Prethermalization and nonthermal fixed point in the Hubbard model

8 Dec 2013 @ Kyoto

Naoto Tsuji (University of Tokyo)

Acknowledgments

University of Fribourg
Philipp Werner

University of Hamburg-CFEL
Martin Eckstein

University of Geneva
Peter Barmettler

University of Tokyo
Hideo Aoki
Takashi Oka

University of Augsburg
Marcus Kollar

This talk is based on:

Tsuji, Eckstein, Werner, Phys. Rev. Lett. 110, 136404 (2013).
Tsuji, Werner, Phys. Rev. B 88, 125126 (2013).
Werner, Tsuji, Eckstein, Phys. Rev. B 86, 205101 (2012).
Aoki, Tsuji, Eckstein, Kollar, Oka, Werner, arXiv:1310.5329 (review).
Tsuji, Barmettler, Aoki, Werner, arXiv:1307.5946.

Outline

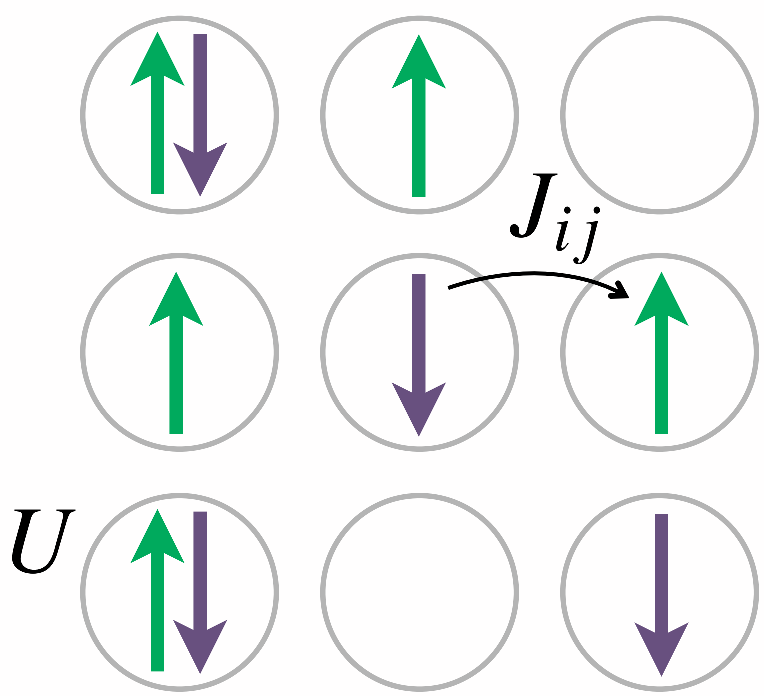
“How” and “in which time scale” does an isolated quantum many-body system thermalize?

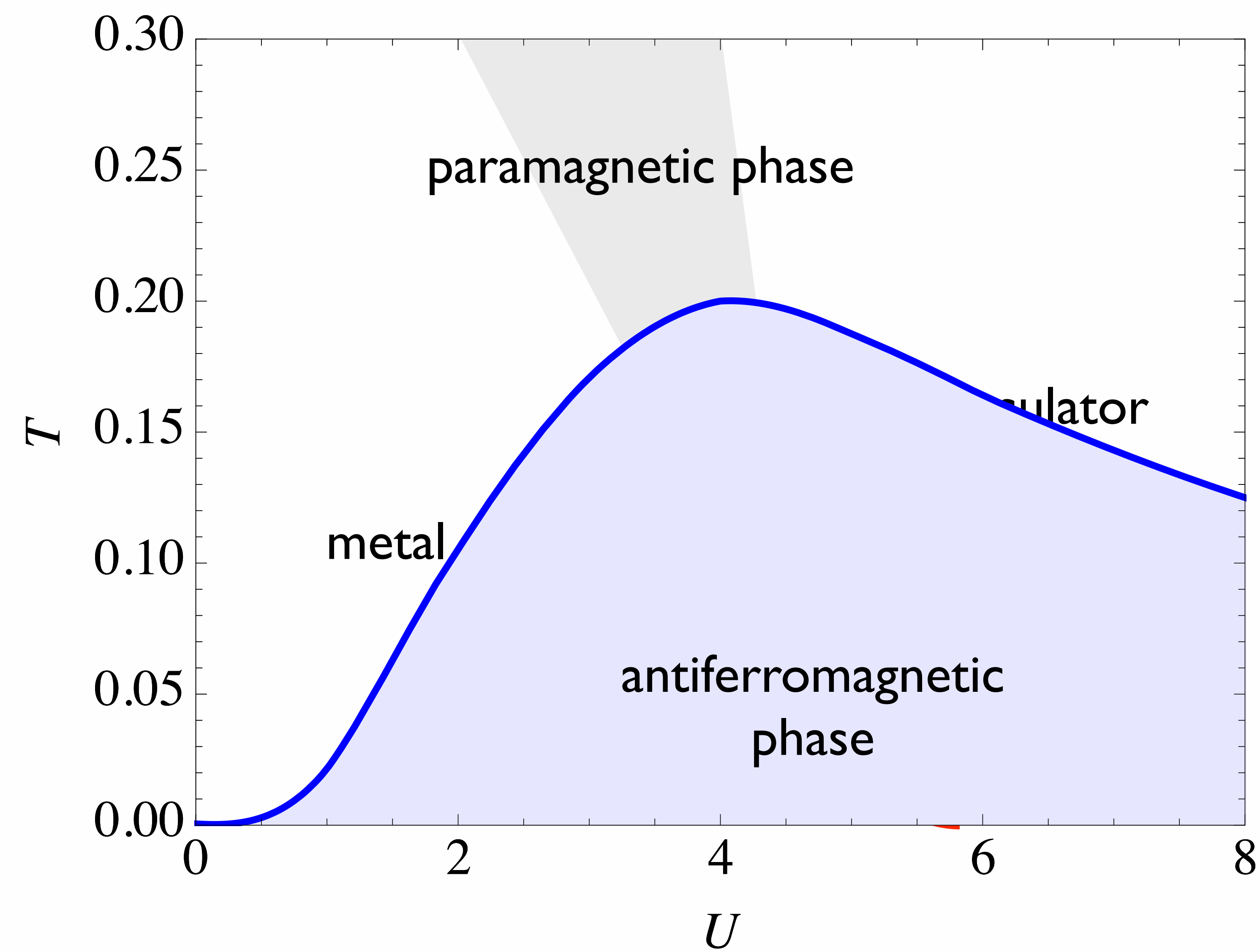
1. Prethermalization in the Hubbard model (overview)
2. Nonthermal fixed point in the Hubbard model
3. Universality of the nonthermal fixed point

Hubbard model

$$H(t) = \sum_{ij\sigma} J_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i c^\dagger_{i\uparrow} c_{i\uparrow} c^\dagger_{i\downarrow} c_{i\downarrow}$$

- DMFT (d=∞) phase diagram at half filling.

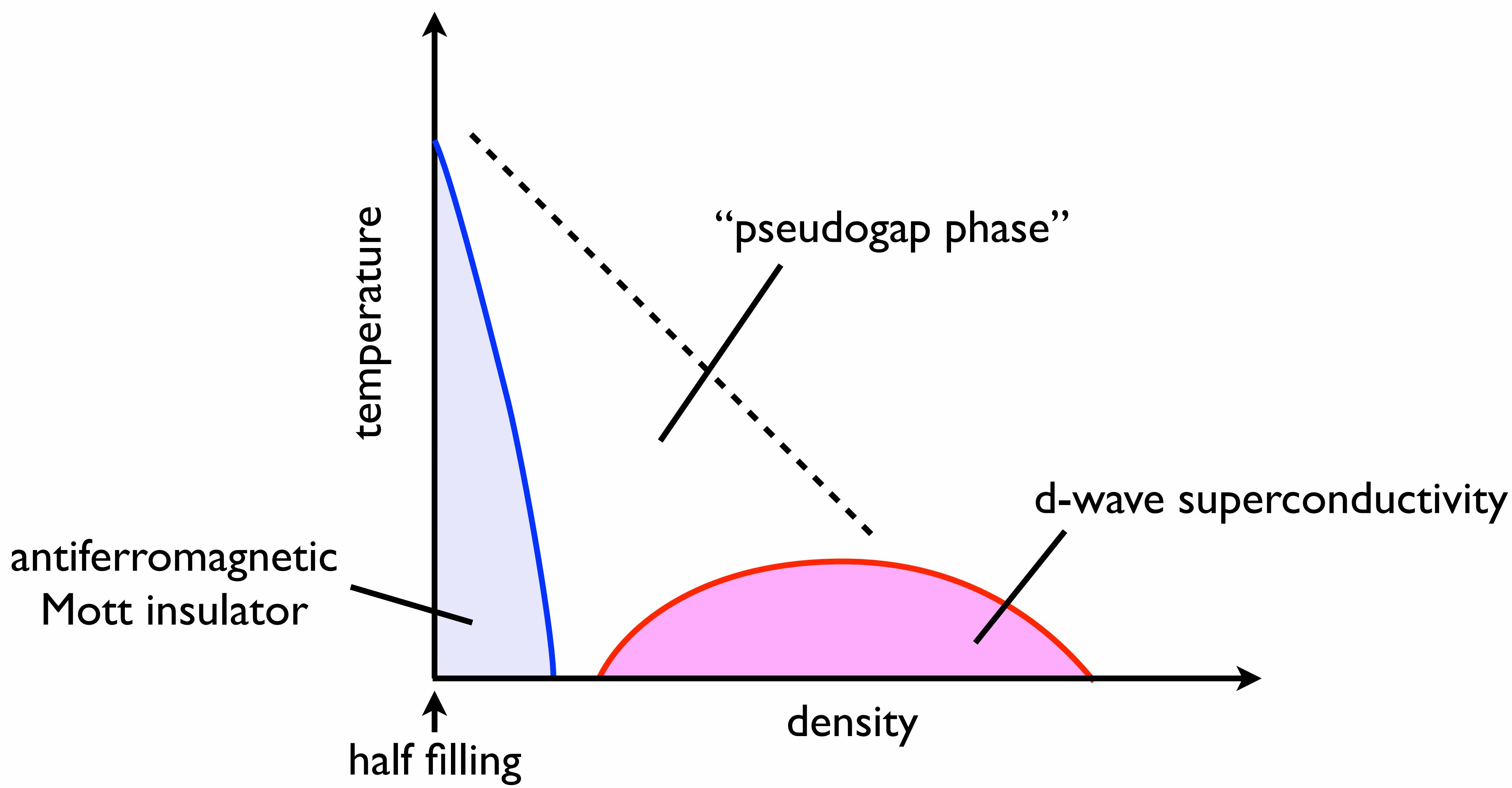




Hubbard model

$$H(t) = \sum_{ij\sigma} J_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow}$$

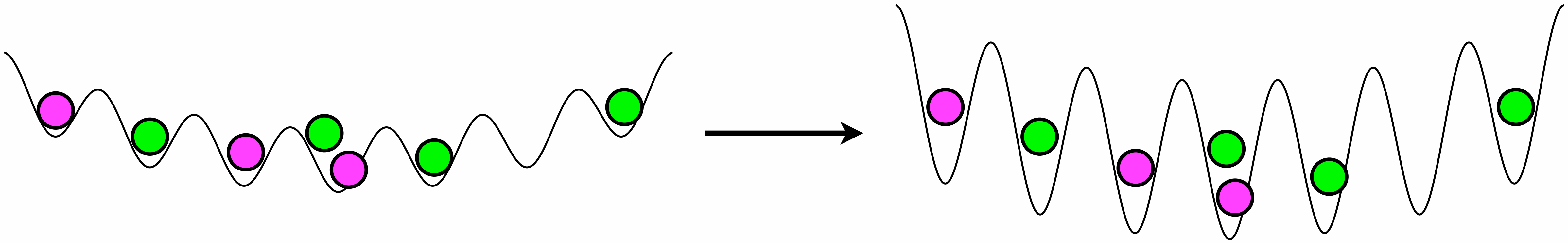
➤ Possible phase diagram in "two dimensions".



Interaction quench

➤ An abrupt change of the interaction parameter in an isolated quantum system.

$$H(t) = \sum_{ij\sigma} J_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U(t) \sum_{i} c_{i\uparrow}^{\dagger} c_{i\uparrow} c_{i\downarrow}^{\dagger} c_{i\downarrow}$$

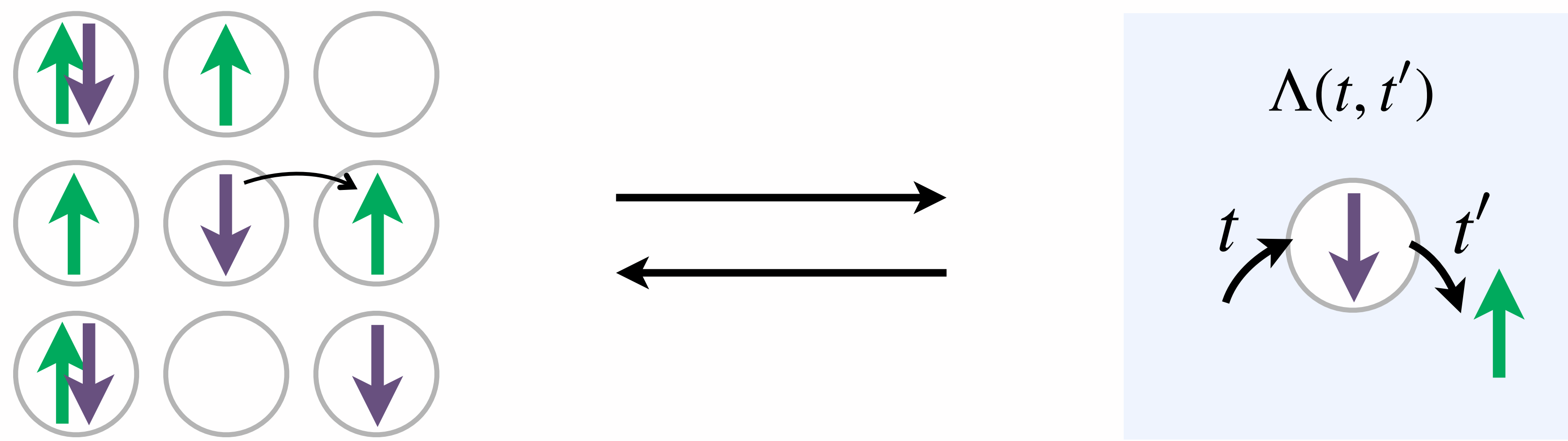


➤ Experimentally realized in cold-atom systems:
by changing the optical lattice potential depth or by using Feshbach resonance.
Greiner, et al., Nature (2002), Bloch, Dalibard, Zwerger, RMP (2008).

Nonequilibrium DMFT

Schmidt, Monien (2002), Freericks, Turkowski, Zlatić (2006), Aoki, Tsuji et al., arXiv:1310.5329.

Lattice model



Impurity model

DMFT self-consistency

$$G^{latt}_{loc}(t, t') \equiv G^{imp}[\Lambda](t, t')$$

$$G^{latt}_{loc} = \sum_{\boldsymbol{k}} \frac{1}{i\partial_t + \mu - \epsilon_{\boldsymbol{k}} - \Sigma^{latt}_{\boldsymbol{k}}}$$

impurity solver
(QMC, IPT, NCA, ED, ...)

DMFT approximation

$$\boxed{\Sigma^{latt}_{\boldsymbol{k}}(t, t') \equiv \Sigma^{imp}(t, t')}$$

➤ DMFT scheme becomes exact in d→∞ limit of lattice models. Metzner, Volhardt (1989).

arXiv.org > cond-mat > arXiv:1310.5329

Search or Article-id (Help | Advanced search)

All papers Go!

Condensed Matter > Strongly Correlated Electrons

Nonequilibrium dynamical mean-field theory and its applications

Hideo Aoki, Naoto Tsuji, Martin Eckstein, Marcus Kollar, Takashi Oka, Philipp Werner

(Submitted on 20 Oct 2013)

The study of nonequilibrium phenomena in correlated lattice systems has developed into an active and exciting branch of condensed matter physics. This research field provides rich new insights that could not be obtained from the study of equilibrium situations, and the theoretical understanding of the physics often requires the development of new concepts and methods. On the experimental side, ultra-fast pump-probe spectroscopies enable studies of excitation and relaxation phenomena in correlated electron systems, while ultra-cold atoms in optical lattices provide a new way to control and measure the time-evolution of interacting lattice systems with a vastly different characteristic timescale compared to electron systems. A theoretical description of these phenomena is challenging because, firstly, we have to compute the quantum-mechanical time-evolution of many-body systems out of equilibrium, and secondly, deal with strong-correlation effects which can be of nonperturbative nature. In this review, we discuss the nonequilibrium extension of the dynamical mean field theory (DMFT), which treats quantum fluctuations in the time domain and works directly in the thermodynamic limit. The method reduces the complexity of the calculation via a mapping to a self-consistent impurity problem. Particular emphasis is placed on a detailed derivation of the formalism, and on a discussion of numerical techniques, which enable solutions of the effective nonequilibrium DMFT impurity problem. We summarize the insights gained into the properties of the infinite-dimensional Hubbard model under strong non-equilibrium conditions. These examples illustrate the current ability of the theoretical framework to reproduce and understand fundamental nonequilibrium phenomena, such as the dielectric breakdown of Mott insulators, photo-doping, and collapse-and-revival oscillations in quenched systems.

