doubly-closed spherical nuclei (such as 40-Ca) have only high-energy ($\Delta N \approx 2$) collective quadrupole excitations.

Spherical vibrating nuclei have both low- and high-energy collective quadrupole excitations. The low-energy ($\Delta N \approx 0$) IS quadrupole modes have enhanced E2 transitions due to the attractive quadrupole interaction, but carry less than (0.10) $S(E2)_{class}$.

ex. an extra contribution to S(E1) from an exchange interaction

Increase of energy-weighted sum-rules, *S(E1)*, from *S(E1)*$_{class}$
due to the presence of attractive Majorana space-exchange interaction

ex. A proton-neutron pair with 2-body space-exchange ($\vec{r}_i \leftrightarrow \vec{r}_j$) interaction

$$v_{ij} = f(|\vec{r}_i - \vec{r}_j|)P^M$$

P^M : Majorana space-exchange operator

$$P^M = -\frac{1+(\vec{\tau}_i \cdot \vec{\tau}_j)}{2}\frac{1+(\vec{\sigma}_i \cdot \vec{\sigma}_j)}{2}$$

$$\left[v_{ij}, z_i\right] = fP^M z_i - z_i fP^M = (z_j - z_i)fP^M$$

z_i : proton coordinate
z_j : neutron coordinate

$$\left[z_i, \left[v_{ij}, z_i\right]\right] = \left[z_i, (z_j - z_i)fP^M\right]$$

$$= z_i(z_j - z_i)fP^M - (z_j - z_i)fP^M z_i$$

$$= z_i(z_j - z_i)fP^M - (z_j - z_i)z_j fP^M$$

$$= -(z_i - z_j)^2 fP^M$$

$$= -\frac{1}{3}(r_{ij})^2 fP^M$$

$\therefore$ An attractive Majorana interaction makes an extra contribution to S(E1) .

7.2.2. Sum rules for ($t_\pm$) charge-exchange excitations

There is no sum-rule, which corresponds to the classical oscillator strength for IS operators.

Instead, model-independent and non-energy weighted sum-rules,

for the difference between t_- and t_+ transitions.

Isospin of nucleon, $t = \frac{1}{2}$ $\quad t_z|n\rangle = \frac{1}{2}|n\rangle$ $\quad t_z|p\rangle = -\frac{1}{2}|p\rangle$ $\quad t_\pm \equiv t_x \pm it_y$

$t_-|n\rangle = |p\rangle$ $\quad t_+|p\rangle = |n\rangle$ $\quad t_+|n\rangle = 0$ $\quad t_-|p\rangle = 0$

$$\left[\sum_k^A t_+(k), \sum_j^A t_-(j)\right] = 2\sum_k^A t_z(k) \qquad 2\langle N,Z|\sum_i^A t_z(i)|N,Z\rangle = N - Z$$

$$\sum_{\mu=1}^3 \sigma_\mu^{\,2} \;\left(= 4\sum_{\mu=1}^3 s_\mu^{\,2} = 4(\vec{s})^2 = 4\frac{1}{2}\left(\frac{1}{2}+1\right)\right) = 3$$

: Basic formulas

(a) charge-exchange non-spin-flip excitations : $\hat{O}_\pm = \sum_k t_\pm(k) f(r_k)$

$$\sum_m |\langle m|\hat{O}_-|0\rangle|^2 - \sum_n |\langle n|\hat{O}_+|0\rangle|^2 = N\langle (f(r))^2\rangle_n - Z\langle (f(r))^2\rangle_p$$

(b) charge-exchange spin-dependent excitations : $\hat{O}_\pm = \sum_\mu \sum_k t_\pm(k)\sigma_\mu(k) f(r_k)$

$$\sum_m \left|\langle m|\hat{O}_-|0\rangle\right|^2 - \sum_n \left|\langle n|\hat{O}_+|0\rangle\right|^2 = 3\left[N\left\langle (f(r))^2\right\rangle_n - Z\left\langle (f(r))^2\right\rangle_p\right]$$

7.3. Giant resonance of isoscalar (IS) or t_z type

7.3.1. Isovector giant dipole resonance (IVGDR)

: the oldest and best known Giant Resonance

Systematics of observed IVGDR frequency

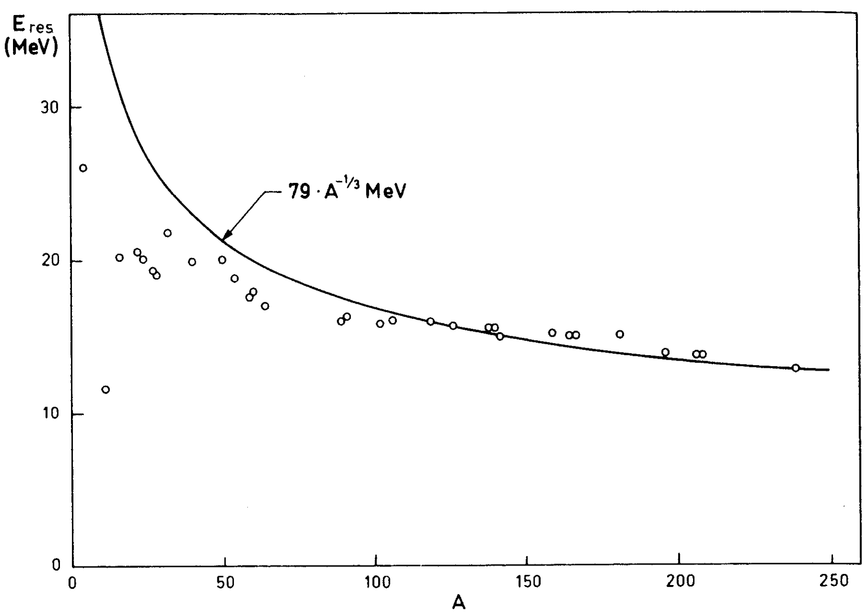


Fig. 6-19 of A.Bohr & B.R.Mottelson, Nuclear Structure, vol.II

Observed $79\ A^{-1/3}$ MeV is in good agreement with the value estimated in the harmonic-oscillator model.

For light nuclei with $A < 50$ a deviation from the systematics is observed.
--- Other types of GR show the same tendency.

Note that unpertrubed $I^{\pi} = 1^{-}$ p-h energies in realistic potentials are approximately degenerate and close to $41\ A^{-1/3}$ MeV (!) also in drip-line nuclei !

Well-established IVGDR – observed peak(s) in photo absorption cross section

Photo absorption cross section of ^{197}Au

Figs.6-18 and 6-26 of A.Bohr & B.R.Mottelson, Nuclear Structure, vol.II

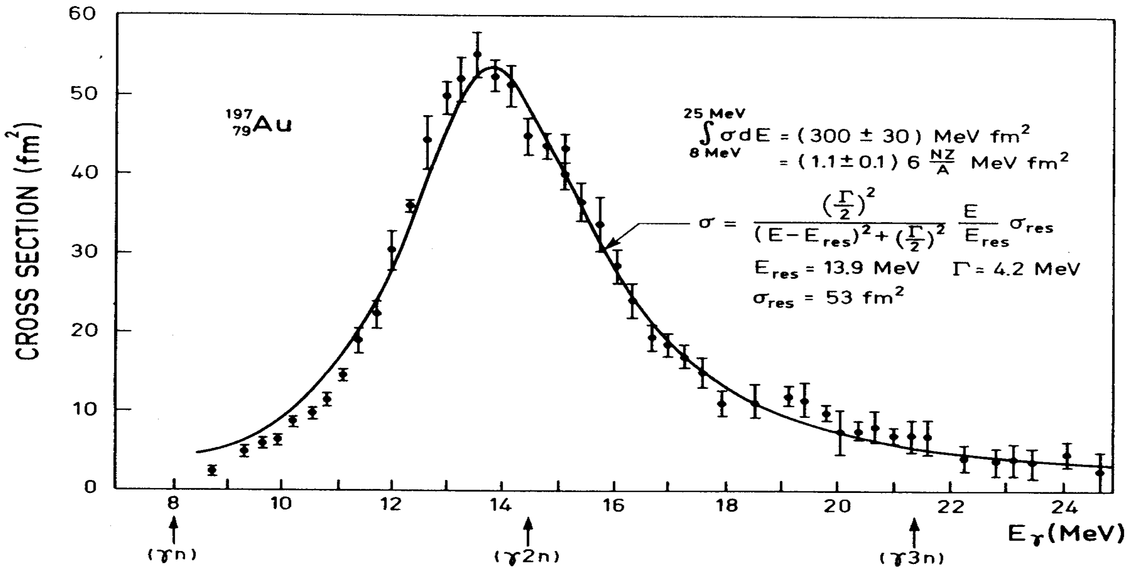
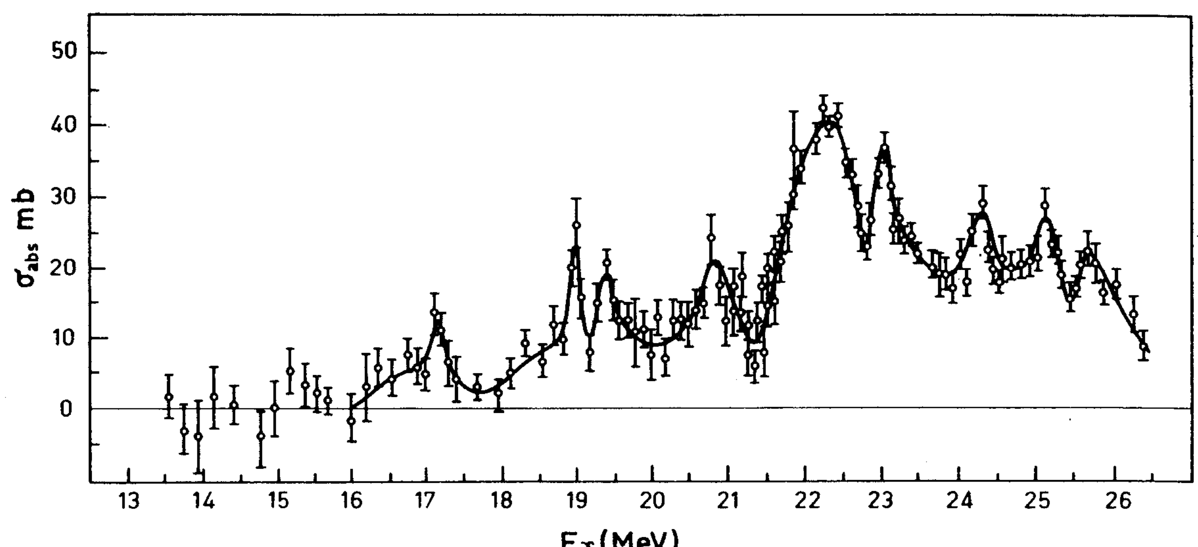


Photo absorption cross section of ^{16}O



Additional dipole strength is observed on the high-energy side, that appears to be associated with short-range (velocity-dependent) correlations between nucleons.

pion threshold

$$\int_0^{140 MeV} \sigma_{photoabs} dE_\gamma \rightarrow 2S(E1)_{class}$$

$$\int_0^{25 MeV} \sigma_{photoabs} dE_\gamma \longrightarrow S(E1)_{class}$$

where

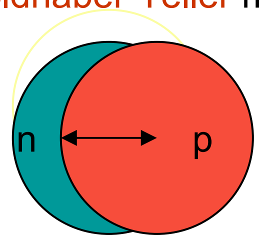
$$S(E1)_{class} = \frac{9}{4\pi} \frac{\hbar^2}{2M} \frac{NZ}{A} e^2$$

$$S(E1)_{obs} = S(E1)_{class}(1 + x)$$

x comes from the velocity- and $(\tau \cdot \tau)$ - dependent terms in the nucleonic interactions.

IVGDR is the giant resonance, of which the semi-classical picture is possible.

Goldhaber-Teller model



Steinwedel-Jensen model

Neutron and proton fluids are oscillating within a sphere, keeping the total density constant.

For simplicity, assuming [IVGDR ~ a standing wave in a nucleus with a fixed boundaries],

the frequency $\omega \propto R^{-1}$

Then, in contrast to one-peak structure in spherical nuclei,

in axially-symmetric quadrupole-deformed nuclei

$$\omega_z \propto R_z^{-1}$$

$$\omega_\perp \propto R_\perp^{-1}$$

the strength distribution would have two peaks corresponding to an oscillation of neutrons vs protons along the long and short axes, as observed in $^{150}Nd_{90}$.

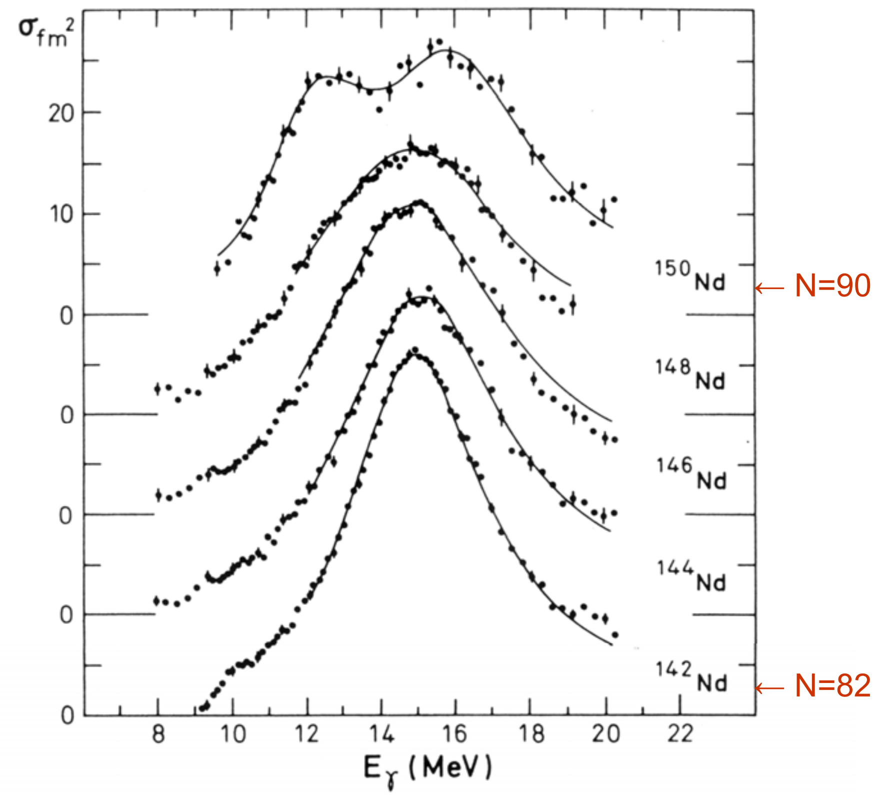
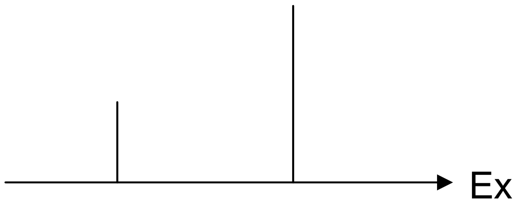
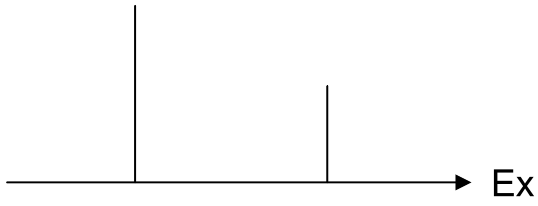


Figure 6-21 Photoabsorption cross section for even isotopes of neodymium.

