Dynamical mean field approach to correlated lattice systems in and out of equilibrium

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Overview

- *Dynamical mean field approximation applied to quantum field theory*

  in collaboration with: O. Akerlund, P. de Forcrand, A. Georges

- *Dynamical mean field theory for bosonic lattice systems*

  in collaboration with: P. Anders, L. Pollet, M. Troyer

- *Dynamical mean field theory for fermionic lattice systems*

  in collaboration with: E. Gull, A. Millis

- *Nonequilibrium extension of dynamical mean field theory*

  in collaboration with: M. Eckstein, M. Kollar, N. Tsuji, T. Oka, H. Aoki

- *Some applications: Hubbard model in strong electric fields*

  in collaboration with: M. Eckstein, N. Tsuji, T. Oka, H. Aoki

DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

  Akerlund, de Forcrand, Georges & Werner (2013)

- Lagrangian density

$$\mathcal{L}[\varphi(x)] = \frac{1}{2}(\partial_\mu \varphi(x))^2 - \frac{1}{2} m_0^2 \varphi(x)^2 - \frac{g_0}{4!}\varphi(x)^4$$

- Spontaneous $\mathbb{Z}_2$ symmetry breaking for negative (renormalized) m^2

- After Wick rotation, discretization and variable transformation

$$S = \sum_x \left(-2\kappa \sum_\mu \varphi_{x+\widehat{\mu}}\varphi_x + \varphi_x^2 + \lambda(\varphi_x^2 - 1)^2 \right)$$

- In the limit $\lambda \to \infty, \quad \varphi_x = \pm 1$ (Ising model)

DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

  Akerlund, de Forcrand, Georges & Werner (2013)

- Mean field theory: mapping to a 0-dimensional effective model

  → replace all interactions by an interaction with a constant background field v

- Partition function of the lattice model

$$Z = \int \mathcal{D}[\varphi] \prod_x \exp\left(-\varphi_x^2 - \lambda(\varphi_x^2 - 1)^2 + 2\kappa \sum_{\mu=1}^{d} \varphi_x \varphi_{x+\widehat{\mu}}\right)$$

becomes $Z_{\mathrm{MF}} = \int_{-\infty}^{\infty} d\varphi \, \exp\left(-\varphi^2 - \lambda(\varphi^2 - 1)^2 + 2\kappa(2d) v\varphi\right)$

- Self-consistency condition

$$v \equiv \langle \varphi \rangle = \frac{1}{Z_{MF}} \int_{-\infty}^{\infty} \varphi \, \varphi \exp\left(- \varphi^2 - \lambda(\varphi^2 - 1)^2 + 2\kappa(2d) v\varphi\right)$$

DMFT for quantum field theories

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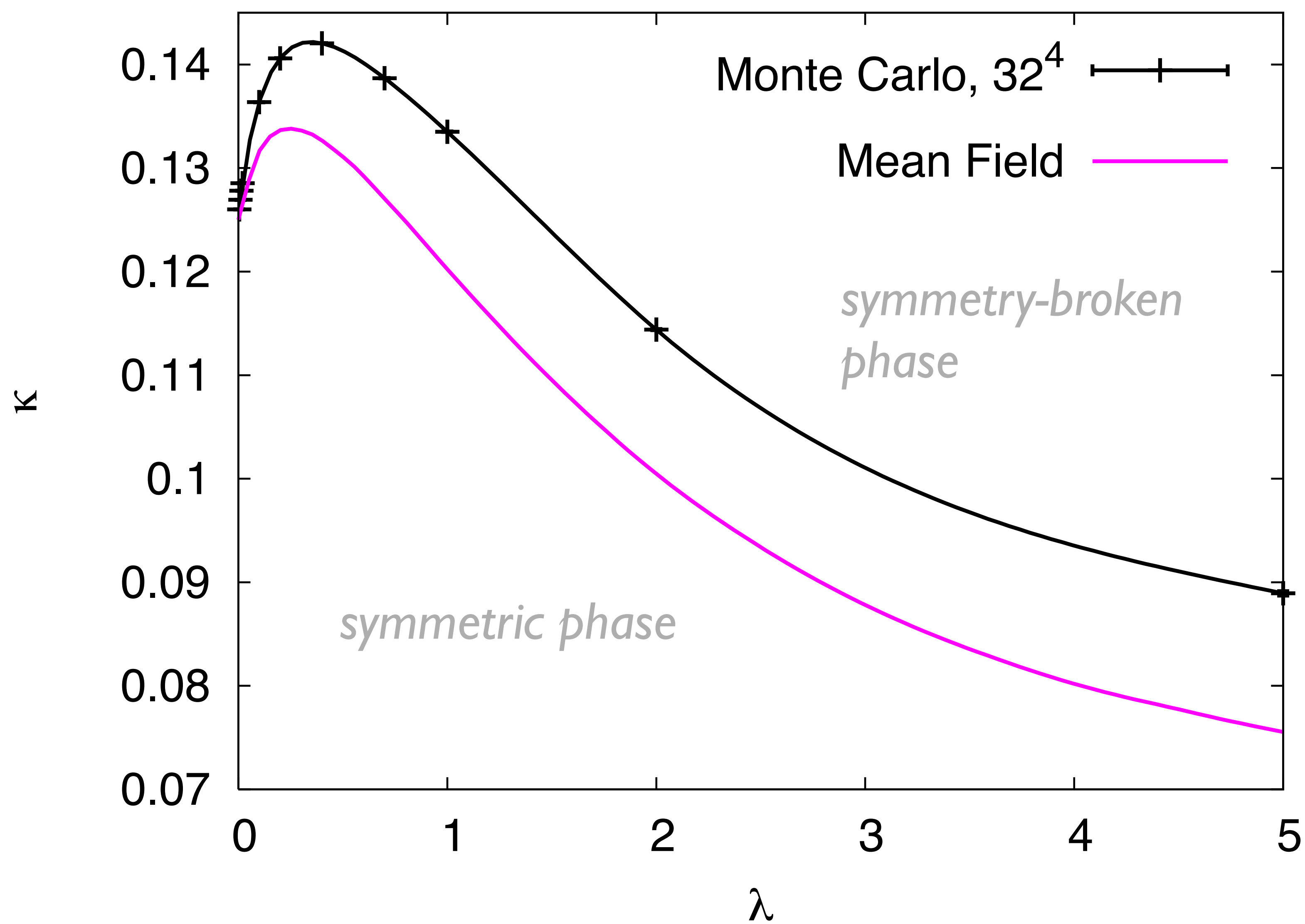
- Mean field theory: Phase diagram (d=3+1)



DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

  Akerlund, de Forcrand, Georges & Werner (2013)

- Dynamical mean field theory: mapping to a (0+1)-dimensional effective model

  $\longrightarrow$ explicitly treat fluctuations in one dimension, freeze fluctuations in the (d-1) other dimensions

- Dynamical dimension: t (with conjugate momentum ω)

  Frozen dimensions: $x_1, \ldots, x_{d-1}$ (conjugate momenta $k_1, \ldots, k_{d-1}$)

  This convention allows to study finite-temperature behavior by varying the extent of the dynamical dimension

- Breaks Lorentz invariance, but let's try it anyhow ...

DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

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- Dynamical mean field theory: mapping to a (0+1)-dimensional effective model

  $\longrightarrow$ explicitly treat fluctuations in one dimension, freeze fluctuations in the (d-1) other dimensions

- Schematically:

$$Z = \int \mathcal{D}[\varphi] \exp\left(2\kappa \sum_x \sum_\mu \varphi_x \varphi_{x+\widehat{\mu}} - \sum_x V(\varphi_x)\right)$$

non-zero in the symmetry-broken phase

$$Z_{\mathrm{DMFT}} = \int \mathcal{D}[\varphi] \exp\left(-\sum_{t,t'} K^{-1}(t-t')\varphi_t \varphi_{t'} - \sum_t V(\varphi_t) + h \sum_t \varphi_t\right)$$

effect of the frozen dimensions on the local dynamics

DMFT for quantum field theories

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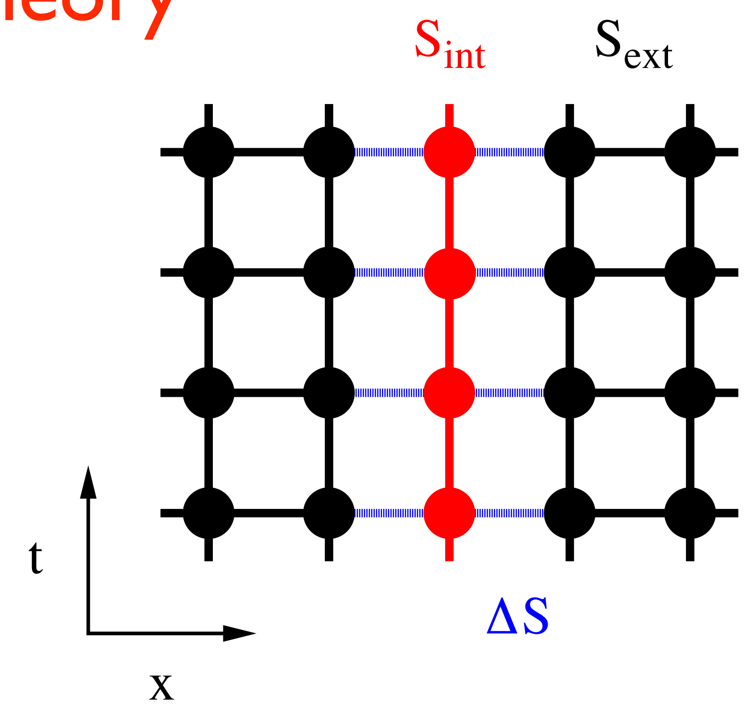


- Cavity construction: separate action into *internal* $x = (\vec{0}, t)$ and *external* $x \neq (\vec{0}, t)$ degrees of freedom

$$S = S_{\text{int}} + \Delta S + S_{\text{ext}}$$

$$S_{\text{int}} = \sum_t \Big[-2\kappa \varphi_{\text{int},t+1}\varphi_{\text{int},t} + \varphi^2_{\text{int},t} + \lambda(\varphi^2_{\text{int},t} - 1)^2 \Big]$$

$$\Delta S = -2\kappa \sum_t \sum_{\langle \text{int,ext} \rangle} \varphi_{\text{int},t}\varphi_{\text{ext},t}$$

$$S_{\text{ext}} = \sum_{x \neq (\vec{0},t)} \Big[-2\kappa \sum_\nu \varphi_{x+\hat{\nu}}\varphi_x + \varphi_x^2 + \lambda(\varphi_x^2 - 1)^2 \Big]$$

- Expand $\exp(-\Delta S)$ and integrate out the external degrees of freedom

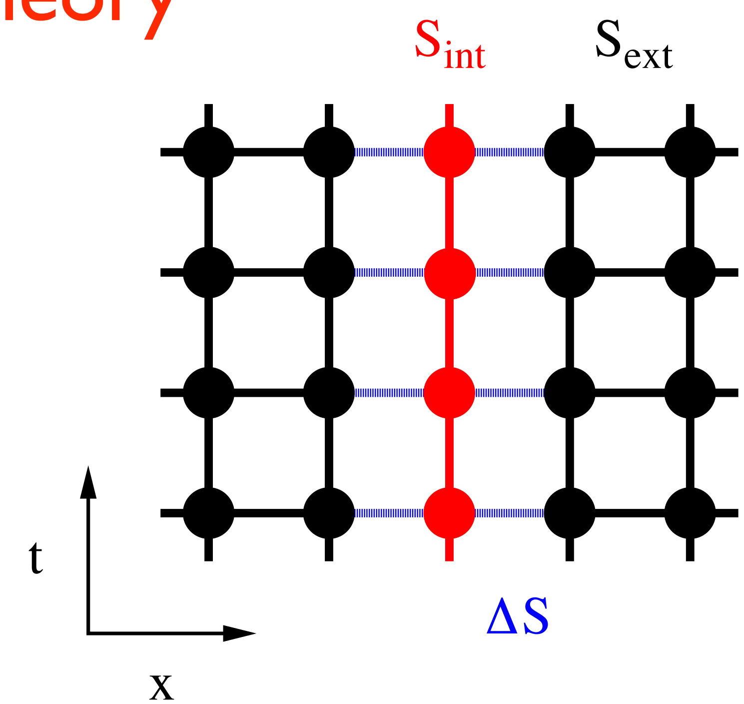
DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

  Akerlund, de Forcrand, Georges & Werner (2013)

- Cavity construction: to allow for symmetry breaking, we write

$$\varphi_{\text{ext},t} = \phi_{\text{ext}} + \delta\varphi_{\text{ext},t}, \quad \langle\varphi_{\text{ext}}\rangle = \phi_{\text{ext}}$$
$$\varphi_{\text{int},t} = \phi_{\text{int}} + \delta\varphi_{\text{int},t}, \quad \langle\varphi_{\text{int}}\rangle = \phi_{\text{int}}$$



$$\Delta S = -\kappa \sum_t \left(2(d-1)\phi^\dagger_{\text{ext}}\delta\varphi_{\text{int},t} + \sum_{\langle\text{int,ext}\rangle} \delta\varphi^\dagger_{\text{int},t}\delta\varphi_{\text{ext},t} \right) + f(\varphi_{\text{ext}})$$
$$\equiv S_1 + \sum_t \delta S + f(\varphi_{\text{ext}})$$

can be included in S_{int} *can be included in S_{ext}*

- Expand $\exp(-\delta S)$

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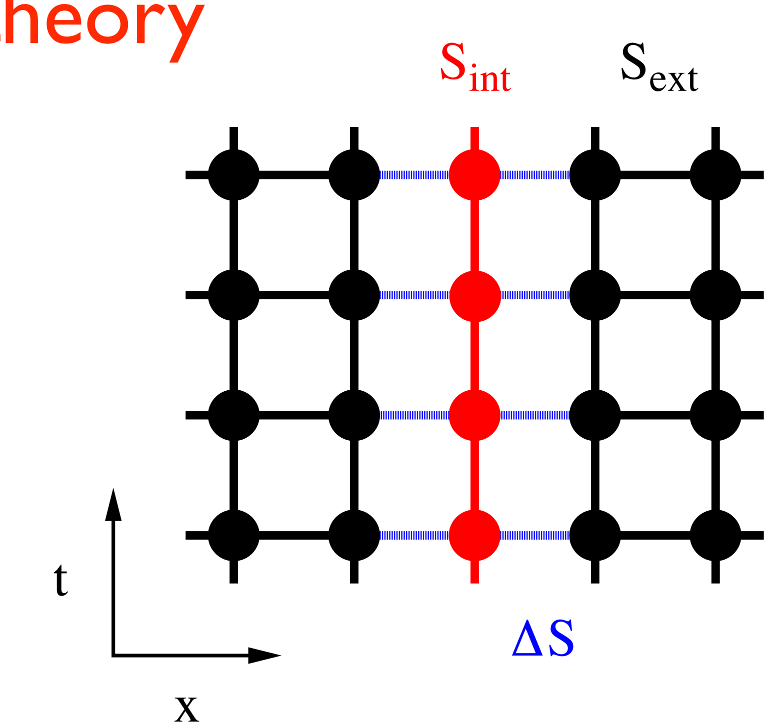

- Cavity construction: Terms up to second order yield

$$Z = Z_{\text{ext}} \int \mathcal{D}\varphi_{\text{int}} \exp(-S_{\text{int}} - S_1)$$
$$\times \left(1 - \sum_t \langle \delta S(t) \rangle_{\text{ext}} + \frac{1}{2} \sum_{t,t'} \langle \delta S(t) \delta S(t') \rangle_{\text{ext}} + \dots \right)$$

- First order term is zero because proportional to $\langle \delta\varphi_{\text{ext}} \rangle_{\text{ext}} = 0$

- Second order term is non-zero:

"hybridization function"

$$\langle \delta S(t) \delta S(t') \rangle_{\text{ext}} \equiv \delta\varphi^\dagger_{\text{int},t} \Delta(t - t') \delta\varphi_{\text{int},t'}$$

DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

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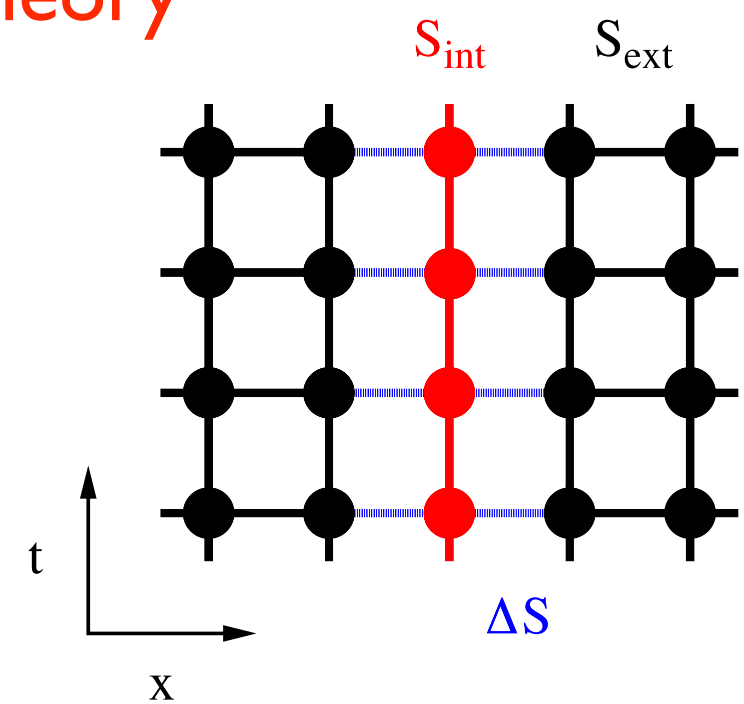
- Cavity construction: After re-exponentiation and switching back from $\delta\varphi$ to φ we find the effective single-site action



$$S_{\text{imp}} = \sum_{t,t'} \varphi_t K^{-1}_{\text{imp},c}(t-t')\varphi_{t'} + \lambda \sum_t (\varphi_t^2 - 1)^2 - h \sum_t \varphi_t$$

$$\widetilde{K}^{-1}_{\text{imp},c}(\omega) = 1 - 2\kappa\cos(\omega) - \widetilde{\Delta}(\omega)$$

Fourier transform of nn interaction in time

$$h = 2\phi_{\text{ext}}(2\kappa(d-1) - \widetilde{\Delta}(0))$$

because action written in terms of phi

- We call it "impurity action" (condensed-matter convention)

DMFT for quantum field theories

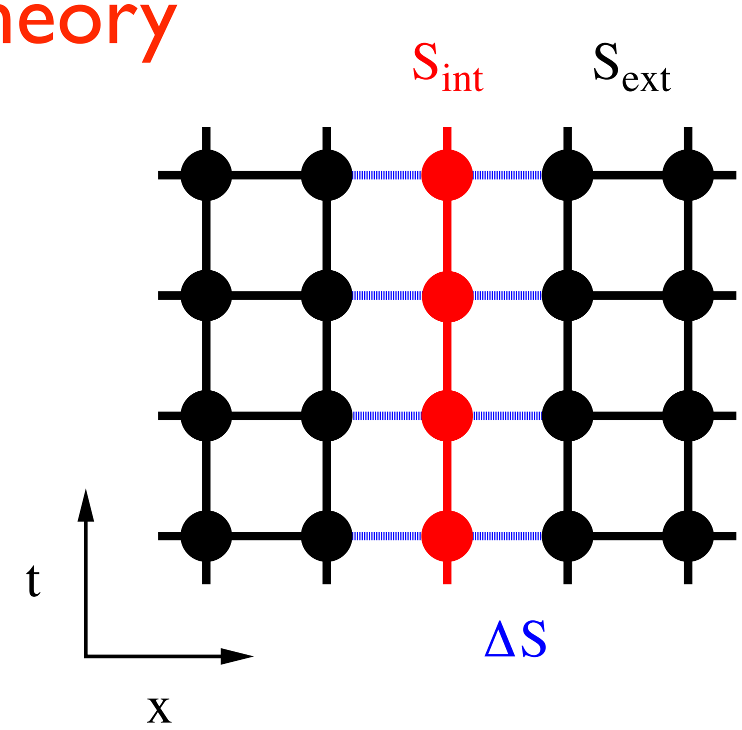
- Simple example: real, scalar φ^4 quantum field theory

  Akerlund, de Forcrand, Georges & Werner (2013)

- Dynamical mean field equations:

  Hybridization function Δ and ϕ_{ext}

  are fixed by self-consistency conditions

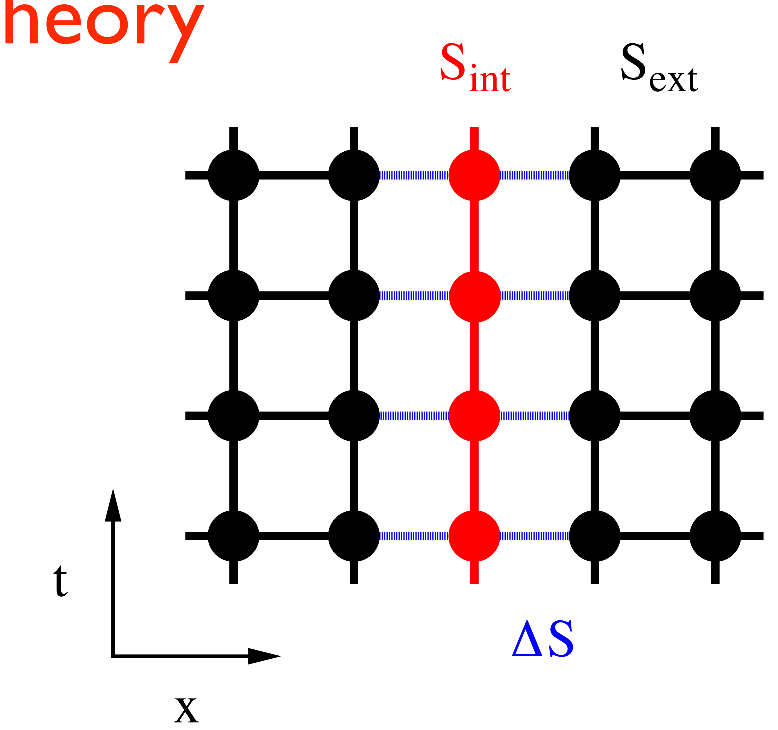


$$
\begin{aligned}
S_{\mathrm{imp}} &= \sum_{t,t'} \varphi_t K^{-1}_{\mathrm{imp},c}(t-t')\varphi_{t'} + \lambda \sum_t (\varphi_t^2 - 1)^2 - h \sum_t \varphi_t \\
\widetilde{K}^{-1}_{\mathrm{imp},c}(\omega) &= 1 - 2\kappa \cos(\omega) - \widetilde{\Delta}(\omega) \\
h &= 2\phi_{\mathrm{ext}}(2\kappa(d-1) - \widetilde{\Delta}(0))
\end{aligned}
$$

DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

  Akerlund, de Forcrand, Georges & Werner (2013)



- Dynamical mean field equations:

  Hybridization function Δ and ϕ_{ext}

  are fixed by self-consistency conditions

- Local lattice Green's function

$$\widetilde{G}_{\text{loc}}(\omega) = \sum_k \frac{1}{\widetilde{G}_0^{-1}(k,\omega) + \widetilde{\Sigma}(k,\omega)}$$

DMFT approximation: identify lattice and impurity self-energy

$$\widetilde{G}_0^{-1}(k,\omega) = 1 - 2\kappa \sum_{i=1}^{d} \cos(k_i)$$

GF of the d-dimensional free theory

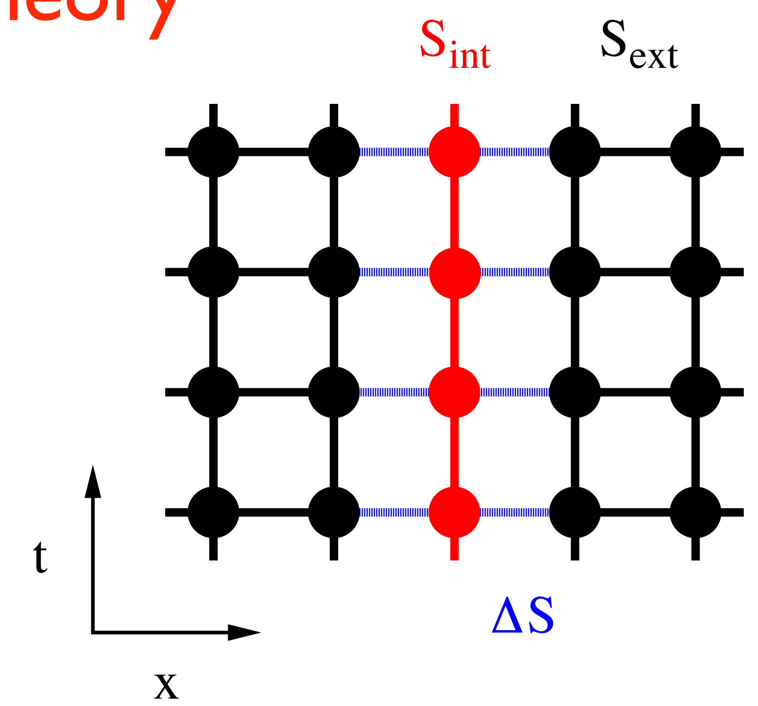
- Impurity Green's function

$$\widetilde{G}_{\text{imp}}(\omega) = \frac{1}{\widetilde{K}_{\text{imp},c}^{-1}(\omega) + \widetilde{\Sigma}_{\text{imp}}(\omega)} = \frac{1}{1 - 2\kappa\cos(\omega) - \widetilde{\Delta}(\omega) + \widetilde{\Sigma}_{\text{imp}}(\omega)}$$

DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

  Akerlund, de Forcrand, Georges & Werner (2013)



- Dynamical mean field equations:

  Hybridization function Δ and ϕ_{ext}

  are fixed by self-consistency conditions

- Local lattice Green's function

$$\widetilde{G}_{\text{loc}}(\omega) = \sum \frac{1}{\widetilde{G}_{\text{imp}}^{-1}(\omega) + \widetilde{\Delta}(\omega) - 2\kappa \sum_{i=1}^{d-1} \cos k_i}$$

- Self-consistency equations:

$$\begin{aligned} \widetilde{G}_{\text{imp}}(\omega) &= \widetilde{G}_{\text{loc}}(\omega) \\ \langle \varphi \rangle_{S_{\text{imp}}} &= \phi_{\text{ext}} \end{aligned}$$

substitution yields an implicit equation for the hybridization function

DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

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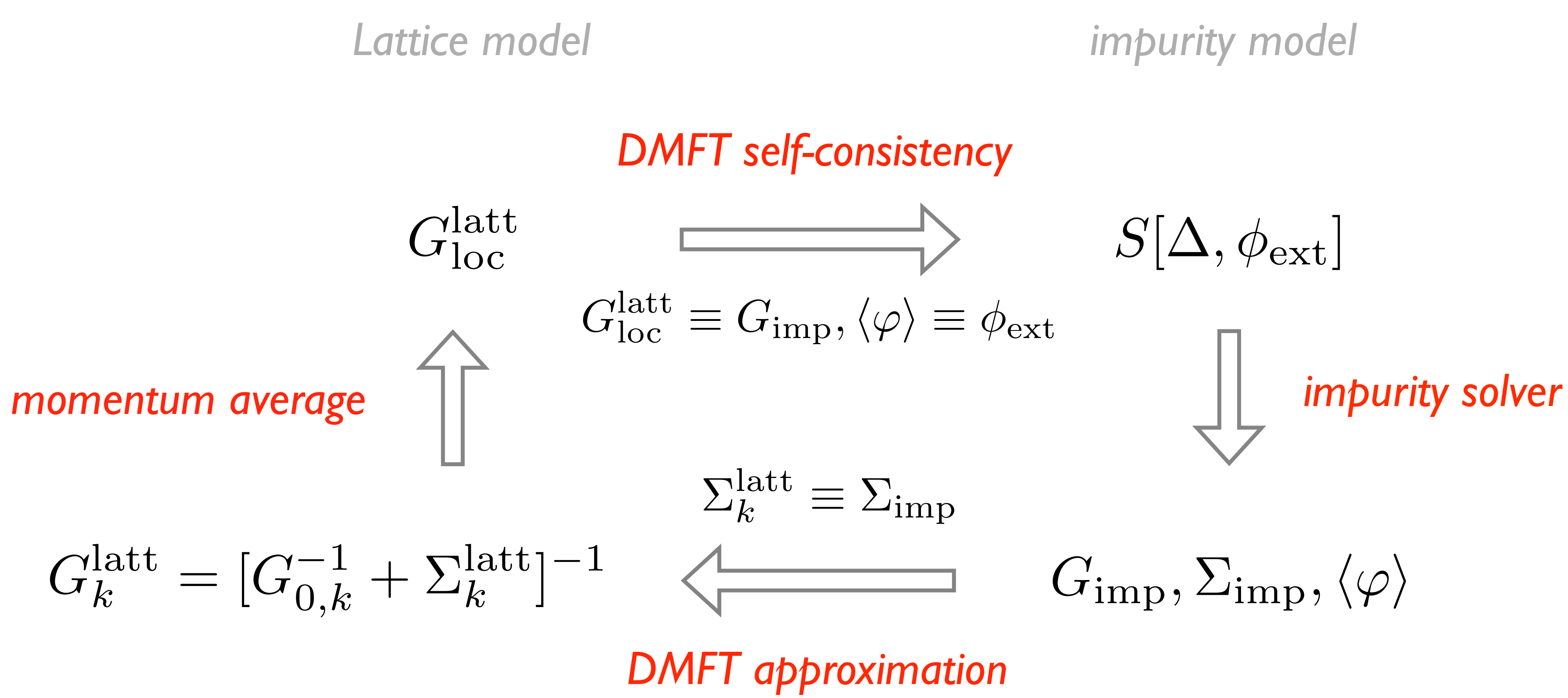
- Dynamical mean field equations:

  Hybridization function Δ and ϕ_{ext}

  are fixed by self-consistency conditions



DMFT for quantum field theories

- Simple example: real, scalar φ^4 quantum field theory

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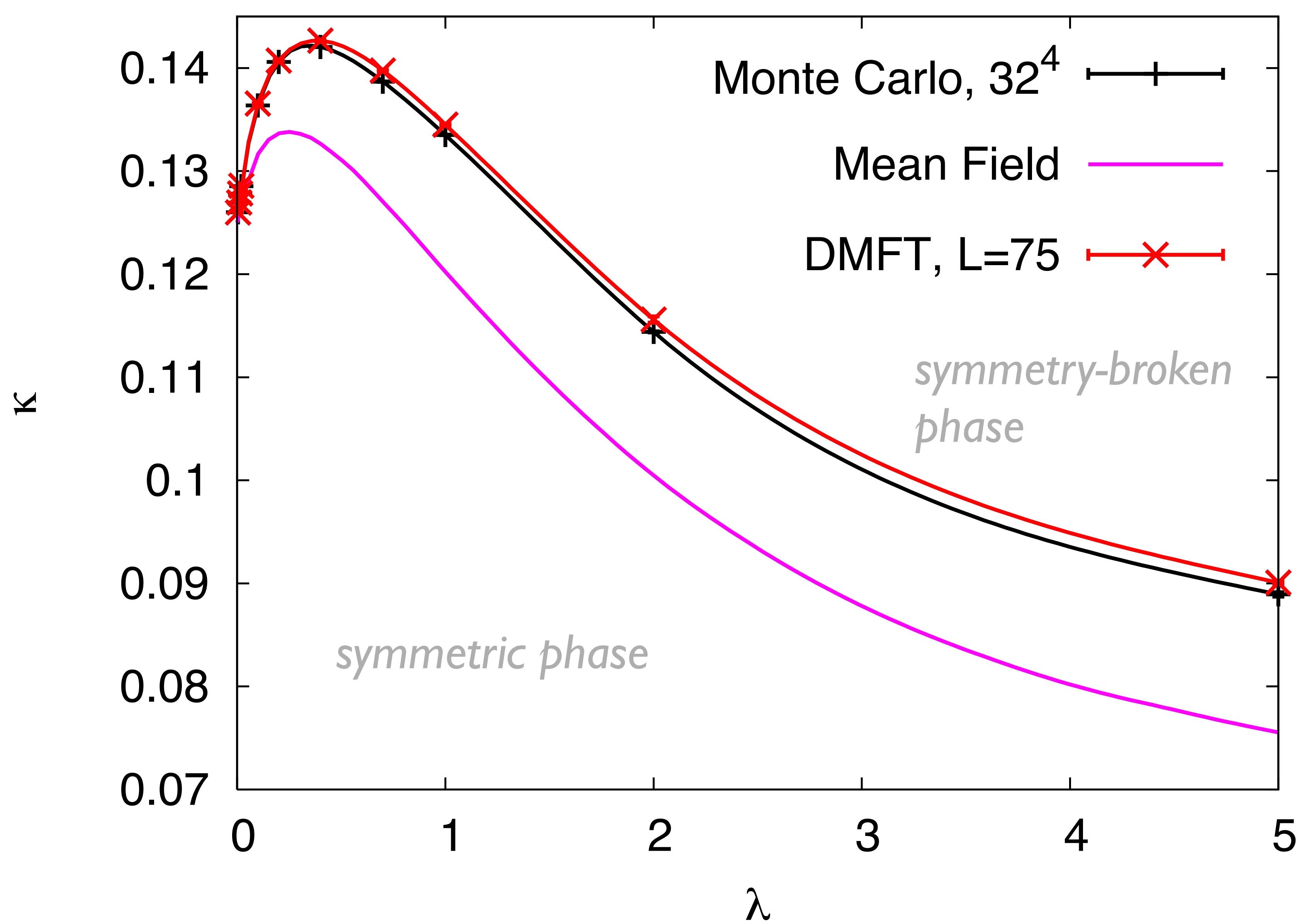
- Dynamical mean field phase diagram (d=3+1):

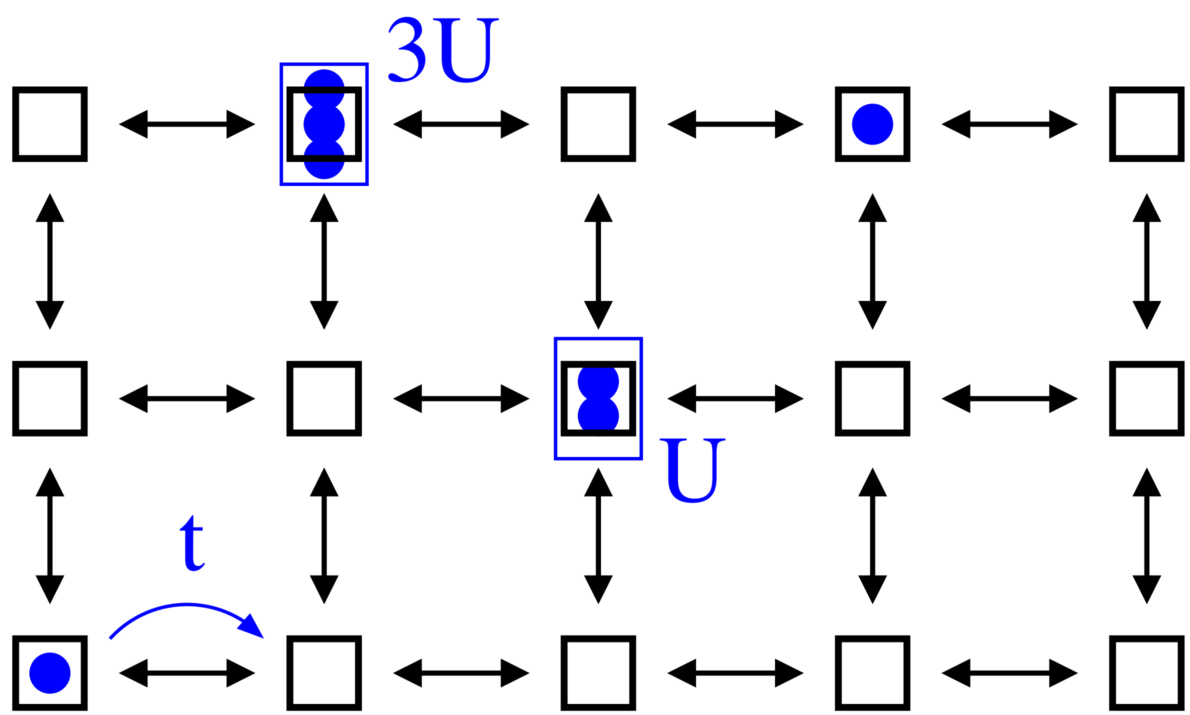


DMFT for Bosons

- Simple example: Bose-Hubbard model

Fisher, Grinstein, Weichmann & Fisher (1989)

$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$



- Mott-Insulator to superfluid transition at low temperature

DMFT for Bosons

- Simple example: Bose-Hubbard model
- Derivation of DMFT formalism analogous to the φ^4 case

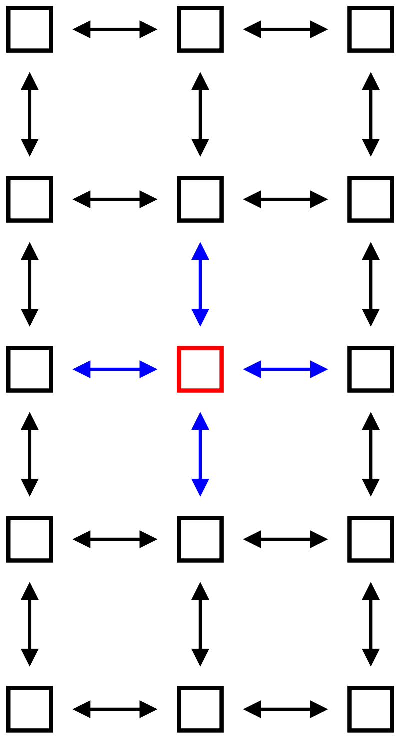
Anders, Pollet, Gull, Troyer & Werner (2011)

- Split action of the lattice model into $S = S_{\text{int}} + \Delta S + S_{\text{ext}}$

$$S_{\text{int}} = \int_0^\beta d\tau \Big[-\mu b_{\text{int}}^\dagger b_{\text{int}} + \frac{U}{2} n_{\text{int}}(n_{\text{int}} - 1) \Big]$$

$$\Delta S = -t \int_0^\beta \sum_{\langle \text{int},\text{ext} \rangle} (b_{\text{int}}^\dagger b_{\text{ext}} + b_{\text{ext}}^\dagger b_{\text{int}})$$

$$S_{\text{ext}} = \int_0^\beta d\tau \Big[-\mu b_{\text{ext}}^\dagger b_{\text{ext}} + \frac{U}{2} n_{\text{ext}}(n_{\text{ext}} - 1) \Big]$$



- To allow for symmetry breaking (condensation), write

$$b_{\text{ext}} = \phi_{\text{ext}} + \delta b_{\text{ext}}, \quad b_{\text{int}} = \phi_{\text{int}} + \delta b_{\text{int}}$$

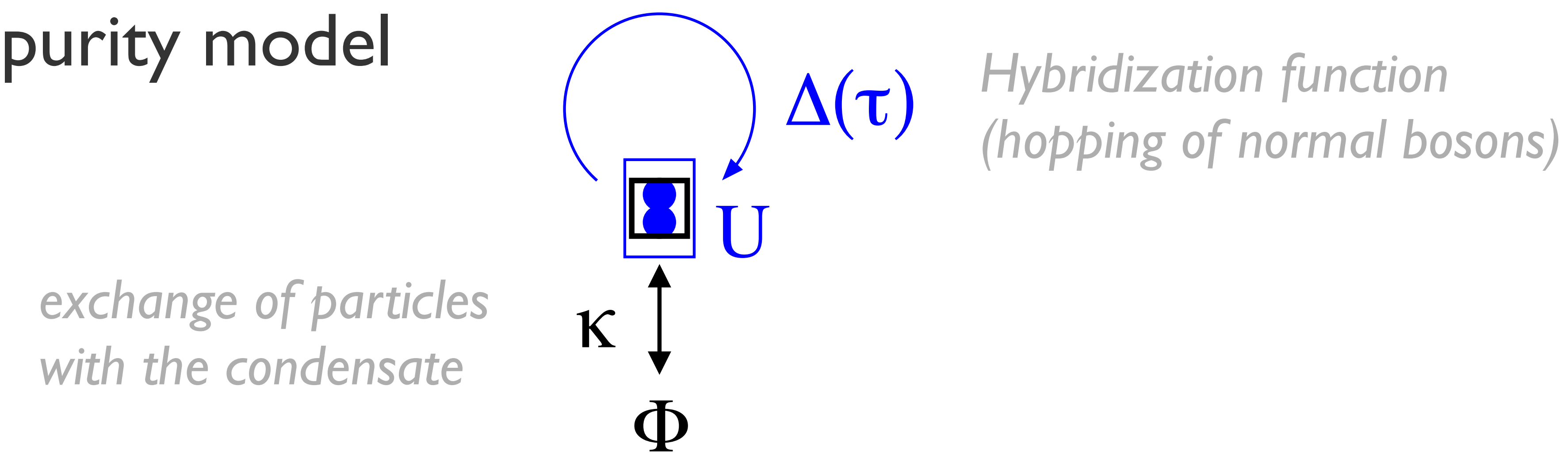
DMFT for Bosons

- Simple example: Bose-Hubbard model
- Derivation of DMFT formalism analogous to the φ^4 case

  Anders, Pollet, Gull, Troyer & Werner (2011)

- Expand $\exp(-\Delta S)$, integrate out external degrees of freedom

- End up with an impurity model



$$S_{\text{imp}} = -\frac{1}{2}\int_0^\beta \mathbf{b}^\dagger(\tau)\mathbf{\Delta}(\tau-\tau')\mathbf{b}(\tau') + \int_0^\beta d\tau\Big[-\mu n(\tau) + \frac{U}{2}n(\tau)[n(\tau)-1]\Big]$$
$$-\kappa\mathbf{\Phi}^\dagger\int_0^\beta d\tau\,\mathbf{b}(\tau)$$

DMFT for Bosons

- Simple example: Bose-Hubbard model
- Derivation of DMFT formalism analogous to the φ^4 case

  Anders, Pollet, Gull, Troyer & Werner (2011)

- Expand $\exp(-\Delta S)$, integrate out external degrees of freedom

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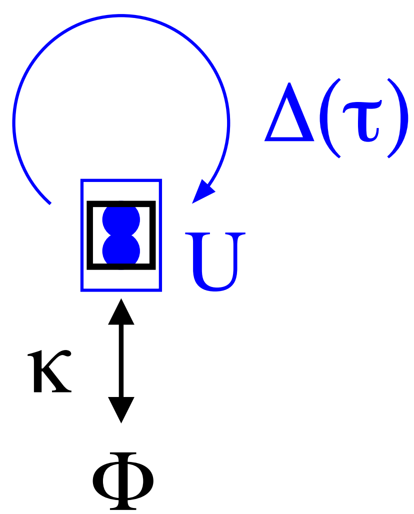


