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(expecting experimentalists as an audience)

One-particle motion in nuclear many-body problem

(The 2nd lecture, V.2)

In this second lectures, V.2, first, the effective one-particle operators with $e_{eff}(E\lambda)$ and $g^{eff}(M\lambda)$ of electromagnetic transitions in the spherical case are reviewed. Then, the energies and electromagnetic moments in the laboratory system are examined, when the shape in the body-fixed system is deformed.

Ikuko Hamamoto

Division of Mathematical Physics, LTH,
University of Lund, Sweden

The figures with figure-numbers but without reference, are taken from

the basic reference : A.Bohr and B.R.Mottelson, Nuclear Structure, Vol. I & II

6. Energy and electromagnetic observables of one-particle configurations

- 6.1. Spherical case – effective (E2, M1, E1) one-particle operators
- 6.2. From Y_{20} deformed intrinsic system to laboratory system
- 6.3. Energies with Y_{20} deformed intrinsic shape
- 6.4. Electromagnetic properties of the system with Y_{20} deformed intrinsic shape (M1, E2, E1)

6. Energy and electromagnetic observables of one-particle states

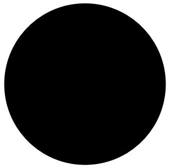
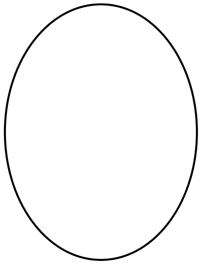
6.1. spherical case – effective one-particle operators (E2, M1, E1)

The deviation of $e_{pol}(E\lambda)/e$ and $g^{eff}(M\lambda)/g^{bare}$ from unity depends on the multipole λ, one-particle orbits, and the size of the configuration space included in the construction of wave functions.

For example, if the wave functions are constructed taking into account the whole degrees of freedom of all nucleons in a given nucleus, the "effective" operators should be the same as the bare operators, except the renormalization coming from possible non-nucleon degree of freedom.

In this section we assume that all possible configuration mixing within one major shell is already taken into account in the construction of wave functions of states.
This includes so-called one-particle states (= one-particle + closed-shell core).
Then, the renormalization of one-particle operators comes from the core polarization involving virtual excitations of giant resonances, besides the possible contribution by non-nucleon degree of freedom.

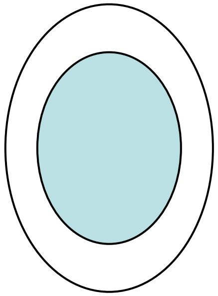
In other words, the major components of wave functions are explicitly taken into account in the construction of wave functions. The effect of some small components on the matrix element of a particular operator, which appreciably contribute to the matrix-element in spite of small admixed probabilities in wave functions, is expressed by renormalizing one-particle operators. → effective operators

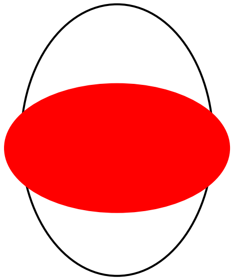
Core  plus particle 

Core polarization

If the relevant interaction is attractive,

If the relevant interaction is repulsive,





one-particle moments increase.

one-particle moments are reduced.

For spin polarization of the core the density above should be replaced by spin density.

If $\Delta E_{tr} << \hbar\omega_{core}$ (ΔE_{tr} : transition energy, $\hbar\omega_{core}$: energy of core excitations), and

[mixed probability of core excitations into one-particle wave-functions] << 1,

the effect of admixed components can be expressed by the renormalization of one-particle operator → static polarization and effective one-particle operators

1) one-particle energy, $\varepsilon_{\ell j}$, obtained for the potential is identified as an observed one-particle energy.

Or, alternatively one-particle energy can be calculated in the Hartree-Fock approximation, if the two-body interaction is sufficiently known, and the one-particle energy is identified as an observed one-particle energy.

In shell model calculations one-particle energies are often just parameters.

2) Electric quadrupole moment operator $$eQ_{op} = e\sum_p r_p^2(3\cos^2\theta_p - 1)$$

For a single-particle in an orbit (nℓj)

$$Q_{sp} = \langle n\ell j, m = j | r^2(3\cos^2\theta - 1) | n\ell j, m = j\rangle \quad = \quad -\frac{2j-1}{2j+2}\langle n\ell j | r^2 | n\ell j\rangle$$

where $$\langle n\ell j | r^2 | n\ell j\rangle \equiv \int r^4 R_{n\ell j}^2(r)dr$$

E2 transition operator

$$M(E2, \mu = 0) = \sqrt{\frac{5}{16\pi}} eQ_{op}$$

The reduced E2 transition probability

$$B(E2; I_1 \to I_2) \quad = \quad \sum_{\mu M_2} \left| \langle I_2 M_2 | M(E2,\mu) | I_1 M_1 \rangle \right|^2 \quad = \quad \frac{1}{2I_1+1} \left| \langle I_2 \| M(E2) \| I_1 \rangle \right|^2$$

For E2 transitions of a single particle

$$B_{sp}(E2; n_1\ell_1 j_1 \to n_2\ell_2 j_2) \quad = \quad \frac{5}{4\pi} e^2 \left(C(j_1 2 j_2; 1/2, 0, 1/2) \langle n_2\ell_2 j_2 | r^2 | n_1\ell_1 j_1 \rangle \right)^2$$

In practice,

$$e \to e_{eff}(E2) = e_{bare} + e_{pol}(E2)$$

For low-energy transitions $$e_{pol}(E2) > 0$$

Estimate of static E2 polarization charge using ISGQR and IVGQR in a harmonic oscillator model

$\hbar\omega_{ISGQR} = 58\ A^{-1/3}$ MeV
$\hbar\omega_{IVGQR} = 135\ A^{-1/3}$ MeV

$$e_{pol}(E2) = e\left(\overset{\text{from ISGQR}}{\frac{Z}{A}} \underbrace{-0.32\frac{N-Z}{A}}_{\text{IV coupling field should not act on the total density at any point}} + \left(\underset{\text{from IVGQR}}{0.32}\overbrace{-0.3\frac{N-Z}{A}}^{\text{n. excess to preserve the local ratio of n \& p in IS GQR}}\right)\tau_z\right)$$

Bohr & Mottelson, Vol.II, eq.(6-386b)

ISGQR increases both $e^{n}_{pol}(E2)$
and $e^{p}_{pol}(E2)$

IVGQR increases $e^{n}_{pol}(E2)$
while decreases $e^{p}_{pol}(E2)$

Neutron excess of the core makes both $e^{n}_{pol}(E2)$ and $e^{p}_{pol}(E2)$ smaller.

For neutrons $(\tau_z = +1)$

$$e^{n}_{pol}(E2) = e\left(\frac{Z}{A} + 0.32 - 0.62\frac{N-Z}{A}\right)$$

→ smaller, as (*N-Z*) becomes larger, for a given A.

For protons $(\tau_z = -1)$

$$e^{p}_{pol}(E2) \approx e\left(\frac{Z}{A} - 0.32\right)$$

ex.

$$e^{p}_{pol}(E2) \approx e\left(\frac{20}{60} - 0.32\right) \approx 0$$

for ${}^{40}_{20}Ca_{40}$

The value of $e_{pol}(E2)$ depends somewhat on nucleon orbits. In particular, the polarization effect decreases for weakly-bound nucleons, since those nucleons being outside the nuclear surface cannot efficiently polarize the core.

A simple approximate correction is to multiply the standard $e_{pol}(E2)$ in the previous page

by $$\frac{\left(\frac{3}{5}\right)R^2}{\langle j_2 | r^2 | j_1 \rangle}$$

Note $\langle \ell | r^2 | \ell \rangle \to \infty$ for ℓ=0 and 1 neutrons, as $\varepsilon_\ell(<0) \to 0$.

For neutrons

$\langle \ell_2 | r^n | \ell_1 \rangle$ with $\ell_1 + \ell_2 \leq n+1$ diverges as $\varepsilon_{\ell_1}, \varepsilon_{\ell_2}(<0) \to 0$

ex. Derivation of the first term of $e_{pol}(E2) = e\frac{Z}{A} + \ \ldots\ldots\ldots$

In the harmonic oscillator model one can show ;
"One particle outside of the closed shell induces a mass quadrupole moment in the closed shell, which is equal to its own mass quadrupole moment."

(B.R.Mottelson, Les Houches, 1958 (Dunod, Paris, 1959) p.283-315.)

