

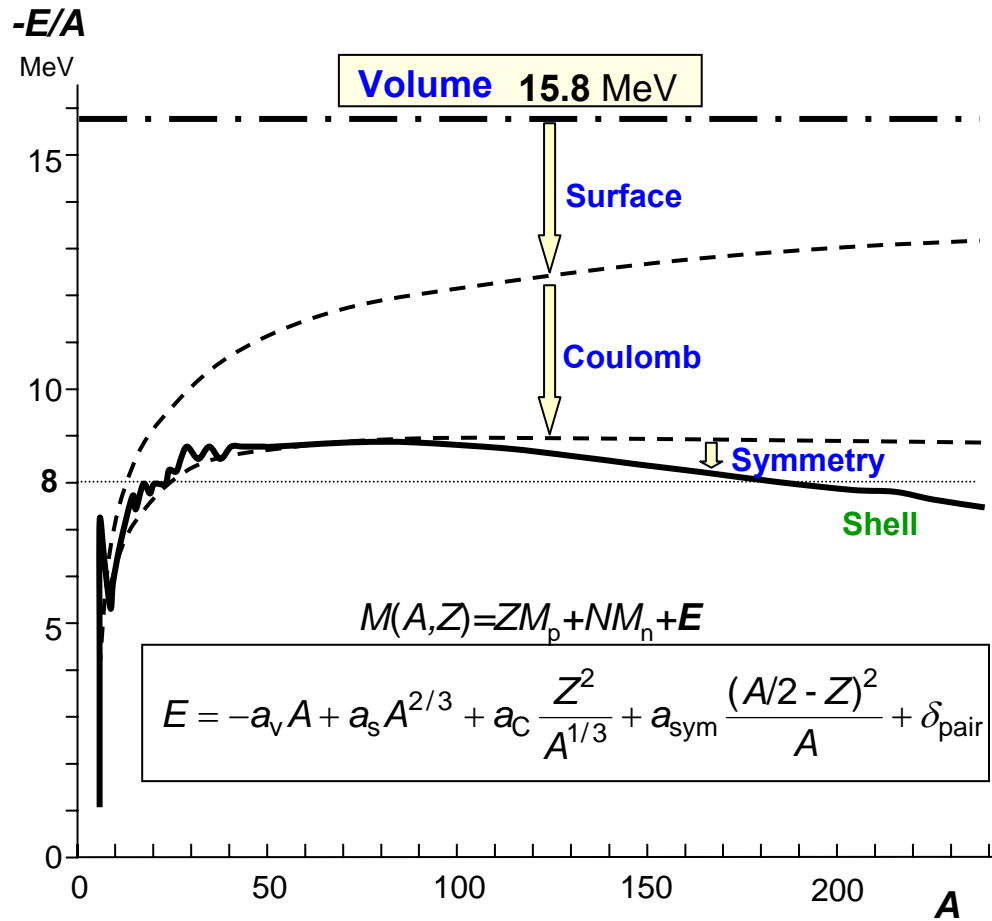
**RIKEN Lecture**  
**April 11, 2006**

# Q & A

**Y. Akaishi**

# ・原子核の saturation とは？

Saturation => 集まっても離れてもあまりエネルギーが変わらない  
 => 原子核の多様性  
 ということの説明をもう少し詳しく。



## Fermi gas model

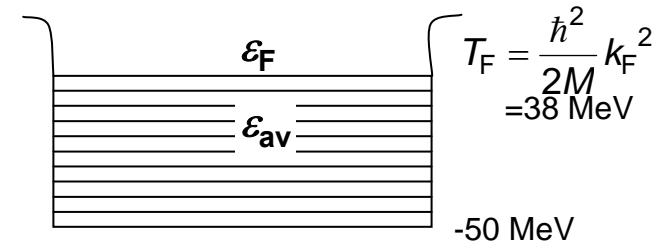
$$A = 4 \sum_{\vec{k}} = 4 \frac{\Omega}{(2\pi)^3} \int_{k \leq k_F} d\vec{k}$$

$$\rho = \frac{2}{3\pi^2} k_F^3 = \frac{1}{\frac{4}{3}\pi r_0^3}$$

$$\rho = 0.17 \text{ fm}^{-3},$$

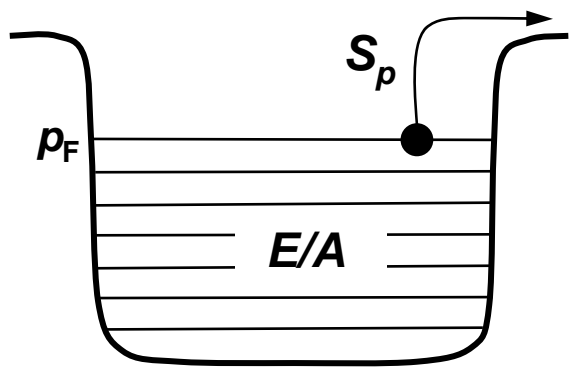
$$r_0 = 1.12 \text{ fm},$$

$$k_F = 1.36 \text{ fm}^{-1}$$

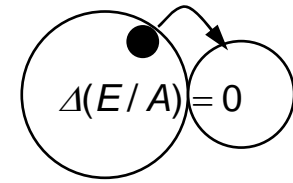


$E/A = \varepsilon_F \leftarrow$  **Saturation**

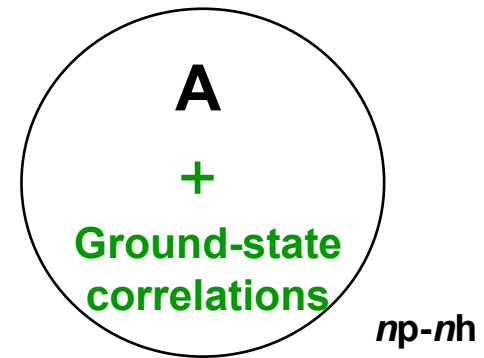
原子核の飽和性  $\Rightarrow$  核構造の多様性



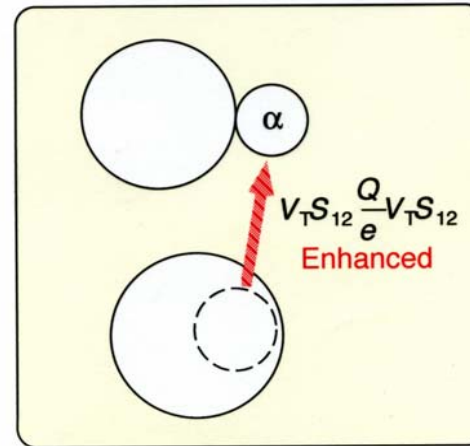
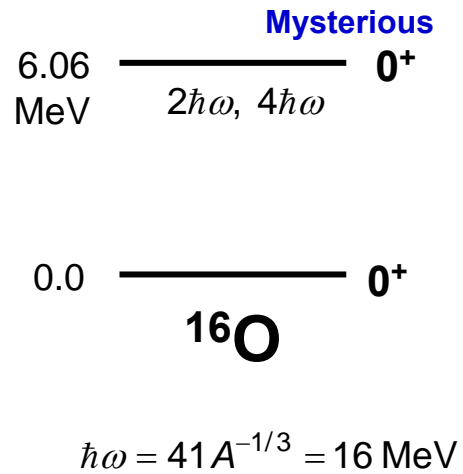
$S_p = -E/A$   
Saturation condition



A, B+b, C+c, D+d, .....  
Almost degenerate



少しエネルギーを与えただけで  
種々のクラスターが成長する。



”相対運動”(回転、反転...)

