原子核物理学連続講義・コースX-2

Baryonic Matter and Neutron Stars
(第2回)

T. Takatsuka (Prof. Emeritus of Iwate Univ.)

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   3-1. Historical overview
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3. Pion condensation (PC)

3-1. historical overview

Pions: $\pi(\pi^0, \pi^+, \pi^-)$, spin (S) =0, isospin (τ)=1, boson,
mass (m_π)≒140 MeV

∼1935 OPEP (Important ingredient of nuclear force since Yukawa's work)

↓

1965 In medium, n→ p + π^- (bose condensation with k=0) when μ_n (chem. Pot.)≥ m_π ; proposed by J.N. Bahcall and R.A. Wolf*)
→ later on, NO! due to the repulsive effects from π-n S-wave int.

↓

1972 π-condensation with k≠0 is OK! by π-N P-wave int., pointed out by A.B. Migdal and independently by R.F. Sawyer and D.J. Scalapino*) ;

explicit introduction of meson degrees of freedom in medium

So many works (including, e.g., ALS*))

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*) A.B. Migdal, Sov. Phys. –JETP34 (1972) 1184.
R.F. Sawyer and D.J. Scalapino, Phys. Rev. D7 (1972) 953.
T. Takatsuka, K. Tamiya, T. Tatsumi and R. Tamagaki, Prog. Theor. Phys. 59 (1978) 1933.

～1980 "No π-cond." "Death to PC"; $\rho_t > \rho_0$ ($\rho_t > (2-3)\rho_0$)

- Stop studies
- Continuous works from a view of NS physics (dense NS cores) (e.g. Kyoto group)

Renewed interests coming from the progress of NS observations (Cooling, Glitches, NS formation mechanism and so on)

1999 Smaller g' (Landau-Migdal parameter), suggested experimentally*)
→ π-condensation is easy to occur

Now Stimulated studies as to "PC and related phenomena"

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*) S. Suzuki and H. Sakai, Phys. Lett. B455 (1999) 25.

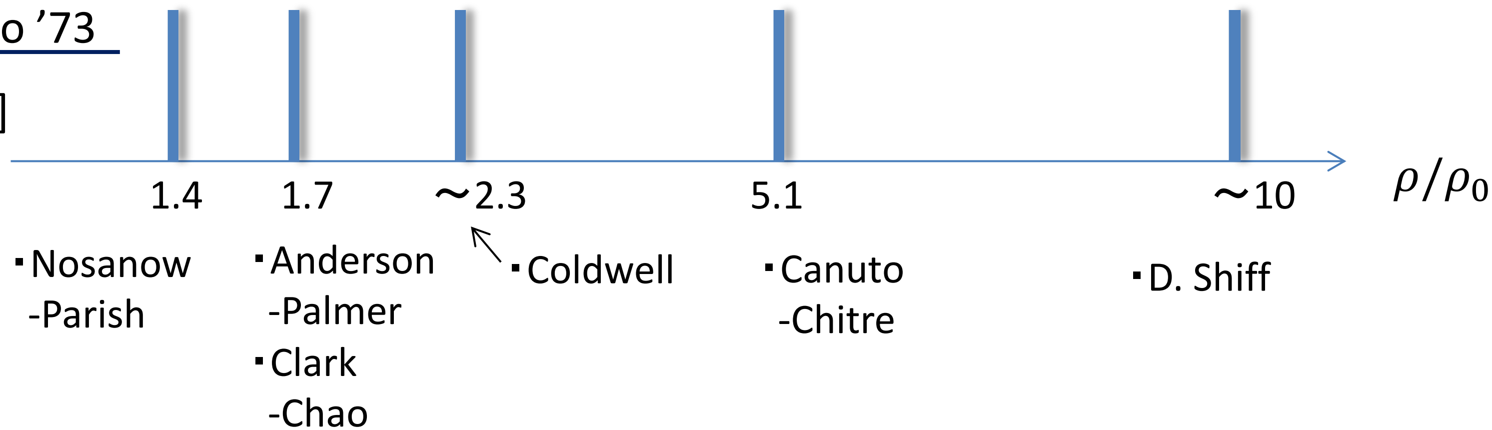
3-2. Alternating-Layer-Spin (ALS) model

Is it possible for neutrons to solidify?

☐ Considering usual mechanism for solidification,
i.e., “Geometrical caging”
------effect of avoiding repulsion
> Zero point energy due to localization

Up to ’73

[Yes]



[No]

- V.R. Pandharipande : up to $\sim 20\rho_0$
- E. ϕstgaard : below $\sim 7\rho_0$

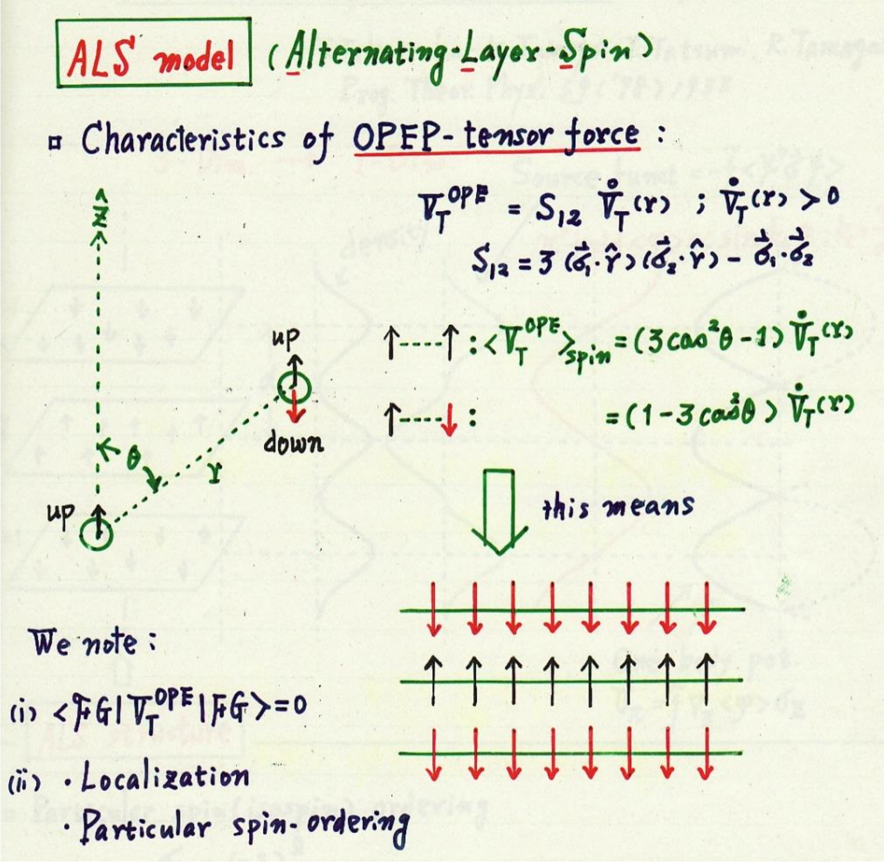
Later on

Possibility of solid neutron matter is denied by more detailed investigation

the reason:

weaker repulsion and stronger quantum effects as compared with those in He system.

☐ Can we have another new mechanism? → Yes

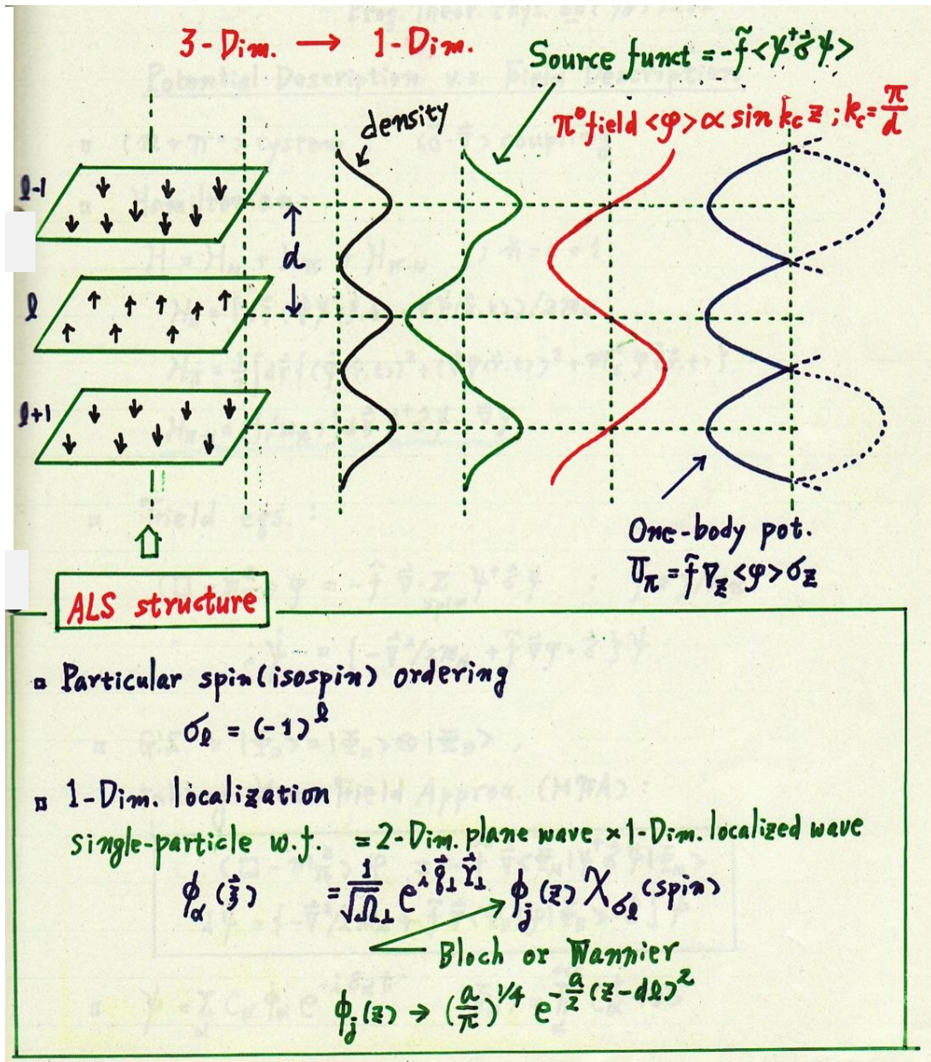


This configuration can utilize the tensor force attractively to reduce the kinetic energy increase ($\frac{3}{4}\hbar\omega > \frac{3}{5}\in_F$), 3-dim. Localization ($\frac{3}{4}\hbar\omega$)

→ 1-dim. Localization ($\frac{3}{4}\hbar\omega$)

Presentation of the ALS model

T. Takatsuka, K. Tamiya, T. Tatsumi, R. Tamagaki
Prg. Theor. Phys. 59 ('78) 1933



3-3. ALS model and PC

□ ALS≡ π^0 condensate*)

① Potential Description (PD) v.s. Field Description (FD)

○ (n+π^0) system ; ($\boldsymbol{\sigma}\cdot\nabla$) coupling

○ Hamiltonian:

$$H = H_N + H_\pi + H_{\pi-N} \; ; \quad \hbar\text{=c=1} \tag{3-3}$$

$$H_N = \int d\boldsymbol{\xi}\, (\nabla\psi^\dagger(\boldsymbol{\xi},t)\cdot\nabla\psi(\boldsymbol{\xi},t))/2m_N \; ; \; \xi\equiv\{\boldsymbol{r},\text{spin}\} \tag{3-4}$$

$$H_\pi = \frac{1}{2}\int d\boldsymbol{r}\{(\dot{\varphi}(\boldsymbol{r},t))^2 + (\nabla\varphi(\boldsymbol{r},t))^2 + m_\pi{}^2\varphi^2(\boldsymbol{r},t)\} \tag{3-5}$$

$$H_{\pi-N} = (f/m_\pi)\int d\,\boldsymbol{\xi}\psi^\dagger\boldsymbol{\sigma}\psi\cdot\nabla\varphi \tag{3-6}$$

○Field eqs.:

$$(\Box - m_\pi{}^2)\varphi = -\tilde{f}\nabla\cdot\psi^\dagger\boldsymbol{\sigma}\psi \quad ; \tilde{f}\equiv f/m_\pi \tag{3-7}$$

$$i\,\dot{\psi} = \{-\nabla^2/2m_N + \tilde{f}\nabla\varphi\cdot\boldsymbol{\sigma}\}\psi \tag{3-8}$$

○ G.S. → $|\Phi_0> = |\Phi_N>\otimes|\Phi_B>$, (3-9)

Taking Mean Field Approx. (MFA):

$$(\Box - m_\pi{}^2)\varphi = -\tilde{f}\nabla < \Phi_N|\psi^\dagger\boldsymbol{\sigma}\psi|\Phi_N > \tag{3-10}$$

$$i\dot{\psi} = \{-\nabla^2/2m_N + \tilde{f}\nabla < \Phi_B|\varphi|\Phi_B > \cdot\,\boldsymbol{\sigma}\}\psi \tag{3-11}$$

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*) ・T. Takatsuka, K. Tamiya, T. Tatsumi and R. Tamagaki ; Prog. Theor. Phys. 59 ('78) 1933

・T. Takatsuka and J. Hiura; Prog. Theor. Phys. 60 ('78) 1234

○ $\psi = \sum_\alpha C_\alpha \phi_\alpha e^{-i\varepsilon_\alpha t}$, $|\Phi_N> = \prod_\alpha^{occ} {C_\alpha}^\dagger |0>$ (3-12)

○ Sol. of π field:

$$(\Box - {m_\pi}^2)\varphi = -\tilde{f}\nabla < \psi^\dagger \boldsymbol{\sigma} \psi > \quad (3\text{-}13)$$

$$\varphi = \varphi_c + \varphi_q \quad (3\text{-}14)$$

$$\varphi = \sum_{\boldsymbol{k}} \{ a_{\boldsymbol{k}}(t) e^{i\boldsymbol{kr}} + h.c.\} / \sqrt{2\omega_{\boldsymbol{k}}\Omega} \quad (3\text{-}15)$$

$$\varphi_q = \sum_k \{A_{\boldsymbol{k}} e^{i(\boldsymbol{kr} - \omega_k t)} + h.c.\} / \sqrt{2\omega_{\boldsymbol{k}}\Omega} \quad \text{(non-cond.)} \quad (3\text{-}16)$$

$$\varphi_c = \sum_{\boldsymbol{k}} \{S(\boldsymbol{k}) e^{i\boldsymbol{kr}} + h.c.\} / \sqrt{2\omega_{\boldsymbol{k}}\Omega} \; ; \; \text{static} \quad \text{(cond.)} \quad (3\text{-}17)$$

Where $S(\boldsymbol{k}) \equiv \sum_\alpha^{occ} S_{\alpha\alpha}(\boldsymbol{k})$ (3-18)

$$S_{\alpha\alpha}(\boldsymbol{k}) \equiv \tilde{f} \int d\boldsymbol{\xi} {\phi_\alpha}^* (i\boldsymbol{k\sigma}) \phi_\alpha e^{-i\boldsymbol{kr}} / \sqrt{2{\omega_{\boldsymbol{k}}}^3\Omega} \quad (3\text{-}19)$$

