RIKEN Lecture
January 6, 2006

# Description of Nuclei in Real & Model Spaces

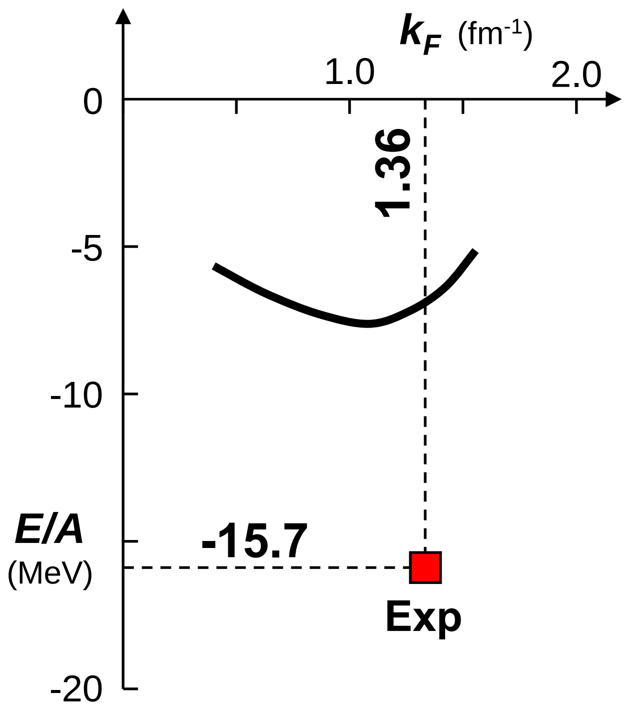
Yoshinori AKAISHI

# Nuclear Physics

**Finite number of nucleons**
**interacting with strong interaction**

**Rich structures**
**on the basis of the nuclear Saturation**

# Nuclear saturation




$E/A = -a_v A + a_s A^{2/3} + \ldots$

Fermi gas $\rho = \frac{2}{3\pi^2} k_F^3$

$A = 4 \sum_{k \le k_F} = 4\left(\frac{L}{2\pi}\right)^3 \frac{4\pi}{3} k_F^3, \quad \rho = A / L^3$

## Infinite nuclear matter

(MeV)

| | H-J |
|---|---|
| *E/A* | -7.8 |
| *KE/A* | 23.9 |
| *PE/A* | -31.7 |
| $^1$S | -15.9 |
| $^3$S | -15.8 |
| $^1$P | 3.2 |
| $^3$P | 0.3 |
| $^1$D | -2.2 |
| $^3$D | -1.3 |

$^1$P, $^3$P, $^1$D, $^3$D: ~0

$^3$S: $v_T \frac{Q}{e} v_T$ is included.

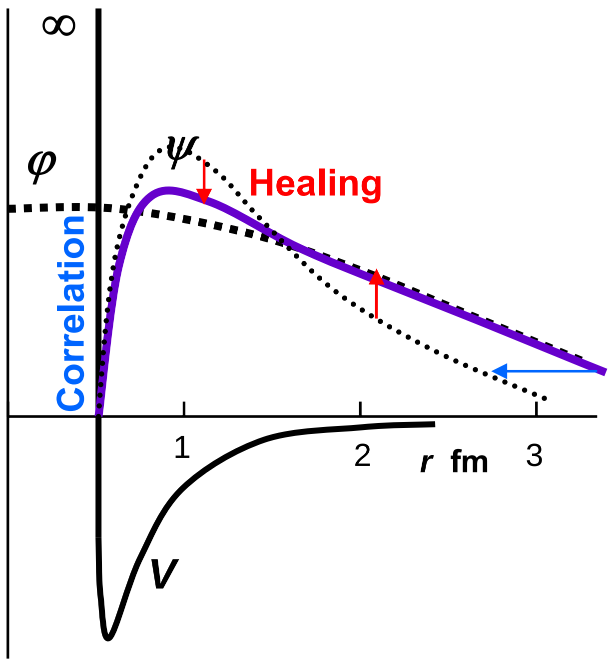
Y. Akaishi, K. Takada & S. Takagi,
Prog. Theor. Phys. 36 (1966) 1135

# Theory of nuclear matter

**K.A. Brueckner & C.A. Levinson**
Phys. Rev. 97 (1955) 1344

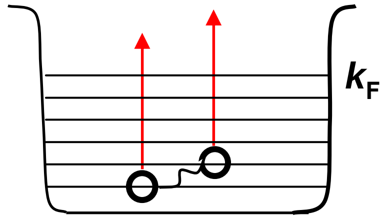
**J. Goldstone**
Proc. Roy. Soc. A239 (1957) 267

**H.A. Bethe**
Phys. Rev. 103 (1958) 241




$$t = v + v\frac{1}{e_0}t \quad \Rightarrow \quad g = v + v\frac{Q}{e}g$$

$$\Delta p \cdot \Delta r \geq \hbar$$




$$e = \varepsilon_1 + \varepsilon_2 - t_1 - t_2$$

***QTQ***

***E/A = aρ + bρ²+...***

**Hole-line expansion method**

**Independent-pair scattering mode**

**⇨ Foundation of Shell Model**

**L.C. Gomes, J.D. Walecka & V.F. Weisskopf**
Ann. Phys. 3 (1958) 241

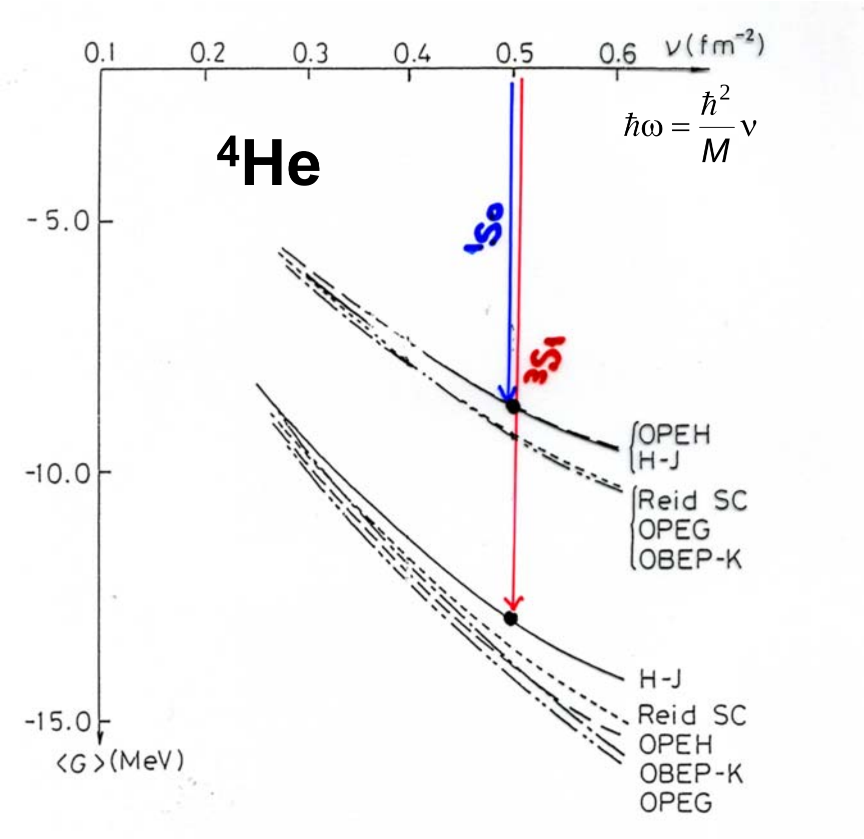
Y. Akaishi, H. Bando, A. Kuriyama & S. Nagata
Prog. Theor. Phys. 40 (1968) 288

**Infinite matter**

# Few-body systems


**Y. Akaishi & S. Nagata,**
**Prog. Theor. Phys. 48 (1972) 133**


### Potential matrix elements



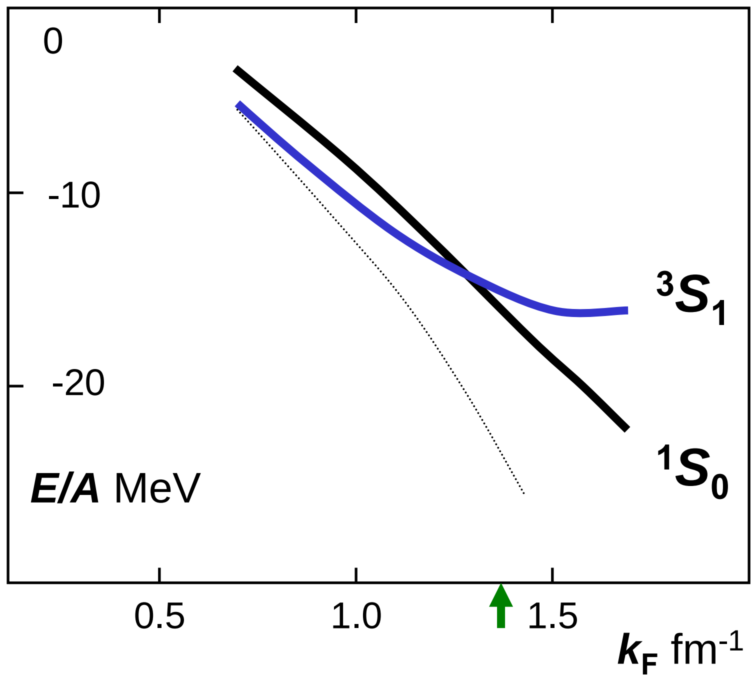

| | Matter | $^4$He |
|---|---|---|
| $^3S/^1S$ ratio | 1.0 | → 1.7 |

**Tensor enhancement**

$$\langle\Phi|V_T|\Phi\rangle + \left\langle\Phi\left|V_T\frac{Q}{e}V_T\right|\Phi\right\rangle$$

# Energy of nuclear matter

H.A. Bethe, Ann. Rev. Nucl. Sci. 21 (1971) 93




## Nuclear saturation

(1) Tensor force
(2) Exchange force
(3) Repulsive core

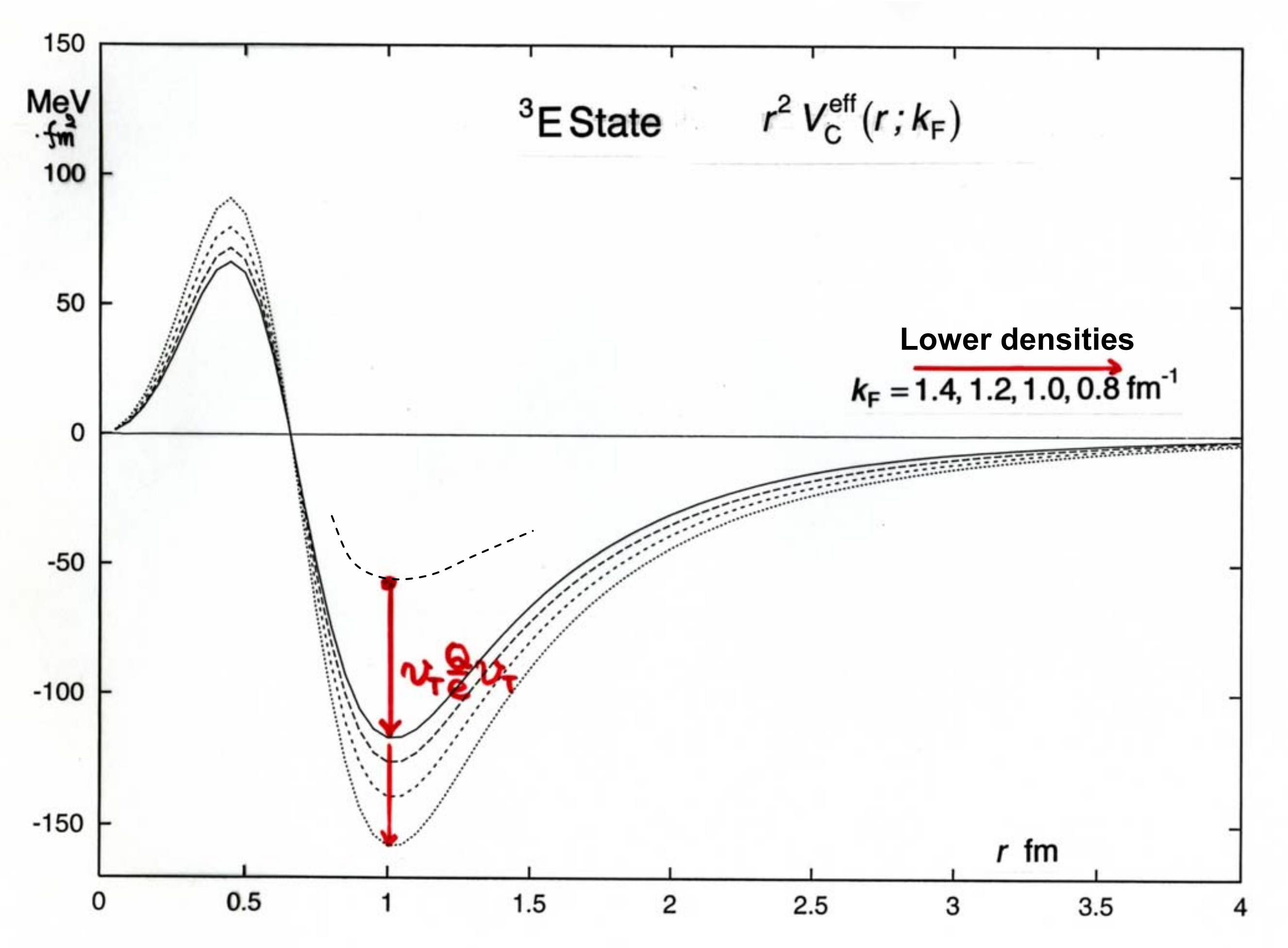
$$\langle \Phi | V_T | \Phi \rangle + \left\langle \Phi \left| V_T \frac{Q}{e} V_T \right| \Phi \right\rangle$$