多様な構造の出現  $\longrightarrow$  新しい質の形成  $\longleftrightarrow$  新しい運動モード

**ダイナミクス**

”パイオンによるテンソル力”

不安定核=(飽和性+非飽和性)の**類層構造**

更なる豊かさの探求

# 核物質中の散乱 => 有限核内の散乱

Healing distance の説明、特に不確定性関係との関連のあたりの説明をもう少し詳しく聞きたい（まだ、狐につままれた感じ）。

## Free NN scattering

### t-matrix

$$t = v + v \frac{1}{e_0} t, \quad e_0 = E_0 - \hat{T}$$

$$t = v \hat{\Omega}$$

$$\hat{\Omega} = 1 + \frac{1}{e_0} v \hat{\Omega}$$

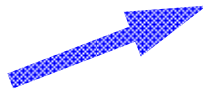
$$|\psi\rangle = \hat{\Omega} |\vec{k}\rangle$$

$$|\psi\rangle = |\vec{k}\rangle + \frac{1}{e_0} v |\psi\rangle, \quad E_0 = \frac{\hbar^2}{2\mu} k^2$$

Lippmann-Schwinger eq.

$$t |\vec{k}\rangle = v |\psi\rangle$$

$$\langle \vec{r} | \vec{k}\rangle = (2\pi)^{-3/2} \exp(i\vec{k}\vec{r})$$



$$f(\theta) = -\frac{1}{4\pi} \frac{2\mu}{\hbar^2} (2\pi)^3 \langle \vec{k}' | t | \vec{k}\rangle \quad \text{Scattering amplitude}$$

$$a = 2\pi^2 \frac{2\mu}{\hbar^2} \langle 0 | t | 0\rangle, \quad k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \dots$$

$$\langle \vec{k}' | t | \vec{k}\rangle = -\frac{1}{2\pi^2} \frac{\hbar^2}{2\mu k} \sum_{\ell} (2\ell + 1) T_{\ell} P_{\ell}(\cos \theta) \quad \text{Scattering length}$$

$$T_{\ell} = e^{i\delta_{\ell}} \sin \delta_{\ell} \quad \text{Phase shifts}$$

## NN scattering in medium

### g-matrix

$$g = v + v \frac{Q}{e} g,$$

$$e = \varepsilon_1 + \varepsilon_2 - \hat{t}_1 - \hat{t}_2, \quad Q = 1 - \sum_{k_i \leq k_F} |\vec{k}_1, \vec{k}_2\rangle \langle \vec{k}_1, \vec{k}_2| \quad \text{Pauli}$$

Dispersion

$$|\psi\rangle = |\phi\rangle + \frac{Q}{e} v |\psi\rangle \quad \text{Bethe-Goldstone eq.}$$

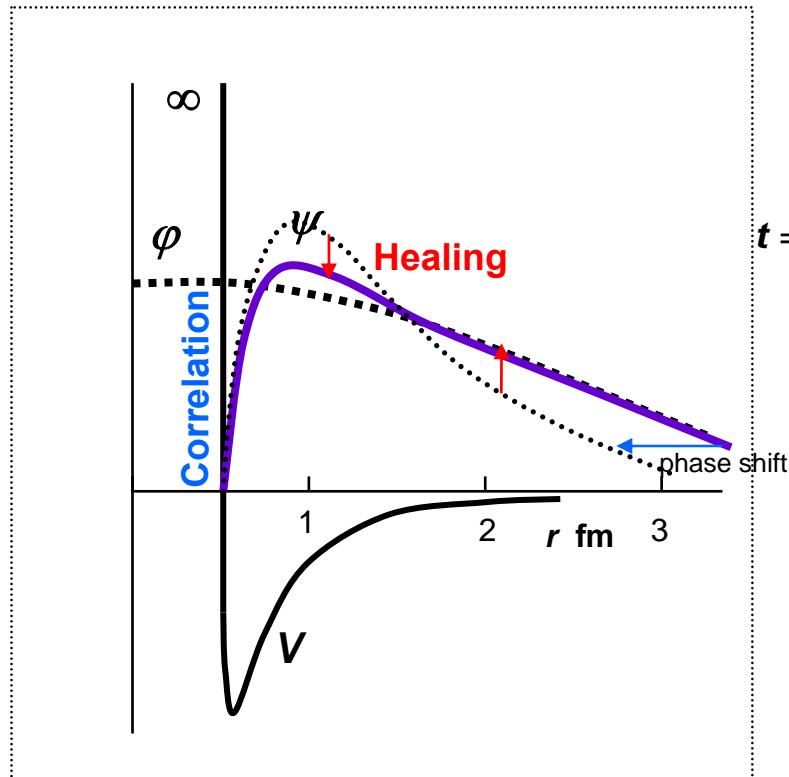
$$g |\phi\rangle = v |\psi\rangle$$

# Theory of nuclear matter

K.A. Brueckner & C.A. Levinson  
Phys. Rev. 97 (1955) 1344

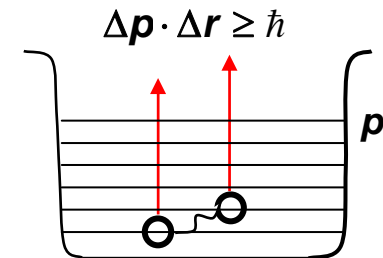
J. Goldstone  
Proc. Roy. Soc. A239 (1957) 267

H.A. Bethe  
Phys. Rev. 103 (1958) 241



$$t = v + v \frac{1}{e_0} t \Rightarrow$$

$$g = v + v \frac{Q}{e} g$$



Independent-pair scattering mode

⇒ Foundation of Shell Model

L.C. Gomes, J.D. Walecka & V.F. Weisskopf  
Ann. Phys. 3 (1958) 241

パウリ原理と核力の  
せめぎ合い

人間—魔法数—カイラル対称性

# Bethe-Goldstone equation

$$|\psi\rangle = |\phi\rangle + \frac{Q}{e} v |\psi\rangle$$

$$\psi(\vec{r}) = \exp(i\vec{k}\vec{r}) + \int d\vec{r}' \langle \vec{r} | \frac{Q}{e} | \vec{r}' \rangle v(r') \psi(\vec{r}')$$

**A simple case:**  $\vec{k}_1 + \vec{k}_2 = 0$ , Relative momentum  $\vec{k} = \frac{M_2 \vec{k}_1 - M_1 \vec{k}_2}{M_1 + M_2} = \vec{k}_1 = -\vec{k}_2$

$$\langle \vec{r}' | \frac{Q}{e} | \vec{r} \rangle = \sum_{\vec{k}} \langle r | \vec{k} \rangle \langle \vec{k} | \frac{Q(k_1, k_2 \rightarrow k)}{e} | \vec{k} \rangle \langle \vec{k} | \vec{r} \rangle,$$



Partial-wave expansions

$$R_\ell(r) = kr j_\ell(kr) + \int_0^\infty dr' G_\ell(r, r') v(r') R_\ell(r')$$

$$\frac{\hbar^2}{2\mu} G_\ell(r, r') = \frac{\gamma}{2} rr' h_\ell^{(1)}(i\gamma r_>) \{ h_\ell^{(1)}(i\gamma r_<) + h_\ell^{(2)}(i\gamma r_<) \} + \frac{2}{\pi} rr' \int_0^{k_F} dk' j_\ell(k'r) \frac{k'^2}{\gamma^2 + k'^2} j_\ell(k'r')$$