Mass quadrupole moment

$$m(IS,\lambda=2) = m_{sp}(IS,\lambda=2) + m_{core-pol}(IS,\lambda=2)$$

Equilibrium shape for a system of a single-particle outside of closed shell
← self-consistency condition of potential and density

Then, in the harmonic oscillator model one obtains

$$m_{core-pol}(IS,\lambda=2) = m_{sp}(IS,\lambda=2)$$

$\therefore$ For E2 operator (Z : proton number of the core, A : nucleon number of the core)

$$e_{pol}(E2) = \frac{Z}{A}e$$ for both protons and neutrons

Note : this harmonic oscillator model produces the frequency of ISGQR

$$\hbar\omega_{ISGQR} = \sqrt{2}\hbar\omega_0 = 58A^{-1/3} \text{ MeV}$$

which is consistent with the observed systematics.

3) Magnetic dipole moment of a single nucleon

$$\vec{\mu} = g_\ell \vec{\ell} + g_s \vec{s} \qquad g_\ell = \begin{cases} 1 \\ 0 \end{cases} \qquad g_s = \begin{cases} 5.58 & \text{for proton} \\ -3.82 & \text{for neutron} \end{cases}$$

$$\mu = \langle j, m=j | g_\ell \ell_z + g_s s_z | j, m=j \rangle = j\left\{ g_\ell \pm (g_s - g_\ell)\frac{1}{2\ell+1} \right\} \quad \text{for} \quad j = \ell \pm 1/2$$

M1 transition operator $M(M1,\mu) = \sqrt{\frac{3}{4\pi}}\frac{e\hbar}{2Mc}\mu_\mu$

In practice,

$g_s \to g_s^{eff}$ and $g_\ell \to g_\ell^{eff}$

For low-energy transitions

$(g_s^{eff} / g_s) < 1$

since the relevant $(\tau\sigma)(\tau\sigma)$ type interaction is repulsive.

Empirical values in medium-heavy nuclei are

$(g_s^{eff} / g_s) = 0.6 \sim 0.7$ for both protons and neutrons,

while those in lighter nuclei are somewhat closer to unity.

The spin-saturated core (i.e. ℓ-s closed nuclei such as ^{16}O and ^{40}Ca) cannot spin-polarize in the lowest order

$\longrightarrow$ $(g_s^{eff} / g_s^{free}) \approx 1$ for one-particles outside the spin-saturated core.

Writing

$$g_\ell^{eff}(p) = 1 + \delta g_\ell(p) \qquad \text{and} \qquad \delta g_\ell^{eff}(n) = \delta g_\ell(n)$$

Empirical values are $\delta g_\ell(p) \approx +0.1$ and $\delta g_\ell(n) \approx -0.05$

(S.Nagamiya and T.Yamazaki, Phys.Rev.C4(1971)1961)

Those δg_ℓ values are compatible with the effect of the meson-exchange current, while they are also consistent with the modification in the current implied by the velocity-dependent effective interaction.

(Bohr &Mottelson, Vol.II, p.484)

Core polarization effect may not simply be described in terms of a renormalization of bare one-particle operators.

Thus, effective magnetic moment operator may have, for example, a term like

$$(\delta\mu)_\nu = \underline{f(r)}(Y_2 s)_{\lambda=1,\nu}$$

radial distribution of the polarizing particle

4) E1 transition operator, which should be orthogonal to the center of mass motion that must not create an excitation,

$$M(E1,\mu=0)=\sqrt{\frac{3}{4\pi}}e\sum_{i}^{(p)}z_i$$

$$e\sum_{i}^{(p)}z_i \rightarrow e\sum_{i}^{(p)}\left(z_i-\frac{1}{A}(\sum_{j}^{(p)}z_j+\sum_{k}^{(n)}z_k)\right) = e\sum_{i}^{(p)}z_i-\frac{e}{A}Z(\sum_{j}^{(p)}z_j+\sum_{k}^{(n)}z_k)$$

$$= \frac{N}{A}e\sum_{i}^{(p)}z_i-\frac{Z}{A}e\sum_{k}^{(n)}z_k$$

In practice, in stable nuclei

$$|e_{eff}^{p}(E1)|<\frac{N}{A}e \qquad \text{and} \qquad |e_{eff}^{n}(E1)|<\frac{Z}{A}e$$

due to the polarization effect associated with IVGDR (Iso Vector Giant Dipole Resonance).

$$\begin{cases} e_{eff}^{p}(E1)=e(1+\chi)\dfrac{N}{A} \\ e_{eff}^{n}(E1)=-e(1+\chi)\dfrac{Z}{A} \end{cases}$$

where $\chi \approx -0.7$ (estimate in B&M VoL.II).

ex. Empirical values obtained in the Pb region are $\left|e_{eff}^{p}(E1)\right|^2 \sim (0.10)e^2 \quad > \quad \left|e_{eff}^{n}(E1)\right|^2$

(from the analysis of E1 decays of octupole multiplet members in ^{209}Bi and ^{207}Pb.)

I.H., Physics Reports,10C (1974) 63-105.

In very light **halo** nuclei such as ^{11}Be, one may expect

$$\left| e^{p}_{eff}(E1) \right| \approx \frac{N}{A} e \qquad \text{and} \qquad \left| e^{n}_{eff}(E1) \right| \approx \frac{Z}{A} e$$

$$\begin{cases} \text{weakly-bound orbits} \rightarrow \text{a change of shell structure and wave-functions} \\ \text{halo particles} \rightarrow \text{difficult to polarize the core} \end{cases}$$

Observed low-energy E1 transitions in stable spherical nuclei are usually very much hindered;

In medium-heavy nuclei $B(E1) < (10^{-5})\, B_W(E1)$

∵) In addition to the small $e_{eff}(E1)$ values, due to the nuclear shell-structure there is no close-lying one-particle configurations that can be connected by E1 operators in either light or medium-heavy nuclei;

ex.

20

$2s_{1/2}$, $1d_{3/2}$, $1d_{5/2}$

8

82

$3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1h_{11/2}$

50

The strong hindrance of low-energy E1 transitions makes it almost impossible to obtain any nuclear structure information from the B(E1) values.

6.2. From the Y_{20} deformed intrinsic system to laboratory system

The intrinsic wave functions are not eigenstates of angular momentum, while the states observed in the laboratory system are the eigenstates.
Thus, one has to construct the total wave functions using respective intrinsic wave functions.

Angular momentum projection from a deformed intrinsic wave function is one way of getting back an eigenstate of angular momentum. However, the projection includes no possible rotational perturbation of intrinsic states.
Particle-rotor model with particles (or some intrinsic degrees of freedom) referred to the body-fixed system is another model, in which angular momentum is a good quantum number.

In the following the simplest and practical (though approximate) way of getting back total angular momentum (Bohr &Mottelson, Vol.II), which is generally expected to work better in heavier nuclei.

In 6.2. a general form of the total wave function for a given intrinsic wave function with Y_{20} deformed intrinsic shape (i.e. axially symmetric and R-invariant shape) is derived.
The formulas can be used not only for intrinsic one-particle configurations but also for more complicated intrinsic configurations.

In 6.3. energies with Y_{20} deformed intrinsic shape are described.

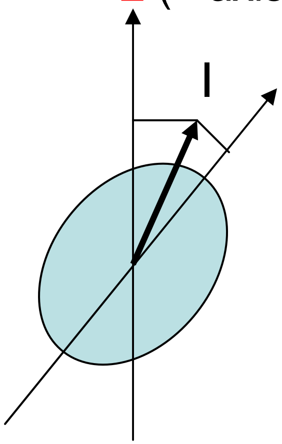
In 6.4. electromagnetic properties of the system with Y_{20} deformed intrinsic shape are described.