Majorana exchange

$(1-m)+mP_r$,

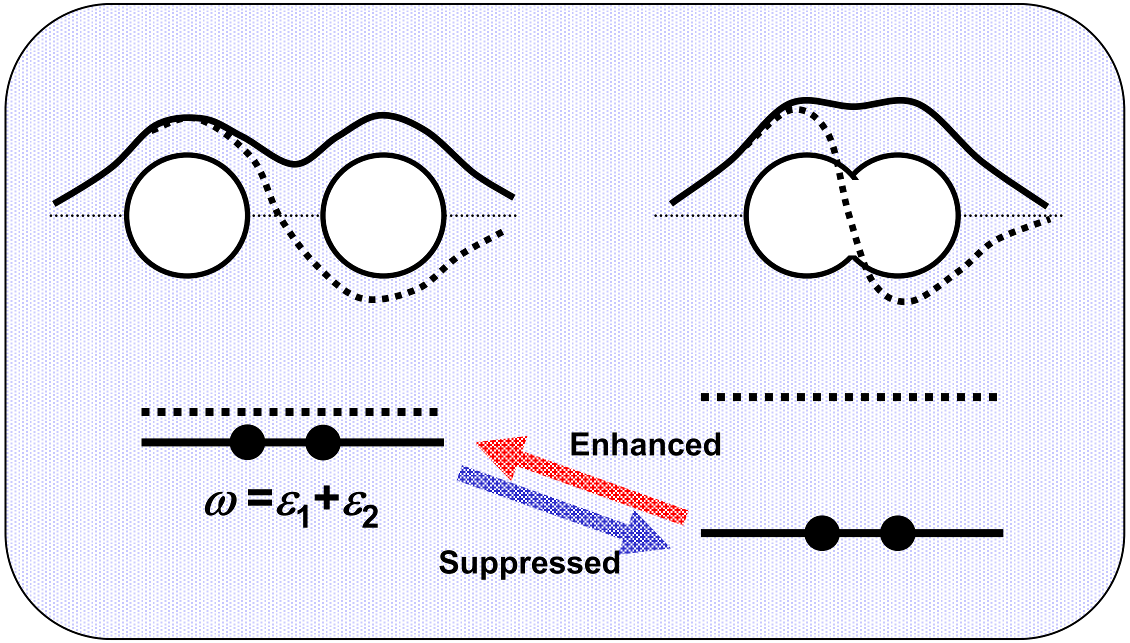
Saturation condition, $m > 0.8$,

is not satisfied.




M. Serra, T. Otsuka et al., Prog. Theor. Phys. 113 (2005) 1009:
*g*-matrix→Relativistic Mean Field

# Tensor force effects on clusterization



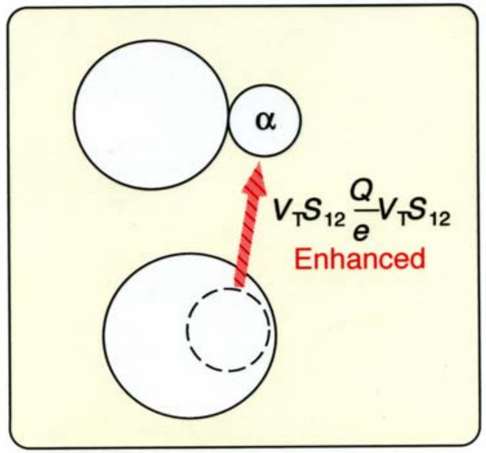

Y. Akaishi, H. Bando & S. Nagata,
Prog. Theor. Phys. Suppl. 52 (1972) 339

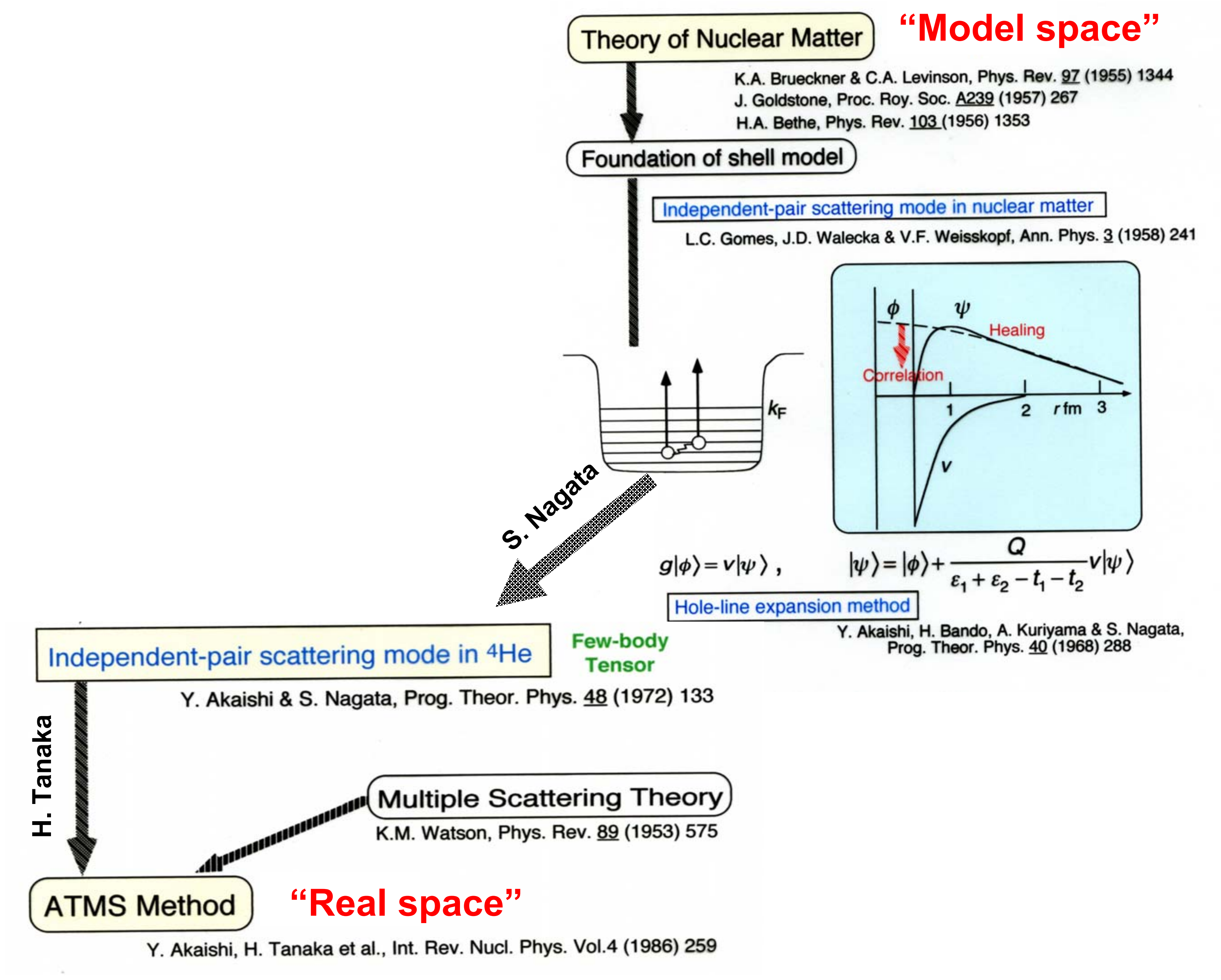
$$V_T S_{12} \frac{Q}{e} V_T S_{12} \approx -8 \frac{V_T^2}{|\Delta|} + 2 \frac{V_T^2}{|\Delta|} S_{12}$$

$$\Delta = \omega - (t_1 + t_2)_{av} \approx -200 \text{ MeV}$$

Central ~ -100 MeV at 1 fm

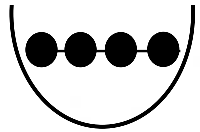
"Essential 4-body"




Theory of Nuclear Matter
"Model space"
K.A. Brueckner & C.A. Levinson, Phys. Rev. 97 (1955) 1344
J. Goldstone, Proc. Roy. Soc. A239 (1957) 267
H.A. Bethe, Phys. Rev. 103 (1956) 1353
Foundation of shell model
Independent-pair scattering mode in nuclear matter
L.C. Gomes, J.D. Walecka & V.F. Weisskopf, Ann. Phys. 3 (1958) 241
$k_F$
$\phi$
$\psi$
Healing
Correlation
1
2
r fm
3
v
S. Nagata
$g|\phi\rangle = v|\psi\rangle$ ,
$|\psi\rangle = |\phi\rangle + \frac{Q}{\varepsilon_1 + \varepsilon_2 - t_1 - t_2} v|\psi\rangle$
Hole-line expansion method
Y. Akaishi, H. Bando, A. Kuriyama & S. Nagata, Prog. Theor. Phys. 40 (1968) 288
Independent-pair scattering mode in $^4He$
Few-body
Tensor
Y. Akaishi & S. Nagata, Prog. Theor. Phys. 48 (1972) 133
H. Tanaka
Multiple Scattering Theory
K.M. Watson, Phys. Rev. 89 (1953) 575
ATMS Method
"Real space"
Y. Akaishi, H. Tanaka et al., Int. Rev. Nucl. Phys. Vol.4 (1986) 259

# Alpha particle

**Model**



$(0s)^4$

$\hbar\omega = 21.6\ \text{MeV}$

$KE = 3 \times \frac{3}{4}\hbar\omega = 48.6\ \text{MeV}$

**Real**

(MeV)

| | | H-J | RSC v8 |
|---|---|---|---|
| Energy | | -20.6 | -21.9 |
| Kin. E | | 131.1 | 103.6 |
| Pot. E | | -151.7 | -125.4 |
| C | 1E | -51.3 | -37.2 |
| | 3E | -26.2 | -0.6 |
| | 1O+3O | -0.4 | 0.5 |
| T | 3E | -69.7 | -89.4 |
| | 3O | -0.5 | -0.7 |
| LS+QLS | | -3.6 | 1.9 |
| P(D) % | | 12.8 | 11.0 |

are 2~2.5 times larger than

The largest contribution

D-state correlation due to tensor force

| Volkov | |
|---|---|
| -29.0 | E |
| 48.6 | KE |
| -77.6 | PE |
| -38.8 | 1E central |
| -38.8 | 3E central |
| 0.0 | |
| 0 | |
| 0 | |
| 0 | |
| 0 | |

