原子核物理学連続講義・コースX-1

Baryonic Matter and Neutron Stars
（第1回）

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1. Introduction

1. Introduction

1-1. Discovery of NSs

前史　―― 架空の天体と考えられていた
1932 チャドウィック　中性子の発見、　1934 ツウィッキーとバーデ；超新星の残骸に
1939 ランダウ、オッペンハイマーとボルコフ；理論上の存在を証明

パルサー発見
1967.11 ジョスリン・ベルとヒウィシュ
こぎつね座　周期T=1.337秒のパルス源を発見、　パルサーと命名

パルサーの正体は？
＊ 周期が秒程度――電波源は小さい
＊ 周期が一定　――周期運動のE大
（候補：白色矮星or中性子星）

何故パルスがでるか―――考え方
1. 白色矮星or中性子星の連星公転運動――重力波の放出が大きすぎる
2. これらの星の脈動
3. これらの星の自転運動
（1968年秋までは2、3の可能性が残っていた）

パルサー＝中性子星である！

1968.10 おうし座のカニ星雲の中心部にパルサー発見、Crabパルサーと命名される
T=0.033秒！（短い！）――2の可能性は消える、また、3のうち白色矮星はダメ
（遠心力で重力不安定）、 従って、パルサー＝中性子星が残る。 加えて、
イ. カニ星雲は超新星の残骸であり（記録あり）、その中心に中性子星があることは星の一生のシナリオと合致する
ロ. カニ星雲全体のエネルギー10^{38}エルグ/秒がピッタリと合う
ハ. 中心部に小さな星のあることが高速カメラによる写真撮影の成功でたしかめられた（1969.1）

パルサー＝中性子星であることが確定、　現在　～1600個が観測されている。

Observation of a Rapidly Pulsating Radio Source

by

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Unusual signals from pulsating radio sources have been recorded at the Mullard Radio Astronomy Observatory. The radiation seems to come from local objects within the galaxy, and may be associated with oscillations of white dwarf or neutron stars.

In July 1967, a large radio telescope operating at a frequency of 81·5 MHz was brought into use at the Mullard Radio Astronomy Observatory. This instrument was designed to investigate the angular structure of compact radio sources by observing the scintillation caused by the irregular structure of the interplanetary medium[1]. The initial survey includes the whole sky in the declination range $-08° < \delta < 44°$ and this area is scanned once a week. A large fraction of the sky is thus under regular surveillance. Soon after the instrument was brought into operation it was noticed that signals which appeared at first to be weak sporadic interference were repeatedly observed at a fixed declination and right ascension; this result showed that the source could not be terrestrial in origin.

Systematic investigations were started in November and high speed records showed that the signals, when present, consisted of a series of pulses each lasting $\sim$0·3 s and with a repetition period of about 1·337 s which was soon found to be maintained with extreme accuracy. Further observations have shown that the true period is constant to better than 1 part in 10^7 although there is a systematic variation which can be ascribed to the orbital

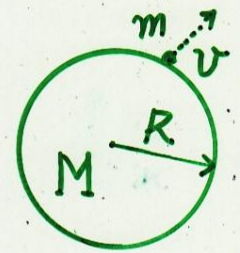
of three others having remarkably similar properties which suggests that this type of source may be relatively common at a low flux density. A tentative explanation of these unusual sources in terms of the stable oscillations of white dwarf or neutron stars is proposed.

Position and Flux Density

The aerial consists of a rectangular array containing 2,048 full-wave dipoles arranged in sixteen rows of 128 elements. Each row is 470 m long in an E.–W. direction and the N.–S. extent of the array is 45 m. Phase-scanning is employed to direct the reception pattern in declination and four receivers are used so that four different declinations may be observed simultaneously. Phase-switching receivers are employed and the two halves of the aerial are combined as an E.–W. interferometer. Each row of dipole elements is backed by a tilted reflecting screen so that maximum sensitivity is obtained at a declination of approximately +30°, the overall sensitivity being reduced by more than one-half when the beam is scanned to declinations above +90° and below −5°. The beamwidth of the array to half intensity is about $\pm\frac{1}{2}°$ in right ascension and ±3° in declination; the phasing arrangement is

パルサー発見の論文（Nature, 217 (1968) 709)

□ Most compact object?



$\frac{1}{2}mv^2 - G\frac{mM}{R} \geq 0$, $v=c$ (光速)

$Rc^2 = 2GM = 2G\bar{\rho}\frac{4}{3}\pi R^3$

$= 2G\frac{m_n}{\frac{4}{3}\pi r_0^3}\frac{4}{3}\pi R^3 = 2Gm_n\left(\frac{R}{r_0}\right)^3$

$\therefore R^2 = \frac{r_0^3 c^2}{2Gm_n}$; r_0, m_n ←原子核の知識!!

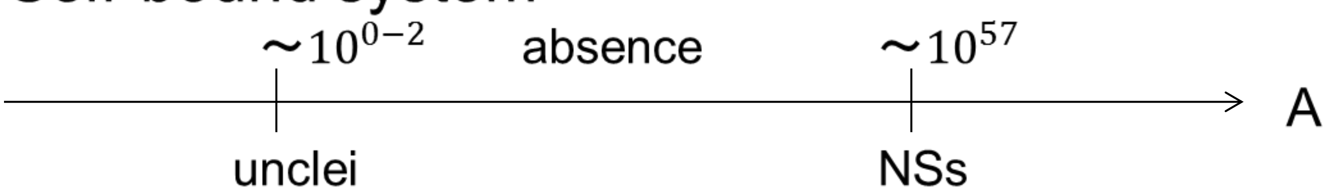
$0.5\,\mathrm{fm} = 0.5\times10^{-15}\,\mathrm{m}$, $1.7\times10^{-27}\,\mathrm{Kg}$

半径: $R = \left(\frac{r_0}{1\,\mathrm{fm}}\right)^{3/2}\times 2.0\times10^{4}\,\mathrm{m} \simeq 7.1\,\mathrm{Km}$

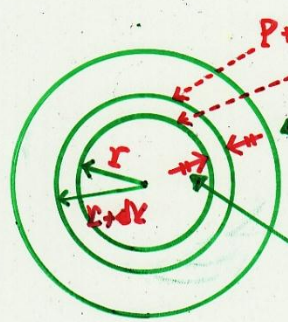
質量: $M = \frac{Rc^2}{2G} \simeq 2.4\,M_\odot$ ($M_\odot$=太陽質量 $2.0\times10^{30}\,\mathrm{kg}$)

粒子数: $A = \frac{M}{m_n} \simeq 2.8\times10^{57}$

□ Self-bound system



中性子星モデル ---> TOV方程式+EOS



$$-dP\times 4\pi r^2 = \frac{GM(r)}{r^2}\rho(r)\times 4\pi r^2 dr$$

- $\dfrac{dP}{dr} = -\dfrac{GM(r)\rho(r)}{r^2}$
- $M(r) = \int_0^r 4\pi r^2 \rho(r)\,dr$

もう1ッ方程式必要

- $P = f(\rho)$: P-ρ 関係*

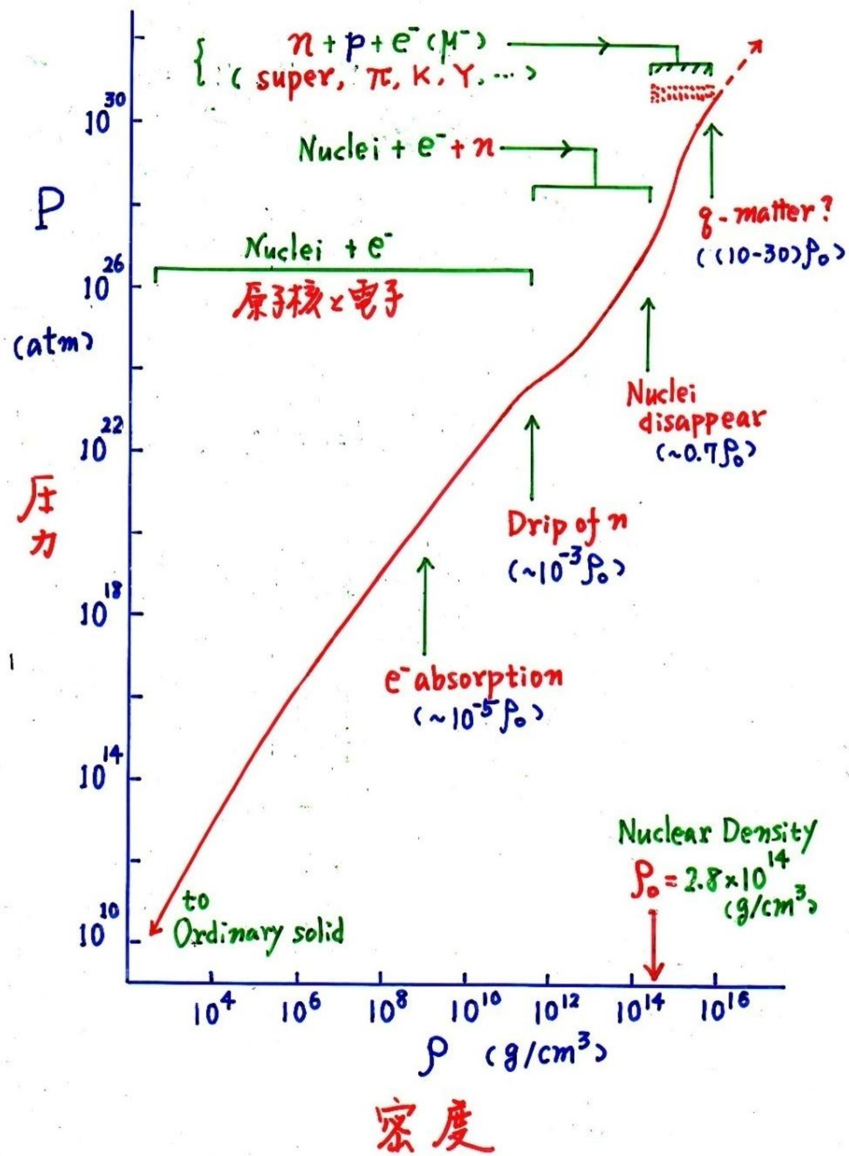
(or. $P = \rho^2 \dfrac{\partial E}{\partial \rho}$; E-ρ 関係)

* 状態方程式 (Equation of State) という。 $\boxed{\text{EOS}}$

⟶ 一般相対論の効果を含めて

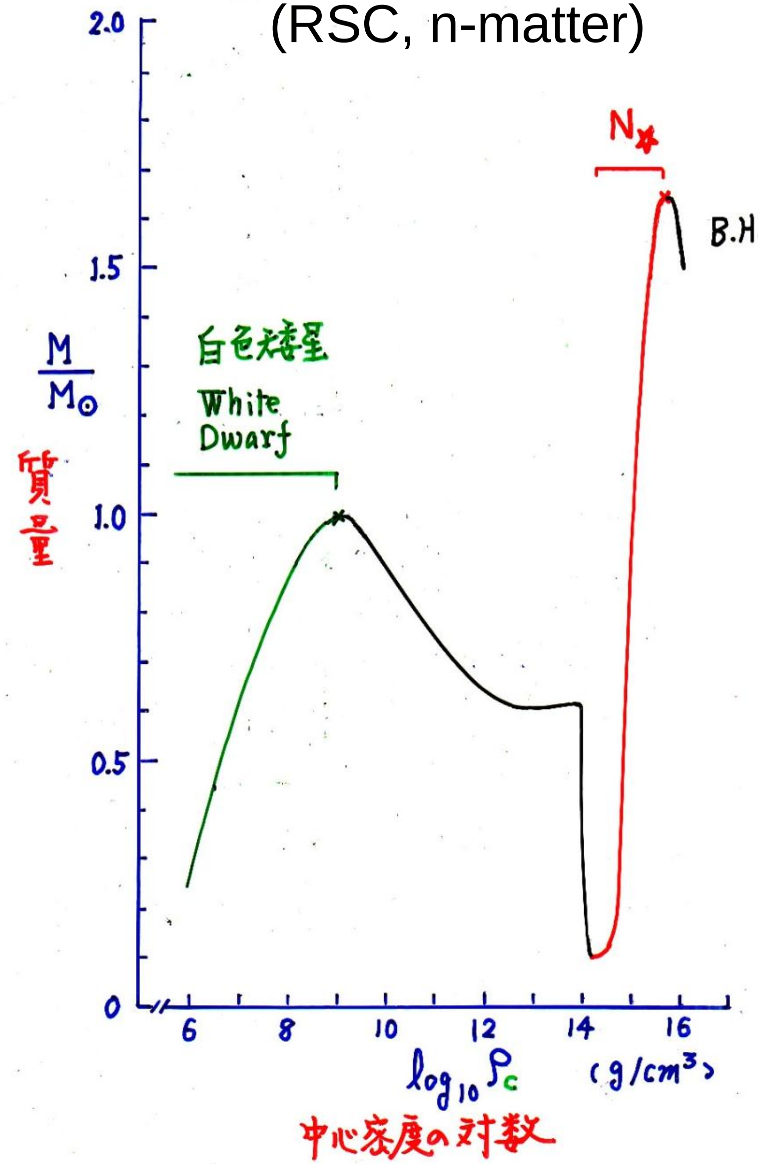
$$\text{TOV:}\quad \boxed{\frac{dP}{dr} = -\frac{G\{M(r)+4\pi P r^3/c^2\}\{\rho(r)+P/c^2\}}{r^2\{1-2GM(r)/c^2 r\}}}$$