For a prolate (oblate) shape the integrated cross section associated with the vibration along the symmetry axis (= longer (shorter) axis), which has lower (higher) frequency, should be about a half of the one along two shorter (longer) axes.



prolate shape



oblate shape

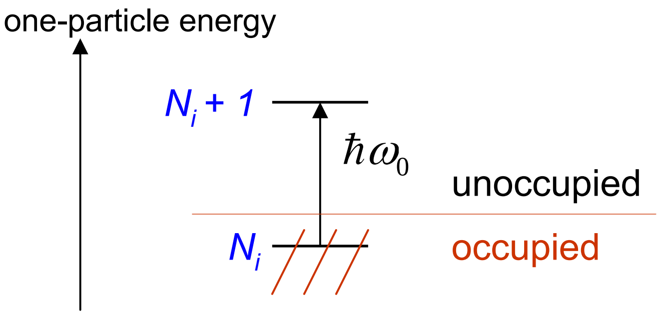
The energy splitting is proportional to the deformation δ

$$\frac{\omega_\perp - \omega_z}{\overline{\omega}} \approx \frac{\Delta R}{\overline{R}} \approx \delta$$

Thus, the ground state of $^{150}Nd_{90}$ is prolately deformed !

Harmonic oscillator potential

$$\langle N_f | x, y, z | N_i \rangle \neq 0 \qquad \text{only for } N_f = N_i \pm 1$$



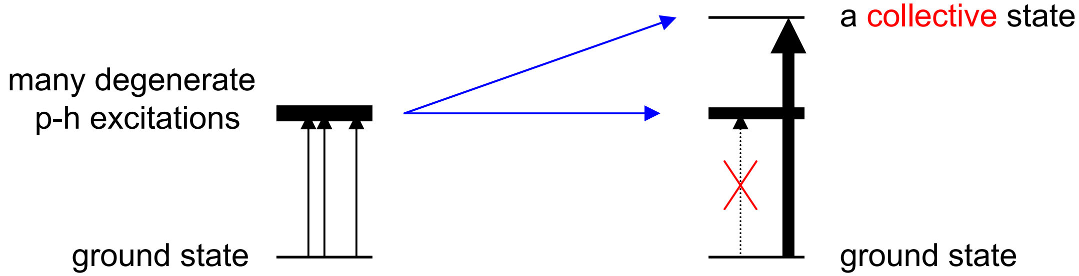
All excitations are from the last-filled major shell to the next major shell, with excitation energies,

$$\Delta E = \hbar\omega_0 \approx 41 A^{-1/3} \text{ MeV}.$$

Many degenerate particle-hole (p-h) excitations, especially in heavier nuclei.

A schematic model : (for IVGDR)

G.E.Brown, *Unified Theory of Nuclear Models,* p.29-32 and p.47-49.



After including a separable repulsive interaction between the p-h excitations, only one collective state is pushed up, while all other states remain at the unperturbed excitation-energy, and the collective state absorbs all transition strength, if one takes

separable interaction $\longleftrightarrow$ relevant transition operator

Taking the strength of IV dipole-dipole interaction from the symmetry term of the phenomenological nuclear one-body potential, in the harmonic oscillator potential model we obtain

unperturbed p-h energy, $\hbar\omega_0 = 41A^{-1/3}$ MeV

$\longrightarrow$ $80A^{-1/3}$ MeV for the excitation energy of the collective state, (= IVGDR)

in agreement with the observed systematics in medium-heavy nuclei.

$\Bigg[$ p-h energies, $41A^{-1/3}$ MeV $\rightarrow$ collective IVGDR at $80A^{-1/3}$ MeV, which consumes the major part of *E1* strength. $\Bigg]$

due to the repulsive p-h interaction

means, $|e^p_{eff}(E1)|$ and $|e^n_{eff}(E1)|$ for low-energy E1 transitions are much smaller than the values of *(N/A) e* and *(Z/A) e ,* respectively (see Sect. 6.1).

In the self-consistent calculations plus RPA for spherical nuclei,
the strength of IVGDR is split into several peaks even for heavier nuclei.
The transition density of lower-lying peaks appears to be closer to the Steinwedel-Jensen prediction, while that of higher-lying peaks looks more like the Goldhaber-Teller one.

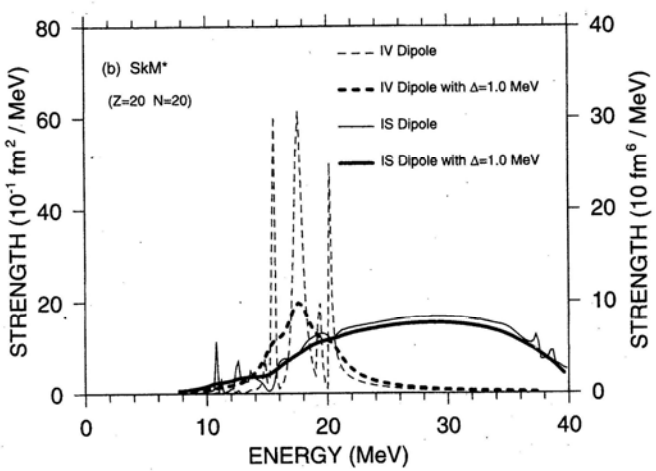
Radial transition density :

Goldhaber-Teller model $$\rho_{GT}^{tr}(r) \propto \frac{d\rho_0(r)}{dr}$$

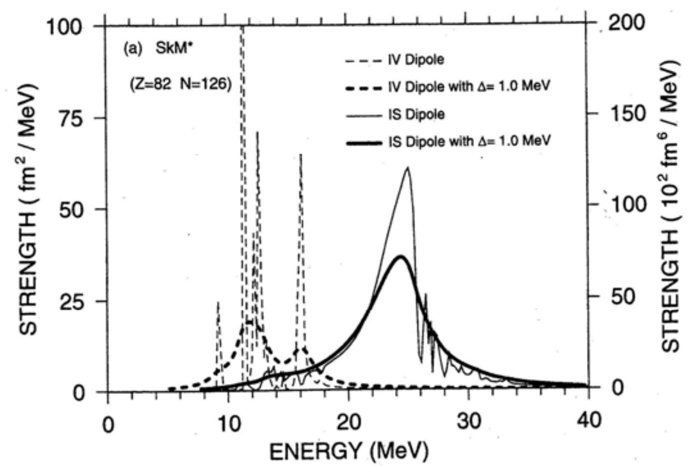
Steinwedel-Jensen model $$\rho_{SW}^{tr}(r) \propto r\rho_0(r)$$

Examples of self-consistent calculations plus RPA (- - - - - IVGDR, ——— ISGDR)

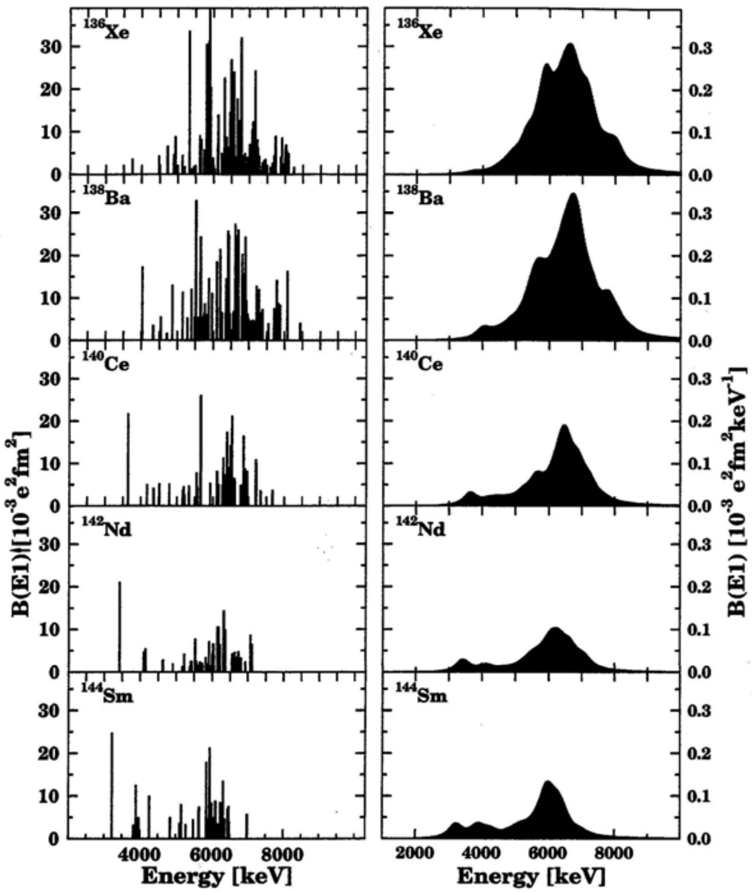
${}^{40}_{20}Ca_{20}$

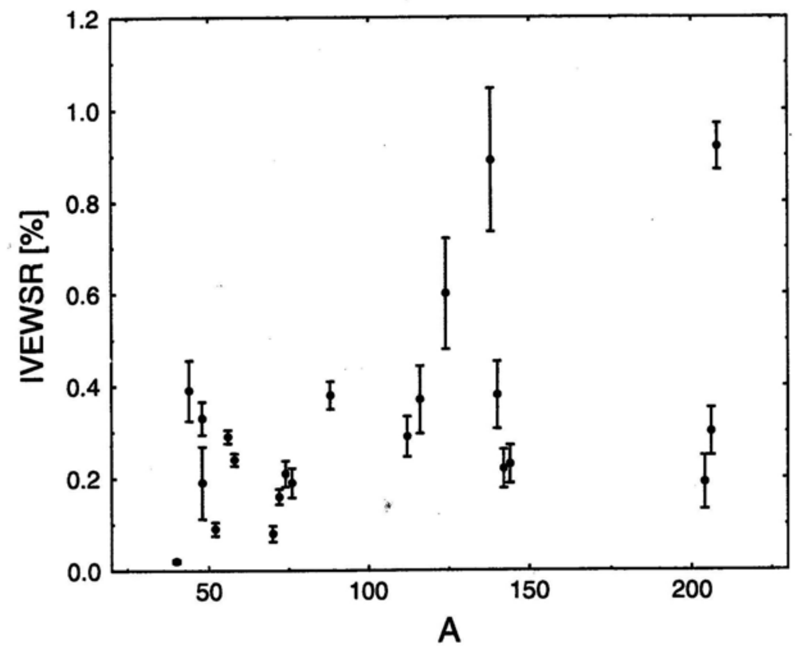


${}^{208}_{82}Pb_{126}$



I.H., H.Sagawa and X.Z.Zhang, PRC 57, R1064 (1998) ex. “Pigmy dipole resonances” observed at much lower energy than IVGDR of the A ≈ 140 region consume less than 1 % of $S(E1)_{class}$.





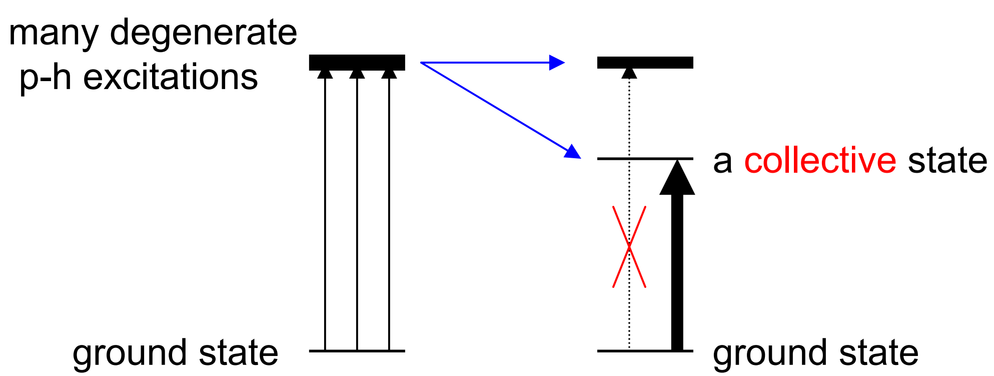
obtained from by folding with a Lorentzian
with a width of 500 keV

(from A. Zilges, 2007)

7.3.2. Isoscalar and isovector giant quadrupole (ISGQR and IVGQR) resonance

	Operator	spin-parity	observed peak energy
ISGQR	$\sum_k r_k^2 Y_{2\mu}(\hat{r}_k)$	2+	64 $A^{-1/3}$ MeV
IVGQR	$\sum_k \tau_z(k) r_k^2 Y_{2\mu}(\hat{r}_k)$	2+	(130 $A^{-1/3}$ MeV ?)

A schematic model : (for ISGQR)



After including a separable attractive interaction between the p-h excitations, only one collective state is pushed down, while all other states remain at the unperturbed excitation-energy, and the collective state obtains all transition strength, if one takes

separable interaction $\longleftrightarrow$ relevant transition operator

Taking the strength of IS quadrupole-quadrupole interaction from the self-consistent condition that the eccentricity of the potential is the same as that of the density, in the harmonic oscillator potential model we obtain

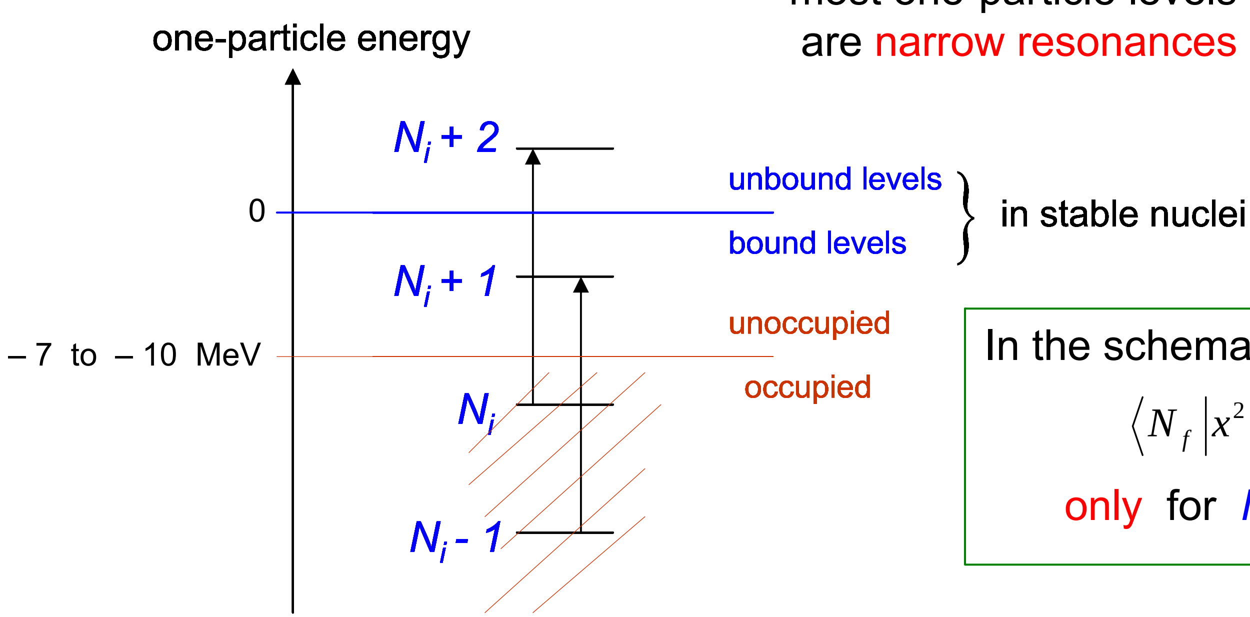
A.Bohr and B.R.Mottelson, *Nuclear Structure*, vol.II, p.509

unperturbed p-h energy, $2\hbar\omega_0$ = 82 $A^{-1/3}$ MeV

$\longrightarrow$ $\sqrt{2}\ \hbar\omega_0$ = 58 $A^{-1/3}$ MeV

for the excitation energy of the collective state (= ISGQR)

In stable nuclei the estimate based on the above h-o potential model works well, because most one-particle levels in the major shell ($N_i + 2$) are narrow resonances in realistic potentials.



In the schematic harmonic oscillator model

$$\langle N_f | x^2, y^2, z^2 | N_i \rangle \neq 0$$

only for $N_f = N_i \pm 2$ or $N_f = N_i$

Using (7.1),

$$S(IS,\lambda=2)_{class} = \frac{50}{4\pi}\frac{\hbar^2}{2m}A\langle r^2\rangle$$

Energy Weighted Sum Rule (EWSR)

The classical sum-rule for IS giant resonances should work when the interaction is $\cancel{\vec{p}_k}$

In the harmonic-oscillator potential model,

$\langle N_f | x^2, y^2, z^2 | N_i \rangle \neq 0$ only for $N_f = N_i$ and $N_f = N_i \pm 2$

Thus,

in contrast to IVGDR, the quadrupole operator has $N \rightarrow N$ matrix elements, in addition to $N \rightarrow N+2$ matrix elements. And, the IS (attractive) coupling between the two kinds of modes, $\Delta N = 0$ and 2, shifts some transition strength to lower-energy modes.