○ This means:

$$a_{\boldsymbol{k}}(t) = A_{\boldsymbol{k}} e^{-i\omega_k t} + S(\boldsymbol{k}) \; ; \quad \text{Displaced} \quad (3\text{-}20)$$

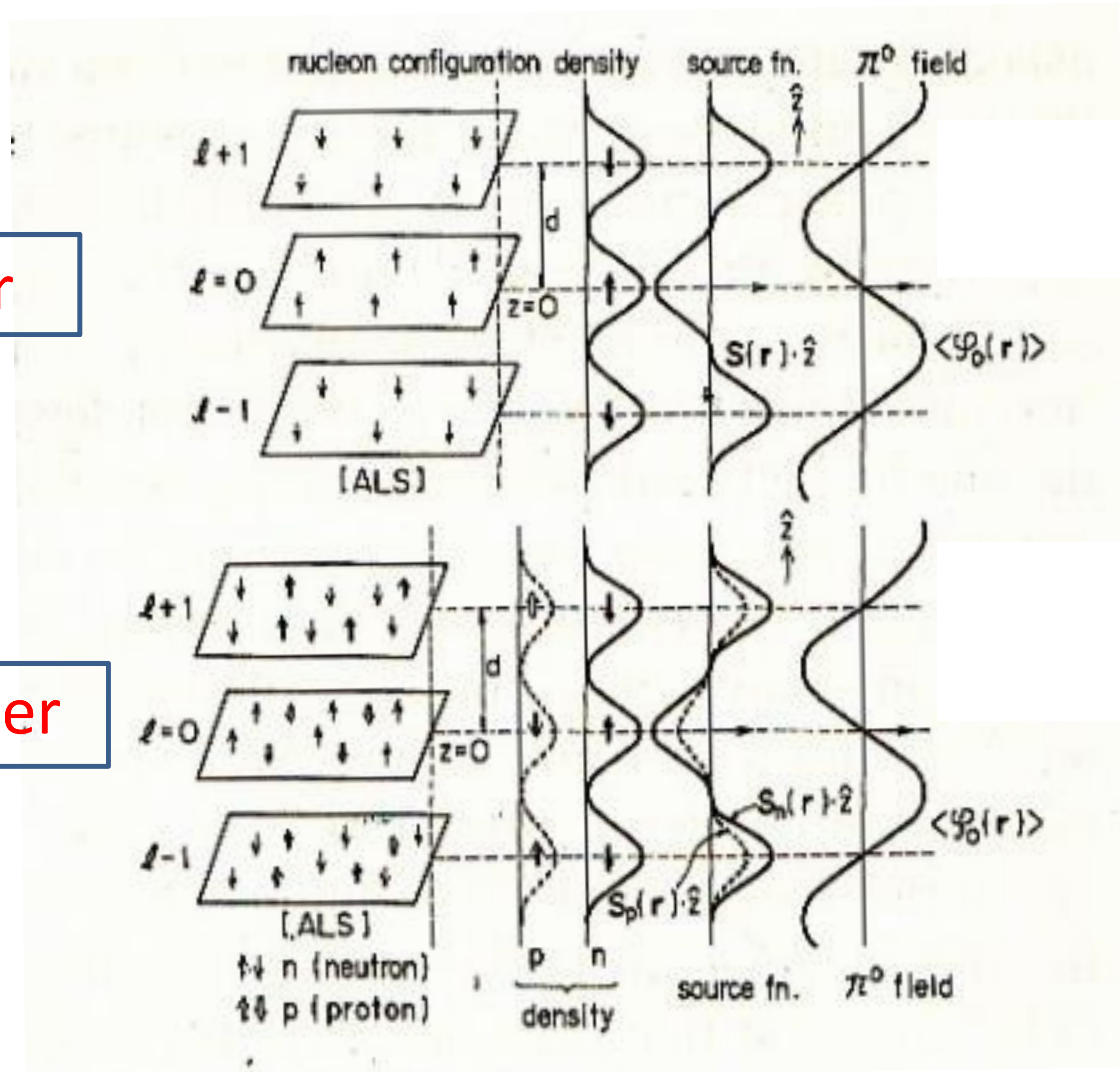
○ Rewrite *H* by using field eq. :

$$H = \int \frac{d\xi(\nabla\psi^\dagger \cdot \nabla\psi)}{2m_N} + \underline{\frac{1}{2}\int d\boldsymbol{r}\,\{\dot{\varphi_q}^2 + (\nabla\varphi_q)^2 + {m_\pi}^2{\varphi_q}^2\}} - \frac{1}{2}\int d\boldsymbol{r}\{(\nabla\varphi_c)^2 + {m_\pi}^2{\varphi_c}^2\}$$

Positive definit $\sum_{\boldsymbol{k}} \omega_{\boldsymbol{k}} {A_{\boldsymbol{k}}}^\dagger A_{\boldsymbol{k}}$ (3-21)

Pure n-matter

Nucleon matter



→ G.S. should be the vaccum with respect to φ_q :

$$0 = A_{\boldsymbol{k}}(0)|\Phi_B > = (a_{\boldsymbol{k}}(0) - S(k))|\Phi_B > \tag{3-22}$$

$$\text{i.e.,} \quad a_{\boldsymbol{k}}|\Phi_B >= S(k)|\Phi_B > \tag{3-23}$$

→ $|\Phi_B >$ is the coherent state of $a_{\boldsymbol{k}}$

○ Then, by the Glauber transformation:

$$|\Phi_B > = e^{\sum_{\boldsymbol{k}} S(\boldsymbol{k})(a_{\boldsymbol{k}}{}^{\dagger} - a_{-\boldsymbol{k}})}|0 > \tag{3-24}$$

i.e.,
$$\boxed{|\Phi_0 > = |\Phi_B > \otimes |\Phi_N > = e^{\sum_{\boldsymbol{k}} S(k)(a_{\boldsymbol{k}}{}^{\dagger} - a_{-\boldsymbol{k}})} \prod_{\alpha}^{occ} c_{\alpha}{}^{\dagger}|0 >} \tag{3-25}$$

○ Number of pions with **k**

$$N_{\pi}(\boldsymbol{k}) =< \Phi_0 \left| a_{\boldsymbol{k}}{}^{\dagger} a_{\boldsymbol{k}} \right| \Phi_0 > = |S(\boldsymbol{k})|^2 \tag{3-26}$$

π^0 condensation → Macroscopic N_{π} (i.e., S(**k**)) for specific **k**,
That is, actualization of “pion cloud” depending on
the structure of nucleon system.

○ Two expressions of total energy:

condensation energy

$$E = (\text{K.E. of Nucleons}) - \frac{1}{2}\int d\boldsymbol{r}\{(\boldsymbol{\nabla}\varphi_c)^2 + {m_\pi}^2{\varphi_c}^2\} \leftarrow \text{[FD]} \qquad (3\text{-}27)$$

$$= (\qquad \text{''} \qquad) + \frac{1}{2}\sum_{\alpha\beta}^{occ} < \alpha\beta|V^{OPE}|\alpha\beta > \leftarrow \text{[PD]} \qquad (3\text{-}28)$$

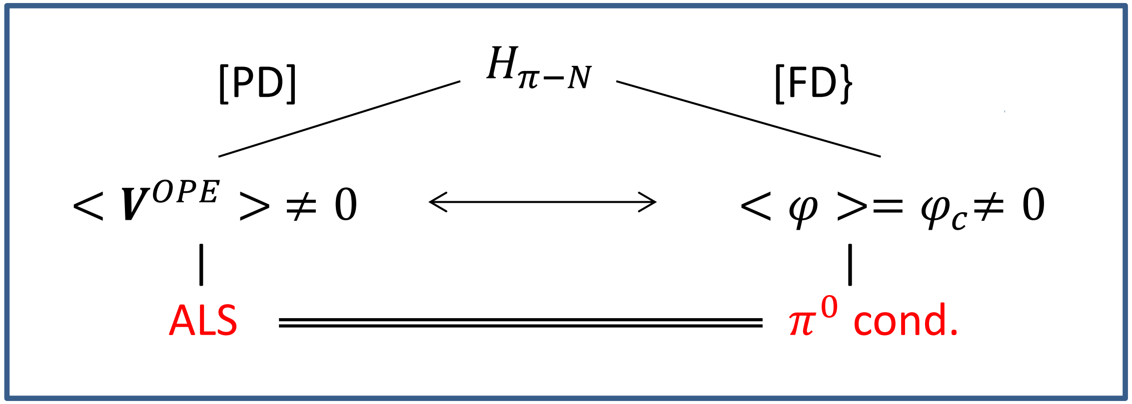
OPEP int. energy

→ equivalence of PD and FD

-- ○ --

$$V^{OPE}(1,2) = m_\pi \frac{f^2}{4\pi}\frac{\tau_1\cdot\boldsymbol{\tau}_2}{3}\{S_{12}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x} + \boldsymbol{\sigma}_1\cdot\frac{e^{-x}}{x}\}$$
$$-\frac{1}{3}m_\pi f^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\delta(x), \qquad x \equiv m_\pi r_{12} \qquad (3\text{-}29)$$

○ Therefore we can see:



(3-30)

② Mechanism for the realization of ALS phase

○ $(\boldsymbol{\nabla}^2 - {m_\pi}^2) < \overset{\varphi_c}{\overset{\parallel}{\varphi}} > = -\tilde{f}\boldsymbol{\nabla} <\text{ALS}|\psi^+\boldsymbol{\sigma}\psi|\text{ALS}> = -\tilde{f}\boldsymbol{\nabla}_z \underset{\underset{\rho d}{\uparrow}}{\underline{\rho_\perp}} \sum_l (-1)^l |\Phi_l(z)|^2$ (3-31)

$\psi = \sum_\alpha C_\alpha \phi_\alpha e^{-i\varepsilon_\alpha t}$, $|\text{ALS}> = \prod_\alpha^{(occ)} {C_\alpha}^\dagger |0>$, (3-32) (3-33)

$\phi_\alpha(\boldsymbol{\xi}) = \frac{1}{\sqrt{\Omega_\perp}} e^{i q_\perp r_\perp} \phi_l(z) \chi_{\sigma_l}(\text{spin})$, $\sigma_l = (-)^l$ (3-34)

$\phi_l(z) = (\frac{a}{\pi})^{1/4} e^{-\frac{a}{2}(z-dl)^2}$

Sol. → $< \varphi(z) > = -2\tilde{f}\rho \sum_{n=1,\ \text{odd}} (\frac{k_n}{{\omega_n}^2}) e^{-\pi^2 n^2/4\Gamma}$ sin $k_n z$ (3-35)

$(k_n = \frac{n\pi}{d}, \Gamma \equiv a d^2, {\omega_n}^2 = {k_n}^2 + {m_\pi}^2)$

Single-mode dominance →

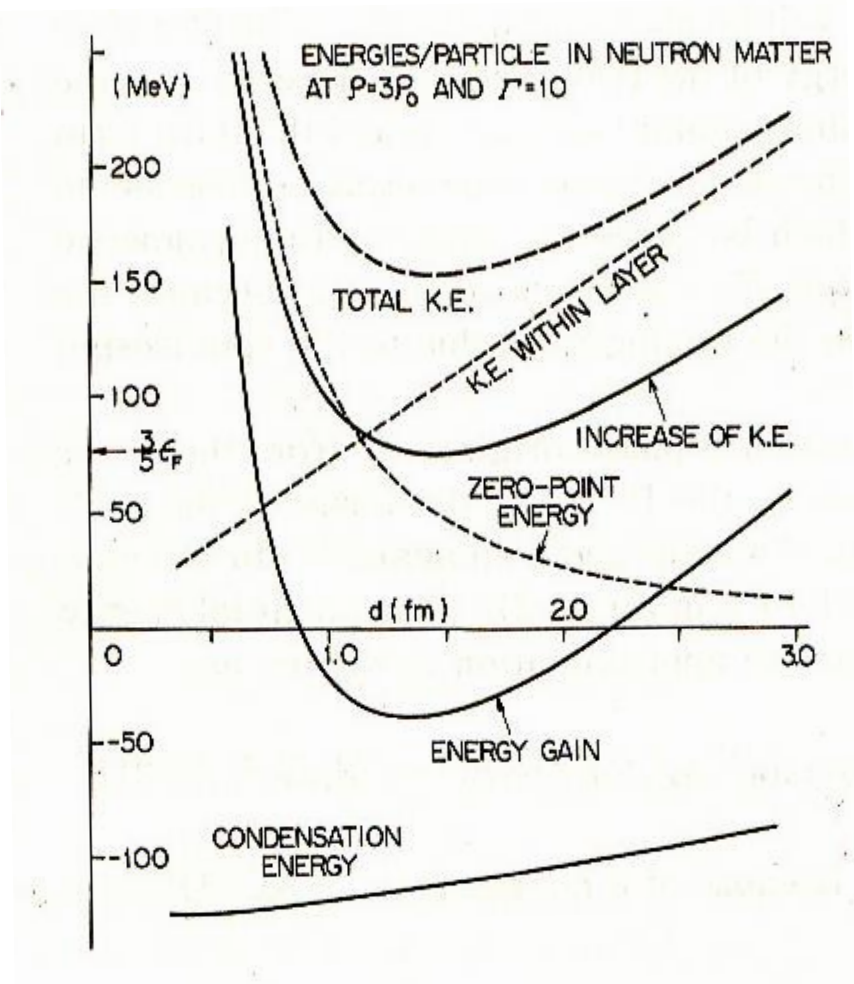
$\simeq -2\tilde{f}\rho \frac{k_c}{{\omega_c}^2} e^{-\pi^2/4\Gamma}$ sin $k_c z$; $k_c \equiv \pi/d$ (3-36)

○ E = (K. E. of neutrons)-$\frac{1}{2}\int d\boldsymbol{r}\{(\boldsymbol{\nabla}<\varphi>)^2 + {m_\pi}^2 <\varphi>^2\}$ (3-37)