From now on:

(1, 2, 3) : body-fixed system
(x, y, z) : laboratory system

μ : components referred to the laboratory system

ν : components referred to the body-fixed system

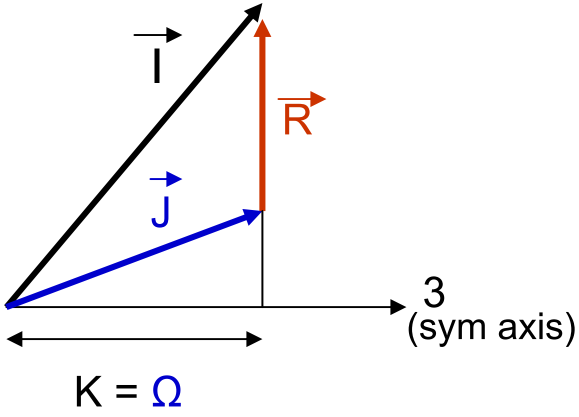


z (= axis fixed in the laboratory)

3 (= symmetry axis fixed in the body)

$K \leftarrow I_3$

$M \leftarrow I_z$



Total angular momentum $\vec{I}$

$$\vec{I} = \vec{R} + \vec{J}$$

$\vec{R}$: angular momentum of collective rotation

$\vec{J}$: intrinsic angular momentum

axially-sym shape → $K\ (\leftarrow I_3) = \Omega\ (\leftarrow J_3)$

No collective rotation about symmetry axis ; $R_3 = 0$

(OBS. No collective rotation in spherically-symmetric nuclei)

Total (= intrinsic x rotational) wave functions and consequences of symmetry

If the intrinsic and rotational parts of the Hamiltonian are separated, the eigenstates of the Hamiltonian are the product form

$$\Psi_{\alpha,I} = \Phi_\alpha(q)\, \varphi_{\alpha,I}(\omega)$$

$\Phi_\alpha(q)$: intrinsic wave-function

$\varphi_{\alpha,I}(\omega)$: rotational wave-function

where α : quantum number specifying intrinsic states,
q : intrinsic variable,
ω : angular variables specifying the orientation of the deformed body with respect to the laboratory system,
I : angular-momentum quantum-numbers.

Rotational wave functions ;

(1) In 2-dimensional rotation (a rotation about a fixed axis)

$$\varphi_{\alpha,I}(\omega) \sim \exp(iM\theta)$$

$\omega \rightarrow \theta$

$I \rightarrow M$

(2) In 3-dimensional rotation

$$\varphi_{\alpha,I}(\omega) \sim D^I_{MK}(\omega)$$

$\omega \rightarrow$ 3 Euler angles (Φ,θ,ψ), to specify the orientation of the body.

$I \rightarrow$ 3 quantum numbers:

$(\vec{I})^2$, M ($\leftarrow I_z$), K ($\leftarrow I_3$)

$$\vec{I}^2 D^I_{MK} = I(I+1)D^I_{MK}$$

$$I_z D^I_{MK} = M D^I_{MK}$$

$$I_3 D^I_{MK} = K D^I_{MK}$$

$$\omega = (\phi, \theta, \psi)$$

$$\int d\omega \equiv \int \sin\theta d\theta \int d\phi \int d\psi$$

$$\int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\psi D^I_{MM'}(\omega)^* D^{I_1}_{M_1 M_1'}(\omega) = \frac{8\pi^2}{2I+1}\delta(I, I_1)\delta(M, M_1)\delta(M', M_1')$$

$$[I_x, I_y] = iI_z$$

$$[I_1, I_2] = -iI_3$$

$$[I_{x,y,z}, I_{1,2,3}] = 0$$

I_x, I_y, I_z ：referred to the lab.system

I_1, I_2, I_3 ：referred to the body-fixed system

$$\langle I, M | I_x \pm iI_y | I, M \mp 1 \rangle = \left(I(I+1) - M(M \mp 1)\right)^{1/2}$$

$$\langle I, K | I_1 \pm iI_2 | I, K \pm 1 \rangle = \left(I(I+1) - K(K \pm 1)\right)^{1/2}$$

I_x, I_y, I_z ; give the change in the state vector when the lab system is rotated about one of its own axes.

I_1, I_2, I_3 ; describe the change in the state vector when the lab system is rotated about an axis of the body-fixed system.

Rotational degrees of freedom ← restricted by the symmetry of deformation

ex. Spherically symmetric nuclei → No collective rotation

ex. Axially-symmetric deformed nuclei → No collective rotation about the symmetry axis

ex. *R*-invariant axially-symmetric deformation

→ rotation $R_\perp(\pi)$ (≡ rotation π about the axis $\perp$ symmetry axis)
must not be included in the rotational degrees of freedom

Correspondingly,
the form of total wave function (in general, a sum of products of intrinsic and rotational wave-functions) is governed by the symmetry of deformation.

Total wave function for Y_{20} deformed intrinsic shape

(a) axially-symmetric shape ⟶ no collective rotation about the sym axis (=3-axis)

$$\longrightarrow K\,(\leftarrow I_3) \;=\; \Omega\,(\leftarrow J_3)$$

$$\Psi_{KIM} = \Phi_{\underset{\overset{\uparrow}{J_3}}{K}}(q)\,D^{I}_{M\underset{\overset{\uparrow}{I_3}}{K}}(\phi,\theta,\psi)\sqrt{\frac{2I+1}{8\pi^2}}$$

$R_\perp(\pi) \rightarrow R_2(\pi)$ ≡ rotation π about the 2-axis

(b) *R*-invariant shape, in addition to axial symmetry (taking $K > 0$)

$$\boxed{\Psi_{KIM} = \sqrt{\frac{2I+1}{16\pi^2}}\left\{\Phi_K(q)D^I_{MK}(\phi,\theta,\psi) + \underline{(-1)^{I+K}\Phi_{\overline{K}}(q)D^I_{M,-K}(\phi,\theta,\psi)}\right\}} \qquad (\$)$$

Rotation by $R_2(\pi)$ does not belong to collective rotation (quantum effect !).

i.e. from the two intrinsic states with *K* and –*K*, only a single rotational state can be formed for a given I.

Note $\Phi_{\overline{K}}(q) \propto \Phi_{-K}(q)$

Obs. The 1st and 2nd term in ($) can be connected by the operator with $\Delta K = 2K$.

$\rightarrow$ $(-1)^I$ dependent term in observables

ex. For K=1/2 bands → the term $(\propto (-1)^I)$ in the energy

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For a Hamiltonian with a coupling between intrinsic and rotational motion, a set of wave functions ($) can be used as a basis for diagonalization. ex. particle-rotor model (Bohr &Mottelson, vol.II, Chap. 4A) .

R-invariance : deformation is invariant under *$R_2(\pi)$* (≡ rotation π about the 2-axis)

Then, $R_2(\pi)$ is not included in collective rotational degrees of freedom.

$R \equiv R_2(\pi)$ can be expressed as

$R_e \equiv R_2(\pi)$, rotation π of the lab system (x, y, z) about the 2-axis
$R_i \equiv R_2(\pi)$, rotation π of the body about the 2-axis

Ψ : total wave-function

$$\underline{R_e}\Psi = \underline{R_i}\Psi$$

is determined by

$R_e\Psi = R_i\Psi \longrightarrow R_i^{-1}R_e\Psi = \Psi$ and

$$R_i^{-1}R_e(1+R_i^{-1}R_e)\Psi = (R_i^{-1}R_e+1)\Psi$$

Then, for $^\forall\Psi$, $\Psi' \equiv (1+R_i^{-1}R_e)\Psi$ satisfies $R_e\Psi' = R_i\Psi'$

$$(1+R_i^{-1}R_e)\Phi_K(q)D^I_{MK}(\phi,\theta,\psi) = \Phi_K(q)D^I_{MK}(\phi,\theta,\psi) + (-1)^{I+K}\Phi_{\overline{K}}(q)D^I_{M-K}(\phi,\theta,\psi) \quad \rightarrow \quad (\$)$$

$\because$)
$$R_e D^I_{MK}(\phi,\theta,\psi) = e^{-i\pi I_2}D^I_{MK}(\phi,\theta,\psi) = (-1)^{I+K}D^I_{M-K}(\phi,\theta,\psi)$$

$\Phi_{\overline{K}}(q) \equiv R_i^{-1}\Phi_K(q)$: Intrinsic state with *–K, which is degenerate with* $\Phi_K(q)$

In fact, $\Phi_{\overline{K}}(q)$ = *T* $\Phi_K(q)$ where *T* : time reversal operator

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$R|K\rangle \propto |-K\rangle$ since R_i inverts the direction of the 3-axis.

R-inv ⟶ Total wave function is a definite combination of two degenerate states with *K* and *–K*.

$$\Psi_{KIM} = \sqrt{\frac{2I+1}{16\pi^2}}\left\{\Phi_K(q)D^I_{MK}(\phi,\theta,\psi) + (-1)^{I+K}\Phi_{\overline{K}}(q)D^I_{M,-K}(\phi,\theta,\psi)\right\}$$

Euler angles :
$\omega \equiv (\phi,\theta,\psi)$

R-inv shape →

the cross term of the first and second terms in the above { … } can produce ;

ex.1 $(-1)^I$ dependent term in the expectation value of the operator $j_\pm I_\mp$ (~ Coriolis coupling)

$$\propto (-1)^{I+K}\left\langle \Phi_{\overline{K}}(q)D^I_{M,-K}(\omega)\right| j_\pm I_\mp \left|\Phi_K(q)D^I_{MK}(\omega)\right\rangle$$

$$\propto (-1)^{I+K}\left\langle \Phi_{-K}(q)\right| j_\pm \left|\Phi_K(q)\right\rangle \int d\omega D^I_{M,-K}{}^*(\omega) I_\mp D^I_{MK}(\omega)$$

that is non-zero only for *K*=1/2 .