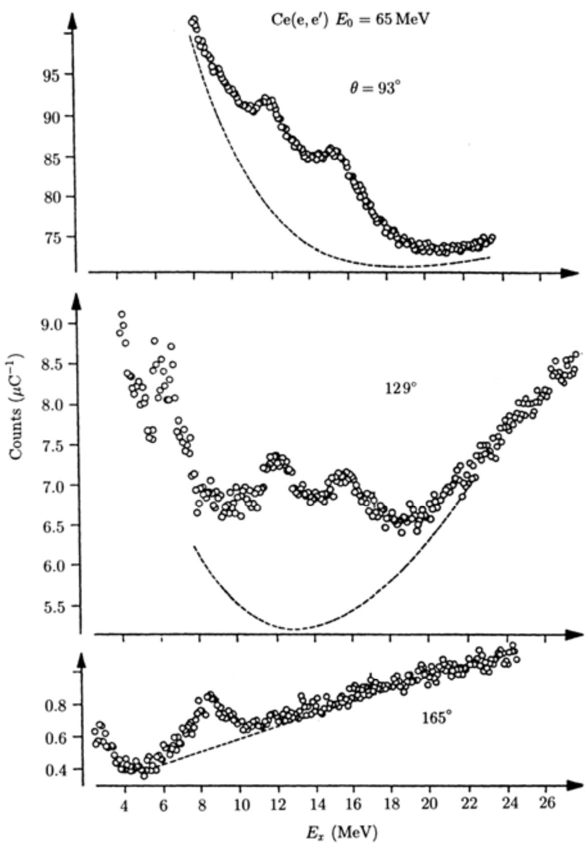
In open-shell nuclei the $N \rightarrow N$ transitions are possible within the last filled major shell, while in medium-heavy nuclei the transitions are present even in the closed-shell nuclei, due to the presence of the spin-orbit splitting.
For example, in the doubly-closed shell nucleus $_{82}Pb_{126}$ one finds 4 low-energy excitations; 2 proton-excitations, $1h_{11/2} \rightarrow 1h_{9/2}$, $2f_{7/2}$ and 2 neutron-excitations, $1i_{13/2} \rightarrow 1i_{11/2}$, $2g_{9/2}$.

Nevertheless, since the sum-rule considered here is the energy-weighted sum-rule, the observed IS lower-lying quadrupole excitations exhaust only up till 15 percent of EWSR.

Some summary of the observed properties of ISGQR of medium-heavy nuclei

M.N.Harakeh & A. van der Woude, *Giant Resonances, 2001*

Observation of the IS giant quadrupole resonance (ISGQR)
- one of the first observations of a giant resonance other than the well-known IVGDR



R.Pitthan, Z. Phys. **260** (1973) 283

Table 4.4 *ISGQR parameters.*

Nucleus	E_x (MeV)	Γ (MeV)	EWSR (%)	Reference	$E_x A^{1/3}$ (MeV)
^{90}Zr					
	14.05 ± 0.25	4.0 ± 0.25	49	(BUE84)	63
	14.0 ± 0.2	3.4 ± 0.2	66 ± 17	(YOU81)	63
	14.0 ± 0.4	3.0 ± 0.5	111 ± 25	(BOR89)	63
^{112}Sn					
	13.65 ± 0.2	3.6 ± 0.2	55	(BUE84)	66
	13.51 ± 0.13	3.15 ± 0.23	123 ± 26	(SHA88)	63
^{116}Sn					
	13.15 ± 0.25	3.6 ± 0.3	60	(BUE84)	64
	13.2 ± 0.3	3.3 ± 0.2	84 ± 25	(YOU81)	64
	13.39 ± 0.14	2.94 ± 0.31	134 ± 28	(SHA88)	65
^{120}Sn					
	12.75 ± 0.25	3.75 ± 0.3	82	(BUE84)	63
	12.7 ± 0.4	3.5 ± 0.4	80	(YOU81)	63
	13.24 ± 0.13	2.88 ± 0.20	135 ± 27	(SHA88)	65
^{124}Sn					
	12.35 ± 0.25	3.6 ± 0.3	88	(BUE84)	61
	12.3 ± 0.4	3.1 ± 0.3	78 ± 25	(YOU81)	61
	13.02 ± 0.13	2.80 ± 0.30	127 ± 31	(SHA88)	65
fit 1	12.7 ± 0.35	3.4 ± 0.5	190 ± 75	(BOR90)	63
fit 2	12.3 ± 0.35	3.8 ± 0.5	225 ± 75	(BOR90)	61
^{144}Sm					
	12.25 ± 0.2	2.5 ± 0.2	50	(BUE84)	64
	12.2 ± 0.2	2.4 ± 0.2	45 ± 15	(YOU81)	64
	12.70 ± 0.14	2.62 ± 0.20	123 ± 29	(SHA88)	66
^{150}Sm					
	12.3 ± 0.2	3 ± 0.2	76	(BUE84)	65
	12.75 ± 0.17	2.85 ± 0.36	132 ± 50	(SHA88)	68
^{152}Sm					
	11.95 ± 0.2	3 ± 0.2	81	(BUE84)	64
	12.78 ± 0.17	3.63 ± 0.42	183 ± 50	(SHA88)	68
^{208}Pb					
	10.60 ± 0.25	2.8 ± 0.25	100	(BUE84)	63
	11 ± 0.2	2.7 ± 0.3	105 ± 25	(YOU81)	65
	10.9 ± 0.3	3.1 ± 0.3	120–170	(BRA85)	64
	11.0 ± 0.3	3.3 ± 0.3	100–150	(BRA85)	65

Experimental information on isovector giant quadrupole resonance (IVGQR) is very limited.

The reason for this can be ;

(a) Due to the high frequency mode, large background and possible overlap with many other excitations;
(b) Large width and relatively small excitation cross section;
(c) Lack of a selective experimental tool to excite IVGQR

Some experimental evidence :
D.Sims et al., Phys.Rev.**C55** (1997) 1288; interference (E1/E2) effects in reactions involving photons.
T.Ichihara et al., Phys.Rev.Lett. **89** (2002) 142501; ^{60}Ni (^{13}C, ^{13}N) ^{60}Co reaction

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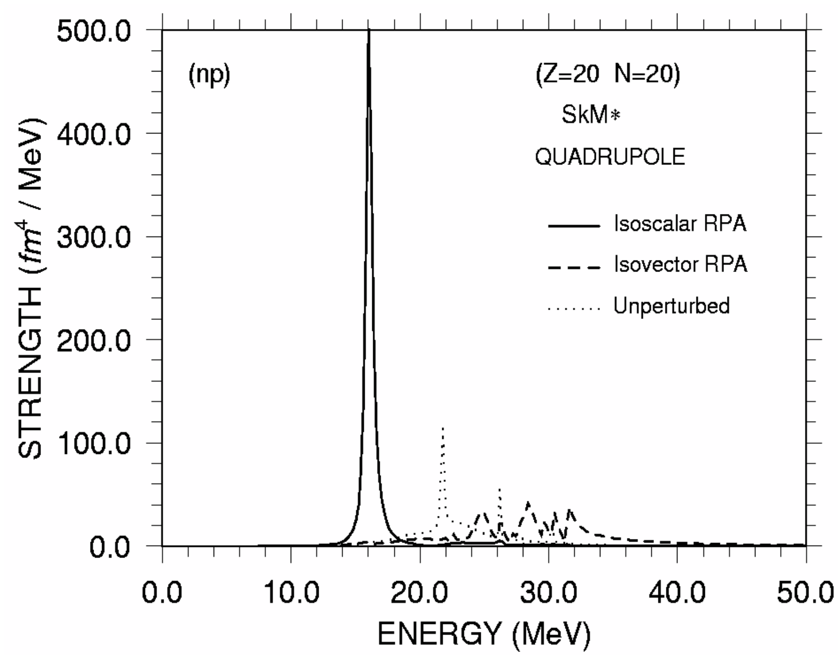
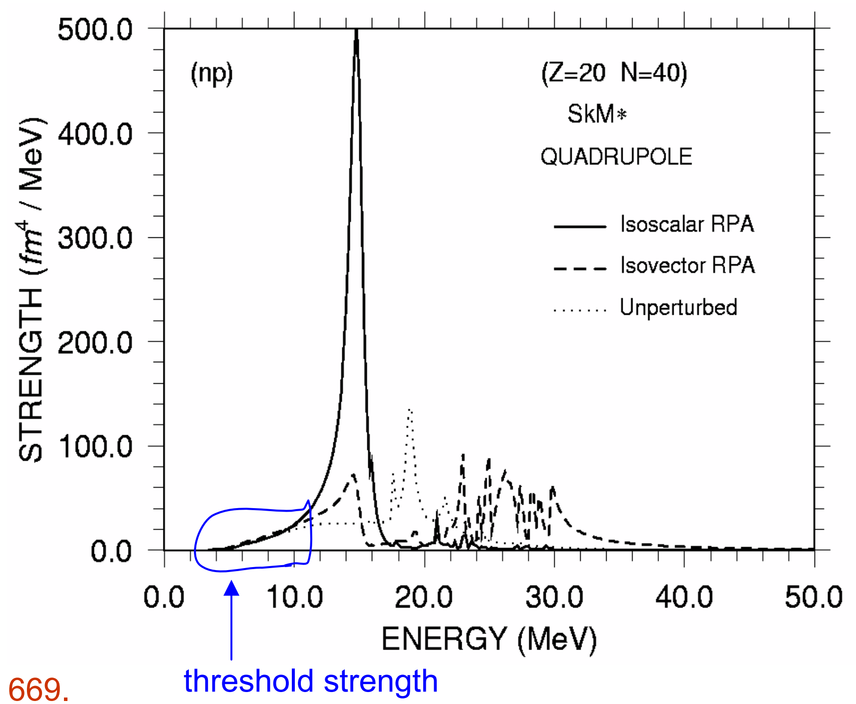
For reference, the result of a self-consistent HF+RPA calculation is shown below.

$_{20}Ca_{20}$ is a stable nucleus, while $_{20}Ca_{40}$ is possibly a neutron-drip-line nucleus.

In both nuclei ISGQR appears as a clean collective peak, while IVGQR spreads over several peaks with varying form factors. The 'threshold strength' in ^{60}Ca comes from the presence of weakly-bound neutrons in the ground state, which are not present in stable nuclei.



I.Hamamoto, H.Sagawa and X.Z.Zhang, Nucl.Phys. **A626** (1997) 669.



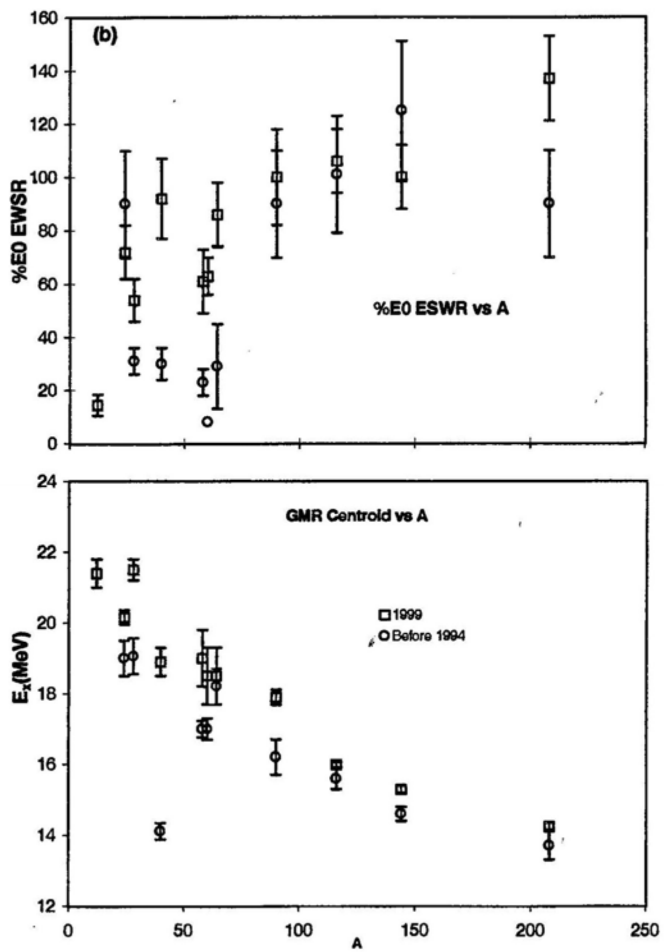
[p-h energies, $82A^{-1/3}$ MeV $\longrightarrow$ collective ISGQR at $58A^{-1/3}$ MeV,
which consumes the major part of IS quadrupole strength.

due to the attractive p-h interaction]

means ; ISGQR makes a considerable amount of positive contribution to $e_{pol}(E2)$ of low-energy E2 transitions.

7.3.3. Isoscalar giant monopole resonance (ISGMR) - compression mode

In ^{208}Pb , observed ISGMR ($E_{ISGMR} \approx 14$ MeV, $\Gamma \approx 3$ MeV) exhausts about 100 % of the energy-weighted sum rule.



Observed properties of ISGMR

D.H.Youngblood, H.L.Clark and Y.W.Lui, RIKEN Review **No.23** (July, 1999) 159.

Examples of experimental data of ISGMR

S.Shlomo and D.H.Youngblood, PRC 47, 529 (1993)

TABLE III. GMR parameters used for compressibility fits.

		Corrected[a] for systematic difference		Values adopted for calculations				
A	Nucleus	E_x (MeV)	$\sigma(E_x)$ (MeV)	E_x (MeV)	$\sigma(E_x)$ (MeV)	Γ (MeV)	$\sigma(\Gamma)$ (MeV)	Ref.[c]
24	Mg	16.71	0.23	16.71		4.73		Gron1
28	Si[b]	19.06	0.50	19.06	0.50	6.30	0.50	TAMU1
40	Ca	14.11	0.23	14.11	0.23			Gron2
58	Ni	17.00	0.40					TAMU2
58	Ni	17.08	0.23	17.06	0.20	3.28	0.18	Gren1
64	Zn	18.20	0.50	18.20	0.50	4.30	0.90	TAMU3
66	Zn	18.40	0.70	18.40	0.70	4.10	1.10	TAMU3
90	Zr	16.20	0.50					TAMU3
90	Zr	15.81	0.36	15.95	0.29	3.29	0.20	Gron3
92	Mo	15.98	0.23	15.98	0.23	4.80	0.30	Gren1
112	Sn	15.70	0.30					TAMU4
112	Sn	15.59	0.27	15.64	0.20	3.67	0.19	Gron4
114	Sn	15.51	0.27	15.51	0.27	3.52	0.29	Gron4
116	Sn	15.60	0.30					TAMU3
116	Sn	15.40	0.28	15.50	0.20	3.96	0.24	Gron4
118	Sn	15.50	0.60	15.50	0.60	4.10	0.70	TAMU3
120	Sn	15.20	0.50					TAMU3
120	Sn	15.23	0.27					Gron4
120	Sn	15.18	0.41	15.21	0.21	3.98	0.21	Gren1
124	Sn	14.80	0.40					TAMU3
124	Sn	15.06	0.28	14.98	0.23	3.50	0.30	Gron4
142	Nd	14.80	0.30	14.80	0.30	3.30	0.20	TAMU5
144	Sm	14.60	0.20					TAMU3
144	Sm	14.84	0.27	14.69	0.16	3.23	0.17	Gron4
146	Nd	15.10	0.20	15.10	0.30	3.30	0.30	TAMU5
148	Sm	14.60	0.20					TAMU6
148	Sm	14.66	0.27	14.62	0.16	3.08	0.23	Gron4
150	Nd	15.40	0.30	15.40	0.30	3.40	0.20	TAMU5
150	Sm	14.68	0.29	14.68	0.29	2.86	0.50	Gron4
152	Sm	15.27	0.29	15.27	0.29	3.13	0.52	Gron4
154	Sm	14.90	0.30	14.90	0.30	2.60	0.40	TAMU3
208	Pb	13.70	0.40					TAMU3
208	Pb	13.63	0.38					Gron5
208	Pb	13.80	0.30	13.73	0.20	2.58	0.20	Juli
232	Th	13.80	0.40	13.80	0.40	3.00	0.50	Juli
238	U	13.70	0.40	13.70	0.40	3.00	0.50	Juli

Measured energy of ISGMR (= "breathing mode"), E_{ISGMR} ,

$\rightarrow$ information on the compressibility of nuclear matter (K_{nm}).