$\varepsilon_1 + \varepsilon_2 = -\frac{\hbar^2}{2\mu} \gamma^2$  : starting energy ,  $k_F$  : density

$$g_k(r) = v(r) \frac{R_\ell(r)}{kr j_\ell(kr)}$$

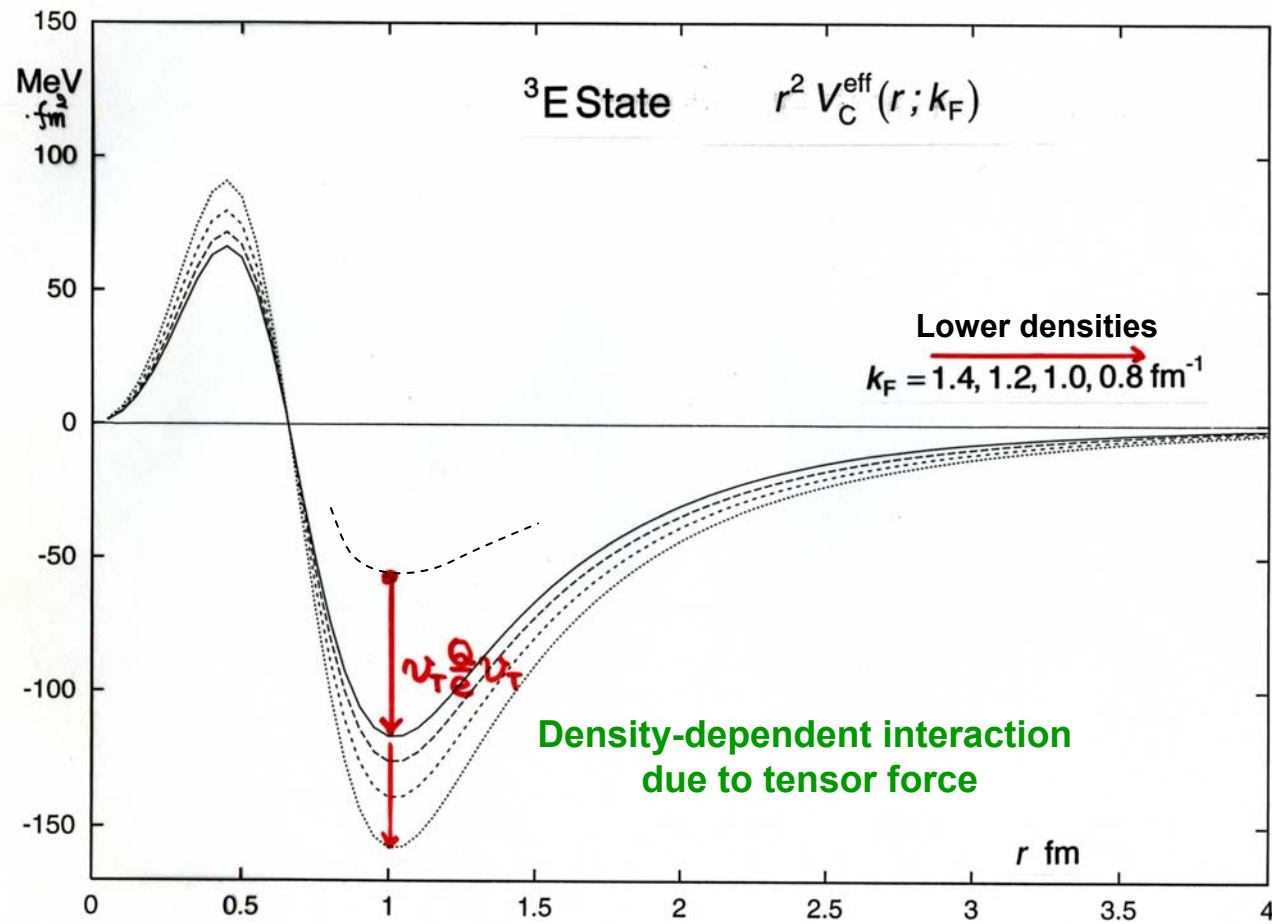
Effective interaction ( $k_F, \gamma$ )

$$\frac{\hbar^2}{2\mu} G_\ell(r, r') = -krr' h_\ell^{(+)}(kr_>) j_\ell(kr_<)$$

Free NN scattering

# ・密度依存力と tensor 力の関係？

Tensor 力を繰り返すと密度依存力になるか？





$$-\frac{1}{r^3} \left\{ 3 \frac{(\vec{\mu}_1 \vec{r})(\vec{\mu}_2 \vec{r})}{r^2} - (\vec{\mu}_1 \vec{\mu}_2) \right\}$$