□ Composition and EOS

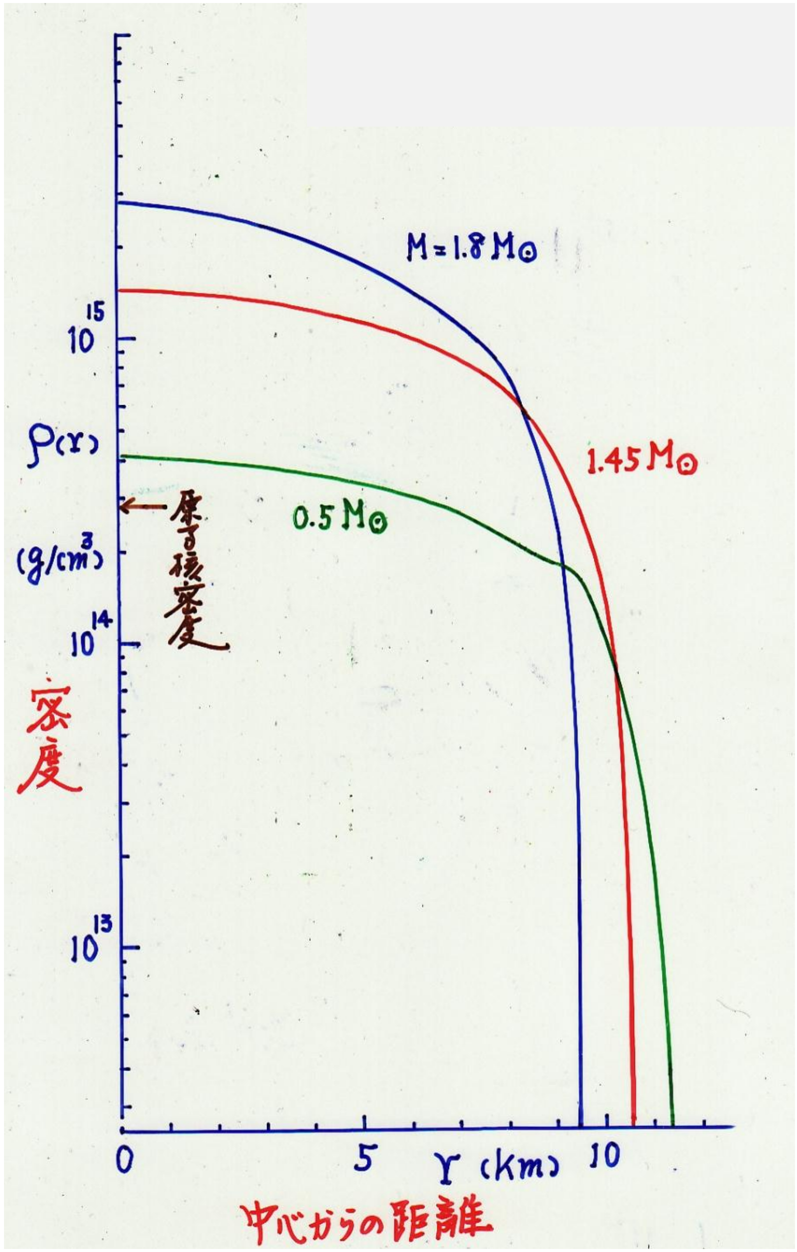


□ Solution of TOV eq.

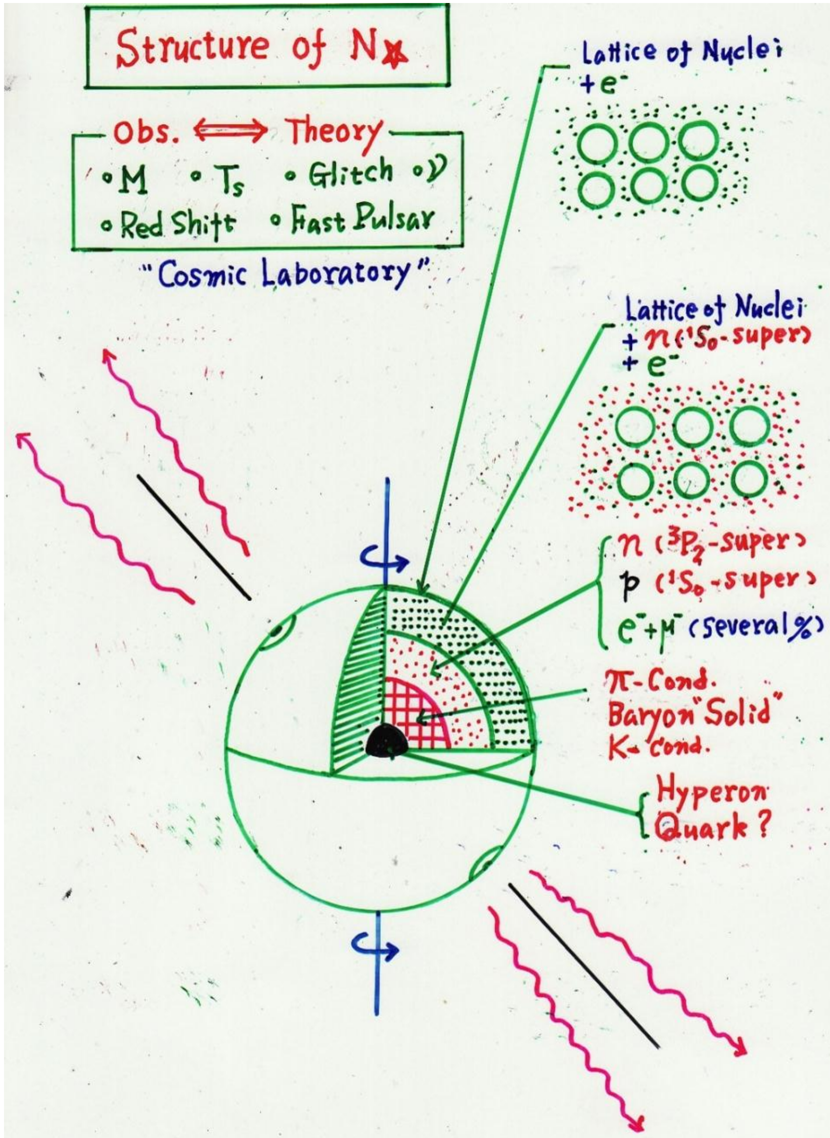
(RSC, n-matter)



□ Example of
density profile
(BJ-1H EOS)



1-2. Profile and structure of NSs



Mass	$(1\sim2)\,M_\odot$
Radius	$(10\sim20)$ Km
Temperature	$\sim10^6$ K (surface), $\sim10^8$ K (internal)
Pressure	$(10^{29}\sim10^{31})$ atm (center)
Density	$\sim10^6$ g/cc (surface), $\sim10^{15}$ g/cc (center)

(5.5g/cc for earth, 1.4g/cc for sun)

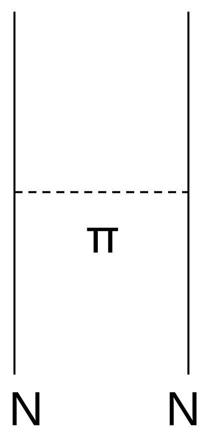
1-3. Characteristics of nuclear force

- Non-central character (Spin-orbit V_{LS} , Tensor V_T)

$$V(1,2) = V_C(1,2) + V_{LS}(r)\boldsymbol{L}\,\mathbf{S} + V_T(r)S_{12}$$

$$S_{12} = 3(\boldsymbol{\sigma}_1\hat{r})(\boldsymbol{\sigma}_2\hat{r}) - \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$$

$$V_T{}^{OPE}(1,2) = \frac{1}{3}\frac{f^2}{\hbar c}m_\pi c^2(\boldsymbol{\tau}_1\boldsymbol{\tau}_2) \times \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^3}\right)\frac{e^{-\mu r}}{\mu r}\, S_{12}$$



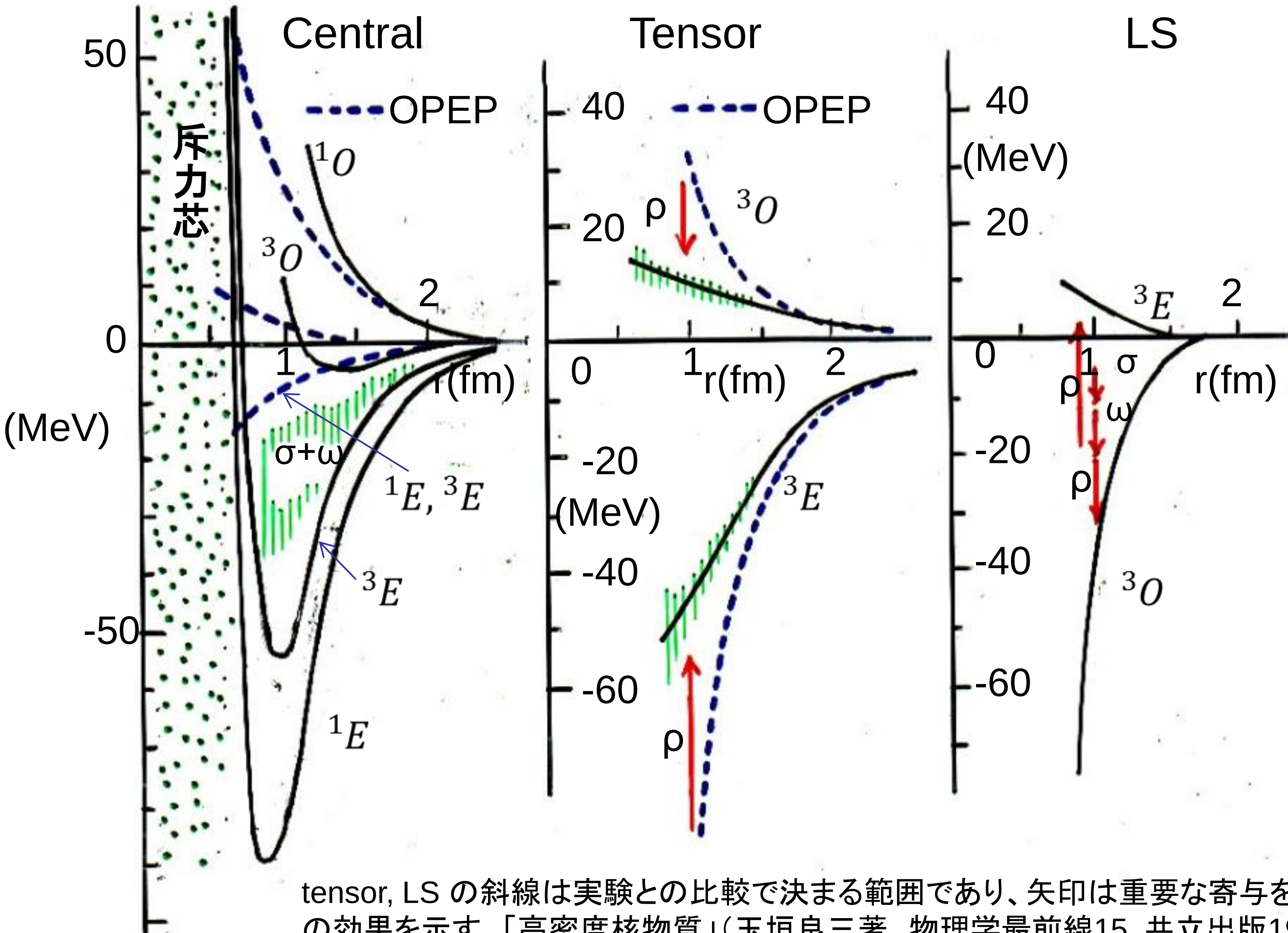
$(\mu = \frac{m_\pi c}{\hbar} = 0.70 fm^{-1}, \quad \frac{f^2}{\hbar c} = 0.08,$

$m_\pi c^2 = 140$ MeV)

OPEP	→	OBEP
π		π, σ, ρ, ω

□ 4つのスピン・パリティ状態と核力

$^3O(\tau=1,\ S=1),\ \ ^1E(\tau=1, S=0),\ \ ^1O(\tau=0, S=0),\ \ ^3E(\tau=0, S=1)$



tensor, LS の斜線は実験との比較で決まる範囲であり、矢印は重要な寄与をする中間子の効果を示す。「高密度核物質」（玉垣良三著、物理学最前線15, 共立出版1980）