Nuclear compressibility is an important information on the equation of state of nuclear matter.

ex. shape of the density distribution, values of the radii, the strength of shock wave following the collapse of supernovae, etc.

However, the relation, $E_{ISGMR} \leftrightarrow K_{nm}$ is model-dependent !

K_{nm} is defined by

$$K_{nm} \equiv 9\rho_0^2 \frac{d^2(E/A)}{d\rho^2}\Big|_{\rho=\rho_0}$$

An effective compression modulus, K_A , for a nucleus with mass number A in terms of $E_{ISGMR}(A)$ is defined by

$$K_A = \frac{m(E_{ISGMR}(A))^2 \langle r^2 \rangle_m}{\hbar^2}$$

where $\langle r^2 \rangle_m$ is the mean-square mass radius.

Writing

$$K_A = K_{vol} + K_{surf} A^{-1/3} + K_{sym} \left(\frac{N-Z}{A} \right)^2 + K_{Coul} \frac{Z^2}{A^{4/3}} + \cdots\cdots \qquad (\$)$$

$$\begin{cases} \lim\limits_{A\to\infty} K_A = K_{vol} = K_{nm} & \text{if the mode corresponds to a radial scaling of the ground-state density.} \\ \lim\limits_{A\to\infty} K_A = (7/10) K_{nm} & \text{from a Hartree-Fock calculation with a constraint on the r.m.s. radius.} \end{cases}$$

Moreover, various K_i values in ($) are poorly determined, since the variations of K_A with N and Z are very small for available nuclei.

Thus, some experts state (for example, Blaizot et al., NP**A 591** (1995) 435) :

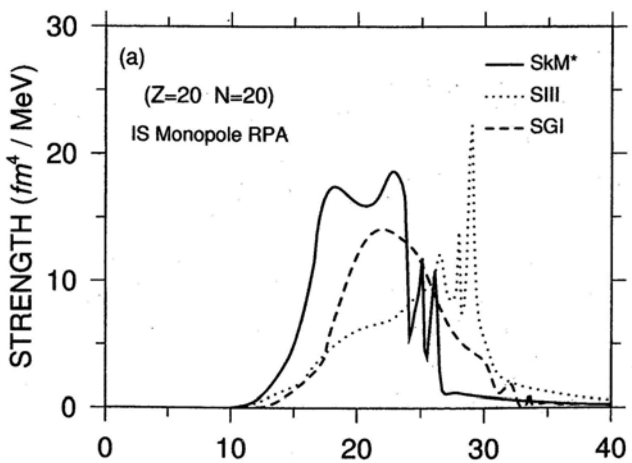
Phenomenological expansion ($) using measured $E_{ISGMR}(A)$ values cannot be used to obtain K_{nm} .
Microscopic calculations remain the most reliable tool for determining K_{nm} from measured $E_{ISGMR}(A)$ values.

K_{nm} = 210 ± 30 MeV

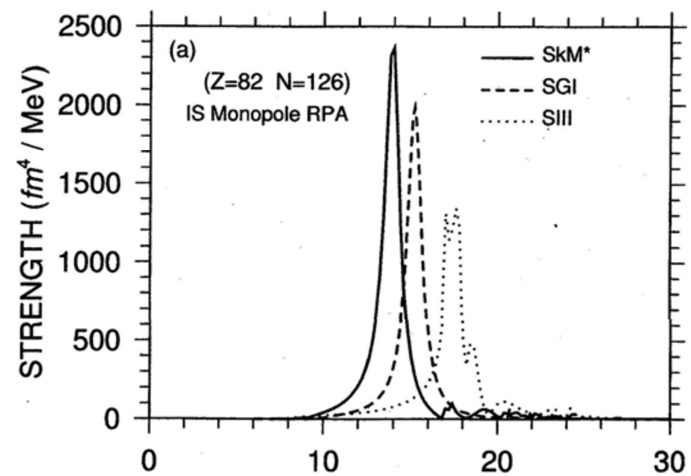
Comparison of calculated ISGMR using self-consistent Hartree-Fock calculations plus RPA with various Skyrme interactions, which have different K_{nm} values. K_{nm} = 217, 256 and 355 MeV for SkM*, SGI and SIII, respectively.

I.H., H.Sagawa & X.Z.Zhang, PR**C 56**, 3121 (1997)

$^{40}_{20}Ca_{20}$



$^{208}_{82}Pb_{126}$



Calculated ISGMR in medium weight and light nuclei usually does not have a clean one-peak shape.

Calculated ISGMR in heavy nuclei is obtained as a well defined resonance and exhausts the sum rule.

(In the above calculation the particle decay width of GR is fully taken into account, while the spreading width, coming from the coupling to 2p-2h configurations, is not included.)

The effective charge of E0 transitions, $e_{eff}(E0)$, for low-energy E0 transitions has not really been studied.

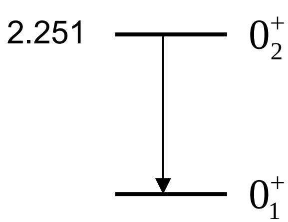
In heavier nuclei self-consistent calculations plus RPA produce a relatively clean resonance peak. Nevertheless, the calculated peak energy is not so different from averaged unperturbed p-h energies in the potential based on harmonic-oscillator.

$\rightarrow$ Calculated values of *E0* polarization charge, $e_{pol}(E0)$, for low-energy *E0* transitions due to ISGMR may not be large and may depend sensitively on the models and parameters used.

---

ex. A recent information on $e^{n}_{eff}(E0)$ from the data on ${}^{12}_{4}Be_{8}$

S.Shimoura et al., Phys. Lett. **B654** (2007) 87;
I.H. and S.Shimoura, J. of Phys. **G34** (2007) 2715.



Measured partial life $\tau(0_2^+ \rightarrow 0_{g.s.}^+)$ = 402 ± 16 ns

$\rightarrow$ $\langle 0_2^+ | e^n_{eff}(E0) r^2 | 0_1^+ \rangle$ = 0.87 e fm²

$\rightarrow$ $e^n_{eff}(E0) = e^n_{pol}(E0) = 0.076\, e$

The presence of weakly-bound neutrons in the deformed potential is duly taken into account.

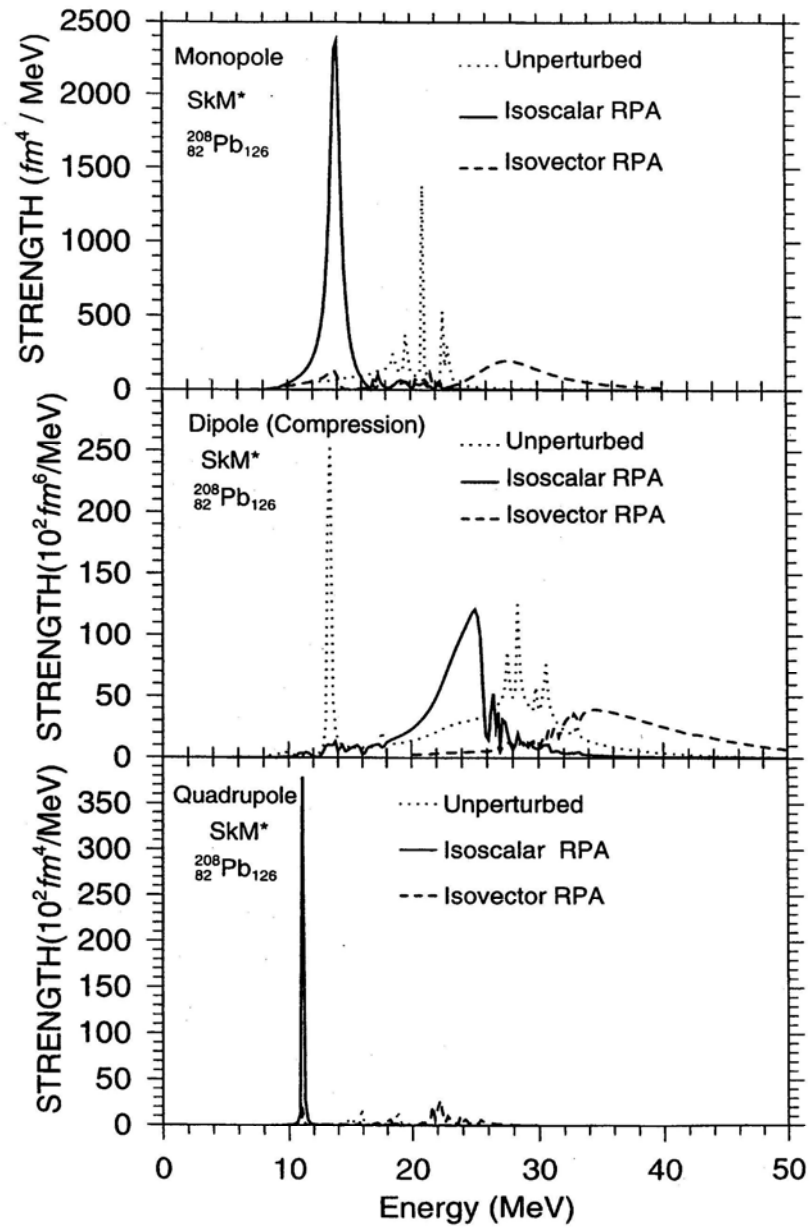
OBS. The polarization charge for *E0* transitions obtained from subtracting the center of mass motion is analogous to that of E2 transitions described in Sect.7.2.1. and is

$$e^n_{eff}(E0) = (Z/A^2)e = (0.028)\, e \quad \text{for} \quad {}^{12}Be$$

Comparison of IS and IVGR in ^{208}Pb calculated by self-consistent Hartree-Fock plus RPA using SKM* interaction.

I.Hamamoto., H.sagawa and X.Z.Zhang, J.Phys.G **24** (1998)1417.

> Particle decay width is fully taken into account, though spreading width coming from the coupling to 2p-2h configurations is not included.



r^2

$r^3\, Y_{1\mu}$

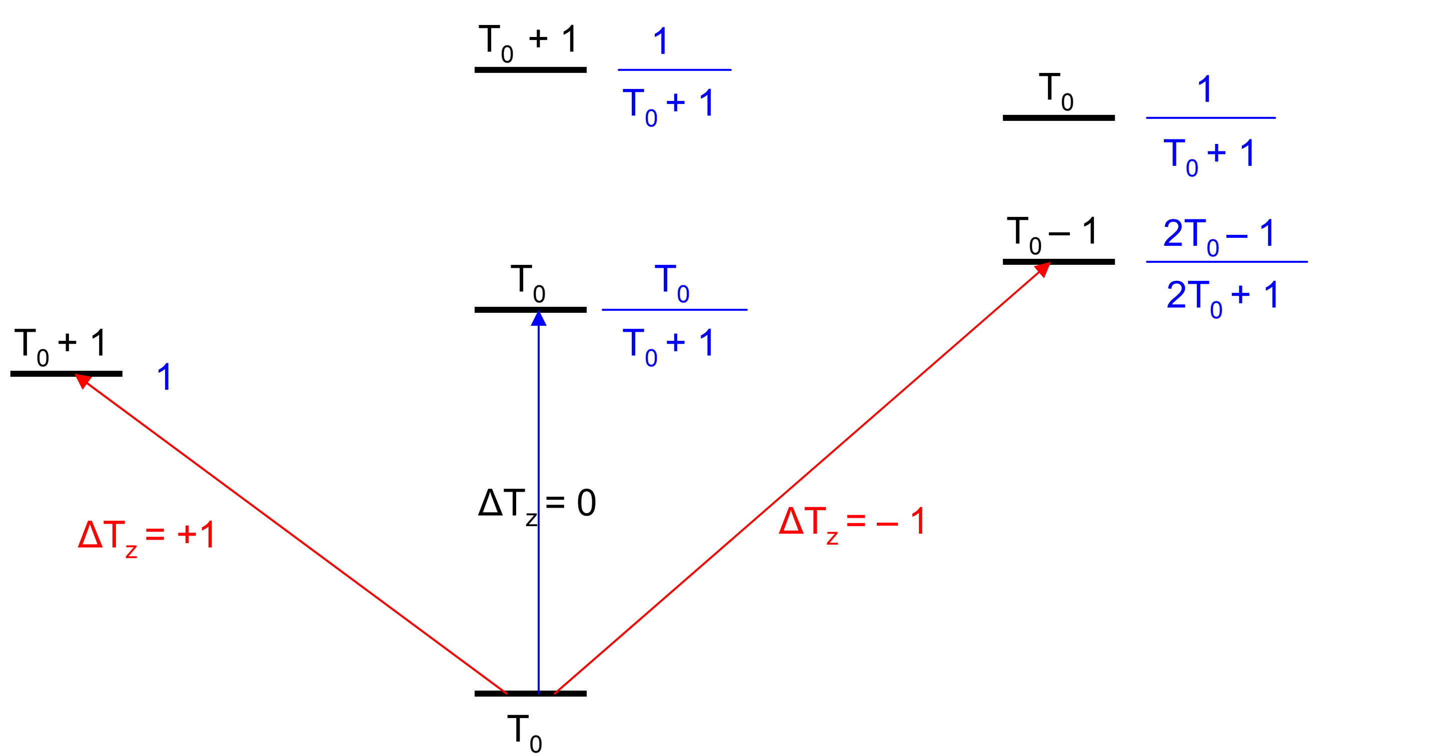
$r^2\, Y_{2\mu}$

Figure 3. Unperturbed and RPA response functions of ^{208}Pb to monopole (r^2Y_{00} and $\tau_z r^2 Y_{00}$ for the IS and IV operator, respectively), compression dipole (r^3Y_{10} and $\tau_z r^3 Y_{10}$) and quadrupole (r^2Y_{20} and $\tau_z r^2 Y_{20}$) operators. The SkM* interaction is used both in HF and RPA.

7.4. Giant Resonances of charge-exchange type ($\Delta T_z = \pm 1$)

Various isospin states, which can be excited by acting an isovector excitation operator on a nucleus with $T = T_z \neq 0$.