Dipole-dipole interaction

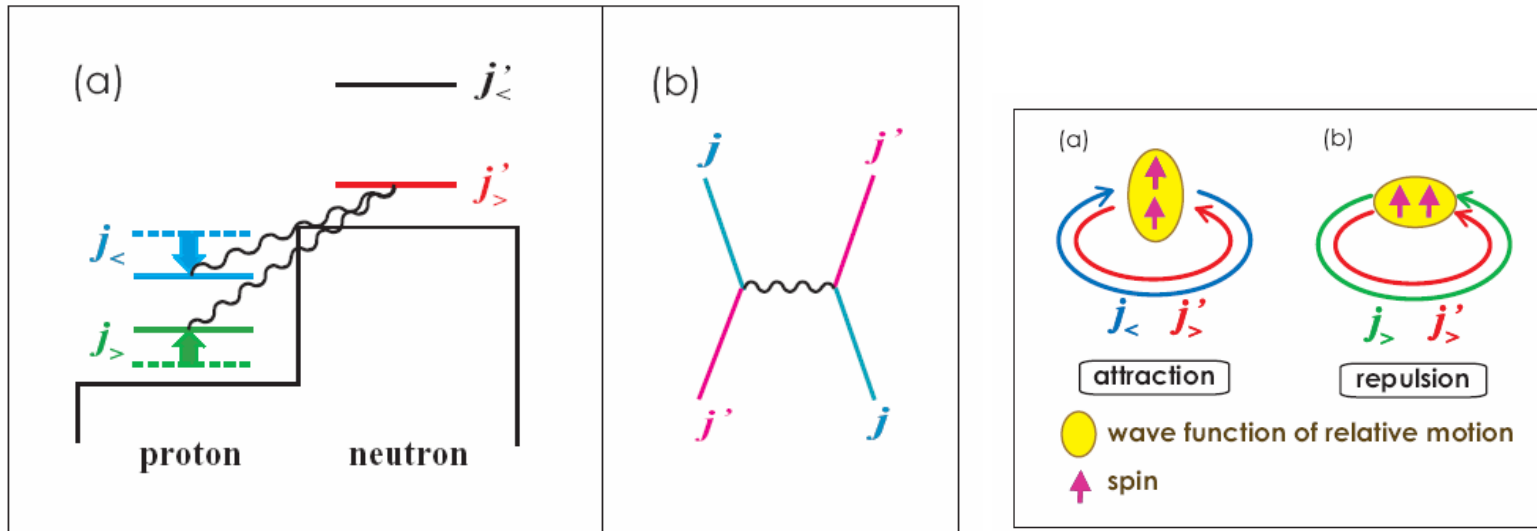
テンソル力の形

## ・中性子過剰核では

p-n pair が少なく、テンソル力が弱まる、という効果はないか。

## Evolution of nuclear shells due to the tensor force

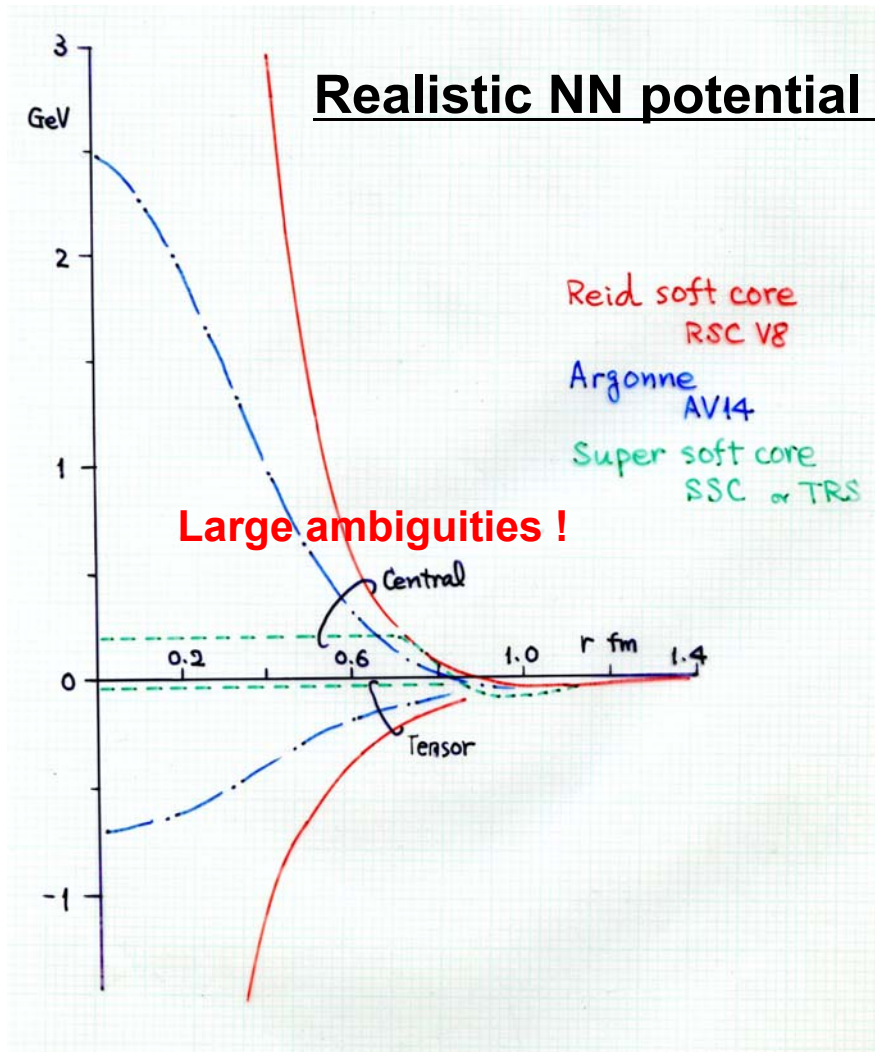
T. Otsuka *et al.*, Phys. Rev. Lett. 95 (2005) 232502



The tensor force plays a key role.

# Short range correlation とは？

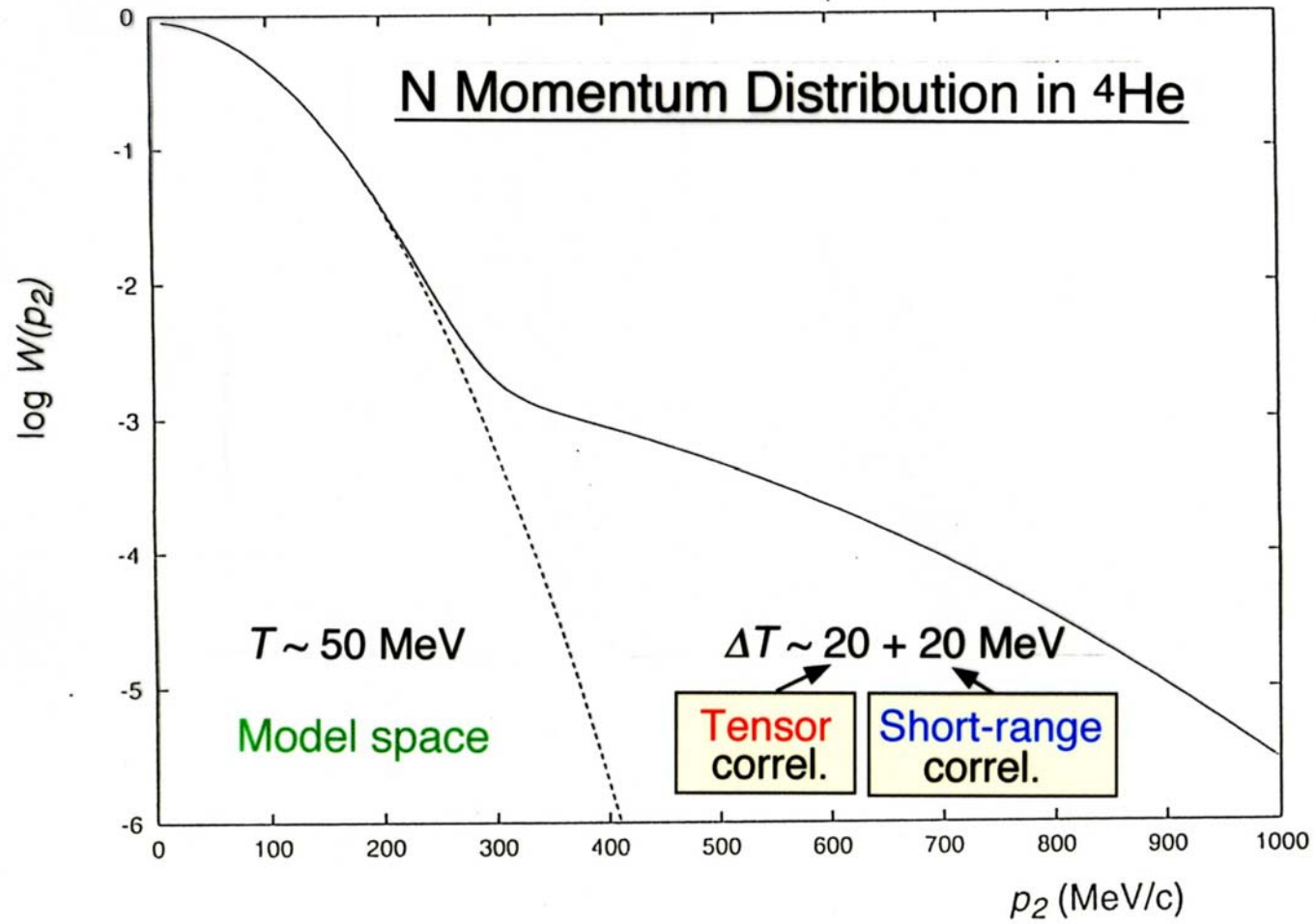
Short range correlation と tensor 力は観測で区別できるか？  
(momentum distribution を見ればわかる？)



Minimum

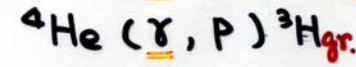
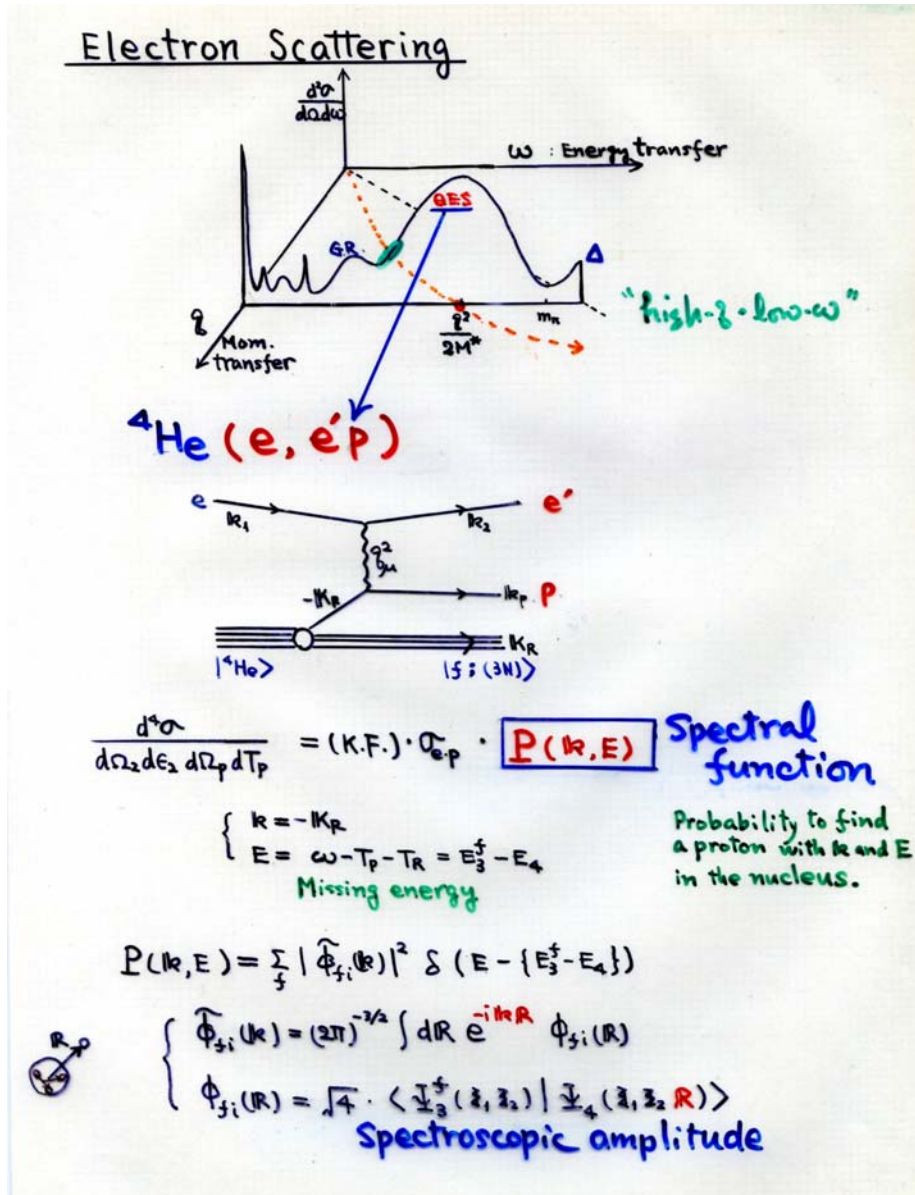
OPEP  
+  
phase shift data  
( 0~660 MeV )

