Controlling Quantum Systems with Spatial Adiabatic Passage

Thomas Busch
Motivation: Complex Quantum System Dynamics

**Step 1:** learn how to control *small* quantum systems (one or two particles)

\[ i\hbar \frac{\partial}{\partial t} |\psi(x_1, x_2; t)\rangle = H(t)|\psi(x_1, x_2; t)\rangle \]

**Step 2:** learn how to control *large* quantum systems (more than two particles)

\[ i\hbar \frac{\partial}{\partial t} |\psi(x_1, x_2, x_3, \ldots; t)\rangle = H(t)|\psi(x_1, x_2, x_3, \ldots; t)\rangle \]

→ bottom-up: following experimental progress, clean and controllable systems
Guiding Principles

Understand Fundamental QM
- Entanglement
- Non-locality
- Decoherence

Develop Applications of QM
- Quantum Computing
- Quantum Simulators
- Quantum Metrology

Find QM in real life
- Energy Transport
- Charge Transfer
- Quantum Phase Transitions

Why
quantum systems are very fragile and have a massive Hilbert space

identify systems which can be engineered

develop techniques for quantum engineering

find techniques for scaling quantum systems up

do this in close collaboration with experimentalists

System of Choice: ultracold atoms
Examples of Projects @ OIST

- superfluid vortices as topological matter
- creation of non-classical states
- quantum walks as quantum memories
- nano-sensors for single atom states
- spin-orbit coupled superfluids
- multicomponent superfluids
- atom-ion hybrid systems
- adiabatic engineering techniques & shortcuts
Spatial Adiabatic Passage

How to move an atom?

Solution: Tunneling (increase and decrease the distance between traps)

Problem: Fragile Process (Rabi Oscillations)

- success depends on good control of three parameters
  \[ t_i \quad \text{interaction time} \]
  \[ t_a \quad \text{approach time} \]
  \[ a_{\text{min}} \quad \text{minimum distance between traps} \]

- precise experimental control necessary for high fidelities ( > 99.9999%)
Spatial Adiabatic Passage

How to move an atom?

Solution: Tunneling (increase and decrease the distance between traps)

Problem: Fragile Process (Rabi Oscillations)

- success depends on good control of three parameters:
  - $t_i$: interaction time
  - $t_a$: approach time
  - $a_{\text{min}}$: minimum distance between traps

- precise experimental control necessary for high fidelities (> 99.9999%)
Spatial Adiabatic Passage

Sequential Tunneling

same problem, only twice
Quantum Systems Unit: Spatial Adiabatic Passage

Counterintuitive Tunneling

100% transfer! (STIRAP)

K Eckert, M Lewenstein, R Corbalán, G Birkl, W Ertmer, J Mompart
Why does this work?

\[
|1\rangle \\
\begin{pmatrix}
\epsilon & \Omega_{12} & 0 \\
\Omega_{12} & \epsilon & \Omega_{23} \\
0 & \Omega_{23} & \epsilon
\end{pmatrix}
\begin{align*}
|1\rangle & \quad |2\rangle & \quad |3\rangle \\
\end{align*}
\]

|1\rangle \quad |2\rangle \quad |3\rangle

\[
|\Psi\rangle = \cos \theta |1\rangle - \sin \theta |3\rangle
\]

\[
\tan \theta = \frac{\Omega_{12}}{\Omega_{23}}
\]

**TRANSFER:**

\[
\begin{align*}
\cos \theta &: 1 \rightarrow 0 \\
\sin \theta &: 0 \rightarrow 1 \\
\theta &: 0 \rightarrow \frac{\pi}{2} \\
\tan \theta &: 0 \rightarrow \infty
\end{align*}
\]
All good. Now what?