M. Sakai, I. Shimodaya, Y. Akaishi, J, Hiura & H. Tanaka, Prog. Theor. Phys. 56 (1974) 32.

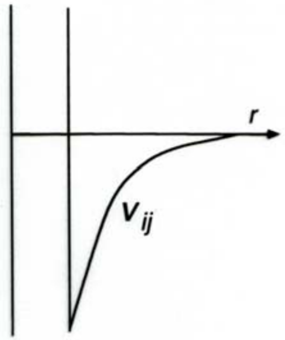
**ATMS**

# Real space vs. model space

**Real space**

$$H|\Psi\rangle = E_0|\Psi\rangle$$

$$H = T + V, \qquad V = \sum_{(ij)} v_{ij}$$




**Transformation**

$$|\Psi\rangle = \hat{F}\,|\Phi\rangle$$

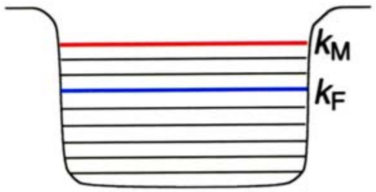
$$\hat{F} = 1 + \frac{Q}{e} V \hat{F}$$

$$e = E_0 - QTQ$$

Now we define $V_M$ so as to satisfy the relation;

$$PV_M|\Phi\rangle = PV|\Psi\rangle\,.$$

**Model space**



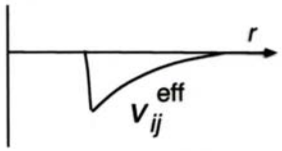

Plane wave basis

Truncated to

$$P = \sum_{k}^{k \le k_M} |\vec{k}\rangle\langle\vec{k}|\,, \quad Q = 1 - P$$

$$H_M|\Phi\rangle = E_0|\Phi\rangle$$

$$H_M = P(T + V_M)P, \quad V_M = \sum_{(ij)} v_{ij}^{eff}$$



## Transformation

$$|\Psi\rangle = \hat{F}\,|\Phi\rangle$$

$$\hat{F} = 1 + \frac{Q}{e} V \hat{F}$$
$$e = E_0 - QTQ$$

i) $P|\Psi\rangle = |\Phi\rangle$ , ii) $\langle\Phi|\Psi\rangle = 1$

---

$$(E_0 - QTQ)|\Psi\rangle = (E_0 - QTQ)|\Phi\rangle + QV|\Psi\rangle$$
$$E_0|\Psi\rangle - TQ|\Psi\rangle = E_0|\Phi\rangle + QV|\Psi\rangle$$
$$E_0|\Psi\rangle - T(1-P)|\Psi\rangle = E_0|\Phi\rangle + (1-P)V|\Psi\rangle$$
$$E_0|\Psi\rangle - T|\Psi\rangle + T|\Phi\rangle = E_0|\Phi\rangle + V|\Psi\rangle - PV|\Psi\rangle$$
$$E_0|\Psi\rangle - H|\Psi\rangle = E_0|\Phi\rangle - T|\Phi\rangle - PV|\Psi\rangle$$

Now we define $V_M$ so as to satisfy the relation;

$$PV_M|\Phi\rangle = PV|\Psi\rangle\ .$$

Then, $(E_0 - H)|\Psi\rangle = 0$

## Reaction matrix

Def. of $g$

$$g_{ij} = v_{ij} + v_{ij}\frac{Q}{e}g_{ij}$$

**Two-body scattering in medium**

Def. of $\hat{F}_{ij}$

$$g_{ij}\hat{F}_{ij} = v_{ij}\hat{F}$$

$$= v_{ij}\left(1+\frac{Q}{e}g_{ij}\right)\hat{F}_{ij}$$

$$= v_{ij}\left(1+\frac{Q}{e}\sum_{(kl)}v_{kl}\hat{F}\right)$$

$$= v_{ij}\left(1+\frac{Q}{e}\sum_{(kl)}g_{kl}\hat{F}_{kl}\right)$$

$$\hat{F} = 1+\sum_{(ij)}\frac{Q}{e}g_{ij}\hat{F}_{ij}$$

$$\hat{F}_{ij} = 1+\sum_{(kl)}{}'\frac{Q}{e}g_{kl}\hat{F}_{kl}$$

**Multiple scattering process**

$$PV|\Psi\rangle = P\sum_{(ij)}v_{ij}\hat{F}|\Phi\rangle = P\sum_{(ij)}g_{ij}\hat{F}_{ij}|\Phi\rangle$$

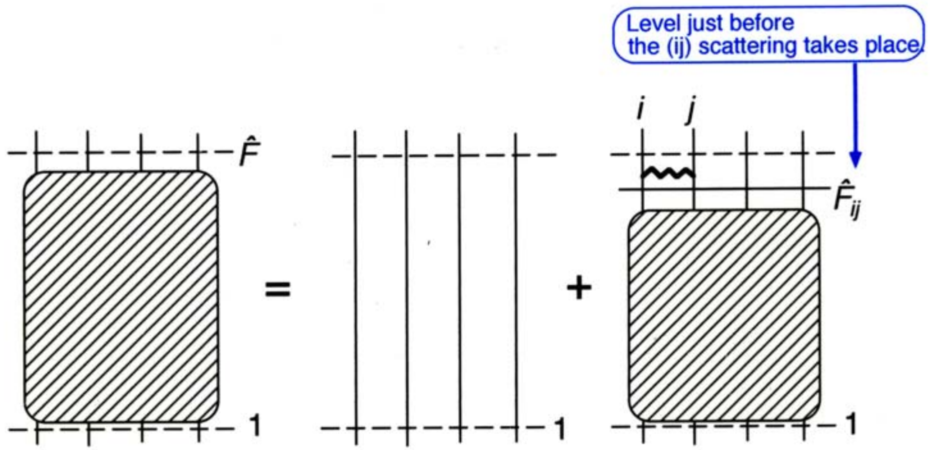
$$= P\sum_{(ij)}g_{ij}|\Phi\rangle + P\sum_{(ijk)}g_{ij}\frac{Q}{e}g_{jk}|\Phi\rangle + .......$$

$$PV_M|\Phi\rangle = P\sum_{(ij)}v_{ij}^{\text{eff}}|\Phi\rangle + .......$$

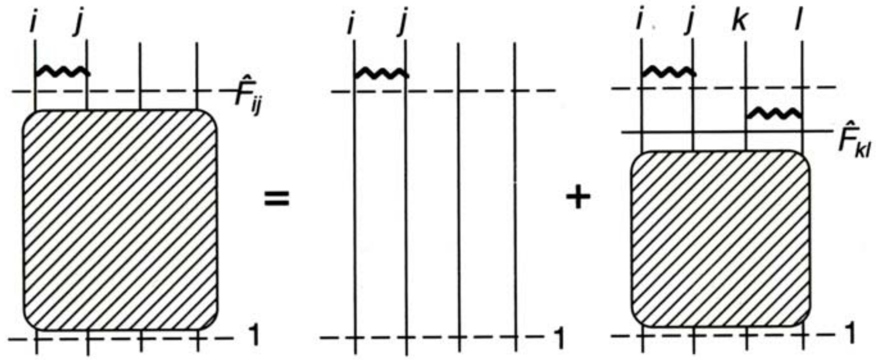
$$v_{ij}^{\text{eff}} = g_{ij}\ , \quad v_{ijk}^{\text{eff}} = g_{ij}\frac{Q}{e}g_{jk}$$

Effective interaction

# Multiple scattering process




$$\hat{F} \quad = \quad 1 \quad + \quad \sum_{(ij)} \frac{Q}{e} g_{ij} \hat{F}_{ij}$$




$$\hat{F}_{ij} \quad = \quad 1 \quad + \quad {\sum_{(kl)}}' \frac{Q}{e} g_{kl} \hat{F}_{kl}$$

**Two-body scattering in medium**

$$g_{ij} = v_{ij} + v_{ij}\frac{Q}{e}g_{ij}$$

## Representation of Multiple Scattering Operator

$$\hat{F} = \hat{F}_{ij} + \frac{Q}{e}g_{ij}\hat{F}_{ij}$$

$$n_{\text{pair}}\hat{F} = \sum_{(ij)}\hat{F}_{ij} + \sum_{(ij)}\frac{Q}{e}g_{ij}\hat{F}_{ij}$$

$$= \sum_{(ij)}\hat{F}_{ij} + \left(\hat{F} - 1\right)$$

$$\hat{F} = 1 + \frac{1}{n_{\text{pair}} - 1}\sum_{(ij)}\left(\hat{F}_{ij} - 1\right)$$

$$\left\{\hat{F} = \left(1 + \frac{Q}{e}g_{ij}\right)\left(\hat{F}_{ij} - 1\right) + \left(1 + \frac{Q}{e}g_{ij}\right)\right\}|\Phi_0\rangle$$

off-shell $\overline{u}_{ij}$

on-shell corell. fn. $u_{ij}$

$$F = \overline{u}_{ij}\left(F_{ij} - 1\right) + u_{ij}$$

$$\left(F_{ij} - 1\right) = \overline{u}_{ij}^{-1}\left(F - u_{ij}\right)$$

$$F = 1 + \frac{1}{n_{\text{pair}} - 1}\sum_{(ij)}\overline{u}_{ij}^{-1}\left(F - u_{ij}\right)$$

$$F = \frac{1}{D}\left[\prod_{(kl)}\overline{u}_{kl}\right]\left[\sum_{(ij)}\frac{1}{\overline{u}_{ij}}u_{ij} - \left(n_{\text{pair}} - 1\right)\right]$$

$$D = \left[\prod_{(kl)}\overline{u}_{kl}\right]\left[\sum_{(ij)}\frac{1}{\overline{u}_{ij}} - \left(n_{\text{pair}} - 1\right)\right]$$

**ATMS**

**Amalgamation of Two-body correlations into Multiple Scattering process**

## $0^{th}$ ATMS

$$\hat{F}_{ij} = \sum_{(kl)}{}' \underbrace{\frac{Q}{e} g_{kl}}_{(u_{kl}-1)} \hat{F}_{kl}$$

Day's approx. ⇒

$$F = \prod_{(kl)} u_{kl}$$

**Jastrow**

## $1^{st}$ ATMS

$$(\hat{F}_{ij} - 1) = \sum_{(kl)}{}' \underbrace{\frac{Q}{e} g_{kl}}_{(u_{kl}-1)} + \sum_{(kl)}{}' \underbrace{\frac{Q}{e} g_{kl}}_{(\bar{u}_{kl}-1)} (\hat{F}_{kl} - 1)$$

## $2^{nd}$ ATMS

$$(\hat{F}_{ij} - 1 - \sum_{(kl)}{}' \frac{Q}{e} g_{kl})$$

$$= \sum_{(kl)}{}' \underbrace{\frac{Q}{e} g_{kl}}_{(\bar{u}_{kl}-1)} \sum_{(mn)}{}' \underbrace{\frac{Q}{e} g_{mn}}_{(u_{mn}-1)} + \sum_{(kl)}{}' \underbrace{\frac{Q}{e} g_{kl}}_{(\tilde{u}_{kl}-1)} (\hat{F}_{kl} - 1 - \sum_{(mn)}{}' \frac{Q}{e} g_{mn})$$

ATMS can improve the wave function
in a systematic way.

# Few-Body Methods

K. Varga and Y. Suzuki,
RIKEN-AF-NP-205 (1995)

| $N$ | $(L,S)J^\pi$ | Method | $E$ (MeV) | | $\langle r^2\rangle^{1/2}$ (fm) | |
|---|---|---|---|---|---|---|
| 3 | $(0,1/2)1/2^+$ | Faddeev [1,2] | −8.25273 | | | Gibson (1981) |
| | | ATMS [4] | −8.26 | | 1.68 | Akaishi (1981) |
| | | CHH [5] | −8.240 | | | Rosati (1989) |
| | | GFMC [9] | −8.26 | ±0.01 | 1.682 | Zabolitzky (1981) |
| | | VMC [34][a] | −8.2689 | ±0.03 | 1.68 | Wiringa (1984) |
| | | SVM | −8.2527 | | 1.682 | Suzuki (1995) |
| 4 | $(0,0)0^+$ | FY [33] | −31.36 | | | Kamada (1992) |
| | | ATMS [4] | −31.36 | | 1.40 | Akaishi (1981) |
| | | CRCG [6] | −31.357 | | | Kamimura (1990) |
| | | GFMC [9] | −31.3 | ±0.2 | 1.36 | Zabolitzky (1981) |
| | | VMC [34][a] | −31.3 | ±0.05 | 1.39 | Wiringa (1984) |
| | | SVM | −31.360 | | 1.4087 | Suzuki (1995) |
| 5 | $(1,1/2)3/2^-$ | VMC [34][a] | −42.98 | ±0.16 | 1.51 | |
| | | SVM | −43.48 | | 1.51 | |
| 6 ($^6$He) | $(0,0)0^+$ | VMC [34][a] | −66.34 | ±0.29 | 1.50 | |
| | | SVM | −66.30 | | 1.52 | |
| 7 ($^7$Li) | $(1,1/2)3/2^-$ | SVM | −83.4 | | 1.68 | |

Malfliet-Tjon potential (Phys. Lett. 56B (1975) 217)

$\hbar^2/M = 41.47$ MeV fm$^2$

$V(r) = \{1458.05 \exp[-3.11\, r) / r - 578.09 \exp[-1.55\, r) / r\}$ in fm, MeV

central potential.