$$T = \frac{\hbar^2}{2\mu} k^2 = -\frac{\hbar^2}{2\mu} \nabla^2$$

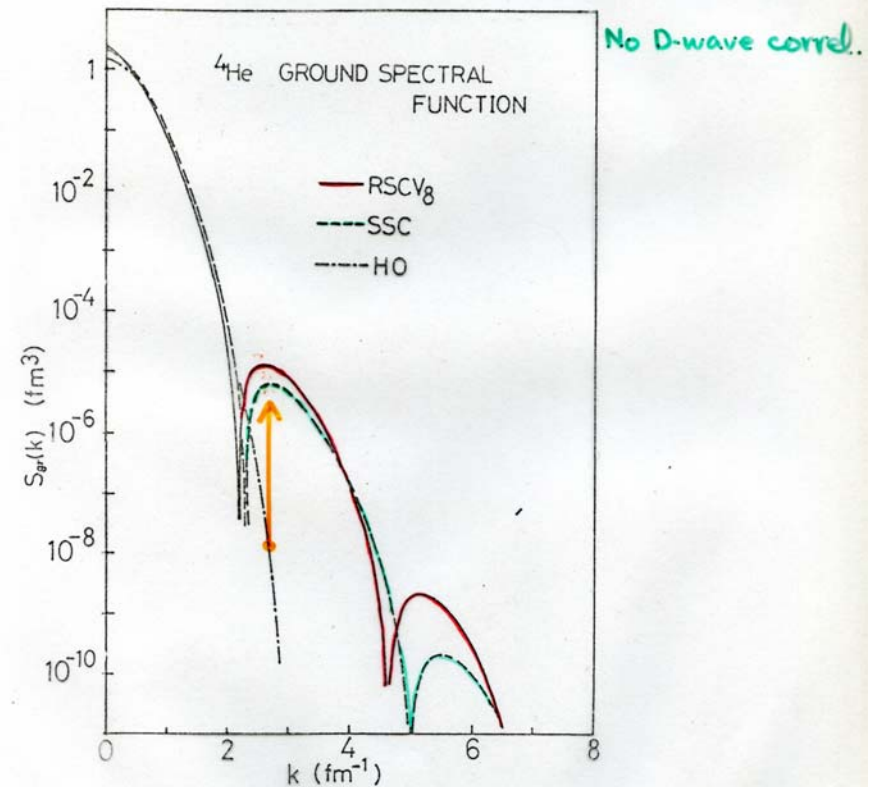
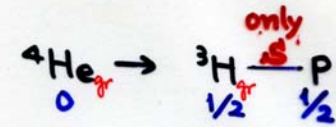


# Detection of short-range correlation

Prog. Theor. Phys. 78 (1987) 732



$\pi$



Backward  $p$ -A collision

Inverse kinematics: Suda

## ・模型どうして比較する場合、

”調理する”前に大体結果がわかるものなのか、調理してみないと何とも言えないのか。

標準的な調理法の修業



独自の調理法の考案

**Tagging, Cut  
etc.**

**Faddeev-Yakubovsky calculation of  ${}^4_{\Lambda}\text{He}$**

A. Nogga, (2001)

**SC97e**

(MeV)

		<b>0<sup>+</sup></b>	<b>1<sup>+</sup></b>
$P_{\Sigma}$		<b>1.57%</b>	<b>1.08%</b>
<b><sup>1</sup>E</b>	$\langle V_{\Lambda} \rangle$	-5.30	-1.34
	$\langle V_{\Sigma} \rangle$	-0.43	-0.12
	$\langle V_{\Lambda\Sigma} \rangle + \langle V_{\Sigma\Lambda} \rangle$	-0.17	-0.03
		<b>-5.90</b>	<b>-1.49</b>
<b><sup>3</sup>E</b>	$\langle V_{\Lambda} \rangle$	1.16	1.55
	$\langle V_{\Sigma} \rangle$	1.18	2.00
	$\langle V_{\Lambda\Sigma} \rangle + \langle V_{\Sigma\Lambda} \rangle$	<b>-11.98</b>	<b>-13.63</b>
		<b>-9.64</b>	<b>-10.08</b>

Model space

$$\begin{aligned} \underline{\mathbf{1}^+} &= \frac{1}{2} V_{\Lambda N}(\mathbf{1E}) + \frac{5}{2} V_{\Lambda N}(\mathbf{3E}) \\ \underline{\mathbf{0}^+} &= \frac{3}{2} V_{\Lambda N}(\mathbf{1E}) + \frac{3}{2} V_{\Lambda N}(\mathbf{3E}) \end{aligned}$$

★ “Is the  ${}^1S_0$  YN int. more attractive than the  ${}^3S_1$  YN int.?” **No !**

★ “Is there any evidence for coherently enhanced  $\Lambda$ - $\Sigma$  coupling in  $0^+$ ?”

**No !**

$$(T_\Lambda + V_\Lambda)\psi_\Lambda + V_{\Lambda\Sigma}\psi_\Sigma = E\psi_\Lambda$$

$$(T_\Sigma + V_\Sigma)\psi_\Sigma + V_{\Sigma\Lambda}\psi_\Lambda = (E - \Delta M)\psi_\Sigma$$

$$\langle \psi_\Lambda \psi_\Lambda \rangle + \langle \psi_\Sigma \psi_\Sigma \rangle = 1$$

$$\Delta M = M_\Sigma - M_\Lambda \approx 80 \text{ MeV}$$

$$\langle \psi_\Lambda \psi_\Lambda \rangle E = \langle \psi_\Lambda T_\Lambda \psi_\Lambda \rangle + \langle \psi_\Lambda V_\Lambda \psi_\Lambda \rangle + \langle \psi_\Lambda V_{\Lambda\Sigma} \psi_\Sigma \rangle$$

$$\langle \psi_\Sigma \psi_\Sigma \rangle E = \langle \psi_\Sigma T_\Sigma \psi_\Sigma \rangle + \langle \psi_\Sigma V_\Sigma \psi_\Sigma \rangle + \langle \psi_\Sigma V_{\Sigma\Lambda} \psi_\Lambda \rangle + \langle \psi_\Sigma \psi_\Sigma \rangle \Delta M$$

$$E = \langle T_\Lambda \rangle + \langle V_\Lambda \rangle + \langle V_{\Lambda\Sigma} \rangle \\ + \langle T_\Sigma \rangle + \langle V_\Sigma \rangle + \langle V_{\Sigma\Lambda} \rangle + P_\Sigma \Delta M$$

$$\langle T_\Sigma \rangle + \langle V_\Sigma \rangle + \langle V_{\Sigma\Lambda} \rangle + P_\Sigma \Delta M = P_\Sigma E$$