1-4. Nucleon Matter

- single-particile w.f. $\varphi_\alpha = \frac{1}{\sqrt{\Omega}} e^{ikr} X_{m_S}$

- B.C.→φ(x, y, z)=φ(x+L, y, z)=φ(x, y+L, z)=φ(x, y, z+L)

  $\Omega = L^3$

  $k_x = \frac{2\pi}{L} n_x \;,\; k_y = \frac{2\pi}{L} n_y \;,\; k_z = \frac{2\pi}{L} n_z \;;\; n_x = 0, \pm 1, \pm 2,$ -----

- level density in k-space

$$\mathrm{dn} = 2\frac{\Omega}{(2\pi)^3} d^3k \rightarrow \mathrm{N} = 2\frac{\Omega}{(2\pi)^3} \int_{occ} d^3k$$

Neutron Matter

- Density: $\rho = \frac{N}{\Omega} = \frac{1}{\Omega}\int dn = \frac{1}{\Omega} 2 \frac{\Omega}{(2\pi)^3}\int_0^{k_F} k^2 dk 4\pi = \frac{{k_F}^3}{3\pi^2}$

$$\rightarrow k_F = (3\pi^2\rho)^{\frac{1}{3}} = (3\pi^2\rho_0)^{\frac{1}{3}}(\rho/\rho_0)^{1/3} \simeq 1.71(\rho/\rho_0)^{1/3}\ \mathrm{fm}^{-1}$$

- Fermi energy (kinetic):

$$\mathrm{E}_F = \hbar^2 {k_F}^2/2m = \frac{\hbar^2}{2m}(3\pi^2\rho_0)^{\frac{2}{3}}(\rho/\rho_0)^{2/3} \simeq 60(\rho/\rho_0)^{2/3}\ \mathrm{MeV}$$

$$\rightarrow \ \mathrm{E}_F = 60, 95, 125, 240\ \mathrm{MeV\ for}\ \rho/\rho_0 = 1, 2, 3, 8.$$

- Average K. E.

$$E_K = \frac{1}{N}\left[2\frac{\Omega}{(2\pi)^3}4\pi\int_0^{k_F}\frac{\hbar^2 k^2}{2m}k^2 dk\right] = \frac{1}{\pi^2}\frac{\Omega}{N}\frac{\hbar^2}{2m}\frac{1}{5}{k_F}^5$$

$$= \frac{1}{\pi^2}\frac{1}{\rho}\frac{{k_F}^3}{5}\frac{\hbar^2}{2m}{k_F}^2 = \frac{3\pi^2\rho}{\pi^2\rho}\frac{1}{5}\mathrm{E}_F = \frac{3}{5}\mathrm{E}_F$$

Fermi gas (FG) phase and Non-central force

$$V(1,2) = V_C\ (r) + V_{LS}\boldsymbol{LS} + V_T(r)S_{12}$$

$$S_{12} = 3(\boldsymbol{\sigma}_1\hat{r})(\boldsymbol{\sigma}_2\hat{r}) - (\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)$$

e.g. Neutron Matter and Born (1st order) Term:

$$e^{i\boldsymbol{pr}} = 4\pi\sum_{L=0}(i)^L j_L(pr)\sum_{m_L} Y_{Lm_L}(\hat{r})Y^*{}_{Lm_L}(\hat{p})$$

$$\langle V(1,2)\rangle_{FG} = \frac{4\pi}{\Omega}\sum_{\boldsymbol{p}_1}^{occ}\sum_{\boldsymbol{p}_2}^{occ}[\sum_{L=odd}\sum_{J=L-1}^{L+1}(2J+1) < p\left|V_{LL}{}^{J1}(r)\right|p >$$

$$+\sum_{L=even}(2L+1) < p\left|V_{LL}{}^{J0}(r)\right|p >,$$

Where

$$< p\left|V_{LL}{}^{JS}(r)\right|p > = \int_0^\infty r^2\, dr j_L(pr)V_{LL}{}^{JS}(r)\ \ ;\ \ p = |\boldsymbol{p}_1 - \boldsymbol{p}_2|/2,$$

$$V_{LL}{}^{JS}(r) = \sum_{spin}\int d\hat{r}\, Y^*{}_{LSJm_J}(1,2)V(1,2)Y_{LSJm_J}(1,2)$$

Spin-angular part w.f.

$$Y_{LSJm_J}(1,2) = \sum_{m_S+m_L=m_J} C\left(LSm_Lm_S\middle|Jm_J\right)Y_{Lm_L}(\hat{r})\mathrm{X}_{Sm_S}(1,2)$$

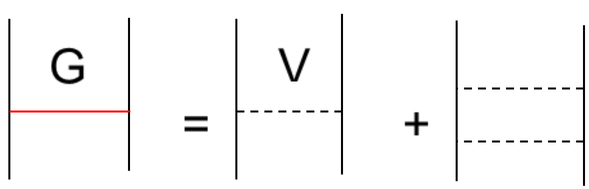
Note,

$$\begin{array}{lcccl} & J=L+1 & J=L & J=L-1 & \\ & \downarrow & \downarrow & \downarrow & \\ V_{LL}{}^{J1}(r) = \{ & \dfrac{2L}{2L+3}, & 2, & -\dfrac{2(L+1)}{2L-1} & \}\ V_T(r) \\ = \{ & \mathrm{L}, & -1, & -(\mathrm{L}+1) & \}\ V_{LS}(r) \end{array}$$

Therefore, $\sum_{J=L-1}^{L+1}(2J+1) < p\left|V_{LL}{}^{J1}(r)\right|p >=0$

○ Effects of non-central force vanish for each partial-wave state
(非中心力効果を眠らせてしまう相)

○ G-matrix eg. : G=$V + V\frac{Q}{e}V + V\frac{Q}{e}V\frac{Q}{e}V + ---- = V + V - \frac{Q}{e}G; G\varphi = \mathrm{V}\Psi$



+ ·········· ladder diagrams

Correlation: $\Psi = \varphi + \frac{Q}{e}V\Psi$ によって $\varphi \rightarrow \Psi$ となっても大勢は変わらない

☐ E/A for Symmetric Nuclear Matter *
---partial-wave state contribution---

(In MeV)

	H-J	OBEP-K	OPEG	Reid SC	Reid HC
$^1S_0$	-15.9	-16.9	-17.2	-15.57	-15.06
$^3S_1$	-15.8	-23.3	-19.1	-14.99	-14.08
$^1P_1$	3.2	4.2	3.8	2.39	3.38
$^3P_0$	-3.0	-3.5	-3.1	-3.30	-3.96
$^3P_1$	9.5	10.3	9.7	9.93	10.59
$^3P_2$	-6.2	-6.2	-5.8	-7.00	-6.83
$^1D_2$	-2.2	-2.3	-2.2	-2.49	-2.68
$^3D_1$	1.2	1.4	1.2	1.44	1.29
$^3D_2$	-3.1	-3.5	-3.1	-4.19	-4.43
$^3D_3$	0.6	0.5	0.2	-0.69	-0.69
higher ℓ	(-1.2)	(0.7)	(0.7)	0.69	0.69
Pot. total	-31.9	-38.6	-34.9	-33.78	-31.78
Kinetic	23.85	23.85	23.85	23.85	23.85
-B.E.	-9.0	-14.7	-11.0	-9.93	-7.93

Contribution from non-central forces almost cancels out for each partial-wave state

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* Y. Akaishi & S. Nagata. P.T.P. Suppl. Extra Number (1968), 476.

2. Nucleon Superfluidities

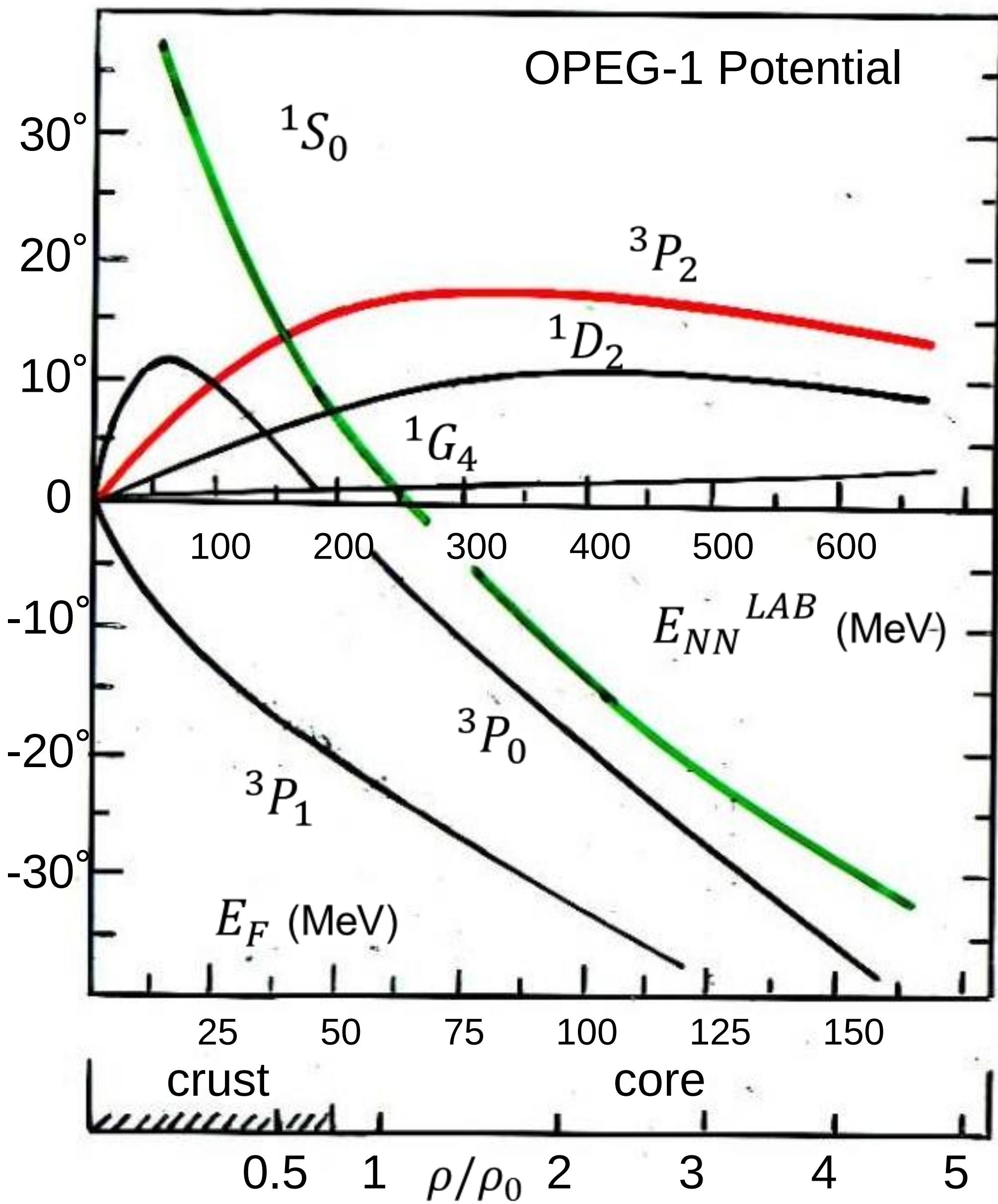
2. Nucleon Superfluidity

2-1. History and $^3P_2$ –superfluid

1957: BCS-Bogoliubov Theory
1958: Application to nuclei
(Bhor-Mottelson-Pines)
→ symmetric nuclear matter (N=Z)
1959: Possibility of superfluidity of neutron matter in the core of NS
by A.B. Migdal
1967: Discovery of NS

1970: ○ New-type superfluid ($^3P_2$; J=2) ; not $^1S_0$
by R. Tamagaki ('70) ;
Hofferberg-Glassgold-Richardson-Ruderman ('70)
○ Non-zero Ang.-Mom. Pairing Theory
R. Tamagaki ('70), T. Takatsuka ('72)
○ Realistic results by Takatsuka ('72, '73)
1972: Obserbation of triplet-P state superfluid in ^3He system
1990〜 Renewed Interests ← Glitches, NS cooling
(many works on $^3P_2$; $^1S_0$ –superfluids in NSs)

$\tau = 1$ Phase shift in Degrees



Most attractive pair-state:

$^1S_0$ (usual type) in crust region

$^3P_2$ (new type) in core region

$$E_{NN}{}^{LAB} = 2E_{NN}{}^{CM} = 4E_F$$

$$E_F = 60(\rho/\rho_0)^{2/3} \text{ MeV}$$

$$\rho_0 = 0.17 \text{ nucleons/f}m^3$$

$$\simeq 2.8 \times 10^{14} \text{ g/cc}$$

2-2. Outline of BCS-Bogoliubov Theory

BCS Theory (variational method)*

Model Hamiltonian: $H_{BCS} = H_0 + H_{pair}$ (2.1)

$$H_0 = \sum_{\boldsymbol{k}} \tilde{\varepsilon}_k (C^*_{\boldsymbol{k}\uparrow} C_{\boldsymbol{k}\uparrow} + C^*_{-\boldsymbol{k}\downarrow} C_{-\boldsymbol{k}\downarrow}); \quad \tilde{\varepsilon}_k = \varepsilon_k - \varepsilon_F \qquad (2.2),\ (2.3)$$

$$H_{pair} = \sum_{\boldsymbol{k}\boldsymbol{k}'} \frac{1}{\Omega} < \boldsymbol{k}' |V| \boldsymbol{k} > C^*_{\boldsymbol{k}'\uparrow} C^*_{-\boldsymbol{k}'\downarrow} C_{-\boldsymbol{k}\downarrow} C_{\boldsymbol{k}\uparrow} \qquad (2.4)$$