$\because$) $j_\pm$ and $I_\mp$ change *K*-value only by ±1.

⟶ $(-1)^I$ dependent term in the rotational energy of *K*=1/2 bands.

ex.2 $(-1)^I$ dependent part of matrix elements of the operator $T^\lambda_\mu = \sum_\nu T^\lambda_\nu D^\lambda_{\mu\nu}(\omega)$

T^λ_μ : operator in the lab system

T^λ_ν : operator in the intrinsic system

$$\propto (-1)^{I+K}\left\langle \Phi_{\overline{K}}(q)D^I_{M,-K}(\omega)\right| \sum_\nu T^\lambda_\nu D^\lambda_{\mu\nu}(\omega) \left|\Phi_K(q)D^I_{MK}(\omega)\right\rangle$$

$$\propto (-1)^{I+K}\sum_\nu \left\langle \Phi_{-K}(q)\right| T^\lambda_\nu \left|\Phi_K(q)\right\rangle \int d\omega D^I_{M,-K}{}^*(\omega) D^\lambda_{\mu\nu}(\omega) D^I_{MK}(\omega)$$

can be non-zero for *ν* = 2*K* .

For example, in B(M1) within a given *K*=1/2 band, and
in B(E2) within a given *K*=1 band, but
not in B(E2) within a given *K*=1/2 band.

λ = 1 and |*ν*| ≤ 1 for M1

λ = 2 and |*ν*| ≤ 2 for E2

K=0 band

$R_e \equiv R_2(\pi)$, rotation π of lab system (x, y, z) about the 2-axis
= equivalent to invert the 3-axis for the fixed lab system (x, y, z)

$$\Psi_{K=0,IM} = D^I_{M,K=0}(\phi,\theta,\psi)\Phi_{K=0}(q) = \sqrt{\frac{4\pi}{2I+1}}Y_{IM}(\theta,\phi)\Phi_{K=0}(q)$$

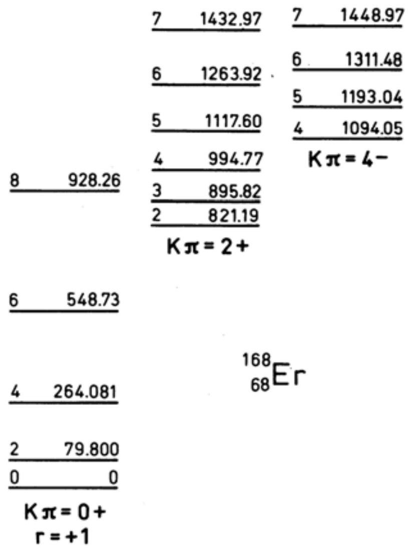
$$\underline{R_e} Y_{IM}(\theta,\phi) = Y_{IM}(\pi-\theta,\phi+\pi) = (-1)^I Y_{IM}(\theta,\phi)$$

inverts the direction of the sym axis (=3-axis)

$$R_i\Phi_{K=0} = r\Phi_{K=0}$$

$$R_e\Psi = R_i\Psi \longrightarrow (-1)^I = r$$

$$\longrightarrow \boxed{\begin{array}{c} I = \text{even for } r = +1 \\ I = \text{odd for } r = -1 \end{array}}$$



The ground state of even-even nuclei has *K*=0 and *r* = +1
(Pairwise-occupied (±*Ω*) nucleon states have *r* = +1.)

∵) $\Phi(1,2) = \frac{1}{\sqrt{2}}\left(\phi_\Omega(1)\phi_{\bar{\Omega}}(2) - \phi_{\bar{\Omega}}(1)\phi_\Omega(2)\right)$ where $\phi_{\bar{\Omega}} \equiv R_i^{-1}\phi_\Omega = -R_i\phi_\Omega$ for Ω = half integer.

(or $R_i^2\phi_\Omega = -\phi_\Omega$)

$$R_i\Phi(1,2) = \frac{1}{\sqrt{2}}\left(-\phi_{\bar{\Omega}}(1)\phi_\Omega(2) + \phi_\Omega(1)\phi_{\bar{\Omega}}(2)\right) = \Phi(1,2)$$

This explains: the ground-band of even-even nuclei has only $I^\pi = 0^+, 2^+, 4^+, \ldots..$

one-particle states in the many-body system

In spherical case

[closed-shell core with J=0] $\longrightarrow$ spherical potential

{ one-particle + closed-shell core (J=0) } : one-particle states

In Y_{20} deformed case

[pairwise-occupied even-even core with K=0] $\rightarrow$ Y_{20} deformed potetnial

{ one-particle + even-even core (K=0) } : one-particle states

For a moderate deformation,

the values of $e_{pol}(E\lambda)$ and $g^{pol}(M\lambda)$ in one-particle operators due to the virtual excitations of Giant Resonances of the core remain nearly the same as in spherical case.

However, $e_{pol}(E\lambda, |\nu|)$ and $g^{pol}(M\lambda, |\nu|)$ are expected, since the properties of GR in Y_{20} deformed nuclei depend on the tensor components $|\nu|$ in the intrinsic system.

6.3. Energies with Y_{20} deformed intrinsic shape

If the deformation and rotation degrees of freedom can be approximately separated, one expects a rotational band associated with each intrinsic configuration.
In other words, to observe rotational spectra is a simple way to find that the nucleus is deformed.

One-particle energies obtained in a deformed potential correspond to the energies of band-head states with the intrinsic one-particle configurations.

In the present section we describe the properties of the states close to band-head states, without taking into account Coriolis perturbation of the intrinsic structure.

Rotational energy associated with a given one-particle configuration (where $K = \Omega$),

$$E_{rot}(K,I) \approx A\left\{ I(I+1) + a(-1)^{I+\frac{1}{2}}(I+\frac{1}{2})\delta(K,\frac{1}{2}) \right\}$$

where $a \equiv -\langle \Omega | j_+ | \overline{\Omega} \rangle$

decoupling parameter $a \approx -\langle [Nn_3\Lambda\Omega] | j_+ | \overline{[Nn_3\Lambda\Omega]} \rangle$ = $\delta(\Omega,1/2)\ \delta(\Lambda,0)\ (-1)^N$