# Alpha particle

Live fish

"Sashimi"

$\hbar\omega = 21.6$ MeV

| (MeV) | H-J | H-J (D.tr.) | H-J (Sc.tr.) | Volkov |
|---|---|---|---|---|
| Energy | -20.6 | -20.6 | -20.6 | -29.0 |
| Kin. E | 131.1 92.5(S) 38.6(D) | 106.1 | 48.6 | 48.6 |
| Pot. E | -151.7 | -126.7 | -69.1 | -77.6 |
| C 1E | -51.3 | -58.8 | -30.0 | -38.8 |
| C 3E | -26.2 | -67.9 | -39.1 | -38.8 |
| C 1O+3O | -0.4 | | | 0.0 |
| T 3E | -69.7 | | | |
| T 3O | -0.5 | | | |
| LS+QLS | -3.6 | | | |
| P(D) % | 12.8 | | | 0 |

Tuncation of D-state

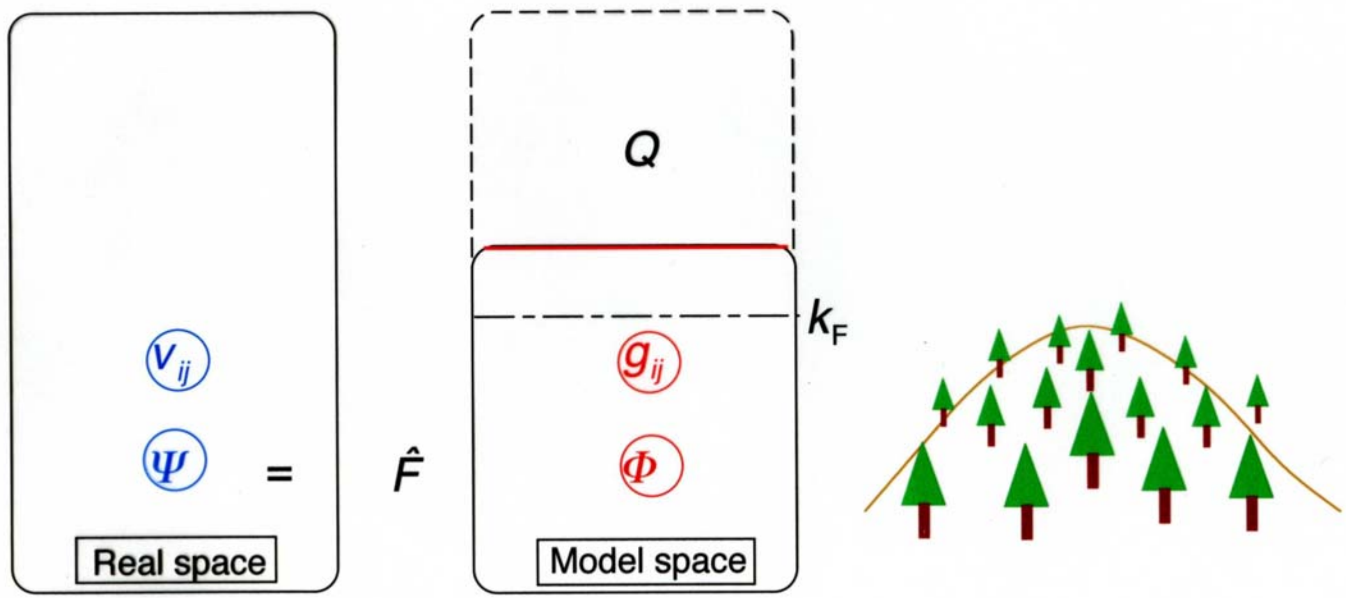
Truncation of short-range correlation

**Structure-dependent $^3S_1$ interaction**

**ATMS**

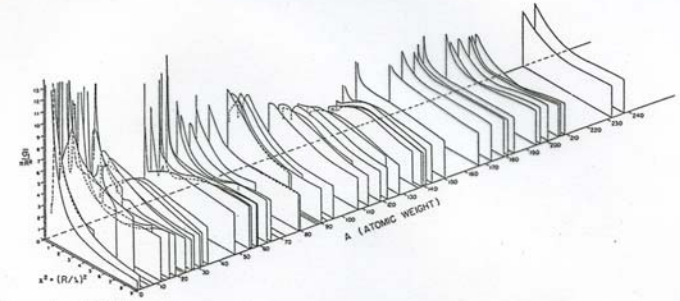
**M. Sakai, I. Shimodaya,Y. Akaishi, J. Hiura & H. Tanaka, Prog. Theor. Phys. Suppl. 56 (1974) 32**

# What is "Model"?




$$g_{ij} = v_{ij} + v_{ij}\frac{Q}{e}g_{ij}$$

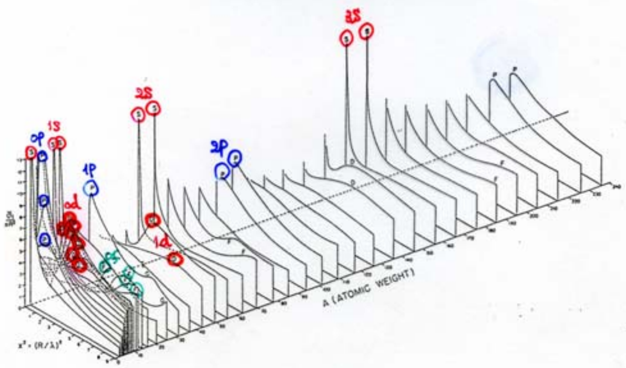
# Optical Model




$E_n = 0 \sim 3\,\mathrm{MeV}$

H.H. Barschall, Phys. Rev. **86** (1952) 431.

$I \approx 500\,\mathrm{keV}$ smearing → Gross structure



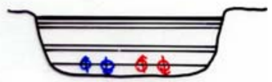

H. Feshbach, C.E. Porter & V.F. Weisskopf,

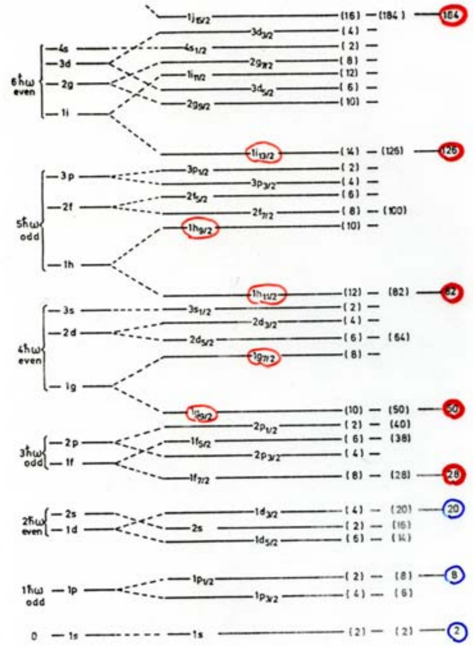
Phys. Rev. **96** (1954) 448.

$$U(r) = \begin{cases} -V_0 - iW_0, & r < R = r_0 A^{-1/3} \\ 0, & r > R \end{cases}$$

$V_0 = 42\,\mathrm{MeV}$, $W_0 = 1.26\,\mathrm{MeV}$, $r_0 = 1.45\,\mathrm{fm}$

# Shell Structure






due to LS splitting

$\hbar\omega \approx 41 A^{-1/3}\,\mathrm{MeV}$

# Recipe for Effective Interaction

Y. Akaishi & K. Takada, Prog. Theor. Phys. 37 (1967) 847

$^1$E central:

$$u_{LL'}^{JS} \quad \Psi / \Phi$$

$$g_C^{L=0}(r) = v_C u_{00}^{00} / j_0(kr) \qquad \to v_C \quad \text{at } r \geq r_{\text{healing}}$$

**Short-range correlation**

$^3$E central:

$$g_C^{L=0}(r) = v_C u_{00}^{11} / j_0(kr) + \sqrt{8} v_T u_{02}^{11} / j_0(kr) \qquad \to v_C$$

$$v_T \frac{Q}{e} v_T$$ **Tensor renormalization**

$^3$E tensor:

$$g_T^{LL'=02}(r) = v_T u_{00}^{11} / j_0(kr) + \frac{1}{\sqrt{8}} \{ v_C - 2v_T - 3v_{LS} - 3v_{QLS} \} u_{02}^{11} / j_0(kr) \qquad \to v_T$$

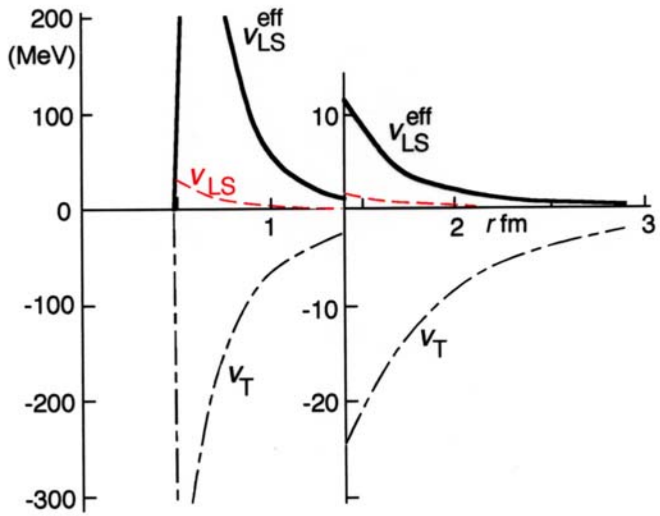
$$g_T^{LL'=22}(r) = \frac{7}{120} [ -3\sqrt{8} v_T u_{20}^{11} / j_2(kr) - 3\{ v_C - 2v_T - 3v_{LS} - 3v_{QLS} \} u_{22}^{11} / j_2(kr) + 5\{ v_C + 2v_T - v_{LS} + 11 v_{QLS} \} u_{22}^{21} / j_2(kr) - 2\{ v_C - \tfrac{4}{7} v_T + 2v_{LS} + 2v_{QLS} \} u_{22}^{31} / j_2(kr) ] \qquad \to v_T + \tfrac{7}{2} v_{QLS}$$

$^3$O spin-orbit:

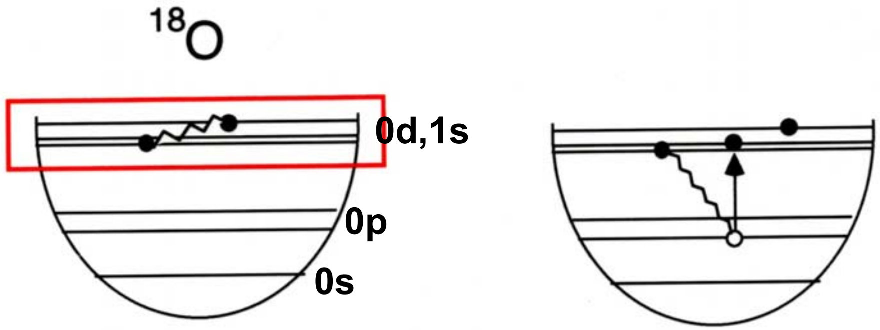
$$g_{LS}^{L=1}(r) = \frac{1}{12} [ -2\{ v_C - 4v_T - 2v_{LS} - 2v_{QLS} \} u_{11}^{01} / j_1(kr) - 3\{ v_C + 2v_T - v_{LS} + 3v_{QLS} \} u_{11}^{11} / j_1(kr) + 5\{ v_C - \tfrac{2}{5} v_T + v_{LS} + v_{QLS} \} u_{11}^{21} / j_1(kr) + 6\sqrt{6} v_T u_{13}^{21} / j_1(kr) ] \qquad \to v_{LS}$$

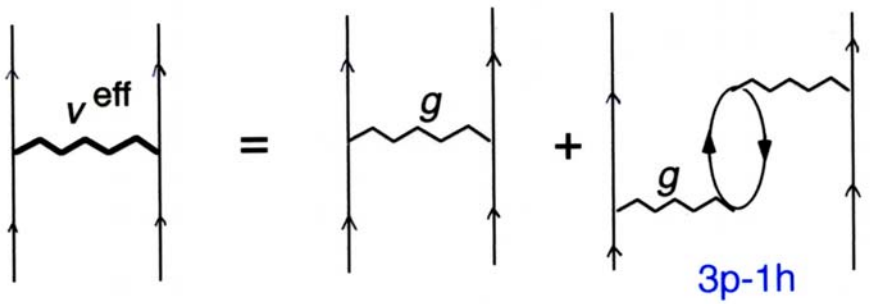
$^3$E spin-orbit

$$g_{LS}^{L=2}(r) = \frac{1}{60} [ -9\sqrt{8} v_T u_{20}^{11} / j_2(kr) - 9\{ v_C - 2v_T - 3v_{LS} - 3v_{LL} \} u_{22}^{11} / j_2(kr) - 5\{ v_C + 2v_T - v_{LS} + 11 v_{LL} \} u_{22}^{21} / j_2(kr) + 14\left\{ v_C - \frac{4}{7} v_T + 2v_{LS} + 2v_{LL} \right\} u_{22}^{31} / j_2(kr) + 24\sqrt{3} v_T u_{24}^{31} / j_2(kr) ] \qquad \to v_{LS}$$