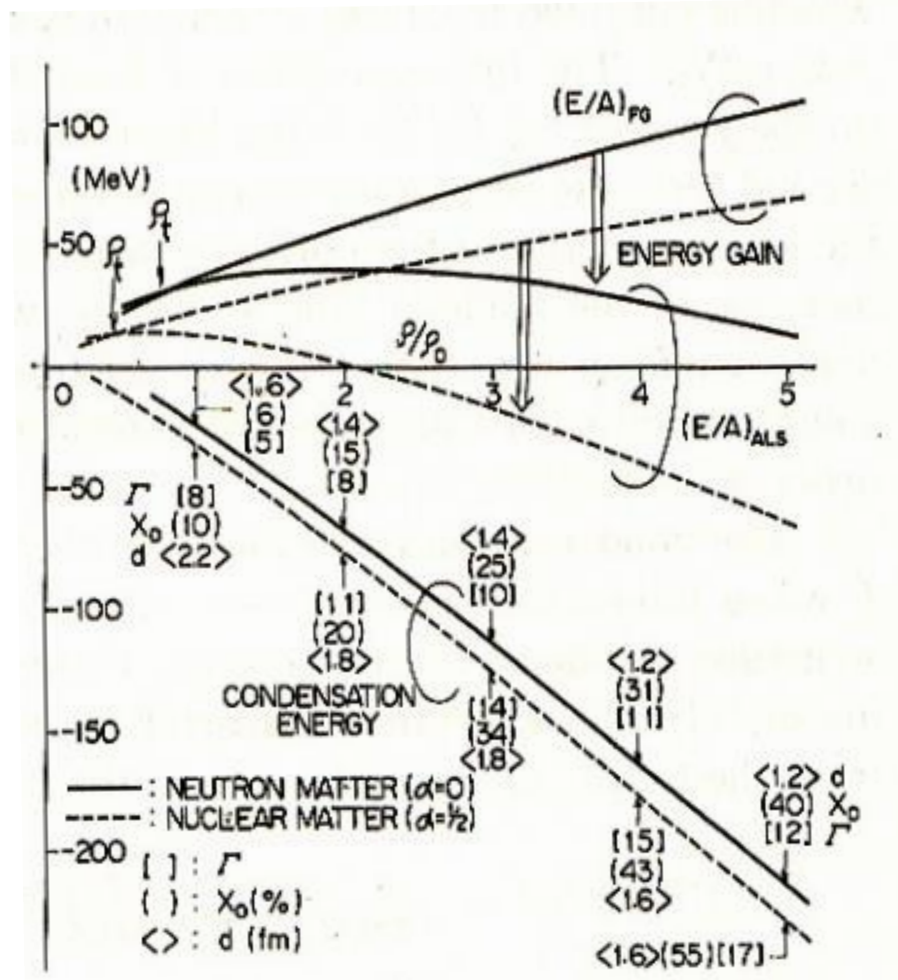
E/N = $\frac{\pi\rho d}{m_N} + \frac{\Gamma}{4 m_N d^2} - \tilde{f}^2 \rho \frac{{k_c}^2}{{\omega_c}^2} e^{-\pi^2/2\Gamma}$; funct. of Γ and d (3-38)

○ ⊿E/N = (ALS)-(FG) = E/N $- \frac{3}{5} \epsilon_F$; $\epsilon_F = \frac{\hbar^2 {q_F}^2}{2 m_N}$; $q_F = (3\pi^2\rho)^{1/3}$ (3-39)

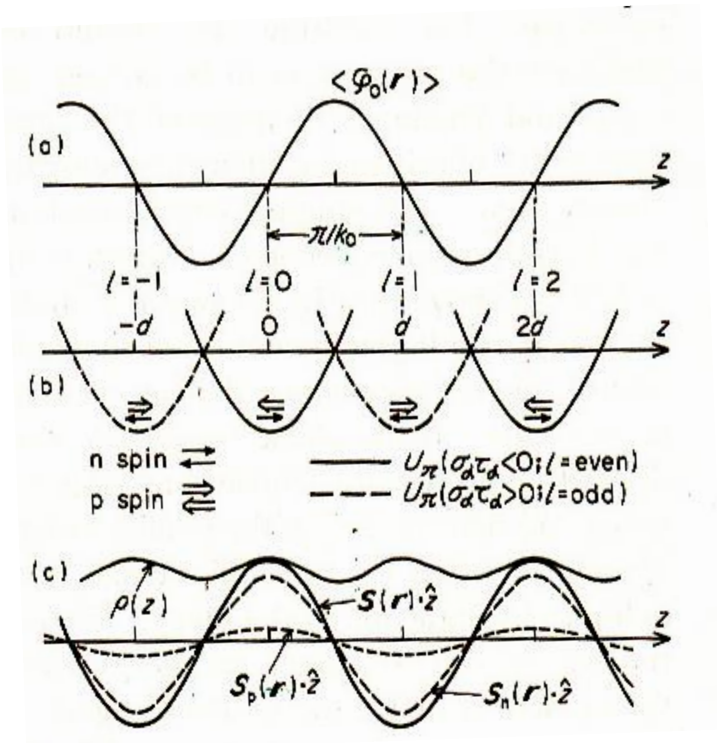
Realization of ALS



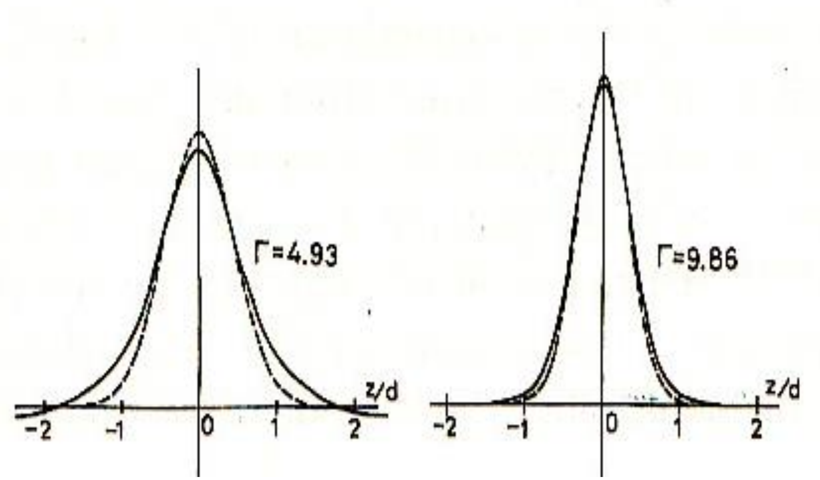
Energy Gain



Selfconsistent Aspects



Wannier v.s. gaussian Functions



□ Charged Pion (π^c) Condensation

- Simple Model (SM) with : $H_{\pi-N}$ only ; MFA, $|\Phi_0>=|\Phi_N>\otimes|\Phi_B>$ (3-40)
- Field eqs. :

$$(\Box - {m_\pi}^2)\varphi_\pm = \sqrt{2}\tilde{f}\boldsymbol{\nabla} < \Phi_N \left|\psi^\dagger \boldsymbol{\sigma}\tau_\pm \psi\right| \Phi_N > \tag{3-41}$$

$$i\dot{\psi} = \left[-\frac{\boldsymbol{\nabla}^2}{2m_N} - \sqrt{2}\tilde{f}\{\tau_+ \boldsymbol{\nabla} < \Phi_B|\psi_-|\Phi_B > \boldsymbol{\sigma} + h.c.\}\right]\psi \tag{3-42}$$

To solve these eqs. Self-consistently under the conditions; charge (Q) and baryon number (N) conservations

$$(\psi \equiv (\psi_p,\ \psi_n),\ \ \tau_\pm = (\tau_1 \pm i\tau_2)/2,\ \ \ \varphi_\pm = (\varphi_1 \pm i\varphi_2)/\sqrt{2}) \tag{3-43}$$

- Source funct. → Isospin flip operator ($\tau_\pm$)
  → good nucleon mode should be

$$\eta_\beta(t) \equiv \eta_\beta e^{-iE_\eta(\beta)t} = {u_\beta}^* \tilde{n}_\beta(t) - {v_\beta}^* \tilde{p}_{\beta_-}(t) \tag{3-44}$$

$$\zeta_\beta(t) \equiv \zeta_\beta\ e^{-iE_\xi(\beta)t} = u_\beta\ \tilde{p}_{\beta_-}(t) + v_\beta \tilde{n}_\beta(t) \tag{3-45}$$

$$(\beta \equiv (\boldsymbol{q}, \sigma),\ \ \ \beta_- \equiv (\boldsymbol{q} - k_c\hat{z}, \sigma),\ \ |u_\beta|^2 + |v_\beta|^2 = 1) \tag{3-46}$$

- $|\Phi_N>=\prod_\beta^{occ} {\eta_\beta}^+ |0> \xrightarrow{\text{No } \pi^c\text{- cond.}}$ FG of pure n-matt. (3-47)
- π^c- cond. of running wave type ($<\varphi_+>\propto e^{ik_c z}$) with the condensed momentum $k_c\hat{z}$

○ coherence of $|\Phi_B>$ can be shown quite analogously with π^0 case :

$$|\Phi_B>=|\Phi_{\pi^-}>\otimes|\Phi_{\pi^+}> \qquad (3\text{-}48)$$

$$|\Phi_{\pi^-}>=exp\{S_{\pi^-}(k_c)(b_{\boldsymbol{k}_c}{}^{\dagger}-b_{k_c})\}|0> \qquad (3\text{-}49)$$

$$|\Phi_{\pi^+}>=exp\{S_{\pi^+}(k_c)(d_{-\boldsymbol{k}_c}{}^{\dagger}-d_{-\boldsymbol{k}_c})\}|0> \qquad (3\text{-}50)$$

$$\begin{bmatrix} S_{\pi^-}(k_c) \\ S_{\pi^+}(k_c) \end{bmatrix} = A_c\sqrt{\Omega\omega_c/2} \times \begin{bmatrix} 1+\mu_\pi/\omega_c \\ 1-\mu_\pi/\omega_c \end{bmatrix} \qquad (3\text{-}51)$$

$$\mu_\pi=\mu_n-\mu_p\,, \qquad \omega_c=(k_c{}^2+m_\pi{}^2)^{1/2}, \qquad (3\text{-}52)$$

$$A_c\equiv-\sqrt{2}\tilde{f}k_c\Omega^{-1}\sum_{\boldsymbol{q}}^{occ}2\,u_{\boldsymbol{q}}v_{\boldsymbol{q}}(\omega_c{}^2-\mu_\pi{}^2) \qquad (3\text{-}53)$$

○ aspect of the condensate :

π^-: $(\boldsymbol{k}_c = k_c\ \hat{z}\ , \mu_\pi)$: coherent state

π^+: $(-\boldsymbol{k}_c\ , -\mu_\pi\ )$: coherent state

$$N_{\pi^-} = S_{\pi^-}{}^2, \qquad N_{\pi^+} = S_{\pi^+}{}^2 \tag{3-54}$$

$$N_p = N_{\pi^-} - N_{\pi^+} (= N_{\pi^c}) \text{ : charge neutrality} \tag{3-55}$$

$\mu_\pi > 0 \rightarrow$ "π^- −dominant" condensate

○ $\mathrm{E/N} = \frac{3}{5} \epsilon_F + (3{\mu_\pi}^2 - {\omega_c}^2){A_c}^2/\rho$ (3-56)

☐ Coexistent Pion Condensation*)

○ π^0 and π^c condensations are made to coexist by taking their condensed momenta as

$$\pi^0: \quad \boldsymbol{k}_0 = k_0\,\hat{z}$$
$$\pi^c: \quad \boldsymbol{k}_c = k_c \hat{r}_\perp$$

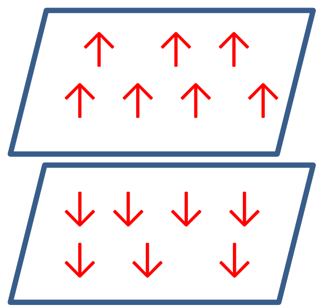
$>$ perpendicular

η composed of

(n↑, p↓) ------------------->

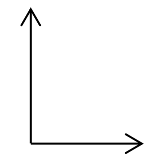
($\eta = u^*\tilde{n} - v^*\tilde{p}$)

(n↓, p↑) ------------------->



$\pi^0 -$cond. in

$\boldsymbol{k}_0$: z-direction



$\boldsymbol{k}_c$

π^c −cond. In x-y plane (2-Dim. FG)

ALS structure of η-particles

○ By this coexistent condensation, the energy gains from $\pi^0$and π^ccondensations become additive

Most probable type of pion condensation:

⟶ $\pi^0\pi^c$Combined condensation

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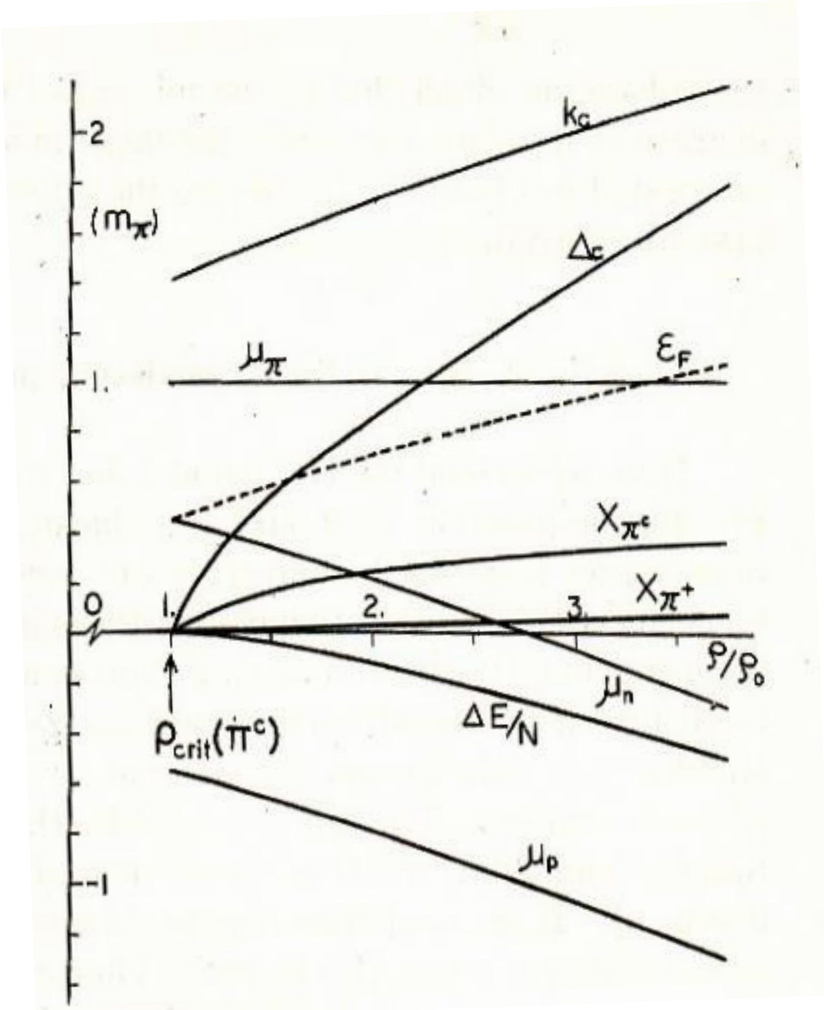
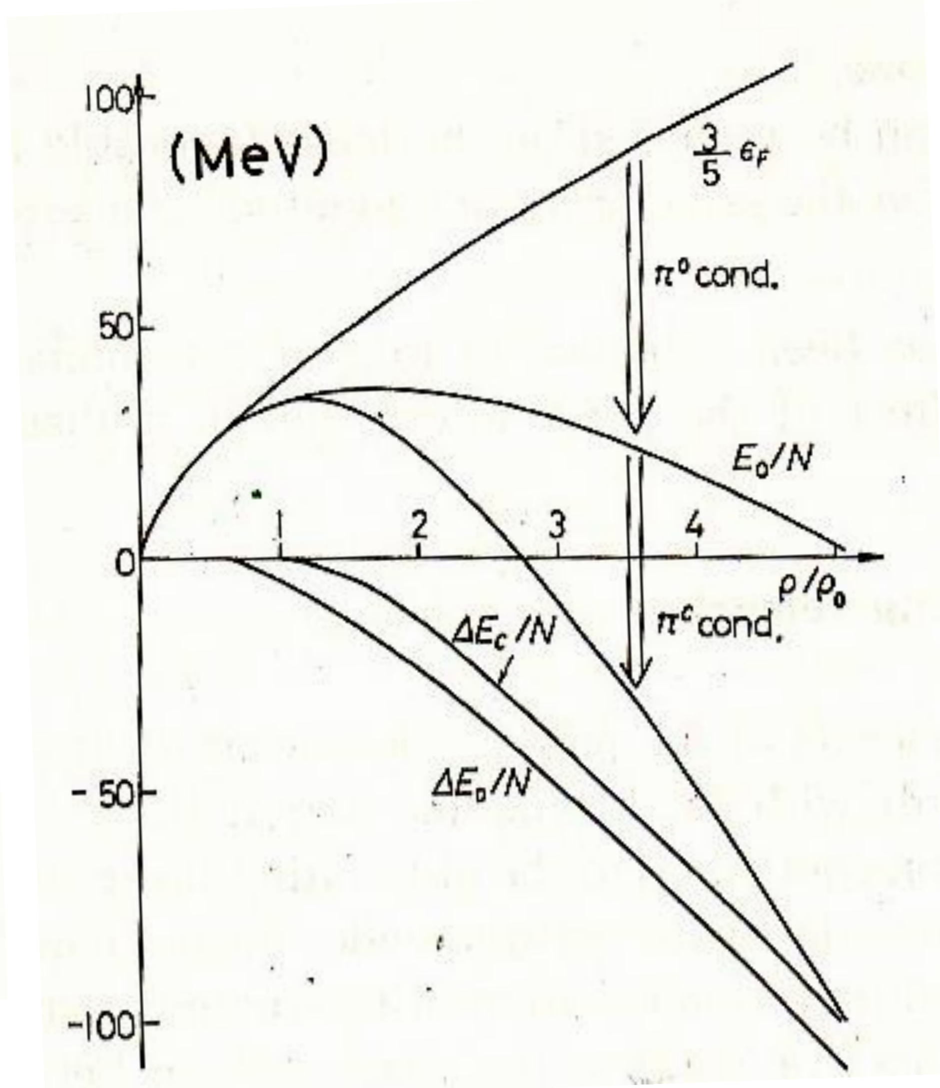
*) K. Tamiya and R. Tamagaki; Prog. Theor. Phys. 60 (1978) 1753