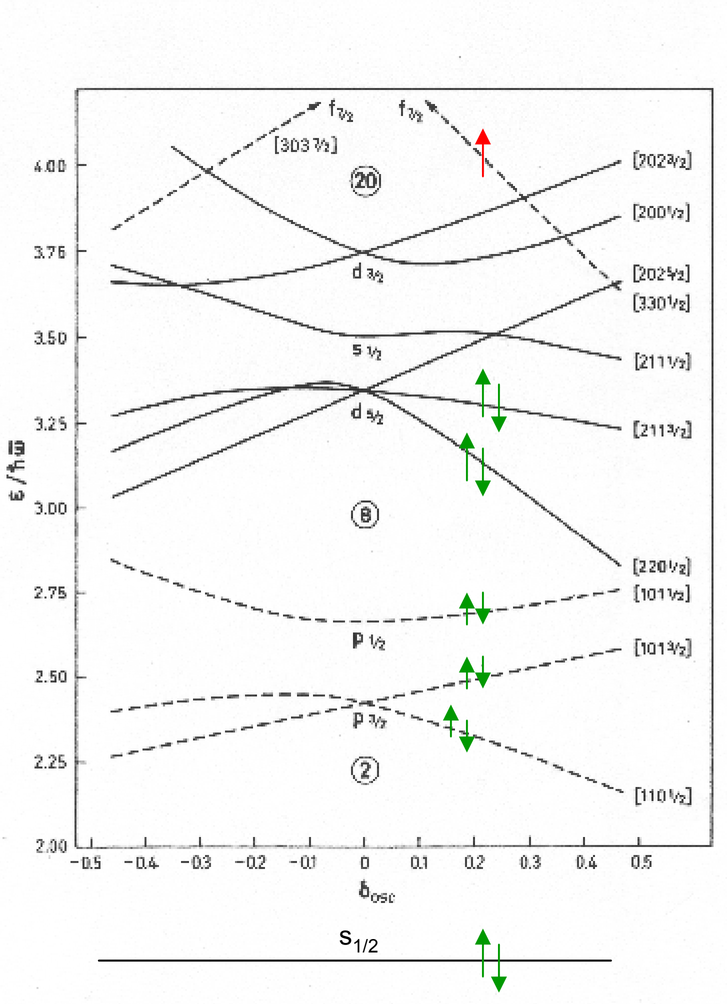
for normal-parity orbits

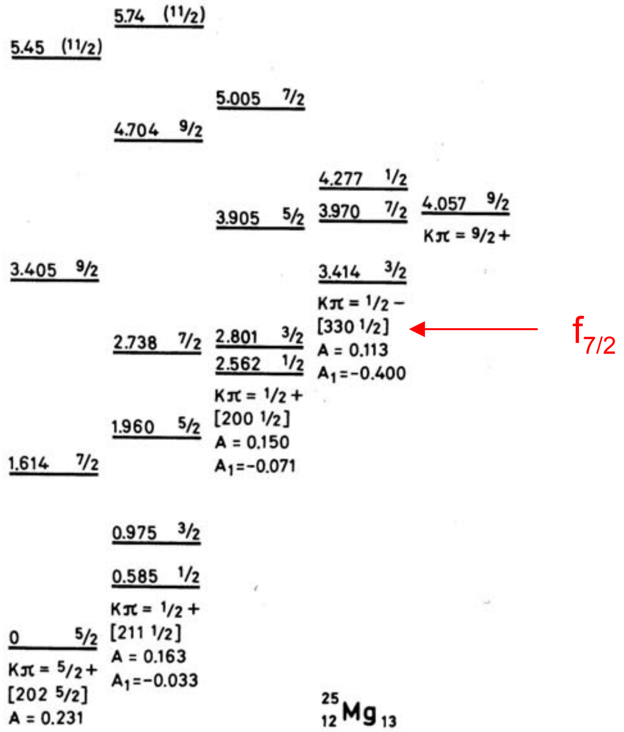
$$a = (-1)^{j-1/2}\left(j + \frac{1}{2} \right)$$

for a single-j configuration

Thus, for normal-parity orbits the band-head state with Ω=1/2 is almost always I=1/2, though the rotational spectra may deviate from I(I+1) .

ex. The N=13 th neutron orbit is seen in low-lying excitations in ^{25}Mg$_{13}$





Note (a) $I \geq K$ ($\leftarrow I_3$)

(b) the bandhead state has I=K.
Exception may occur for K=1/2 bands.

(c) some irregular rotational spectra are observed for K=1/2 bands.

1) Leading-order E2 and M1 intensity relation works pretty well

$\rightarrow$ $Q_0 \approx +50$ fm^2 $\rightarrow$ $\delta \approx 0.4$

$(g_K - g_R) \approx 1.4$ for [202 5/2] etc.

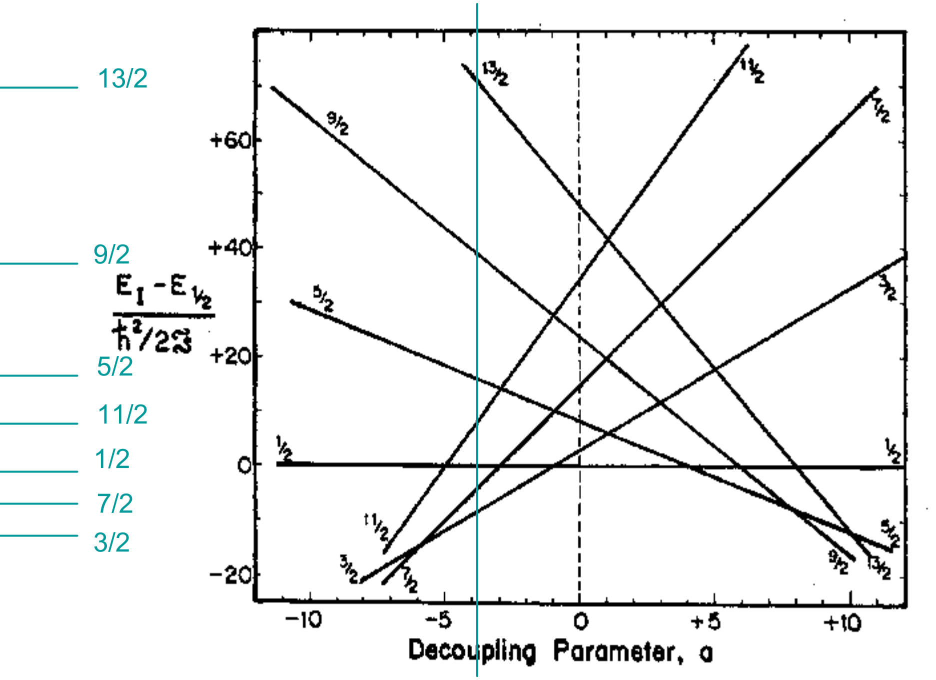
Rotational spectra unique in the intrinsic configuration with Ω=1/2

$$E_{rot}\left(K=\Omega=\frac{1}{2},I\right)=\frac{\hbar^2}{2\Im}\left\{I(I+1)+a(-1)^{I+\frac{1}{2}}(I+\frac{1}{2})\right\}$$

For one-particle in a single j-shell (≈ high-j shell)

decoupling parameter $a=(-1)^{j-1/2}(j+\frac{1}{2})$ =

		I_{lowest} of the Ω=1/2 band
+1	for j=1/2	1/2
-2	for j=3/2	3/2
+3	for j=5/2	1/2
-4	for j=7/2	3/2
+5	for j=9/2	5/2
-6	for j=11/2	3/2 and 7/2
+7	for j=13/2	5/2



M.E.Bunker and C.W.Reich,Rev.Mod.Phys.43 (1971)348.

In rotational bands with high-j configuration
[*I = j mod 2*] levels are pushed down
relative to
[*I = j-1 mod 2*] levels,
also after including the full Coriolis coupling.

$$a=-\langle j,m=1/2|\,j_+\,|\overline{j,m=1/2}\rangle=(-1)^{j-1/2}\langle j,m=1/2|\,j_+\,|\,j,m=-1/2\rangle$$
$$=(-1)^{j-1/2}(j+\frac{1}{2})$$

6.4. Electromagnetic properties of the system with Y_{20} deformed intrinsic shape

Writing $|KIM\rangle$ for the state with the wave function Ψ_{KIM} in ($) ,

$$\langle K_2 I_2 M_2 | T_{\lambda\mu} | K_1 I_1 M_1 \rangle = \frac{1}{\sqrt{2I_2+1}} C(I_1 \lambda I_2; M_1 \mu M_2) \langle K_2 I_2 \| T_\lambda \| K_1 I_1 \rangle$$

Wigner-Eckart theorem
on M-components.

the reduced transition probability is written as

$$B(\lambda; I_1 \to I_2) = \frac{1}{2I_1+1} \left| \langle K_2 I_2 \| T_\lambda \| K_1 I_1 \rangle \right|^2$$

Using Bohr and Mottelson, Vol.II, eqs.(4-91) and (4-92) for the expressions of $\langle K_2 I_2 \| T_\lambda \| K_1 I_1 \rangle$

$$B(\lambda; K_1 I_1 \to K_2 I_2) \;=\; \{ C(I_1 \lambda I_2; K_1, K_2 - K_1, K_2) \langle K_2 | T_{\lambda, K_2 - K_1} | K_1 \rangle$$

$$+ (-1)^{I_1 + K_1} C(I_1 \lambda I_2; -K_1, K_1 + K_2, K_2) \langle K_2 | T_{\lambda, K_1 + K_2} | \bar{K}_1 \rangle \}^2$$

for ($K_1 \neq 0$ and $K_2 \neq 0$)

For matrix elements within a band, the second term inside { } vanishes for

$c(-1)^{2K} = +1$ where c = -1 (+1) for electric (magnetic) transitions

If $K_1 = 0$,

$$B(\lambda, K_1 = 0, I_1 \to K_2 I_2) = C(I_1 \lambda I_2; 0 K_2 K_2)^2 \langle K_2 | T_{\lambda, K_2} | K_1 = 0 \rangle^2 \begin{cases} 2 & \text{for } K_2 \neq 0 \\ 1 & \text{for } K_2 = 0 \end{cases}$$

For matrix elements within a K=0 band, $\langle K = 0 | T_{\lambda,0} | K = 0 \rangle = 0$, for magnetic operators.