Hybridization function (hopping of normal bosons)

exchange of particles with the condensate

Nambu notation

- $\boldsymbol{\Delta}$ and $\boldsymbol{\Phi}$ fixed by the DMFT self-consistency condition

  approximation: identification of lattice and impurity self-energy

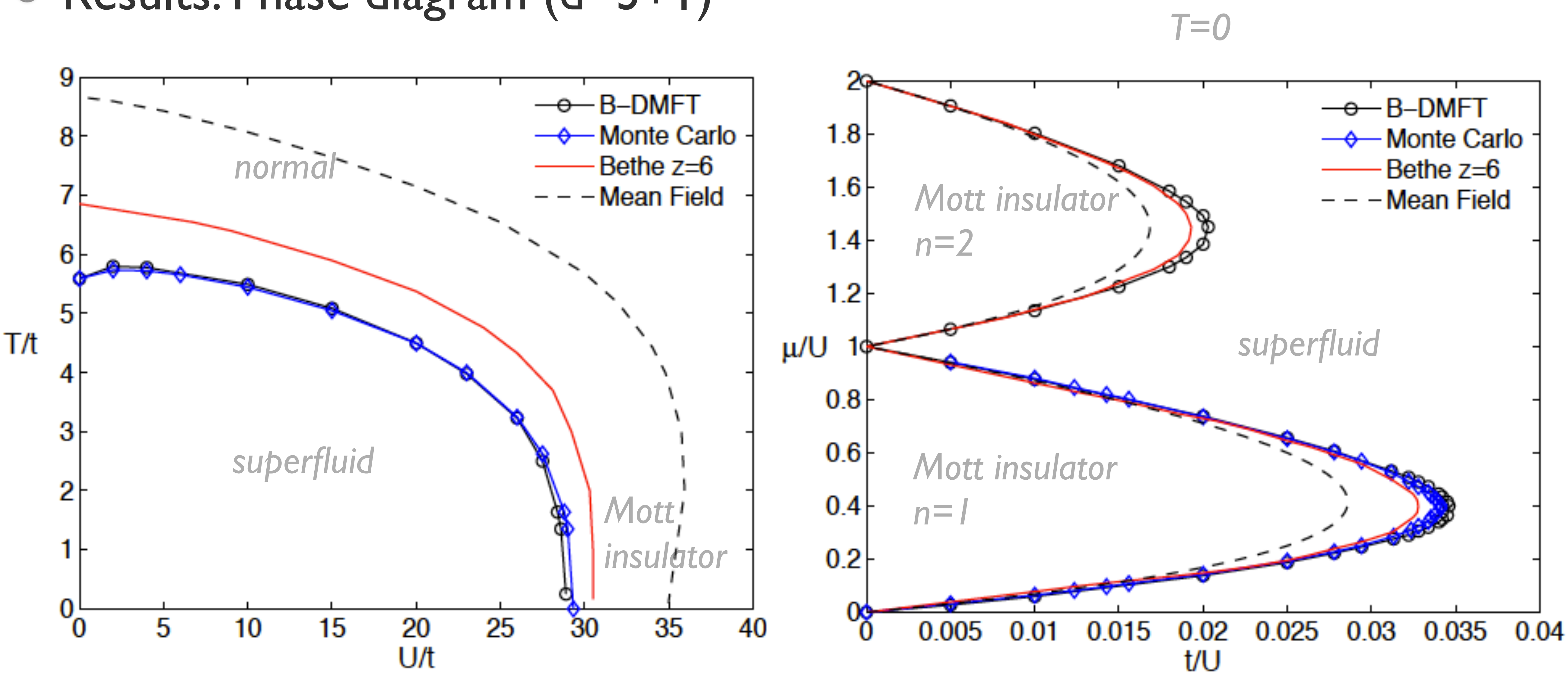
$$\mathbf{\Phi} = \langle \mathbf{b}(\tau) \rangle_{S_{\rm imp}}, \quad \mathbf{G}^{\rm latt}_{\rm loc} = \mathbf{G}_{\rm imp,c} \quad [= -\langle T\mathbf{b}(\tau)\mathbf{b}^\dagger(0)\rangle_{S_{\rm imp}} + \mathbf{\Phi}\mathbf{\Phi}^\dagger]$$

DMFT for Bosons

- Simple example: Bose-Hubbard model
- Derivation of DMFT formalism analogous to the φ^4 case

  Anders, Pollet, Gull, Troyer & Werner (2011)

- Results: Phase diagram (d=3+1)

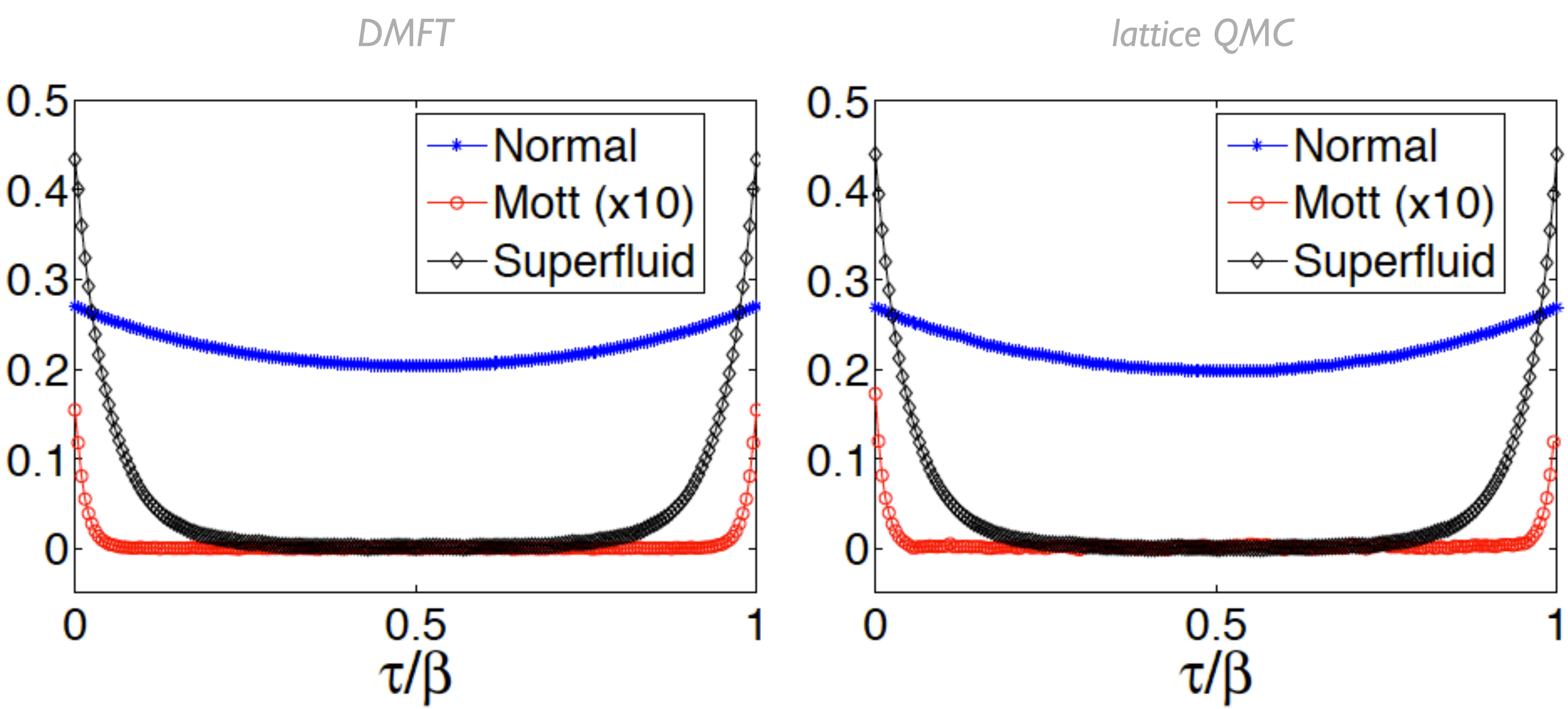


DMFT for Bosons

- Simple example: Bose-Hubbard model
- Derivation of DMFT formalism analogous to the φ^4 case

  Anders, Pollet, Gull, Troyer & Werner (2011)

- Results: Connected density-density correlation functions (d=3+1)

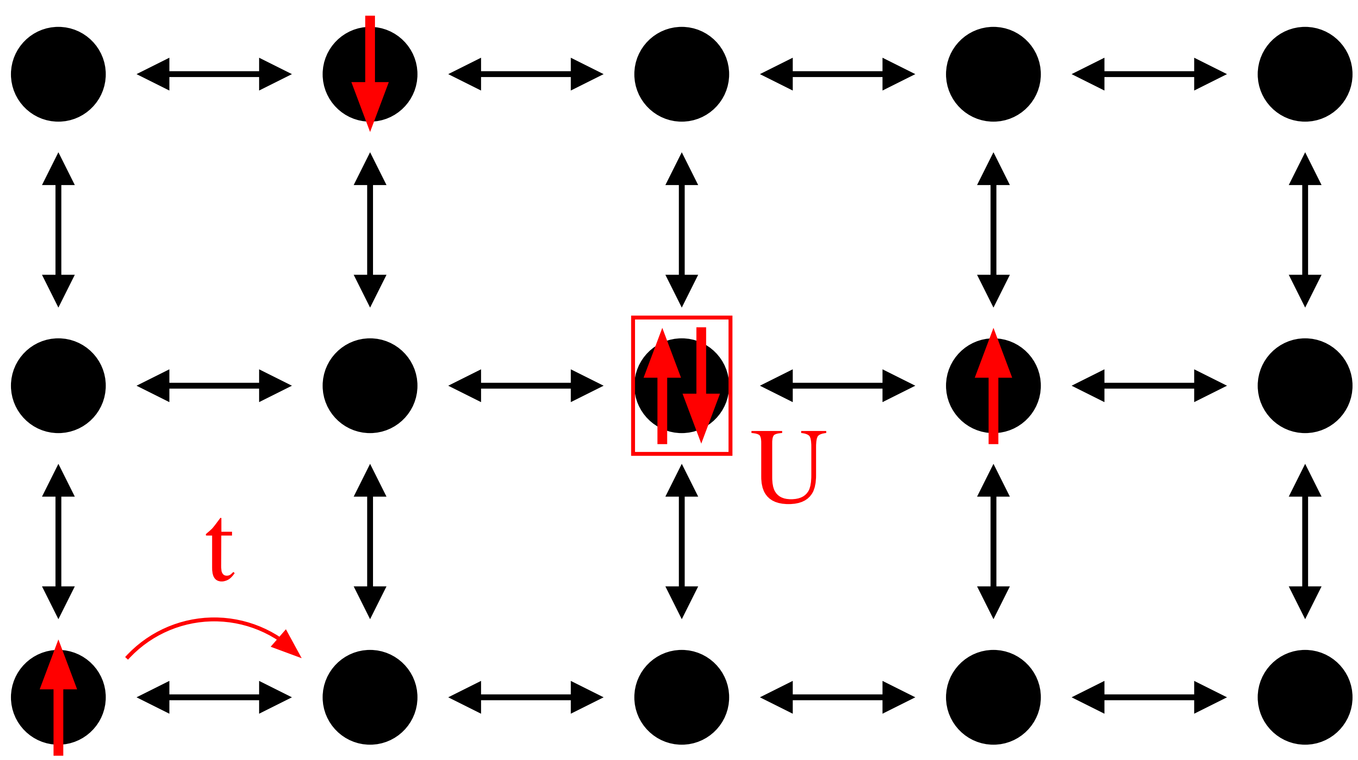


DMFT for Fermions

- Simple example: single-band Hubbard model

  Gutzwiller, Kanamori, Hubbard (1963)

$$H = -t \sum_{\langle i,j \rangle, \sigma} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow} - \mu \sum_{i,\sigma} n_{i,\sigma}$$



- Describes Mott transition, magnetic and superconducting transitions

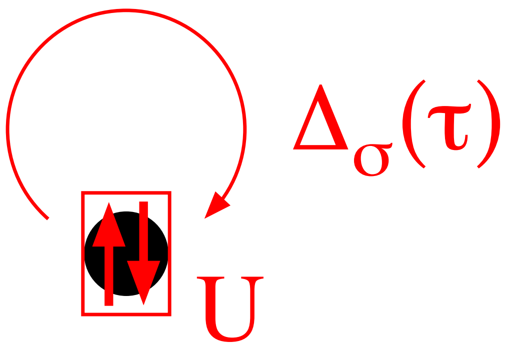
DMFT for Fermions

- Simple example: single-band Hubbard model

  Gutzwiller, Kanamori, Hubbard (1963)

- Dynamical mean field theory: Mapping to a quantum impurity model

  Georges & Kotliar (1992)



Hybridization function (describes hopping of electrons into the lattice and back)

- Hybridization functions Δ_σ fixed by DMFT self-consistency condition

  approximation: identification of lattice and impurity self-energy

$$G^{\mathrm{latt}}_{\mathrm{loc},\sigma} = G_{\mathrm{imp},\sigma}$$

DMFT for Fermions

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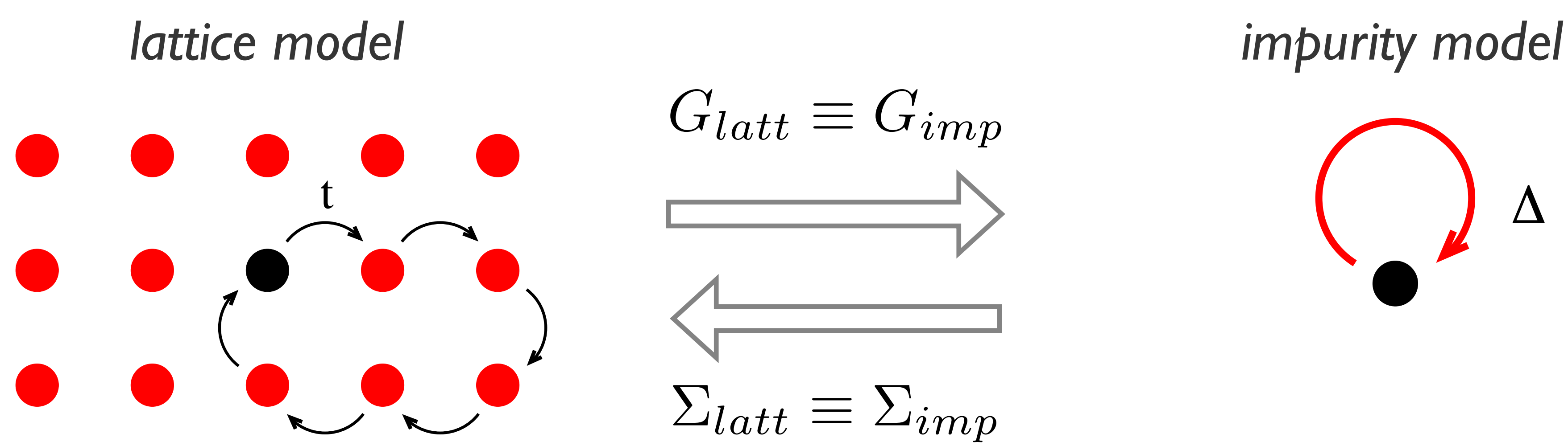

- Various numerical approaches to solve the impurity problem:

  weak-coupling perturbation theory, strong-coupling perturbation theory, exact diagonalization, NRG, QMC, ...

DMFT for Fermions

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  Gutzwiller, Kanamori, Hubbard (1963)

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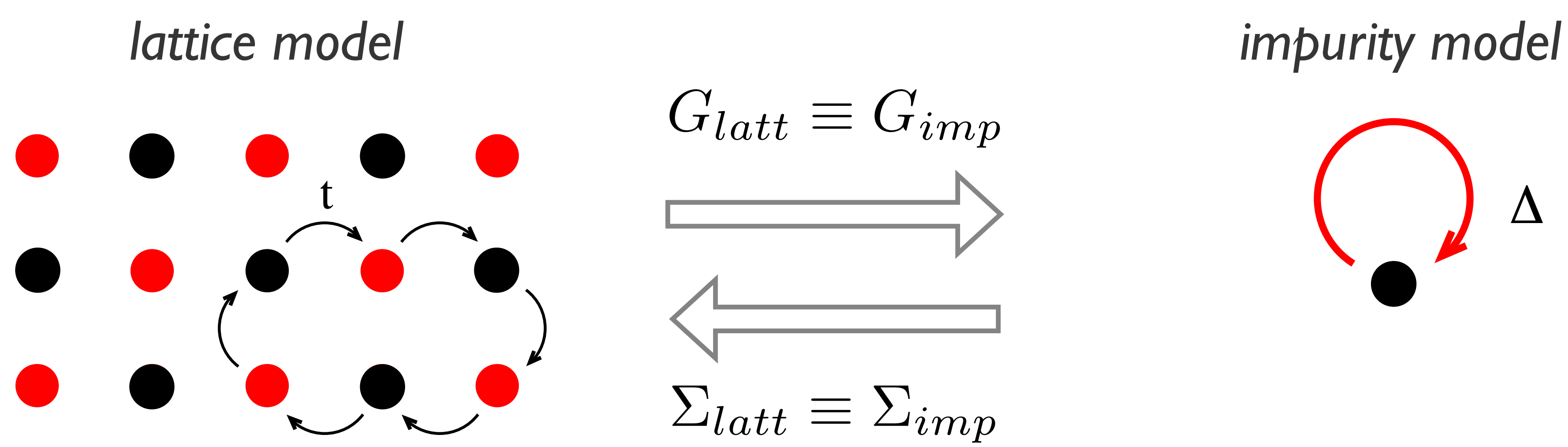


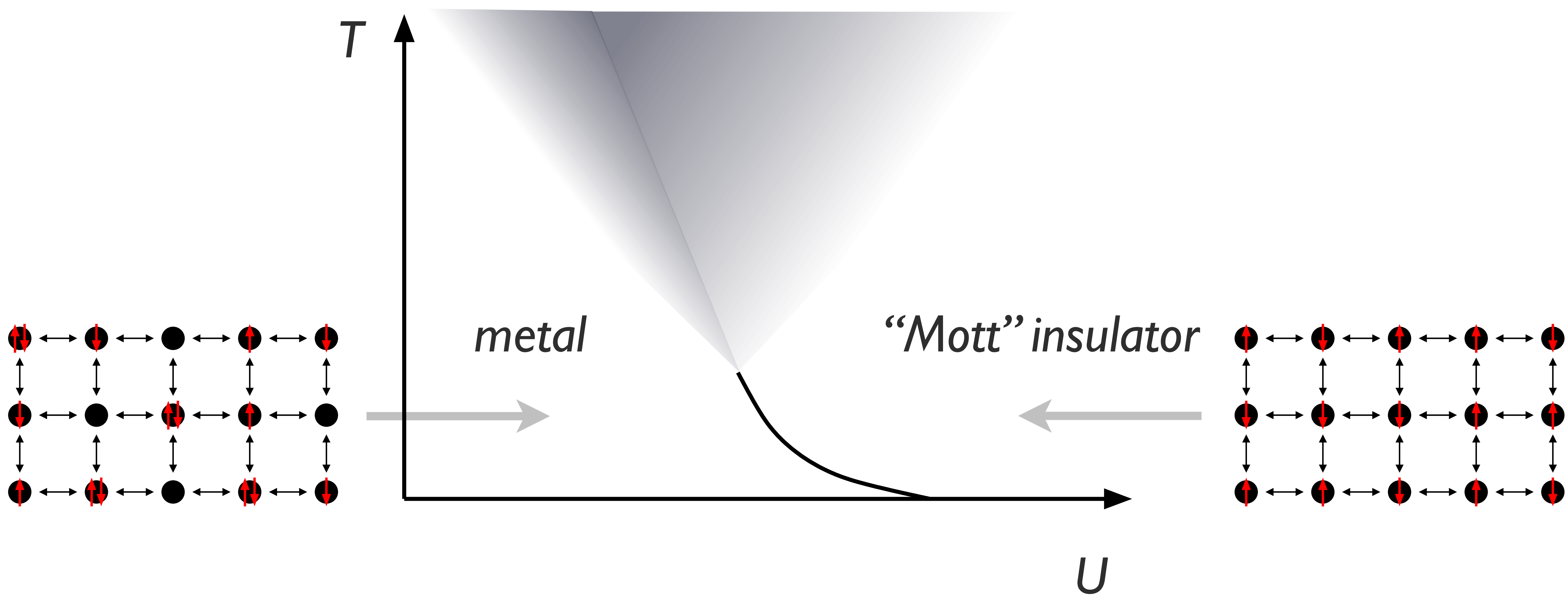
- Single-site DMFT can treat two-sublattice order (e. g. AFM)

$$\mathrm{Bath}_{B,\sigma}[G_{A,\sigma}], \quad \mathrm{Bath}_{A,\sigma}[G_{B,\sigma}]$$

- Pure Neel order: $\mathrm{Bath}_{B,\sigma} = \mathrm{Bath}_{A,\bar{\sigma}}$

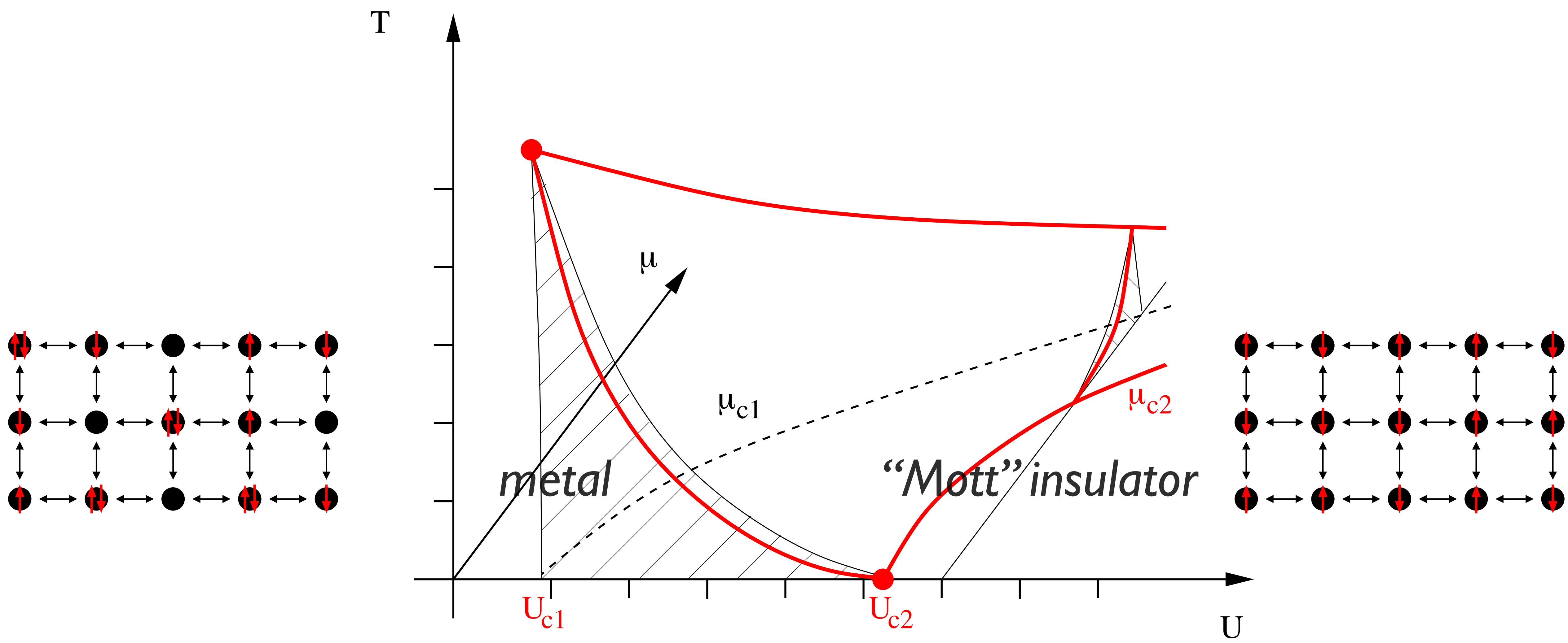
DMFT for Fermions

- Equilibrium DMFT phase diagram (half-filling)
- Paramagnetic calculation: Metal - Mott insulator transition at low *T*
- Smooth crossover at high *T*