# Theory of Effective Interaction




T.T.S. Kuo & G.E. Brown, Nucl. Phys. 85 (1966) 40

K. Ando, H. Bando, S. Nagata et al., Prog. Theor. Phys. Suppl. No.65 (1979)

## Since then, 20 years have passed.

Y. En'yo:

**What is "Tensor Force"?**

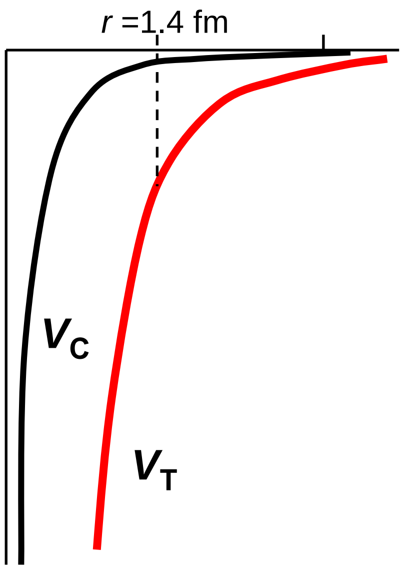
## Realistic NN interaction

$$V = V_{\mathrm{C}}(r) + \boxed{V_{\mathrm{T}}(r) S_{12}} + V_{\mathrm{LS}}(r)\vec{L}\vec{S} + V_{\mathrm{W}} W_{12} + V_{\mathrm{LL}}\vec{L}^2$$

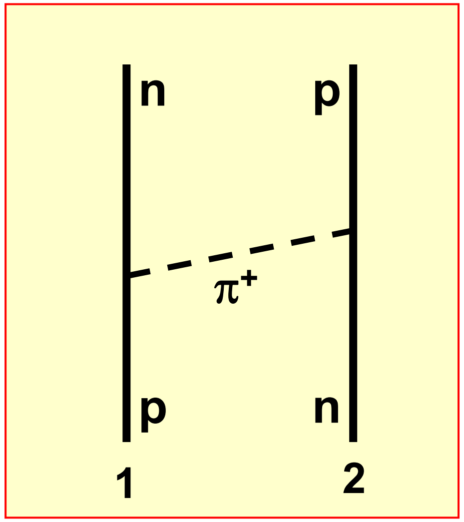
$$S_{12} = 3\frac{(\vec{\sigma}_1\vec{r})(\vec{\sigma}_2\vec{r})}{r^2} - \vec{\sigma}_1\vec{\sigma}_2$$

$$S_{12}\begin{pmatrix} Y_{J-1,1,J} \\ Y_{J,1,J} \\ Y_{J+1,1,J} \end{pmatrix} = \frac{1}{2J+1}\begin{bmatrix} -2(J-1) & 0 & 6\sqrt{J(J+1)} \\ 0 & 2(2J+1) & 0 \\ 6\sqrt{J(J+1)} & 0 & -2(J+2) \end{bmatrix}\begin{pmatrix} Y_{J-1,1,J} \\ Y_{J,1,J} \\ Y_{J+1,1,J} \end{pmatrix}$$

**OPEP**



# OPEP




$$\left(\vec{\nabla}_2^2-\kappa^2\right)\phi^{(+)}=-4\pi\left\{i\frac{f}{\kappa}\left(\vec{\sigma}_1\vec{\nabla}_1\right)\tau_1^{(-)}\right\}\delta\left(\vec{r}_2-\vec{r}_1\right)$$

$$\tau^{(-)}\mathbf{p}=\sqrt{2}\,\mathbf{n}$$

$$\phi^{(+)}\left(\vec{r}_2-\vec{r}_1\right)=\left\{i\frac{f}{\kappa}\left(\vec{\sigma}_1\vec{\nabla}_1\right)\tau_1^{(-)}\right\}\frac{\exp\left(-\kappa\left|\vec{r}_2-\vec{r}_1\right|\right)}{\left|\vec{r}_2-\vec{r}_1\right|}$$

$$\vec{\nabla}_2^2\,\phi=-4\pi\,e_1\delta\left(\vec{r}_2-\vec{r}_1\right)$$

$$\phi=e_1\frac{1}{\left|\vec{r}_2-\vec{r}_1\right|}$$

$$V_{\mathrm{Coul}}\left(\vec{r}_2-\vec{r}_1\right)=e_2e_1\frac{1}{\left|\vec{r}_2-\vec{r}_1\right|}$$

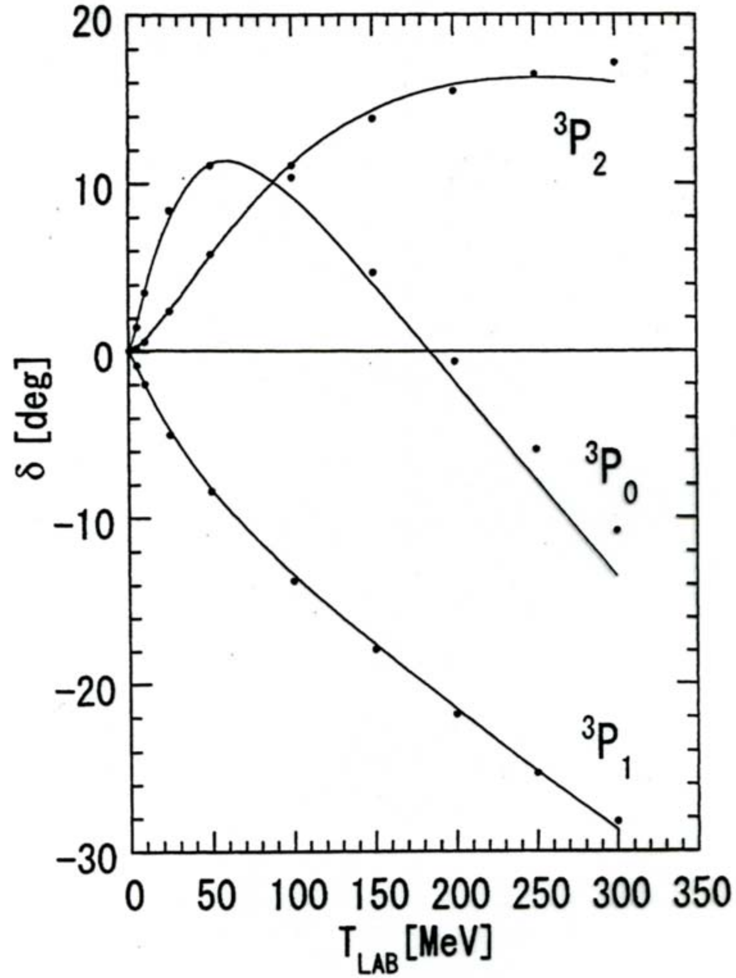
$$V_{\mathrm{OPEP}}\left(\vec{r}_2-\vec{r}_1\right)=\left\{i\frac{f}{\kappa}\left(\vec{\sigma}_2\vec{\nabla}_2\right)\right\}\left\{\tau_2^{(+)}\tau_1^{(-)}+\tau_2^{(0)}\tau_1^{(0)}+\tau_2^{(-)}\tau_1^{(+)}\right\}\left\{i\frac{f}{\kappa}\left(\vec{\sigma}_1\vec{\nabla}_1\right)\right\}\frac{\exp\left(-\kappa\left|\vec{r}_2-\vec{r}_1\right|\right)}{\left|\vec{r}_2-\vec{r}_1\right|}$$

$$\vec{r}=\left|\vec{r}_2-\vec{r}_1\right|,\quad \kappa=\frac{m_\pi c}{\hbar}$$

$$V_{\mathrm{OPEP}}(r)=\left(\frac{f^2}{\hbar c}\right)m_\pi c^2\frac{1}{3}\left(\vec{\tau}_1\vec{\tau}_2\right)\left\{\left(\vec{\sigma}_1\vec{\sigma}_2\right)+S_{12}\left(1+\frac{3}{\kappa r}+\frac{3}{(\kappa r)^2}\right)\right\}\frac{\exp(-\kappa r)}{\kappa r}$$

$$-\left\{3\frac{(\vec{\mu}_1\vec{r})(\vec{\mu}_2\vec{r})}{r^2}-\vec{\mu}_1\vec{\mu}_2\right\}\frac{1}{r^3}$$

# An evidence for OPEP




| | |
|---|---|
| $^3P_0$ | $V_C - 4V_T$ $\quad -2V_{LS}$ |
| $^3P_1$ | $V_C + 2V_T$ $\quad -V_{LS}$ |
| $^3P_2$ | $V_C - (2/5)\,V_T + (6\sqrt{6}/5)\,V_T^* + V_{LS}$ |

$(\vec{\tau}_1\vec{\tau}_2)V_T S_{12}$, $\quad (\vec{\tau}_1\vec{\tau}_2) = -3$ for $^3E$

1 for $^3O$

# The AV8' Potential

R.B. Wiringa, V.G.J. Stoks & R. Schiavilla, Phys. Rev. C**51** (1995) 38.

$$V = v^{\pi} + v_{ST}^{R}$$

OPEP

$$v^{\pi} = f^2 \left( \frac{m}{m_c} \right)^2 \frac{1}{3} mc^2 (\vec{\tau}_1 \vec{\tau}_2) \{ (\vec{\sigma}_1 \vec{\sigma}_2) Y_m(r) + T_m(r) S_{12} \}$$

$$Y_m(r) = \frac{e^{-mr}}{mr} \left( 1 - e^{-cr^2} \right)$$

$$T_m(r) = \left\{ 1 + \frac{3}{mr} + \frac{3}{(mr)^2} \right\} \frac{e^{-mr}}{mr} \left( 1 - e^{-cr^2} \right)^2$$

$$f^2 = 0.075, \quad m = \frac{1}{3}(m_0 + 2m_c), \quad c = 2.1\,\mathrm{fm}^{-2}$$

$$v_{ST}^{R} = v_{ST}^{C} + v_{ST}^{T} S_{12} + v_{ST}^{LS} \vec{L}\vec{S}$$

# Alpha particle

| (MeV) | AV8' |
|---|---|
| Energy | -25.9 |
| Kin. E | 102.4 |
| Pot. E | -128.3 |
| C: 1E, 3E, 1O+3O | -55.3 |
| T: SD, DD | -68.4 |
| LS | -4.7 |
| *P*(D) % | 13.9 |

Phys. Rev. C64 (2001) 044001
Benchmark test calculation of 4N

**FY**
H. Kamada, W. Gloeckle et al.
**CRCGV**
M. Kamimura, E. Hiyama et al.
**SVM**
Y. Suzuki, K. Varga et al.
**HH**
M. Viviani, A. Kievsky et al.
**GFMC**
J. Carlson, R.B. Wiringa et al.
**NCSM**
P. Navratil, B.R. Barrett et al.

| AV8' (Sc&D tr.) |
|---|
| -25.3 |
| 54.1 |
| -79.4 |
| -32.0 |
| -47.4 |

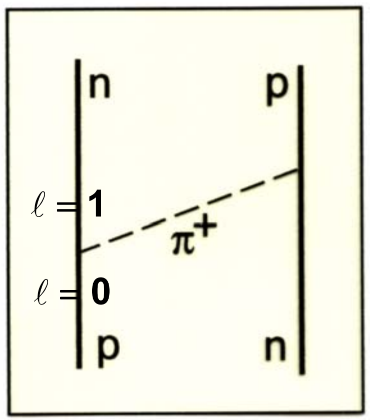
Ordinary
single particle model

# “Is the effective model unique?”

**2002**

**K. Ikeda**

**Tensor ⇨ pion**



## Tensor BHF Calculation of $^4$He

A challenge @ the citadel of standard S. M.

$$\Phi_{\text{intr}} = \prod_{k=1}^{4} F(\vec r_k; \vec\sigma_k, \vec\tau_k)\,\chi_{\text{spin-isospin}}$$

$$F(\vec r_k; \vec\sigma_k, \vec\tau_k) = \left\{ f_s(r_k) - i\underline{(\vec\sigma_k \vec r_k)} f_p(r_k)\underline{\hat g(\vec\tau_k)} \right\}$$

**Parity** $(\vec\sigma\,\vec r)\alpha Y_{00} = -r\left|(\ell = 1, s = \tfrac{1}{2}) j = j_z = \tfrac{1}{2}\right\rangle$