For reference,

If the intrinsic moments $T_{\lambda\mu}$ does not depend on $I_\pm$, the matrix element between the two states with the form of the wave function, ($), is given by

$$\langle K_2 I_2 \| T_\lambda \| K_1 I_1 \rangle = (2I_1+1)^{1/2} \{ C(I_1 \lambda I_2; K_1, K_2-K_1, K_2) \underline{\langle K_2 | T_{\lambda,\nu=K_2-K_1} | K_1 \rangle}$$

$$+ (-1)^{I_1+K_1} C(I_1 \lambda I_2; -K_1, K_1+K_2, K_2) \underline{\langle K_2 | T_{\lambda,\nu=K_1+K_2} | \overline{K}_1 \rangle} \}$$

BM,Vol.II, eq.(4-91)

for $(K_1 \neq 0, K_2 \neq 0)$

If one of the bands, or both, has *K=0*,

$$\langle K_2 I_2 \| T_\lambda \| K_1 = 0, I_1 \rangle = (2I_1+1)^{1/2} C(I_1 \lambda I_2; 0 K_2 K_2) \underline{\langle K_2 | T_{\lambda,\nu=K_2} | K_1 = 0 \rangle} \begin{cases} \sqrt{2} & K_2 \neq 0 \\ 1 & K_2 = 0 \end{cases}$$

BM Vol.II, eq.(4-92)

When the intrinsic states are one-particle configurations, the intrinsic matrix elements of M1, E1 and E2 operators

$$\underline{\langle K_2 | T_{\lambda,\mu} | K_1 \rangle} \qquad \text{and} \qquad \underline{\langle K_2 | T_{\lambda,\mu} | \overline{K}_1 \rangle}$$

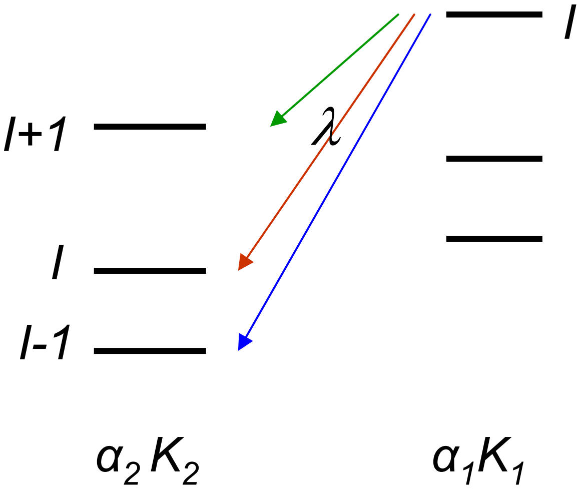
can be evaluated using Tables 1 and 2 appended in the end of Chap.4, depending on whether the wave function of the one-particle configuration is approximated by an [N n_3 Λ Ω] representation or a single-j configuration.

Transitions between two bands with intrinsic configurations α_1,Ω_1 (= K_1) and α_2,Ω_2 (= K_2)

ex. If $(-1)^{I+K}$ term is absent or negligible,

$$B(\lambda;\alpha_1K_1I_1 \to \alpha_2K_2I_2) = \underbrace{C(I_1\lambda I_2;K_1,K_2-K_1,K_2)^2}_{\text{kinematical factor}} \underbrace{\langle\alpha_2K_2|T_\lambda|\alpha_1K_1\rangle^2}_{\text{intrinsic matrix element, common in all transitions}}$$

= 0 for $|I_1 - I_2| > \lambda$ or $|K_1 - K_2| > \lambda$



The ratio of $B(\lambda)$ values between the members of given two bands is obtained from the Clebsch-Gordan coefficients,;

$$C(I_1\lambda I_2;K_1,K_2-K_1,K_2)^2$$

$B(\lambda)$: $B(\lambda)$: $B(\lambda)$

$$\approx C(I\lambda I+1;K_1,K_2-K_1,K_2)^2 : C(I\lambda I;K_1,K_2-K_1,K_2)^2 : C(I\lambda I-1;K_1,K_2-K_1,K_2)^2$$

1) Magnetic dipole (M1) moments and transitions

(One-particle) M1 operator in the intrinsic (= body-fixed) system

$$\overrightarrow{M1} \propto g_R \vec{R} + g_\ell \vec{\ell} + g_s \vec{s}$$

$$(M1)_\nu = \sqrt{\frac{3}{4\pi}}\frac{e\hbar}{2Mc}(g_R R_\nu + g_\ell \ell_\nu + g_s s_\nu)$$

$$\vec{I} = \underline{\vec{R}} + \vec{\ell} + \vec{s}$$

rotational angular momentum of the even-even core

$$\sqrt{\frac{3}{4\pi}}\frac{e\hbar}{2Mc}(\underbrace{g_R I_\nu}_{\text{magnetic moment}} + \underbrace{(g_\ell - g_R)\ell_\nu + (g_s - g_R)s_\nu}_{\text{M1 transition}})$$

$\because)$ The operator I_ν does not make any transitions.

$g_R = Z / A$: a uniform rotation of a charged body

g_R values obtained from observed magnetic moments of 2_1+ states of even-even nuclei using $\mu = g_R I$ are somewhat smaller than Z/A.

$g_R \approx \dfrac{\Im_p}{\Im_p + \Im_n}$ where $\Im$ (= moments of inertia) → larger for Δ → smaller

ex. In even-even rare-earth nuclei the pairing gap $\Delta_p > \Delta_n \longrightarrow g_R < Z / A$

In odd-A nuclei one may expect

$$\begin{cases} g_R > Z / A & \text{for odd-Z nuclei where } \Delta_p \rightarrow smaller \text{ and } \Im_p \rightarrow larger \\ g_R < Z / A & \text{for odd-N nuclei where } \Delta_n \rightarrow smaller \text{ and } \Im_n \rightarrow larger \end{cases}$$

Indeed, one observes $(g_R)_{odd-Z} > (g_R)_{odd-N}$

In practice,

$g_s \rightarrow g_s^{eff}$ and $g_\ell \rightarrow g_\ell^{eff}$

Furthermore, in axially-symmetric deformed nuclei one generally expects

$g_{s_3} \neq g_{s_1} = g_{s_2}$

For one-particle configuration with Ω in Y_{20} deformed shape potential, we have K=Ω, and static magnetic dipole moments and M1 transition probabilities within a given one-particle configuration (i.e. within a given band) can be written

$$\mu = g_R I + (g_K - g_R)\frac{K^2}{I+1} + \delta(K,1/2)\frac{g_K - g_R}{4(I+1)}(2I+1)(-1)^{I+1/2} b$$

$$B(M1; K, I_1 \rightarrow K, I_2 = I_1 \pm 1) = \begin{cases} \frac{3}{4\pi}\left(\frac{e\hbar}{2Mc}\right)^2 (g_K - g_R)^2 K^2 (C(I_1 1 I_2; K0K))^2 & \text{for } K > ½ \\ \frac{3}{16\pi}\left(\frac{e\hbar}{2Mc}\right)^2 (g_K - g_R)^2 \left\{1 + (-1)^{I_> + \frac{1}{2}} b\right\}^2 \big(C(I_1 1 I_2; 1/2, 0, 1/2\big)^2 & \text{for } K = ½ \end{cases}$$

where $I_>$ denotes the greater of I_1 and I_2 ,

$$g_K K = \langle \Omega | g_\ell \ell_3 + g_s s_3 | \Omega \rangle$$

and b (= magnetic decoupling parameter) is defined by

$$(g_K - g_R) b = \langle \Omega = 1/2 | (g_\ell - g_R)\ell_+ + (g_s - g_R) s_+ | \overline{\Omega = 1/2} \rangle$$

which can be rewritten

$$j_+ = \ell_+ + s_+$$

$$(g_K - g_R) b = -(g_\ell - g_R) a - \frac{1}{2}(-1)^\ell (g_s + g_K - 2g_\ell)$$

Observed g_R factors from the $2+$ first rotational states of even-even nuclei

g_R and g_K factors in odd-Z and odd-N nuclei obtained by combining a measured magnetic moment with a measured B(M1) value