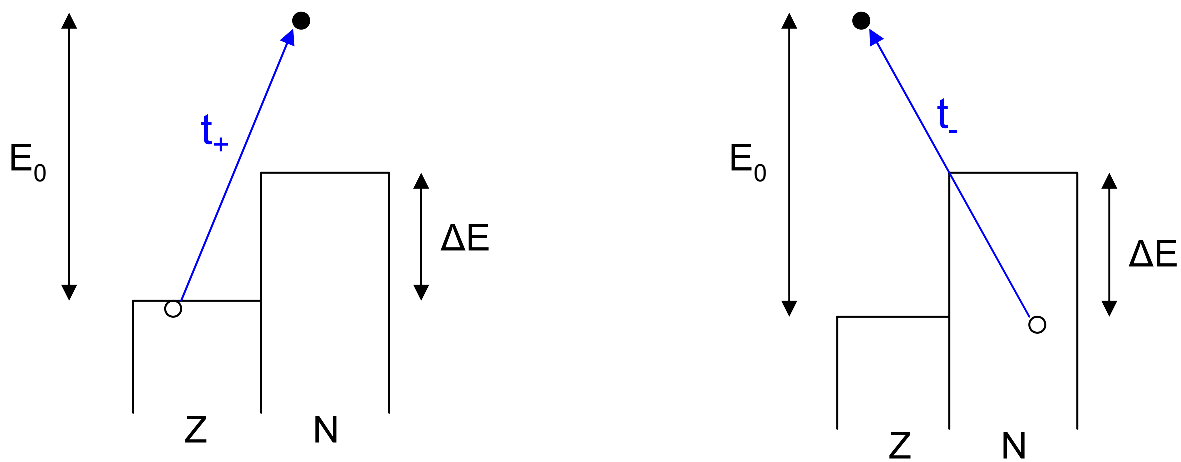
Excitation strengths $\propto$ $[\, C(T_0\, 1\, T\, ;\, T_0\, ,\, \Delta T_z\, ,\, T_0 + \Delta T_z)\,]^2$



Z–1, N+1	$Z \neq N$	Z+1, N–1
$T_z = \frac{N - Z}{2} + 1$	$T_0 = T_z = \frac{N - Z}{2}$	$T_z = \frac{N - Z}{2} - 1$

$\boxed{E_X(t_+ \, GR) < E_X(t_- \, GR)}$ in the presence of neutron excess

p-h excitation energy E_X measured from the ground state of mother nuclei



$E_X(t_+) = E_0 - \Delta E$ $E_X(t_-) = E_0 + \Delta E$

$$\begin{cases} E_0 \sim \alpha \hbar \omega_0 \sim A^{-1/3} \\ \Delta E \sim A^{2/3} \text{ for neutron-excess in stable neutron-rich nuclei} \end{cases}$$

$\therefore$ $E_X(t_+)$ becomes monotonically smaller as $A \rightarrow$ larger.

This relation, $E_X(t_+ \, GR) < E_X(t_- \, GR)$, is present in all charge-exchange ($t_\pm$) GRs.

In N>Z nuclei towards neutron-drip-line

$E_x(t_+ \, GR) \ll E_x(t_- \, GR)$ E_x in respective final nuclei

Some expected features unique in spin-isospin ($\sigma_\mu t_\pm$) Giant Resonances

1) [$t_\pm \sigma_\mu$] $\longrightarrow$ Not almost all strength under the GR peak.

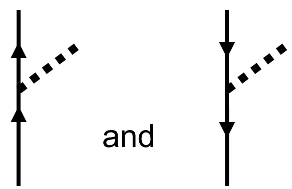
Instead, a considerable amount of high-energy tail above the peak is expected, with the tensor correlation responsible for the highest energy components.

Dependence of the high-energy tail on respective GRs ?

2) [σ_μ] $\longrightarrow$ Relatively large width (or large spread) of GR

$\because$) a) Unperturbed 1p-1h excitations have already an energy spread of $2\ \Delta E_{\ell s}$
where the spin-orbit splitting of high-j orbit is expressed by $\Delta E_{\ell s}$ ($\approx$ 7-9 MeV),
except for GTGR and some IVSGDR where the spread $\approx\ \Delta E_{\ell s}$

b) Due to the same sign of the couplings to a particle and a hole;
the coupling of 1p-1h to 2p-2h configurations is strong,
in contrast to spin-independent modes.



Examples of charge-exchange Giant Resonances studied in β-stable nuclei

spin-isospin modes

* compression mode

	spin-parity	operator $(\hat{O}_{\pm})$
IAS	0+	$\sum_k t_{\pm}(k)$
GT GR	1+	$\sum_k t_{\pm}(k)\vec{\sigma}_k$
IV GQR	2+	$\sum_k t_{\pm}(k) r_k^2 Y_{2\mu}(\hat{r}_k)$
IV spin GMR*	1+	$\sum_k t_{\pm}(k)\vec{\sigma}_k(r_k^2 - \langle r^2 \rangle_{excess})$
IV spin GDR	0–, 1–, 2–	$\sum_k t_{\pm}(k) r_k (Y_1(\hat{r}_k)\vec{\sigma}_k)_{J\pi}$

Direct and systematic experimental data are available only for IAS and GTGR.

IAS = Isobaric Analogue State

IV = IsoVector

GMR = Giant Monopole Resonance

GDR = Giant Dipole Resonance

Allowed β decay; $\sum_k^A t_\pm(k) \equiv F_\pm$ Fermi transitions : $(\ell, j)_p \longleftrightarrow (\ell, j)_n$

$\sum_k^A \vec{\sigma}(k) t_\pm(k) \equiv GT_\pm$ Gamow-Teller transitions :

$$\left(\ell, j=\ell\pm\frac{1}{2}\right)_p \longleftrightarrow \left(\ell, j=\ell\pm\frac{1}{2}\right)_n \text{ and } \left(\ell, j=\ell\mp\frac{1}{2}\right)_n$$

Isospin of nucleon, $t=\frac{1}{2}$ $\quad t_z|n\rangle = \frac{1}{2}|n\rangle$ $\quad t_z|p\rangle = -\frac{1}{2}|p\rangle$

$t_-|n\rangle = |p\rangle$ $\quad t_+|p\rangle = |n\rangle$ $\quad t_+|n\rangle = 0$ $\quad t_-|p\rangle = 0$

ex. In the L-S doubly-closed-shell $N=Z$ nucleus, ${}^{40}_{20}Ca_{20}$, one expects

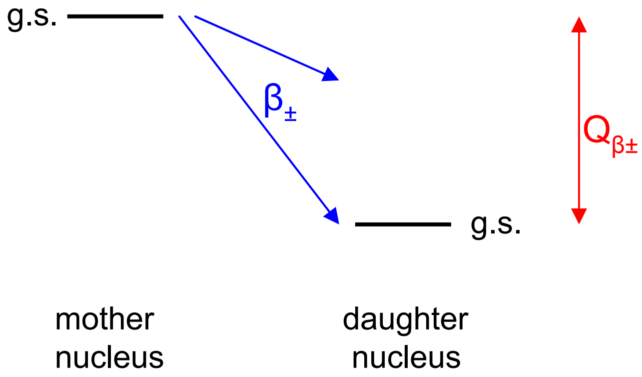
$\sum_k^A t_\pm(k)|gr\rangle \approx 0$ and $\sum_k^A \sigma_\mu(k) t_\pm(k)|gr\rangle \approx 0$

$F_\pm$ operators are raising and lowering operators of the z-component of total isospin (T) without changing the total isospin, $\Delta T=0$;

$\sum_k^A t_\pm(k) = T_\pm$ $\quad T_\pm|T, T_z\rangle = |T, T_z \pm 1\rangle$ $\quad$ In particular, $T_\pm|T=T_z=0\rangle = 0$

$GT_\pm$ operators may change the total isospin, $\Delta T = -1, 0, +1$, but $\cancel{T=0 \rightarrow T=0}$

β-decay can populate only the states with $Ex \le Q_{\beta\pm}$ in daughter nuclei.



That means, in β-stable nuclei
β-decays of ground states can populate only
the low-energy tail of GTGR in daughter nuclei.
Thus, those β-decays are considerably hindered.

In contrast, using charge-exchange reactions on mother nuclei,

(p, n), (^{3}He, t) for t_- ($n \to p$ in target nuclei)

(n, p), (d, ^{2}He), (t, ^{3}He) for t_+ ($p \to n$ in target nuclei)

the response is obtained up till high excitation energy in daughter nuclei.
The price which one must pay is ; the analysis of data to obtain nuclear matrix elements is much more complicated than in β decays.
In those charge-exchange reactions, Gamow-Teller Giant Resonance (GTGR) was found !

7.4.1. Fermi transitions ; $(F_{\pm} =)\ \hat{O}_{\pm} = \sum_k t_{\pm}(k)$

spin-parity of the operator = 0^+

$$\sum_m |\langle m|\hat{O}_-|0\rangle|^2 - \sum_n |\langle n|\hat{O}_+|0\rangle|^2$$

$$(\equiv S_- - S_+) = (N - Z)$$

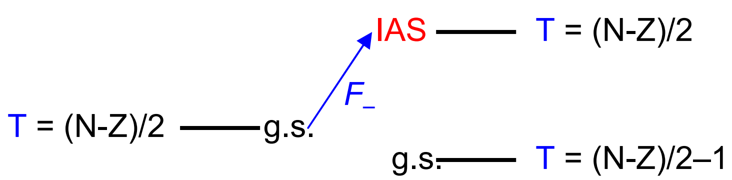
The sum rule for Fermi transitions is usually exhausted by the transition to the Isobaric Analogue State (IAS), which has a very narrow width.

$$|IAS\rangle = T_{\pm}|0\rangle$$

That means, Isospin is a good quantum number, in general, in both light nuclei and medium-heavy nuclei with neutron excess.

Isospin of the ground state is maximum broken for N=Z nuclei with $Z \rightarrow$ large.

ex. For N>Z



(N,Z)　　(N-1, Z+1)

T_z = (N-Z)/2　　T_z = (N-Z)/2–1

In this example $F_+|T = T_z = (N-Z)/2\rangle = 0$

$\because$) $|T = (N-Z)/2\rangle$ cannot have $T_z = (N-Z)/2+1$ component.

$$\therefore \quad S_- - \cancel{S_+} = N - Z$$

7.4.2. Gamow-Teller resonance ; (GT$_\pm$ =) $\hat{O}_{\pm} = \sum_k t_{\pm}(k)\sigma_{\mu}(k)$

spin-parity of the operator = 1^+

$$\sum_{\mu=1}^{3}\sum_{m} |\langle m|\hat{O}_{-}|0\rangle|^2 - \sum_{\mu=1}^{3}\sum_{n} |\langle n|\hat{O}_{+}|0\rangle|^2 \quad (\equiv S_{-} - S_{+}) \; = \; 3(N-Z)$$

Some experimental observation

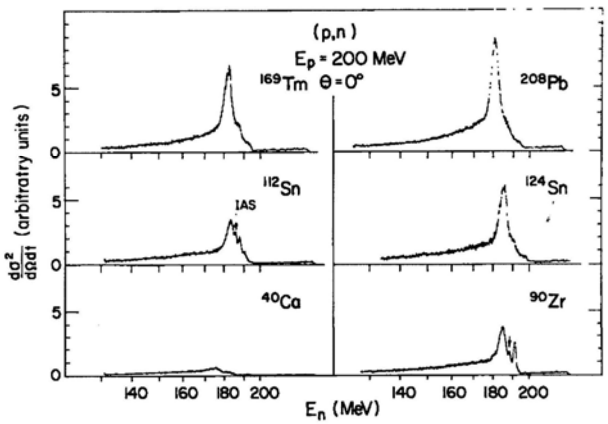
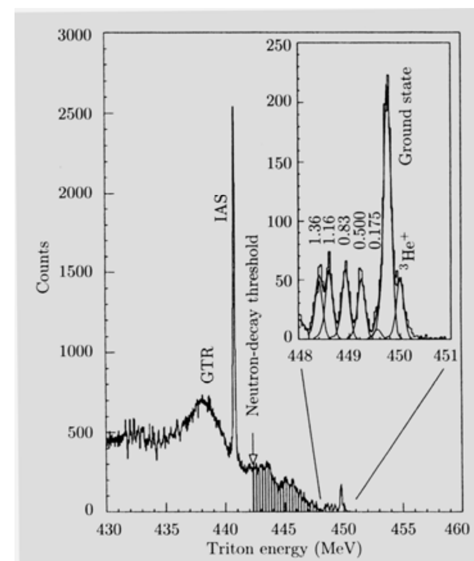


Fig. 5. Neutron t.o.f. spectra at 200 MeV and $\theta = 0°$ for the indicated targets. The spectra are normalized to show relative cross sections.

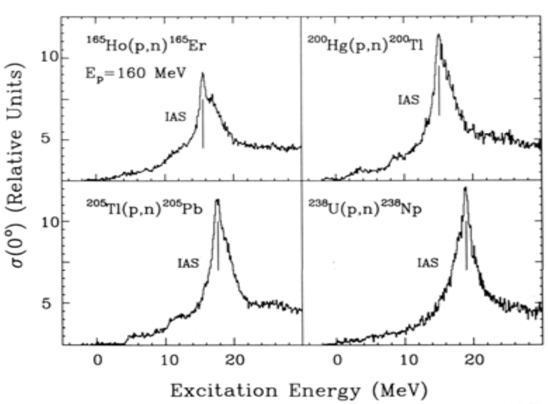
C.Gaarde et al., Nucl.Phys.A369 (1981) 258.

In order to observe GTGR, the incident energy of proton or ^{3}He beams must be chosen carefully. (The population of spin-isospin modes relative to excitations of other types depends on the incident energy.)



The 0° ^{71}Ga(^{3}He,t)^{71}Ge spectrum at 450 MeV.

M.Fujiwara et al., Nucl.Phys.A599 (1996)223c.



J.Rapaport and E.Sugarbaker, Ann.Rev.Nucl.Part.Sci., 44 (1994) 109.

ex. Observed properties of IAS and GTGR in $^{208}_{82}Pb_{126}\,(^3He,t)\,^{208}_{83}Bi_{125}$ with $E(^3He)$ = 450 MeV

H.Akimune et al., PRC **52,** 604 (1995).

	(T = 22) "IAS"	(T ≈ 21) "GTGR"
Ex (MeV)	18.8	19.2
Width (MeV)	0.232	3.7
Sum rule (%)	100	~ 60

"IAS" = T_- |gr of ^{208}Pb>

"GTGR" = GT_- |gr of ^{208}Pb>

From the (^{3}He,t) reaction; only the GTGR peak region is included and $S_+ = 0$ was assumed due to Pauli blocking.

Missing (GT) _ strength used to be a problem in 1980s.

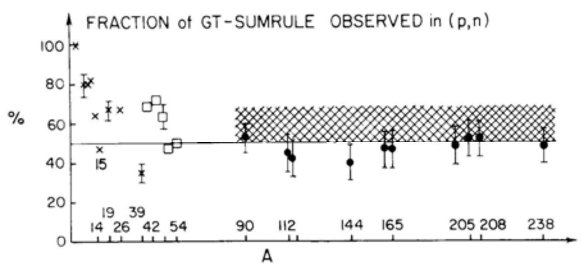


FIGURE 4
Fraction of Gamow-Teller sumrule strength observed in (p,n) reactions. Three different regions in A are discussed in the text. In the p- and sd-shell the strength is most often in a few sharp states. In the fp-shell a multipole decomposition is attempted. For heavier nuclei the dots (with error bars) represent strengths in peaks (low lying + giant), whereas the cross hatched region also includes strength under the collective state. Possible strength above (larger E_x) the collective state is not included.

C.Gaarde, Niels Bohr centennial Conf., 1985.

1) Back-ground subtraction problem ;
- broad GT bump is located on top of a continuum. Including this continuum or not makes a large difference in the extracted strength.
- GTGR has a clean resonance shape ?

2) S_+ may not be negligible even for medium-heavy nuclei.

3) Possible missing GT strength is carried by the excitation, [nucleon → Δ resonance at 1232 MeV] ?

The direct measurement of S_- and S_+ , performing both (p,n) and (n,p) reactions;

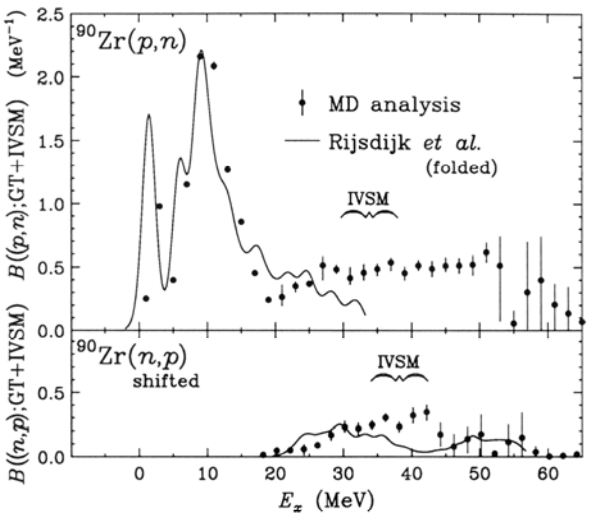


Fig. 3. GT plus IVSM strength distributions obtained by the MD analysis of the ^{90}Zr(p, n) and ^{90}Zr(n, p) reactions (in GT unit). The ^{90}Zr(n, p) spectrum is shifted by +18 MeV. The curves are taken from Ref. [29]. The energy regions of IVSM excitation are indicated by braces. See text for details.