DMFT for Fermions

- Equilibrium DMFT phase diagram
- Paramagnetic calculation: Metal - Mott insulator transition at low *T*
- Doping-driven metallization upon increasing μ



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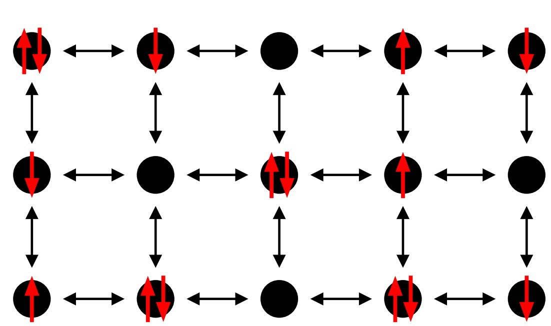
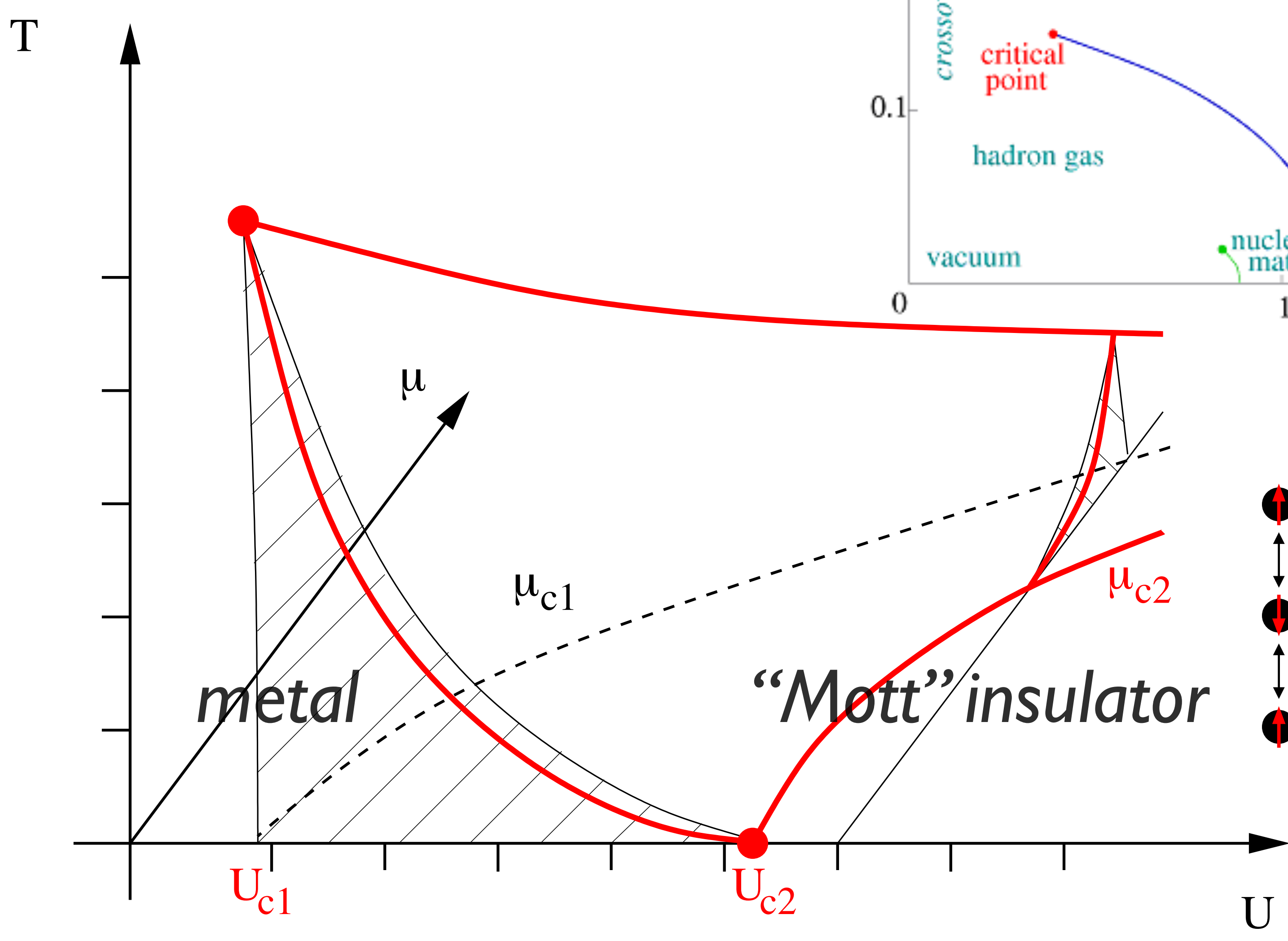
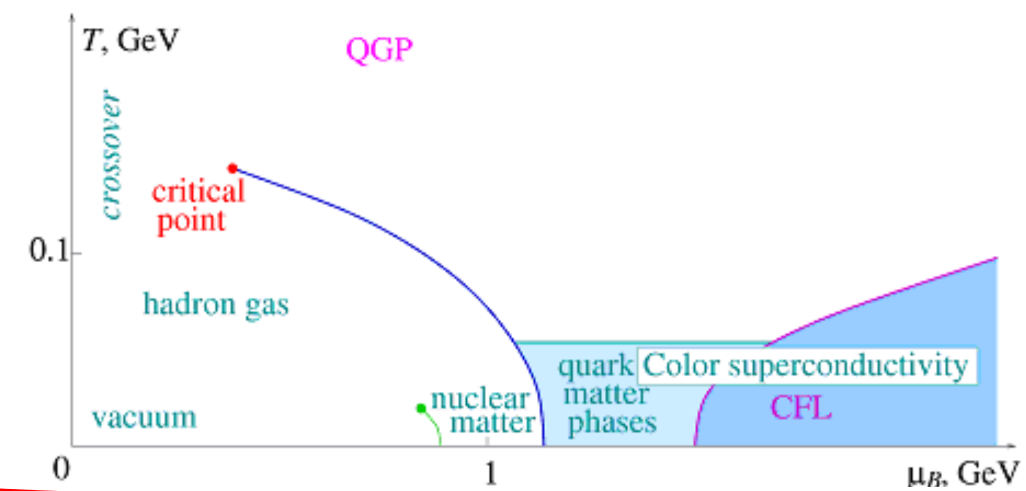
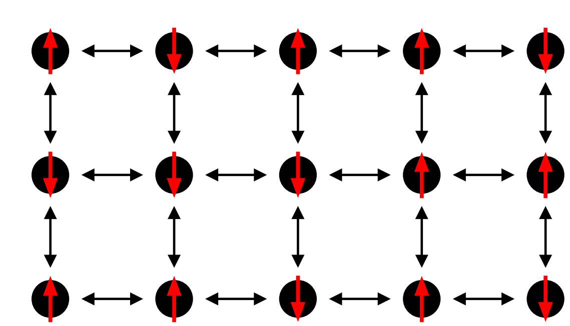


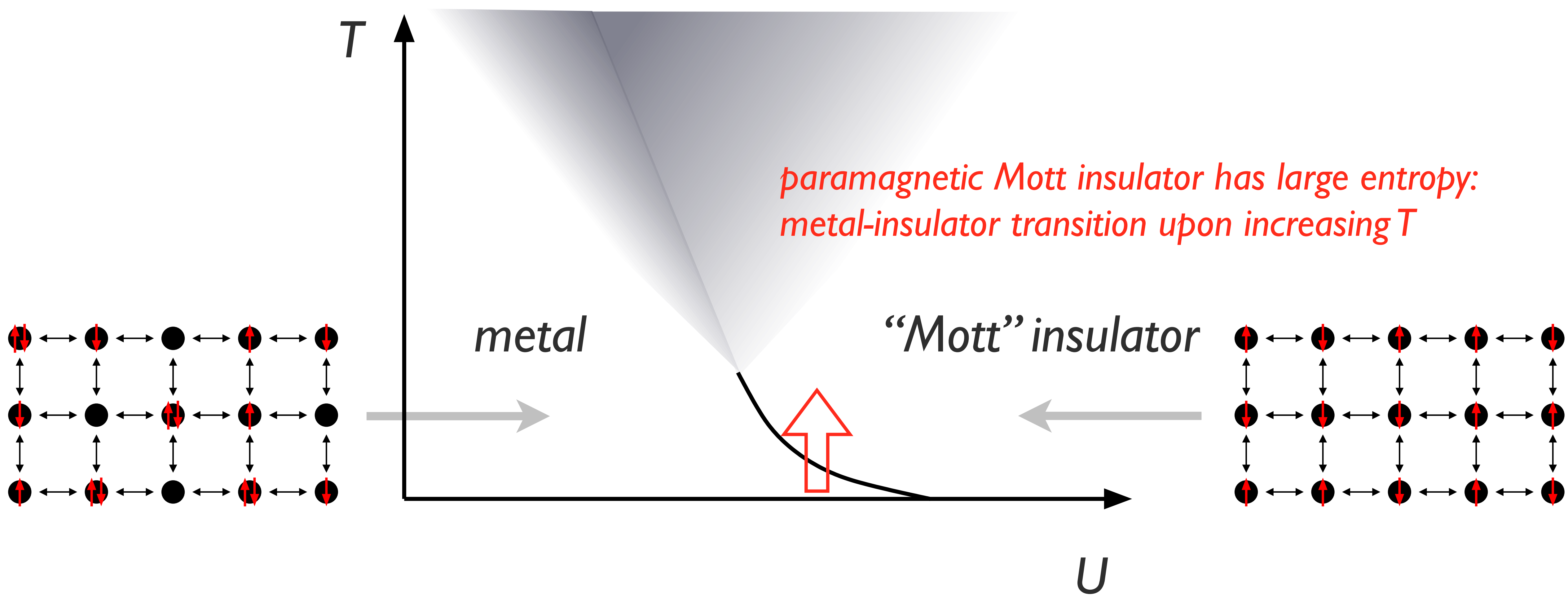






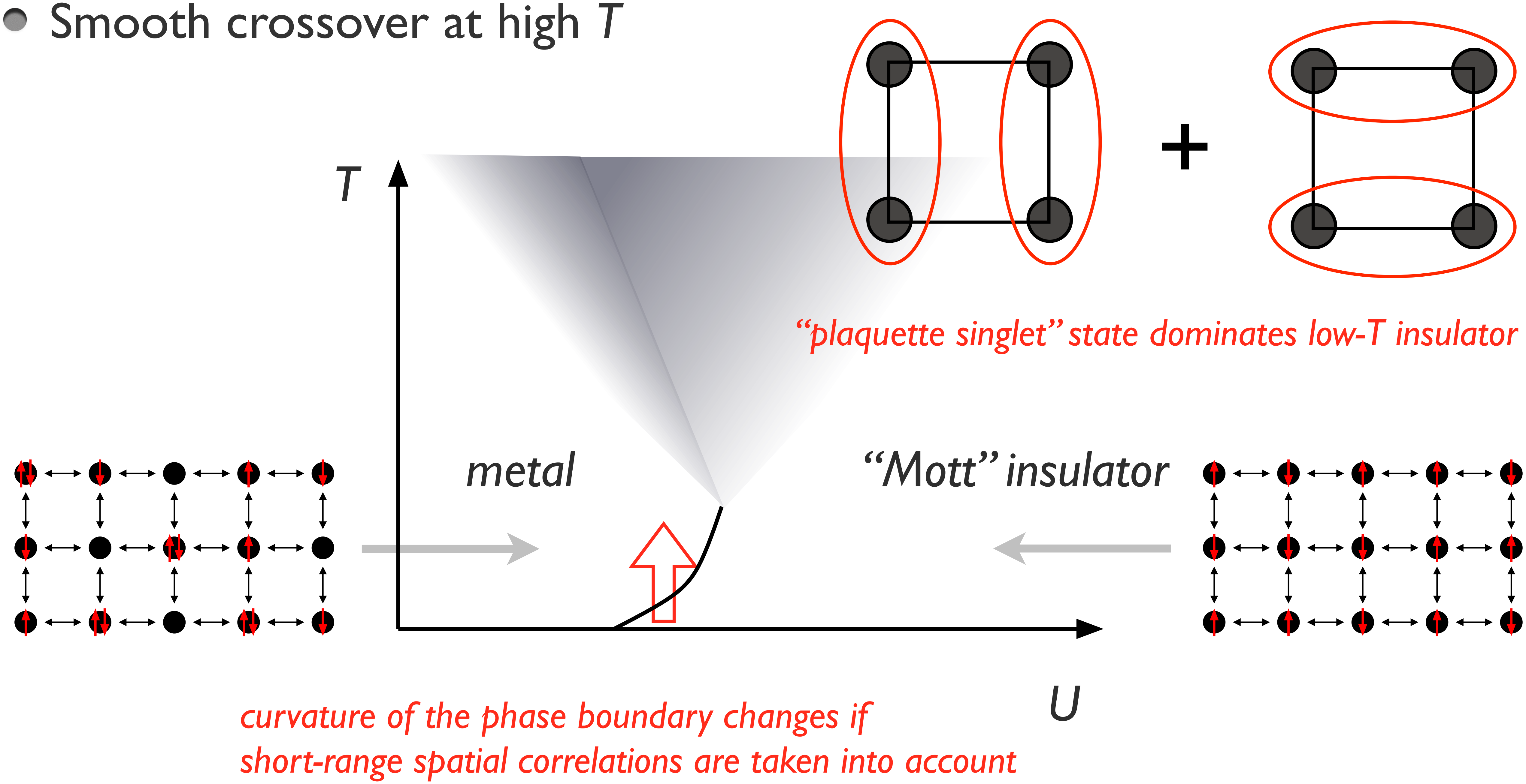
DMFT for Fermions

- Equilibrium DMFT phase diagram (half-filling)
- Paramagnetic calculation: Metal - Mott insulator transition at low *T*
- Smooth crossover at high *T*



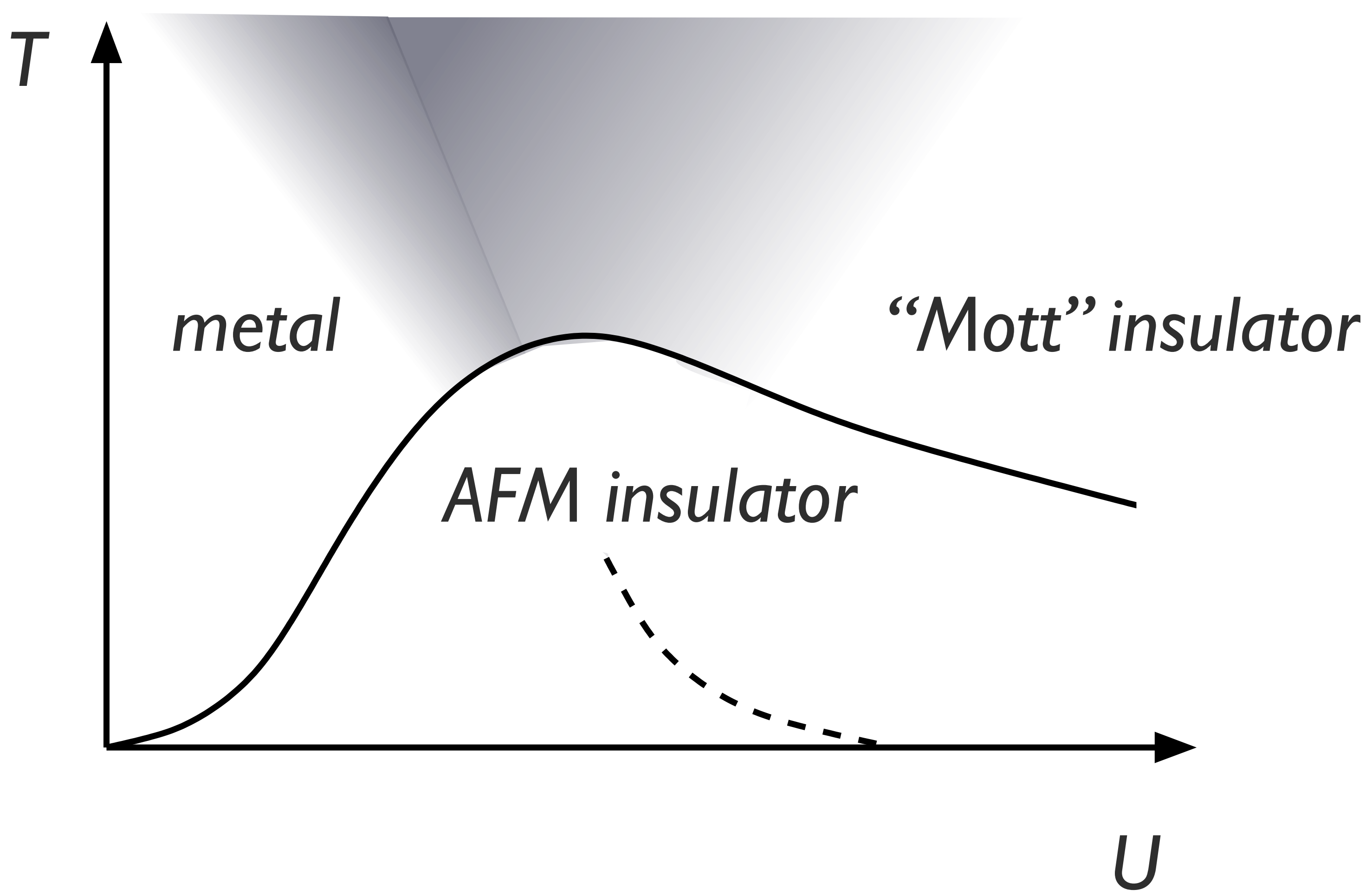
DMFT for Fermions

- Equilibrium DMFT phase diagram (half-filling)
- Paramagnetic calculation: Metal - Mott insulator transition at low T
- Smooth crossover at high T



DMFT for Fermions

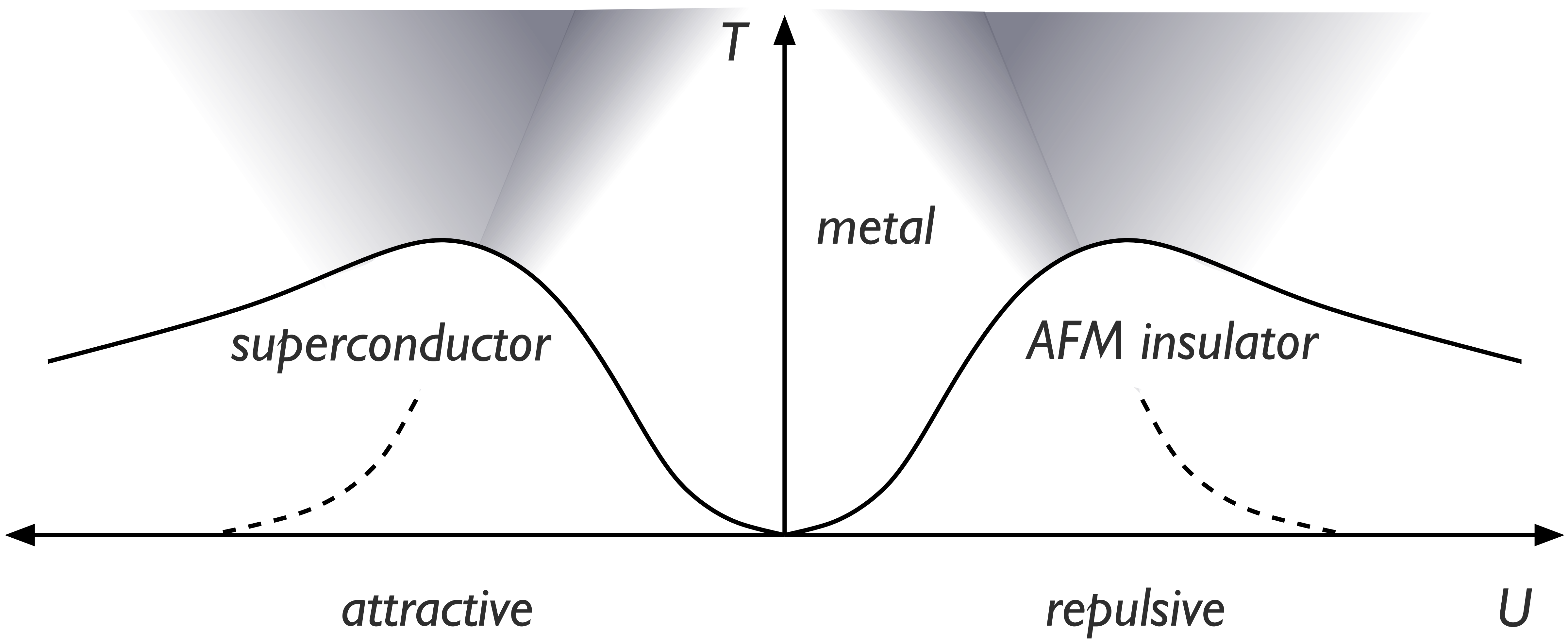
- Equilibrium DMFT phase diagram (half-filling)
- With 2-sublattice order: Antiferromagnetic insulator at low T
- Smooth crossover at high T



DMFT for Fermions

- Equilibrium DMFT phase diagram (half-filling)
- Transformation $c_{i\uparrow} \to c_{i\uparrow}^\dagger \quad (i \in A), \quad c_{i\uparrow} \to -c_{i\uparrow}^\dagger \quad (i \in B)$