$$(1 - P_\Sigma) E = \langle T_\Lambda \rangle + \langle V_\Lambda \rangle + \langle V_{\Lambda\Sigma} \rangle$$

$$E = \frac{1}{1 - P_\Sigma} \langle T_\Lambda \rangle + \frac{1}{1 - P_\Sigma} \langle V_\Lambda \rangle + \frac{1}{1 - P_\Sigma} \langle V_{\Lambda\Sigma} \rangle$$



$$E = \langle T_\Lambda \rangle_{\text{sc}} + \langle V_\Lambda \rangle_{\text{sc}} + \langle V_{\Lambda\Sigma, \Sigma\Lambda} \rangle_{\text{sc}}$$

$$(T_\Lambda + V_\Lambda)\Psi_\Lambda + V_{\Lambda\Sigma} \frac{1}{E - \Delta M - T_\Sigma - V_\Sigma} V_{\Sigma\Lambda} \Psi_\Lambda = E\Psi_\Lambda$$

$$\Psi_\Lambda = \frac{1}{\sqrt{1 - P_\Sigma}} \psi_\Lambda$$

Single-channel description of  ${}^4_{\Lambda}\text{He}$

SC97e

(MeV)

		0 <sup>+</sup>	1 <sup>+</sup>
1E	$\langle V_{\Lambda} \rangle_{sc}$	-5.38	-1.35
	$\langle V_{\Lambda\Sigma, \Sigma\Lambda} \rangle_{sc}$	-0.09	-0.02
		-5.47	-1.37
3E	$\langle V_{\Lambda} \rangle_{sc}$	1.18	1.57
	$\langle V_{\Lambda\Sigma, \Sigma\Lambda} \rangle_{sc}$	-6.09	-6.89
		-4.91	-5.32

★ Coherently enhanced

x3/5  
-4.13

“Cooking”!

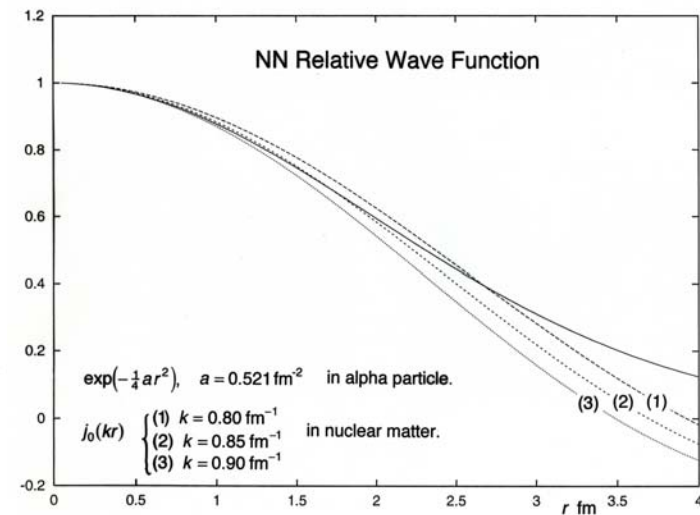
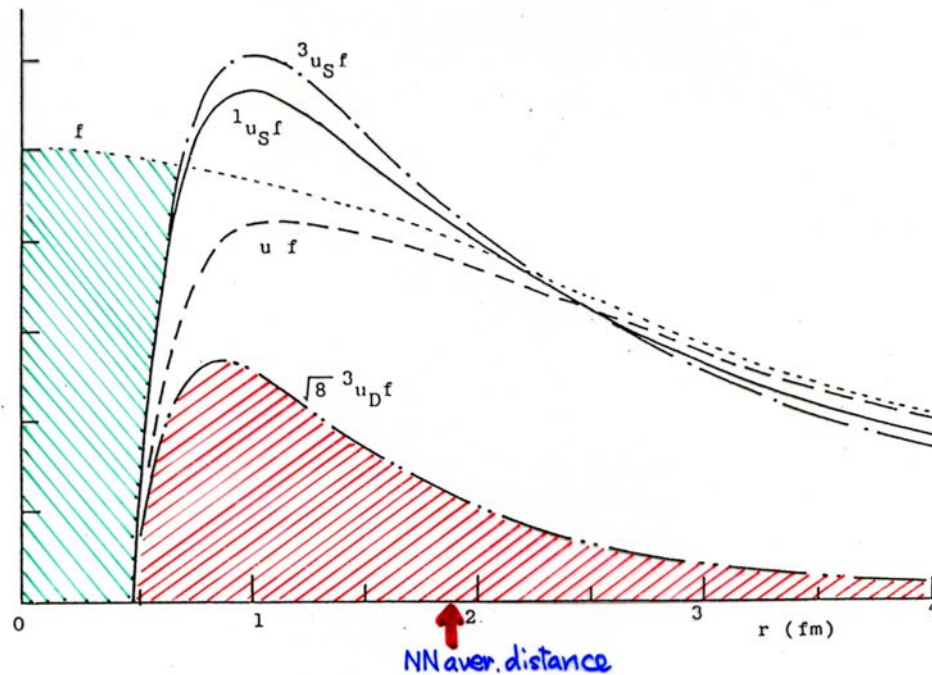
★  $\langle {}^1S_0 \rangle_{sc}$  is more attractive than  $\langle {}^3S_1 \rangle_{sc}$ .

$$\begin{aligned} \langle V_{\Lambda\Sigma, \Sigma\Lambda} \rangle_{sc} &= \langle V_{\Lambda\Sigma} \rangle + \langle V_{\Sigma\Lambda} \rangle + \langle V_{\Sigma} \rangle \\ &+ \langle \{T_{\Sigma} + P_{\Sigma}\Delta M\} \rangle - \frac{P_{\Sigma}}{1-P_{\Sigma}} \langle \{T_{\Lambda} + V_{\Lambda}\} \rangle \\ &= \frac{1}{1-P_{\Sigma}} \langle V_{\Lambda\Sigma} \rangle \end{aligned}$$



• Infinite nuclear matter は現実のものとう結びつけるのか？

Local density approximation



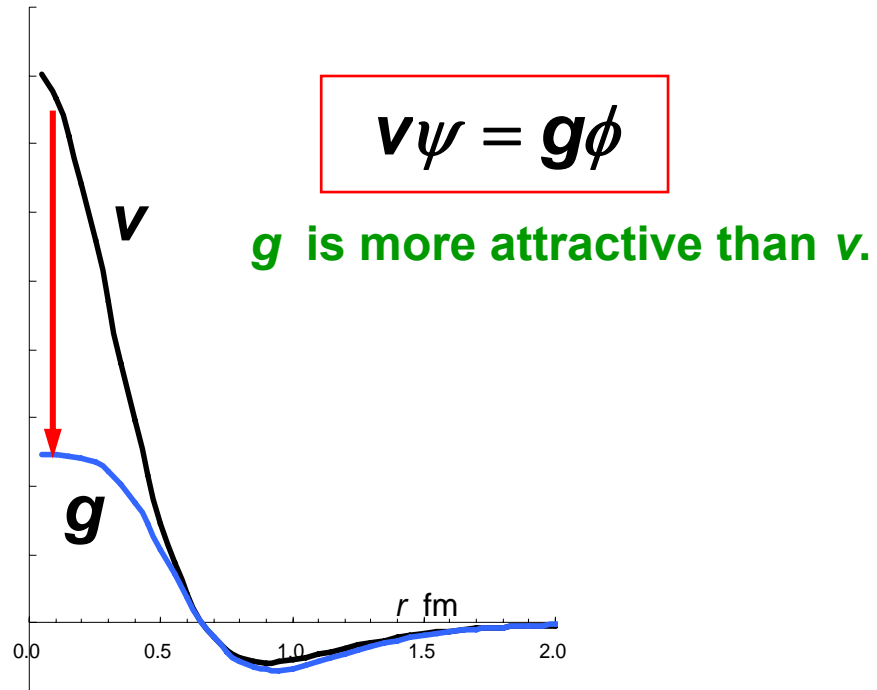
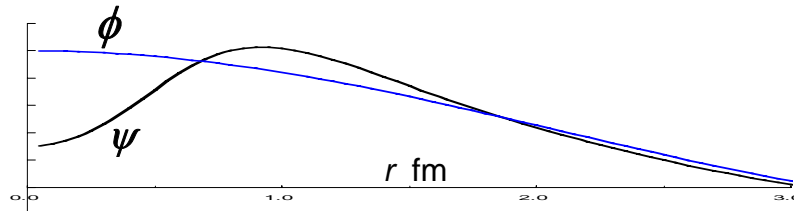
Y. Akaishi & S. Nagata, P. T. P. 48 (1972) 133