Realization of π^c-cond. (SM)



Additive Energy Gain (SM)



□ Toward Realistic Treatment

- $H_{\pi-N}$ only:

$$\rho_t(\pi^0) \simeq \rho_t(\pi^c) \simeq \rho_t\ (\pi^0\pi^c) \simeq \rho_0$$

- Other effects :
  - ° short-range correlation
  - ° ρ-meson contribution
  - ° quantum correction (exch. Effect)
  - ° Isobar $\Delta(1232)$ effect
  - ° N-N int. other than $H_{\pi-N}$

  (short-range correlation, ρ-meson contribution, quantum correction ← Act against; Isobar $\Delta(1232)$ effect ← Act for)

- Results :

	Authors	ρ_t
π^0	T. Kunihiro and T. Tatsumi ('81)	$\sim 2\ \rho_0$
	K. Tamiya and R. Tamagaki ('81)	(2-3) ρ_0
	T. Takatsuka and J. Hiura ('82)	(1.5-2.6) ρ_0
	O. Benhar ('83, '85)	(3-4) ρ_0
	A. Akmal and V.R. Pandharipande ('98)	$\sim 1.2\ \rho_0$
π^c	W. Weise and G.E. Brown ('74)	$\sim 2.1\ \rho_0$
	T. Tatsumi ('82)	(1.5-2.2) ρ_0
$\pi_0\pi_c$	T. Muto and T. Tatsumi ('87)	(3-5) ρ_0

4. Baryonic superfluidity under PC*)

☐ Motivation:

Neutrons in NS interior are in the superfluid state of $^3P_2$-type at densities $\rho \simeq (1-3)\rho_0$.

On the other hand, pion condensations are considered to set in or develop somewhere in this density region.

The there arises a question:

Whether the nucleon superfluid, shown to be realizable from ordinary Fermi gas, persist or not when pion condensations come into play.

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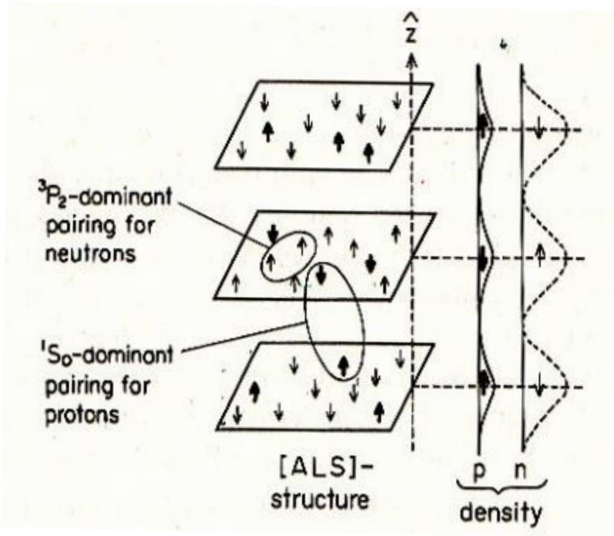
*) As review articles,

T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. Suppl. No. 112 (1993) 107.

T. Takatsuka, Int. Journal of Modern Phys. : Conference Series 11 (2012) 133.

4.1 Pairing Correlation under PC

(1) Under π^0 condensation*)



$$\phi_\alpha{}^{rel}(\boldsymbol{\xi}) = \frac{1}{\sqrt{\Omega_\perp}} e^{i\boldsymbol{q}_\perp \boldsymbol{r}_\perp} \phi_l{}^{rel}(z) \chi_{Sm_S}{}^{(1,2)} \quad (4\text{-}1)$$

$$\sum_{m_L} (i)^{m_L} J_{m_L}(q_\perp r_\perp) e^{i m_L (\varphi_{q_\perp} - \varphi_{r_\perp})} \quad (4\text{-}2)$$

$S = 1, \quad m_S = (-1)^l$

Density localization

○ Remarks

(i) 1-Dim. Localization → pairing correlation is operative in 2-Dim. FG space, and predominant for the pair in the same layer;

- $(\boldsymbol{q}_\perp, \ l; -\boldsymbol{q}_\perp, \ l)$-Cooper pair
- superfluid of 2-Dim. character

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*) T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. 62 ('79) 1655; 64 ('80) 2270; 65 ('81) 1333; 67 ('82) 1649.
R. Tamagaki, T. Takatsuka and H. Furukawa Prog. Theor. Phys. 64 ('80) 1865.

(ii) Pair state is specified by $\tilde{\lambda} \equiv (S, m_S, m_L)$ instead of $\lambda \equiv (S, L, J, m_J)$.

(iii) $\tilde{\lambda}_1 \equiv (S=1,\ \underline{m_S = m_L = (-1)^l})$ is most effective, where $^3P_2$ interaction dominates

$|m_J| = 2$

⟶ $^3P_2$-dominant pairing.

Gap Equation

: 2-Dim. $\tilde{\lambda}_1 = (S=1,\ m_S = m_L = (-1)^l)$

$$\varDelta_{\tilde{\lambda}_1}(q_\perp) = -\frac{1}{2}\int_0^\infty q_\perp' dq_\perp' < q_\perp' \left|V_{\lambda_1}(r_\perp)\right| q_\perp > \varDelta_{\tilde{\lambda}_1}(q_\perp')/\sqrt{\tilde{\varepsilon}^2(q_\perp') + \varDelta_{\tilde{\lambda}_1}{}^2(q_\perp')} \quad \text{(4-3)}$$

$$< q_\perp' | V_{\tilde{\lambda}_1}(r_\perp) | q_\perp > \equiv \int_0^\infty r_\perp d\, r_\perp J_1(q_\perp' r_\perp) V_{\tilde{\lambda}_1}(r_\perp) J_1(q_\perp r_\perp) \quad \text{(4-4)}$$

$$V_{\tilde{\lambda}_1}(r_\perp) \equiv \left(\frac{a}{\pi}\right)^{1/2} \int dz\, e^{-\frac{a}{2}z^2} V_{\tilde{\lambda}_1}(r) \quad \text{(4-5)}$$

$$V_{\tilde{\lambda}_1}(r) \equiv V_c(r) + V_T(r)\left(\frac{3z^2 - r^2}{r^2}\right) + V_{LS}(r) m_S m_L \quad \text{(4-6)}$$

$$\tilde{\varepsilon}(q_\perp) = \hbar^2 (q_\perp{}^2 - q_{\perp F}{}^2)/2m_N{}^* \quad \text{(4-7)}$$

(2) Under π^c condensation

- No localization, 3-Dim. Nature holds. But one important difference arises: superfluid is described by quasineutron basis.

$$\eta = u^* \tilde{n} - v^* \tilde{p},\ \zeta = u\tilde{p} + v\tilde{n} \qquad \text{(4-8)}$$

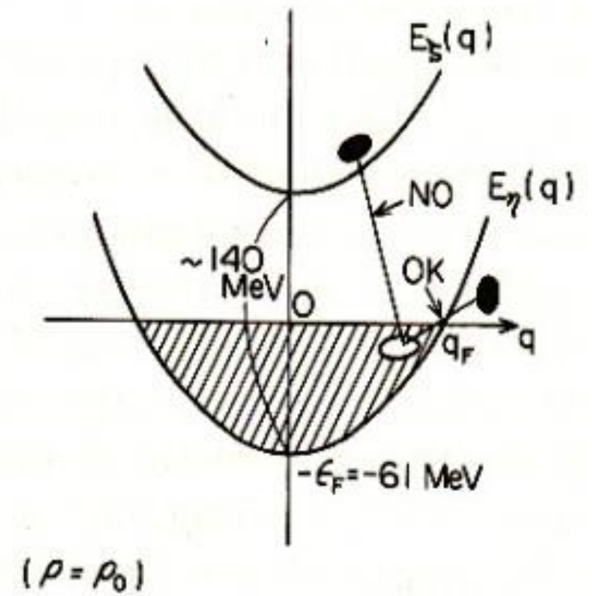
$$(|u|^2 + |v|^2 = 1)$$

- Remarks:

(i) Large band gap

$$\Rightarrow \ |\Phi_0> = \prod_{\beta}^{occ} \eta_\beta{}^\dagger |0> \qquad \text{(4-9)}$$

Excitation of $(\mathbf{q}\sigma; -\mathbf{q}\sigma')$
Cooper pair from η–particle states to ζ-particle ones are stately neglected
→ we can restrict ourselves to η–particle (quasineutron) space.



(ii) Isospin is not a good quantum number
→ pair state is specified by $\lambda' \equiv (S, L, J)$
→$\lambda_1{}' \equiv (S = 1, L = 1, J = 2)$ –pair state is most attractive, which includes $^3P_2$-int. $(\tau = 1)$ and $\tau = 0$-int. with $^3P_2$-kinematical factor
→ means "attenuation" of $^3P_2$–int.

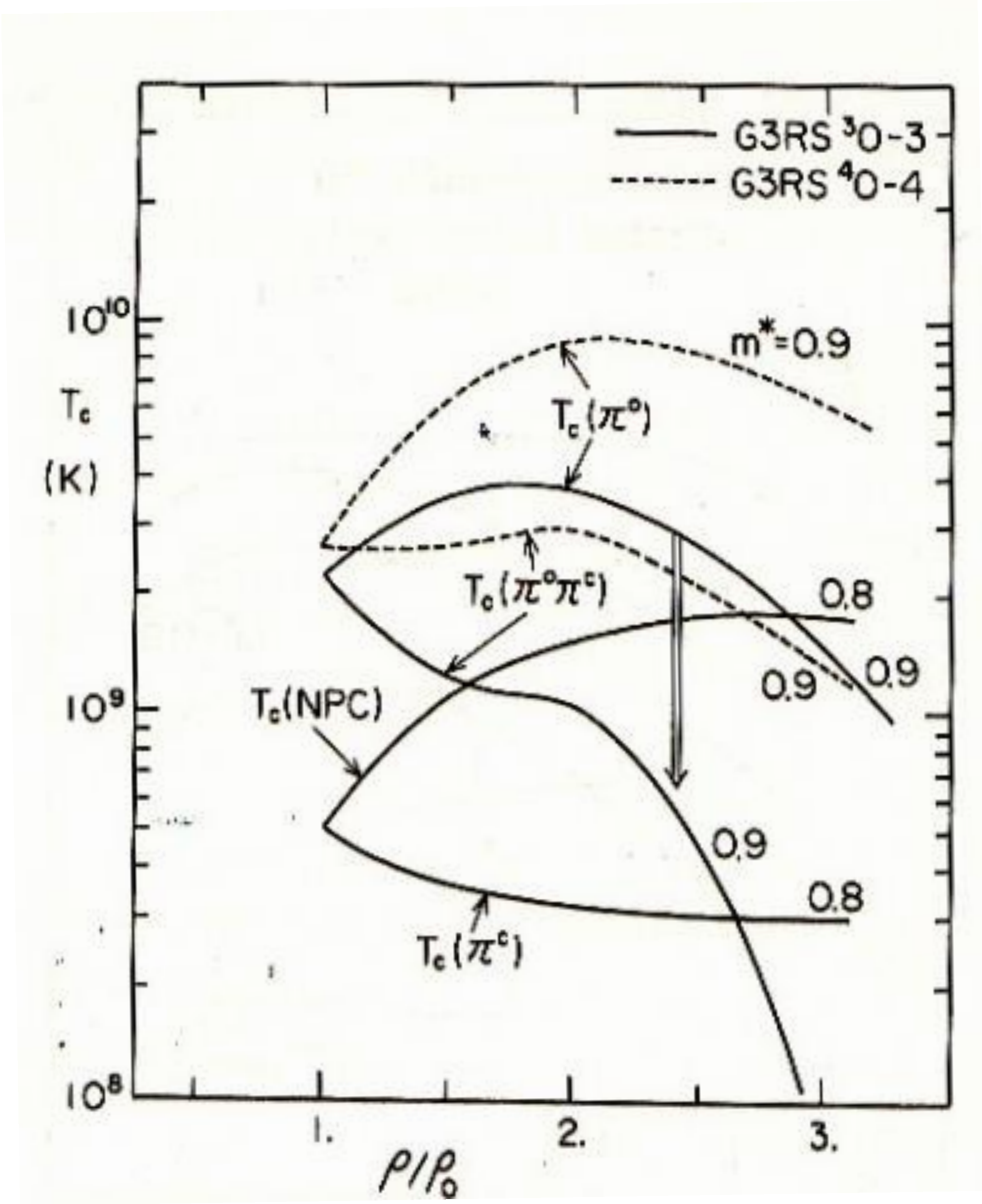
Attenuation factor <1

$^3P_2$-int. ⟶ $^3P_2$-int. $\times \Lambda$

(3) Under $\pi^0\pi^c$ condensation

- The characteristics of (1) and (2) join together:
  - 2-Dim. Nature due to π^0-cond.
  - quasineutron superfluid and “attenuation” due to π^c-cond.
- Most probable type of superfluid at higher densities.