Comments: 57 pages, 32 figures
Subjects: **Strongly Correlated Electrons (cond-mat.str-el)**
Cite as: **arXiv:1310.5329 [cond-mat.str-el]**
(or **arXiv:1310.5329v1 [cond-mat.str-el]** for this version)

Submission history

From: Philipp Werner [view email]
[v1] Sun, 20 Oct 2013 13:36:45 GMT (8013kb,A)

Which authors of this paper are endorsers? | Disable MathJax (What is MathJax?)

Link back to: arXiv, form interface, contact.

Download:

- PDF
- PostScript
- Other formats

Ancillary files (details):

- cxx/makefile
- cxx/parm.sh
- cxx/src/const.h
- cxx/src/dmft.cxx
- cxx/src/dmft.h

(23 additional files not shown)

Current browse context:
cond-mat.s
< prev | next
new | recent | 310

Change to browse by:
cond-mat

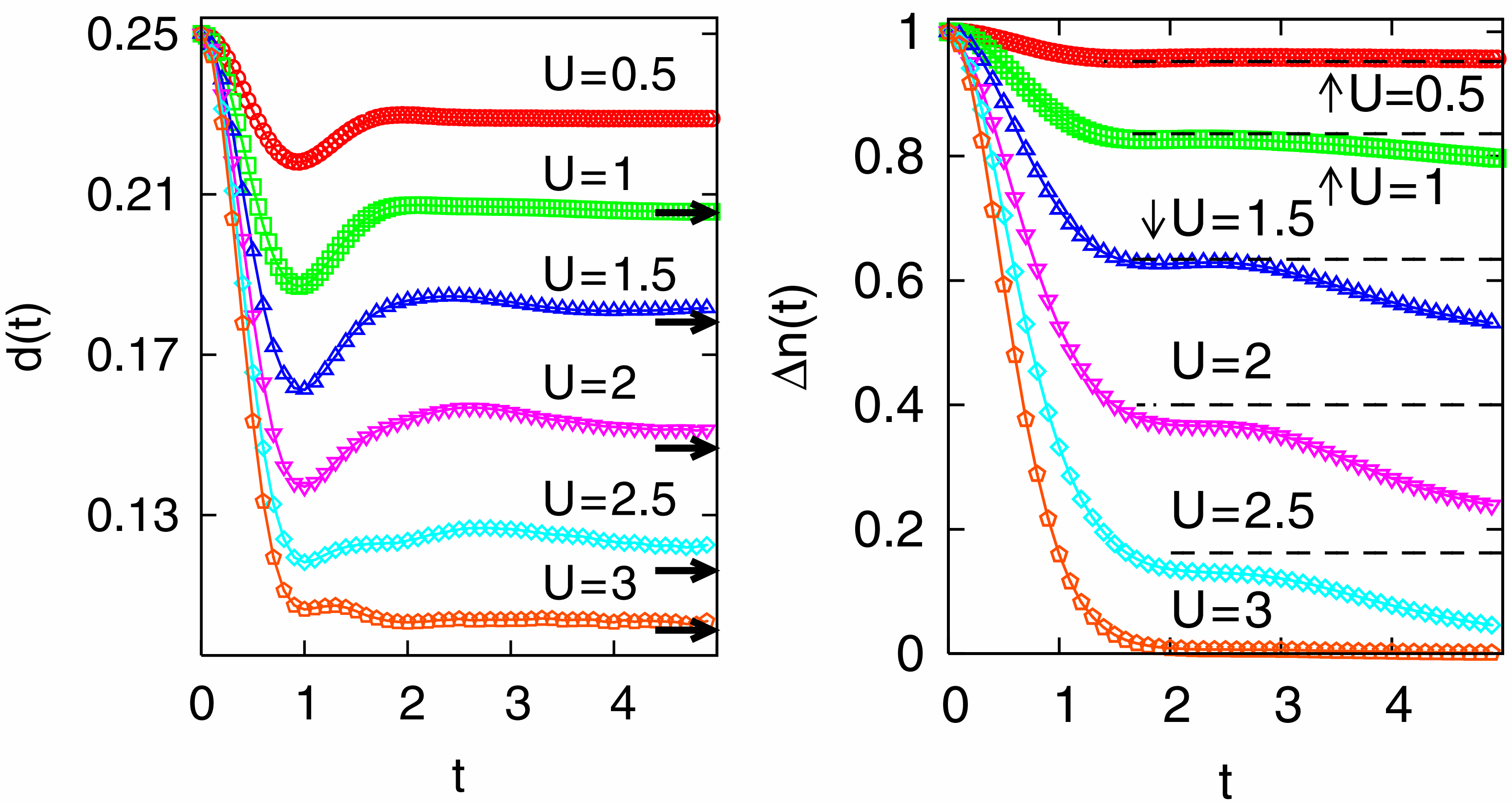
References & Citations

- NASA ADS

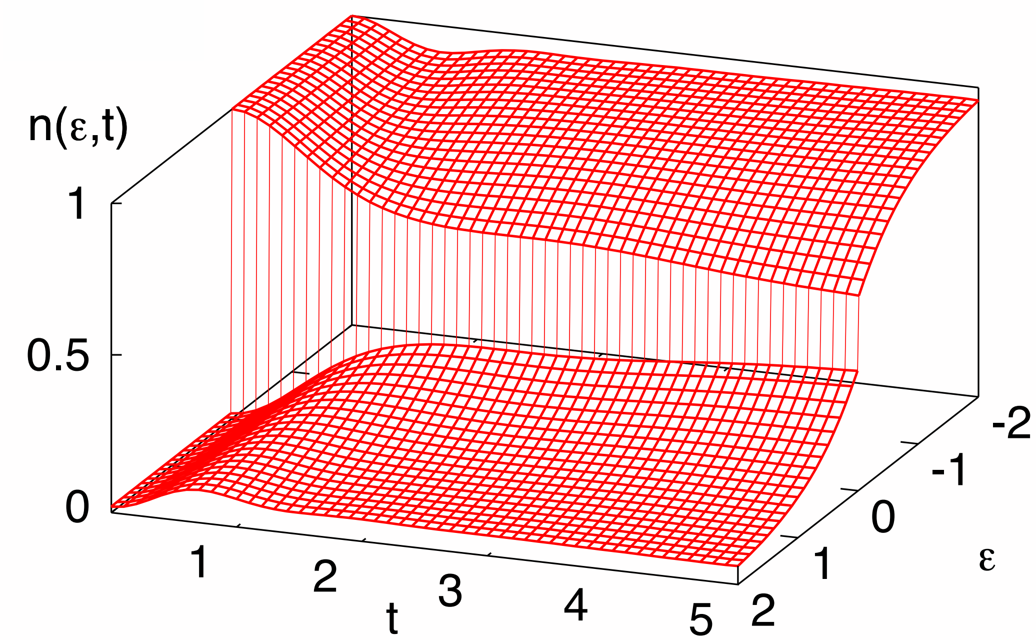
Bookmark (what is this?)

Prethermalization in d=∞

- $d(t)=\langle n_\uparrow n_\downarrow \rangle$: the double occupancy → “mode-integrated”
- $n(\epsilon_k,t)=\langle c_k^\dagger(t)c_k(t)\rangle$: the full momentum distribution → “mode-resolved”

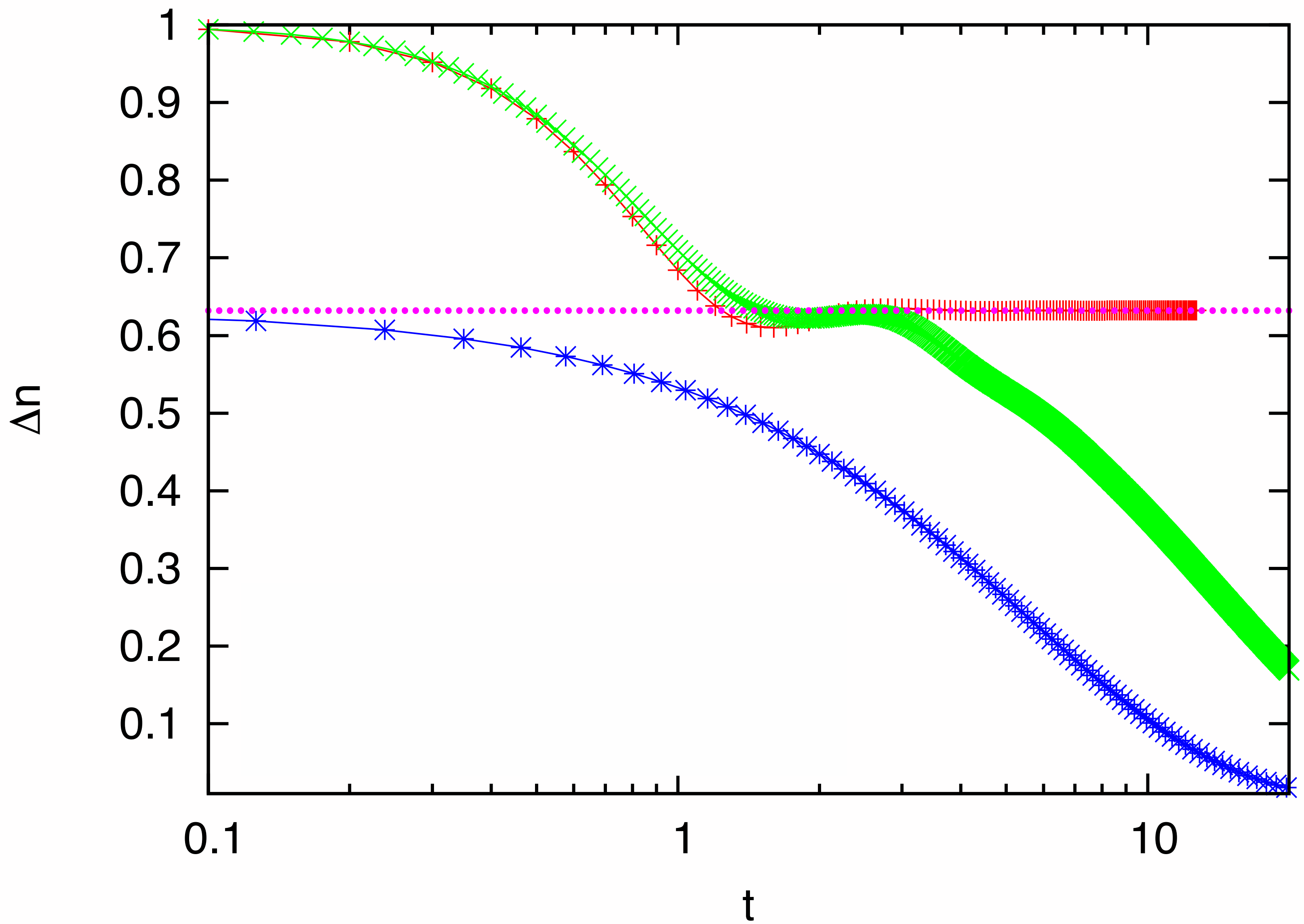


Eckstein, Kollar, Werner, PRL (2009).



Eckstein, Kollar, Werner, PRB (2010).

Prethermalization in d=∞



Short-time approximation: Moeckel, Kehrein, PRL (2008)

Nonequilibrium DMFT : Aoki, Tsuji et al., arXiv:1310.5329.

Quantum Boltzmann equation: Stark, Kollar, arXiv:1308.1610

Generalized Gibbs ensemble (GGE): Kollar, Wolf, Eckstein, PRB (2011)

Generalized Gibbs ensemble (GGE)

Kollar, Wolf, Eckstein, PRB (2011)

- Weak interaction: $H = H_0 + gH_1 \quad (|g| \ll 1)$

$$H_0 = \sum_{\alpha} \epsilon_\alpha c_\alpha^\dagger c_\alpha, \quad H_1 = \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta$$

- Approximate constants of motion:

$$\tilde{n}_\alpha = n_\alpha + g \sum_{\beta\gamma\delta} \frac{2V_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + \text{h.c.}}{\epsilon_\alpha + \epsilon_\beta - \epsilon_\gamma - \epsilon_\delta} + O(g^2)$$

- Construct GGE with them:

$$\rho_{\widetilde{\mathrm{GGE}}} \propto \exp\left(-\sum_\alpha \lambda_\alpha \tilde{n}_\alpha\right) \quad \text{with} \quad \langle \tilde{n}_\alpha \rangle_{\widetilde{\mathrm{GGE}}} \overset{!}{=} \langle \tilde{n}_\alpha \rangle_0$$

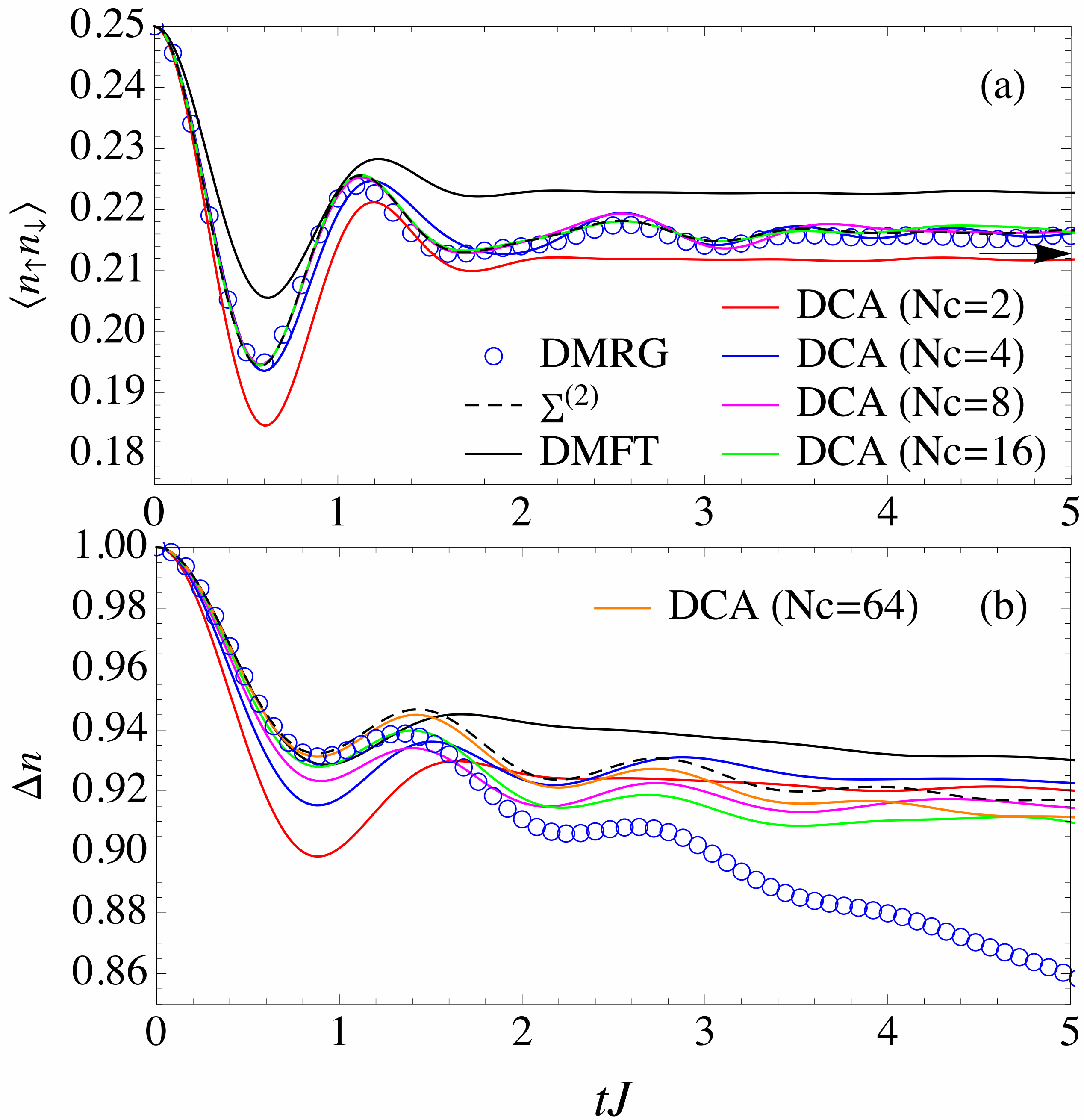
- Prethermalization plateau is described by GGE:

$$\langle n_\alpha \rangle_{\widetilde{\mathrm{GGE}}} = \langle n_\alpha \rangle_{\mathrm{pretherm}} + O(g^3)$$