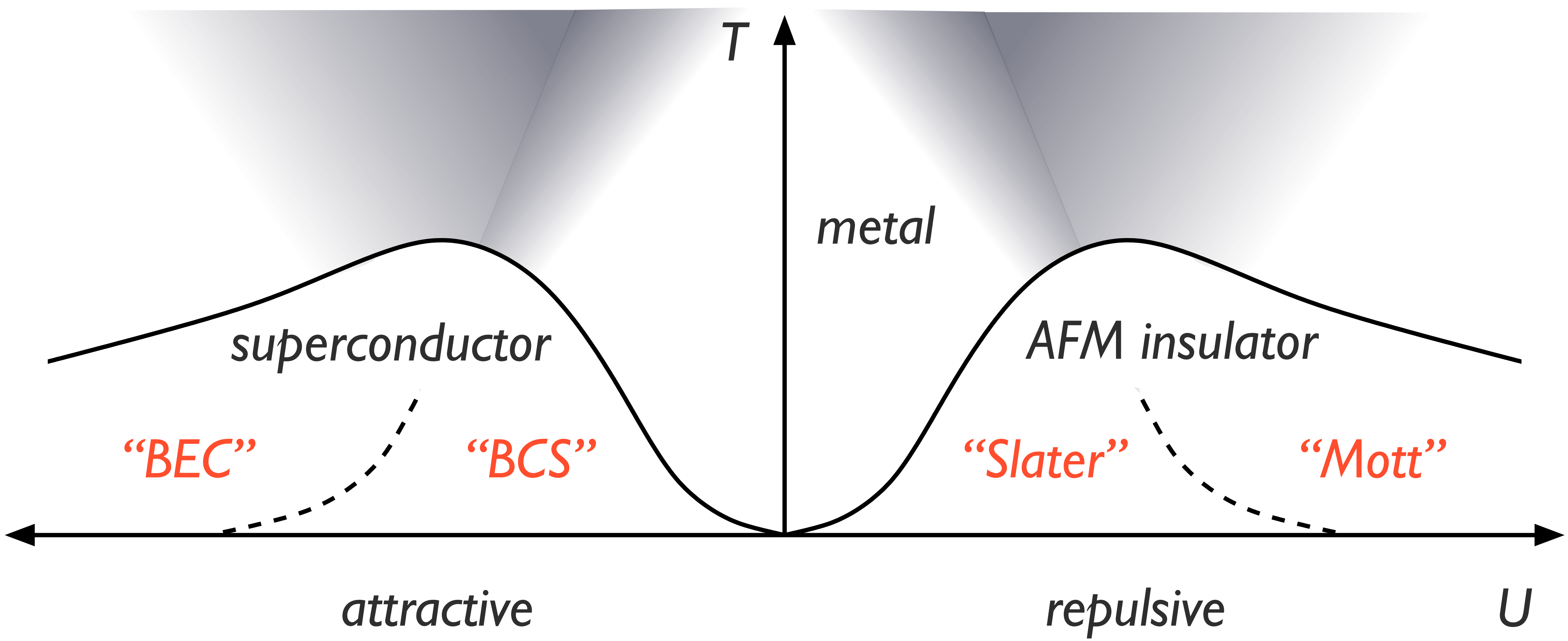
  maps repulsive model onto attractive model



DMFT for Fermions

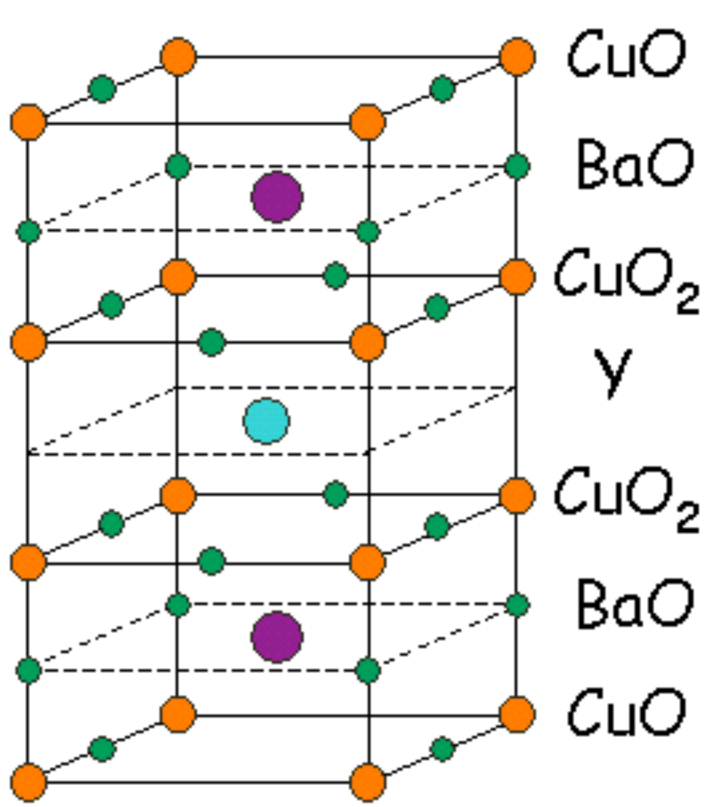
- Equilibrium DMFT phase diagram (half-filling)
- Half-filling: transformation $c_{i\uparrow} \to c_{i\uparrow}^\dagger \quad (i \in A), \quad c_{i\uparrow} \to -c_{i\uparrow}^\dagger \quad (i \in B)$

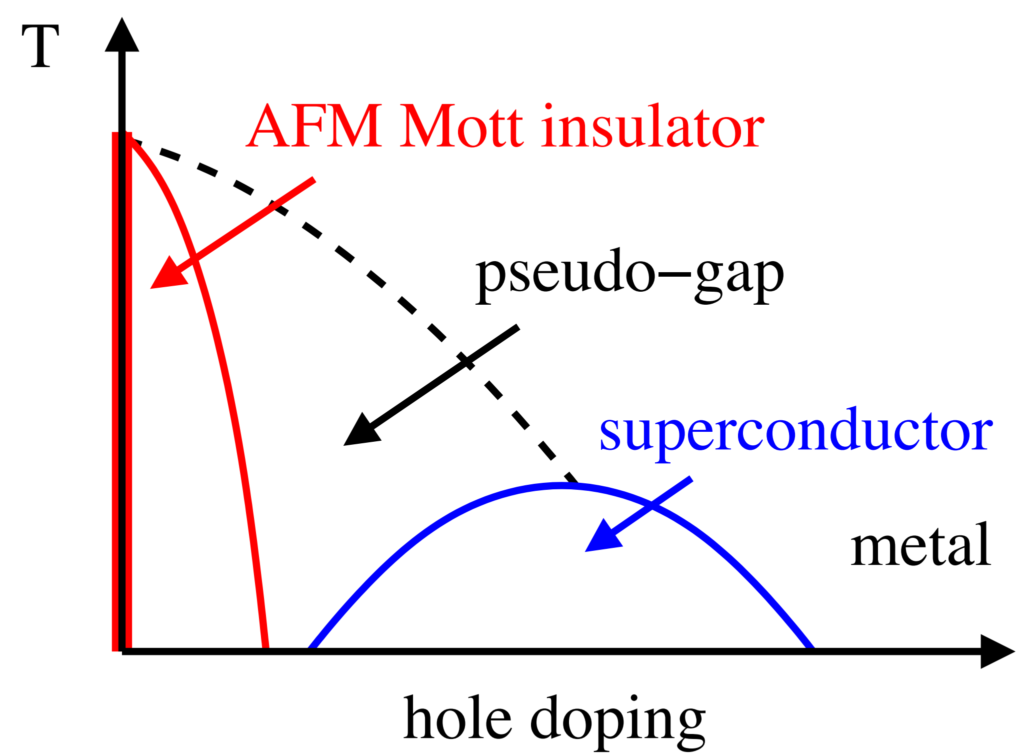
  maps repulsive model onto attractive model

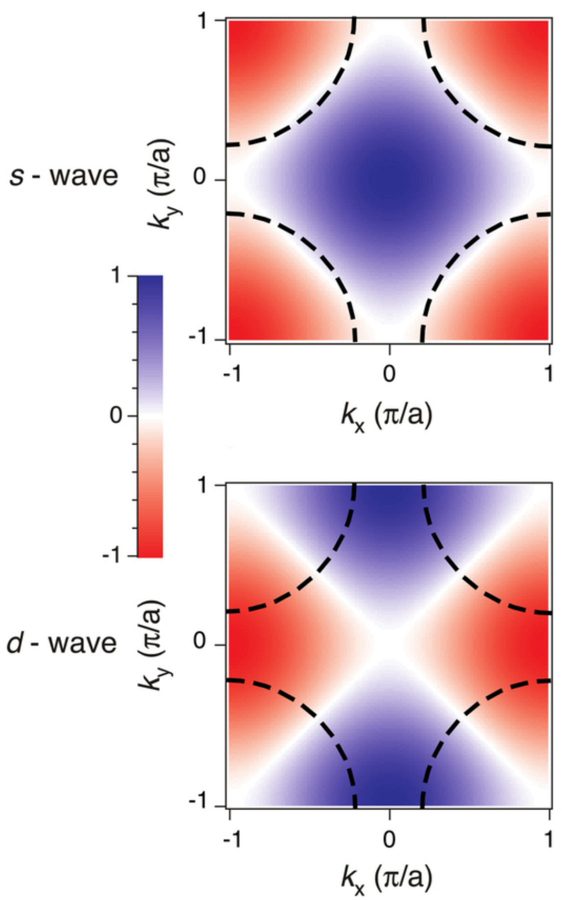


DMFT for Fermions

- Low-dimensional systems
- DMFT is exact in $d = \infty$ *Metzner & Vollhardt (1989)*
- Neglect of spatial fluctuations problematic in $d < 3$
- $d = 2$ Hubbard model is believed to describe the physics of high-*Tc* (cuprate) superconductors







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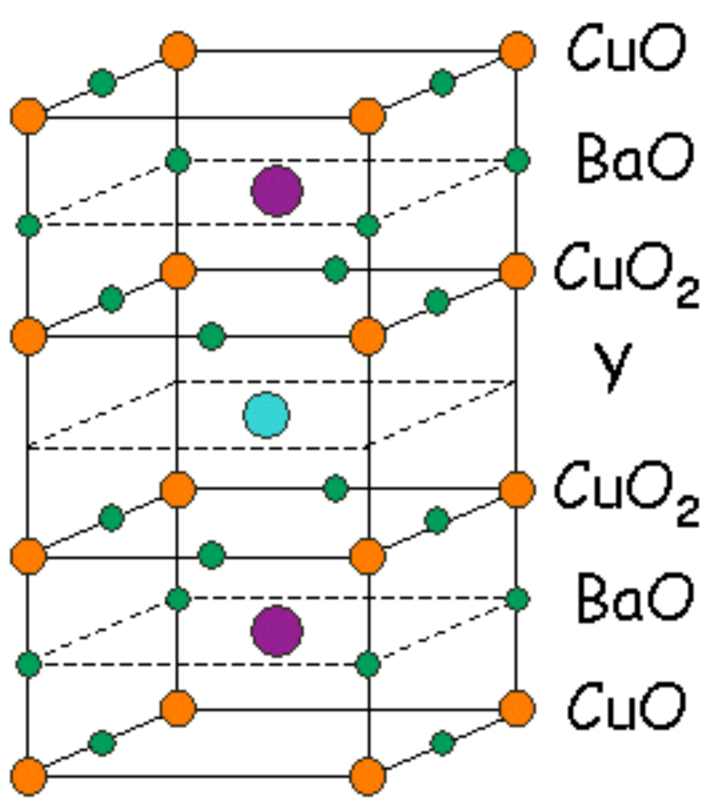
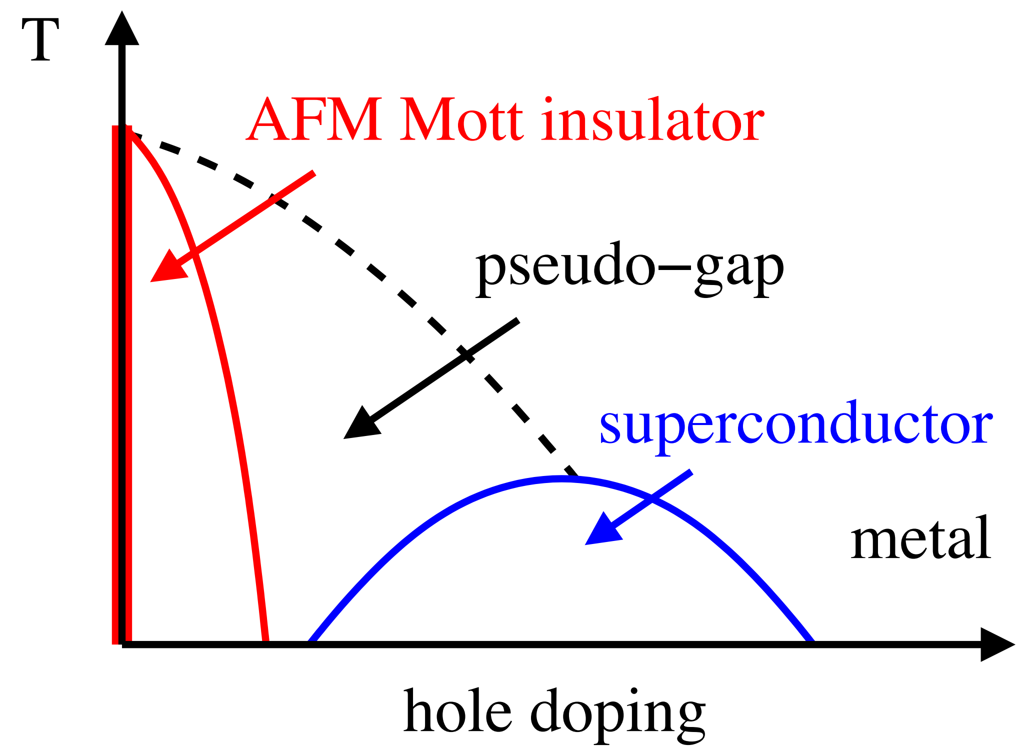
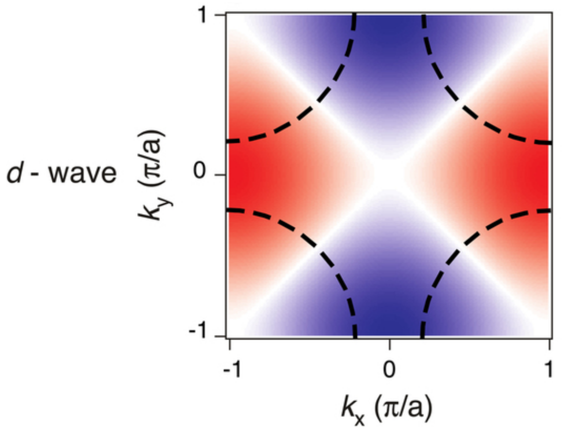


to describe the physics of the cuprates, we need a cluster extension of DMFT, with at least 4 sites



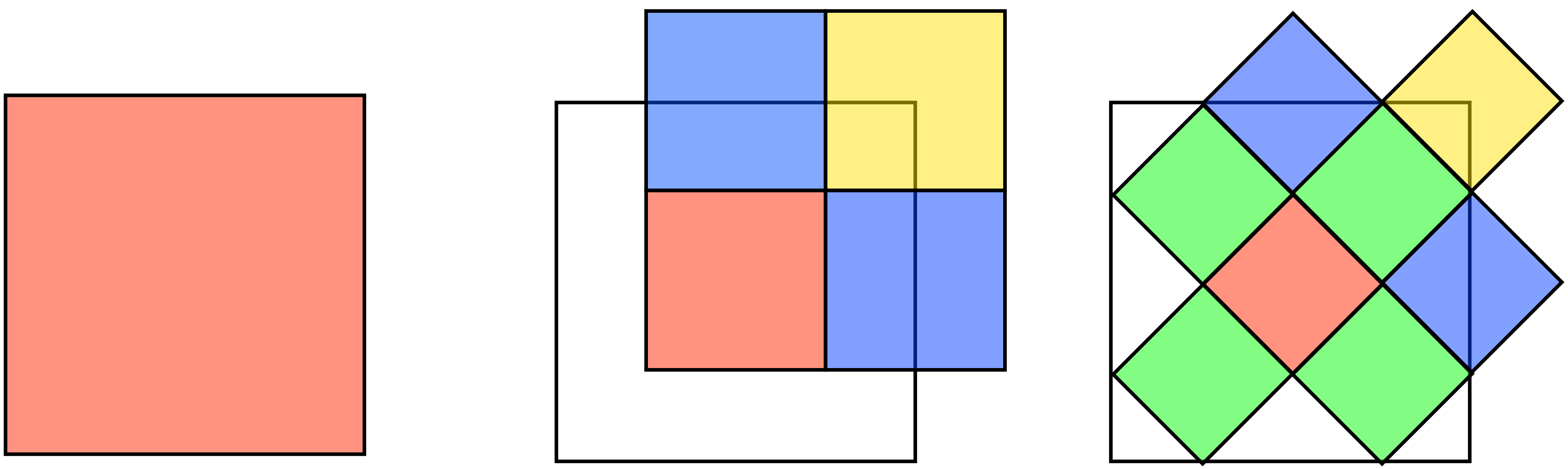


DMFT for Fermions

- Low-dimensional systems
- Cluster DMFT self-consistently embeds a cluster of N_c sites into a fermionic bath *Hettler, Prushke, Krishnamurthy & Jarrell (1998)*
- If cluster is periodized: coarse-graining of the momentum-dependence

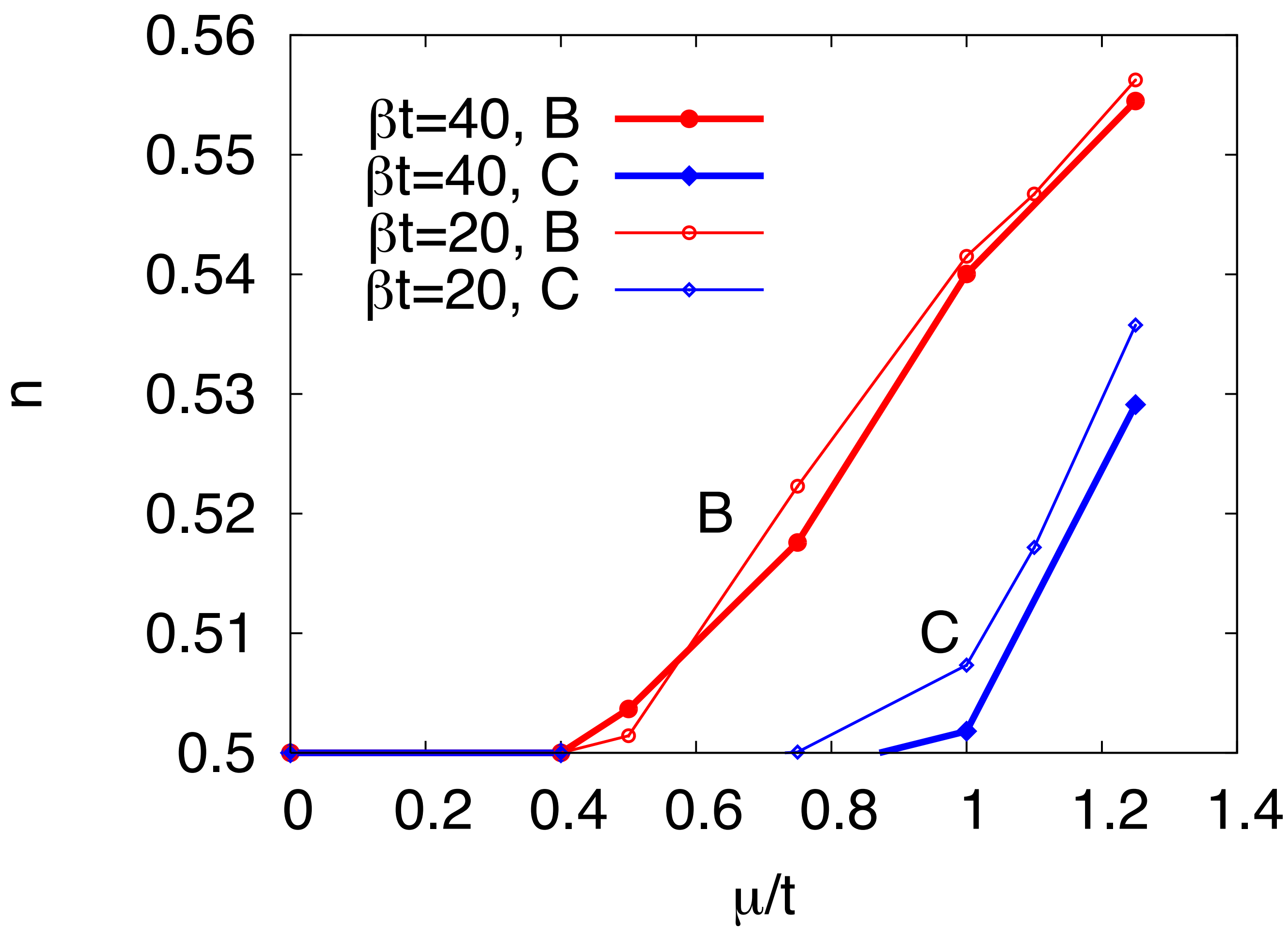
$$\Sigma(p,\omega) = \sum_a \phi_a(p)\Sigma_a(\omega)$$

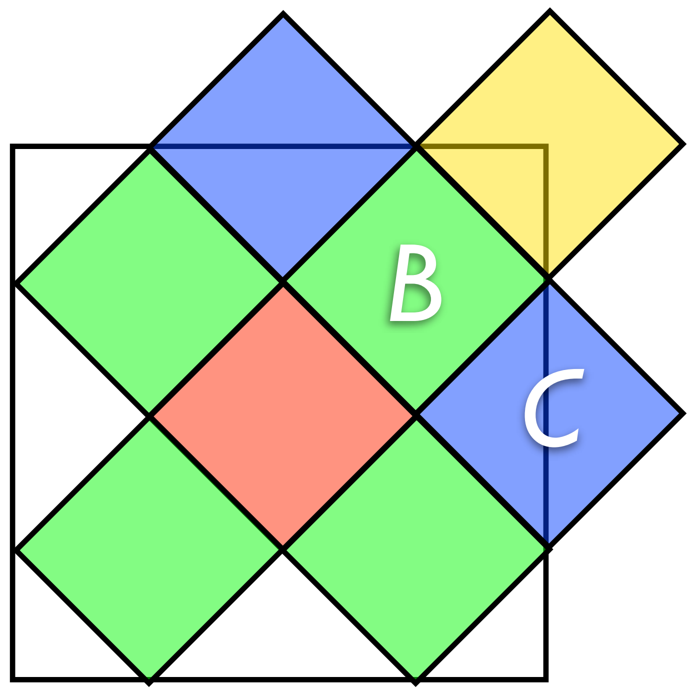
- "Tiling" of the Brillouin zone



DMFT for Fermions

- Low-dimensional systems
- Cluster DMFT self-consistently embeds a cluster of N_c sites into a fermionic bath *Hettler, Prushke, Krishnamurthy & Jarrell (1998)*
- Doping driven insulator-metal transition in the 8-site cluster DMFT
  - first 8% of dopants go into the *B* sector *Werner, Gull, Parcollet & Millis (2010)*