Critical Temperature T_c of Nucleon Superfluids under π^0, π^c, $\pi^0\pi^c$-condensates (Simple Model)



4-2. Baryonic superfluidity with ⊿ effects

□ Under π^0 condensation with ⊿(1232)

○ π-Cond. ← ⊿-mixing is essential

○ n-Super → (n+⊿^{0})-Super

① Interaction in N-Space → in (N+⊿)-Space

$$|N\rangle \equiv \begin{bmatrix} |p\rangle \\ \circ|n\rangle \end{bmatrix} \rightarrow |B\rangle \equiv \begin{bmatrix} |p\rangle \\ |n\rangle \\ |\Delta^{++}\rangle \\ |\Delta^{+}\rangle \\ \circ|\Delta^{0}\rangle \\ |\Delta^{-}\rangle \end{bmatrix} \begin{matrix} \leftarrow \tau/2 = 1/2 \\ \leftarrow -1/2 \\ \leftarrow \theta = 3/2 \\ \leftarrow = 1/2 \\ \leftarrow -1/2 \\ \leftarrow -3/2 \end{matrix} \quad i\text{-spin} \qquad (4\text{-}10)$$

Extended Operator :

$$\mathbf{1} \equiv \begin{bmatrix} 1 & 0 \\ 0 & \kappa_1 \end{bmatrix}, \qquad \boldsymbol{s} \equiv \begin{bmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \kappa_\sigma \boldsymbol{\Sigma} \end{bmatrix}$$

← Spin op. for ⊿ (4-11)

$$\boldsymbol{t} \equiv \begin{bmatrix} \boldsymbol{\tau} & 0 \\ 0 & \kappa_\tau \boldsymbol{\theta} \end{bmatrix}, \qquad \boldsymbol{S} \equiv \begin{bmatrix} \boldsymbol{\sigma\tau} & \lambda_{\sigma\tau} \boldsymbol{S}^\dagger T^\dagger \\ \lambda_{\sigma\tau} \boldsymbol{ST} & \kappa_{\sigma\tau} \boldsymbol{\Sigma\theta} \end{bmatrix}$$

$\kappa_\tau \boldsymbol{\theta}$ ← *i*-spin of. for ⊿

$\boldsymbol{ST}$ ← transition *i*-spin op., transition spin op. (N → ⊿)

SU(4) quark model → $\kappa_1 = 1,\ \kappa_\sigma = 2,\ \kappa_e = 2,\ \kappa_{\sigma e} = \frac{4}{5},\quad \lambda_{\sigma c} = \sqrt{72/25}$ (4-12)

② Quasi-Neutron $^3P_2$ − dominant Pairing

$$|\widetilde{N}_\alpha> = u_\alpha|n_\alpha> - v_\alpha|\varDelta_\alpha{}^0> \qquad \text{(quasi-n)} \qquad (4\text{-}13)$$

$$|\widetilde{\varDelta}_\alpha> = u_\alpha\big|\varDelta_\alpha{}^0> + v_\alpha\big|n_\alpha> \qquad \text{(quasi-}\varDelta^0\text{)} \qquad (4\text{-}14)$$

$$|\Phi_F> = \Big|\Phi_{ALS}> = \prod_\alpha^{(occ)} \widetilde{N}_\alpha{}^\dagger\Big|0> \qquad (4\text{-}15)$$

$$\alpha \equiv \{\boldsymbol{q}_\perp, l\};\ \ spin \to\ \sigma_\alpha/2 = \Sigma_\alpha = 1/2$$

$$i\text{-spin} \to \tau_\alpha/2 = \theta_\alpha = -1/2$$

basis function:

$$\phi_\alpha(\boldsymbol{\xi}) = \phi_{lq_\perp}(\xi) = \frac{1}{\sqrt{\Omega_\perp}} e^{i\boldsymbol{q}_\perp.\boldsymbol{r}_\perp}(a/\pi)^{1/4} e^{-a(z-dl)^2} \chi_l{}^{(B)} \qquad (4\text{-}16)$$

$$\chi_l{}^{(B)}(\text{spin, isospin}) = u_l\,\chi_l{}^{(n)} - v_l\,\chi_l{}^{(\Delta^\circ)} \qquad (4\text{-}17)$$

$$u_l \simeq u, \qquad v_l \simeq (-1)^l\,v$$

$$H_{BB}{}^{(\text{pair})} = \frac{1}{2}\Sigma_{lq_\perp{}'q_\perp} < l\boldsymbol{q}_\perp{}', l-\boldsymbol{q}_\perp{}'|V_{BB}(1,2)|l\boldsymbol{q}_\perp, l-\boldsymbol{q}_\perp> \qquad (4\text{-}18)$$

$$\times N_{l\boldsymbol{q'}_\perp}{}^\dagger N_{l-\boldsymbol{q'}_\perp}{}^\dagger N_{l-\boldsymbol{q}_\perp} N_{l\boldsymbol{q}_\perp}$$

↓

Most attractive pair

$$\Lambda \equiv \{S = 1,\ m_S = (-1)^l, m_L\}$$

$$H_{BB}{}^{(\Lambda-\mathrm{pair})} = \frac{(2\pi)^2}{\Omega_\perp} \sum_{l} \sum_{q_\perp{}'} \sum_{q_\perp} \sum_{m_L} < q_\perp{}' | V_{BB}{}^{(\Lambda)} | q_\perp > b_{lm_L}{}^\dagger(q_\perp{}') b_{lm_L}(q_\perp) \qquad (4\text{-}19)$$

Pair Operator: $b_{lm_L}{}^\dagger(q_\perp) = \frac{1}{\sqrt{2}} \int d\,\varphi_q \frac{1}{\sqrt{2}} e^{im_L\varphi_q} \tilde{N}_{lq_\perp}{}^\dagger \tilde{N}_{l-q_\perp}{}^\dagger$ (4-20)

2-dim. Matrix Elements:

$$< q_\perp{}' \, | V_{BB}{}^{(\Lambda)} | q_\perp > \equiv \int_0^\infty d\, r_\perp r_\perp J_{m_L}(q_\perp{}' r_\perp) V_{BB}{}^{(\Lambda)}(r_\perp) J_{m_L}(q_\perp r_\perp) \qquad (4\text{-}21)$$

2-dim. Pot.

$$\begin{aligned} V_{BB}{}^{(\Lambda)}(r_\perp) \equiv & \int_{-\infty}^{\infty} dz\, e^{-im_L\varphi_r} (\frac{a}{2\pi})^{1/4} e^{-az^2/4} \\ & \times < \chi_{1m_S}{}^{(BB)} \, | V_{BB}(1,2) | \chi_{1m_S}{}^{(BB)} > \\ & \times (\frac{a}{2\pi})^{1/4} e^{-az^2/4} e^{im_L\varphi_r} \end{aligned} \qquad (4\text{-}22)$$

⇩ In the same way as the case without $\varDelta$

Gap eq. :

$$\varDelta_1(q_\perp) = -\frac{1}{2} \int_0^\infty d\, q_\perp{}' q_\perp < q_\perp{}' | V_{BB}{}^{(\Lambda)} | q_\perp > \times \frac{\varDelta_1(q_\perp{}')}{\sqrt{\tilde{\varepsilon}_{q_\perp{}'}{}^2 + \varDelta_1{}^2(q_\perp{}')/2\pi}} \qquad (4\text{-}23)$$

$$\varDelta_{ALS} \equiv \varDelta_1(q_{\perp F})/\sqrt{2\pi} \;, \quad \kappa_B T_c \simeq 0.57 \varDelta_{ALS} \qquad (4\text{-}24)$$

☐ Under π^c condensation with $\varDelta$*)

○ Quasibaryon basis

$$|\tilde{n}_\beta> = \frac{1}{\sqrt{N}}\{|n_\beta> + y_1|\varDelta_{\beta+}{}^{-}>\} \qquad \text{(quasi-n)} \tag{4-25}$$

$$|\tilde{p}_{\beta-}> = \frac{1}{\sqrt{N}}\{|p_{\beta-}> + y_1|\varDelta_{\beta--}{}^{++}\} \qquad \text{(quasi-p)} \tag{4-26}$$

$$N = 1 + |y_1|^2, \beta \equiv \{\boldsymbol{q}, \sigma\}, \beta_\pm \equiv \{\boldsymbol{q} \pm \boldsymbol{k}_c, \sigma\}, \beta_{--} \equiv \{\boldsymbol{q} - 2\boldsymbol{k}_c, \sigma\} \tag{4-27}$$

○ BCS-quasiparticles

$$|\eta_\beta> = u_\beta|\tilde{n}_\beta> - v_\beta|\tilde{p}_{\beta-}>, \qquad |\zeta_\beta> = v_\beta{}^*|\tilde{n}_\beta> + u_\beta{}^*|\tilde{p}_{\beta--}> \tag{4-28}$$

☐ Under $\pi^0\pi^c$ condensation with $\varDelta$**)

○ Quasibaryon basis

$$|\tilde{n}_\gamma> = \frac{1}{\sqrt{N}}\{|n_\gamma> + z_1|\varDelta_\gamma{}^0> + z_2|\varDelta_{\gamma+}{}^{-}>\} \tag{4-29}$$

$$|\tilde{p}_{\gamma-}> = \frac{1}{\sqrt{N}}\{|p_{\gamma-}> + z_1|\varDelta_{\gamma-}{}^{+}> + z_2|\varDelta_{\gamma--}{}^{++}>\} \tag{4-30}$$

$$N = 1 + |z_1|^2 + |z_2|^2, \qquad \gamma \equiv \{l, q_\perp, \sigma\}, \qquad \gamma_\pm \equiv \{l, q_\perp \pm k_c, \sigma\}, \qquad \gamma_{--} \equiv \{l, q_\perp - 2k_c, \sigma\} \tag{4-31}$$

○ BCS-quasiparticles

$$|\eta_\gamma> = u_\gamma|\tilde{n}_\gamma> - v_\gamma|\tilde{p}_{\gamma-}>, \qquad |\zeta_\gamma> = v_\gamma{}^*|\tilde{n}_\gamma> + u_\gamma{}^*|\tilde{p}_{\gamma-}> \tag{4-32}$$

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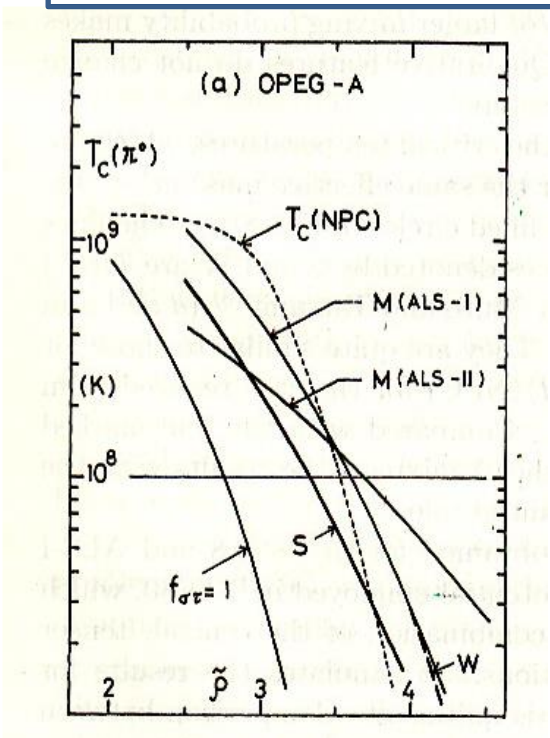
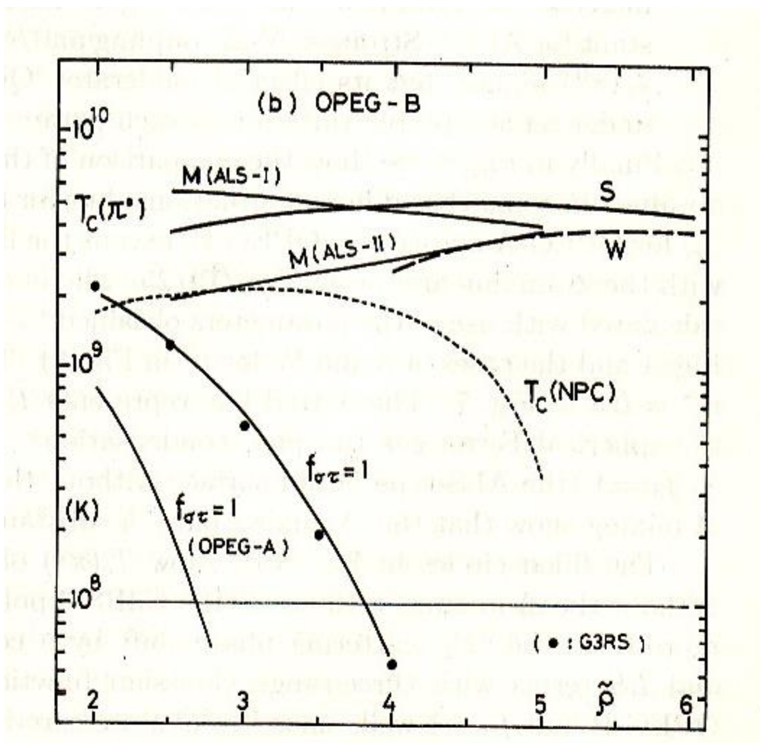
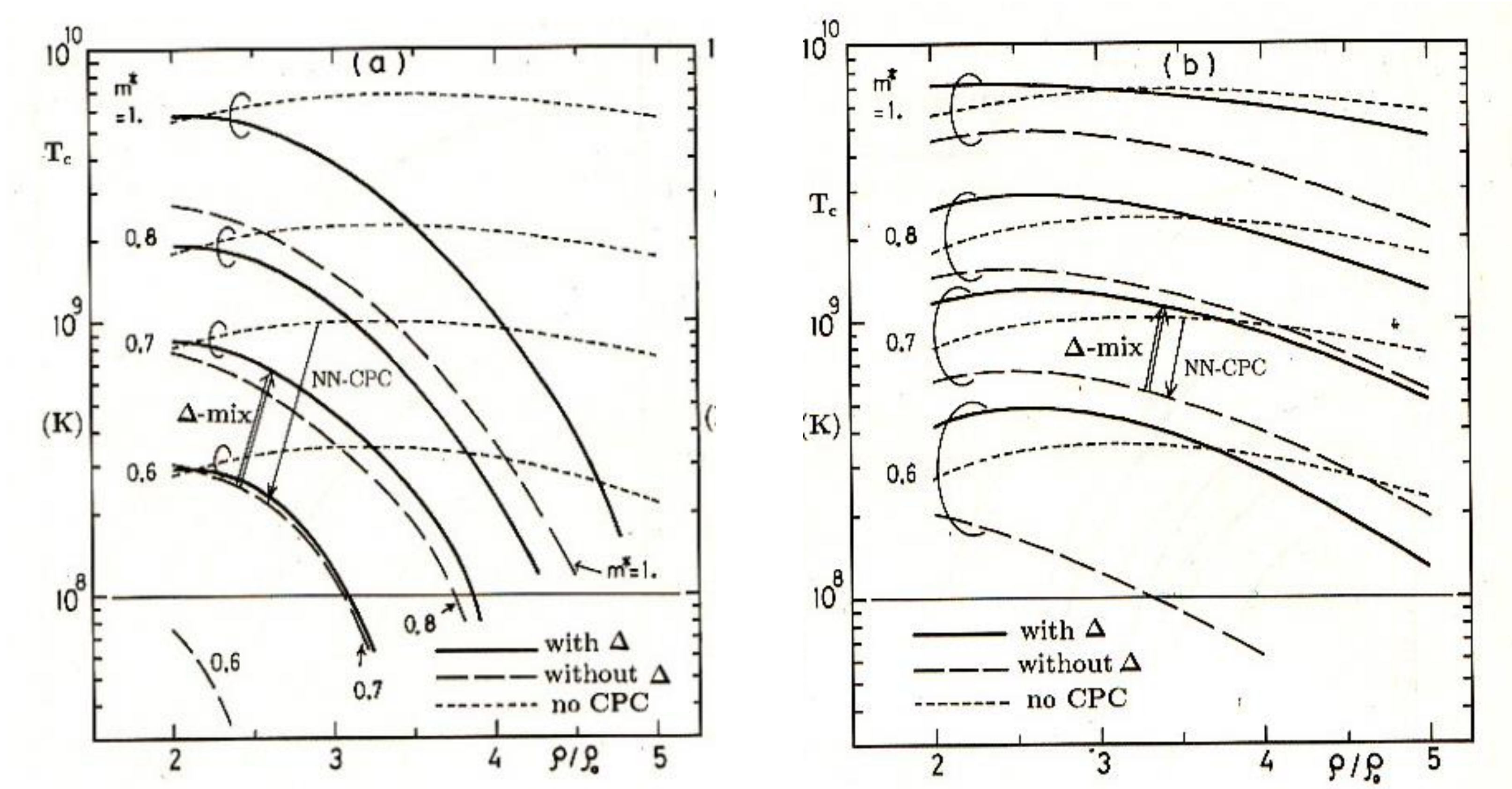
*) T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. 101 (1999) 1043.
**) R. Tamagaki and T. Takatsuka, Prog. Theor. Phys. 110 (2006) 573; 117 (2007) 861.