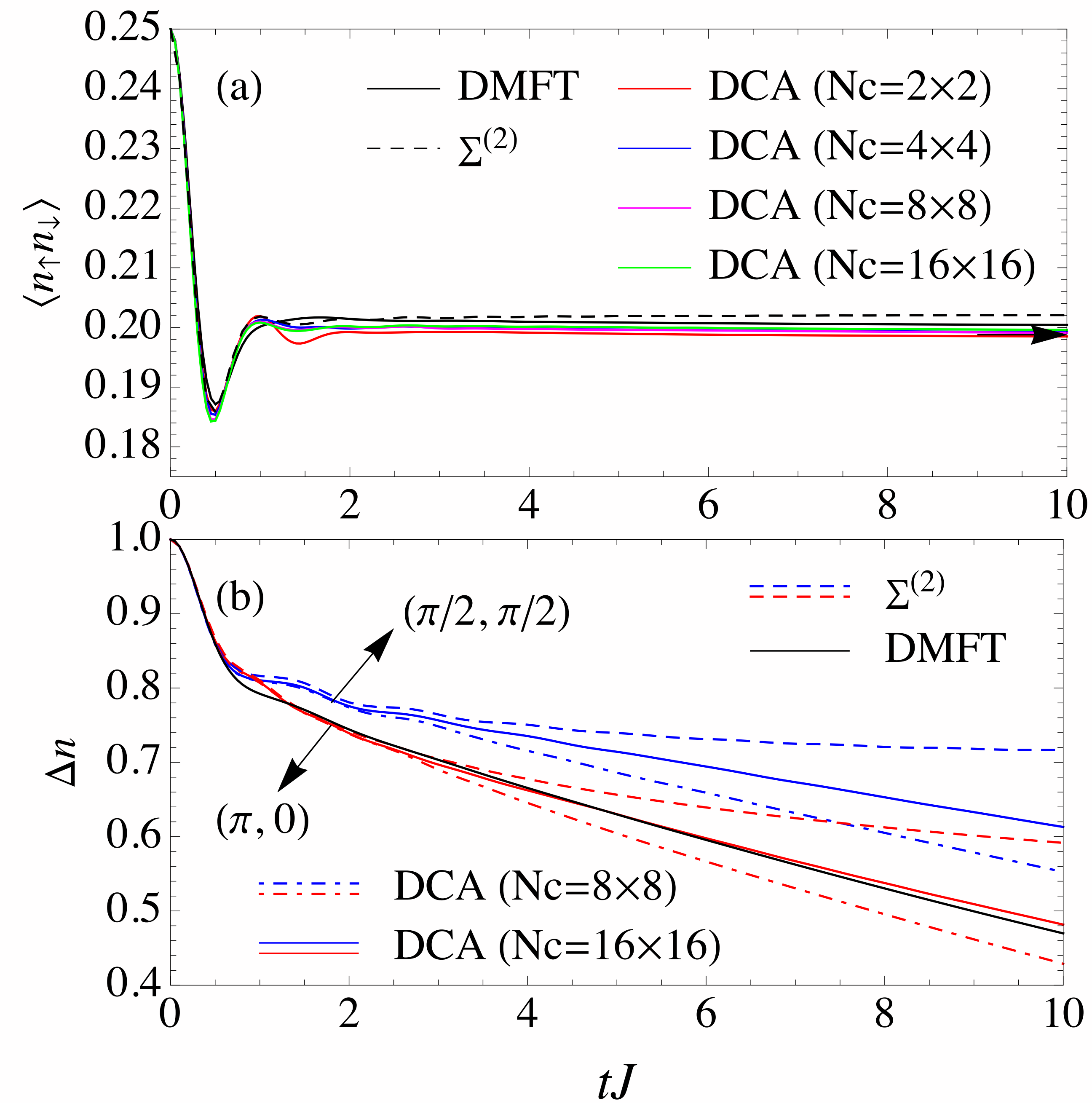
Prethermalization in d=1 and 2

Tsuji, Barmettler, Aoki, Werner, arXiv:1307.5946.

➤ d=1, U/J=0 → 1



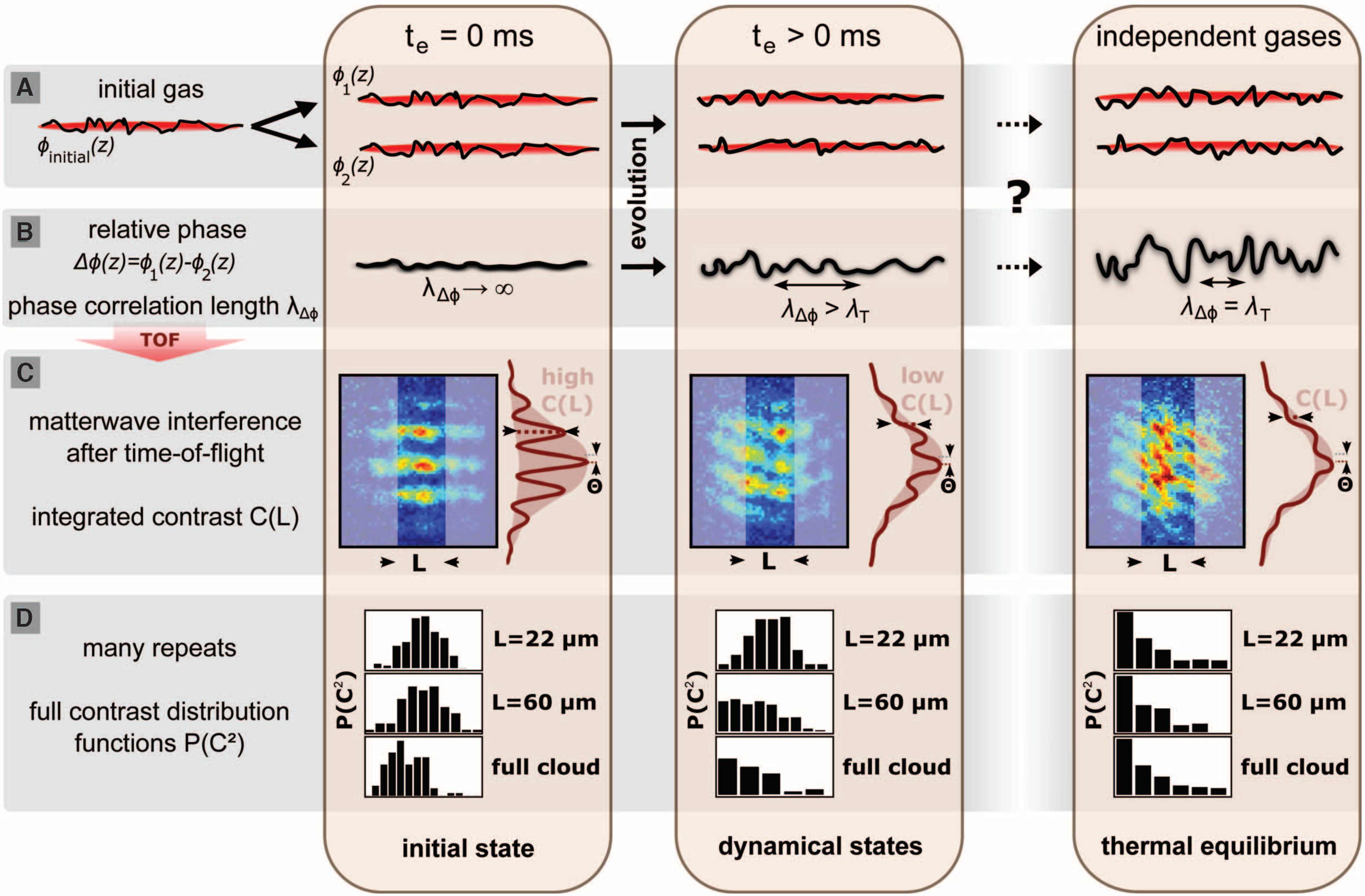
➤ d=2, U/J=0 → 2



Relaxation and Prethermalization in an Isolated Quantum System

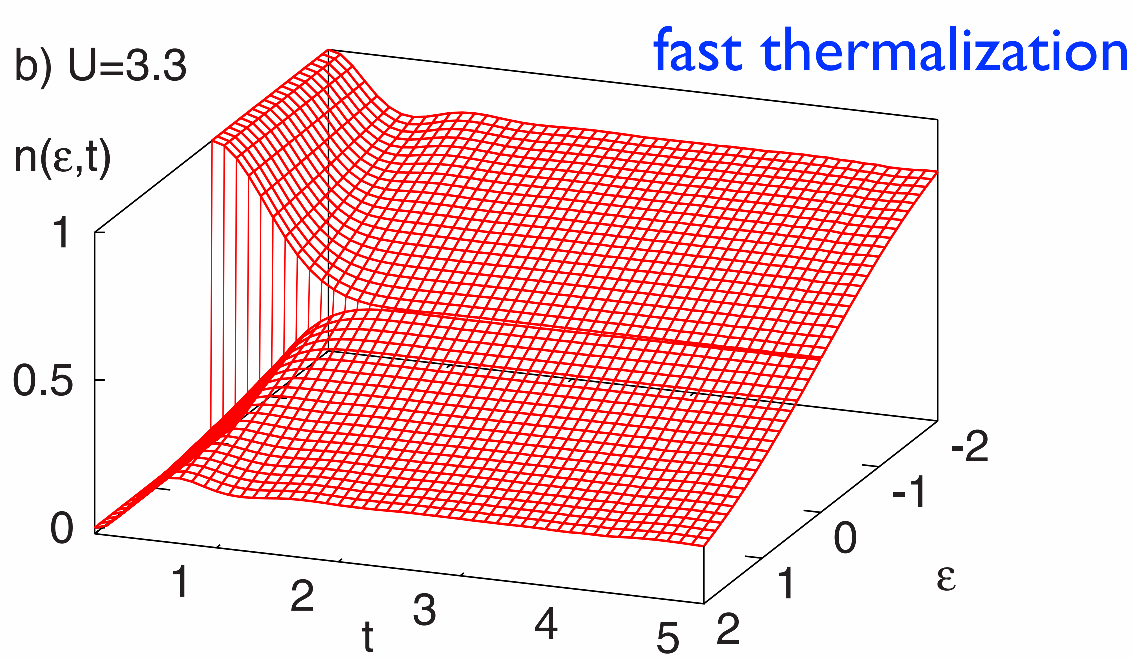
M. Gring,[1] M. Kuhnert,[1] T. Langen,[1] T. Kitagawa,[2] B. Rauer,[1] M. Schreitl,[1] I. Mazets,[1,3] D. Adu Smith,[1] E. Demler,[2] J. Schmiedmayer[1,4]*

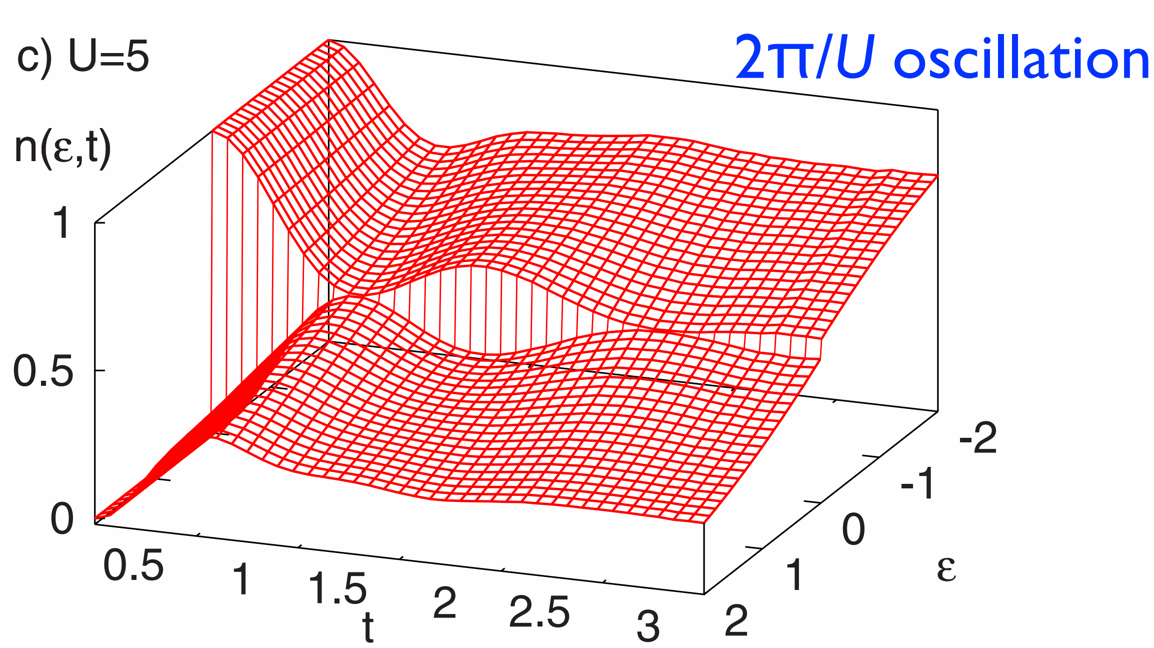
Science 337, 1318 (2012)

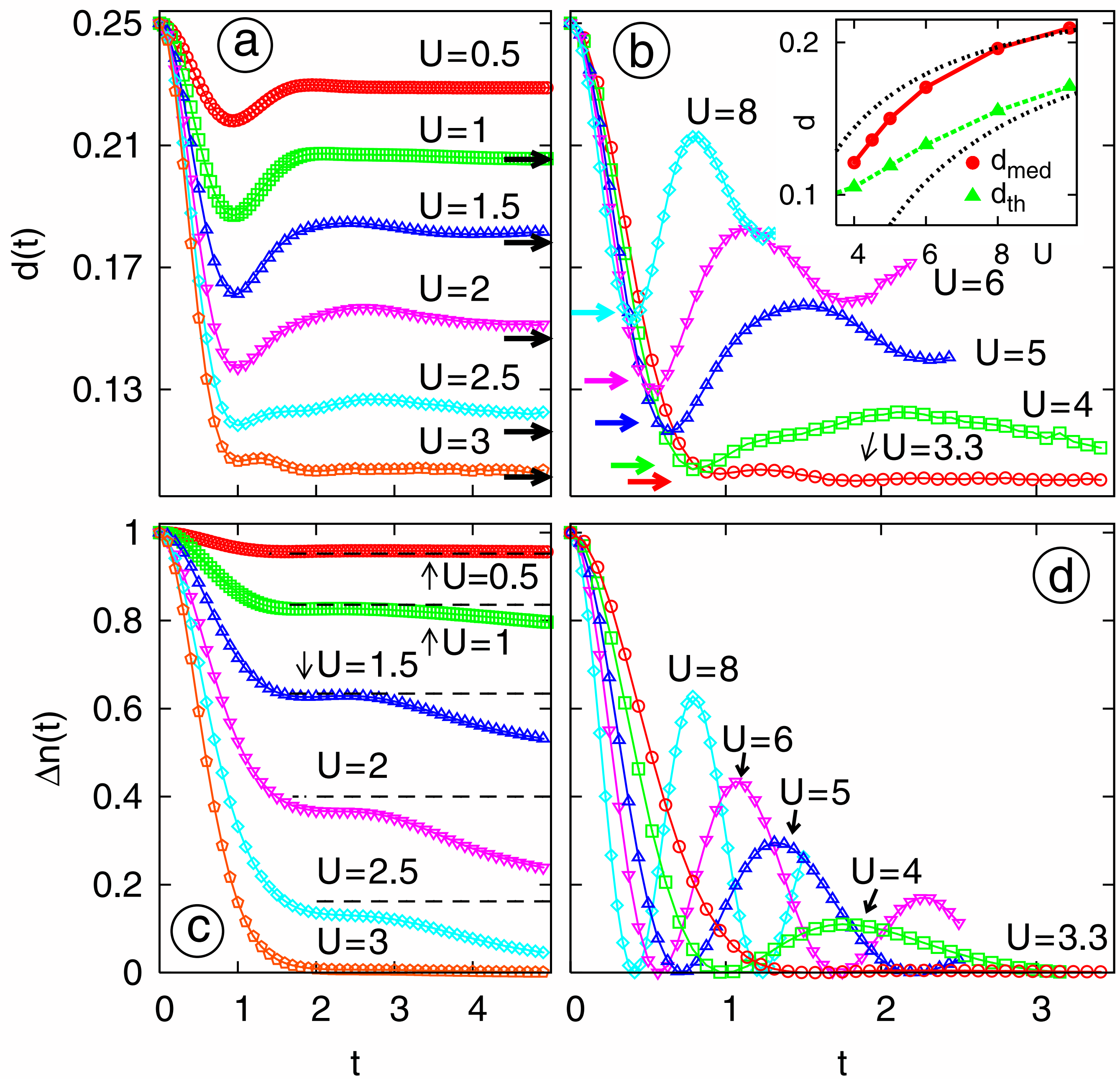


Dynamical transition in d=∞

➤ Sharp change of the relaxation behavoir between weak- and strong-coupling regimes.