Variational w.f.: $|\Psi>_{BCS} = \Pi_{\boldsymbol{k}} (u_{\boldsymbol{k}} + v_{\boldsymbol{k}} C^*_{\boldsymbol{k}\uparrow} C^*_{-\boldsymbol{k}\downarrow}) |0>$ (2.5)

$$u_{\boldsymbol{k}}^2 + v_{\boldsymbol{k}}^2 = 1 \qquad (2.6)$$

$$\frac{\delta}{\delta u_{\boldsymbol{k}}} < \Psi|H_{BCS}|\Psi >_{BCS} = 0 \to 2\tilde{\varepsilon}_{\boldsymbol{k}} u_{\boldsymbol{k}} v_{\boldsymbol{k}} = -\frac{1}{\Omega} (u_{\boldsymbol{k}}^2 - v_{\boldsymbol{k}}^2) \sum_{\boldsymbol{k}'} < \boldsymbol{k}'|V|\boldsymbol{k} > u_{\boldsymbol{k}'} v_{\boldsymbol{k}'} \qquad (2.7)$$

$$\Delta_{\boldsymbol{k}} = -\frac{1}{\Omega} \sum_{\boldsymbol{k}'} < \boldsymbol{k}'|V|\boldsymbol{k} > u_{\boldsymbol{k}'} v_{\boldsymbol{k}'} \qquad (2.8)$$

$$2\tilde{\varepsilon}_k u_{\boldsymbol{k}} v_{\boldsymbol{k}} = (u_{\boldsymbol{k}}^2 - v_{\boldsymbol{k}}^2) \Delta_{\boldsymbol{k}} \qquad (2.7')$$

(2.6), (2.7') $\to u_{\boldsymbol{k}}^2 = \frac{1}{2}\left(1 + \frac{\tilde{\varepsilon}_k}{\sqrt{\tilde{\varepsilon}_{k'}{}^2 + \Delta_{\boldsymbol{k}}{}^2}}\right), \quad v_k{}^2 = \frac{1}{2}\left(1 - \frac{\tilde{\varepsilon}_k}{\sqrt{\tilde{\varepsilon}_k{}^2 + \Delta_{\boldsymbol{k}}{}^2}}\right)$ (2.9)

(2.9), (2.8) $\to \Delta_{\boldsymbol{k}} = -\frac{1}{\Omega} \sum_{\boldsymbol{k}'} < \boldsymbol{k}'|V|\boldsymbol{k} > \Delta_{\boldsymbol{k}'} / 2\sqrt{\tilde{\varepsilon}_{k'}{}^2 + \Delta_{\boldsymbol{k}'}{}^2}$ (2.10)

V: central only $\to < \boldsymbol{k}'|V|\boldsymbol{k} > = 4\pi \int_0^\infty r^2 dr j_0(k'r) V(^1S_0) j_0(kr) = 4\pi < k'|V(^1S_0)|k >$

BCS gap eg. ($^1S_0$ –type) $\boxed{\Delta_k = -\frac{1}{\pi} \int k'^2 dk' < k'|V(^1S_0)|k > \Delta_{k'} / \sqrt{\tilde{\varepsilon}_{k'}{}^2 + \Delta_{k'}{}^2}}$ (2.11)

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*) J. Barden, L.N. Cooper and J.R. Schrieffer, Phys. Rev. 108 (1957) 1105

Bogoliubov Theory (quasiparticle method)*

Bogoliubov trans.: $\alpha_{\boldsymbol{k}0} = U_{\boldsymbol{k}} C_{\boldsymbol{k}\uparrow} - V_{\boldsymbol{k}} C^*_{-\boldsymbol{k}\downarrow}$, $\alpha_{\boldsymbol{k}1} = U_{\boldsymbol{k}} C_{-\boldsymbol{k}\downarrow} + V_{\boldsymbol{k}} C^*_{\boldsymbol{k}\uparrow}$ (2.12)

$$U_{\boldsymbol{k}}^{\,2} + V_{\boldsymbol{k}}^{\,2} = 1 \tag{2.13}$$

Inverse trans.: $C_{\boldsymbol{k}\uparrow} = U_{\boldsymbol{k}} \alpha_{\boldsymbol{k}0} + V_{\boldsymbol{k}} \alpha_{\boldsymbol{k}1}$, $C^*_{-\boldsymbol{k}\downarrow} = U_{\boldsymbol{k}} \alpha_{\boldsymbol{k}1} - V_{\boldsymbol{k}} \alpha_{\boldsymbol{k}0}$, etc. (2.14)

Rewrite H_{BCS}: $H_{BCS} = H_{00} + H_{11} + H_{20} + H_{higher}$ (2.15)

$$H_{00} = 2\sum_{\boldsymbol{k}} \tilde{\varepsilon}_k V_k^{\,2} + \frac{1}{\Omega} \sum_{\boldsymbol{k}\boldsymbol{k}'} < \boldsymbol{k}' |V| \boldsymbol{k} > U_{\boldsymbol{k}'} V_{\boldsymbol{k}'} U_{\boldsymbol{k}} V_{\boldsymbol{k}} \tag{2.16}$$

$$H_{11} = \sum_{\boldsymbol{k}} [2\tilde{\varepsilon}_k (U_{\boldsymbol{k}}^{\,2} - V_{\boldsymbol{k}}^{\,2}) - \frac{1}{\Omega} \sum_{\boldsymbol{k}'} < \boldsymbol{k}' |V| \boldsymbol{k} > 2U_{\boldsymbol{k}'} V_{\boldsymbol{k}'} U_{\boldsymbol{k}} V_{\boldsymbol{k}}] \times (\alpha^*_{\boldsymbol{k}0} \alpha_{\boldsymbol{k}0} + \alpha^*_{\boldsymbol{k}1} \alpha_{\boldsymbol{k}1}) \tag{2.17}$$

$$H_{20} = \sum_{\boldsymbol{k}} \left[2\tilde{\varepsilon}_k U_{\boldsymbol{k}} V_{\boldsymbol{k}} + \frac{1}{\Omega} (U_{\boldsymbol{k}}^{\,2} - V_{\boldsymbol{k}}^{\,2}) \sum_{\boldsymbol{k}'} < \boldsymbol{k}' |V| \boldsymbol{k} > U_{\boldsymbol{k}'} V_{\boldsymbol{k}'} \right] \times (\alpha^*_{\boldsymbol{k}0} \alpha_{\boldsymbol{k}1} + \alpha^*_{\boldsymbol{k}1} \alpha_{\boldsymbol{k}0}) \tag{2.18}$$

$$H_{higher} = \{\alpha^*\alpha^*\alpha^*\alpha^*,\ \alpha^*\alpha^*\alpha^*\alpha,\ \alpha^*\alpha^*\alpha\alpha,\ \alpha^*\alpha\alpha\alpha,\ \alpha\alpha\alpha\alpha\} \tag{2.19}$$

Dangerous term $H_{20} = 0 \rightarrow$ the same gap eg. as in BCS.

Bogoliubov vaccum $|\Psi>_B$: quasiparticle vaccum

$$\alpha\,|\Psi>_B = 0 \tag{2.20}$$

$$H_{11} = \sum_{\boldsymbol{k},i} \sqrt{\tilde{\varepsilon}_k^{\,2} + \Delta_k^{\,2}}\, \alpha^*_{\boldsymbol{k}i} \alpha_{\boldsymbol{k}i} \tag{2.21}$$

Excitation energy

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*) N.N. bogoliubov, Sov. Phys. –JETP 7 (1958) 41.

$\boxed{\text{Yoshida}}$ → equivalence of two theories*

Difinition of Unitary Operator:

$$S_0 = -i \sum_{\boldsymbol{k}} \theta_{\boldsymbol{k}} (b^*{}_{\boldsymbol{k}} - b_{\boldsymbol{k}}) \qquad (2.22)$$

$$b^*{}_{\boldsymbol{k}} = C^*{}_{\boldsymbol{k}\uparrow} C^*{}_{-\boldsymbol{k}\downarrow} \quad \text{(pair operator)} \qquad (2.23)$$

$$\alpha_{\boldsymbol{k}o} = e^{iS_0} C^*{}_{\boldsymbol{k}\uparrow} e^{-iS_0} = \overset{\overset{U_k}{\|}}{\cos\theta_{\boldsymbol{k}}}\ C_{\boldsymbol{k}\uparrow} - \overset{\overset{V_k}{\|}}{sin\theta_{\boldsymbol{k}}}\ C^*{}_{-\boldsymbol{k}\downarrow}$$

$$\alpha_{\boldsymbol{k}1} = e^{iS_0} C_{-\boldsymbol{k}\downarrow} e^{-iS_0} = cos\theta_{\boldsymbol{k}}\ C_{-\boldsymbol{k}\downarrow} + sin\theta_{\boldsymbol{k}} C^*{}_{\boldsymbol{k}\uparrow} \qquad (2.24)$$

$$\left(e^{iS} Q e^{-iS} = Q + [iS,\ Q] + \frac{1}{2!}\big[iS,\ [iS, Q]\big] + \ \text{-----------}\ \right)$$

$$|\Psi>_{BCS} = e^{iS_0} |\Psi>_B = \Pi_{\boldsymbol{k}} (cos\theta_{\boldsymbol{k}} + sin\theta_{\boldsymbol{k}} C^*{}_{\boldsymbol{k}\uparrow} C^*{}_{-\boldsymbol{k}\downarrow}) |0> \qquad (2.25)$$

Therefore, putting $U_{\boldsymbol{k}} = u_{\boldsymbol{k}},\ \ V_{\boldsymbol{k}} = v_{\boldsymbol{k}}$,

Two theories are equvalent.

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*) K. Yoshida, Phys. Rev. 111 (1958) 1255.

2-3. Theory of Non-zero Angular-Momentum Pairing

${}^3P_2 + ({}^3F_2)$-pairing $\rightarrow$ Extend BCS-Bogoliubov Theory (J=0 pairing)

$\uparrow$ $\rightarrow$ Partial wave representation

(S=1, L=1, J=2)

Generalized BCS-Bogoliubov Theory *

☐ Generalization of pair operator : $b^*{}_{\boldsymbol{k}} = C^*{}_{\boldsymbol{k}\uparrow} C^*{}_{-\boldsymbol{k}\downarrow}$

$$\rightarrow b^*{}_{\lambda L m_J}(k) = \frac{1}{\sqrt{2}} \sum_{\sigma_1 \sigma_2} (\tfrac{1}{2} \tfrac{1}{2}\ \sigma_1 \sigma_2 | S m_S)(S L m_S m_L | J m_J) \times \int d\hat{k} Y_{L m_L}(\hat{k}) C^*{}_{\boldsymbol{k}\sigma_1} C^*{}_{\boldsymbol{k}\sigma_2}$$

λ=(S, J) (3.1)

☐ BCS Hamiltonian : $H_{BCS} = H_0 + H_{pair}$ (3.2)

$$H_0 = \sum_{\boldsymbol{k}} \tilde{\varepsilon}_k (C^*{}_{\boldsymbol{k}\uparrow} C_{\boldsymbol{k}\uparrow} + C^*{}_{-\boldsymbol{k}\downarrow} C_{-\boldsymbol{k}\downarrow})\ ;\ \ \tilde{\varepsilon}_k = \varepsilon_k - \varepsilon_F \quad (3.3)$$

$$H_{pair} = \sum_{\boldsymbol{k}} \sum_{\boldsymbol{k}'} \frac{1}{\Omega} < \boldsymbol{k}' | V | \boldsymbol{k} > b^*{}_{\boldsymbol{k}} b_{\boldsymbol{k}} \quad (3.4)$$

$$\rightarrow\ H_0 = \sum_{\boldsymbol{k}\sigma} \tilde{\varepsilon}_k C^*{}_{\boldsymbol{k}\sigma} C_{\boldsymbol{k}\sigma} \quad (3.5)$$

$$H_{pair}{}^{(\lambda)} = \frac{(4\pi)^2}{\Omega} \sum_{k'} \sum_{k} \sum_{L'L} (i)^{L'-L} < k' \left| V_\lambda{}^{L'L} \right| k > \sum_{m_J} b^*{}_{\lambda L' m_J}(k') b_{\lambda L m_J}(k) \quad (3.6)$$

$\uparrow$ inclusion of ${}^3P_2 - {}^3F_2$ tensor coupling

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*) R. Tamagaki, Prog. Theor. Phys. 44 (1970) 905.
T. Takatsuka, Prog. Theor. Phys. 48 (1972) 1517.