Critical Temperature (T_c) of ${}^3P_2$-dominant Baryon Superfluid under π^0-cond. with Δ^0 effects

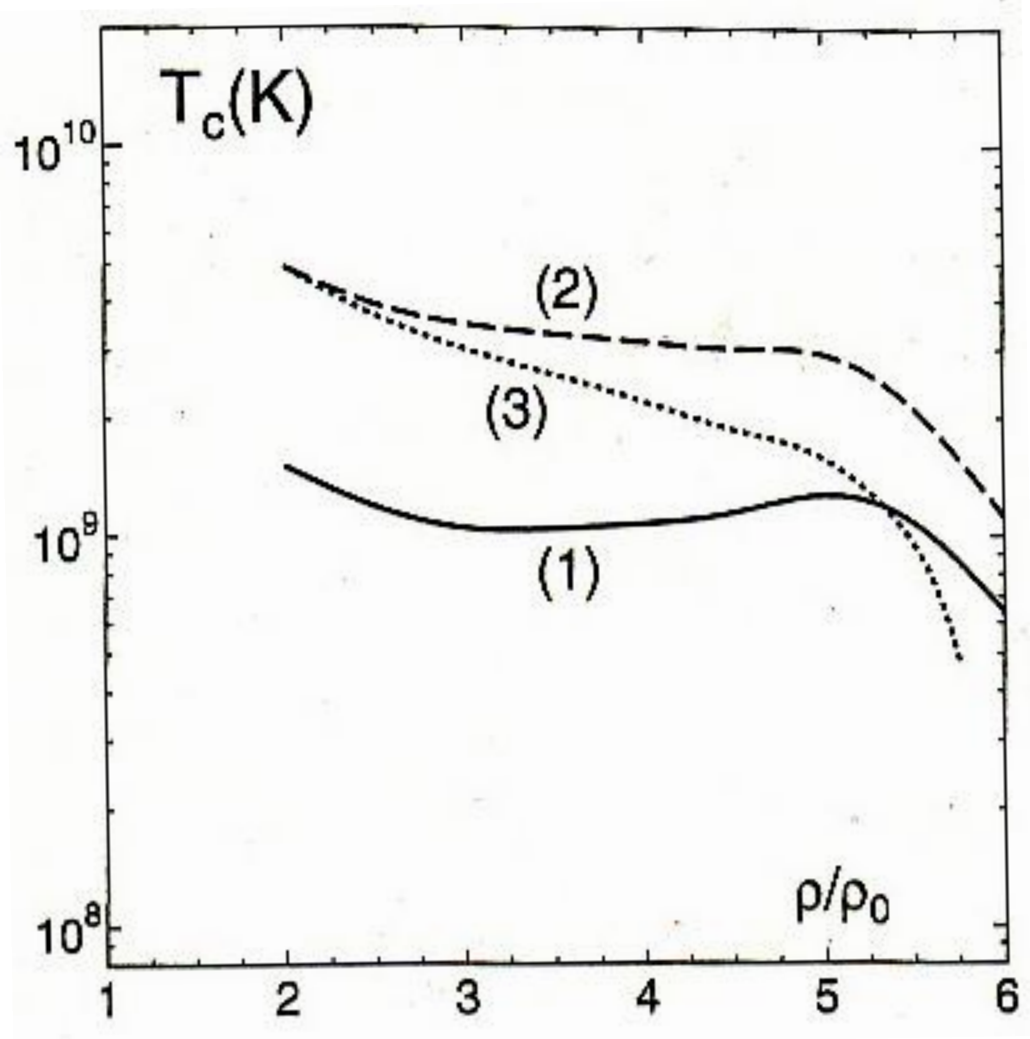




Critical Temperature Baryon $^3P_2$–superfluid under π^0 condensation with Δ effects



Critical Temperature of ${}^3P_2$–dominant Baryon Superfluid under $\pi^0\pi^c$ condensation with $\varDelta$ effects

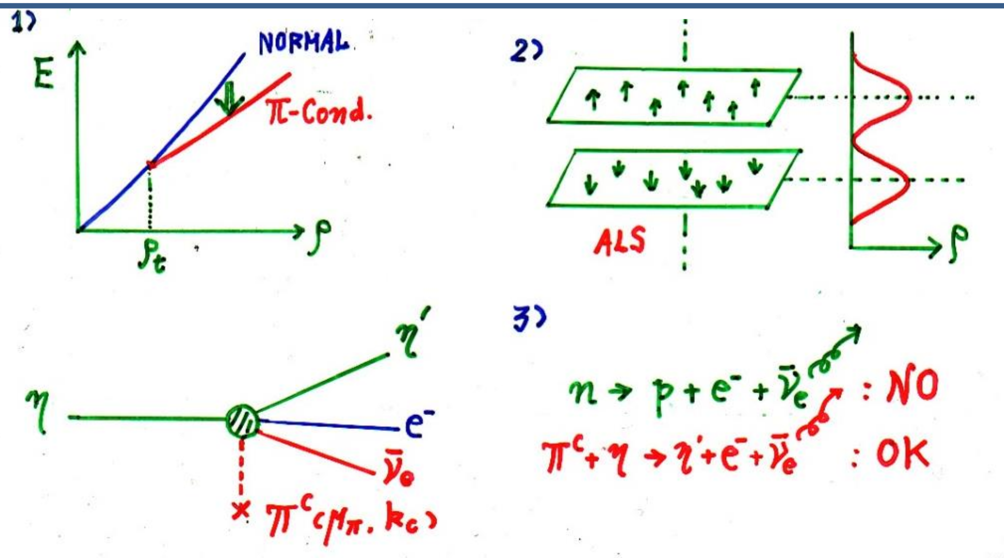


5. Neutron star phenomena with PC

□ Characteristics of Pion-Condensed NS

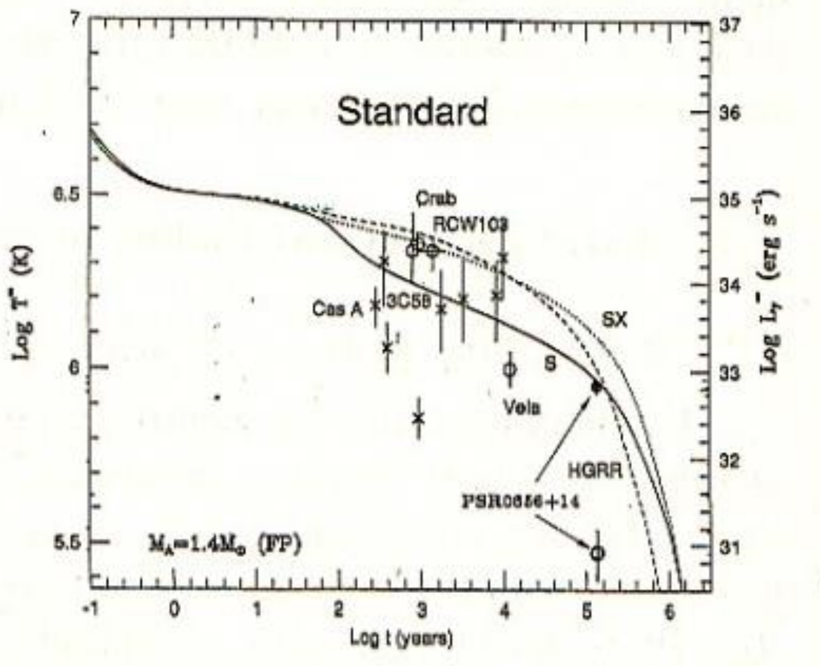
○ 3-points:

1) "Softening":
   EOS is remarkably softened by the energy gain due to π-cond.
   -------- $\pi^0\pi^c$, $\pi^0\,\pi^c$
2) "Solid":
   Solid-like '1-Dim. Localization) state is provided by the ALS structure
   -------- π^0, $\pi^0\,\pi^c$
3) "Pion-Cooling":
   Cooling of NS is dramatically accelerated due to the URCA process mediated by pion condensation --------π^c, $\pi^0\pi^c$

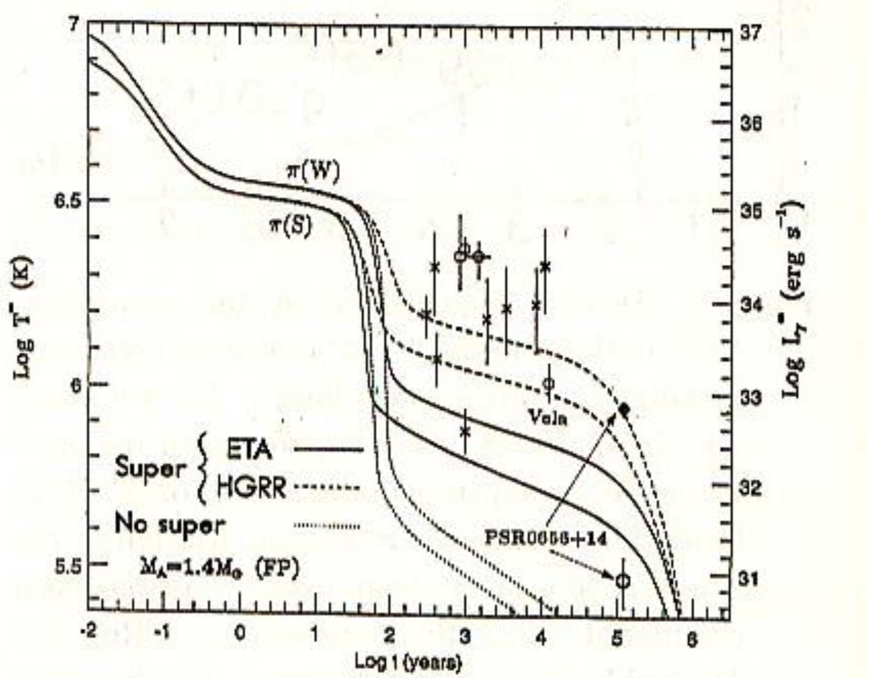


Effect of π condensation on NS cooling

e strength of the classical π^c field, m^*_η the



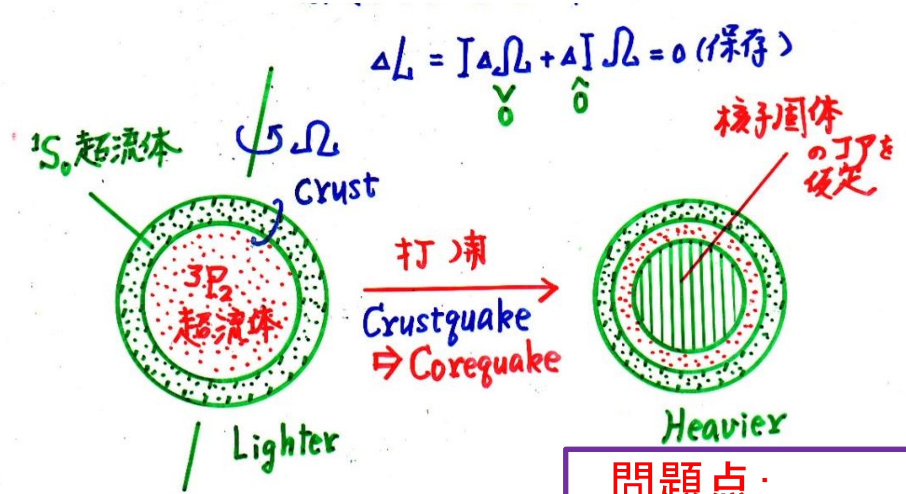
NO PC



With PC

□Problems of starquake model

2成分（中性子超流体とそれ以外）



OK←Crab ($\Delta\Omega_0/\Omega \sim 10^{-8}$)
NO←Vela(〃 $\sim 10^{-6}$)

問題点：
①核子固体は不可能
② Heating
③ Two Exp. Terms (τ_1, τ_2)