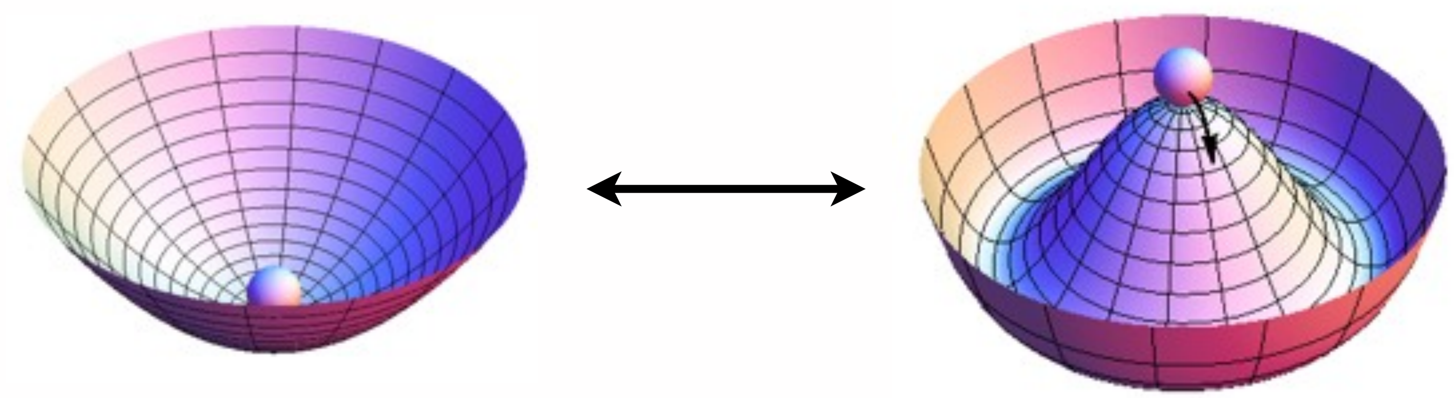




Eckstein, Kollar, Werner (2009, 2010).

Thermalization w/ long-range order

- How does the fermionic condensed-matter system prethermalize and thermalize after the interaction quench in the presence of a long-range order (classical fluctuations)?



- The order parameter dynamics has been described by a macroscopic (sometimes phenomenological) Ginzburg-Landau equation,

$$-\Gamma \frac{\partial m}{\partial t} = \frac{\delta \mathcal{F}_{GL}}{\delta m} = am + bm^3 - \frac{c}{2M}\nabla^2 m$$

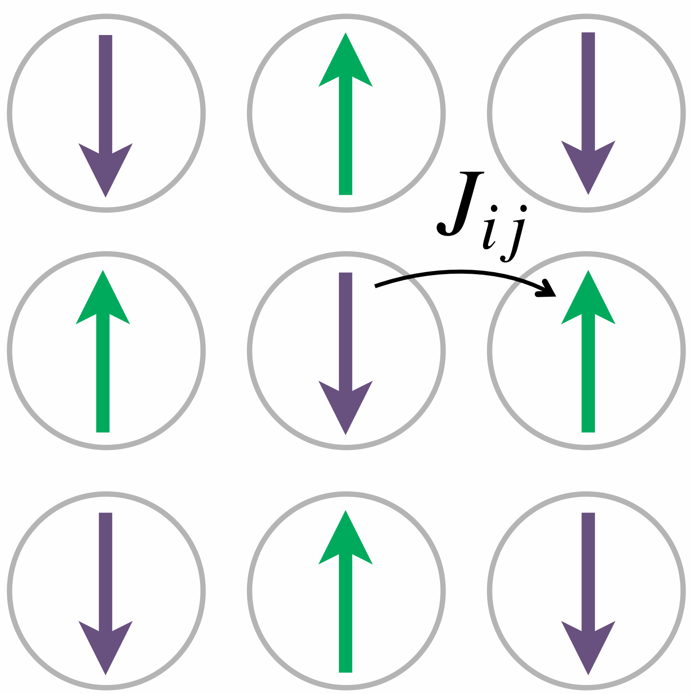
- Validity of the equation:

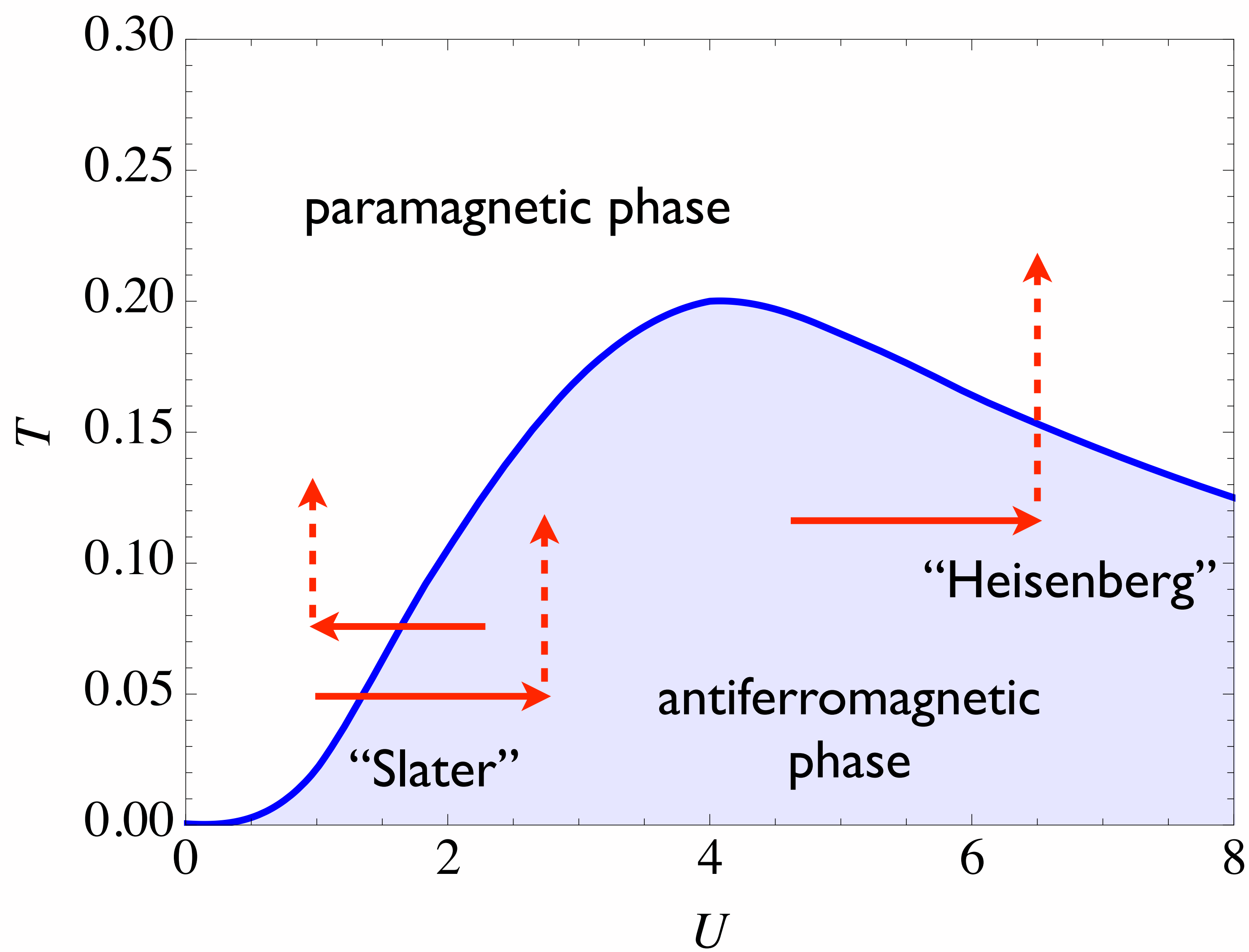
$$\tau_{\text{quasiparticle thermalization}} \ll \tau_{\text{order parameter}}$$

Hubbard model with AFM

- Interaction quench in the Hubbard model with AFM order:

$$H(t) = \sum_{ij\sigma} J_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U(t) \sum_{i} c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow}$$

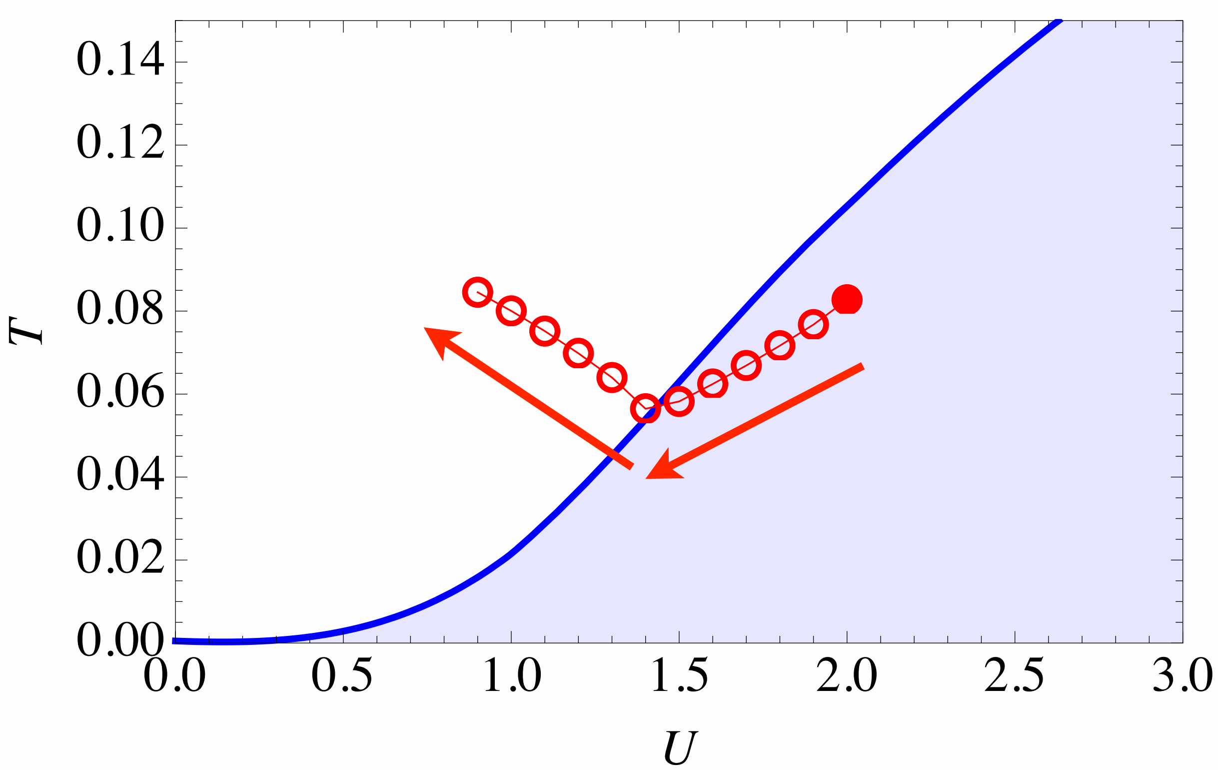


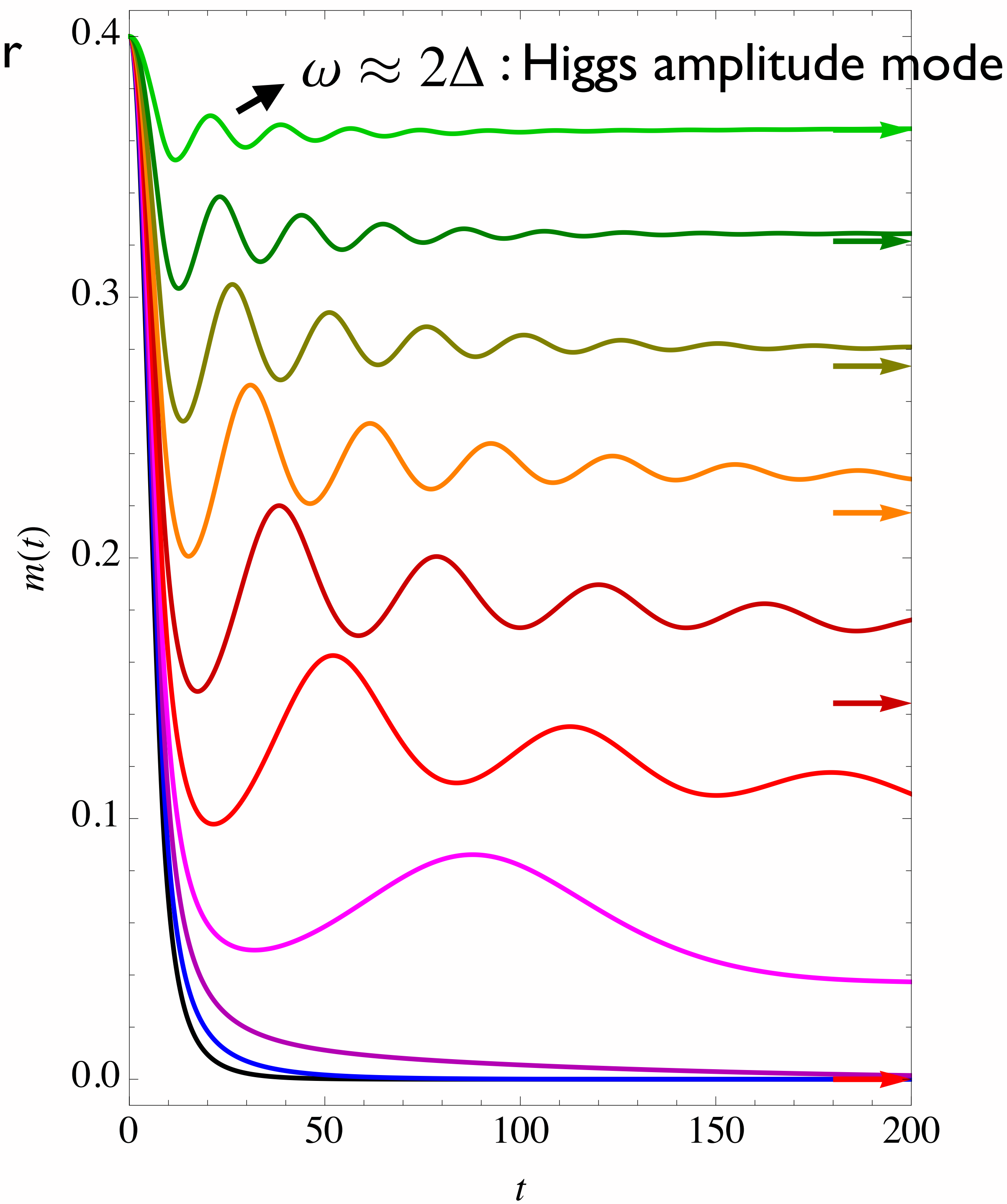


Quench: AFM → PM

Tsuji, Eckstein, Werner, PRL (2013)

- $m(t) = \langle |n_{i\uparrow}(t) - n_{i\downarrow}(t)| \rangle$: AFM order parameter
- The initial U_i is fixed.
- The final U_f ($< U_\mathrm{i}$) is systematically changed.