☐ Extended Bogoliubov Transformation :

$$S_0 = -i \sum_{\boldsymbol{k}} \theta_{\boldsymbol{k}} (b^*{}_{\boldsymbol{k}} - b_{\boldsymbol{k}})$$

$$\rightarrow \quad S_\lambda = -i \sum_{Lm_J} \sum_k \{\varphi_{\lambda Lm_J}(k) b^*{}_{\lambda Lm_J}(k) - \varphi^*{}_{\lambda Lm_J}(k) b_{\lambda Lm_J}(k)\} \tag{3.7}$$

quasiparticle op. $\boldsymbol{\alpha}^*{}_{\boldsymbol{k}} = e^{iS_\lambda} \boldsymbol{C}^*{}_{\boldsymbol{k}} e^{-iS_\lambda}$, etc. (3.8)

$$\boldsymbol{\alpha}^*{}_{\boldsymbol{k}} = \begin{pmatrix} \alpha^*{}_{\boldsymbol{k}0} \\ \alpha^*{}_{\boldsymbol{k}1} \end{pmatrix} , \quad \boldsymbol{C}^*{}_{\boldsymbol{k}} = \begin{pmatrix} C^*{}_{\boldsymbol{k}\uparrow} \\ C^*{}_{\boldsymbol{k}\downarrow} \end{pmatrix} \tag{3.9}$$

☐ Rewrite H_{BCS} by α*, α and dangerous term $H_{20} = 0$ → gap eg. for λ-pairing

☐ Finally

$\Delta(\mathrm{k}) = -\frac{1}{\pi} \int k'^2 dk' < k'|V(^1S_0)|k > \Delta(\mathrm{k}')/\sqrt{\tilde{\varepsilon}_{k'}{}^2 + \Delta^2(k')}$ (BCS $^1S_0$ −gap equation) ⇒

$$\Delta_{\lambda Lm_J}(k) = -\frac{1}{\pi}(-)^{1-S} \int k'^2 dk' \sum_{L'} (i)^{L'-L} < k' \left| V_\lambda{}^{L'L} \right| k > \sum_{L''} \sum_{m'_J} \Delta_{\lambda L' m'_J}(k')$$

$$\times \int d\hat{k}' T_r \left[G_{\lambda^* L' m_J}(\hat{k}') G_{\lambda L'' m'_J}(k') \right] / \sqrt{\tilde{\varepsilon}_{k'}{}^2 + D_\lambda{}^2(\boldsymbol{k}')} \tag{3.10}$$

$$D_\lambda(\boldsymbol{k}') = \frac{1}{2} \sum_{L'L} \sum_{m'_J m_J} \Delta^*{}_{\lambda Lm_J}(k') \Delta_{\lambda L'' m'_J}(k') T_r [G^\dagger{}_{\lambda Lm_J}(\hat{k}') G_{L' m'_J}(\hat{k}')] \tag{3.11}$$

$$G_{\lambda Lm_J}(\hat{k}') = \{(1/2\ 1/2 \sigma_1 \sigma_2 | Sm_S)(SLm_S m_L | Jm_J) Y_{Lm_J - m_S}(\hat{k})\} \tag{3.12}$$

: (2 × 2) matrix in spin-space

generalized gap eg. for λ=(S, J)-pairing

☐ for λ=$^1S_0$ −pairing, $\Delta(\mathrm{k}) = \Delta_{\lambda 00}\ (k)/\sqrt{8\pi}$ ⇒ usual BCS case.

☐ Transformed Hamiltonian :

$$H_{BCS} = H_{00} + H_{11} + \overset{\overset{0}{\parallel}}{H_{20}} + \text{higher order } \mathrm{O}(\tfrac{1}{\Omega}) \tag{3.13}$$

$$H_{00} = \sum_{\boldsymbol{k}} [\tilde{\varepsilon}_k (1 - \tilde{\varepsilon}_k / \sqrt{\tilde{\varepsilon}_k{}^2 + D_\lambda{}^2(\boldsymbol{k})}) - D_\lambda{}^2(\boldsymbol{k})/2\sqrt{\tilde{\varepsilon}_k{}^2 + D_\lambda{}^2(\boldsymbol{k})}\ ] \tag{3.14}$$

$$H_{11} = \sum_{\boldsymbol{k}} \sum_\rho \sqrt{\tilde{\varepsilon}_k{}^2 + D_\lambda{}^2(\boldsymbol{k})}\ \alpha^*{}_{\boldsymbol{k}\rho} \alpha_{\boldsymbol{k}\rho} \tag{3.15}$$

☐ Angular averaged gap function :

$$\overline{D}_\lambda(\boldsymbol{k}) = [\tfrac{1}{4\pi} \int d\hat{k}\ D_\lambda{}^2(\boldsymbol{k})]^{1/2} \tag{3.16}$$

☐ Energy shift :

$$\Delta \mathrm{E} = H_{00} - 2 \sum_{k(<k_F)} \tilde{\varepsilon}_k = -\tfrac{1}{2} N_F \overline{D}_\lambda{}^2(k_F)\ ; \tag{3.17}$$

$$N_F = 3N/4\mathrm{E}_F$$

(level density at the Fermi energy E_F)

2-4. Superfluidity in NSs

Three elements in gap equations

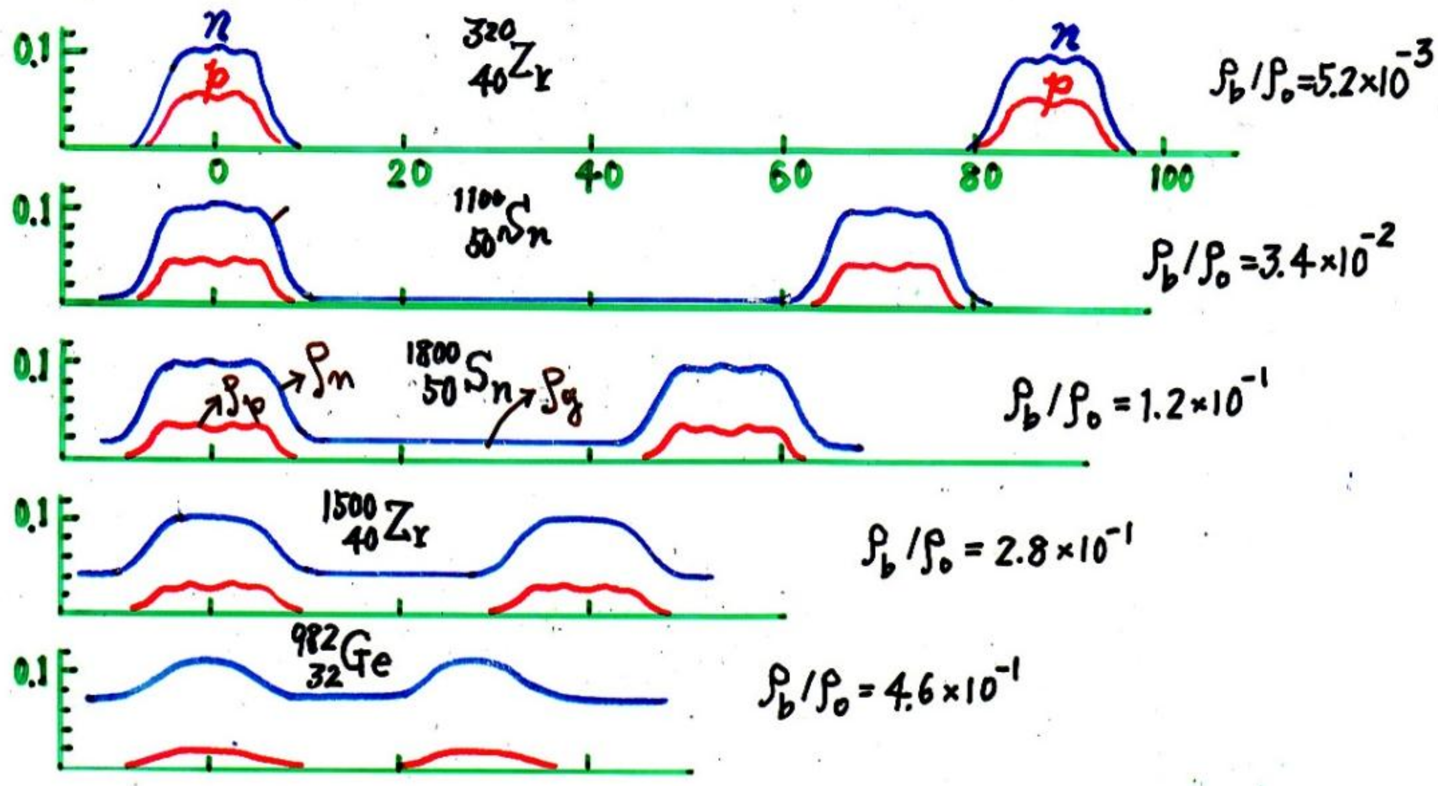
- Here, we note the 3-elements (Fermi momentum k^{FB}, effective mass m^*_B and pairing interaction) to control the energy gap.

$$\Delta_B(k) = -\frac{1}{\pi}\int k'^2 dk' \langle k' | V_{BB}({}^1S_0) | k \rangle \times \frac{\Delta_B(k')}{\sqrt{\tilde{\varepsilon}_B^2(k') + \Delta_B^2(k')}}$$

$$\tilde{\varepsilon}_B(k') \equiv \varepsilon_B(k') - \varepsilon_B(k_{FB}) \simeq \hbar^2(k'^2 - k_{FB}^2)/2m^*_B$$

#) For 3P2 NN pairing, the situation is similar, although the gap equation becomes complex due to the 3P2-3F2 tensor-coupling.

☐ Structure of NS-crust *



○ Approximating nuclei as n-rich nuclear matter with asymmetry α :
$\widetilde{x} = \rho_{pN}/\rho_{nN}$(in nuclei) and ρ_{nG}(n-gass) are given then taking
$\rho_{nN} = 0.1\mathrm{f}m^{-3} \rightarrow \rho_{nN}, \rho_{pN}, \ \rho_{nG}$ and α= $\frac{1-\dot{x}}{1+\dot{x}}$ ⇒ $m^*{}_{nN}, \ m^*{}_{pN}, \ m^*{}_{nG}$

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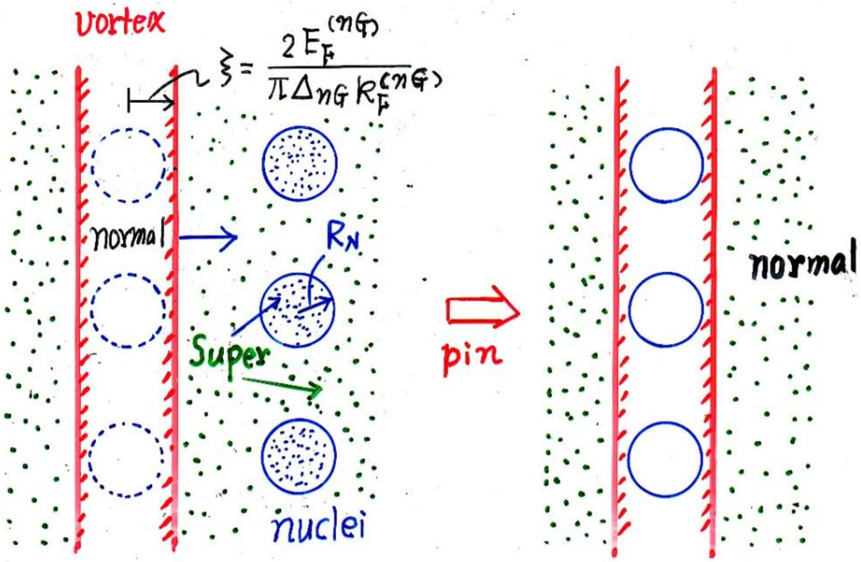
*) J.W. Negele and D. Vautherin, Nucl. Phys. 48 (1993) 298.