**Charge**

$$\hat g(\vec\tau)p = \frac{1}{2}(1-i)p - \sqrt{\frac{1}{2}}n$$

$$\hat g(\vec\tau)n = \frac{1}{2}(1+i)n + \sqrt{\frac{1}{2}}p$$

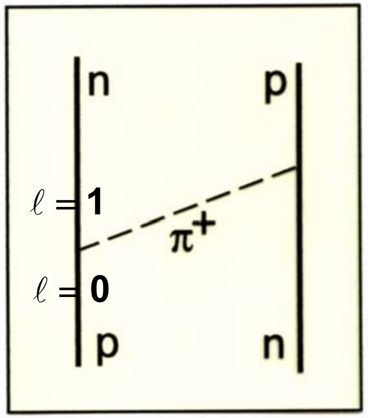
$\pi^+$, $\pi^0$, $\pi^-$ coherence

Projection: $\Psi = P^\tau P^\pi \Phi_{\text{intr}}$

## Ikeda's idea

**Tensor ⇨ pion**

**Parity- & charge-mixed s.p. state**

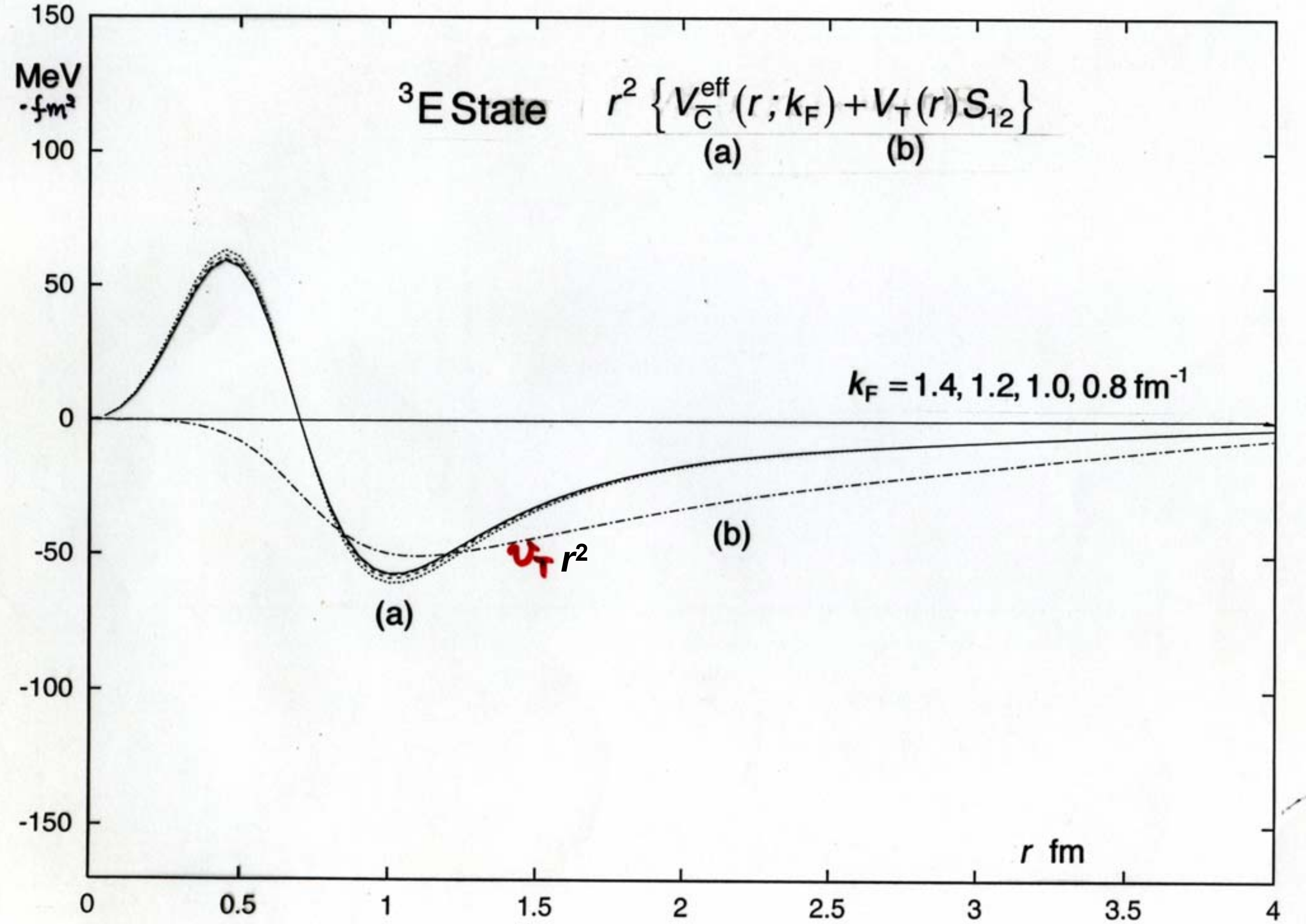



$$f_s(r) = \sum_{n=1}^{12} C_n \exp\left\{-\left(\frac{r}{b_n}\right)^2\right\}, \quad f_p(r) = \sum_{n=1}^{12} D_n \exp\left\{-\left(\frac{r}{b_n}\right)^2\right\}$$

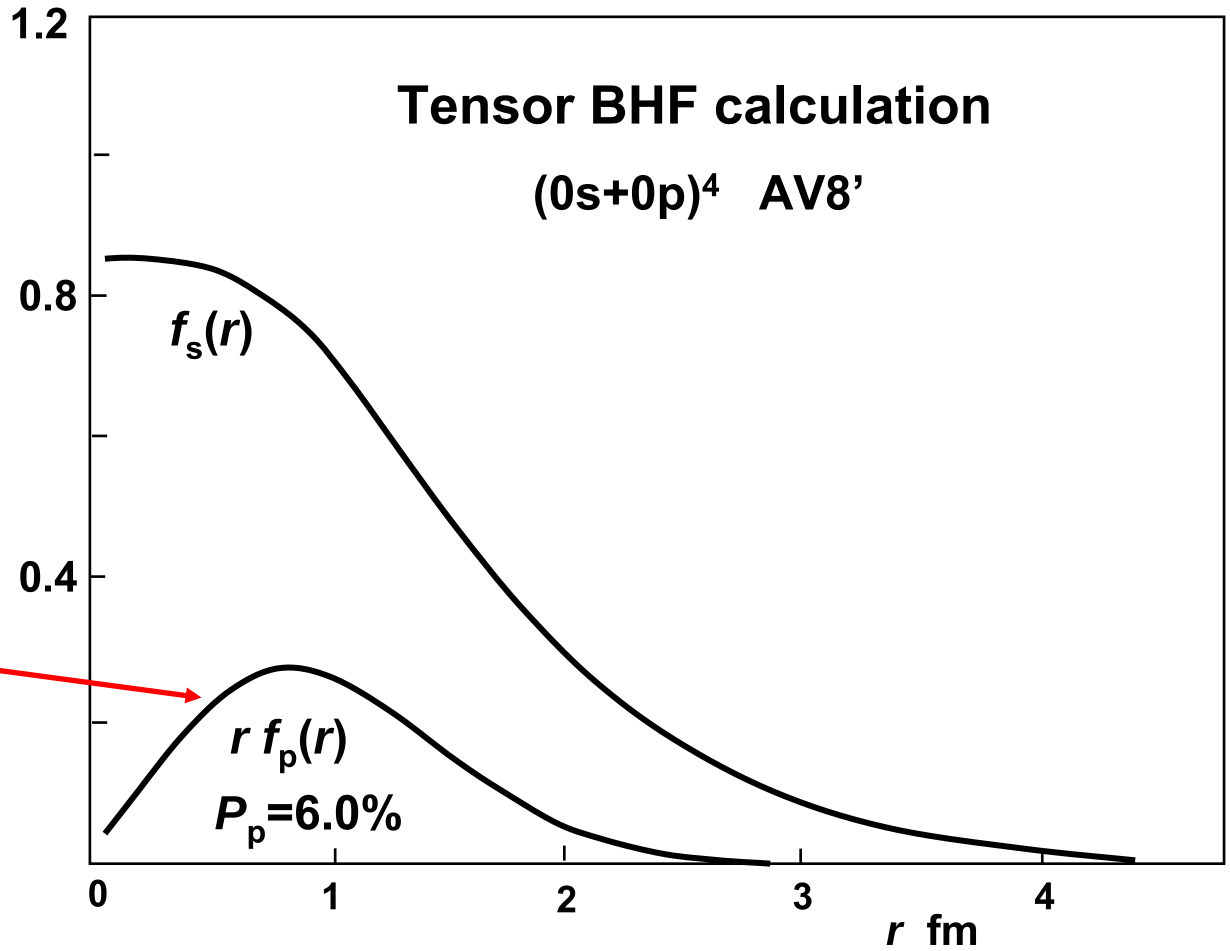
Complex ($D_n$)

$b_n / b_{n-1} = c;\quad b_1 = 0.1\,\text{fm},\quad b_{12} = 6.0\,\text{fm}$

Parameter search: Simplex method



150
MeV
·fm²
$^3$E State
$r^2 \left\{ V_{\overline{C}}^{\mathrm{eff}}(r\,;k_F) + V_T(r) S_{12} \right\}$
(a)
(b)
100
50
$k_F = 1.4, 1.2, 1.0, 0.8\ \mathrm{fm}^{-1}$
0
(b)
-50
$v_T\, r^2$
(a)
-100
-150
r fm
0
0.5
1
1.5
2
2.5
3
3.5
4




Mixed up!

$\hbar\omega_p > \hbar\omega_s$

Y. Kanada-En'yo

# Alpha particle

| (MeV) | | | AV8' |
|---|---|---|---|
| Energy | | | -25.9 |
| Kin. E | | | 102.4 |
| Pot. E | | | -128.3 |
| C | 1E | | -55.3 |
| | 3E | | |
| | 1O+3O | | |
| T | SD | | -68.4 |
| | DD | | |
| LS | | | -4.7 |
| *P*(D) % | | | 13.9 |

Phys. Rev. C64 (2001) 044001
Benchmark test calculation of 4N

| AV8' (Sc tr.) |
|---|
| -7.6 |
| 58.6 |
| -66.2 |
| -25.8 ] -35.1<br>-9.3 |
| -31.1 |
| *P*(p)=6.0 % |

**Parity- & charge-mixed single particle model**

"Copernican"
Heliocentric

| AV8' (Sc&D tr.) |
|---|
| -25.3 |
| 54.1 |
| -79.4 |
| -32.0<br>-47.4 |

**Ordinary single particle model**

"Ptolemaic"
Geocentric

## Wave function of $^4$He

$$\Psi = \Psi_S + \Psi_D$$

$$\Psi_S = f^S\{0,0\}^A + \left(f^M_{m1}\{0,0\}^M_{m2} - f^M_{m2}\{0,0\}^M_{m1}\right)$$

Principal S    Mixed S'

$$\Psi_D = \sum_{M_L}(22M_LM_S|00)\left(g^M_{m1,M_L}\{0,2\}^M_{m2,M_S} - g^M_{m2,M_L}\{0,2\}^M_{m1,M_S}\right)$$

Mixed D

$$g^M_{m,M_L} = \sqrt{\tfrac{10}{3}}\sum_{(ij)} c^{ij}_m w^{TE}_{ij}\left[\vec{r}_{ij}\otimes\vec{r}_{ij}\right]^{(2)}_{M_L}\Phi^S$$

## Spin functions

$S \rightarrow$ partition $\left[\frac{N}{2}+S, \frac{N}{2}-S\right]$

$$\Sigma^2 = \left[[s(1)s(2)]^1[s(3)s(4)]^1\right]^2, \quad S=2 \qquad [4]$$

$$\Sigma^0_{m1} = \left[[s(1)s(2)]^1[s(3)s(4)]^1\right]^0, \quad S=0$$

$$\Sigma^0_{m2} = \left[[s(1)s(2)]^0[s(3)s(4)]^0\right]^0, \quad S=0$$

[22]

## Isospin-spin functions  $[T, S]^R$

$$\{0,0\}^A = \sqrt{\tfrac{1}{2}}\left[T^0_{m1}\Sigma^0_{m2} - T^0_{m2}\Sigma^0_{m1}\right]$$

$$\{0,0\}^M_{m1} = \sqrt{\tfrac{1}{2}}\left[T^0_{m2}\Sigma^0_{m2} - T^0_{m1}\Sigma^0_{m1}\right] \qquad \{0,2\}^M_{m1} = T^0_{m1}\Sigma^2$$

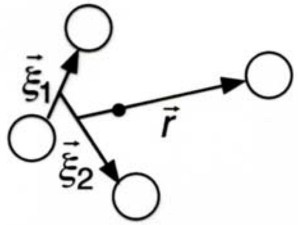
$$\{0,0\}^M_{m2} = \sqrt{\tfrac{1}{2}}\left[T^0_{m1}\Sigma^0_{m2} + T^0_{m2}\Sigma^0_{m1}\right] \qquad \{0,2\}^M_{m2} = T^0_{m2}\Sigma^2$$

# Momentum Distribution of N in $^4$He

"Density correlation"

$$W(\vec{p}) = (2\pi)^{-3} \int d\vec{r} d\vec{r}' \exp(i\vec{p}(\vec{r} - \vec{r}'))\, \rho(\vec{r}, \vec{r}')$$

$$\rho(\vec{r}, \vec{r}') = \left(\frac{4}{3}\right)^3 \int\int d\vec{\xi}_1 d\vec{\xi}_2\, \Psi^*\left(\vec{\xi}_1, \vec{\xi}_2, \frac{4}{3}\vec{r}\right) \Psi\left(\vec{\xi}_1, \vec{\xi}_2, \frac{4}{3}\vec{r}'\right)$$