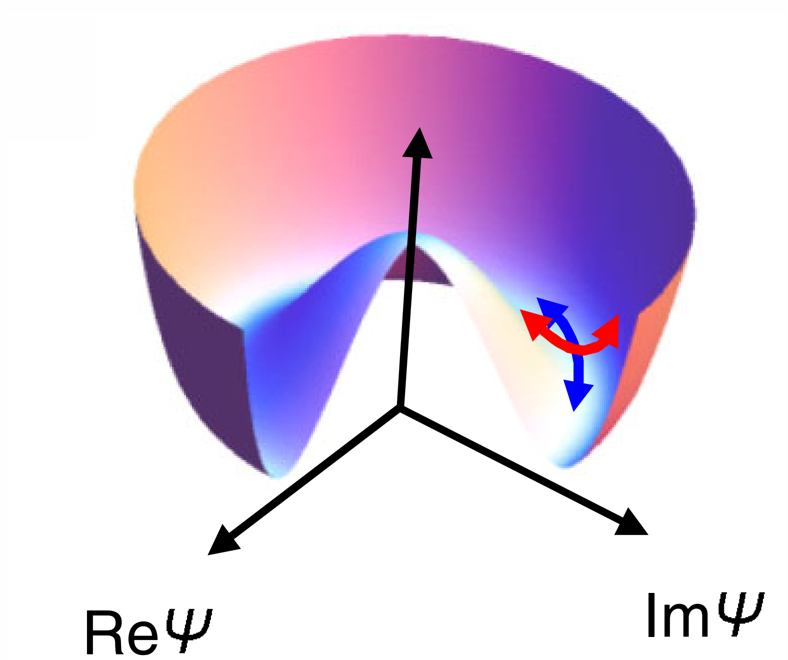
$U_i = 2.0, U_f = 1.0, 1.1, \ldots, 1.9$

Higgs Amplitude Mode in the BCS Superconductors $Nb_{1-x}Ti_xN$ Induced by Terahertz Pulse Excitation

Ryusuke Matsunaga,[1] Yuki I. Hamada,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3] Hirotaka Terai,[2] Zhen Wang,[2] and Ryo Shimano[1]

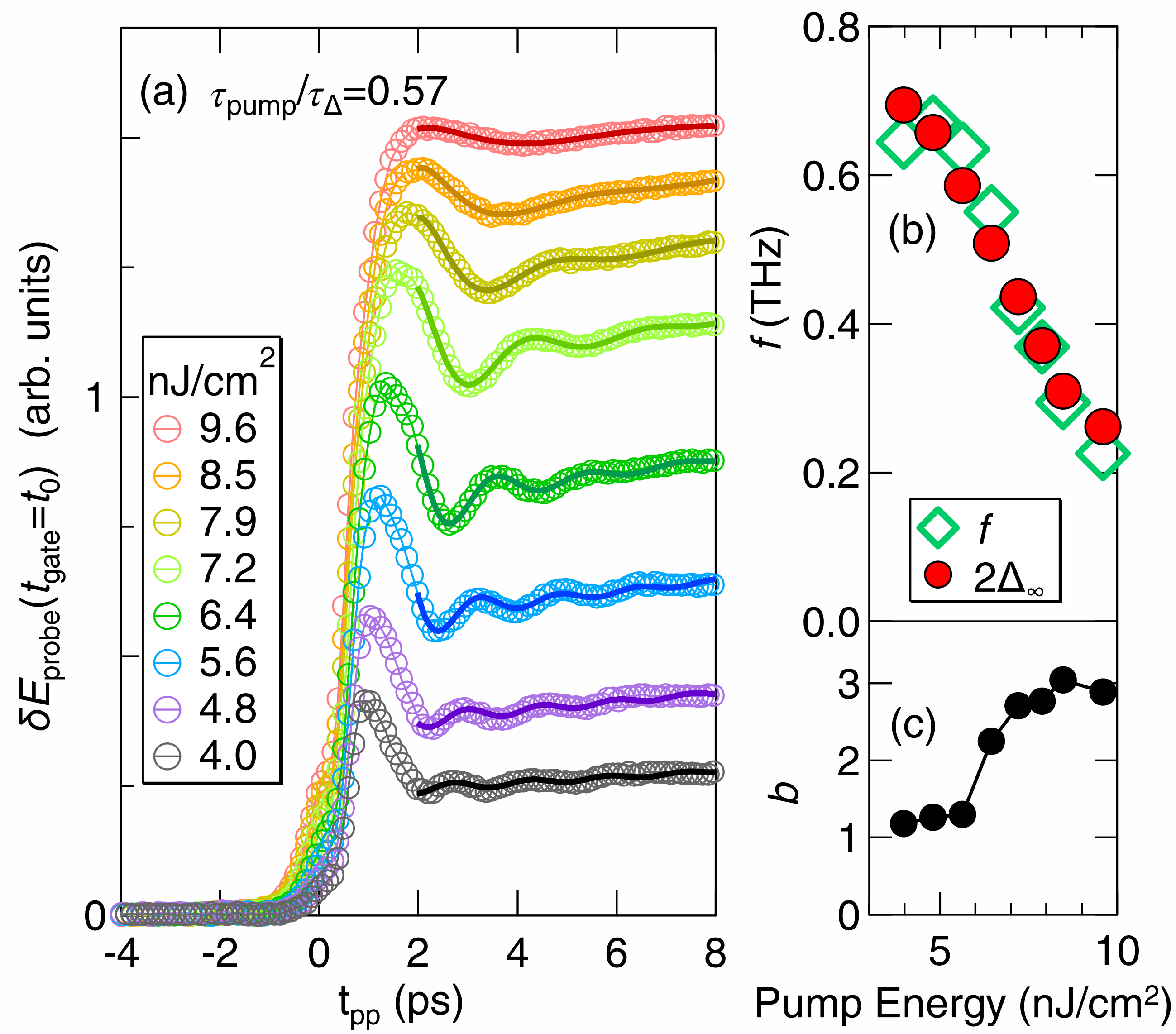
[1]*Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan*
[2]*National Institute of Information and Communications Technology, 588-2 Iwaoka, Nishi-ku, Kobe 651-2492, Japan*
[3]*National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan*

(Received 2 May 2013; published 29 July 2013)



➤ Higgs mode

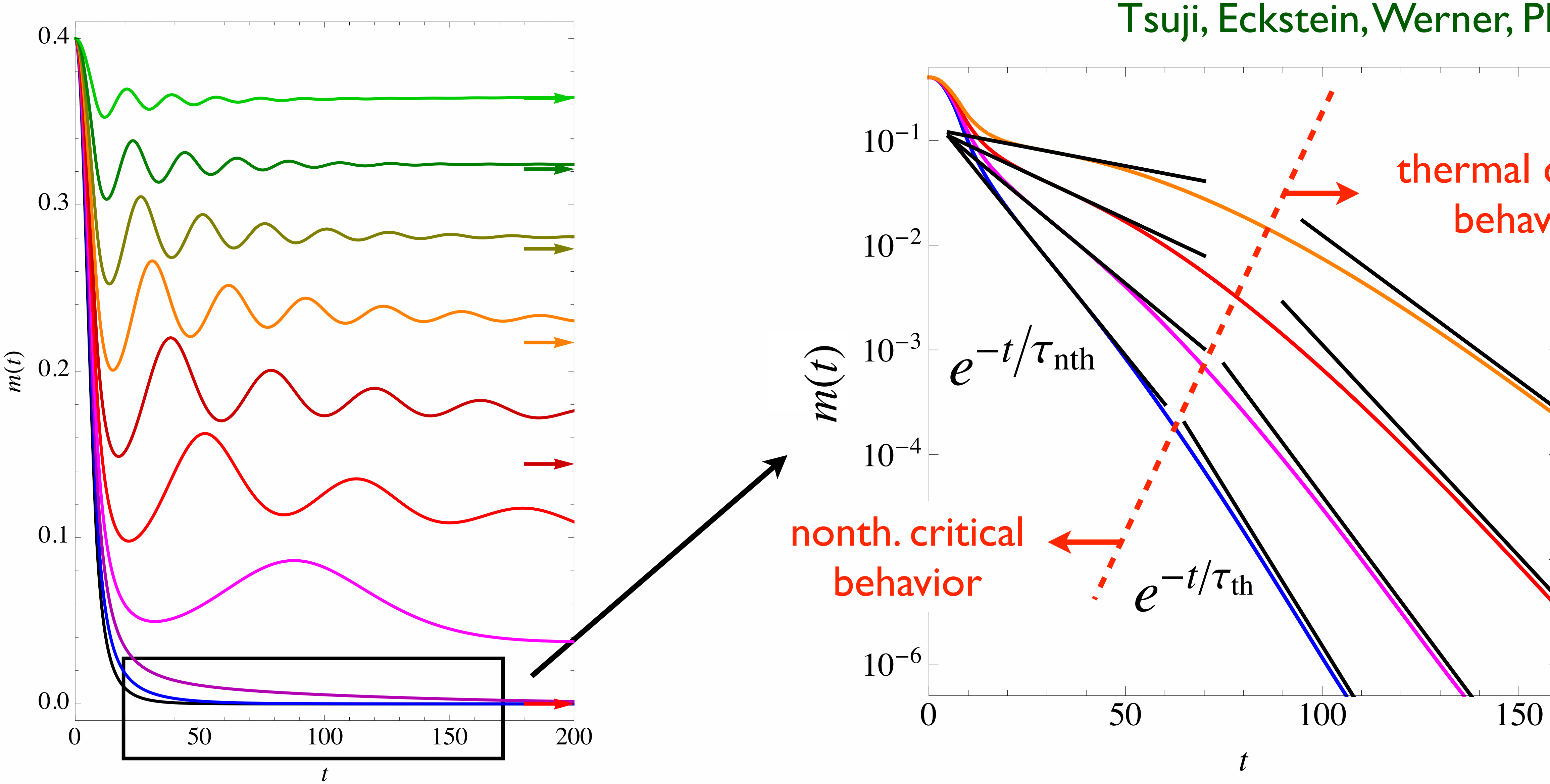
➤ Nambu-Goldstone mode



Two step relaxation

➤ Relaxation crossovers from the nonthermal critical behavior in the intermediate time scale to the thermal critical behavior in the long time scale.

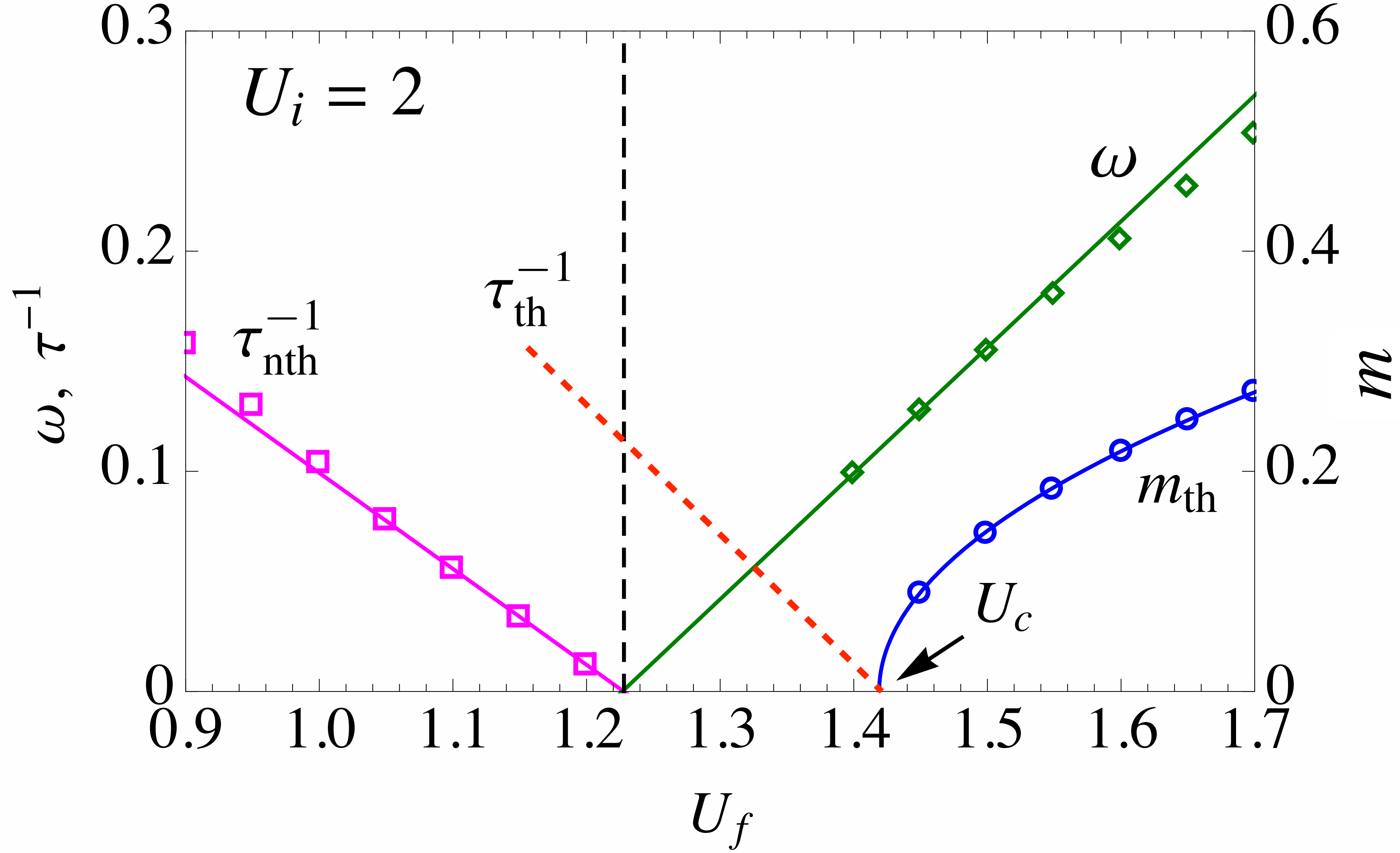
Tsuji, Eckstein, Werner, PRL (2013)



$U_i = 2.5, U_f = 1.6, 1.7, 1.8, 1.9$

Nonthermal criticality

Tsuji, Eckstein, Werner, PRL (2013)

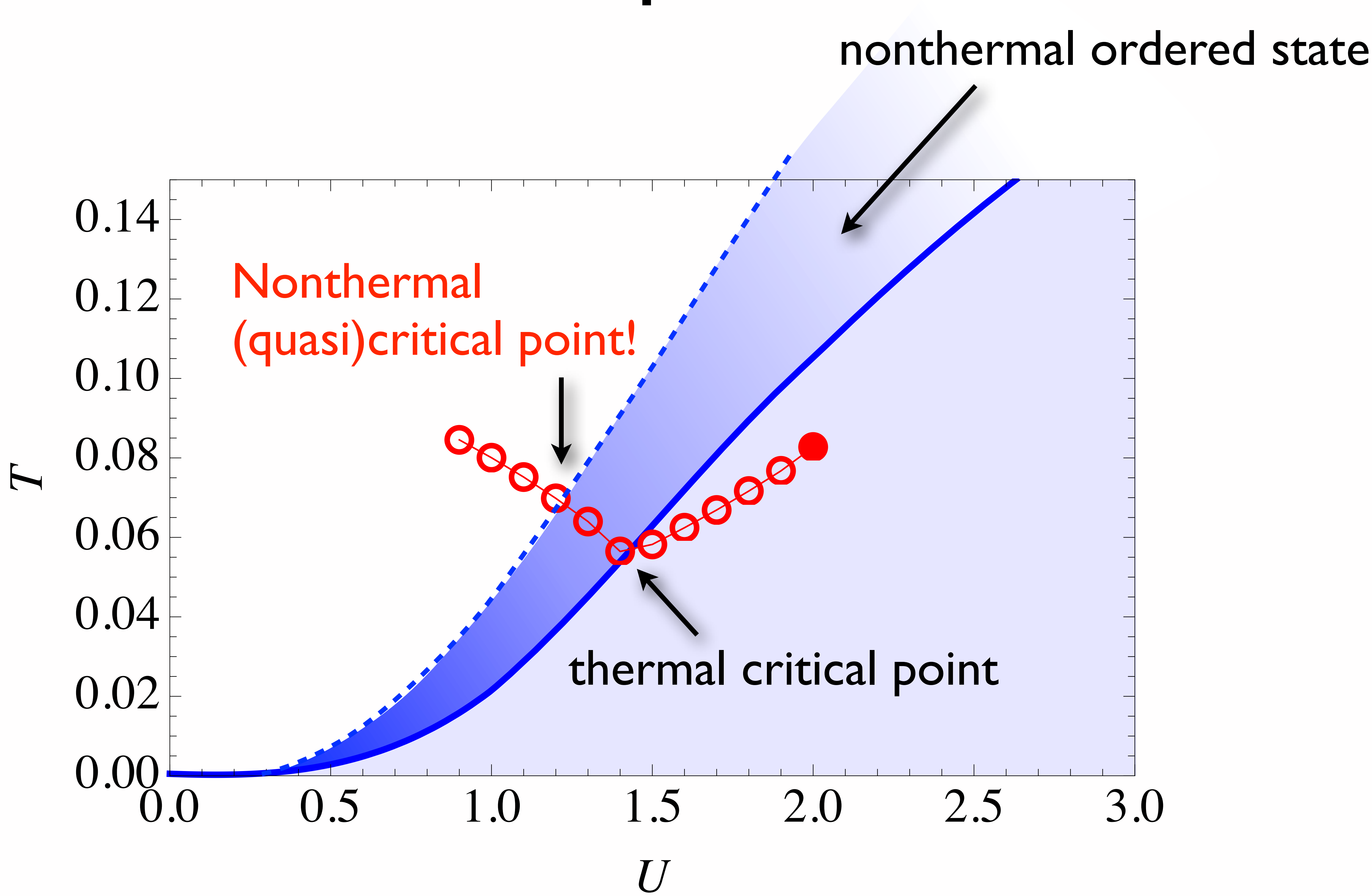


$\tau_{\mathrm{nth}}{}^{-1}$: Nonthermal relaxation time.

ω : Frequency of the Higgs mode.

m_{th} :Thermal value of the order parameter.

This implies...