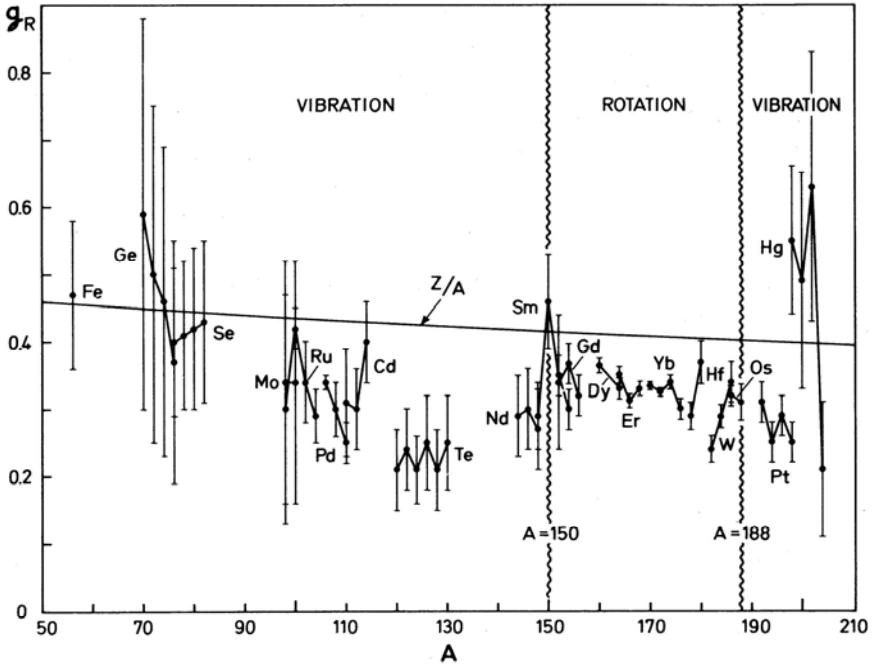


Figure 4-6 g factors for first excited 2+ states in even-even nuclei. The figure is based on

Nucleus	Orbit	g_R	$(g_K)_{obs.}$	$(g_K)_{calc.}$	$(g_s)_{eff}/(g_s)_{free}$
		Odd-proton configurations			
^{153}Eu	413 5/2	0.47	0.67	0.30	0.57
^{159}Tb	411 3/2	0.42	1.83	2.28	0.71
^{165}Ho	523 7/2	0.43	1.35	1.53	0.72
^{169}Tm	411 1/2	0.41	−1.57	−2.44	0.79
			0.32*	−0.05*	0.47*
^{175}Lu	404 7/2	0.31	0.73	0.41	0.55
^{181}Ta	404 7/2	0.29	0.78	0.41	0.48
^{185}Re	402 5/2	0.42	1.61	1.90	0.74
^{187}Re	402 5/2	0.41	1.63	1.90	0.76
		Odd-neutron configurations			
^{155}Gd	521 3/2	0.32	−0.48	−0.61	0.79
^{157}Gd	521 3/2	0.26	−0.53	−0.61	0.87
^{161}Dy	642 5/2	0.21	−0.34	−0.45	0.76
^{161}Dy	523 5/2	0.32	0.17	0.39	0.44
^{163}Dy	523 5/2	0.27	0.25	0.39	0.64
^{167}Er	633 7/2	0.18	−0.26	−0.39	0.67
^{171}Yb	521 1/2	0.28	1.43	1.75	0.82
			−0.48*	−0.79*	0.71*
^{173}Yb	512 5/2	0.28	−0.49	−0.56	0.87
^{177}Hf	514 7/2	0.26	0.21	0.40	0.52
^{179}Hf	624 9/2	0.22	−0.22	−0.35	0.63

Table 5-14 Magnetic g factors for odd-A nuclei ($150 < A < 190$). The experimental data are

ex. Can the measured magnetic moment of the ground state with $I\pi=1/2+$ in ^{11}Be or ^{15}C tell whether the nucleus is spherical or deformed ?

μ_{obs} = -1.6816(8) μ_N in $^{11}\mathrm{Be}_7$ (W.Geithner et al.,PRL,1999)

$|\mu_{obs}|$ = 1.720(9) μ_N in $^{15}\mathrm{C}_9$ (K.Asahi et al.)

The answer is “no”. (I.H. and S.Shimoura, J.Phys.G:**34**(2007)2715.)

For a spherical shape the relevant one-particle orbit must be $s_{1/2}$. Then, $\mu = (0.5)\, g_s^{eff}$ in μ_N.

For a prolately deformed shape the one-particle orbit must be the [220 1/2] orbit.

Then, decoupling parameter $a = 1$,

$$\left.\begin{aligned}
&g_\ell = 0 \quad \text{because of neutron,}\\
&g_K = \langle\Omega|g_\ell \ell_3 + g_s s_3|\Omega\rangle / K \;=\; g_s\\
&(g_K - g_R)b = -(g_\ell - g_R)a - \frac{1}{2}(-1)^\ell (g_s + g_K - 2g_\ell) \;=\; g_R - \frac{1}{2}(g_s + g_K)
\end{aligned}\right\}$$

$$\mu = g_R I + (g_K - g_R)\frac{K^2}{I+1} + \delta(K,1/2)\frac{g_K - g_R}{4(I+1)}(2I+1)(-1)^{I+1/2}b \;=\; (0.5)\, g_s^{eff} \text{ in } \mu_N .$$

(independent of g_R)

2) Electric quadrupole (E2) transitions

With quadrupole deformed intrinsic shape all nucleons collectively contribute to E2 moments.

Intrinsic quadrupole moment with an axially symmetric quadrupole deformation

$$eQ_0 \equiv \left\langle K \left| e\sum_p r_p^2 (3\cos^2\theta_p - 1) \right| K \right\rangle \quad = \left(\frac{16\pi}{5}\right)^{1/2} \left\langle K \left| M(E2, \nu = 0) \right| K \right\rangle$$

where $M(E2,\nu)$ denotes the components referred to the body-fixed system.

The E2 moments referring to the lab. system

$$M(E2,\mu) = \sum_\nu M(E2,\nu) D^2_{\mu\nu}(\omega) \;\Rightarrow M(E2,\nu=0) D^2_{\mu,\nu=0}(\omega)$$

$\omega = (\phi, \theta, \psi)$: Euler angles

The collective E2 moment above connects states belonging to the same rotational band.

$$\boxed{B(E2; KI_1 \to KI_2) = \frac{5}{16\pi} e^2 Q_0^2 C(I_1 2 I_2; K0K)^2}$$

where for $I >> K$,

$$C(I_1 2 I_2; K0K) \approx \begin{cases} \left(\frac{3}{8}\right)^{1/2} & \text{for } I_2 = I_1 \pm 2 \\ \pm\left(\frac{3}{2}\right)^{1/2} \frac{K}{I} & \text{for } I_2 = I_1 \pm 1 \\ -\frac{1}{2} & \text{for } I_2 = I_1 \end{cases}$$

---

ex. In well-deformed rare-earth nuclei,

$$B(E2; K=0, I=2 \to K=0, I=0) \approx 200\ B_W(E2)$$

The static quadrupole moment in the lab system

$$\boxed{Q = C(I2I;K0K)C(I2I;I0I)Q_0 \ = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}Q_0}$$

$$\begin{cases} Q_0 > 0 & : \text{ prolate shape} \\ Q_0 < 0 & : \text{ oblate shape} \end{cases}$$

$I \rightarrow \infty$ keeping a fixed K ;

$$Q \rightarrow -\frac{Q_0}{2}$$

For *I*=*K* (i.e. the band head state in most cases)

$$Q = \frac{I(2I-1)}{(I+1)(2I+3)}Q_0$$

Note $I \rightarrow \infty$ keeping $K = I$;

$Q \rightarrow Q_0$; classical limit

For ellipsoidal shape (or triaxial shape)

K is not a good quantum number,
and the collective E2 moments depend on two intrinsic quadrupole parameters, Q_0 and Q_2 .

$$M(E2,\mu) = \sum_{\nu} M(E2,\nu) D^{2}_{\mu\nu}(\omega) \;\Rightarrow \sqrt{\frac{5}{16\pi}}\, e\left\{ Q_0 D^2_{\mu 0} + Q_2 (D^2_{\mu 2} + D^2_{\mu,-2}) \right\}$$

where

$$Q_0 \equiv \langle \alpha | \sum_p (2x_3^2 - x_1^2 - x_2^2)_p | \alpha \rangle \;\Rightarrow \left(\frac{4}{5} \right) Z R_0^2 \beta \cos\gamma$$

$$Q_2 \equiv \sqrt{\frac{3}{2}} \langle \alpha | \sum_p (x_1^2 - x_2^2)_p | \alpha \rangle \;\Rightarrow \left(\frac{4}{5\sqrt{2}} \right) Z R_0^2 \beta \sin\gamma$$

5 of μ values
(μ = −2, −1, 0, +1, +2)