☐ Energy gap for the composition in NS crust

ρ/ρ_0	$\rho_{nG}(\mathrm{fm}^{-3})$	$\tilde{x}$	A	$R_N(\mathrm{fm})$	m^*_{nG}	m^*_{nN}	$\Delta_{nG}(\mathrm{MeV})$	$\Delta_{pN}(\mathrm{MeV})$
2.4×10^{-3}	9.7×10^{-5}	0.53	200	5.2	1.00	0.71	0.17(0.17)	0.02(0.08)
3.5×10^{-3}	2.6×10^{-4}	0.53	250	5.3	1.00	0.71	0.37(0.36)	0.02(0.08)
5.2×10^{-3}	4.8×10^{-4}	0.53	320	5.4	1.00	0.71	0.57(0.56)	0.02(0.08)
9.4×10^{-3}	1.2×10^{-3}	0.52	500	5.6	1.00	0.72	1.02(0.99)	0.03(0.08)
2.2×10^{-2}	3.0×10^{-3}	0.46	950	6.0	1.00	0.73	1.68(1.63)	0.06(0.15)
3.4×10^{-2}	4.7×10^{-3}	0.45	1100	6.2	0.99	0.73	2.01(1.95)	0.07(0.16)
5.2×10^{-2}	7.8×10^{-3}	0.44	1350	6.4	0.98	0.73	2.38(2.31)	0.07(0.17)
1.2×10^{-1}	1.8×10^{-2}	0.35	1800	6.7	0.96	0.76	2.66(2.59)	0.17(0.31)
2.0×10^{-1}	3.1×10^{-2}	0.32	1645	6.8	0.95	0.76	2.33(2.28)	0.22(0.36)
2.8×10^{-1}	4.4×10^{-2}	0.28	1500	6.8	0.94	0.77	1.79(1.75)	0.28(0.44)
3.6×10^{-1}	5.7×10^{-2}	0.23	1235	6.8	0.93	0.79	1.10(1.14)	0.37(0.53)
4.6×10^{-1}	7.4×10^{-2}	0.16	982	6.7	0.91	0.81	0.45(0.50)	0.46(0.64)

Energy gaps (Δ_{nG}, Δ_{pN}) are for the $^1S_0$ pairing interaction by RSC (OPEG-1) potential.

Vortex Pinning by Lattice Nuclei



○ Energy difference by pinning :

gain → $-\rho_{nG} V \frac{3}{8} \Delta_{nG}^2 / E_F^{(nG)}$

loss → $+(\rho_{nN} V \frac{3}{8} \Delta_{nN}^2 / E_F^{(nN)} + \rho_{pN} V \frac{3}{8} \Delta_{pN}^2 / E_F^{(pN)})$

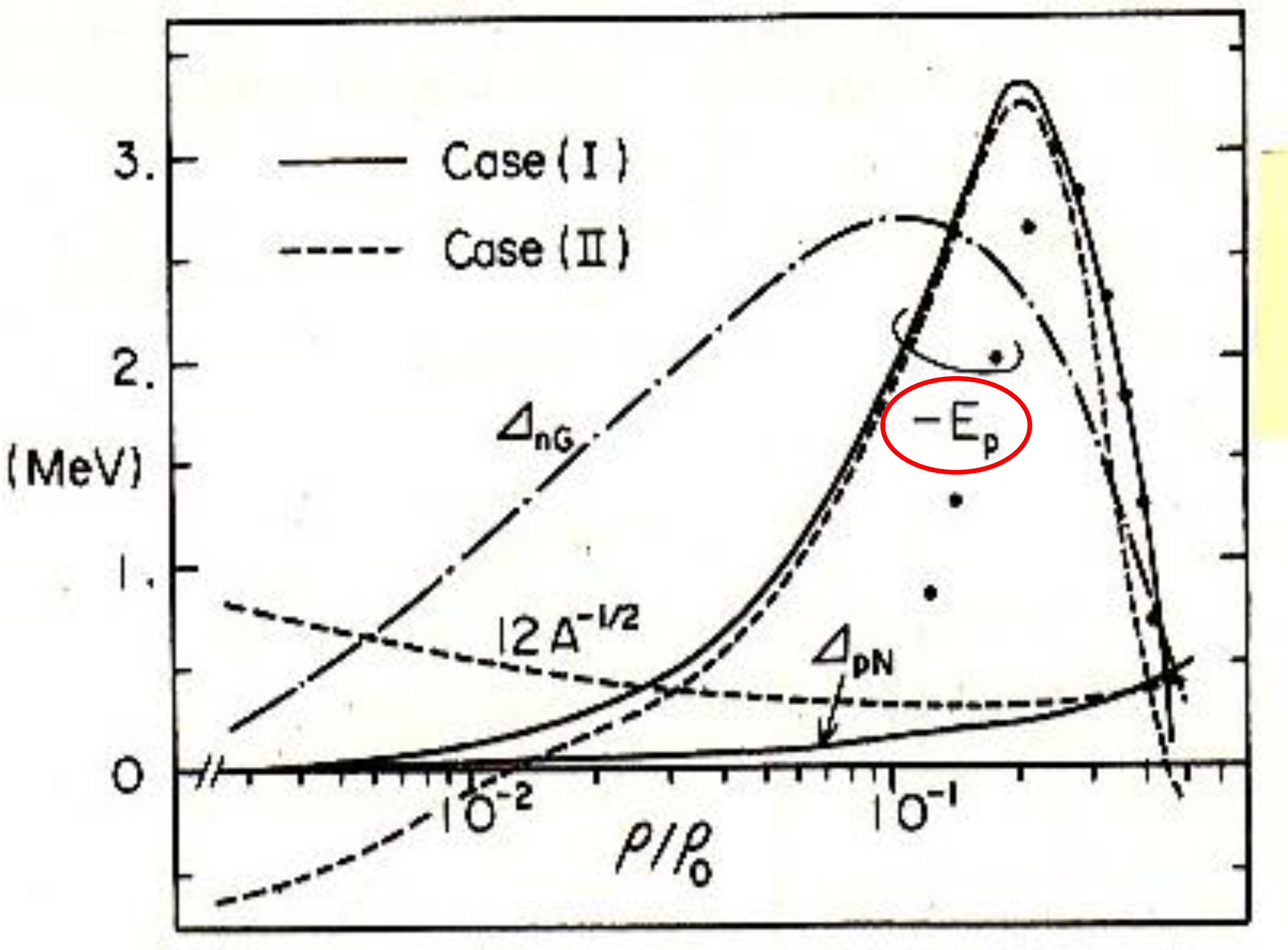
$$E_{pin} = \frac{3}{8} V \left(\frac{\Delta_{nN}^2 \rho_{nN}}{E_F^{(nN)}} + \frac{\Delta_{pN}^2 \rho_{pN}}{E_F^{(pN)}} - \frac{\Delta_{nG}^2 \rho_{nG}}{E_F^{(nG)}} \right)$$

$$V = \frac{4}{3}\pi R_N^3 \quad \text{for } \xi > R_N$$

$$= \frac{4}{3}\pi R_N^3 [1 - (1 - \xi^2/R_N^2)^{3/2}] \quad \text{for } \xi < R_N$$

○ When $E_{pin} < 0$, pinning occurs

☐ Pinning energy E_p in NS-crust versus density ρ



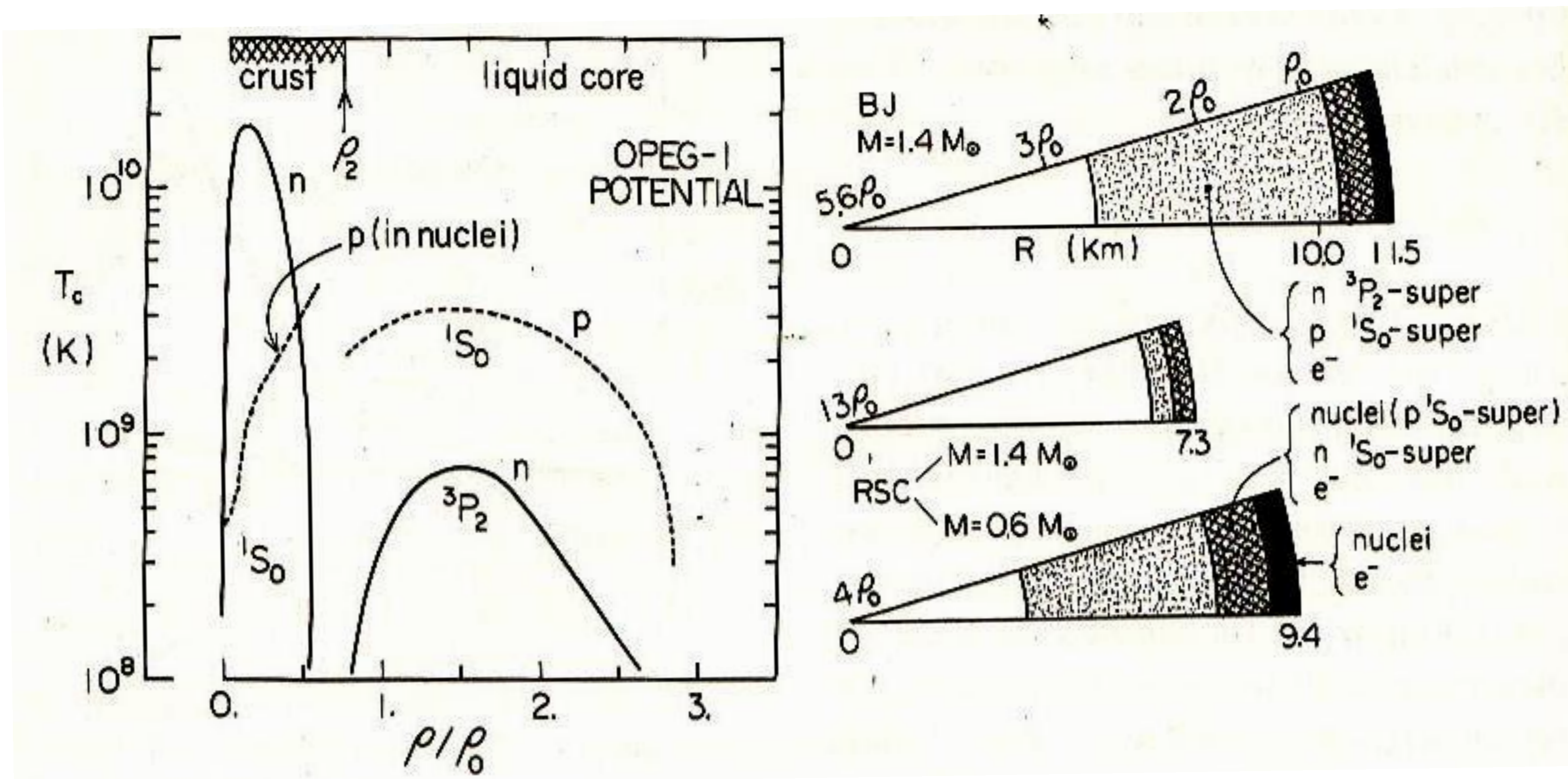
□ Single particle potential U(k) for n and p in the liquid core of NSs

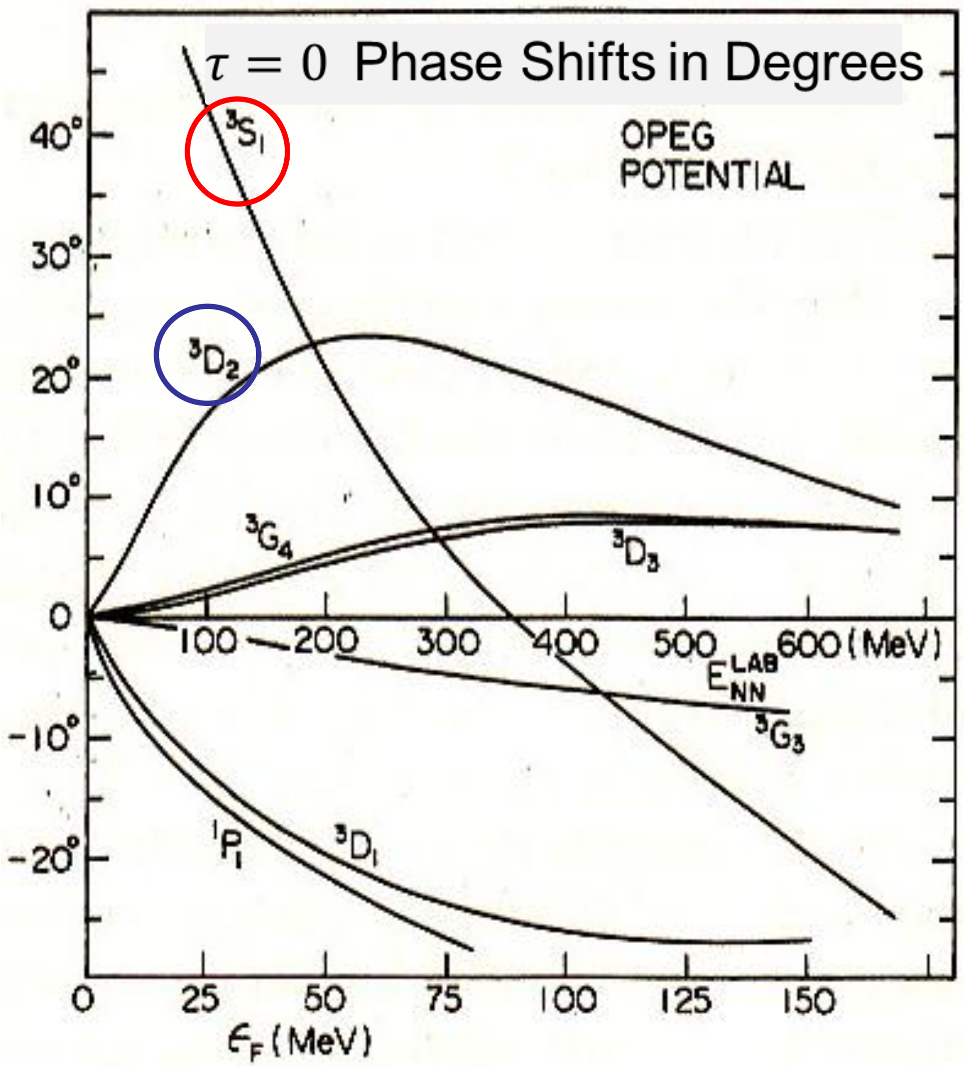
(G-matrix calculations with OPEG-1 potential).