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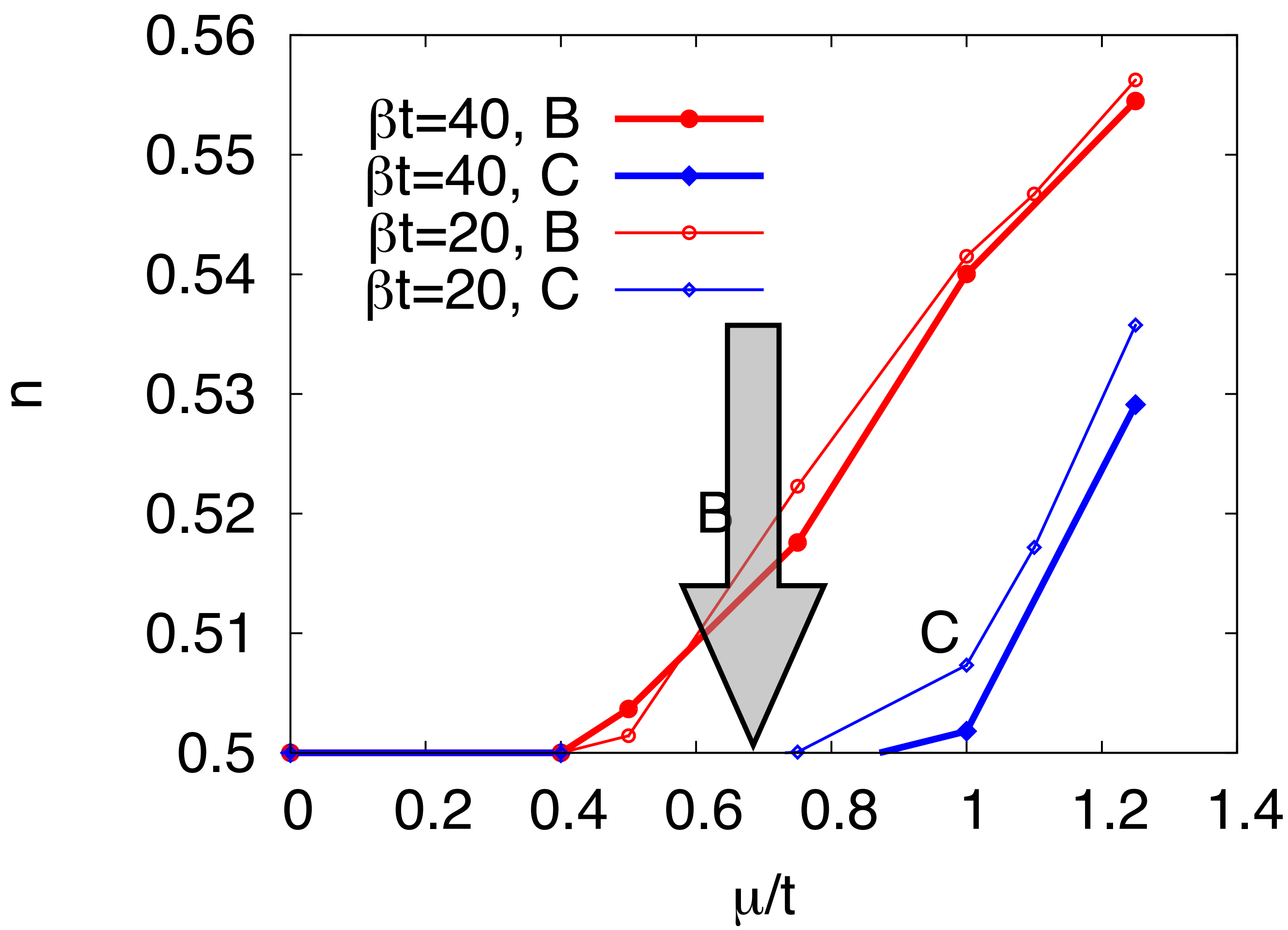
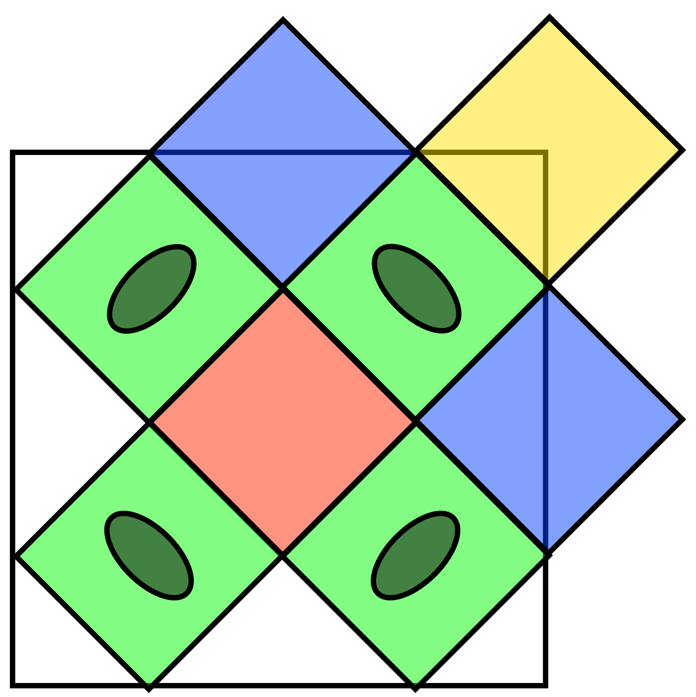


Pseudogap phase as a momentum-selective Mott state



Nonequilibrium DMFT

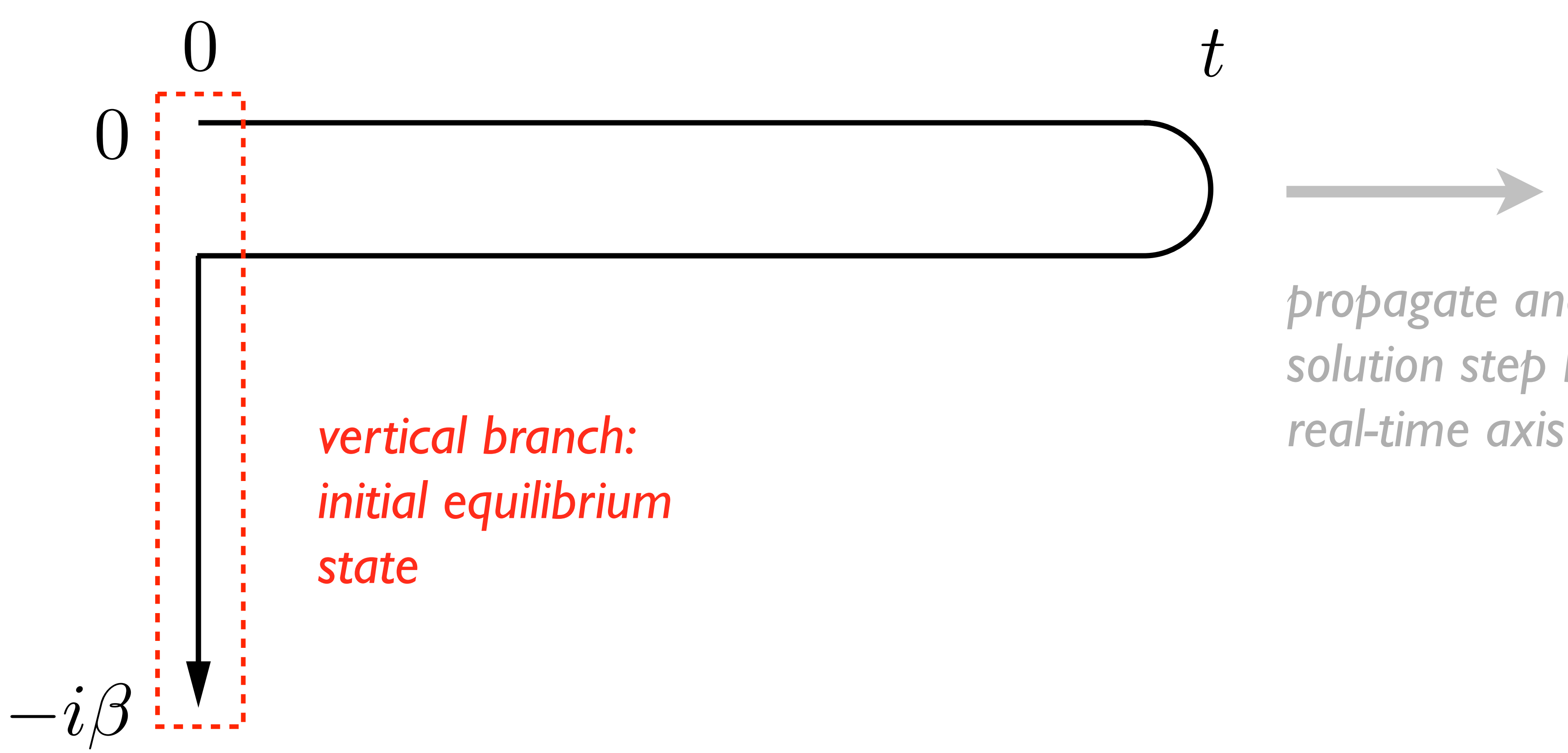
- DMFT accurately treats time-dependent fluctuations

  → can be directly applied to nonequilibrium systems *Freericks et al. (2006)*

- *Equilibrium:* solve DMFT equations on the imaginary-time interval

  Time-translation invariance: can use Fourier transformation

- *Nonequilibrium:* solve DMFT equations on the Kadanoff-Baym contour *Integral-differential equations of Volterra type*

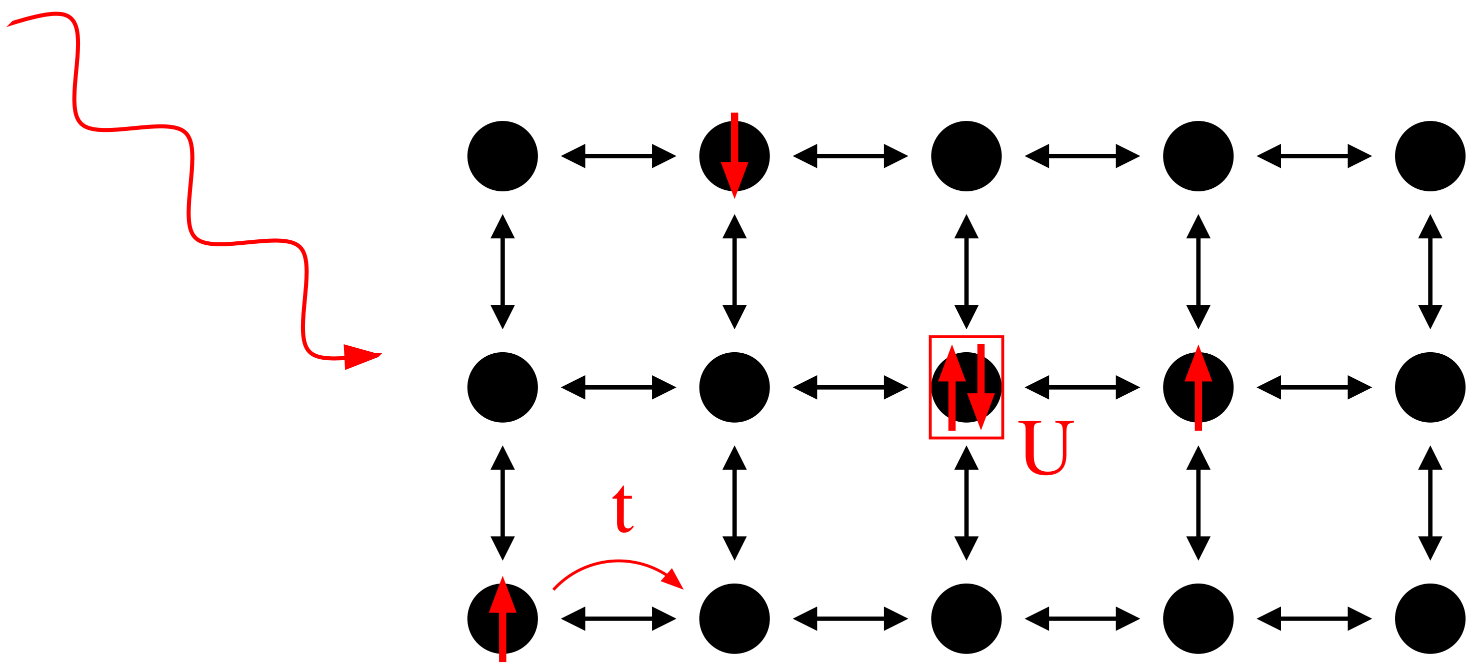


propagate and converge the DMFT solution step by step along the real-time axis

Nonequilibrium DMFT

- Field $E(t)$ applied at t=0
- Choose gauge with pure vector potential: $E(t) = -\partial_t A(t)$

Freericks et al. (2006)



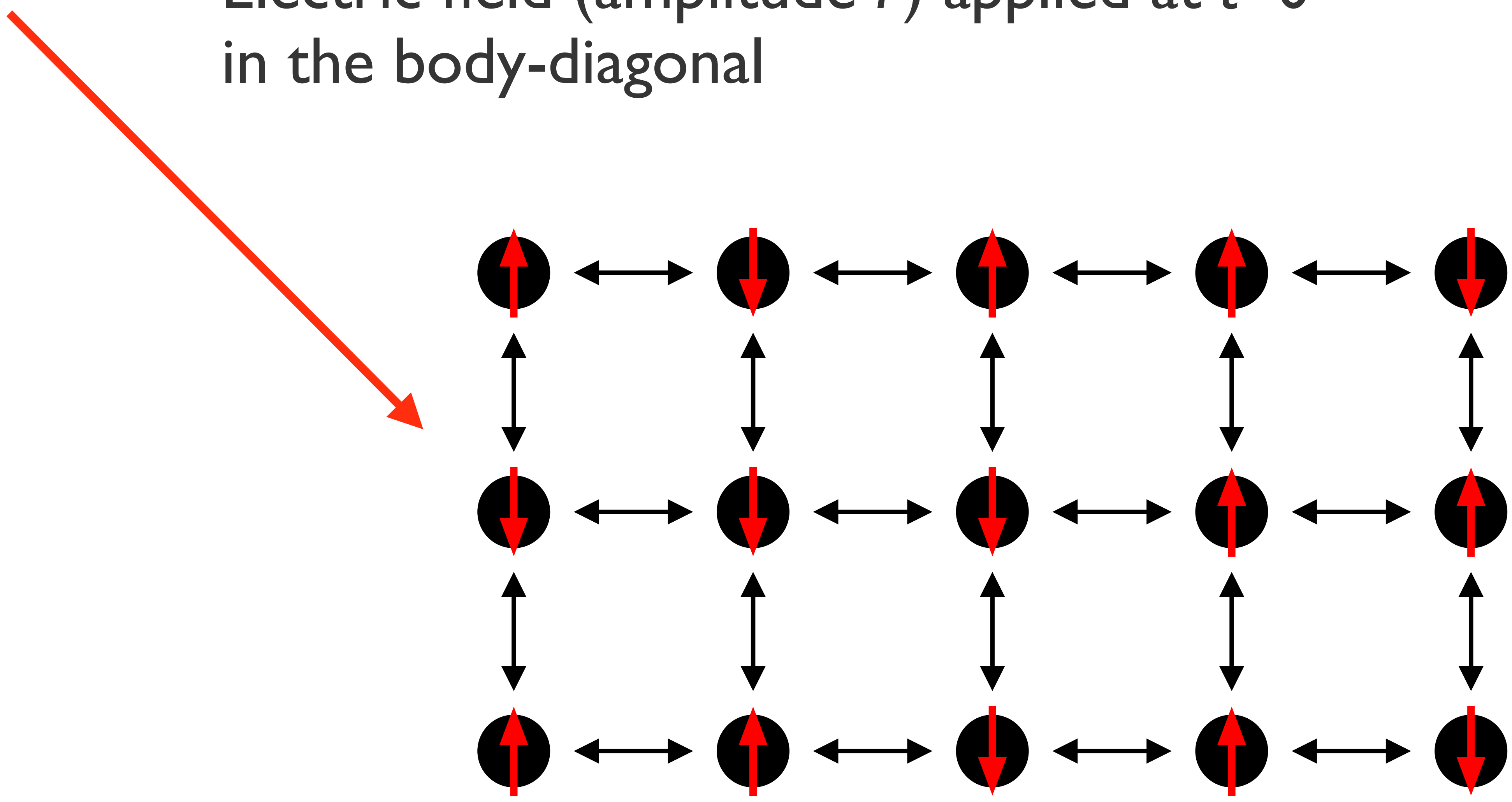
- Peierls substitution: $\varepsilon(k) \rightarrow \varepsilon(k - eA(t))$
- Lattice: hybercubic, infinite-d limit $\rho(\varepsilon) = \frac{1}{\sqrt{\pi}W} \exp(-\varepsilon^2/W^2)$

DC electric fields

- Dielectric breakdown of the Mott insulator

Eckstein, Oka, & Werner (2011); Eckstein & Werner (2013)

Electric field (amplitude *F*) applied at *t*=0
in the body-diagonal



DC electric fields

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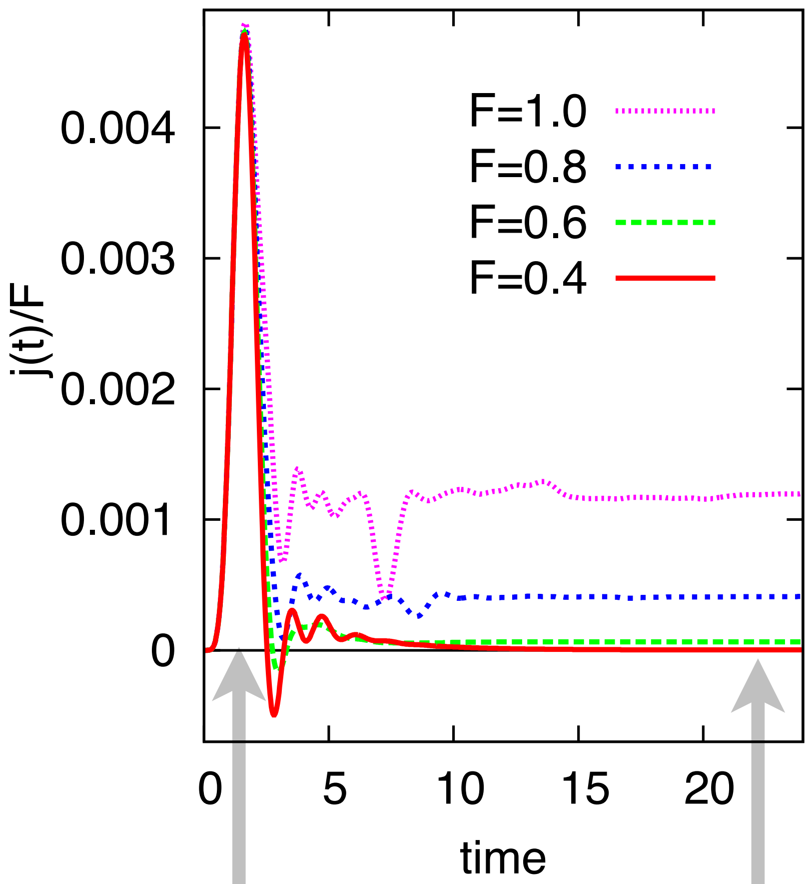
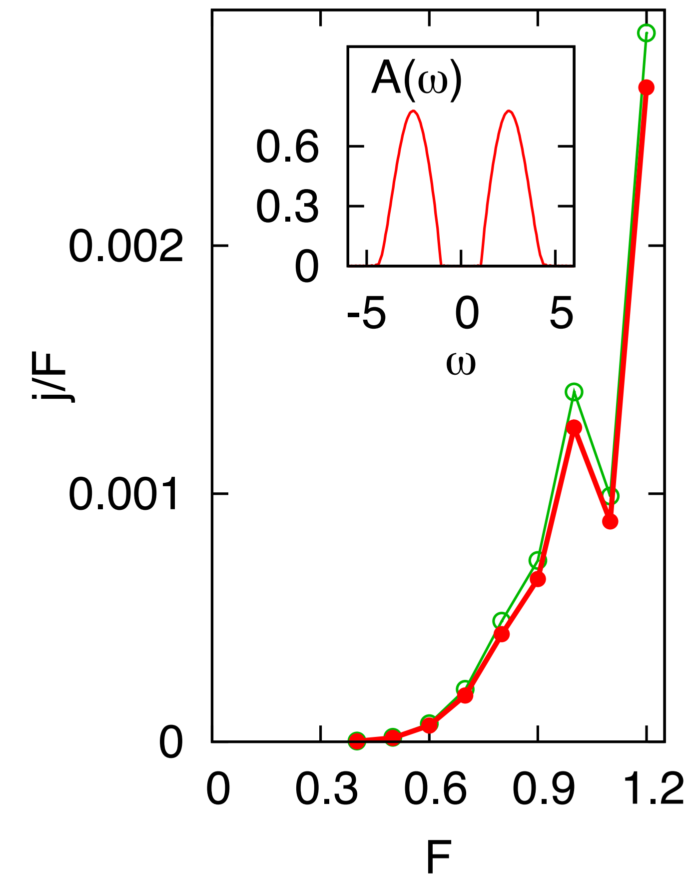


build-up of a polarization

quasi-steady state



quasi-steady current follows a threshold-law:

$$j/F \propto \exp(-F_{\rm th}/F)$$

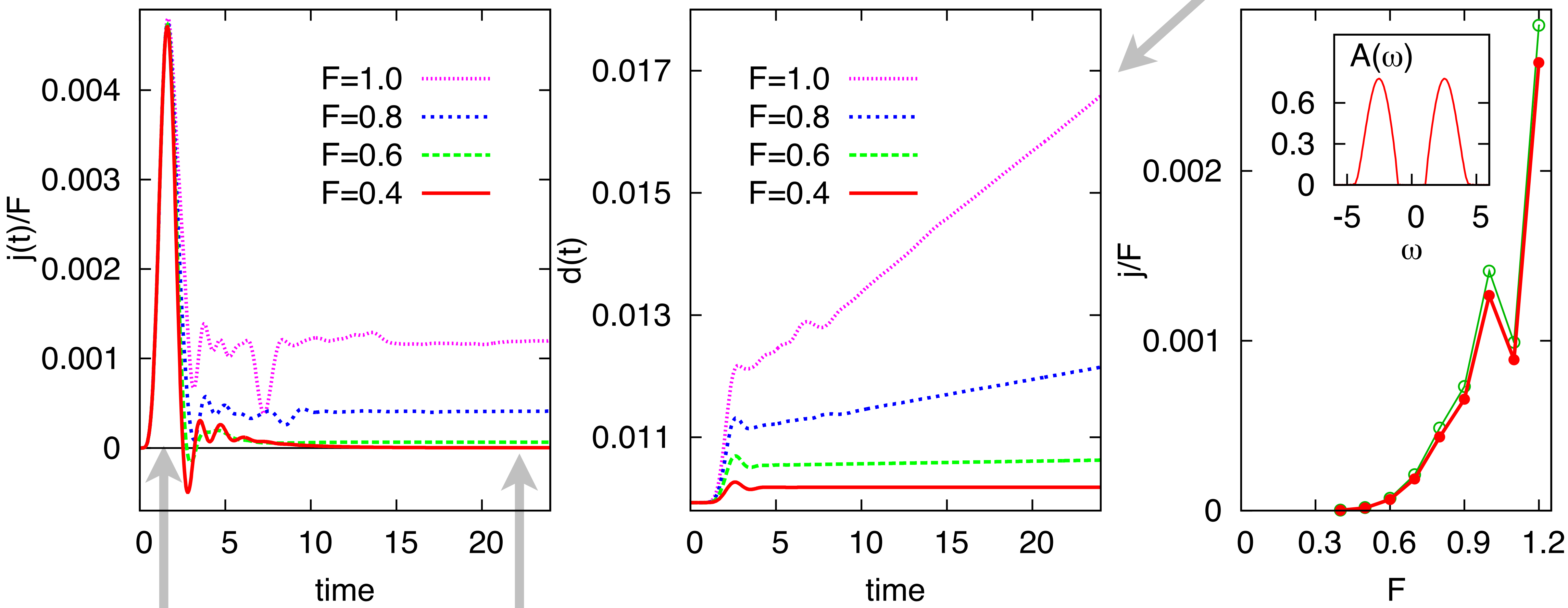
DC electric fields

- Dielectric breakdown of the Mott insulator

  Eckstein, Oka, & Werner (2011); Eckstein & Werner (2013)