→ 3 Euler angles
2 intrinsic quadrupole parameters, Q_0 and Q_2

$|\alpha\rangle$: intrinsic state

$$r^2 Y_{20} = \sqrt{\frac{5}{16\pi}} (2x_3^2 - x_1^2 - x_2^2)$$

$$r^2 Y_{22} = \sqrt{\frac{15}{32\pi}} (x_1 + i x_2)^2$$

$$r^2 Y_{2-2} = \sqrt{\frac{15}{32\pi}} (x_1 - i x_2)^2$$

$$\langle I_2 K_2 \| M(E2) \| I_1 K_1 \rangle = (2I_1 + 1)^{1/2} \left(\frac{5}{16\pi} \right)^{1/2} e\{ Q_0 C(I_1 2 I_2; K_1 0 K_2) + Q_2 \big(C(I_1 2 I_2; K_1 2 K_2) + C(I_1 2 I_2; K_1, -2, K_2) \big) \}$$

3) Electric dipole (E1) transitions

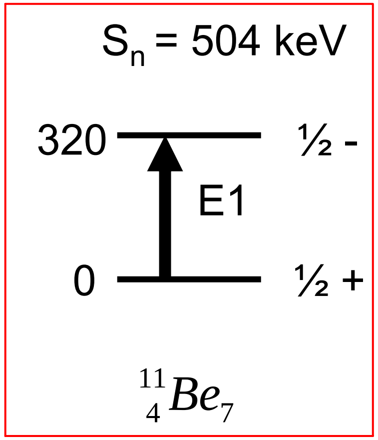
In Y_{20} deformed nuclei one expects

$$e_{pol}(E1, \nu = 0) \neq e_{pol}(E1, \nu = \pm 1)$$

since GDR (Giant Dipole Resonance) in Y_{20} deformed nuclei splits into 2 peaks with $\nu = 0$ and $\nu = \pm 1$

ex.1. In very light **halo** nuclei such as ^{11}Be, one may expect

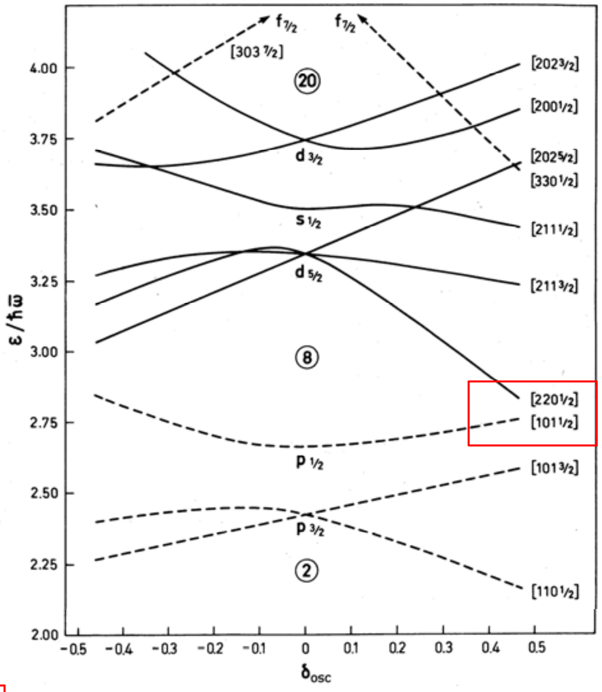
$$\left|e^{p}_{eff}(E1)\right| \approx \frac{N}{A}e \quad \text{and} \quad \left|e^{n}_{eff}(E1)\right| \approx \frac{Z}{A}e \qquad (\%)$$



$\approx p_{1/2}$

$\approx s_{1/2}$

even if the nucleus is deformed.



a) ε($s_{½}$) is pushed down relative to ε($p_{½}$) due to weakly bound

b) {The [220] ½ + wave function ~ $s_{1/2}$} because of halo.

Observed Strong E1 transition,

B(E1;1/2+→1/2-) = (0.115±0.01) e^2 fm^2 = 0.36 B_W(E1) : the largest B(E1) so far observed.

The observed large B(E1) value can be indeed explained by using the value (%) together with a deformation β = 0.7~0.8 . (I.H. and S.Shimoura, J.Phys.G:34(2007)2715.)

Note

½ - at 320 keV ~ [101 1/2]

The ground ½ + ~ [220 1/2]

Asymptotically <[101 1/2]|E1|[2201/2]> = 0

Thus, if it is not a **halo** nucleus,
the E1 transitions are much hindered.

ex.2. Both quadrupole- and octupole deformation → intrinsic dipole moment.

Relatively large $B(E1) = (10^{-2} \sim 10^{-4})\, B_W(E1)$ values are observed between the yrast positive- and negative-parity bands in the Ra-Th region ($N \sim 136$) and Ba-Sm region ($N\sim 88$), especially for high spins.

Those nuclei are supposed to be quadrupole-soft (or deformed) and octupole-soft (or deformed).

Octupole deformation in addition to quadrupole deformation

→ a shift between the center of charge and the center of mass

(Electric charge would move toward the surface region with large curvature.)

→ dipole moment D in the body-fixed frame

In the body-fixed system

c.m. coordinate for neutrons

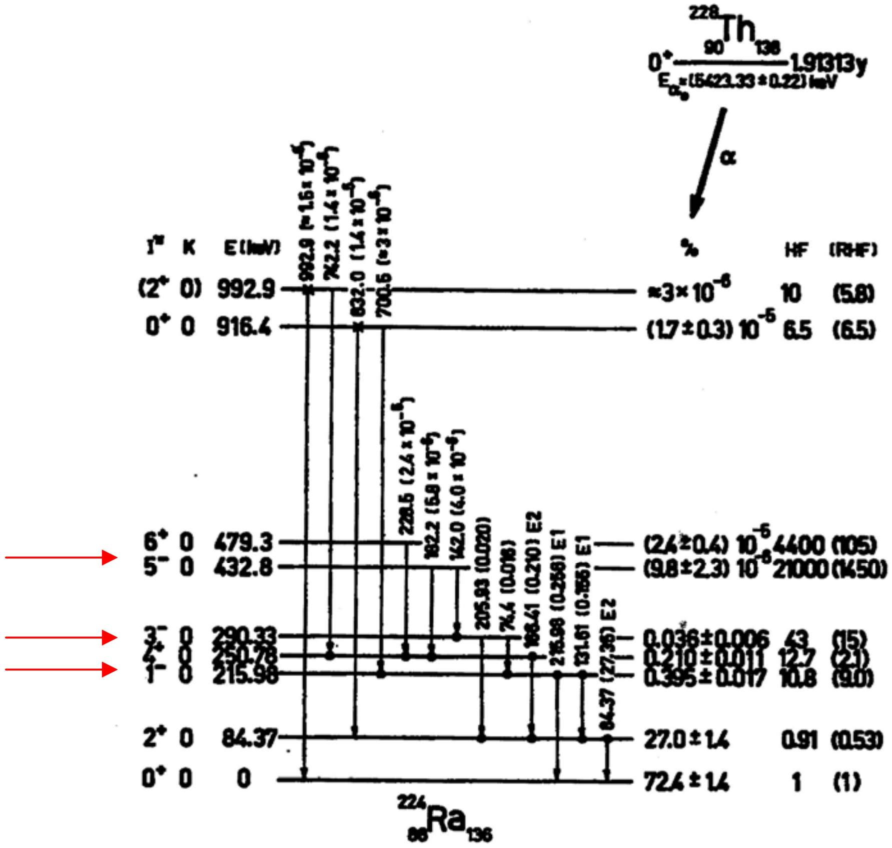
$$e\frac{N}{A}\sum_{i}^{(p)} z_i - e\frac{Z}{A}\sum_{k}^{(n)} z_k = e\frac{NZ}{A}\left(\frac{1}{Z}\sum_{i}^{(p)} z_i - \frac{1}{N}\sum_{k}^{(n)} z_k\right) = e\frac{NZ}{A}\left(\underset{\uparrow}{z_{p-c.m.}} - \overset{\downarrow}{z_{n-c.m.}}\right)$$

c.m. coordinate for protons

Assuming an axially-symmetric shape $\quad D_{\nu=0} \propto (\beta_2\beta_3)_{1-,\nu=0}$

Octupole softness (or deformation) can be seen from observed very low-lying negative-parity levels in even-even nuclei.

Ex. in $^{224}_{88}Ra_{136}$ the lowest 1- state is known only at 216 keV !



If octupole soft in Y_{20} deformation

$K = 0^-$ band :

$I = 1, 3, 5, ,,,,,,$ all with $\pi = -$.

$K = 1^-$ band :

$I = 1,2,3,4,5,,,,$ all with $\pi = -$.

ex.3.

Measured $B(E1) \sim 10^{-5} B_W(E1)$ values in many deformed rare-earth nuclei, which are supposed not to be octupole soft, are difficult to be explained, especially those in odd-A nuclei.