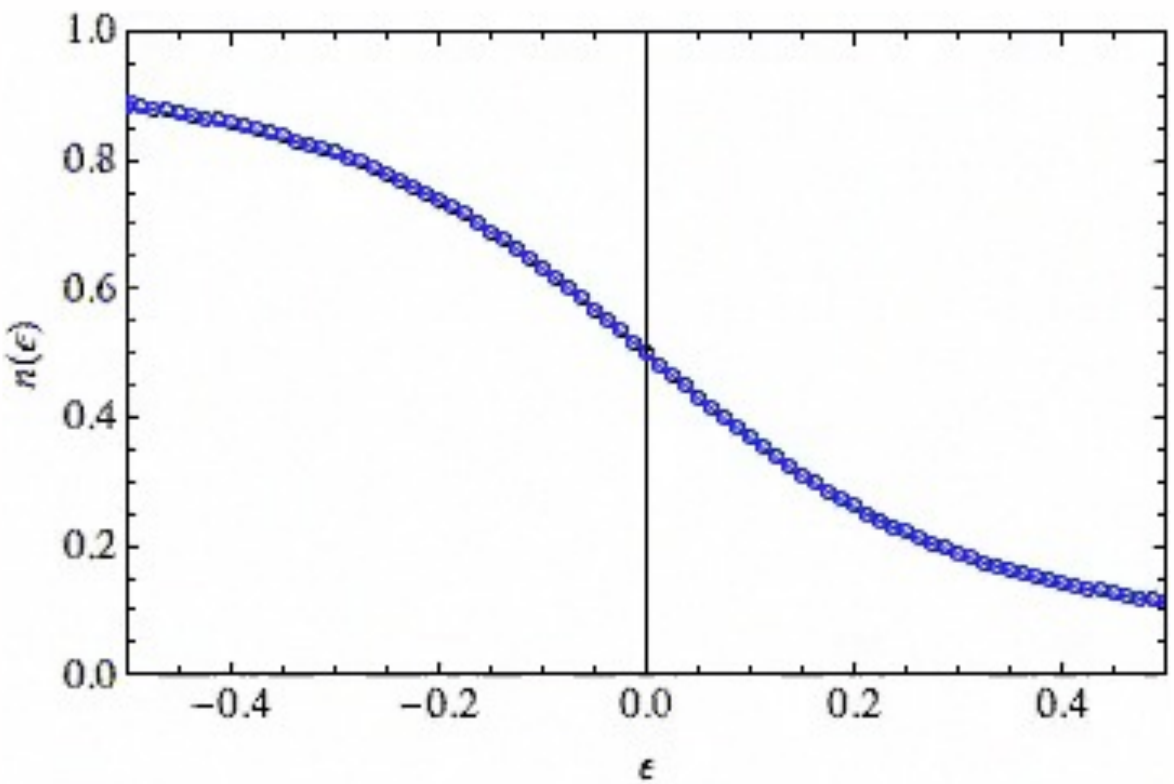
Momentum distribution

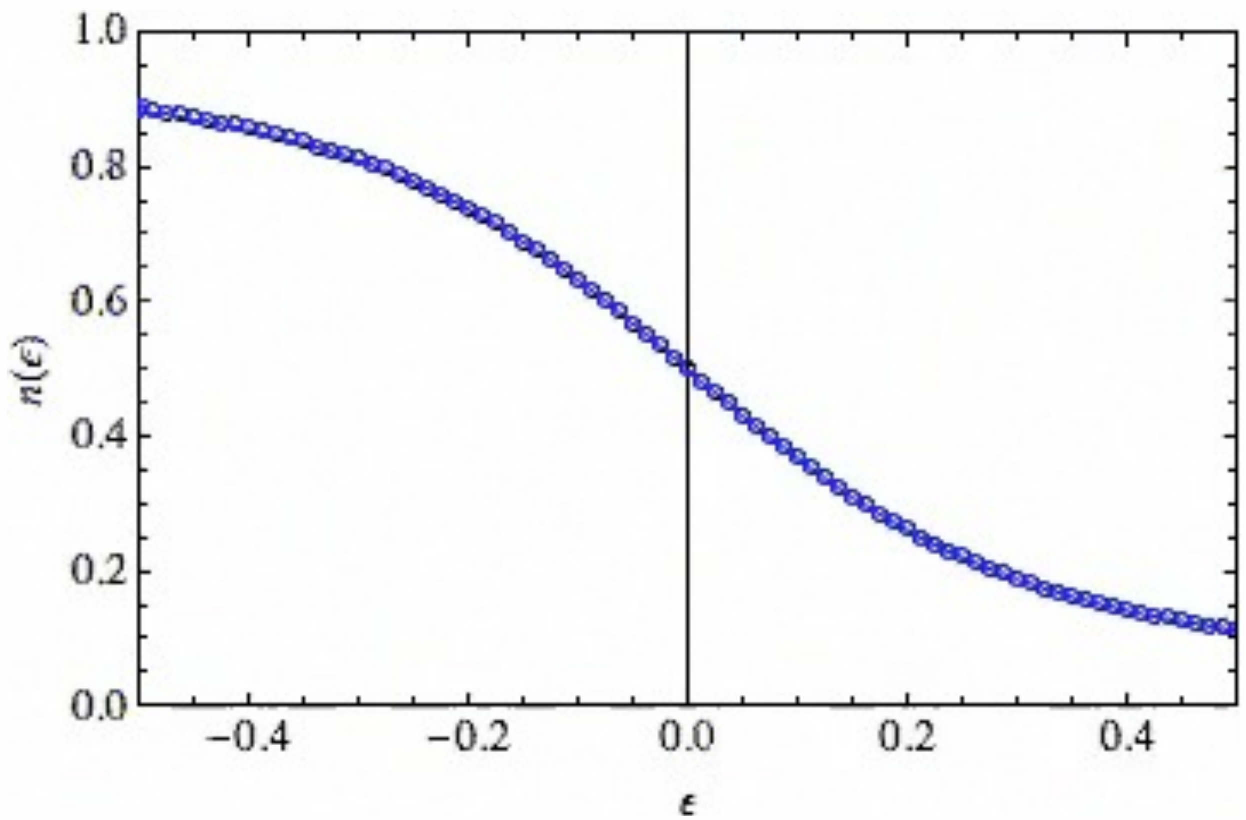
$$n_{\boldsymbol{k}}(t) = N^{-1} \sum_{ij} e^{i\boldsymbol{k}\cdot(\boldsymbol{R}_i - \boldsymbol{R}_j)} \langle c^\dagger_{i\sigma}(t) c_{j\sigma}(t) \rangle$$

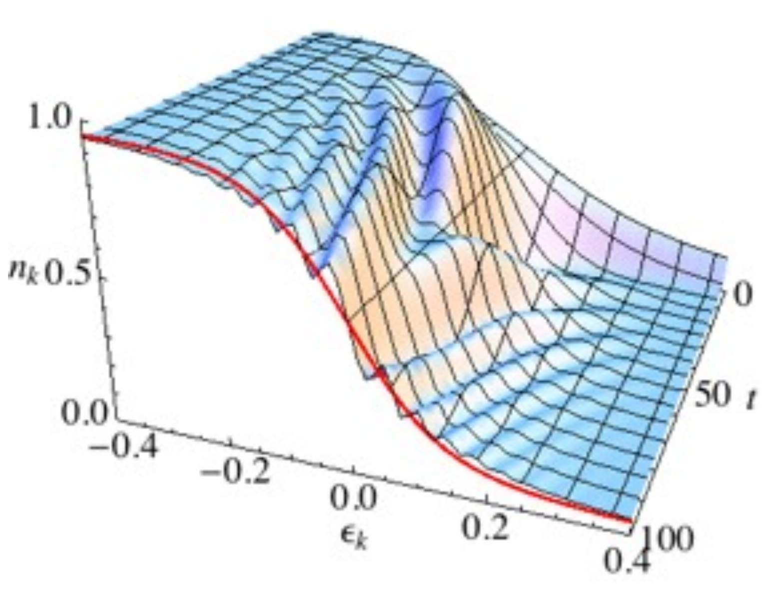
cf. T=0 static mean-field: $n_{\boldsymbol{k}} = \frac{1}{2} - \frac{\epsilon_{\boldsymbol{k}}}{2\sqrt{\epsilon_{\boldsymbol{k}}^2 + \Delta^2}}$

$\sim \epsilon_{\boldsymbol{k}}^{-2} \quad (\epsilon_{\boldsymbol{k}} \to \infty)$

$U_i = 2 \to U_f = 1.4$







➤ The momentum distribution shows a power-law decay:

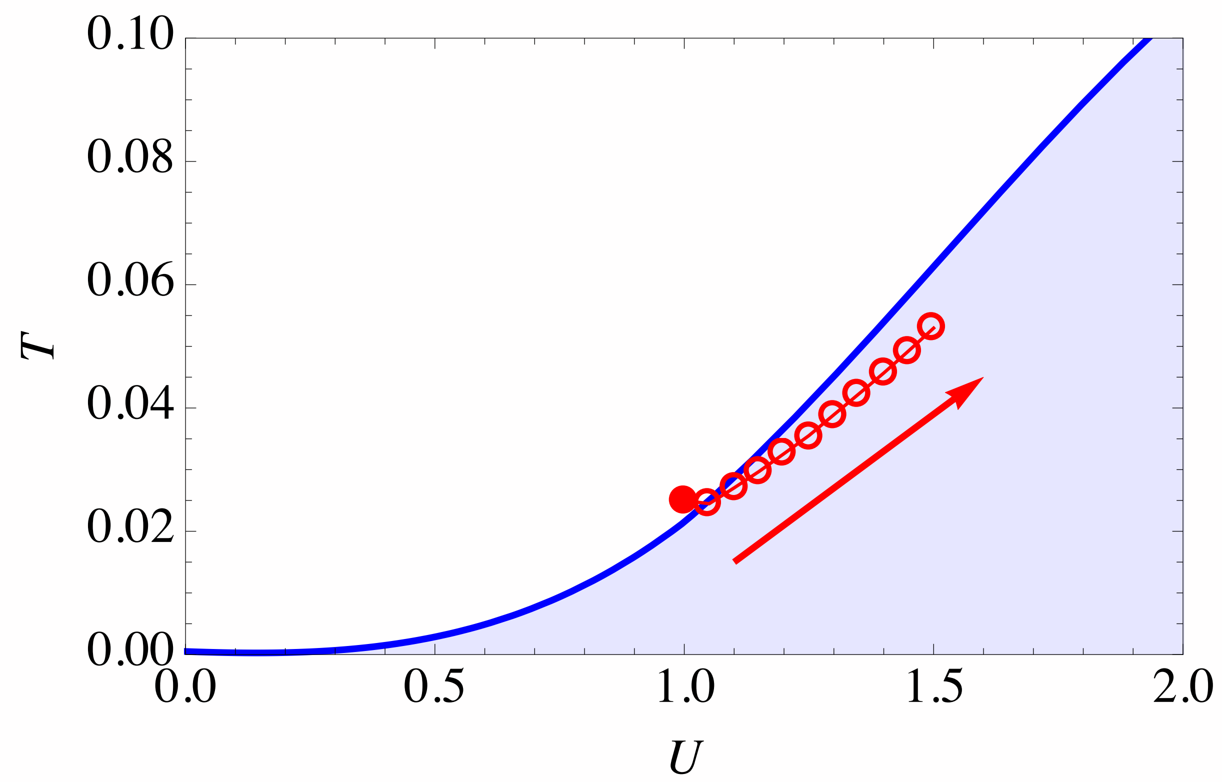
$$n(\epsilon) \sim \epsilon^{-\alpha}$$

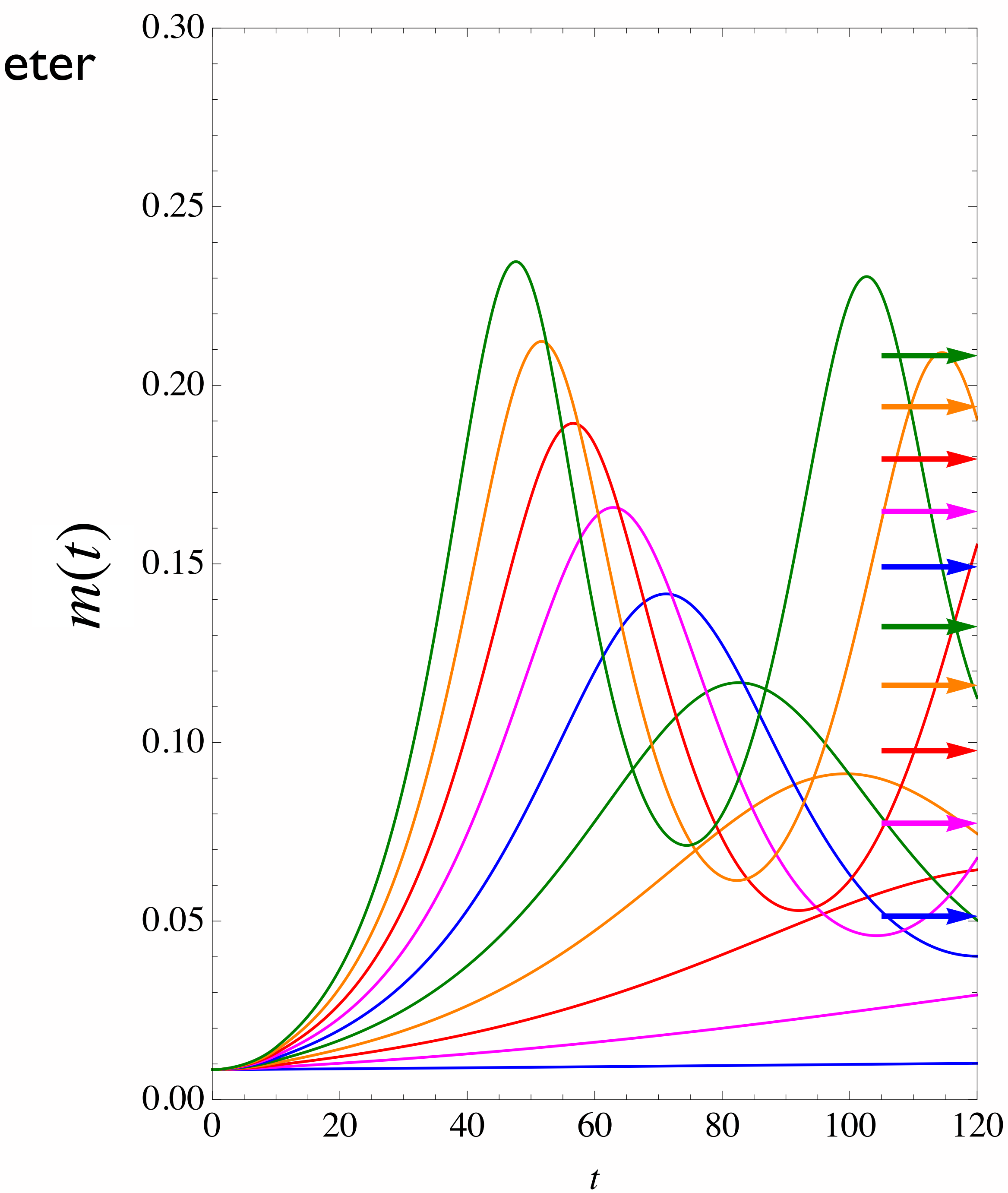
$\alpha \sim 1.17$ (in this case) : non-universal?

Quench: PM → AFM

Tsuji, Werner, PRB (2013)

- $m(t) = \langle |n_{i\uparrow}(t) - n_{i\downarrow}(t)| \rangle$: AFM order parameter
- The initial U_i is fixed.
- The final U_f (> U_i) is systematically changed.