K.Yako and H.Sakai et al., Phys.Lett. **B615** (2005) 193.

^{90}Zr (p,n) E_p = 295 MeV

^{90}Zr (n,p) E_n = 293 MeV

S_+ was carefully measured !

A multipole decomposition technique was applied to extract the GT component from the continuum.

IVSM = IsoVector Spin Monopole modes are expected around the place indicated.
Are IVSM_ or $IVSM_+$ modes populated in these reactions ?

GT quenching factor extracted from Ex < 50 MeV :

$$Q \equiv \frac{S_- - S_+}{3(N-Z)} = 0.88 \pm 0.06$$

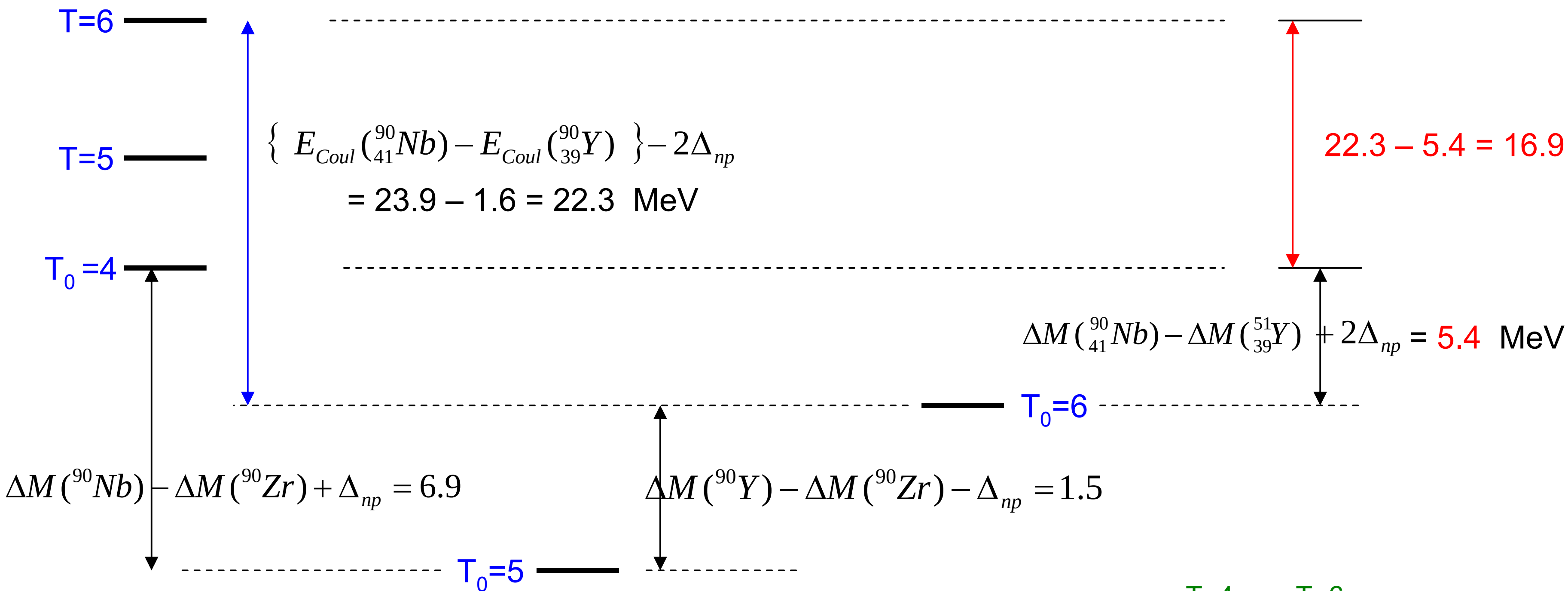
1) The coupling to non-nucleonic degrees of freedom (ex. Δ-resonance !?) in nuclei is presumably very small.

2) An appreciable amount of GT strength is found in the energy region much higher than the peak energy of GTGR.

A prediction by G.F.Bertsch and I.H., PRC **26** (1982) 1323 ;

Due to the spin-isospin character of GT operator, some unperturbed 1p-1h GT strength is shifted to the higher-lying (10-45 MeV) 2p-2h states, with the tensor correlation responsible for the highest energy components.

K.Yako and H.sakai et al., Phys.Lett. B615 (2005) 193.



${}^{90}_{41}Nb_{49}$	${}^{90}_{40}Zr_{50}$	${}^{90}_{39}Y_{51}$
($\Delta M = -82.66$)	($\Delta M = -88.77$)	($\Delta M = -86.49$)
(b.e.=777.006)	(b.e.=783.899)	(b.e.=782.400)

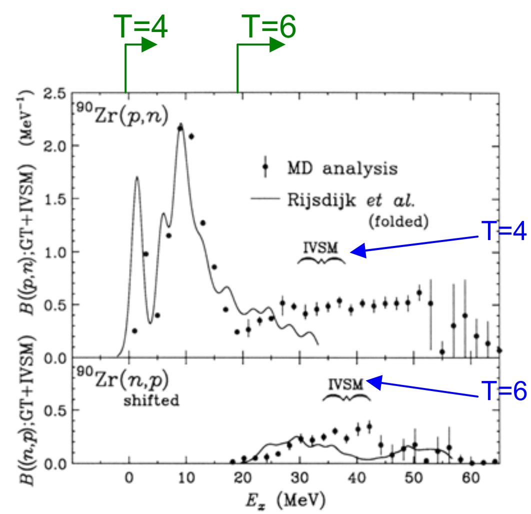


Fig. 3. GT plus IVSM strength distributions obtained by the MD analysis of the ^{90}Zr(p, n) and ^{90}Zr(n, p) reactions (in GT unit). The ^{90}Zr(n, p) spectrum is shifted by +18 MeV. The curves are taken from Ref. [29]. The energy regions of IVSM excitation are indicated by braces. See text for details.

1) The T=6 part of GTGR_ is only a fraction, 1/66, of the total GTGR_ strength.

2) All $GTGR_+$ strength in ^{90}Y has T=6, which is however expected to be very small.

3) The total strength S_+ of $IVSM_+$ (all with T=6) on ^{90}Zr is not small and about 70 % of that S_ of IVSM_ on ^{90}Zr .

$$\Delta_{np} \equiv [\Delta M(n) - \Delta M({}^1H)]c^2 = (m_n - m_p - m_e)c^2 = 0.78 \text{ MeV}$$

The energy of GTGR is pushed up from unperturbed (proton-hole) (neutron) or (proton) (neutron-hole) energies, due to the repulsive interaction in the $\vec{\sigma}\vec{\tau}$ channel.

$\Longrightarrow$ Effective GT operator, $(GT)_{eff} \approx (0.6 - 0.7)\ (GT)_{free}$

M1GR

Spin-dependent part of magnetic dipole (M1) operator is approximately

$$(M1) \propto \sum_{\mu=1}^{3} \sum_{k} t_z(k)\sigma_\mu(k) = [\Delta T_z = 0] \text{ part of GT operator, } (GT)_\pm = \sum_{\mu=1}^{3} \sum_{k} t_\pm(k)\sigma_\mu(k)$$

$\because$) $$O(M1,\mu) = \sqrt{\frac{3}{4\pi}}\frac{e\hbar}{2mc}\left(g_\ell \ell_\mu + g_s s_\mu\right) \qquad g_\ell = \begin{cases} 1 \\ 0 \end{cases} \qquad g_s = \begin{cases} 5.58 & \text{for proton} \\ -3.82 & \text{for neutron} \end{cases}$$

$$g_\ell \ell_\mu + g_s s_\mu = \frac{1}{2}\left(\ell_\mu + \underbrace{(g_s^p + g_s^n)}_{1.76} s_\mu\right) - \frac{1}{2}\left(\ell_\mu + \underbrace{(g_s^p - g_s^n)}_{9.40} s_\mu\right)\tau_z$$

Cf. In ${}^{12}_{6}C_6$ (S_p =15.96, S_n =18.72 MeV)
E_x (1^+, T=0) = 12.7, E_x (1^+, T=1) = 15.1 MeV

In heavy nuclei the strength of M1 GR is highly fragmented.

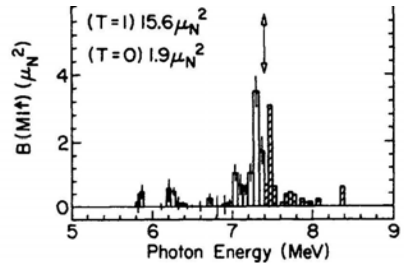
ex. ^{208}Pb (a j-j closed shell nucleus)

$$\begin{cases} \text{neutron :} & \left(i_{13/2}^{-1} i_{11/2}\right)_{1+} & \varepsilon_{p-h} = 5.57 \text{ MeV} \\ \text{proton :} & \left(h_{11/2}^{-1} h_{9/2}\right)_{1+} & \varepsilon_{p-h} = 5.85 \text{ MeV} \end{cases}$$

$\Longrightarrow$ Giant M1 resonance centered around 7.3 MeV, with a full width of about 1 MeV.

$\Longrightarrow$ $g_s^{eff} \approx (0.7)\ g_s^{free}$ for low-energy M1 transitions.

M1 strength for $E_x < S_n$ (= 7.37 MeV) measured by ${}^{208}Pb(\vec{\gamma},\gamma)$ using highly polarized tagged photons



R.M.Laszewski et al., PRL 61, (1988) 1710

7.4.3. IsoVector Spin Giant Monopole Resonance (IVSGMR) ; $\hat{O}_{\pm} = \sum_k t_{\pm}(k)\sigma_{\mu}(k) r_k^{\ 2}$

$$\sum_m |\langle m|\hat{O}_-|0\rangle|^2 - \sum_n |\langle n|\hat{O}_+|0\rangle|^2 \quad = \quad 3\left(N\langle r^4\rangle_n - Z\langle r^4\rangle_p\right)$$

spin-parity of the operator = 1^+

This IVSM operator has the same spin, isospin and parity as those of GT operator, though IVSM mode is a compression mode while GT is not.
Moreover, the GT strength extends to the continuum energy region much higher than that of the main peak, in the high energy region it may be experimentally difficult to differentiate IVSM strength from higher-lying GT strength.

Taking into account the orthogonality to GT operator, theoretically one needs to use

$$\hat{O}_{IVSM} = \sum_k t_{\pm}(k)\sigma_{\mu}(k)\left(r_k^{\ 2} - < r^2 >\right)$$

I.H. and H.Sagawa, PRC **62** (2000) 024319.

in order to obtain only the strength of IV Spin Monopole mode.

However, IVSM mode has a form factor quite different from that of GT transitions.

$\rightarrow$ $({}^3He, t)$ with appropriate incident energies may excite IVSMR more easily than *(p,n)* ?

The dependence of cross sections on incident energies or a comparison of (p,n) with (^{3}He,t) may differentiate the strength of IVSM from that of GT.

In nuclei with a larger neutron excess $\boxed{E_x(GR_-) > E_x(GR_+)}$

less (if not zero) GT_+ strength is expected due to Pauli blocking (namely, the neutron level in p→n by GT_+ operator is already occupied).

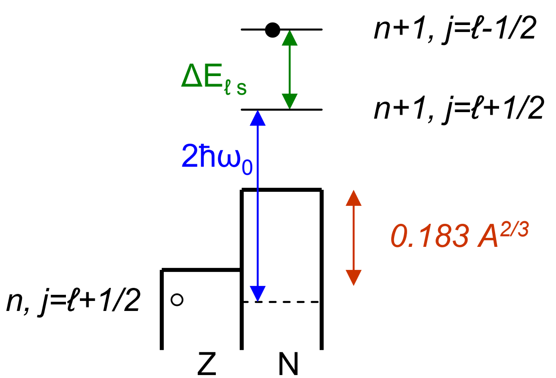
$\leftrightarrow$ Excitation energy of $IVSGMR_+$ in daughter nuclei becomes considerably lower, compared with that of $IVSGMR_-$ in daughter nuclei.

[Maximum energy of relevant p-h configurations estimated from the ground state of mother nuclei]

(The collective peak may appear just above the max p-h energy, when unperturbed p-h excitations are spread over a broad energy region, compared with the strength of relevant p-h interactions.)

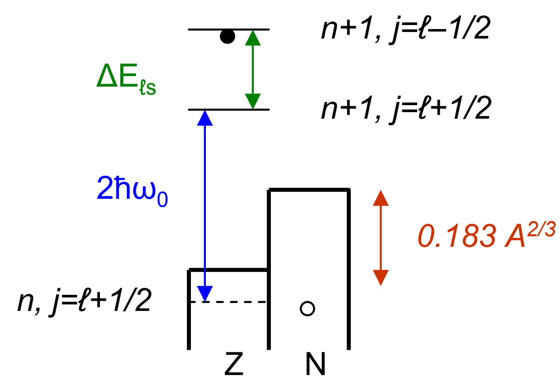
For stable nuclei $(N-Z)_{\beta-stable} \approx 6\times 10^{-3} A^{5/3} \Rightarrow (N_F^n - N_F^p)\hbar\omega_0 \approx 0.183 A^{2/3}$ MeV

$IVSM_+$ p-h excitations.



$$E_x(IVSM_+) = 2\hbar\omega_0 - 0.183A^{2/3} + \Delta E_{\ell s}$$

$IVSM_-$ p-h excitations.



$$E_x(IVSM_-) = 2\hbar\omega_0 + 0.183A^{2/3} + \Delta E_{\ell s}$$

$\boxed{\therefore\ [\,E_x(IVSGMR_-) - E_x(IVSGMR_+)\,] > 2 \times 0.183\, A^{2/3}}$,

since $IVSGMR_-$ is more collective than $IVSGMR_+$ due to the neutron excess.

$0.183A^{2/3} = 6.42$ MeV $\approx 1\hbar\omega_0$ for ^{208}Pb

The relation $[E_x(t_+ GR) < E_x(t_- GR)]$ in nuclei with neutron excess is valid for all types of $t_\pm$ GRs, though the actual energy difference depends also on the collectivity of modes.

In nuclei which are much more neutron-rich than β-stable nuclei, one has

1) $(N-Z) > 6\times10^{-3} A^{5/3}$

2) The ground state of t_+ daughter nuclei becomes much higher than that of mother nuclei.

Then, possible $IVSGMR_+$ may have even lower E_x in daughter nuclei.

Or, some appreciable 1^+ strength may be found at lower E_x , when GT_+ transitions should be forbidden.

One may try reactions such as *(n,p)* or *(t, ^{3}He)* on such neutron-rich nuclei in the inverse kinematics, and find out the lower-lying spin-dependent strength ?

Some comments:

1) Knowing that even the simplest compression mode, ISGMR, has not a simple resonance shape in the light-medium mass region, IVSM strength may not be concentrated on one collective resonance.

In the schematic harmonic oscillator model ;
unperturbed p-h excitations for ISGMR are totally degenerate at $2\hbar\omega_0$, while those for IVSGMR are spread over $2\hbar\omega_0 \pm \Delta E_{\ell s} \approx 80A^{-1/3} \pm 8$ MeV .

2) Similar to GTGR or the GT strength distribution, IVSGMR may have a considerable amount of strength tail at the energy higher than the major peak, since it is also a spin-isospin mode.