- Time evolution of the current and double occupancy

constant production of doublon-hole pairs



build-up of a polarization

quasi-steady state

quasi-steady current follows a threshold-law:

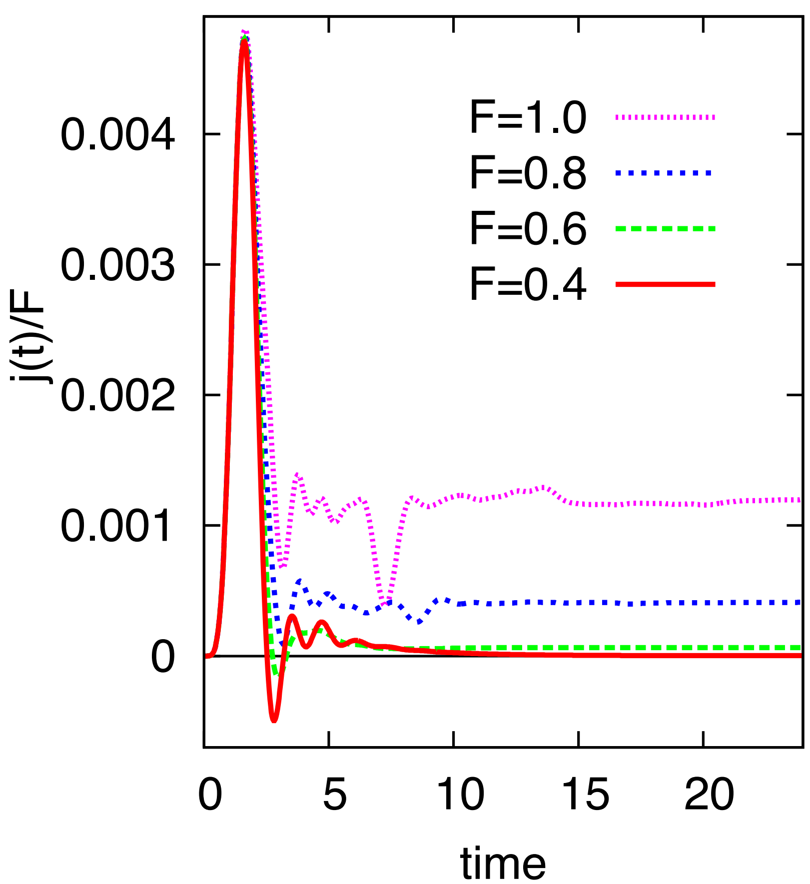
$$j/F \propto \exp(-F_{\rm th}/F)$$

DC electric fields

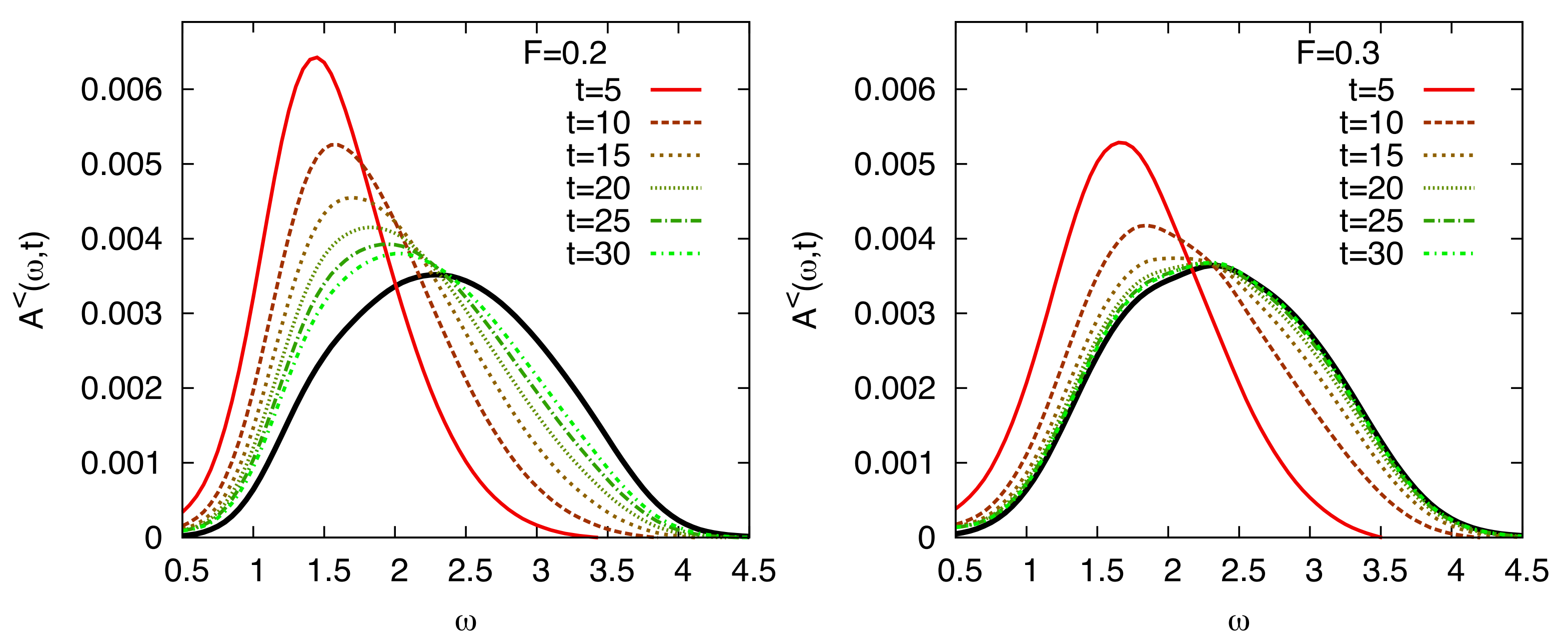
- Dielectric breakdown of the Mott insulator

  Eckstein, Oka, & Werner (2011); Eckstein & Werner (2013)

- Why is there a steady-state current if number of carriers increases?



time-dependent occupation function compared to a rescaled spectral function (black)



- Doublons (and holes) quickly heat up to infinite temperature!

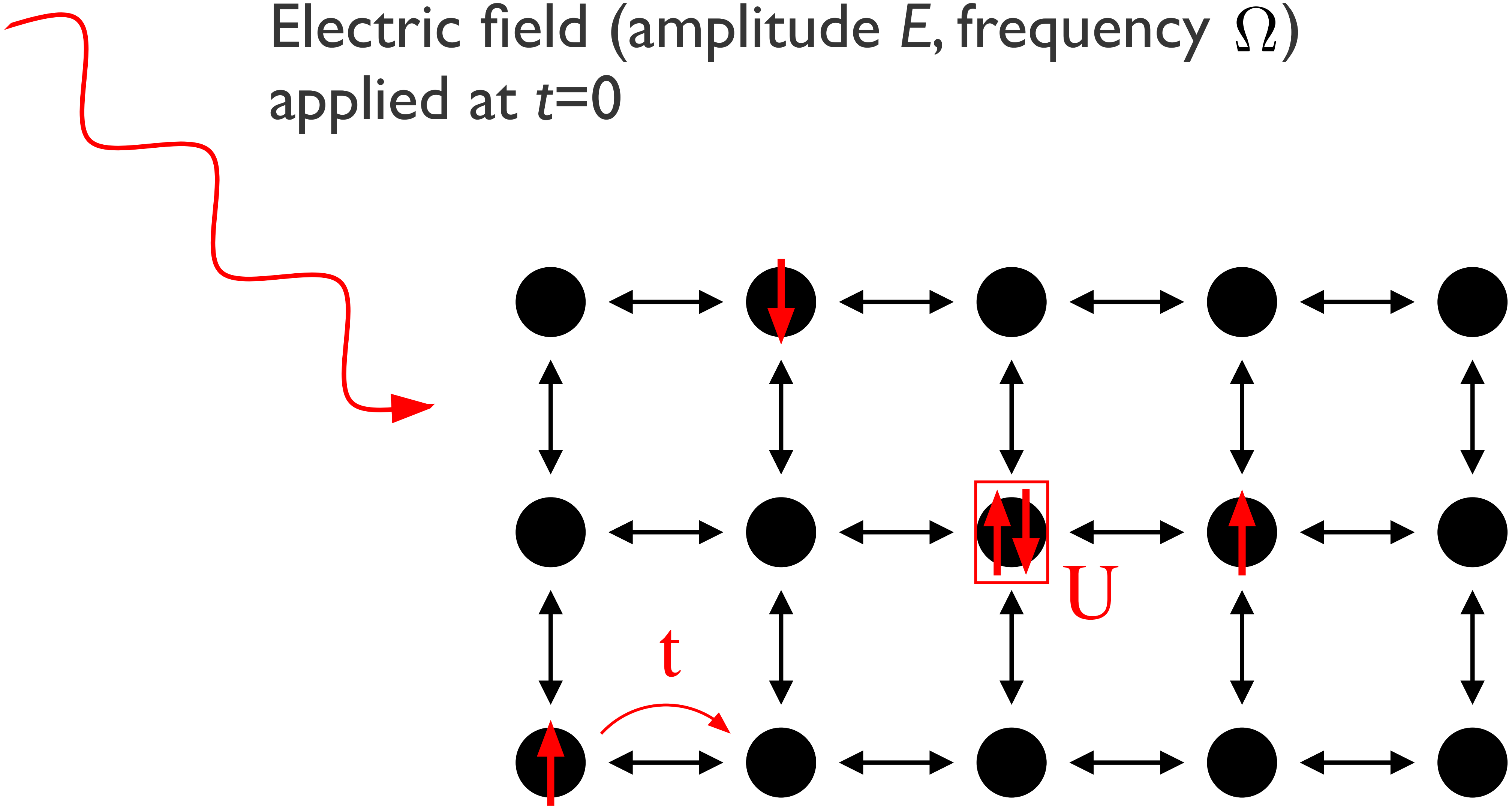
  → no contribution to the current

DC electric fields

- AC-field quench in the Hubbard model (metal phase)

  Tsuji, Oka, Werner and Aoki (2011)

Electric field (amplitude *E*, frequency Ω)
applied at *t*=0



Periodic electric fields

- AC-field quench in the Hubbard model (metal phase)

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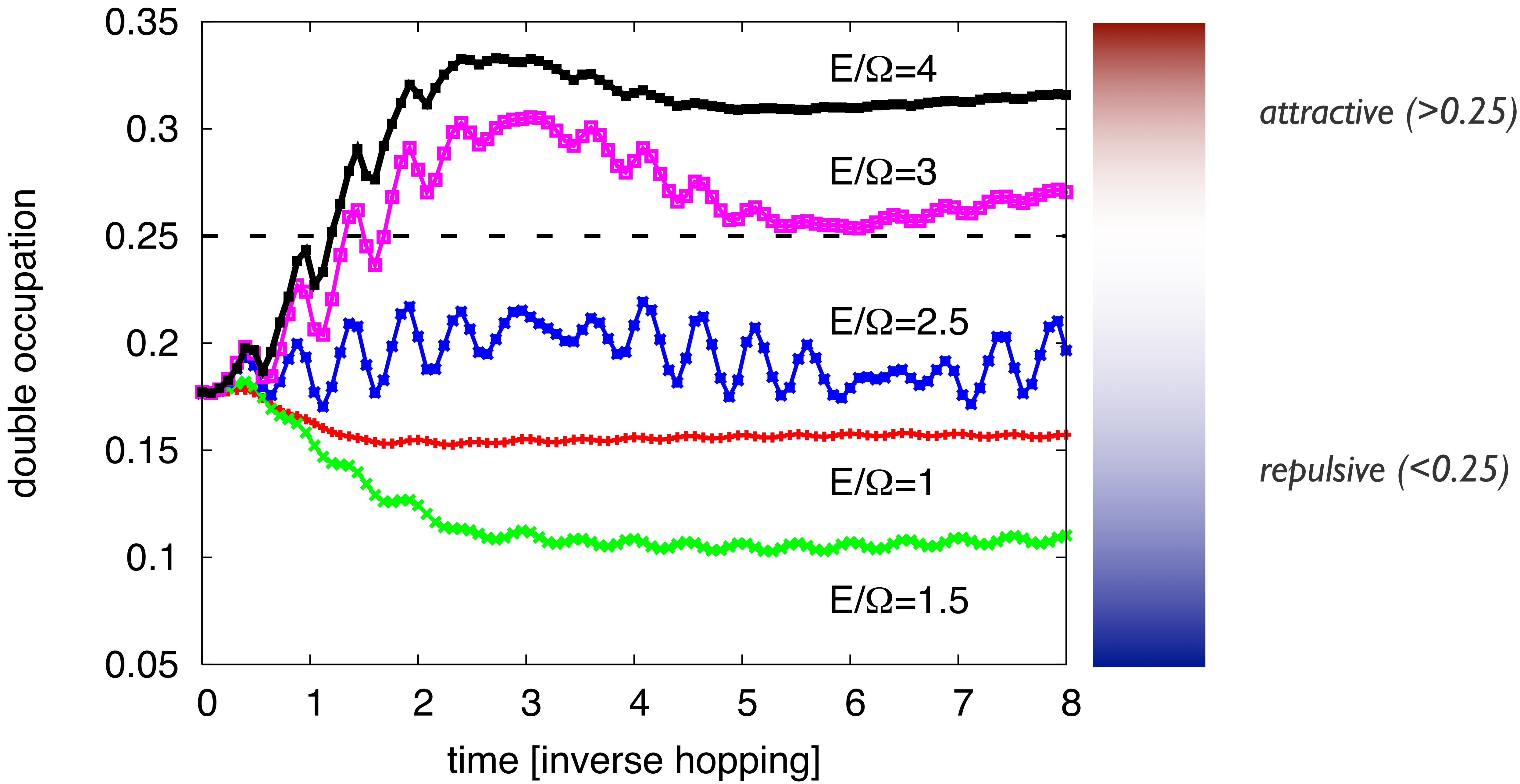
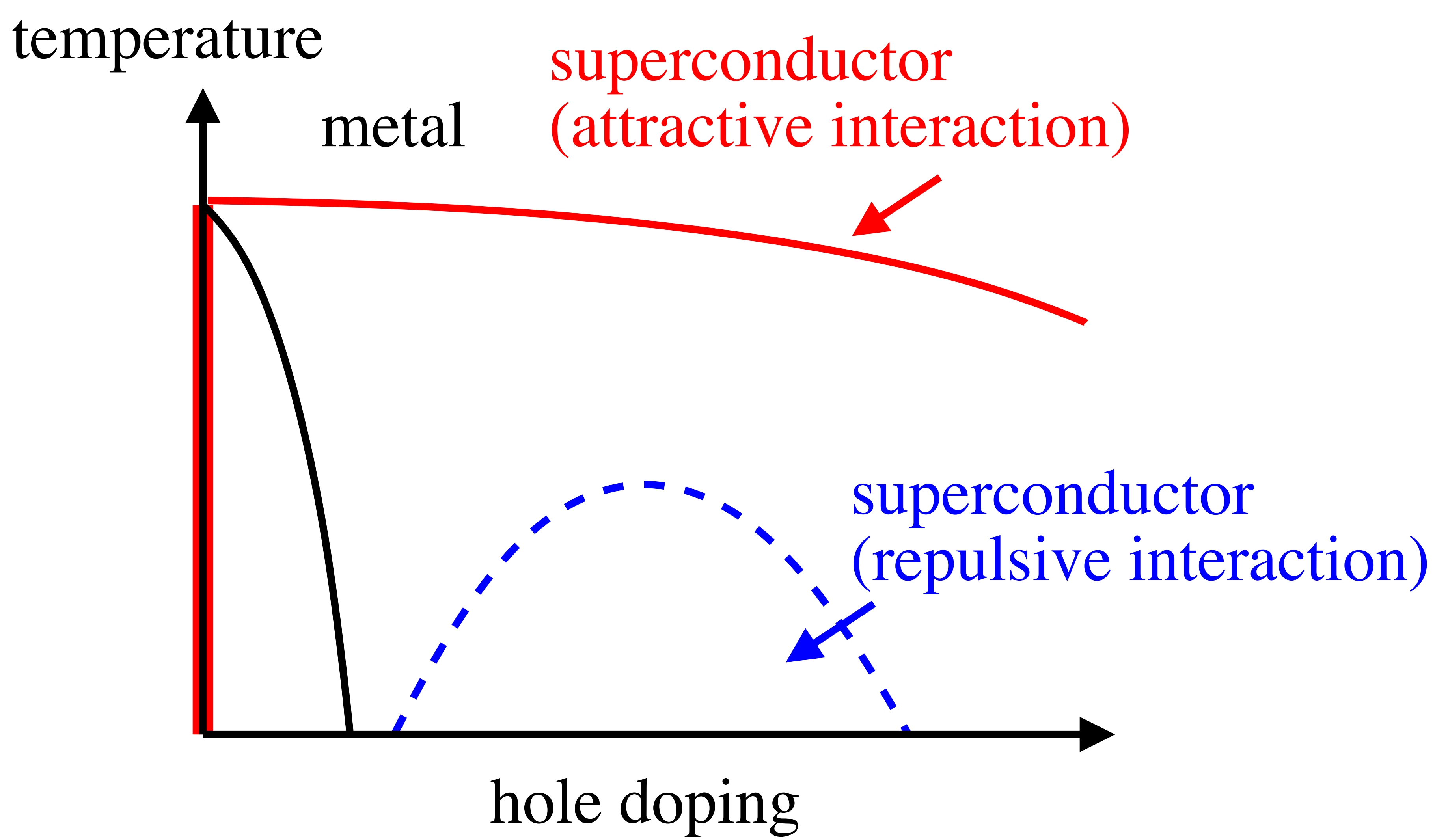


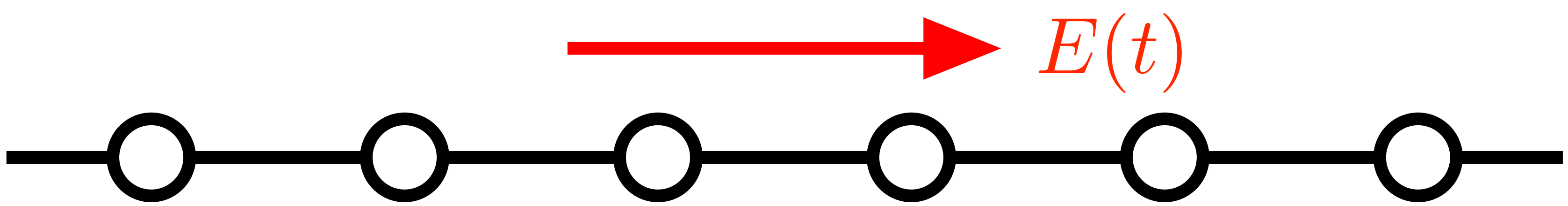
Periodic electric fields

- AC-field quench in the Hubbard model (metal phase)
  - Sign inversion of the interaction: repulsive ↔ attractive
  - *Dynamically generated* high-Tc superconductivity?