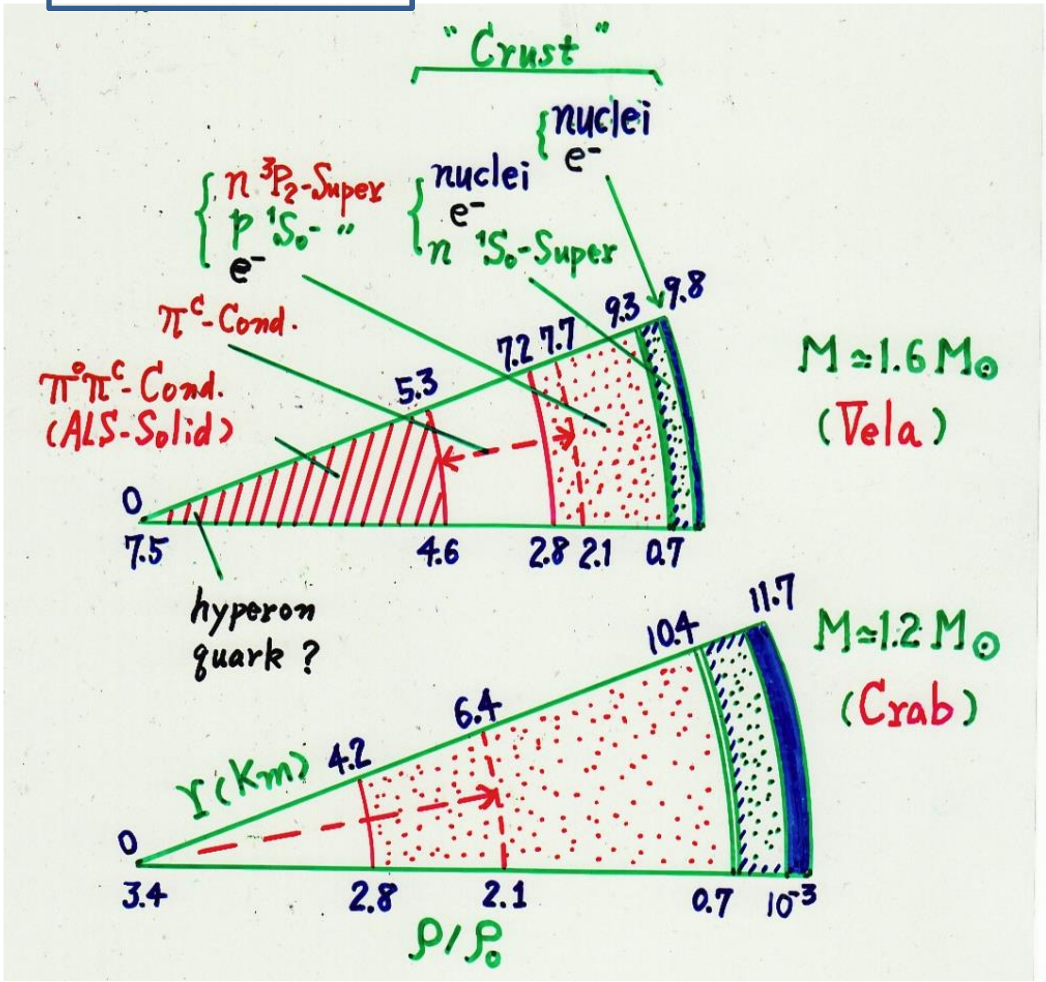
打開が可能（π凝縮星という観点）

New Corequake Model

①→π凝縮による “ALS固体” でOK
②→ 〃 Rapid cooling でOK
(次のグリッチまでに充分冷やされる)
③→2タイプの超流体：$^1S_0 \rightarrow \tau_1$，$^3P_2 \rightarrow \tau_2$と考える
Vela とCrabのグリッチの大きさのちがい→質量の差異で可

☐ M-dep. Of NS structure

BJ-1H+PC

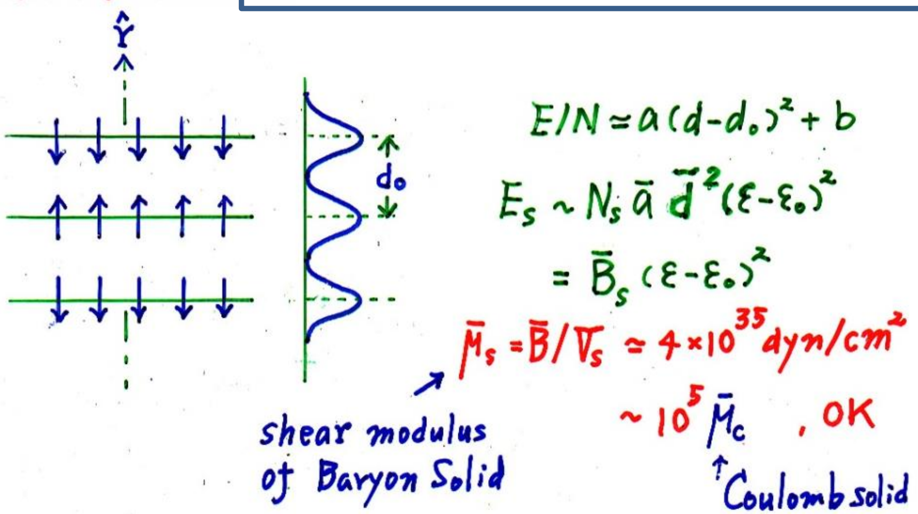


5-1. Pulsar glitch model based on PC*)

□How to overcome the problems

・Crab → Crustquake, Vela→ Corequake

(A) No solid core → "ALS-solid" due to π^0-cond.



(B) Heating → Rapid "Pion Cooling" due to π^c-cond.

By the next glitch, Pion Cooling can get rid of the heat due to the released strain energy

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*) T. Takatsuka and R. Tamagaki , Prog. Theor. Phys. 79 (1988) 274; 82 (1989) 945.

(i) How much energy is released?

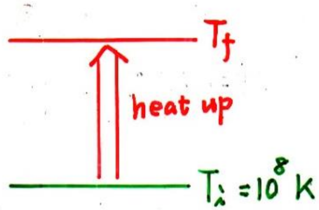
Strain energy : $\Delta E_s \simeq 2\bar{B}_s |(\varepsilon - \varepsilon_0)\Delta\varepsilon|$

$\sim 6 \times \theta_m \times 10^{47}$ erg

↑critical strain angle

$\theta_m = 5\times10^{-4} \sim 5\times10^{-2} \longrightarrow \sim 3\times10^{44-46}$ erg !

(ii) To what extent is NS heated up?



specific heat

$\bar{C} = \bar{C}_0 T$; $\bar{C}_0 = 2.5\times10^{-2}$

(Low. Temp. Approx.)

$$\Delta E_s = N\int_{T_i}^{T_f} \bar{C}\, dT = \frac{1}{2} N \bar{C}_0 (T_f^2 - T_i^2)$$

$\longrightarrow \Delta T (\equiv T_f - T_i) \simeq (0.04 - 2)\times10^8$ K !

(iii) How long is the cooling time Δt for $T_f \rightarrow T_i$?

$$\Delta t = -N\int_{T_f}^{T_i} dT\, \bar{C}/L_\pi \leftarrow \text{Luminosity} \simeq \eta\times10^{57} T^6$$

$$\simeq (1-8)\times10^{-2}\,\eta^{-1}\ \text{yr} \begin{cases} \eta = 1 & 1) \\ \eta \sim 0.1 & 2) \end{cases}$$

< 1yr $< t_g = (2-4)$ yr . OK

Energy Release by Starquake

$E(\varepsilon, \varepsilon_0) = E_0$ (indep. of ε)

oblateness ε, reference ε_0

$-\frac{1}{2} I \Omega^2 \varepsilon$ ---- rot. energy

$+A\varepsilon^2$ ---- grav. 〃 $\left(A = \frac{3}{25}\frac{GM^2}{R}\right)$

$+B(\varepsilon - \varepsilon_0)^2$ ---- strain 〃 $(B = \mu V)$

$$\Delta E = \frac{\partial E}{\partial \varepsilon}\Delta\varepsilon + \frac{\partial E}{\partial \varepsilon_0}\Delta\varepsilon_0$$

$(5\times10^{-4} \sim 5\times10^{-2})$

* $|\Delta E| = 2(A+B)|\varepsilon - \varepsilon_0|\,|\Delta\varepsilon| = 2(A+B)\theta_m|\Delta\varepsilon|$

$(1-Q)\frac{|\Delta\Omega_0|}{\Omega_0}$

* $\left(\Delta E_{rot} = \frac{\partial E_{rot}}{\partial \Omega}\Delta\Omega + \frac{\partial E_{rot}}{\partial I}\Delta I = \frac{1}{2} I \Omega^2 \left(\frac{\Delta\Omega}{\Omega}\right)\right)$

BJ-EOS

$i = c$, or $i = c + s$

	M	$\Delta\Omega/\Omega$	$\Omega\ (s^{-1})$	$I_i\ (10^{45}\ g\cdot cm^2)$	A (erg)	B (erg)
Crab	$1.25 M_\odot$	$\sim10^{-8}$	189	0.0328	1.30×10^{51}	1.93×10^{48}
Vela	$1.61 M_\odot$	$\sim10^{-6}$	70	0.590	2.51×10^{52}	2.56×10^{53}

- Crab → crustquake : $\Delta E \simeq 1.3 \times 10^{39\sim41}$ erg

  $(\Delta E_{rot} \simeq 5.9 \times 10^{39}$ erg$)$

- Vela → corequake : $\Delta E \simeq 2.8 \times 10^{44\sim46}$ erg

  $(\Delta E_{rot} \simeq 1.4 \times 10^{42}$ erg$)$

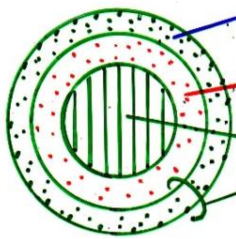
⇨残りは何に使われる？

(1) Heating

(2) γ, X ray burst – like phenomena ?

(C) Two exponential terms ($\tau_1 \sim$ monthes, $\tau_2 \sim$ days)

→ **Extend 2-comp. into 3-comp.**



$^1S_0$-superfluid : I_s , τ_s , Ω_s : 1st

$^3P_2$-superfluid : I_p , τ_p , Ω_p : 2nd

Solid core, Crust } : I_c , Ω : 3rd

Eq. of motion : (external torque: α)

$$I_c \dot{\Omega} = -\alpha - I_c(\Omega - \Omega_s)/\tau_s - I_c(\Omega - \Omega_p)/\tau_p$$

$$I_s \dot{\Omega}_s = I_c(\Omega - \Omega_s)/\tau_s$$

$$I_p \dot{\Omega}_p = I_p(\Omega - \Omega_p)/\tau_p$$

τ_s is responsible for τ_1

τ_p " τ_2

○Solution :

$$\Omega(t) = \Omega^{no}(t) + \Delta\Omega_0 \, [\underbrace{Q_1 e^{-t/\tau_1}}_{\text{long}} + \underbrace{Q_2 e^{-t/\tau_2}}_{\text{short}} + (1-Q_1-Q_2)]$$

$$\Delta\dot{\Omega}(t) \equiv \dot{\Omega}(t) - \dot{\Omega}^{no}(t)$$

$$= \Delta\Omega_0 \, [\frac{Q_1}{\tau_1} e^{-t/\tau_1} + \frac{Q_2}{\tau_2} e^{-t/\tau_2}]$$

$$\Omega^{no}(t) = -\frac{\alpha}{I} t + \text{const.}$$

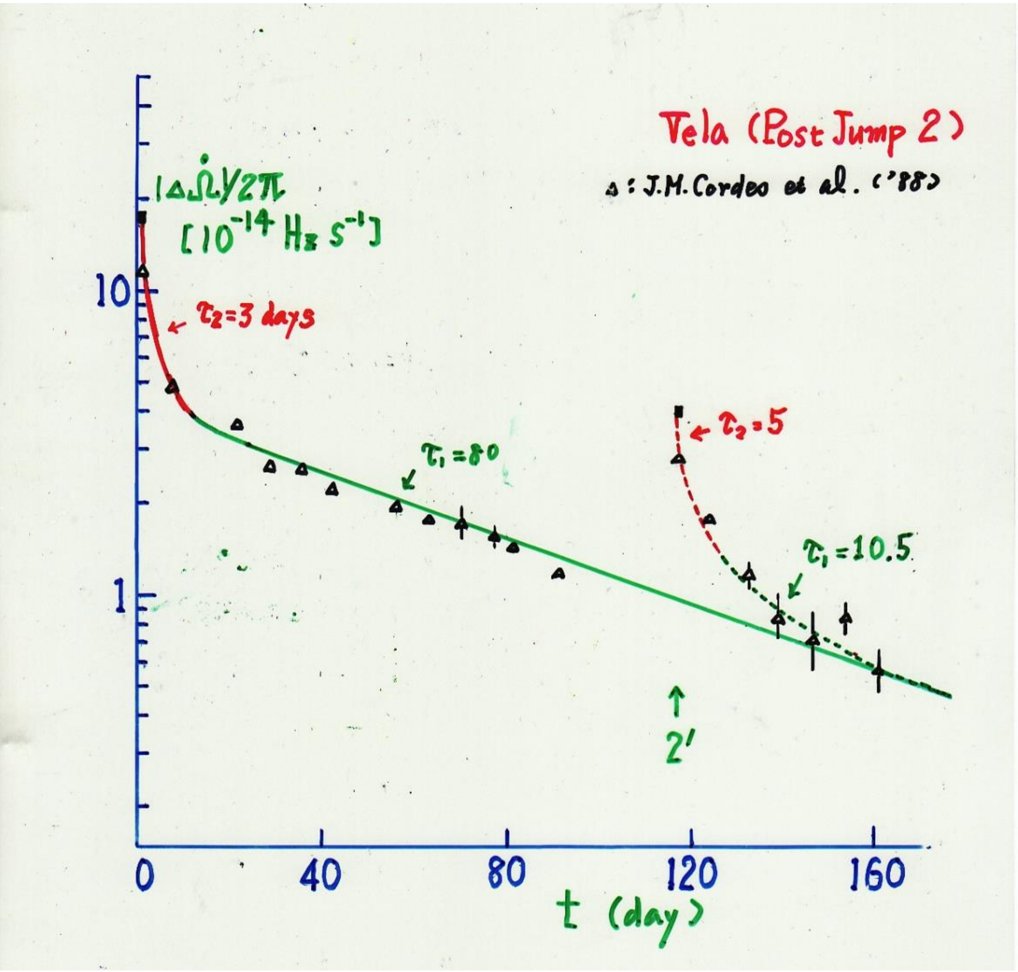
$$\tau_1 \simeq \frac{I_s}{I_c} \tau_s, \quad \tau_2 \simeq \frac{I_p}{I} \tau_p, \quad I = I_c + I_s + I_p$$

$$Q_1 \simeq \frac{I_s}{I} \{ (1-\Delta\Omega_{so}/\Delta\Omega_0) - \frac{I_p}{I} \frac{(1-\Delta\Omega_{po}/\Delta\Omega_0)}{(1-\tau_2/\tau_1)} \}$$