I.H. and H.sagawa, PRC **62**, 024319 (2000)

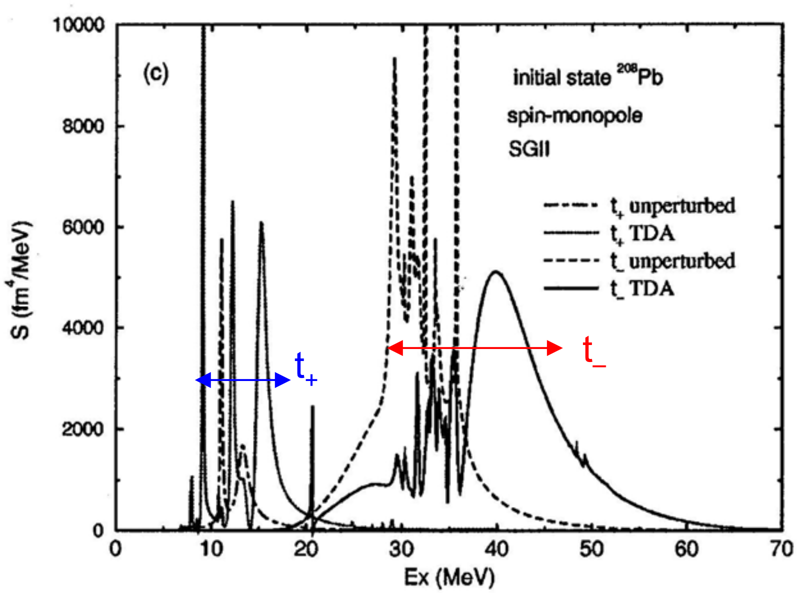
Ex. of calculated charge-exchange spin monopole ($t_\pm$ SMR) modes

$${}^{208}_{82}Pb_{126} \rightarrow {}^{208}_{81}Tl_{127}$$ t_+ mode

$${}^{208}_{82}Pb_{126} \rightarrow {}^{208}_{83}Bi_{125}$$ t_- mode

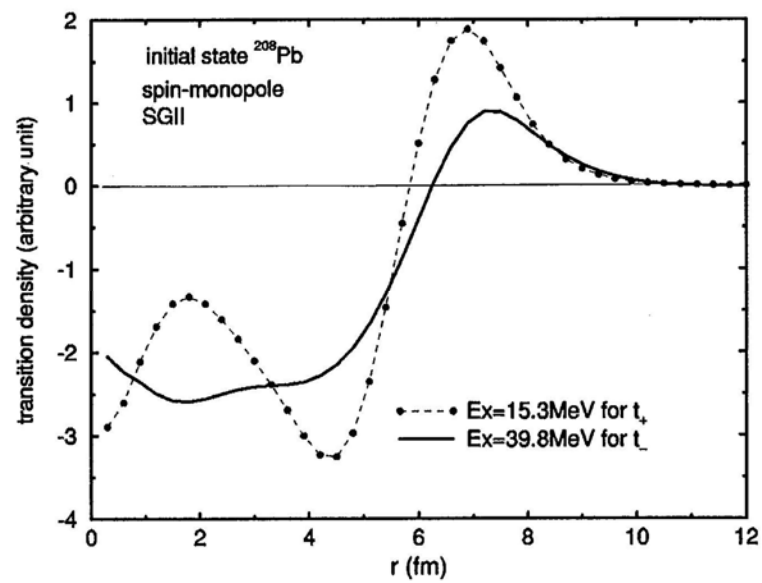
HF plus TDA with a Skyrme interaction

Response functions



E_x is measured from the ground state of the mother nucleus

Radial part of transition density of $IVSGMR_\pm$ (compression modes !)



Possible high-energy tail of the strength is not obtained in this kind of calculations (namely, [HF plus TDA] or [HF plus RPA]).

7.4.4. IsoVector Spin Giant Dipole Resonance (IVSGDR) ; $\hat{O}_{\pm} = \sum_k t_{\pm}(k) r_k \left(Y_1(\hat{r}_k) \otimes \vec{\sigma}(k)\right)_{J\pi}$

(There are a considerable amount of experimental data.)

Defining $\quad S_{\pm}^{J\pi} \equiv \sum_m |\langle m | \sum_k t_{\pm}(k) r_k \left(Y_1(\hat{r}_k) \otimes \vec{\sigma}(k)\right)_{J\pi} |0\rangle|^2 \quad$ where Jπ = 0–, 1– and 2– ,

one obtains $\quad S_{-}^{J} - S_{+}^{J} = \dfrac{2J+1}{4\pi}\left[N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right]$

$$\sum_{J=0,1,2} (S_{-}^{J} - S_{+}^{J}) = \frac{9}{4\pi}\left[N\langle r_n^2\rangle - Z\langle r_p^2\rangle\right] \qquad (\&)$$

---

ex. Using experimental data from ^{90}Zr(p,n) and ^{90}Zr(n,p) on the l.h.s. of (&), the difference between $\langle r_n^2\rangle$ and $\langle r_p^2\rangle$ can be obtained, if $\langle r_p^2\rangle$ is known from the observed charge radius. (Harakeh & Woude, Giant Resonances, 2001)

K.Yako, H.Sagawa and H.Sakai, Phys.Rev.C **74**, 051303(R) (2006)

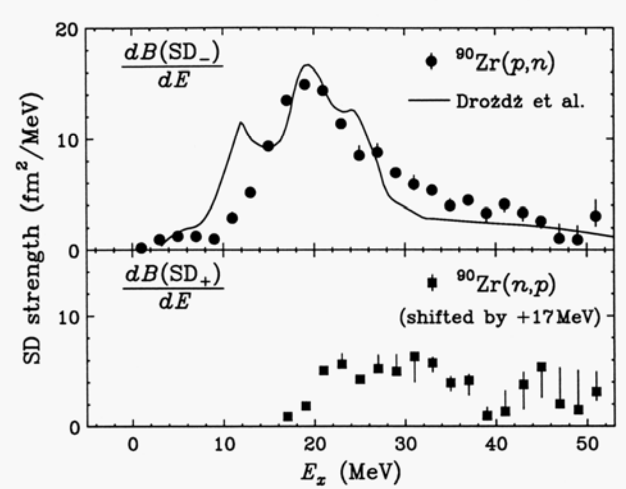


FIG. 1: Charge exchange SD strength $\frac{dB(\mathrm{SD}_-)}{dE}$ (upper panel) and $\frac{dB(\mathrm{SD}_+)}{dE}$ (lower panel). The circles and squares are the experimental data. The $\frac{dB(\mathrm{SD}_+)}{dE}$ spectra are shifted by +17 MeV. The curve is the results of the second RPA calculation by Drożdż *et al.* [33].

The ground state of $^{90}_{40}Zr_{50}$ has

$$T = T_0 (= T_z) = (50-40)/2 = 5$$

IVSD$_-$: T = 4, 5, 6 in $^{90}_{40}Zr_{50}(p,n)^{90}_{41}Nb_{49}$

IVSD$_+$: T = 6 in $^{90}_{40}Zr_{50}(n,p)^{90}_{39}Y_{51}$

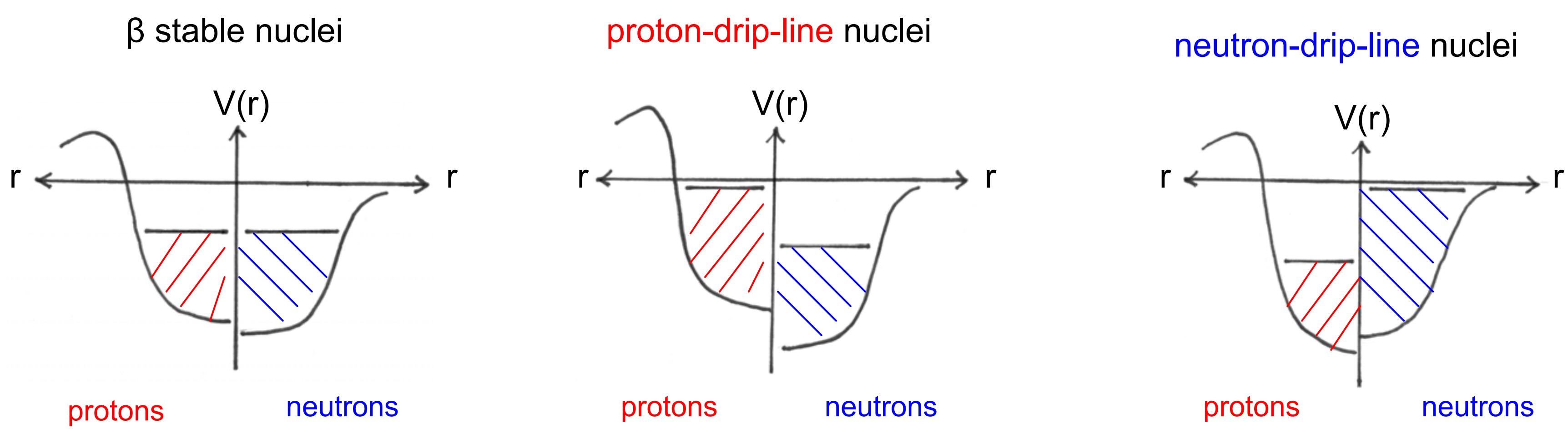
A multipole decomposition analysis at θ=4.6° (= max of SD mode) was performed, and the SD strengths up to 40 MeV in the left figure were included. → $N\langle r_n^2\rangle - Z\langle r_p^2\rangle$ = 207 ± 17 fm^2

neutron skin thickness :

$$\sqrt{\langle r^2\rangle_n} - \sqrt{\langle r^2\rangle_p} = 0.07 \pm 0.04 \text{ fm}$$

7.5. Giant resonances in nuclei far away from the stability line

drip-line nuclei — very different *N*/*Z* ratio, compared to stable nuclei with the same *A*, in addition to the presence of weakly-bound nucleons.

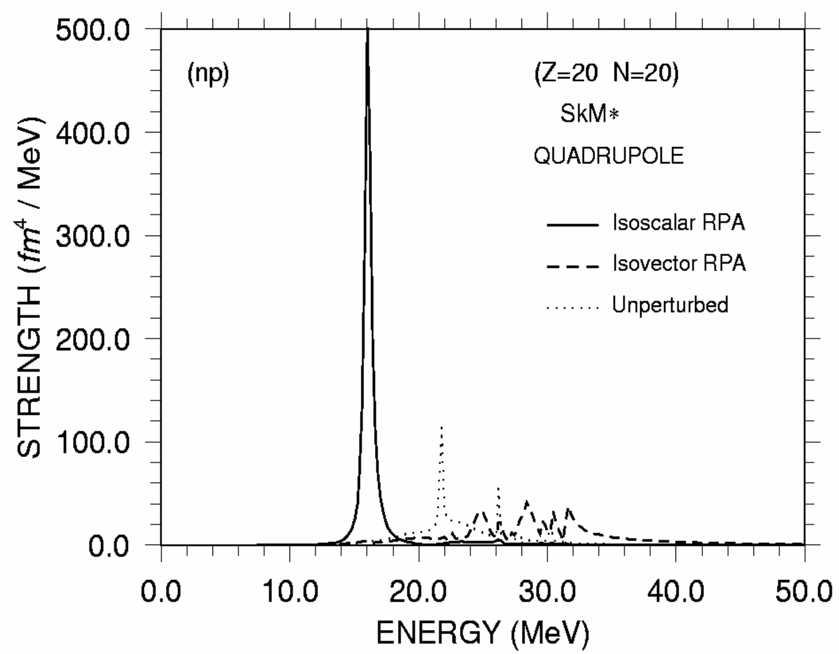


Since the Fermi levels for protons and neutrons are very different in drip line nuclei, this binding energy difference of least-bound protons and neutrons will produce interesting phenomena in charge-exchange reactions or β decays.

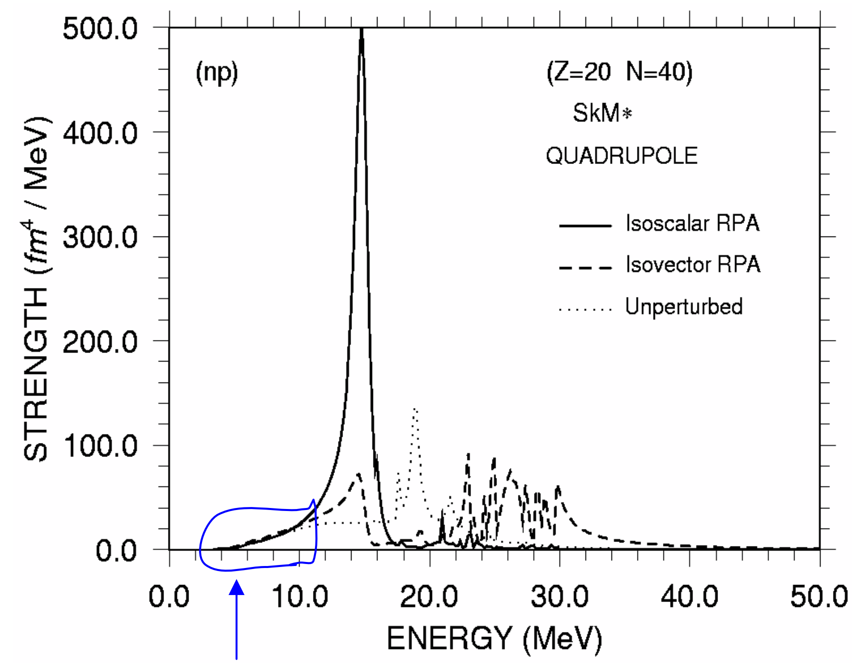
7.5.1. ISGQR of nuclei with weakly-bound neutrons

(an example of weakly-bound neutrons → threshold strength)

Ex. Calculated GQR of β-stable nuclei



Calculated GQR of neutron-drip-line nuclei



threshold strength

Increase of energy-weighted sum-rules, $S(IS,\lambda=2)_{class} = \frac{50}{4\pi}\frac{\hbar^2}{2m}A\langle r^2\rangle$, by the threshold strength

← extra contribution by weakly-bound neutrons in the ground state to $\langle r^2\rangle$.

Threshold strength couples very little with other p-h configurations

→ threshold strength contributes very little to $e_{pol}(E2)$.

Ex. ISGQR of a possibly neutron-drip-line nucleus with weakly-bound neutrons, $^{60}_{20}Ca_{40}$

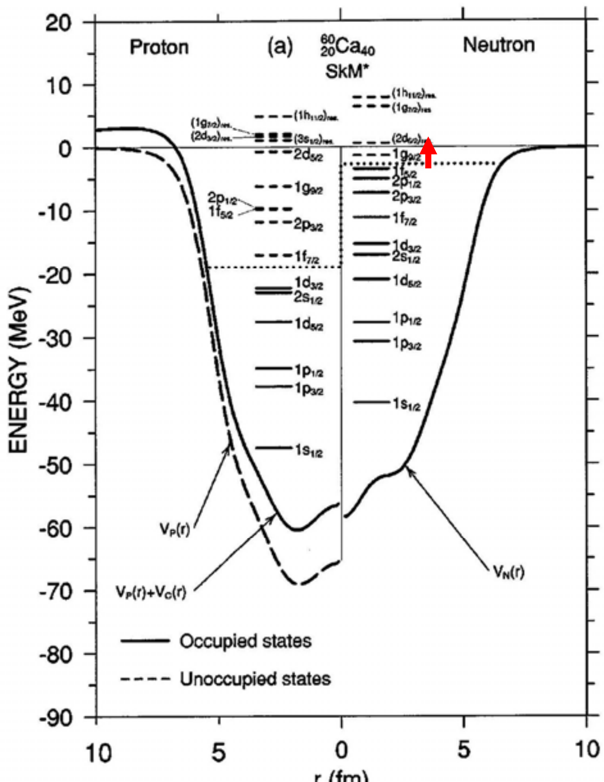
(calculated results only)

Compared with ISGQR in β-stable nuclei, the frequencies of possible neutron p-h configurations are lower, while the frequencies of proton p-h configurations remain nearly the same or become larger.