Akaishi-Bando-Nagata, P. T. P. Suppl. 52 (1972) 339

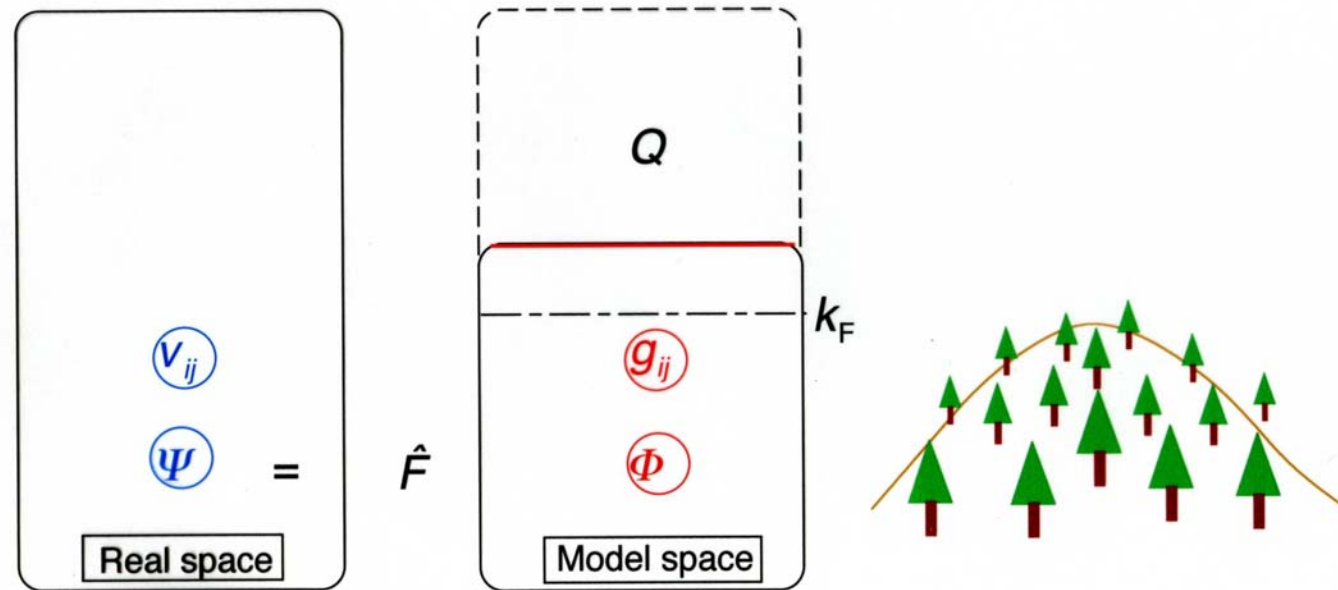
Tensor effects on clusterization

# Effective interaction で

Core を捨てたのに なぜ原子核がつぶれないのか？



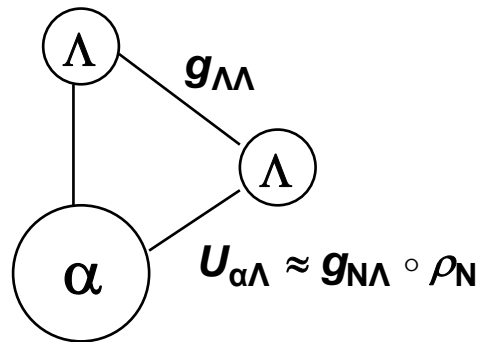
## What is "Model"?



$$g_{ij} = v_{ij} + v_{ij} \frac{Q}{e} g_{ij}$$

# Faddeev calculations for ${}^6_{\Lambda\Lambda}\text{He}$

Phys. Rev. C 68 (2003) 024002



Reference	$l_{\max}$	$l_{\Lambda\alpha}$	$l_{\Lambda\Lambda}$	$B_{\Lambda\Lambda}$ (MeV)
Faddeev	0	0	0	11.15
	1	0,1	0	11.19
	2	0,1,2	0,2	11.21
	4	0,1,2,3,4	0,2,4	11.21
	6	0,1,2,3,4,5,6	0,2,4,6	11.21
Ikeda et al.	a		0	10.3
	a		0,2,4	10.8

More importantly, our Faddeev calculations demonstrate that the Ikeda *et al.* calculation misses our converged value of  $B_{\Lambda\Lambda}$  by about 0.4 MeV (which is a sizable miss in this three-body trade).

Effective interaction を使って、Schroedinger eq. を解くのはdouble counting!

・Fは、本来全てを解かないと導けないような気がするが、

これを導き出せることが不思議。  
Fを簡単な場合で、手で計算できる例を挙げて欲しい。

Transformation

$$|\Psi\rangle = \hat{F} |\Phi\rangle$$

$$\hat{F} = 1 + \frac{Q}{e} V \hat{F}$$
$$e = E_0 - QTQ$$

## Reaction matrix

Def. of  $g$

$$g_{ij} = v_{ij} + v_{ij} \frac{Q}{e} g_{ij}$$

Two-body scattering in medium

Def. of  $\hat{F}_{ij}$

$$g_{ij} \hat{F}_{ij} = v_{ij} \hat{F}$$

$$= v_{ij} \left( 1 + \frac{Q}{e} g_{ij} \right) \hat{F}_{ij}$$

$$= v_{ij} \left( 1 + \frac{Q}{e} \sum_{(kl)} v_{kl} \hat{F} \right)$$

$$= v_{ij} \left( 1 + \frac{Q}{e} \sum_{(kl)} g_{kl} \hat{F}_{kl} \right)$$

$$\hat{F} = 1 + \sum_{(ij)} \frac{Q}{e} g_{ij} \hat{F}_{ij}$$

$$\hat{F}_{ij} = 1 + \sum_{(kl)} \frac{Q}{e} g_{kl} \hat{F}_{kl}$$

Multiple scattering process

$$PV|\Psi\rangle = P \sum_{(ij)} v_{ij} \hat{F} |\Phi\rangle = P \sum_{(ij)} g_{ij} \hat{F}_{ij} |\Phi\rangle$$

$$\Downarrow$$

$$= P \sum_{(ij)} g_{ij} |\Phi\rangle + P \sum_{(ijk)} g_{ij} \frac{Q}{e} g_{jk} |\Phi\rangle + \dots$$

$$PV_M |\Phi\rangle = P \sum_{(ij)} v_{ij}^{\text{eff}} |\Phi\rangle + \dots$$

$$v_{ij}^{\text{eff}} = g_{ij}, \quad v_{ijk}^{\text{eff}} = g_{ij} \frac{Q}{e} g_{jk}$$

Effective interaction

## 0<sup>th</sup> ATMS

$$\hat{F}_{ij} = \sum_{(kl)}' \frac{Q}{e} g_{kl} \hat{F}_{kl} \xrightarrow{\text{Day's approx.}} F = \prod_{(kl)} u_{kl}$$

**Jastrow**

## 1<sup>st</sup> ATMS

$$(\hat{F}_{ij} - 1) = \sum_{(kl)}' \frac{Q}{e} g_{kl} + \sum_{(kl)}' \frac{Q}{e} g_{kl} (\hat{F}_{kl} - 1)$$