→ generalise beyond 1D

→ find shortcuts to avoid adiabatic restrictions

→ identify suitable experimental settings for observation

→ generalise to many particle systems

→ identify non-classical correlations

→ develop into other engineering tools: quantum state preparation, deterministic single atom source ....
**Shortcut To Adiabaticity**

**Idea:** add terms to Hamiltonian that compensate for diabatic excitations when driving is non-adiabatic

\[ \mathcal{H} = H_0 + H_1 \]

\[
H_0(t) = \frac{\hbar}{2} \begin{pmatrix}
0 & \Omega_{12}(t) & 0 \\
\Omega_{12}(t) & 0 & \Omega_{23}(t) \\
0 & \Omega_{23}(t) & 0
\end{pmatrix},
\]

\[
H_1(t) = \frac{\hbar}{2} \begin{pmatrix}
0 & 0 & i\Omega_{13}(t) \\
0 & 0 & 0 \\
-i\Omega_{13}(t) & 0 & 0
\end{pmatrix}
\]

Spatial Adiabatic Passage in 2D

up to now:

\[
\begin{align*}
1 & \quad \Omega_{12} \quad \text{1} \\
2 & \quad \Omega_{23} \quad \text{2} \\
3 & \quad \Omega_{31} 
\end{align*}
\]

→ symmetry breaking gives additional coupling

→ use this degree of freedom to make new states

→ create angular momentum

Tunneling-induced angular momentum for single cold atoms
Shortcut To Adiabaticity

\[ \mathcal{H}(t) = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_{12}(t) & i\Omega_{13}(t) \\ \Omega_{12}(t) & 0 & \Omega_{23}(t) \\ -i\Omega_{13}(t) & \Omega_{23}(t) & 0 \end{pmatrix} \]

but: shortcut Hamiltonian is imaginary!

\[ \Omega_{13} \sim \langle 1|\mathcal{H}|3 \rangle \]

cannot get this phase dynamically for transition between eigenstates

nice idea, but cannot be implemented for SAP…?!

use geometric phase!?
Geometric Phase

**Brief Reminder:** Aharanov Bohm Effect

- assume charged particle moving in a magnetic field that is constant everywhere (or localised)
- phase is added when particle moves from $\vec{r}_i$ to $\vec{r}_j$

$$\phi_{ij} = \frac{q}{\hbar} \int_{\vec{r}_i}^{\vec{r}_j} \vec{A} \cdot d\vec{l}$$

$\vec{r}_i$ are the positions of the wells and $\vec{A}$ is the magnetic vector potential

- total phase in a closed loop:

$$\Phi = \phi_{12} + \phi_{23} + \phi_{31} = \frac{q}{\hbar} \int \vec{A} \cdot d\vec{l} = \frac{q}{\hbar} \Phi_B$$

magnetic flux through the closed path around the triangle
**Geometric Phase**

\[ H_{AB} = -\frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_{12}e^{-i\phi_{12}} & \Omega_{13}e^{i\phi_{31}} \\ \Omega_{12}e^{i\phi_{12}} & 0 & \Omega_{23}e^{i\phi_{23}} \\ \Omega_{13}e^{-i\phi_{31}} & \Omega_{23}e^{-i\phi_{23}} & 0 \end{pmatrix} \]

**what we want:**

\[ H = -\frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_{12} & -i\Omega_{13} \\ \Omega_{12} & 0 & \Omega_{23} \\ i\Omega_{13} & \Omega_{23} & 0 \end{pmatrix} \]

engineer field such that \( \phi_{12} = \phi_{23} = 0 \) and \( \phi_{31} = -\frac{\pi}{2} \)

requires field with specific spatial profile
Geometric Phase

change basis using only local phases

\[ U = \begin{pmatrix}
  e^{\frac{i}{2} (\phi_{12} + \phi_{23})} & 0 \\
  0 & e^{\frac{i}{2} (-\phi_{12} + \phi_{23})} \\
  0 & 0
\end{pmatrix} \]

so that we get

\[ \mathcal{H}'_{AB} = U \mathcal{H}_{AB} U^{-1} = -\frac{\hbar}{2} \begin{pmatrix}
  0 & \Omega_{12} & \Omega_{31} e^{i\Phi} \\
  \Omega_{12} & 0 & \Omega_{23} \\
  \Omega_{31} e^{-i\Phi} & \Omega_{23} & 0
\end{pmatrix} \]

and therefore only need

\[ \Phi = -\frac{\pi}{2} \quad \quad \quad \Phi_B = \oint \vec{A} \cdot d\vec{l} = -\frac{\hbar\pi}{2q} \]

only relevant value is the total phase (or total flux)

can be achieved with homogeneous field distribution
Spatial Adiabatic Passage in 2D