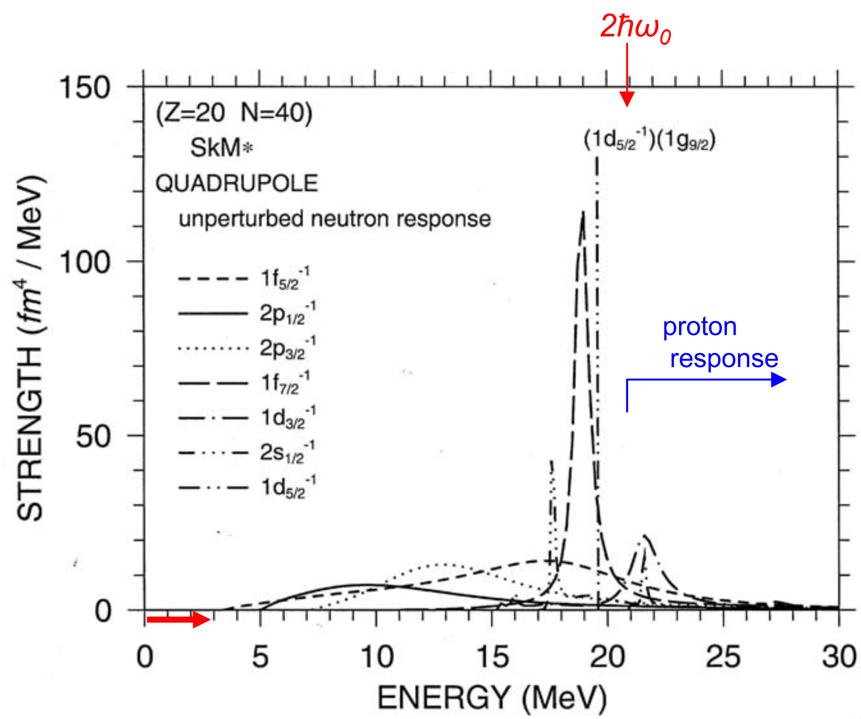
$\Rightarrow$ ISGQR has
- lower frequency
- broader width

However, collective correlation structure and transition density are similar to those of β-stable nuclei.

Hartree-Fock potentials and one-particle energy levels

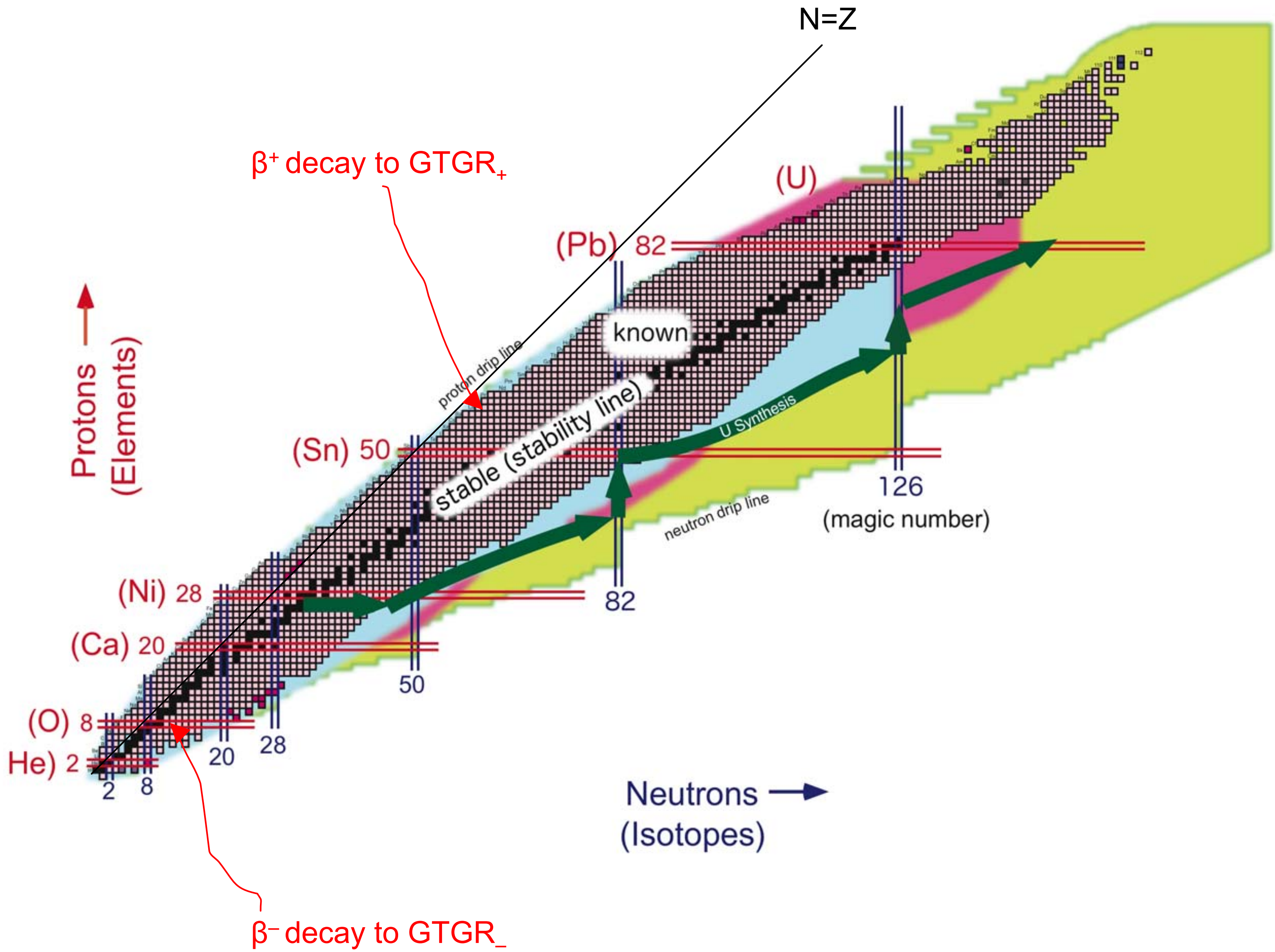


Unperturbed neutron response to $r^2Y_{2\mu}$



I.H., H.Sagawa and X.Z.Zhang, PRC **64**, 024313 (2001).

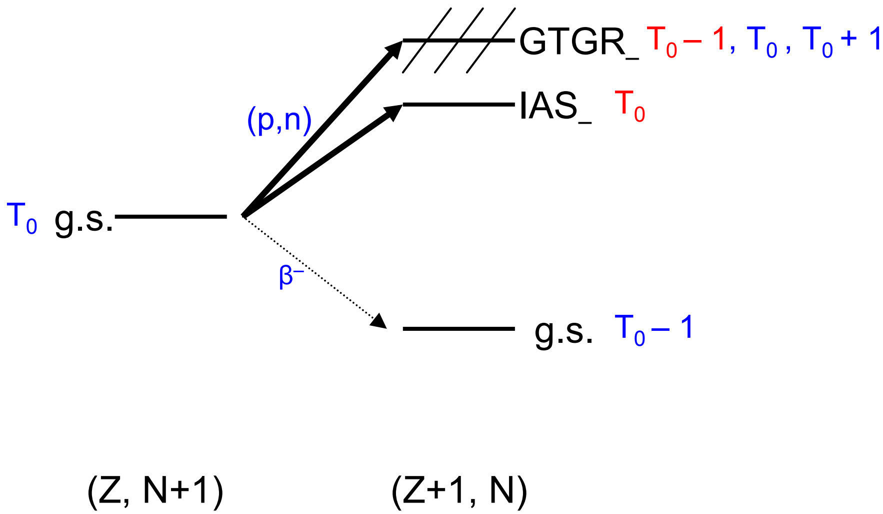
7.5.2. β-decay to GTGR in drip line nuclei



1) β^- decay in nuclei with N > Z

H.Sagawa, I.H. and M.Ishihara, **PLB 303**, 215 (1993)

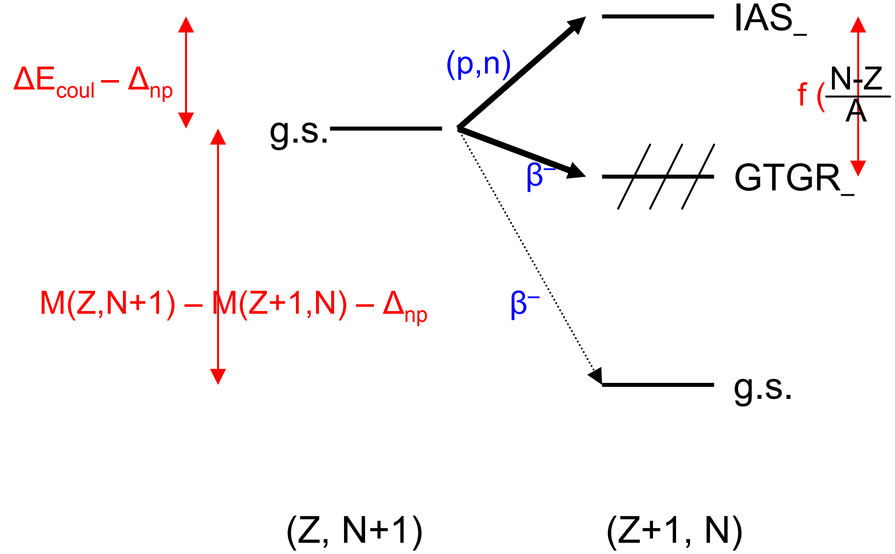
β stable nuclei



$$T_0 = \frac{N+1-Z}{2}$$

very neutron-rich light (Z < 7) nuclei

The relative energy between IAS and GTGR is a function of (N-Z)/A. The larger (N-Z)/A, the lower GTGR.



$$\Delta_{np} = (\Delta M(n) - \Delta M(^1H))\, c^2 = 0.78 \text{ MeV}$$

$$\Delta E_{Coul}(Z+1) = E_{Coul}(Z+1) - E_{Coul}(Z) \propto ((Z+1)^2 - Z^2)\, A^{-1/3} \propto Z\, A^{-1/3}$$

Energy difference of different T states in a given nucleus

$$E(A, T+1, M_T = T) - E(A, T, M_T = T) \approx 4 b_{sym} \frac{T + 1/2}{A} \propto \frac{|N - Z|}{A}$$

Dependence of the energy difference between IAS_ and GTGR_ on (N–Z)/A

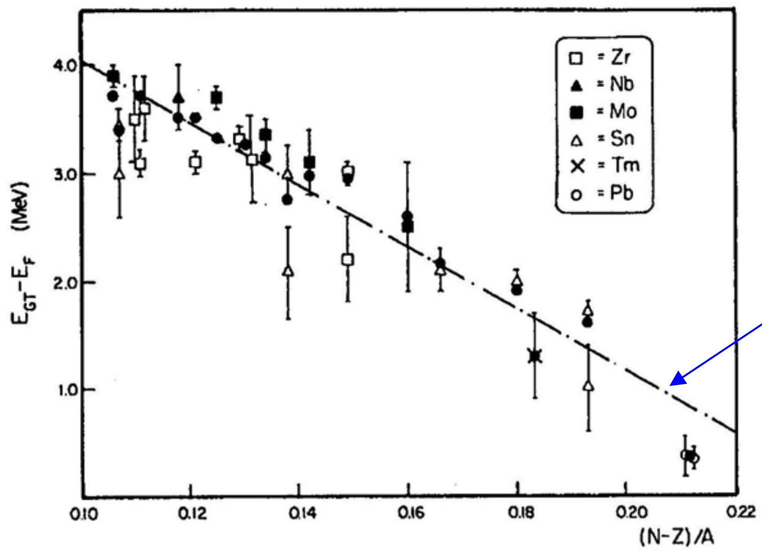


Fig. 1. Plot of $E_{GT} - E_F$ versus $(N - Z)/A$. The experimental data were taken from ref. [8] for ^{90}Zr, from ref. [10] for

K.Nakayama, A.Pio Galeao and F.Krmpotic, PLB**114**, 217 (1982)

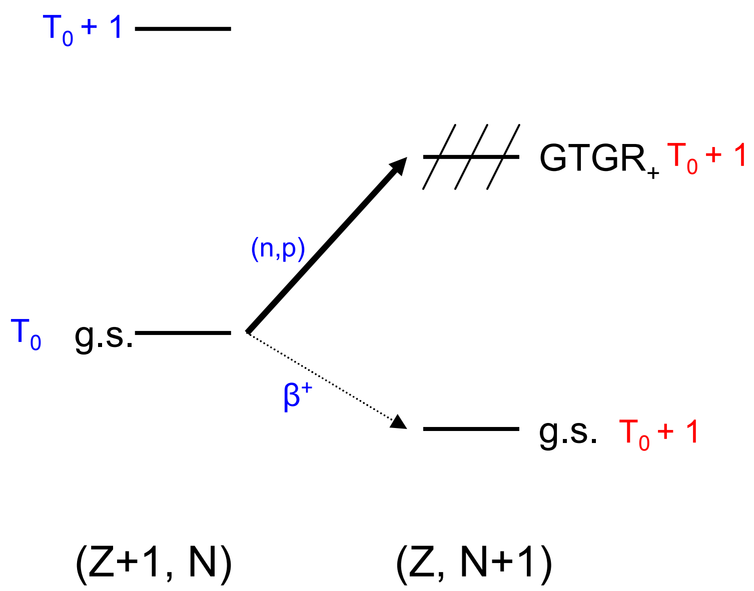
$$E(GT) - E(IAS) = 7.0 - 57.8\,\frac{N-Z}{2A}$$

Note	$\frac{N-Z}{A}$
^{24}O	0.3333
^{20}C	0.4000
^{22}C	0.4545
^{8}He	0.5000
^{208}Pb	0.2115

2) β^+ decay in nuclei with N > Z

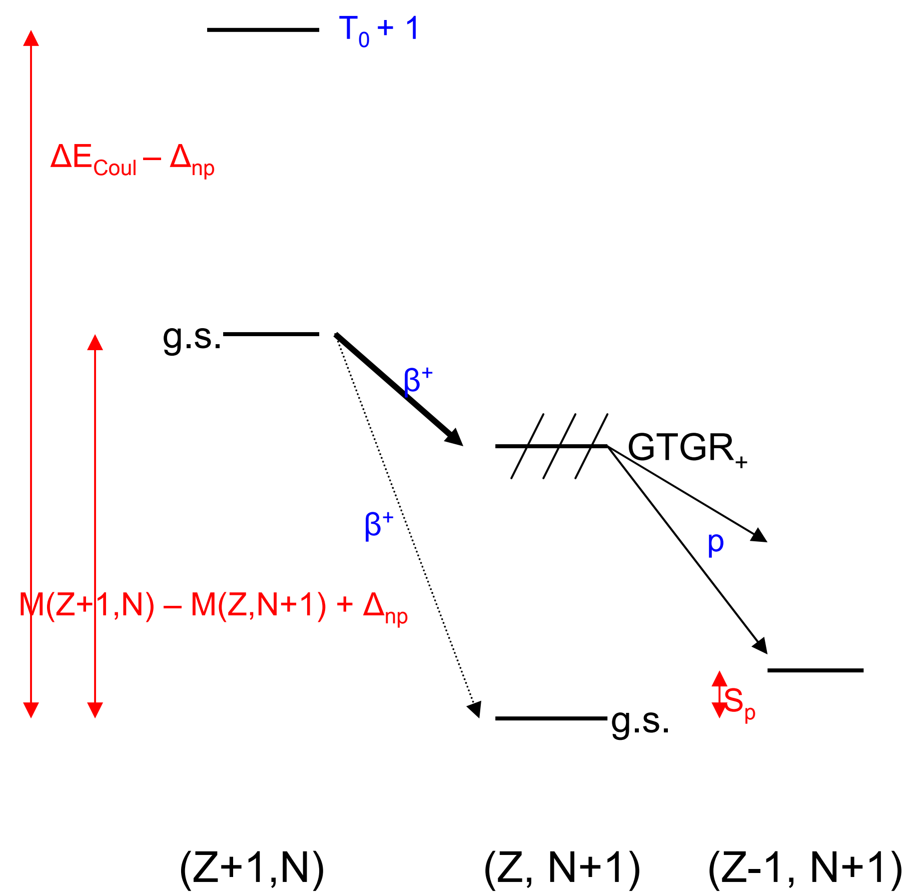
F.Frisk, I.H. and X.Z.Zhang, PRC **52** (1995) 2468.

β stable nuclei



$$T_0 = \frac{N - Z - 1}{2}$$

medium-heavy proton-drip-line nuclei (Z > 50)



The mass difference, M(Z+1,N) – M(Z,N+1), increases rapidly, as stable → proton-drip-line nuclei.

⇒ $GTGR_+$ comes easily into the scope of β^+ decays, namely below the ground state of mother nuclei.