89 % 11%

$$W(\vec{p}) = C\left\{\exp\left(-B\vec{p}^2\right) + s\, \exp\left(-B\vec{p}^2 / t\right)\right\}$$

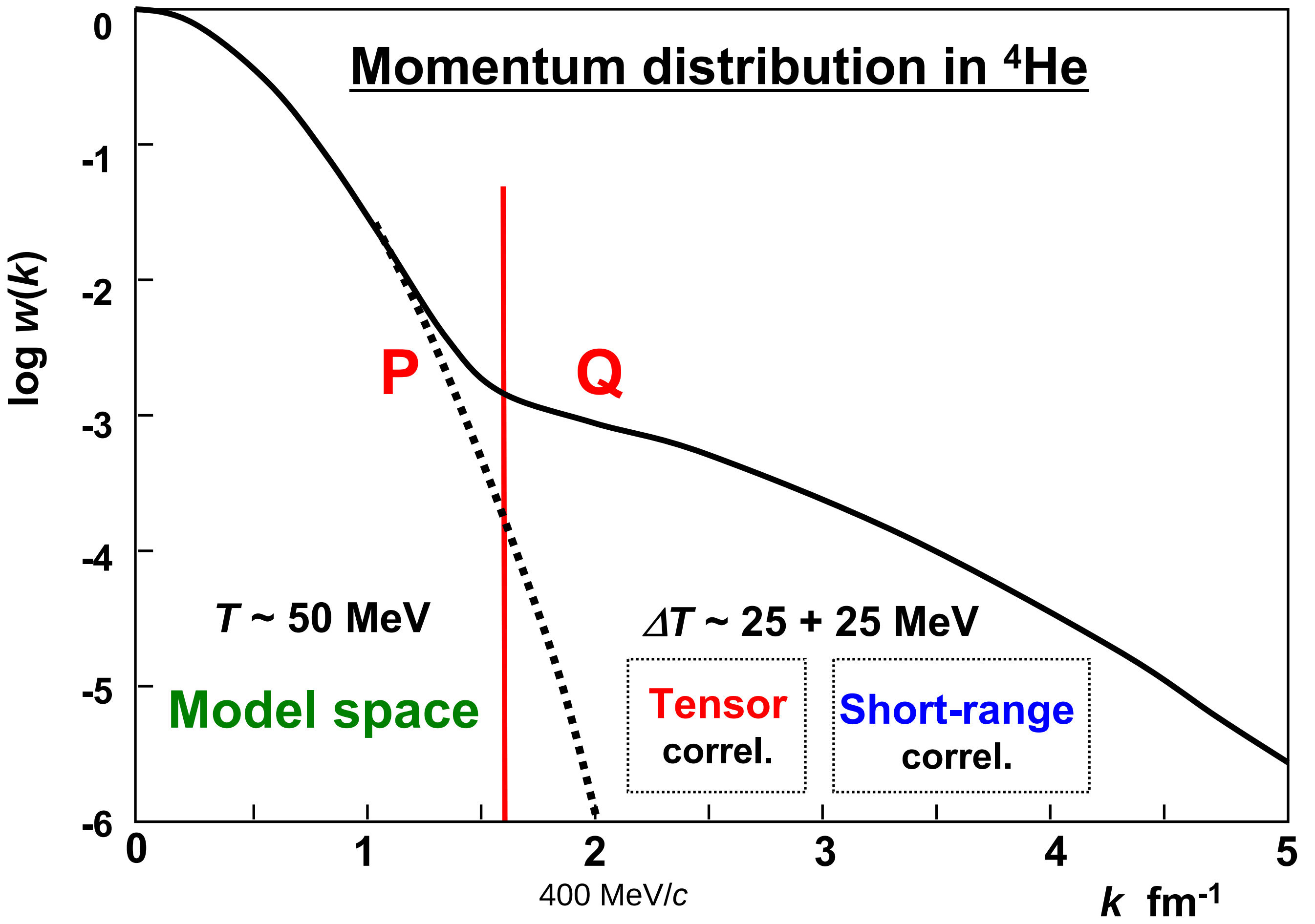
$$B = 1.79\ \text{fm}^2, \quad t = 12, \quad s = 0.00286.$$

$$\text{K.E.} = (A-1)\frac{A-1}{A}\int d\vec{p}\, W(\vec{p})\, \frac{\hbar^2}{2M}\vec{p}^2$$

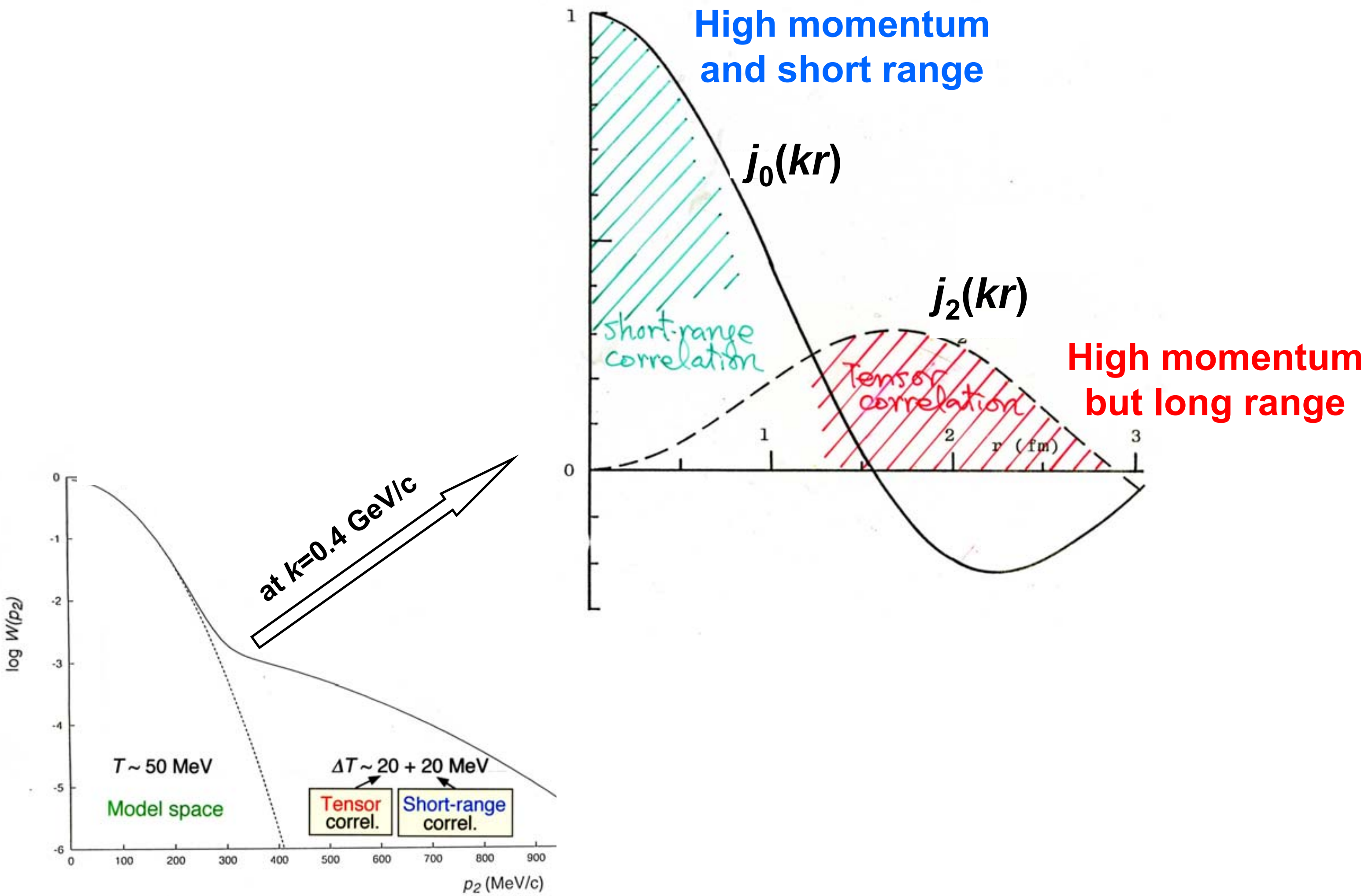
## Form factor

"Density fluctuation"

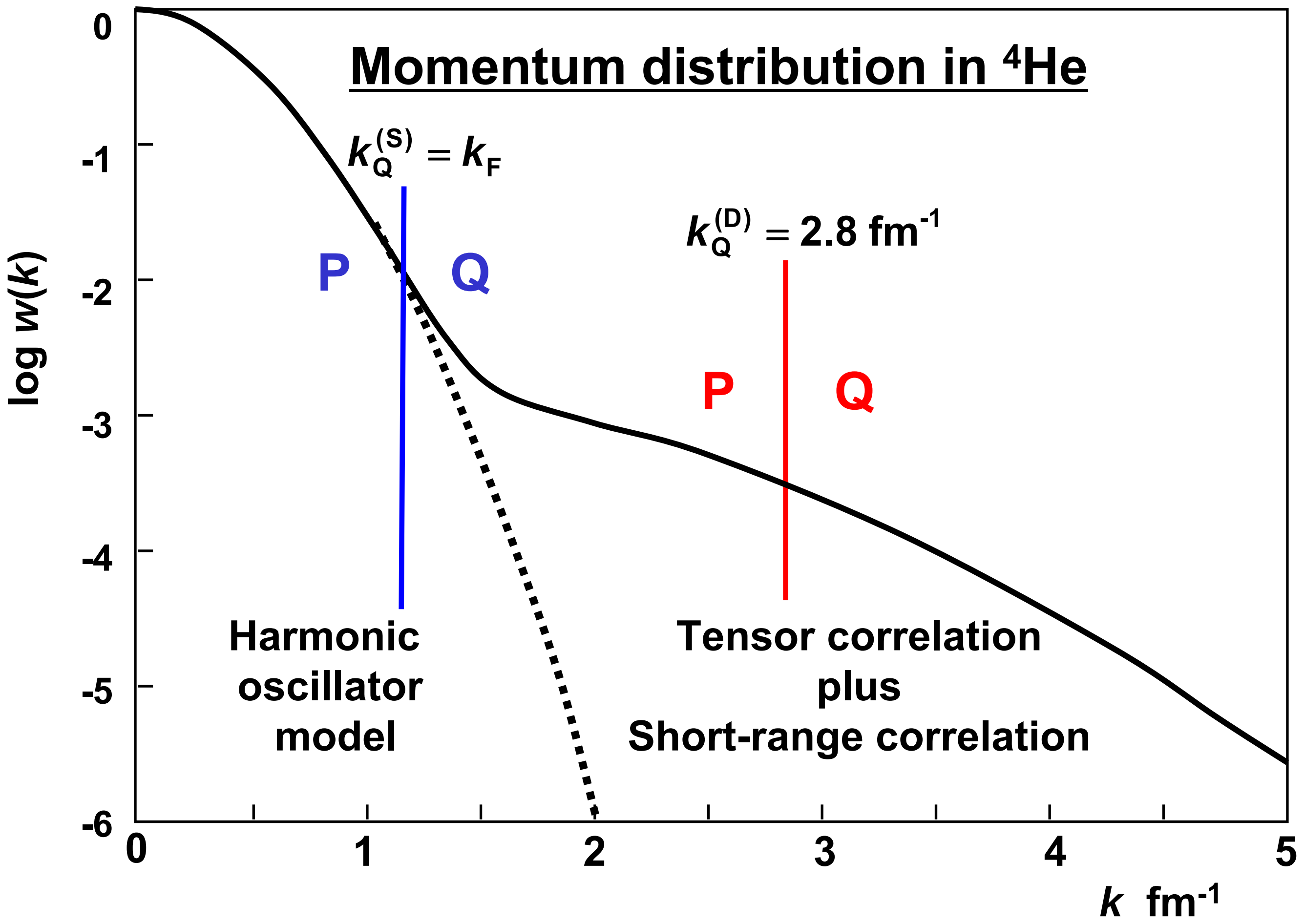
$$F(\vec{p}) = \int d\vec{r}\ \exp(i\vec{p}\vec{r})\, \rho(\vec{r}, \vec{r})$$