- **no shortcut pulse**
  - full transfer (adiabatic)
  - low transfer (fast)

- **with shortcut pulse**
  - full transfer (adiabatic)

---

**can also be inverted to measure magnetic fields!**
All good. Next…

1. generalise beyond 1D
2. find shortcuts to avoid adiabatic restrictions
3. identify suitable experimental settings for observation
4. generalise to many particle systems
5. identify non-classical correlations
6. develop into other engineering tools: deterministic single atom source
Interactions

\[ H = \sum_{k=1}^{2} \left( -\frac{1}{2} \partial_{x_k}^2 + V(x_k) \right) + g \delta(x_1 - x_2), \]

- resonance not guaranteed
- dark state not guaranteed

non-interacting bosons

strongly interacting bosons
(non-interacting fermions)

\[ g = 0 \quad g = \infty \]
Weak Interactions

Three-well Bose-Hubbard model:

\[
H_B = \sum_{j=L,M,R} \left[ \frac{U}{2} n_j(n_j - 1) + \epsilon_0 n_j \right] \\
+ \Omega_{LM} \left( b_L^{\dagger} b_M + b_M^{\dagger} b_L \right) + \Omega_{MR} \left( b_M^{\dagger} b_R + b_R^{\dagger} b_M \right) \\
+ \Omega_{LM}^{(co)} \left( b_L^{\dagger 2} b_M^2 + b_M^{\dagger 2} b_L^2 \right) + \Omega_{MR}^{(co)} \left( b_M^{\dagger 2} b_R^2 + b_R^{\dagger 2} b_M^2 \right)
\]

diagonalise and find two energy bands

\[
E = 1 \quad \text{particles are in different wells}
\]

\[
E = 1 + U \quad \text{particles are in same well}
\]
Weak Interactions

level crossings make following the *dark state* effectively impossible
Three-well Fermi-Hubbard model:

$$H_F = \sum_{j=L,M,R} \left[ U n_{j0} n_{j1} + \sum_{i=0,1} \epsilon_i n_{ji} \right] + \sum_{i=0,1} \left[ \Omega^{(i)}_{LM} a_{Li}^\dagger a_{Mi} + \Omega^{(i)}_{MR} a_{Mi}^\dagger a_{Ri} + \text{h.c.} \right]$$

$$+ \Omega^{(co)}_{LM} a_{L0}^\dagger a_{M0} a_{M1} + \Omega^{(co)}_{MR} a_{M0}^\dagger a_{R0} a_{R1} + \text{h.c.},$$

diagonalise and find two energy bands

$$E' = 2 \quad \text{particles are in different wells}$$

$$E = 2 - |U| \quad \text{particles are in same well}$$
Strong Interactions

\[ |\bar{U}| = 0.15 \]

level crossings make following the dark state effectively impossible
interaction band is isolated, but crossings still exist!

adiabatic and diabatic dynamics can lead to full transfer
Summary

Adiabatic techniques are not necessarily slow or limited to single particles.

Spatial Adiabatic Passage possess an experimentally implementable Shortcut to Adiabaticity

Interactions can lead to band-separation that allow to use single particle ideas for many-particle systems.
Collaborations

Irina Reshodko
Lee O’Riordan
Tara Hennessy
Albert Benseny
Yongping Zhang
Jeremie Gillet
Angela White
Rashi Sachdeva
Thomas Fogarty
James Schloss
TB

Andreas Ruschhaupt
Anthony Kiely