Origin of the attractive interaction

- Periodic E-field leads to a population inversion



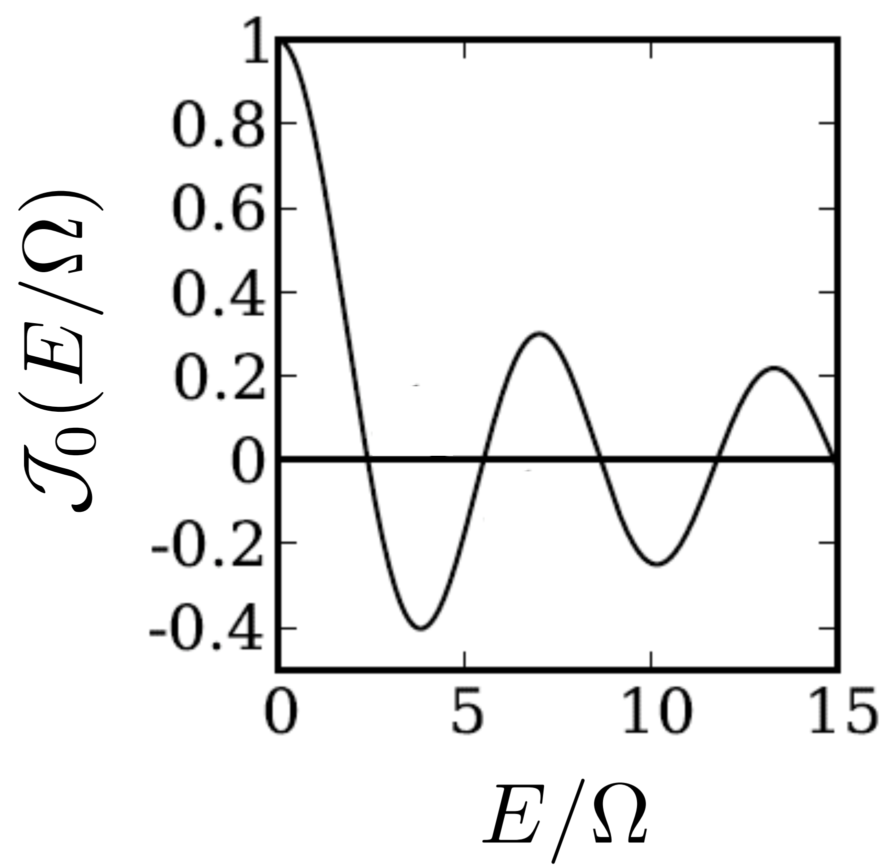
- Gauge with pure vector potential

$$E(t) = E\cos(\Omega t) = -\partial_t A(t)$$
$$\Rightarrow A(t) = -(E/\Omega)\sin(\Omega t)$$

- Peierls substitution $\epsilon_k \longrightarrow \epsilon_{k-A(t)}$

- Renormalized dispersion

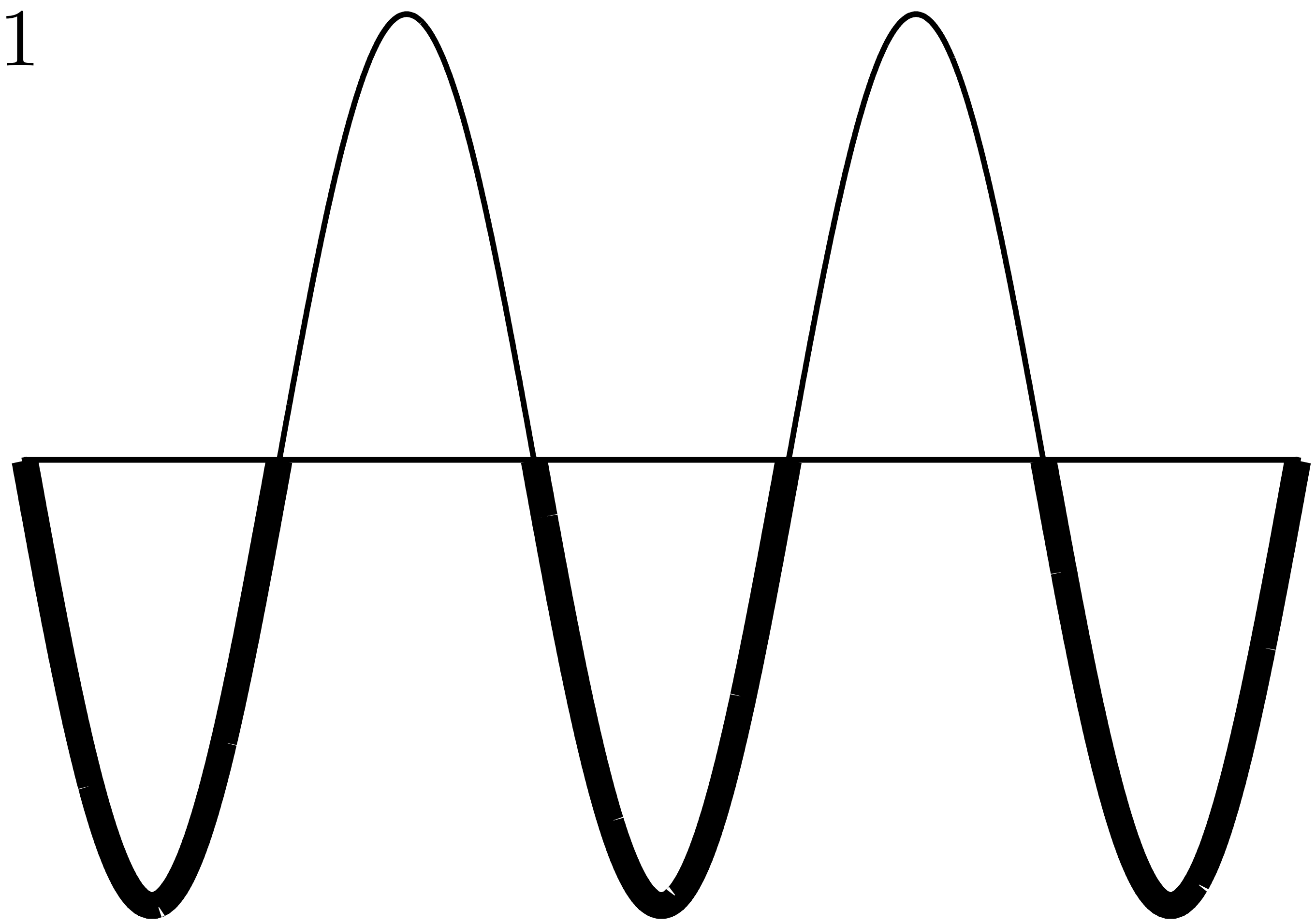
$$\overline{\epsilon_k} = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt\,\epsilon_{k-A(t)} = \mathcal{J}_0(E/\Omega)\epsilon_k$$



Origin of the attractive interaction

- Periodic E-field leads to a population inversion

$$\mathcal{J}_0(E/\Omega) = 1$$



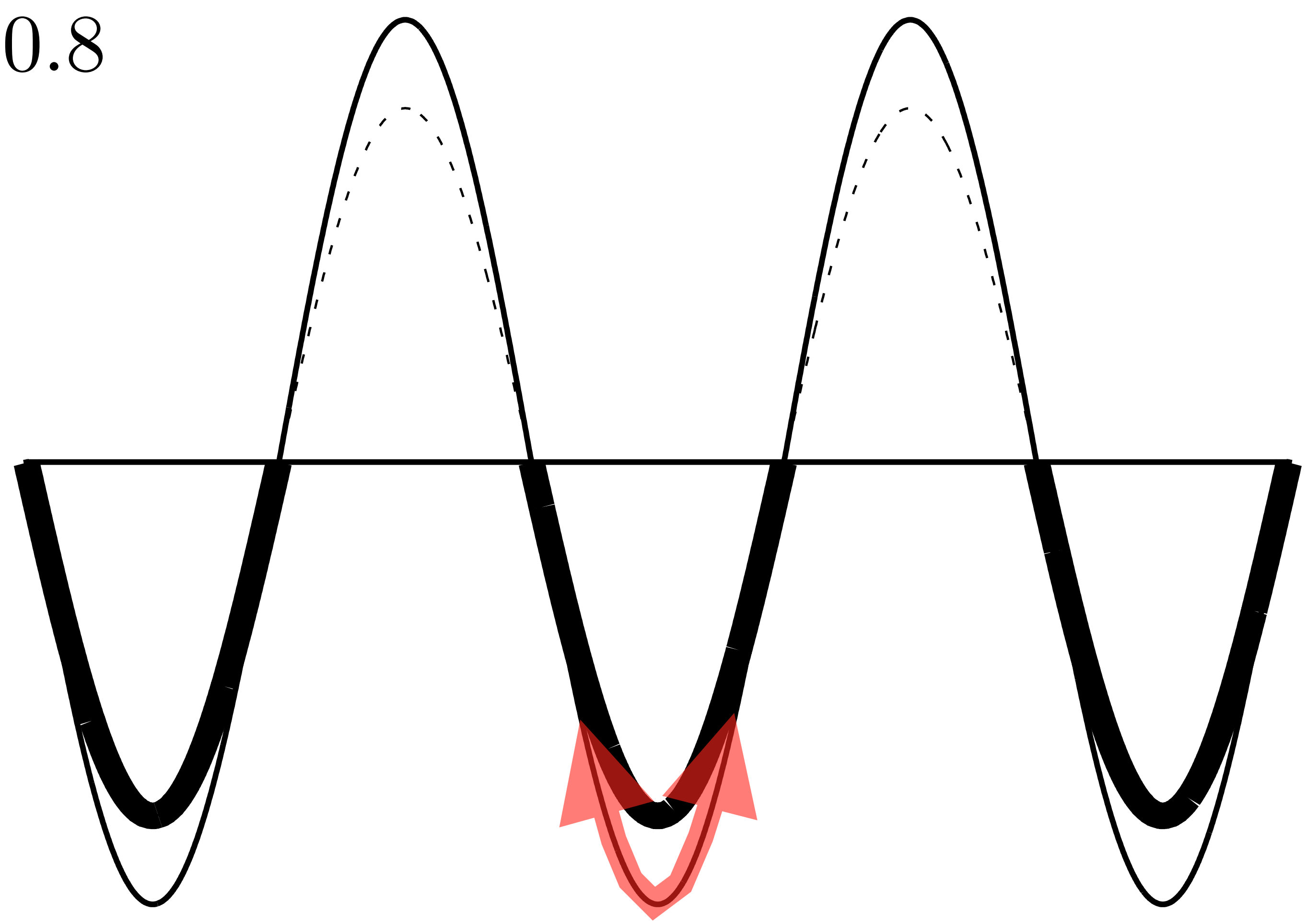
- Renormalized dispersion

$$\overline{\epsilon_k} = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt\,\epsilon_{k-A(t)} = \mathcal{J}_0(E/\Omega)\epsilon_k$$

Origin of the attractive interaction

- Periodic E-field leads to a population inversion

$\mathcal{J}_0(E/\Omega) = 0.8$



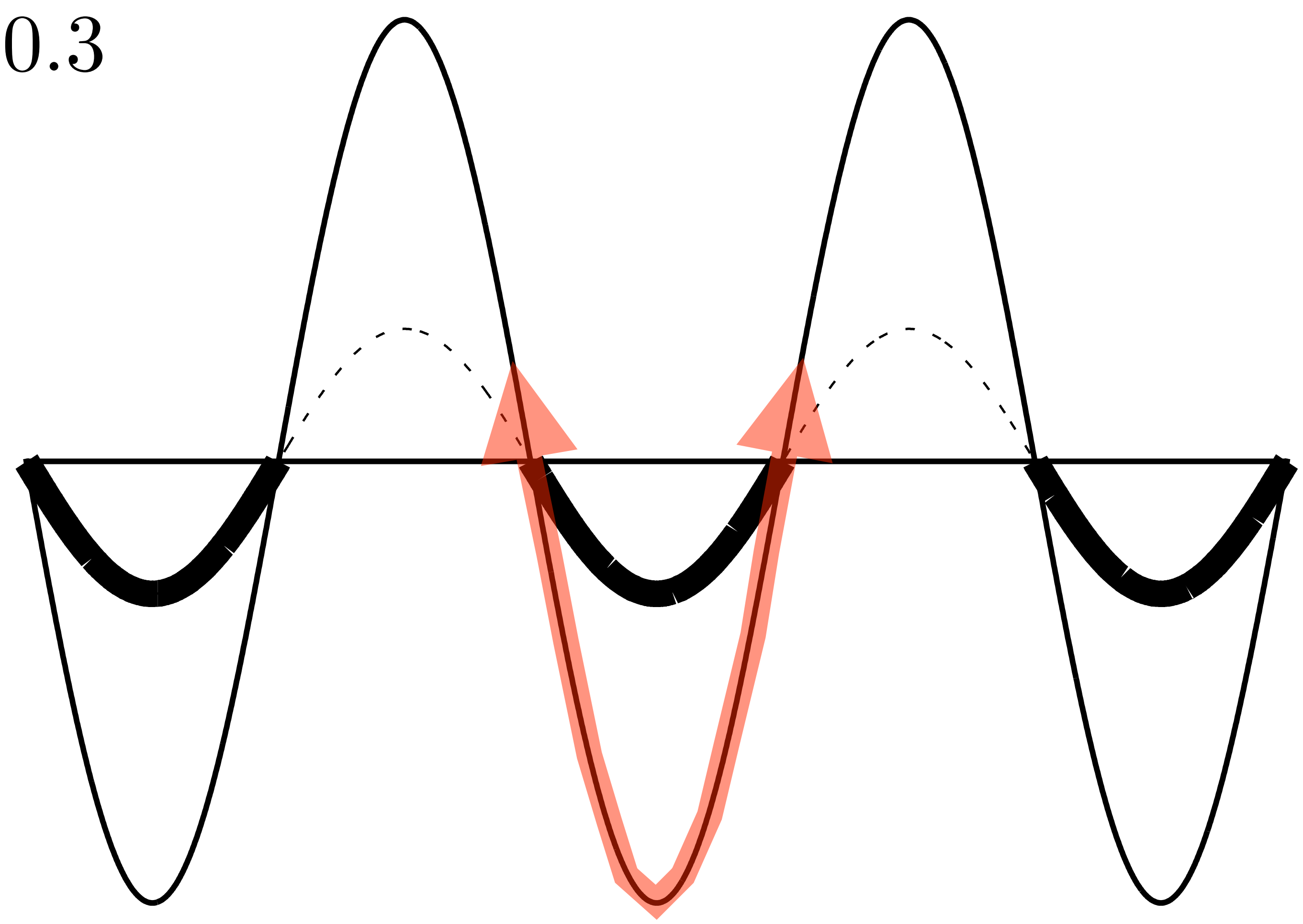
- Renormalized dispersion

$$\overline{\epsilon_k} = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt\,\epsilon_{k-A(t)} = \mathcal{J}_0(E/\Omega)\epsilon_k$$

Origin of the attractive interaction

- Periodic E-field leads to a population inversion

$\mathcal{J}_0(E/\Omega) = 0.3$



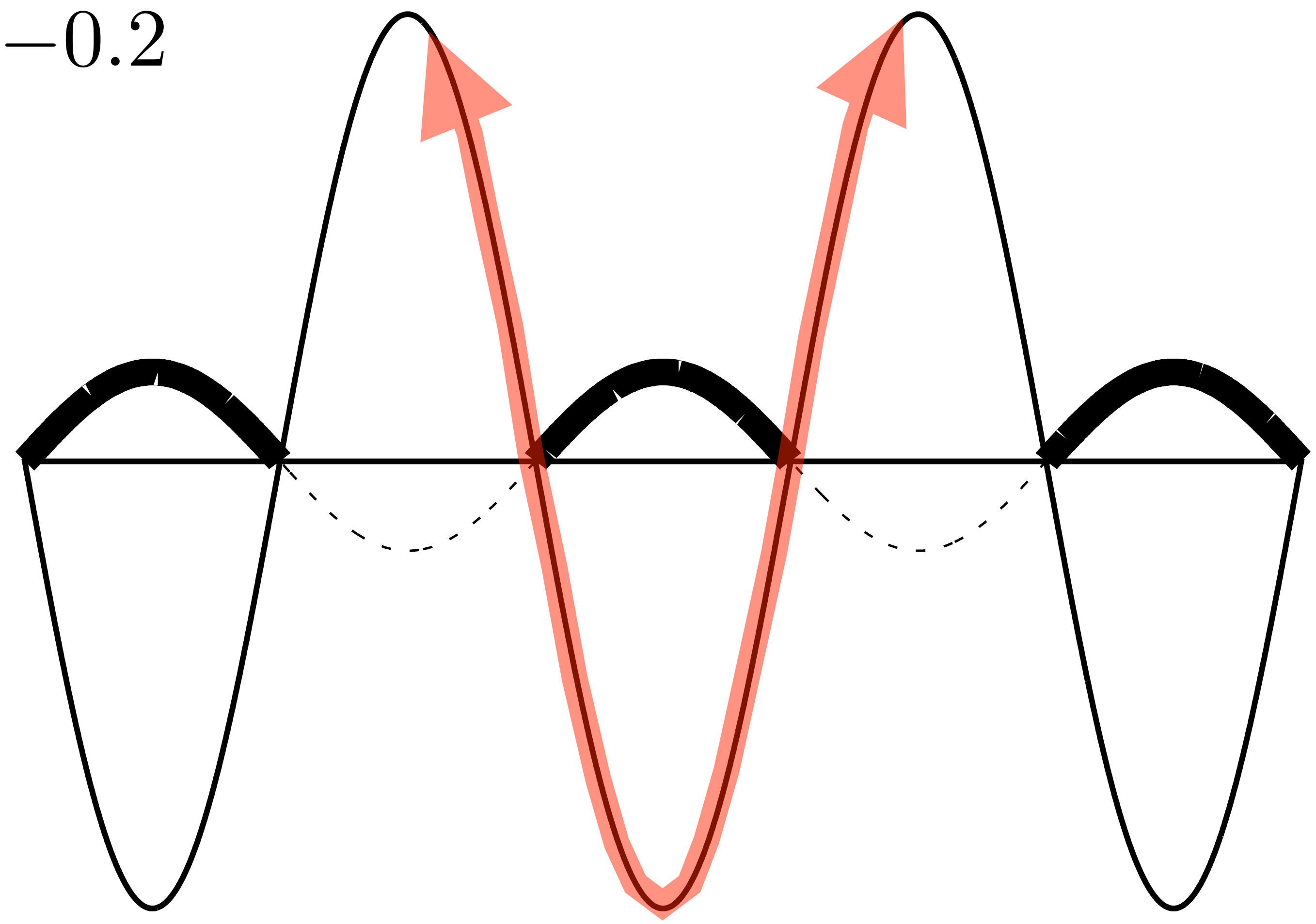
- Renormalized dispersion

$$\overline{\epsilon_k} = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt \epsilon_{k-A(t)} = \mathcal{J}_0(E/\Omega)\epsilon_k$$

Origin of the attractive interaction

- Periodic E-field leads to a population inversion

$\mathcal{J}_0(E/\Omega) = -0.2$



- Renormalized dispersion

$$\overline{\epsilon_k} = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt \epsilon_{k-A(t)} = \mathcal{J}_0(E/\Omega)\epsilon_k$$

Origin of the attractive interaction

- Inverted population = negative temperature

- State with $U > 0, T < 0$ is equivalent to state with $U < 0, T > 0$

$$\tilde{T} < 0, \mathcal{J}_0 < 0 \qquad \rho \propto \exp\left(-\frac{1}{\tilde{T}}\left[\sum_{k\sigma} \mathcal{J}_0 \epsilon_k n_{k\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}\right]\right)$$

$$T_{\text{eff}} = \frac{\tilde{T}}{\mathcal{J}_0} > 0 \qquad = \exp\left(-\frac{1}{T_{\text{eff}}}\left[\sum_{k\sigma} \epsilon_k n_{k\sigma} + \frac{U}{\mathcal{J}_0} \sum_i n_{i\uparrow} n_{i\downarrow}\right]\right)$$

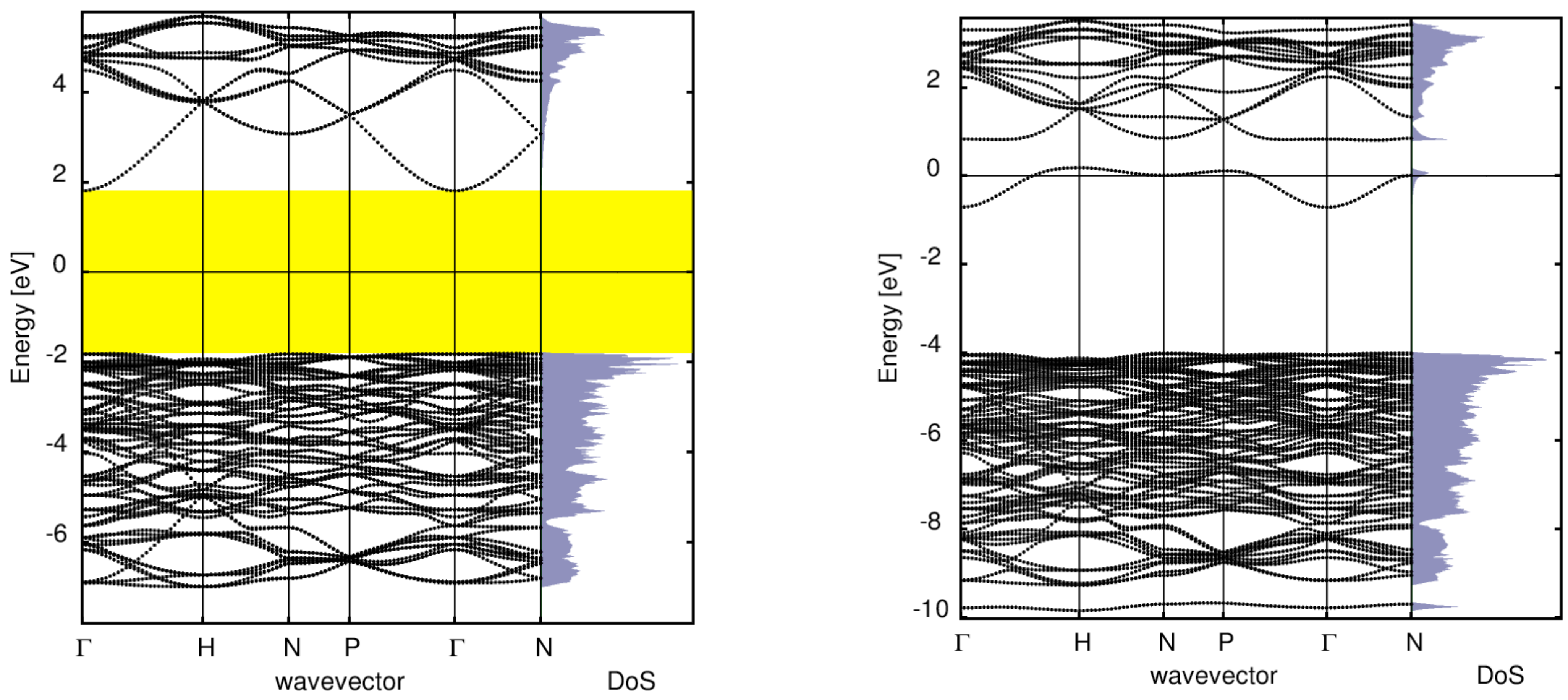
- Effective interaction of the $T_{\text{eff}} > 0$ state

$$U_{\text{eff}} = \frac{U}{\mathcal{J}_0(E/\Omega)}$$

Experimental realization?

- Potentially interesting material: *Sn* doped *In_2O_3*
  - Transparent conductor
  - Single *s*-band crossing the Fermi level

band structure by Bernard Delley (PSI)



Summary

- *Overview of DMFT.*
  - *Basic idea: neglect spatial fluctuations, keep dynamical fluctuations*
  - *Map (d+1) dimensional lattice to a (0+1) dimensional impurity model*

- *Phasediagram of the Hubbard model.*
  - *Importance of short-range spatial correlations in d=2*
  - *Relationship to cuprate physics*

- *Extension to nonequilibrium problems.*
  - *Dielectric breakdown of a Mott insulator in strong DC fields*
  - *Dynamical band-flipping and interaction conversion by AC fields*