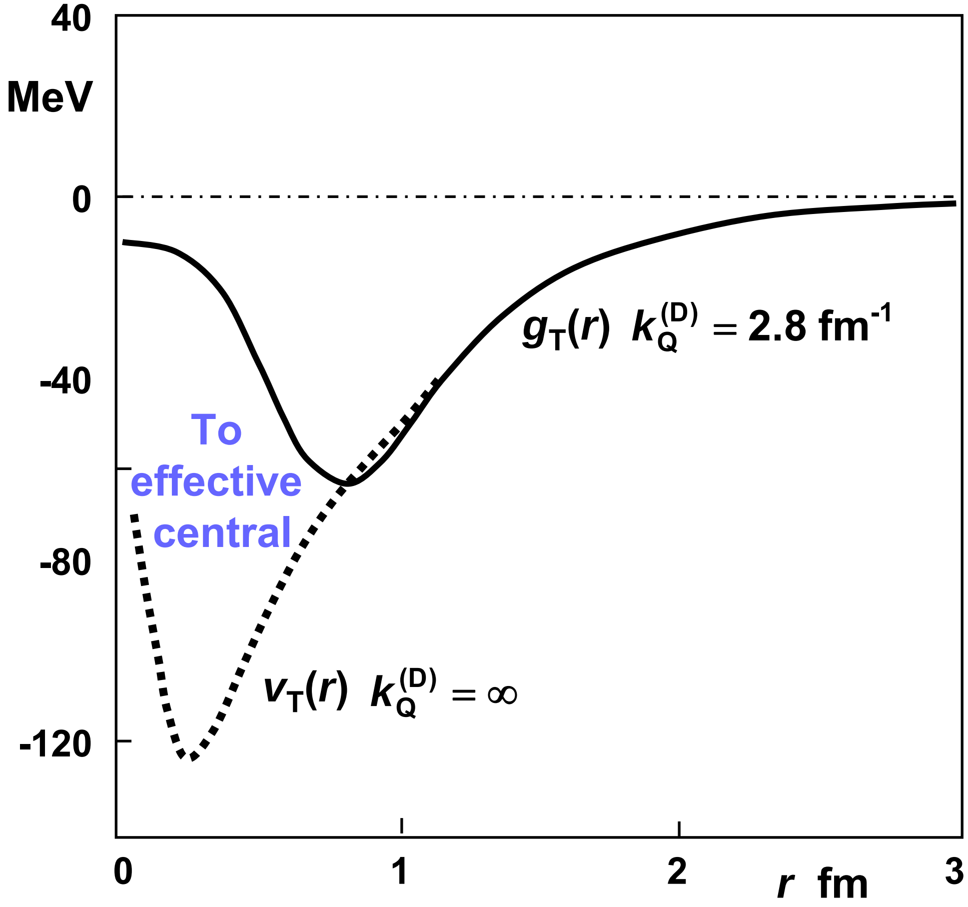


Momentum distribution in 4He
log w(k)
0
-1
-2
-3
-4
-5
-6
P
Q
T ~ 50 MeV
Model space
ΔT ~ 25 + 25 MeV
Tensor
correl.
Short-range
correl.
0
1
2
3
4
5
400 MeV/c
k fm-1

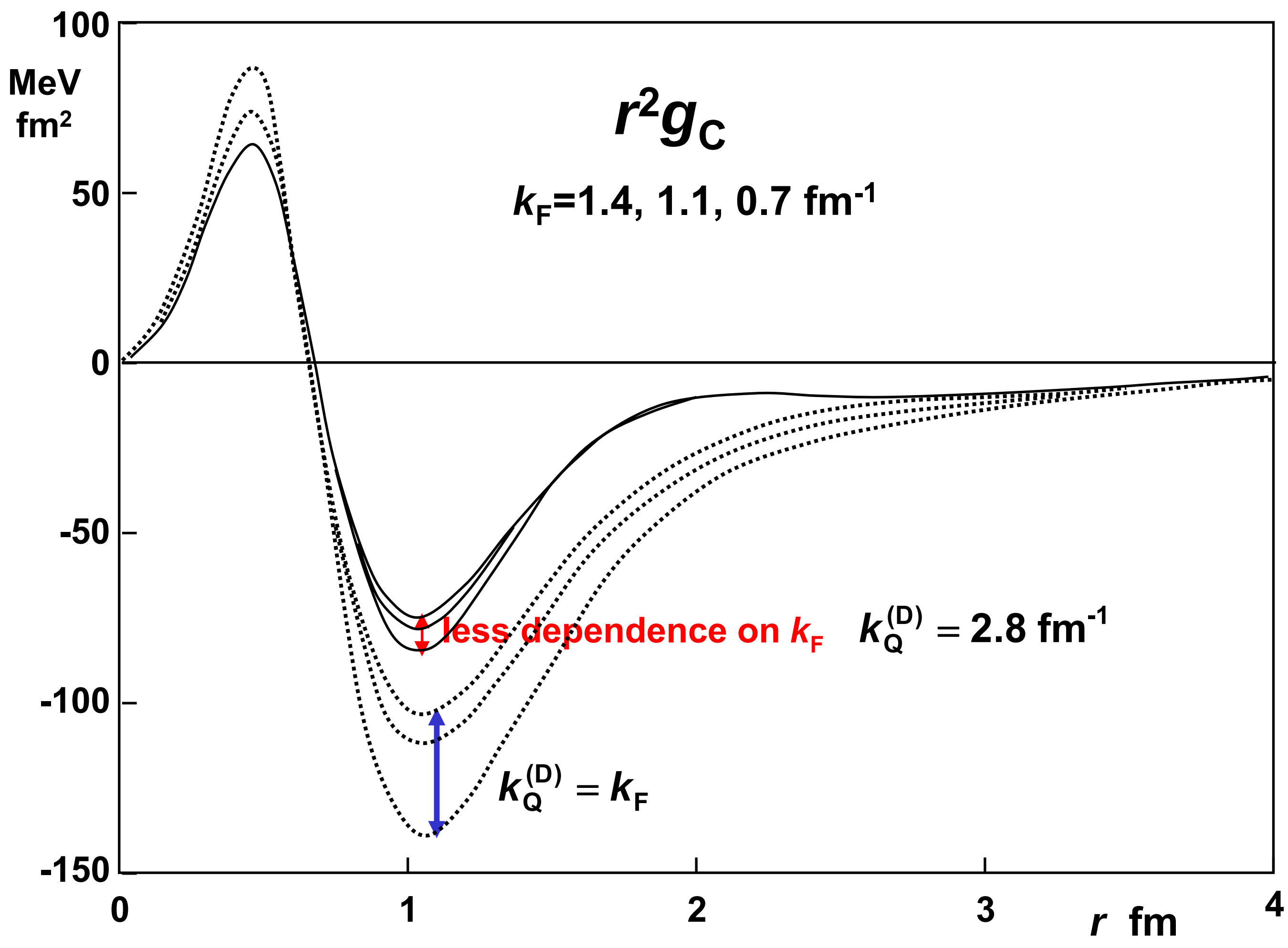


High momentum
and short range
$j_0(kr)$
$j_2(kr)$
High momentum
but long range
short-range
correlation
Tensor
correlation
r (fm)
at k=0.4 GeV/c
log W(p2)
p2 (MeV/c)
T ~ 50 MeV
Model space
ΔT ~ 20 + 20 MeV
Tensor
correl.
Short-range
correl.


Momentum distribution in ⁴He
log w(k)
0
-1
-2
-3
-4
-5
-6
0
1
2
3
4
5
k fm⁻¹
k_Q^(S) = k_F
k_Q^(D) = 2.8 fm⁻¹
P
Q
P
Q
Harmonic oscillator model
Tensor correlation plus Short-range correlation

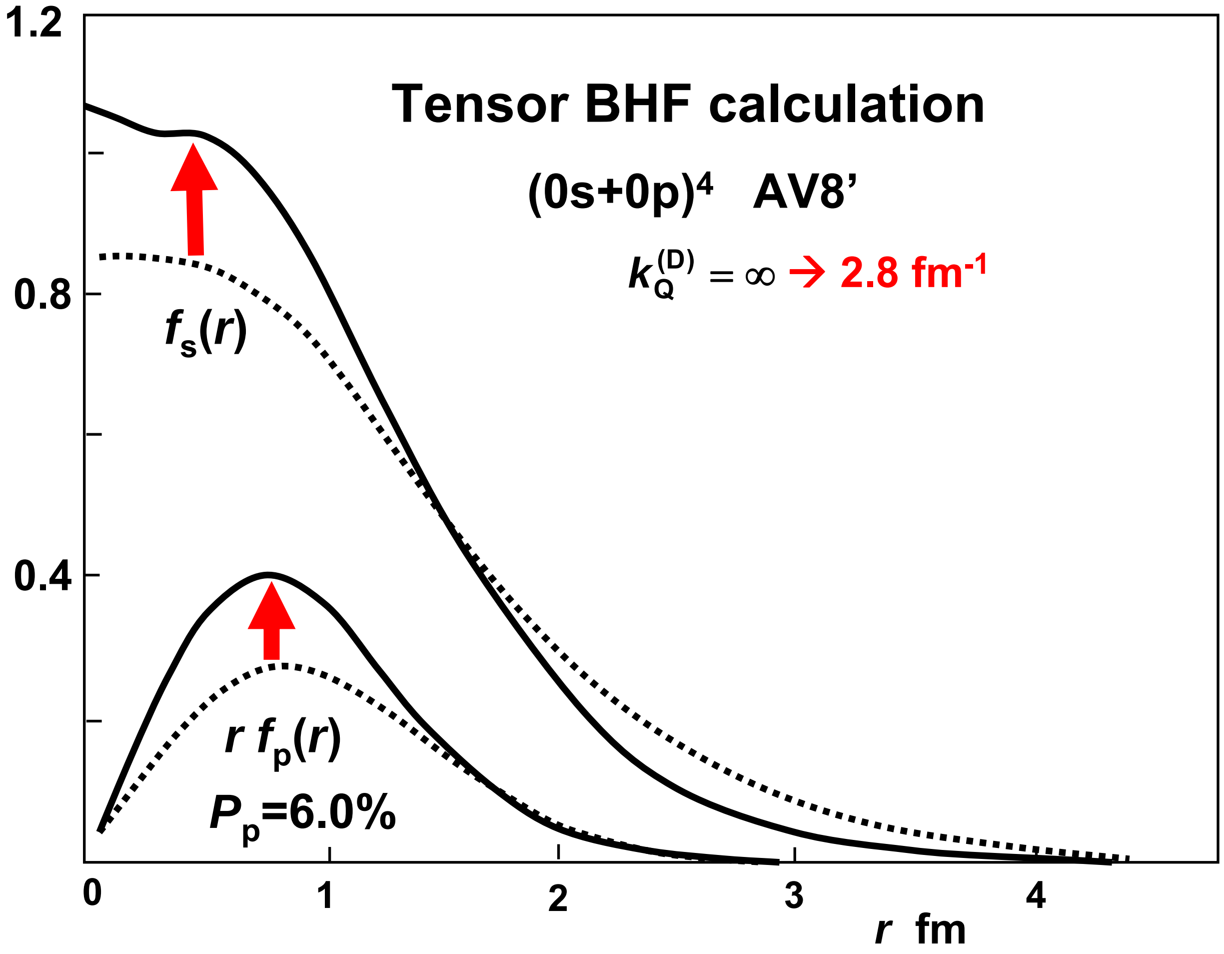


40
MeV
0
-40
-80
-120
0
1
2
3
r fm
$g_T(r)$ $k_Q^{(D)} = 2.8$ fm$^{-1}$
$v_T(r)$ $k_Q^{(D)} = \infty$
To
effective
central



100
50
0
-50
-100
-150
MeV fm²
$r^2 g_C$
$k_F$=1.4, 1.1, 0.7 fm$^{-1}$
less dependence on $k_F$
$k_Q^{(D)} = 2.8$ fm$^{-1}$
$k_Q^{(D)} = k_F$
0
1
2
3
4
$r$ fm

| |
|---|
| $k_Q^{(D)} = 2.8\ \mathrm{fm}^{-1}$ |
| -23.1 |
| 89.1 |
| -112.1 |
| -29.5, -27.4 → -56.9 |
| -55.2 |
| $P$(p)=9.8 % |



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| -29.5 | -56.9 |
| -27.4 | |
| -55.2 | |
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Parity- & charge-mixed single particle model

"Copernican"
Heliocentric

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| 54.1 |
| -79.4 |
| -32.0 |
| -47.4 |

Ordinary single particle model

"Ptolemaic"
Geocentric

# Concluding remarks

**Ordinary single particle model**

State-dependent effective interactions
(Density-, cluster-, halo-, … dependent)

**Charge-parity mixed single particle model** ← Challenging !

High-momentum phenomena
due to long-range tensor correlation

Pions play a leading role in nuclei.

(Restoration of chiral symmetry)

"New generation"

# Models of new generation

Antisymmetrized Molecular Dynamics

Y. Kanada-En'yo, H. Horiuchi et al.

**A wide variety of nuclear structures**

↑ Dynamically determined

**Effective interaction**

must include physical essences
of strong interaction.

"Pion dominance"

# Thank you very much!