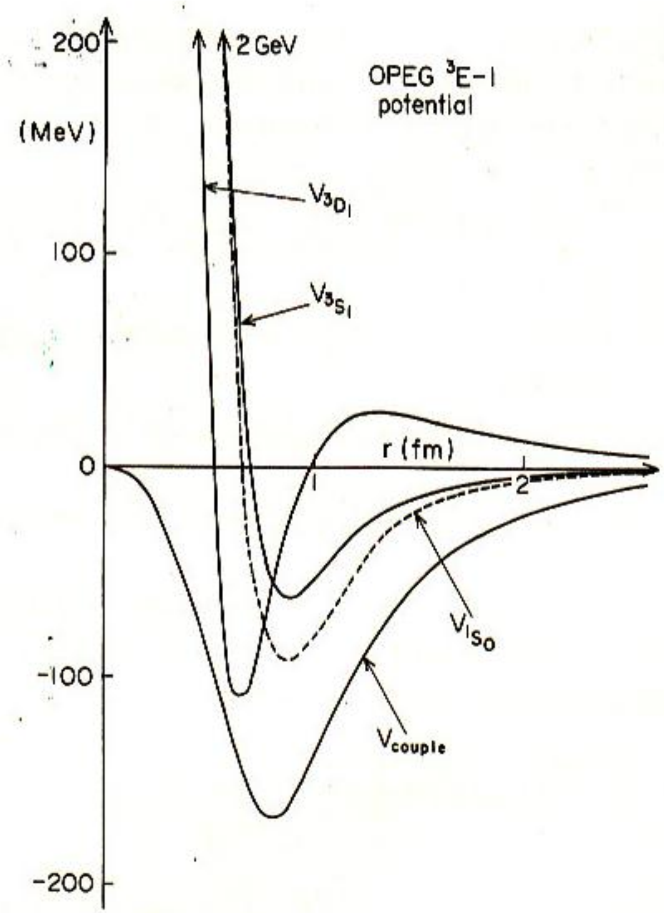
$m^*{}_n(m^*{}_p)$ is the effective mass parameter for n (p).

$\rho(10^{14}\mathrm{g/cc})$	$\epsilon_{Fn}(\mathrm{MeV})$	$U_n(k)(\mathrm{MeV})$; k in fm^{-1}	m_n^*
2.4	54	$-35.0\exp(-0.15k^2)-12.0\exp(-0.10k^2)$	0.82
4.0	76	$-41.0\exp(-0.15k^2)-29.0\exp(-0.10k^2)$	0.78
5.5	94	$-8.6\exp(-0.16k^2)$	0.75
7.9	120	$20.0\exp(-0.704k^2)-123.0\exp(-0.215k^2)$	0.70
$\rho(10^{14}\mathrm{g/cc})$	$\epsilon_{Fp}(\mathrm{MeV})$	$U_p(k)(\mathrm{MeV})$; k in fm^{-1}	m_p^*
2.4	5.4	$-82.80\exp(-0.13k^2)$	0.67
4.0	9.4	$-100.0\exp(-0.15k^2)-11.5\exp(-0.12k^2)$	0.67
6.5	18.0	$-104.0\exp(-0.15k^2)-30.0\exp(-0.20k^2)$	0.53
7.9	23.5	$-125.0\exp(-0.15k^2)-17.3\exp(-0.20k^2)$	0.53

□ Critical temperature T_c for NS constituents (left panel) and multiphase structure of NSs (right panel).




$\tau = 0$ Phase Shifts in Degrees
OPEG
POTENTIAL
40°
30°
20°
10°
0
-10°
-20°
$^3S_1$
$^3D_2$
$^3G_4$
$^3D_3$
100
200
300
400
500
600 (MeV)
E_{NN}^{LAB}
$^3G_3$
$^1P_1$
$^3D_1$
0
25
50
75
100
125
150
ϵ_F (MeV)


200
(MeV)
100
0
-100
-200
2 GeV
OPEG ^3E-1
potential
$V_{^3D_1}$
$V_{^3S_1}$
r (fm)
1
2
V_{1S_0}
V_{couple}

☐ n-p Pairing in symmetric nuclear matter *

$({}^3S_1 + {}^3D_1)$ – coupled gap equation

$$\Delta_{{}^3S_1} \to \Delta_S \quad , \quad \Delta_{{}^3D_1} \to \Delta_D$$

$$\Delta_S(k) = -\frac{1}{\pi}\int k'^2 dk' \langle k'|V_{{}^3S_1}|k\rangle \{\Delta_S(k') f(\theta) + \Delta_D(k') g(\theta)\}/E(k')$$

$$+\frac{1}{\pi}\int k'^2 dk' \langle k'|V_{\text{coupl.}}|k\rangle \int d\hat{k}' \{\Delta_S(k') g(\theta) + \Delta_D(k') h(\theta)\}/E(k'), \quad (1)$$

$$\Delta_D(k) = \frac{1}{\pi}\int k'^2 dk' \langle k'|V_{\text{coupl.}}|k\rangle \int d\hat{k}' \{\Delta_S(k') f(\theta) + \Delta_D(k') g(\theta)\}/E(k')$$

$$-\frac{1}{\pi}\int k'^2 dk' \langle k'|V_{{}^3D_1}|k\rangle \int d\hat{k}' \{\Delta_S(k') g(\theta) + \Delta_D(k') h(\theta)\}/E(k'), \quad (2)$$

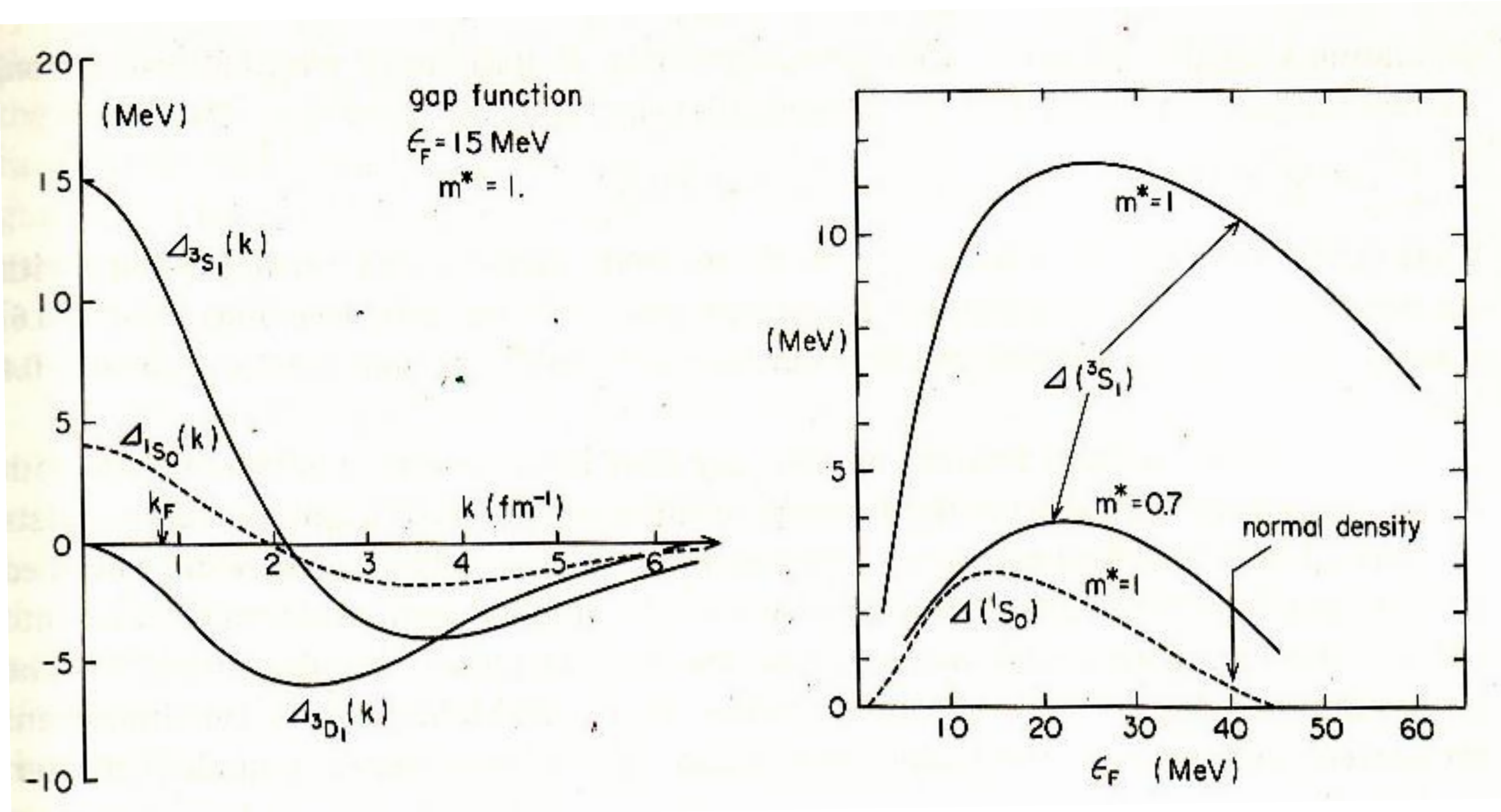
$$\begin{cases} f(\theta) = 1/4\pi, \\ g(\theta) = -\sqrt{2}(3\cos^2\theta - 1)/8\pi, \\ h(\theta) = (3\cos^2\theta + 1)/8\pi, \end{cases} \quad (3)$$

$$E(k') = \sqrt{\tilde{\varepsilon}^2(k') + D_\lambda^2(k')} \quad , \quad (4)$$

$$D_\lambda^2(k') = 4\pi\{\Delta_S^2(k') f(\theta) + 2\Delta_S(k')\Delta_D(k') g(\theta) + \Delta_D^2(k') h(\theta)\} \quad (5)$$

*) T. Takatsuka and R. Tamagaki; Prog. Theor. Phys. Suppl. 112 (1993) 27.

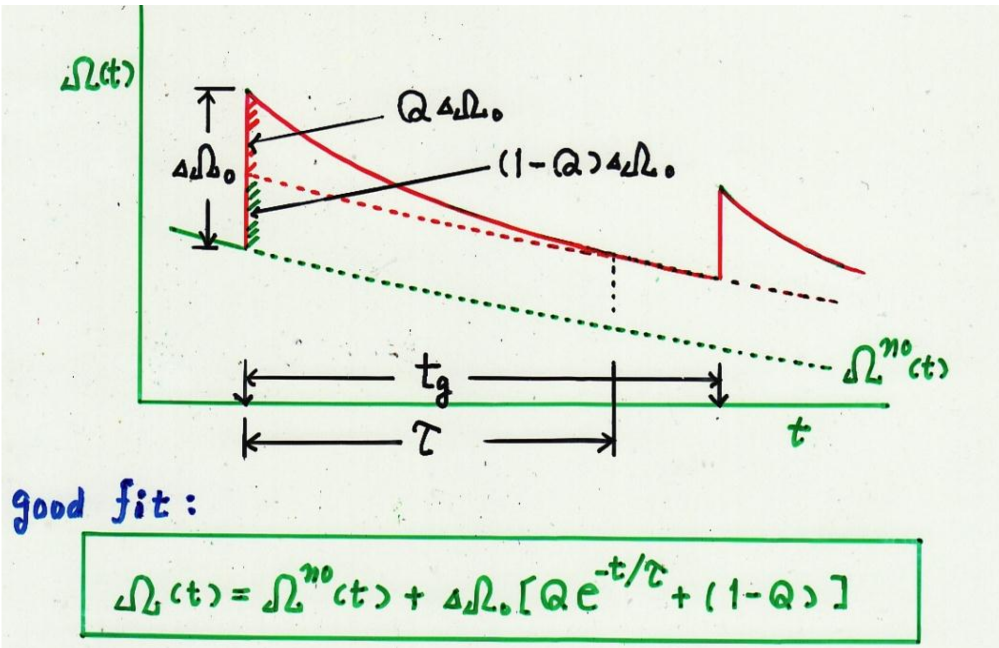
□ Gap functions in for ${}^3S_1 - {}^3D_1$ pairing (left) and corresponding energy gaps (right).