$(u_{kl} - 1) \quad (\bar{u}_{kl} - 1)$

## 2<sup>nd</sup> ATMS

$$\begin{aligned} & (\hat{F}_{ij} - 1 - \sum_{(kl)}' \frac{Q}{e} g_{kl}) \\ &= \sum_{(kl)}' \frac{Q}{e} g_{kl} \sum_{(mn)}' \frac{Q}{e} g_{mn} + \sum_{(kl)}' \frac{Q}{e} g_{kl} (\hat{F}_{kl} - 1 - \sum_{(mn)}' \frac{Q}{e} g_{mn}) \end{aligned}$$

$(\bar{u}_{kl} - 1) \quad (u_{mn} - 1) \quad (\tilde{u}_{kl} - 1)$

**ATMS can improve the wave function  
in a systematic way.**

# ATMS-Euler

Y. Akaishi, Int. Rev. Nucl. Phys. 4 (1986) 259.

## Two-body wave function in ${}^4\text{He}$

Simplest ATMS ( $f = U_s \phi$ )

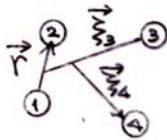
$$\Psi = \prod_{(ij)} f(r_{ij})$$

Variational principle

$$\delta_f [\langle \Psi | H | \Psi \rangle - \lambda \langle \Psi | \Psi \rangle] = 0$$

$$\left[ -\frac{\hbar^2}{M} \left\{ \frac{d^2}{dr^2} + \left( \frac{2}{r} + \frac{1}{S(r)} \frac{dS}{dr} \right) \frac{d}{dr} \right\} + U(r) + v(r) \right] f(r) = \lambda f(r)$$

non-hermitian.



$$S(r) = \int d\vec{x}_3 d\vec{x}_4 |f(r_{13}) f(r_{14}) f(r_{23}) f(r_{24}) f(r_{34})|^2$$

$$g(r) \equiv \sqrt{S(r)} f(r) \quad \text{realistic NN pot.}$$

$$\left[ -\frac{\hbar^2}{M} \left\{ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right\} + U^{\text{av}}(r) + v(r) \right] g(r) = \lambda g(r)$$

ATMS-Euler eq.  
Average pot. coming from 3rd, 4th nucleons

$$\int d\vec{r} |g(\vec{r})|^2 = \langle \Psi | \Psi \rangle = 1$$

$g(r)$  is the two-body function.

2粒子描像を抽出

# Hartree-Fock

Slater determinant

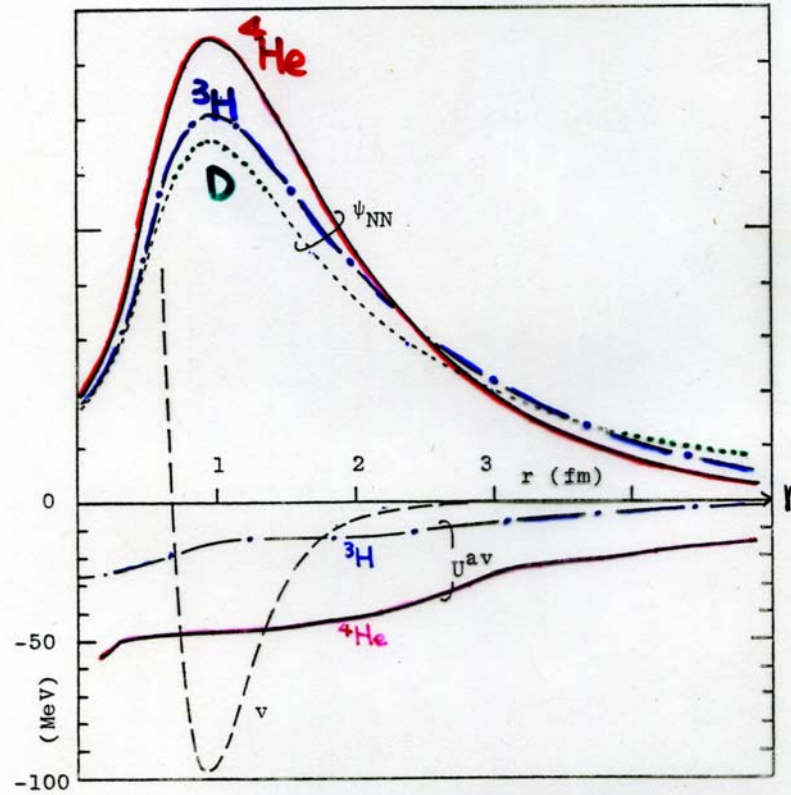
H-F eq.

Single particle wave function

1粒子描像を抽出



# Quasi-deuteron in nuclei



$$-\frac{\hbar^2}{M} \frac{d^2}{dr^2} \cdot r g(r) = \left\{ \lambda - \underbrace{U^{av}(r)}_{\text{confining effect}} - v(r) \right\} r g(r)$$

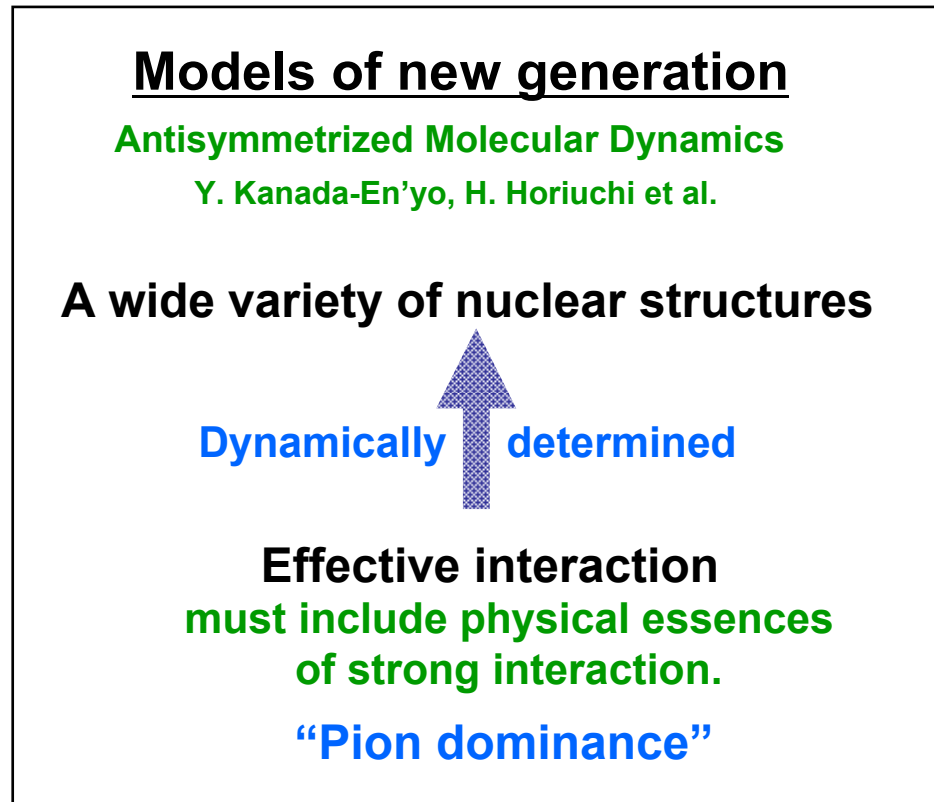
at  $r=1 \text{ fm}$

$$\left\{ \right\} = \begin{cases} 113 \text{ MeV} & \text{for } {}^4\text{He} \\ 97 \text{ MeV} & \text{for } {}^3\text{H} \end{cases}$$

## ・パイオンを交換するモデルを導入したのは、

何か積極的な理由があるのか。従来の枠組みでは記述できない重要な物理があって、それを改善するためにパイオン取り入れたのか？

- ・Deutron, alpha particle の場合に tensor 力が重要なのはわかったが、それら以外で tensor 力を入れないと記述できない物理は？  
(中重核以上、観測量は？)



**池田さんたちが研究中！**

**Thank you!**