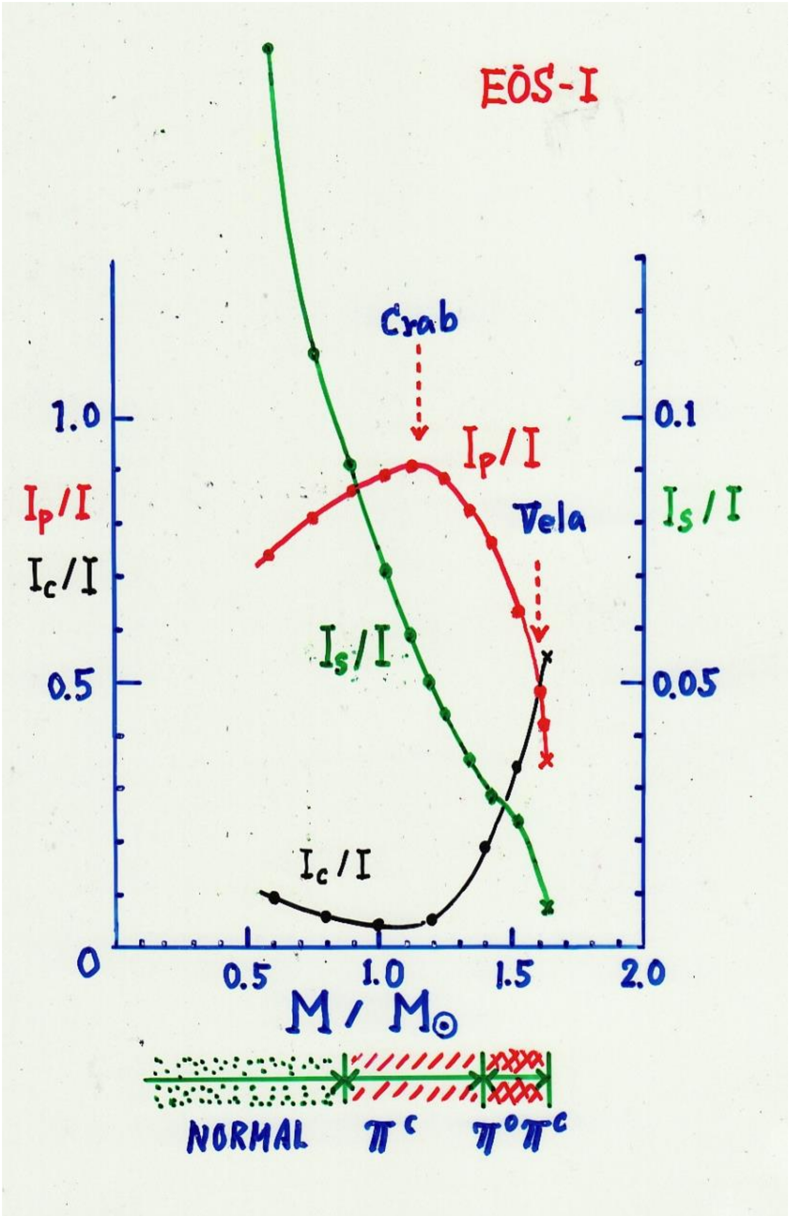
$$Q_2 \simeq \frac{I_p}{I} \{ (1-\Delta\Omega_{po}/\Delta\Omega_0) - \frac{I_s}{I} \frac{(1-\Delta\Omega_{so}/\Delta\Omega_0)}{(1-I_c \tau_1/I \tau_2)} \}$$

(i) short term not visible in Ω8t) because of
$Q_2 \gg Q_1$, becomes visible in $\Delta\dot{\Omega}(t)$; $\tau_2 \ll \tau_1 \rightarrow \frac{Q_1}{\tau_1} \sim \frac{Q_2}{\tau_2}$.

(ii) Assuming ang. Mom. Consv. At glitch (t≃0) for respective component ($\Delta I_i/I_i = -\Delta\Omega_{i0}/\Omega_i$, i=c, s, p) information of internal structure can be extracted.



Vela (Post Jump 2)
Δ : J.M. Cordes et al. ('88)
$|\Delta\dot{\Omega}|/2\pi$
$[10^{-14}$ Hz s$^{-1}]$
τ_2=3 days
τ_1=80
τ_2=5
τ_1=10.5
2'
10
1
0
40
80
120
160
t (day)



EOS-I
Crab
Vela
Ip/I
Ic/I
Is/I
1.0
0.5
0
0.1
0.05
0.5
1.0
1.5
2.0
M / M⊙
NORMAL
π^c
π^0π^c

□ Example :

- Crab : $I_p / I \sim 0.9 \longrightarrow M \sim 1.2\, M_\odot$

  ($(1.1 \sim 1.3) M_\odot$)

  $\longrightarrow$ No Solid Core

  $\longrightarrow$ Crustquake only $\rightarrow \frac{\Delta\Omega_0}{\Omega} \sim 10^{-8}$

- Vela : $I_p / I \sim I_c / I \sim 0.5 \longrightarrow M \sim 1.6\, M_\odot$

  ($(1.4 \sim 1.6) M_\odot$)

  $\longrightarrow$ With Solid Core

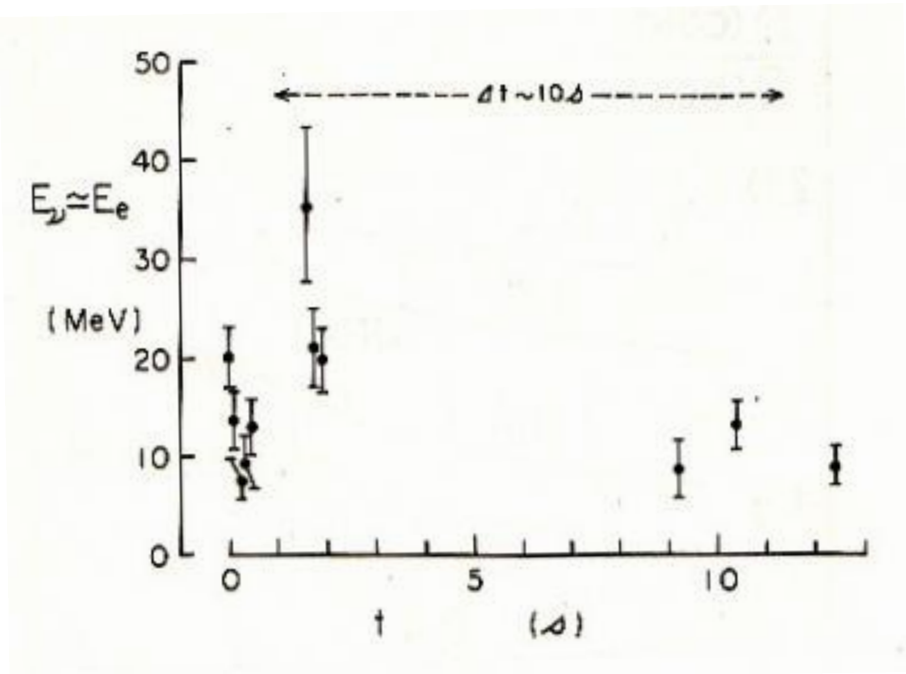
  $\longrightarrow$ Corequake is possible ! $\rightarrow \frac{\Delta\Omega_0}{\Omega} \sim 10^{-6}$

  (triggared by crustquake)

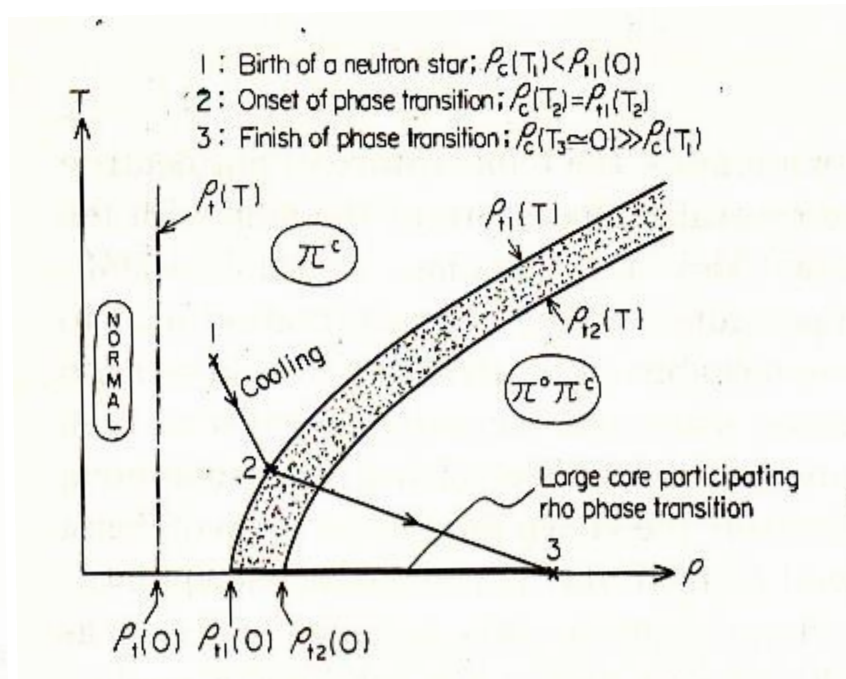
That is, consistent with our model setting.

5-2. PC and ν-burst from SN1987$A^{*)}$

ν-burst from SN1987A



Scenario



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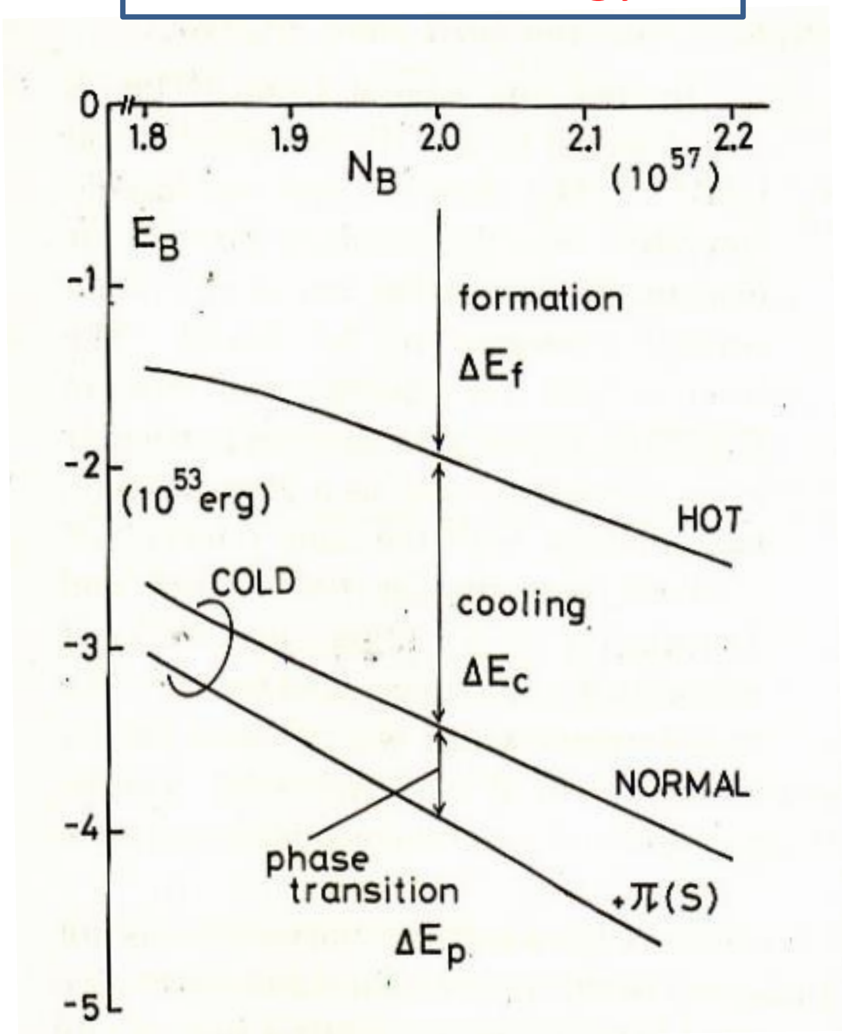
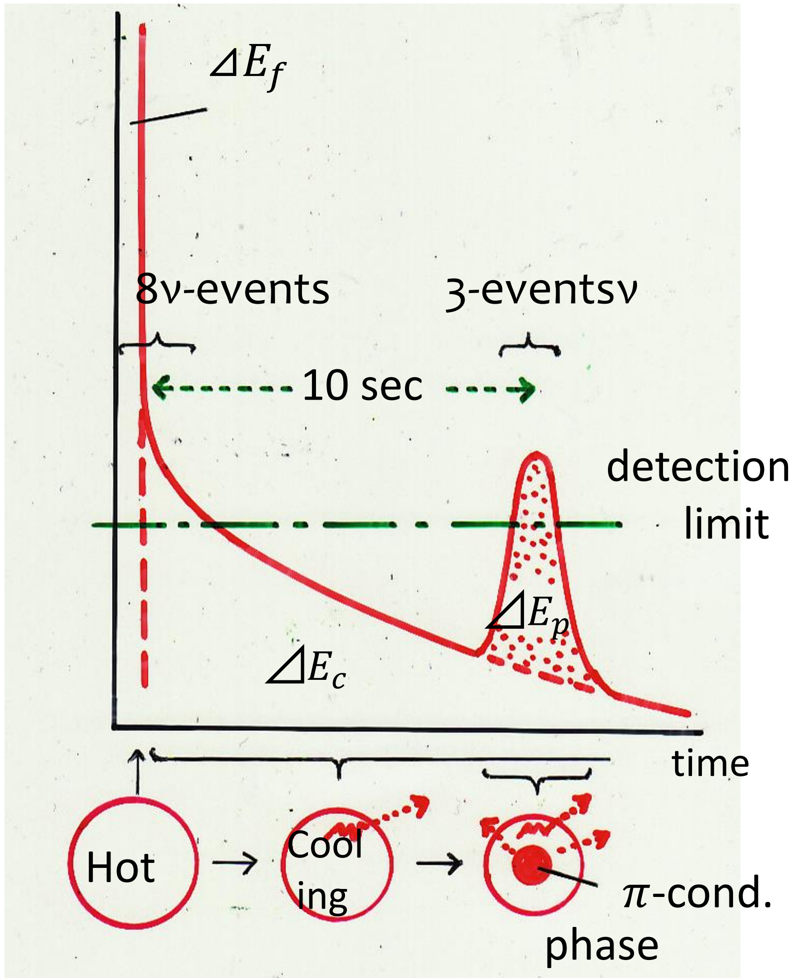
*) T. Takatsuka, Prog. Theor. Phys. 78 (1987) 516; 80 (1988) 361

Releasable Energy



ν-luminosity



□ $\Delta E_{obs} \sim (0.9 - 3.5) \times 10^{53}$ erg (K. Sato and H. Suzuki, Phys. Lett. B196 (1987) 267)

$\sim (1.6 - 3.1) \times 10^{53}$ erg (S.H. Kahana, J. Cooperstein and E. Baron, Phys. Lett. B196 (1987) 259)