2-5. Relevance to NS phenomena

Pulsar glitch

Sudden speed-up and macroscopic relaxation time



	Crab	Vela
Age (yr)	10^3	$\sim 10^4$
Ω(rad/s)	188.5	70.4
$\Delta\Omega_0/\Omega$	$\sim 10^{-8}$	$\sim 10^{-6}$
τ	$\sim$days	a month$\sim$a year
Q	$\sim$0.9	$\leq$0.05
t_g(yr)	(4-5)	(2-3)
$\lvert\dot{\Omega}\rvert(s^{-2})$	2.4×10^{-9}	9.8×10^{-11}

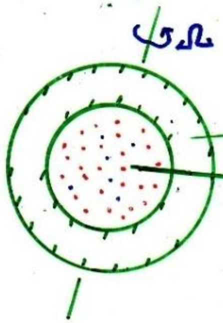
Vela $\rightarrow$ 15 times, Crab $\rightarrow$ 8 (14) times

Macroscopic τ $\rightarrow$ evidence for the existence of superfluid
Q $\rightarrow$ Internal structure (superfluid portion)
Q$\sim$0.9 $\Rightarrow$ necessity of superfluid also in NS cores $\rightarrow$ $^3P_2$ – superfluid

Starquake Model *

▫ Two-component : 1st. → n-superfluid : I_n , Ω_n

2nd. → others (Crust + normal + Charged particles) : I_c , Ω_c



n : $^1S_0$-super

n : $^3P_2$-super
p : $^1S_0$-super
coexist

$2^{\text{nd}}. \rightarrow I_c\dot{\Omega} = -\alpha - I_c(\Omega - \Omega_n)/\tau_c$

$1^{\text{st}}. \rightarrow I_n\dot{\Omega}_n = \frac{I_c(\Omega-\Omega_n)}{\tau_c}$

$\Omega(t) = \Omega^{no}(t) + (\Delta\Omega_0)[Qe^{-\frac{t}{\tau}} + (1-Q)]$

$\tau = \frac{\tau_c I_n}{I}$; $I = I_n + I_c$, $Q \simeq I_n/I$

$\Delta I/I = -(\Delta\Omega)_\infty/\Omega = -(1-Q)\Delta\Omega_0/\Omega$

☐ Scenario:

Slowing down of rotation ($\dot{\Omega} < 0$) →
Accumulation of stress in the crust
→ exceeds
A certain critical value, a crack
occurs (ΔI<0)
→ by ang. Momentum cons.
ΔL= IΔΩ+ΔIΩ=0
sudden speed up ΔΩ----- Glitch
→ because of the existence of
superfluid, macroscopic time is
necessary for the star to corotate
as a whole
--------- τ

▫ Explanation of macroscopic τ :

- n = normal, p = normal → $\sim 10^{-17}$ sec. (n-p scatt.)
- n = super, p = normal → $\sim 10^{-17} \times 10^{18} = \sim 10^{1}$ sec
  ↑ $(\Omega/\Omega_{c2})^{-1}$
- n = normal, p = super → $\sim 10^{-11}$ sec (magnetic int.)
- n = super, p = super → $\sim 10^{-11} \times 10^{18} = \sim 10^{7}$ sec
  OK

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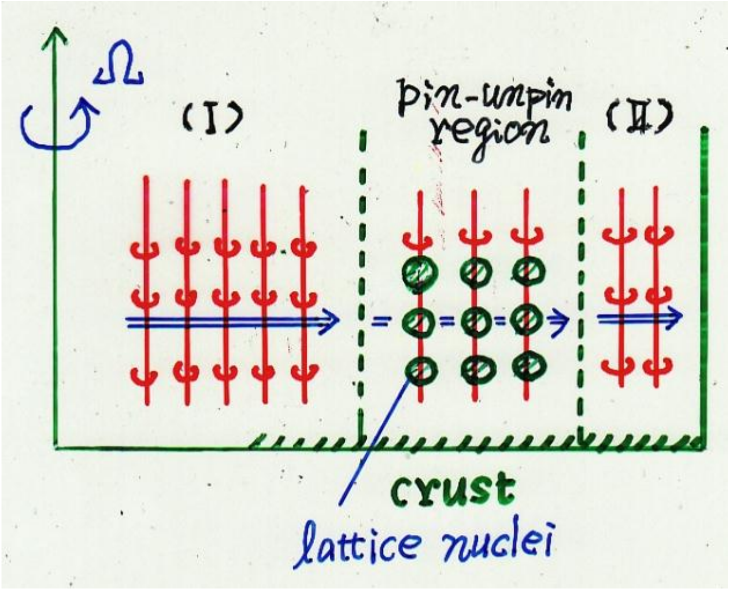
*) G. Baym, C.J. Pethick, D. Pines and M. Ruderman, Nature 224 (1969) 872.
D. Pines, J. Shaham and M. Ruderman, Nature Phys. Sci. 27 (1972) 83.

Vortex Creep Model *



□ Scenario (original "pin-unpin" → thermal creep):

NS-spin down= Crust-spindown
→ Outward motion of vortex lines (I → II)
→ Captured in Pin. Region
→ Ω_s(super)>Ω(crust)
→ when unpinning force (=Magnus Force: f_{magnus}) exceeds the pinning force f_{pin}, catastrophic unpinning occurs
→ transfer excess angular momentum of superfluid to crust, which is the Glitch

- $\Omega = \frac{1}{2}\kappa n_v$; $\kappa = h/2m_N$
- $f_{pin} = E_{pin}/ab$; $a = R_N$ or ξ, b = lattice spacing
- $f_{magnus} = \rho r_{pin} \kappa \omega$; $\omega = \Omega_s - \Omega$: mismatch

$\Rightarrow f_{pin} = f_{magnus} \rightarrow \omega_{cr}$, $t_g = \omega_{cr}/|\dot{\Omega}|$

$I_{pin}\,\omega_{cr} = I_c\,(\Delta\Omega_0) \rightarrow I_{pin}/I \simeq I_{pin}/I_c = \frac{\Delta\Omega_0}{\omega_{cr}}$

Ang. mom. consv. $(I = I_c + I_{pin} \simeq I_c)$

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*) D. Pines, J. Shaham, M.A. Alper and P.W. Anderson, Prog. Theor. Phys. Suppl. 69 (1980) 376.
M.A. Alper; P.W. Anderson, D. Pines and J. Shaham, Ap. J. 249 (1981) L29.

Problems

1) Can unpinning occur?

$$t_g = \omega_{cr}/|\dot{\Omega}| = t_0 \omega_{cr}/\Omega \ , \ t_0 = \Omega/|\dot{\Omega}|$$

	t_g (yr)	Ω (rad/s)	t_0 (yr)
Vela:	2-4	70	2.3×10^4
Crab:	3-11	189	2.5×10^3

Obs. $\omega_{cr} \sim (6-12)\times10^{-3}$ rad/s for Vela
$\sim (2-8)\times10^{-1}$ " for Crab

Cal. $\omega_{cr} = E_{pin}/(ab\rho\kappa r_{pin}) \sim (24-2)$ rad/s.

$\Longrightarrow$ To overcome this defect, they introduced "Super weak pinning region", assuming the unpinning event as occuring by "thermal excitation".

$\longrightarrow$. pin-unpin physics becomes obscure

. ω_{cr} is not calculable.

2) Is it possible for unpinning to occur catastrophically?

$\Omega = \frac{1}{2}\kappa n_v$; $\kappa = h/2m_N$

vortexline spacing $\ell = 1/\sqrt{n_v}$

	Vela	Crab
n_v (fm^{-2})	7.0×10^{-22}	1.9×10^{-21}
ℓ (fm)	3.8×10^{10}	2.3×10^{10}
b (")	$70\sim30$	$70\sim30$

1 vortex per 10^{8-9} lattice -----
$\Rightarrow$ hard to expect collective effect

Obs. $\rightarrow$ unpinning of $10^{11\sim12}$ vortex lines simultaneously

3) Can I_{pin}/I be consistent with N~~s~~-model?

$I_{pin}\,\omega_{cr} = I_c\,\Delta\Omega_0 \simeq I\,\Delta\Omega_0$

$I_{pin}/I \simeq \Delta\Omega_0/\omega_{cr} = \frac{\Delta\Omega_0}{\Omega}\,\frac{t_0}{t_g}$

$\sim (1-4)\times10^{-2}$ for Vela $\leftarrow$ Too Large?

$\sim (4-10)\times10^{-6}$ for Crab

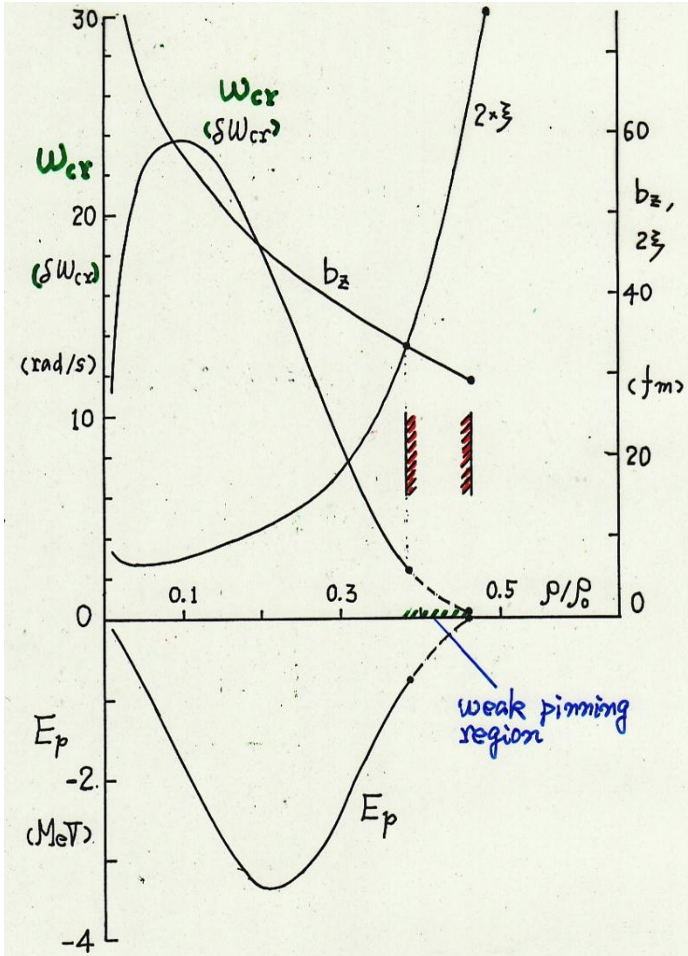
4) Unified explanation for Vela and Crab?

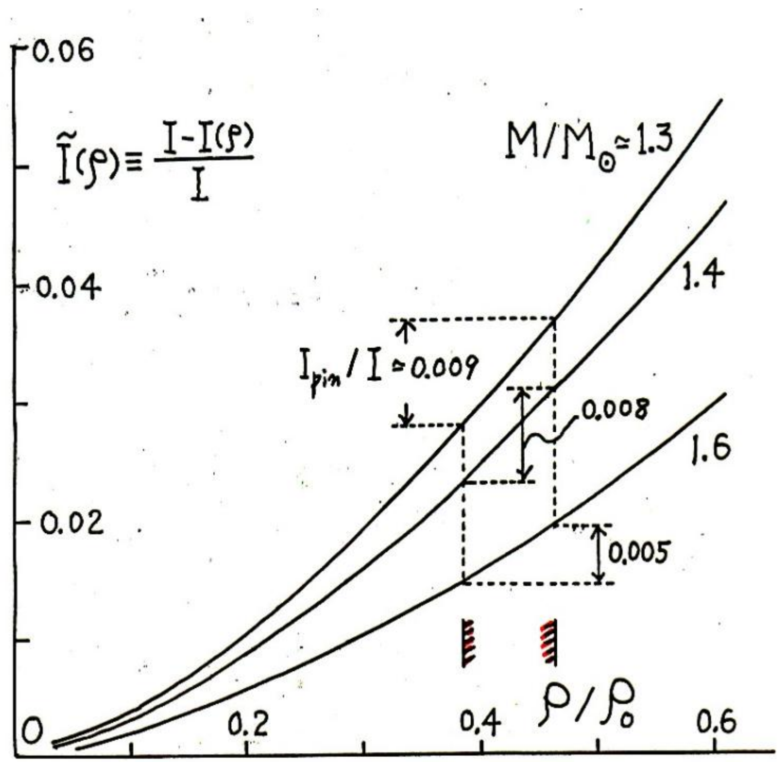
$(I_{pin}/I)_{Crab} / (I_{pin}/I)_{Vela} \sim 10^{-4}$

$\longrightarrow$ different mechanism?

5) How about Q (~0.9 for Crab, ≤ 0.05 for Vela)

Pinning region and the potion





＜捕捉＞

質問への回答（第一回講義関係）

Questions:

○ What is the order parameter for 3P2 - superfluid ?

- Energy gap , but with angle –dependence like He-3 case
- As a “wave function” order parameter introduced by Ginzburg-Landau, distribution function of Cooper pair.

○ What symmetry is broken ?

- Rotational symmetry --- formation of vortex
- Gage symmetry --- Coherent state of Cooper pair , as a bulk state of the system