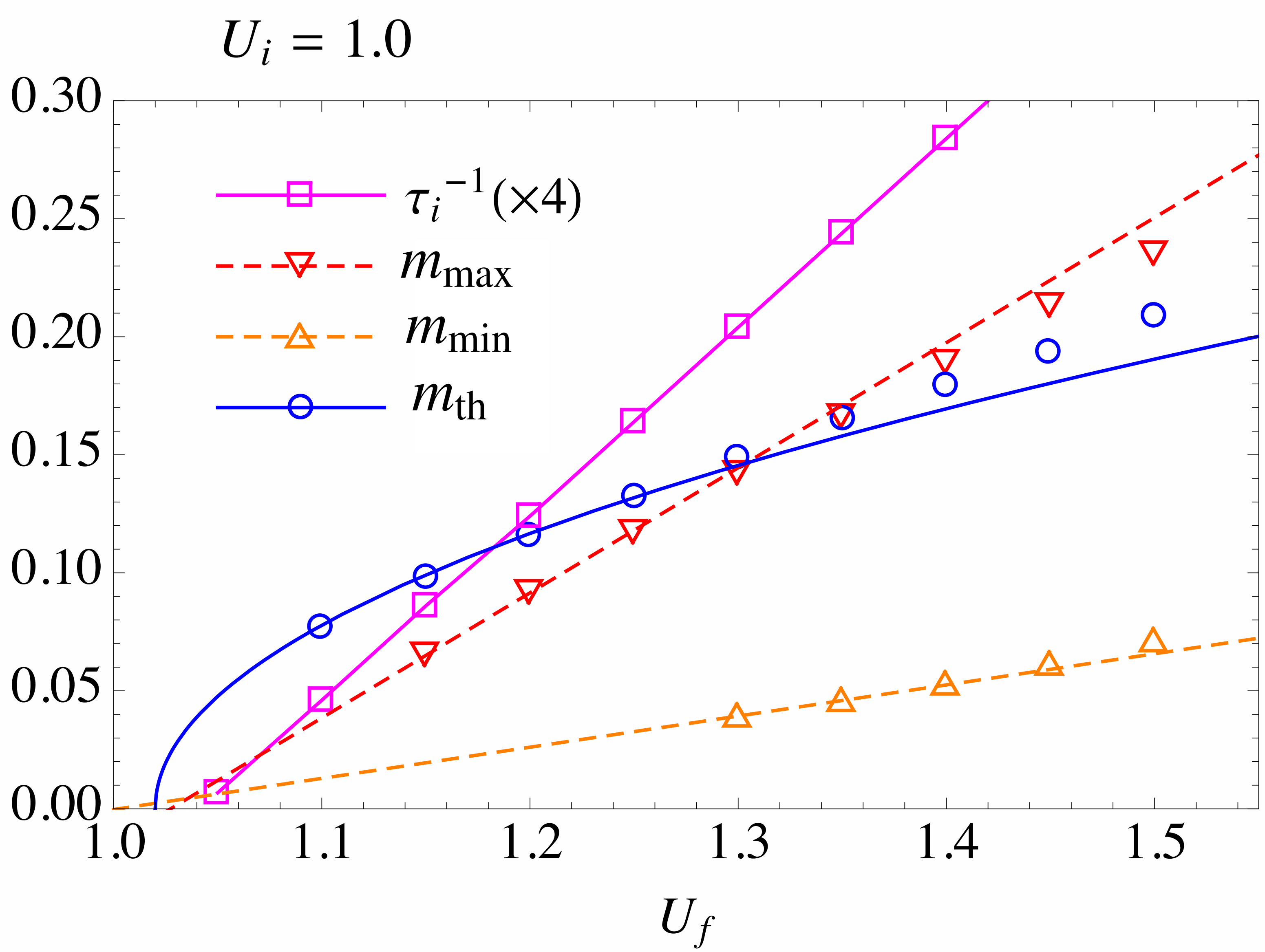




$U_i = 1.0, U_f = 1.05, 1.1, \ldots, 1.5$

Nonthermal criticality

Tsuji, Werner, PRB (2013)



m_{max} : Maximum of the first peak in amplitude oscillation.

m_{min} : Minimum of the first peak in amplitude oscillation.

m_{th} : Thermal values of order parameter reached in the long-time limit.

τ_i : Rate of the initial exponential growth ($\Phi \propto e^{t/\tau_i}$).

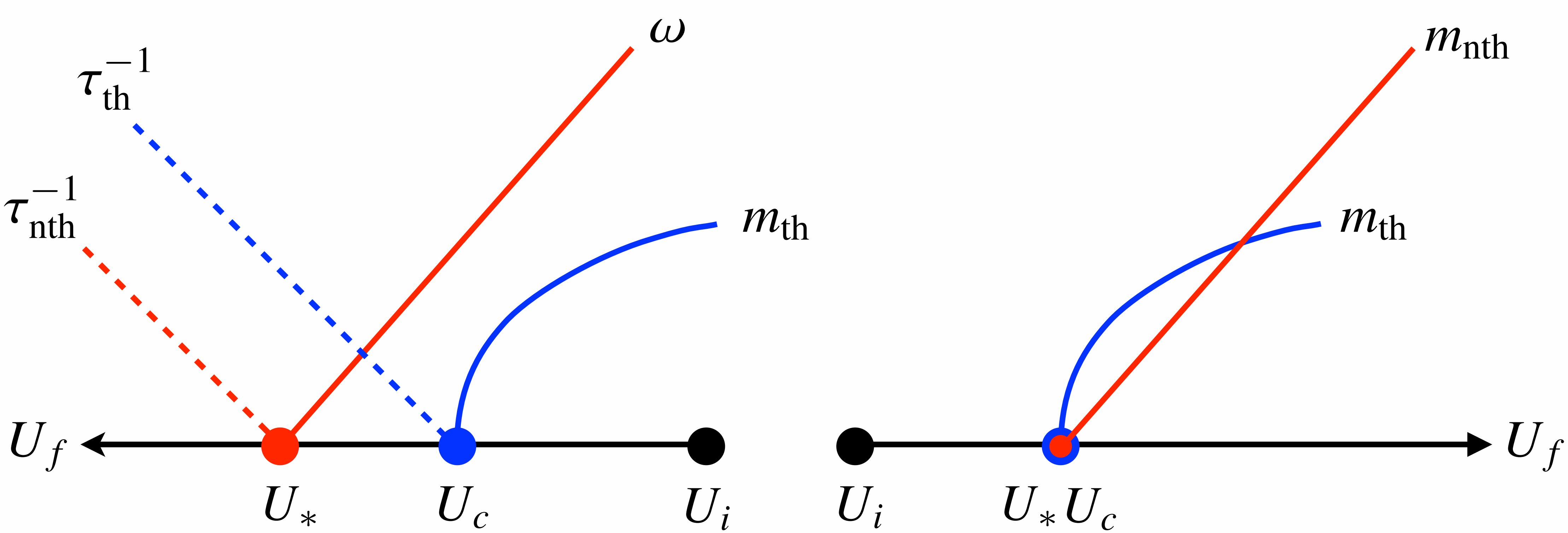
Summary of critical behavior

	intermediate time scale	longer time scale
τ	$\propto \lvert U_f - U_* \rvert^{-1}$	$\propto \lvert U_f - U_c \rvert^{-1}$
m	$\propto \lvert U_f - U_* \rvert^{1}$	$\propto \lvert U_f - U_c \rvert^{1/2}$
ω	$\propto \lvert U_f - U_* \rvert^{1}$	—

$z\nu$

β

Hohenberg, Halperin, RMP (1977)



Classical equation of motion

➤ Time-dep. Hartree approximation:

$$\Sigma_A(t,t') = U(t)n_B(t)\delta_{\mathcal{C}}(t,t')$$
$$\Sigma_B(t,t') = U(t)n_A(t)\delta_{\mathcal{C}}(t,t')$$

➤ Dyson equation:

$$\begin{pmatrix} i\partial_t + \mu - \Sigma_A & -\epsilon_{\boldsymbol{k}} \\ -\epsilon_{\boldsymbol{k}} & i\partial_t + \mu - \Sigma_B \end{pmatrix} \begin{pmatrix} G^{AA}_{\boldsymbol{k}} & G^{AB}_{\boldsymbol{k}} \\ G^{BA}_{\boldsymbol{k}} & G^{BB}_{\boldsymbol{k}} \end{pmatrix} = \begin{pmatrix} \delta_{\mathcal{C}} & 0 \\ 0 & \delta_{\mathcal{C}} \end{pmatrix}$$

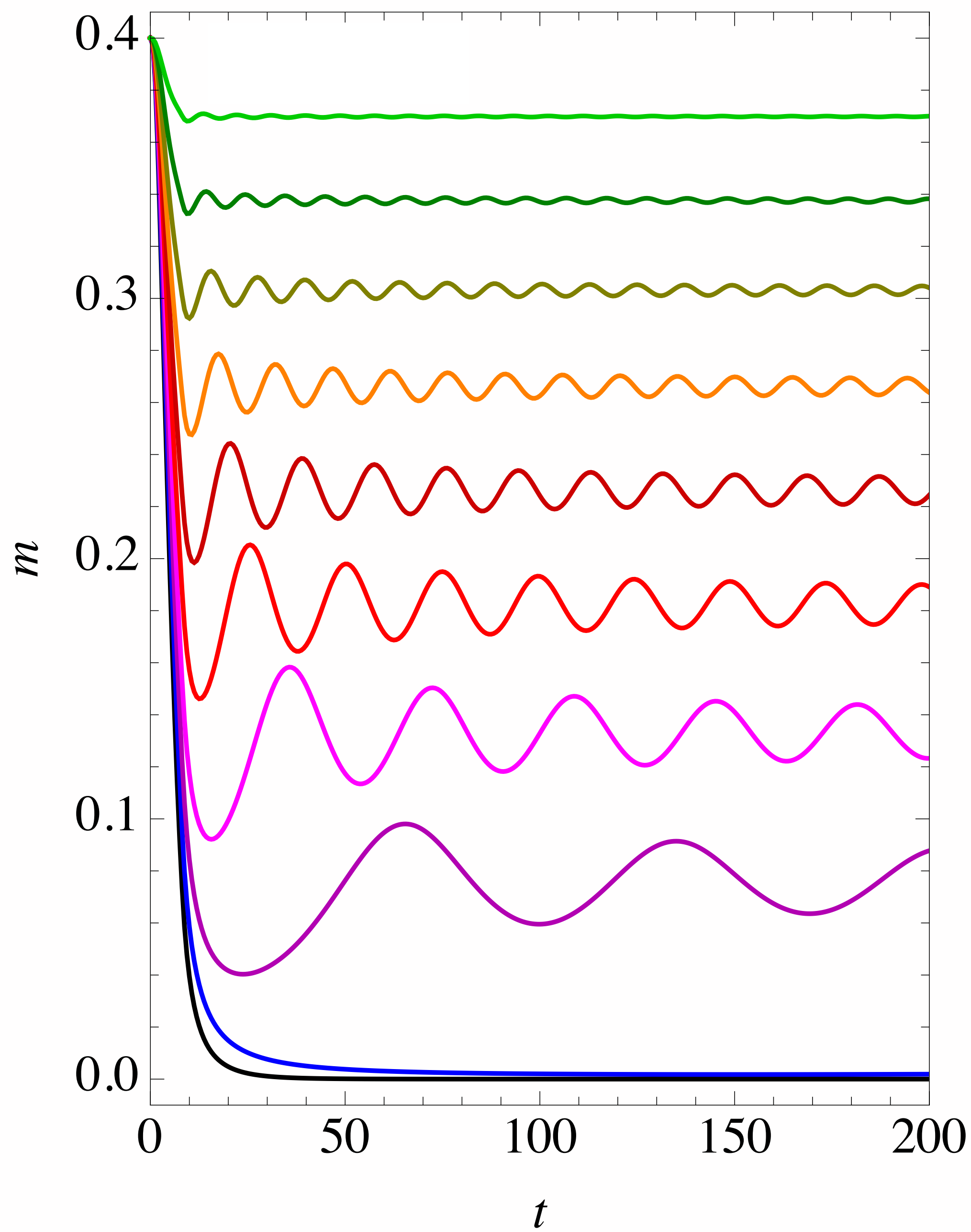
➤ The classical equation of motion :

$$\partial_t \boldsymbol{f}_{\boldsymbol{k}}(t) = \boldsymbol{b}_{\boldsymbol{k}}(t) \times \boldsymbol{f}_{\boldsymbol{k}}(t)$$
$$\boldsymbol{b}_{\boldsymbol{k}}(t) = (-2\epsilon_{\boldsymbol{k}}, 0, U(t)m(t))$$
$$m(t) = \sum_{\boldsymbol{k}} f^z_{\boldsymbol{k}}(t)$$

cf. Time-dep. BCS equation: Barankov, et al. (2004, 2006), Yuzbashyan et al. (2005, 2006), Warner, Leggett (2005).

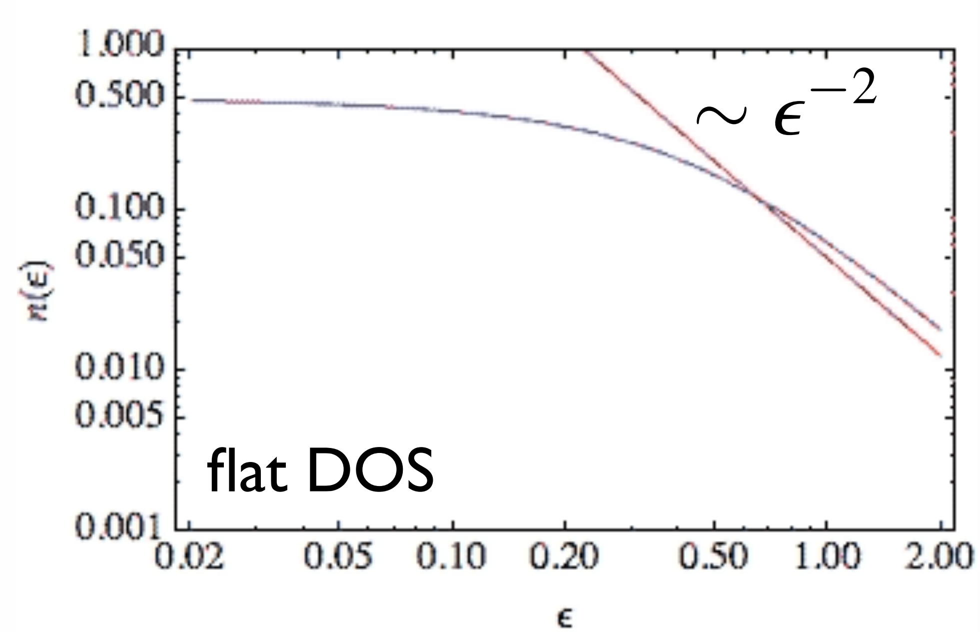
Classical equation of motion

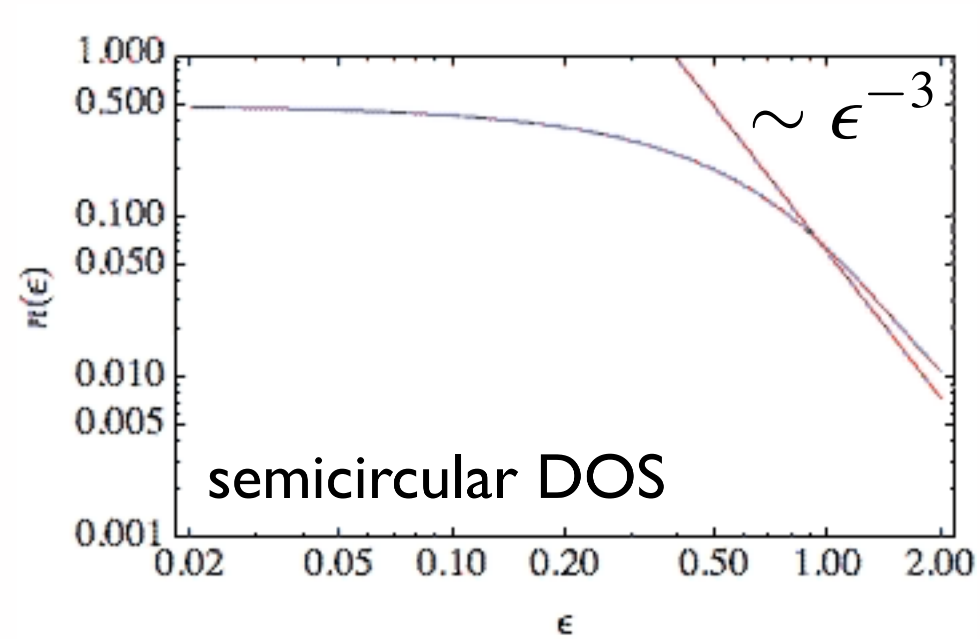
➤ The Hartree results:



$U_i = 2.0, U_f = 1.0, 1.1, \ldots, 1.9$

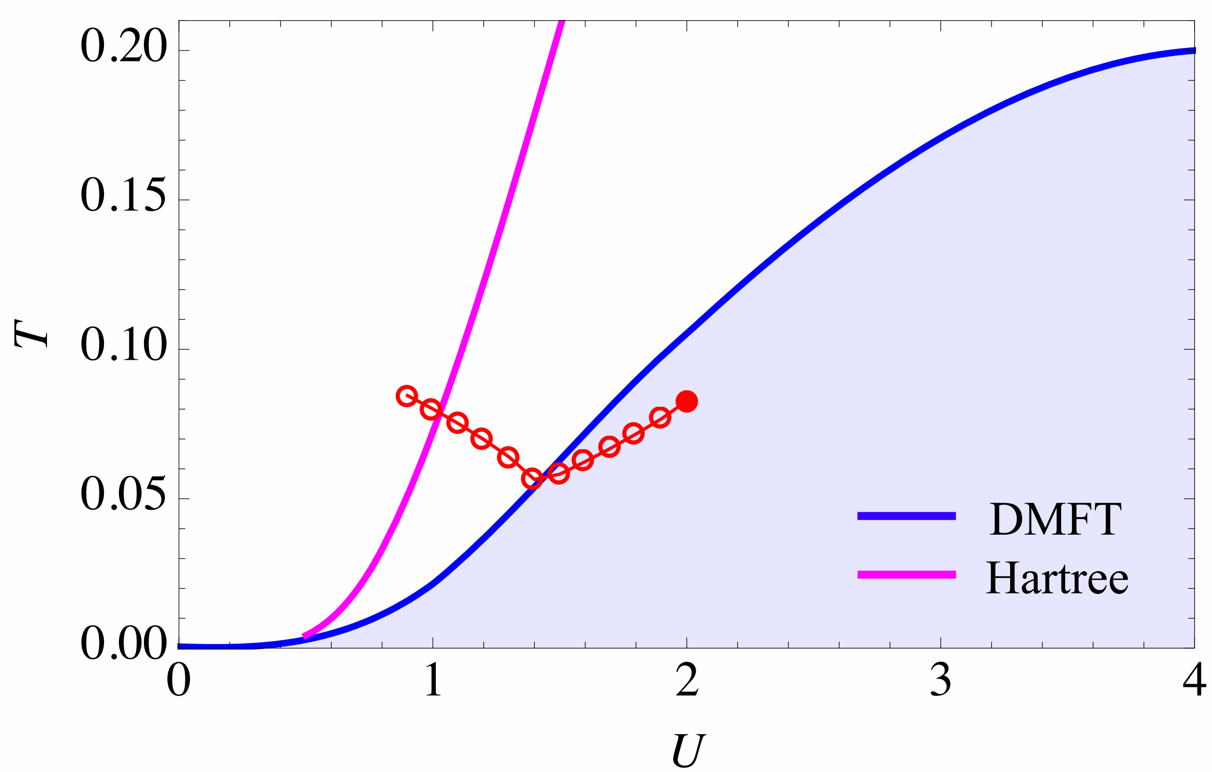
➤ The momentum distribution (U=2→1.8):

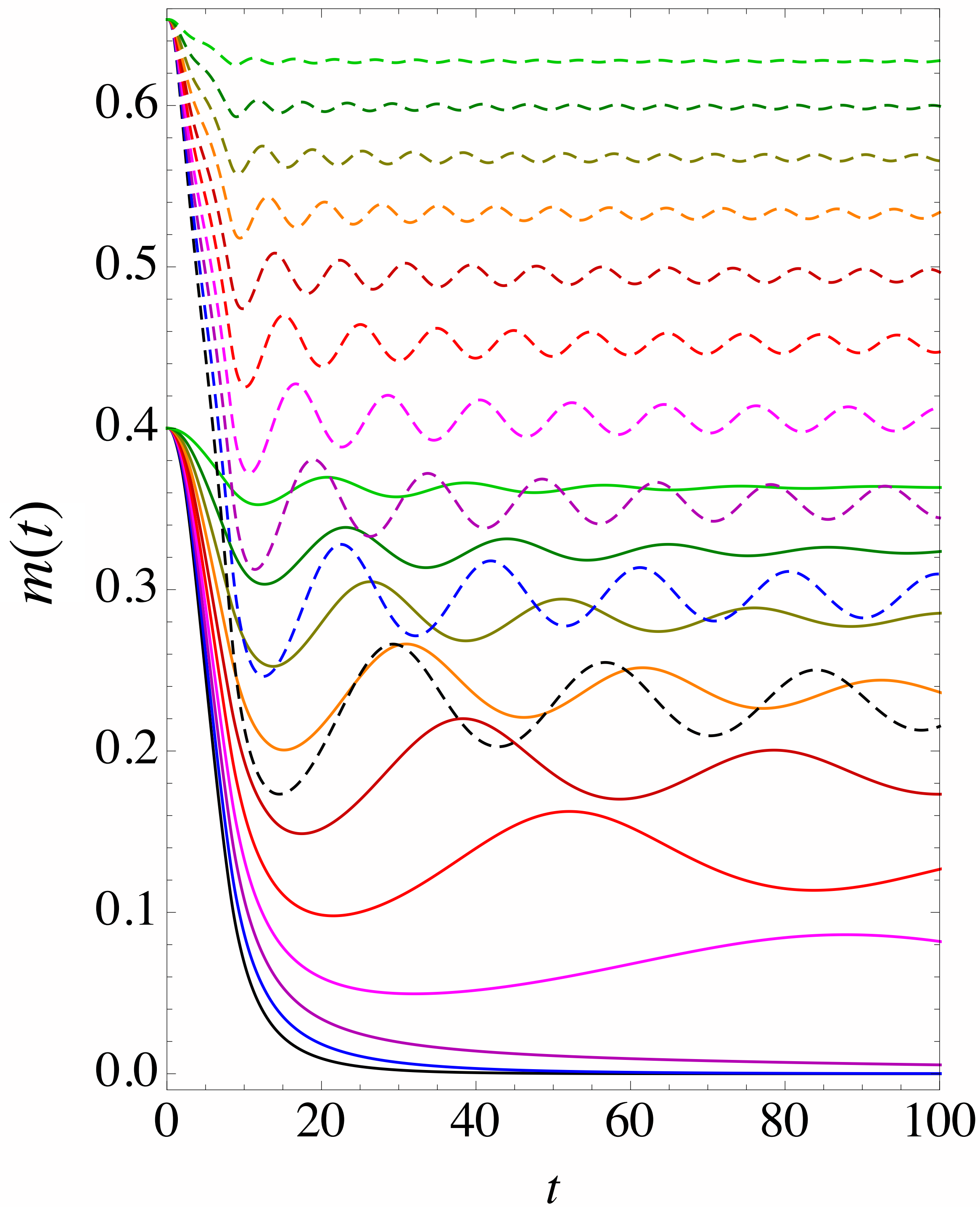




Hartree doesn't agree

- Hartree results: dashed lines (left panel)
- DMFT results: solid lines (left panel)



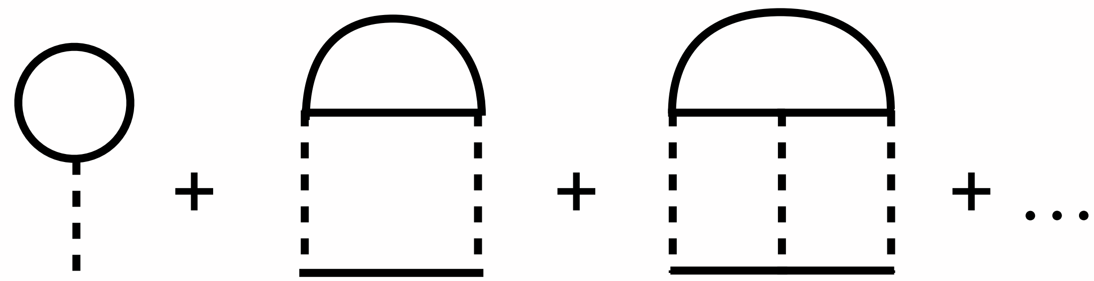


$U_i = 2.0, U_f = 1.0, 1.1, \ldots, 1.9$

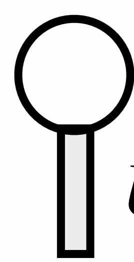
Renormalized interaction

➤ Argument: short-time dynamics is governed by quasiparticles having a renormalized interaction $\tilde{U}(t)$, which has a one-to-one correspondence with the Hartree particles:

$$\Sigma(t, t') =$$

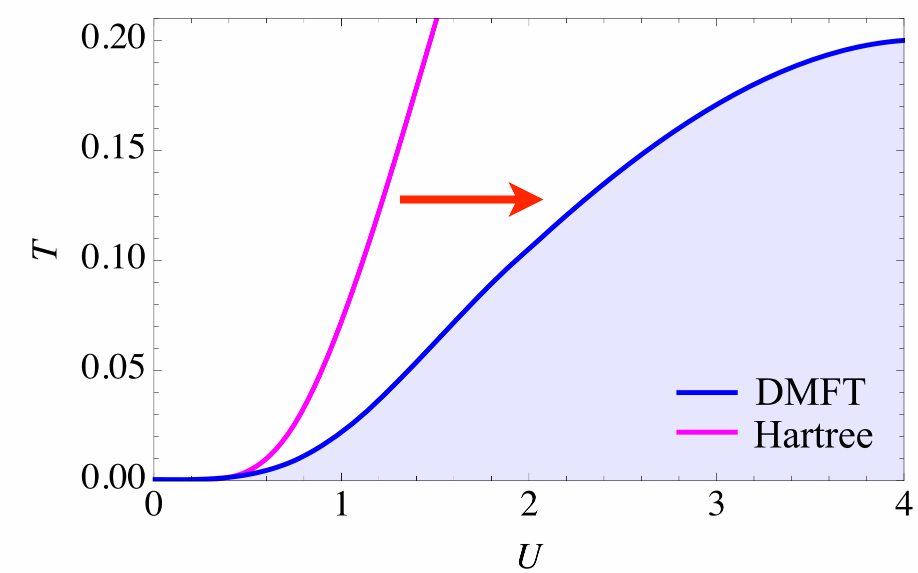


$$\approx$$



$\tilde{U}$

$U \longrightarrow \widetilde{U}(U)$: non-singular

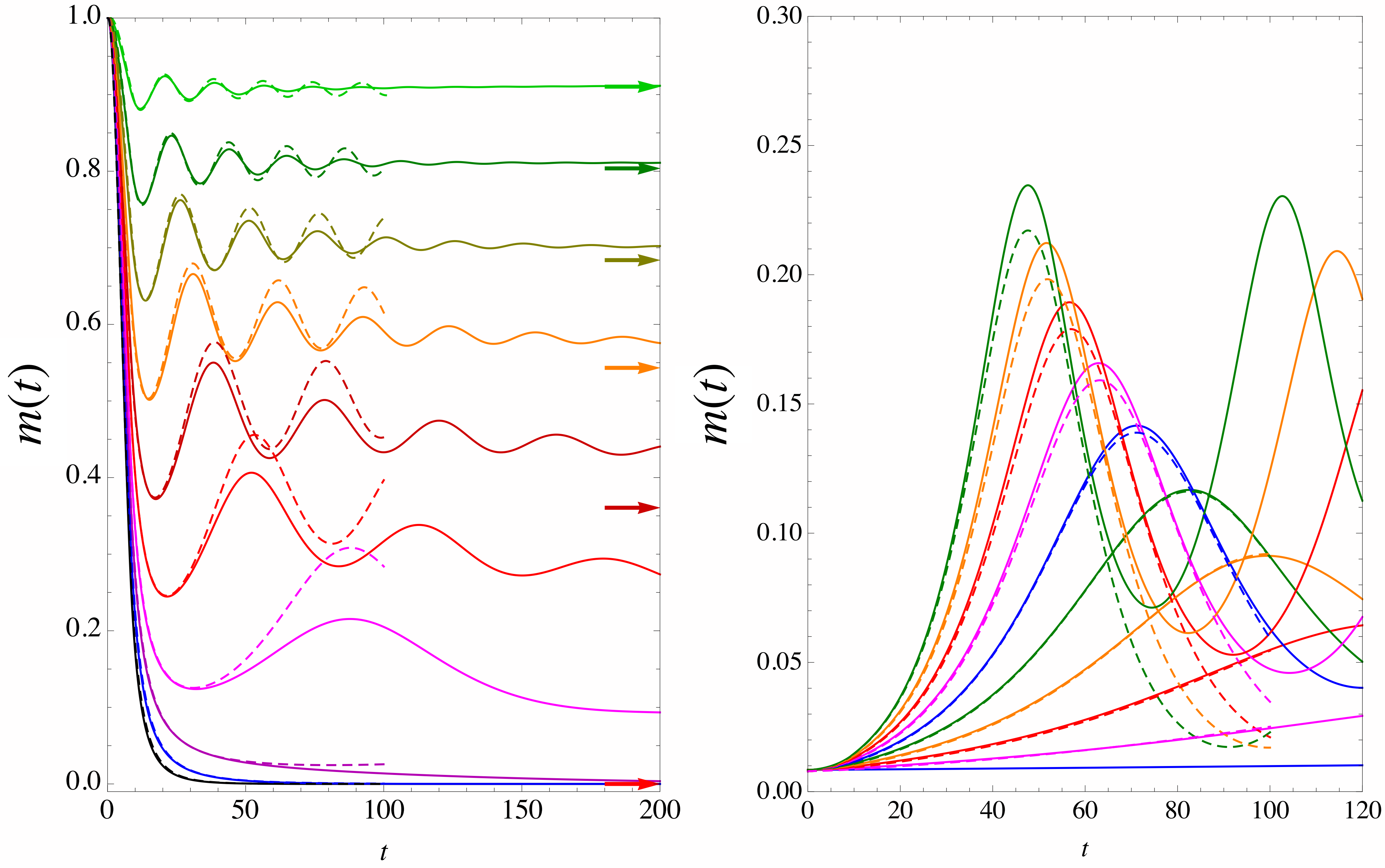


➤ Pairing interaction is effectively reduced due to quantum fluctuations.
e.g. *T*-matrix theory (Nozières, Schmidtt-Rink, '85).

$$\widetilde{U} = \frac{U}{1 + U\Gamma}$$

Renormalized Hartree agrees

- We compare the DMFT results (solid curves) with the Hartree solution for the quasiparticles with the renormalized interaction (dashed).



Nonthermal criticality

- From the argument, it follows that the order parameter satisfies:

$$-\frac{\partial^2 m}{\partial t^2} = \frac{\partial \mathcal{F}_{\mathrm{nth}}}{\partial m} \quad \text{with} \quad \mathcal{F}_{\mathrm{nth}} = -\frac{1}{2}am^2 + \frac{\widetilde{U}_f^2}{8}m^4$$

- The constant a is determined from a condition

$$-\tilde{U}(U_f)\sum_{\boldsymbol{k}} \frac{2\epsilon_{\boldsymbol{k}}}{(2\epsilon_{\boldsymbol{k}})^2 + a} f_0(\epsilon_{\boldsymbol{k}}) = 1$$

where $f_0(\epsilon_k)$ is the initial momentum distribution. From this, one can show that

$$a = a_0(U_f - U_*)^2$$

which contrasts with the conventional GL theory,

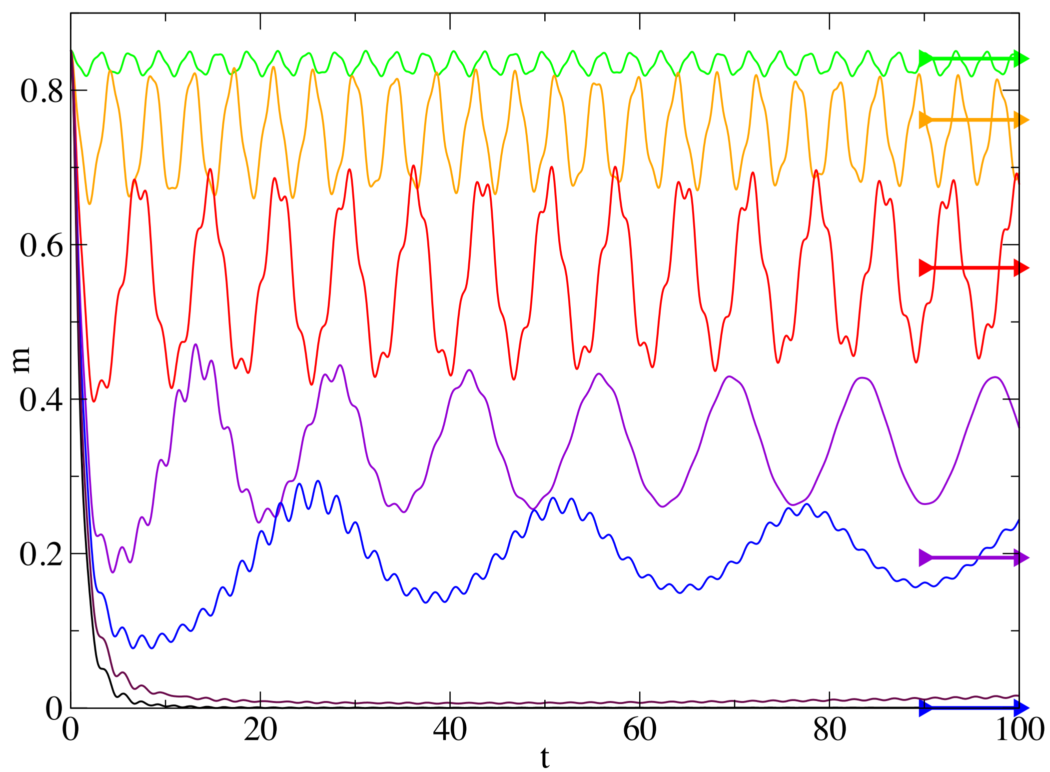
$$a = a_0(U_f - U_c)$$

- This evidences that the nonthermal critical point belongs to a universality class different from the conventional GL.

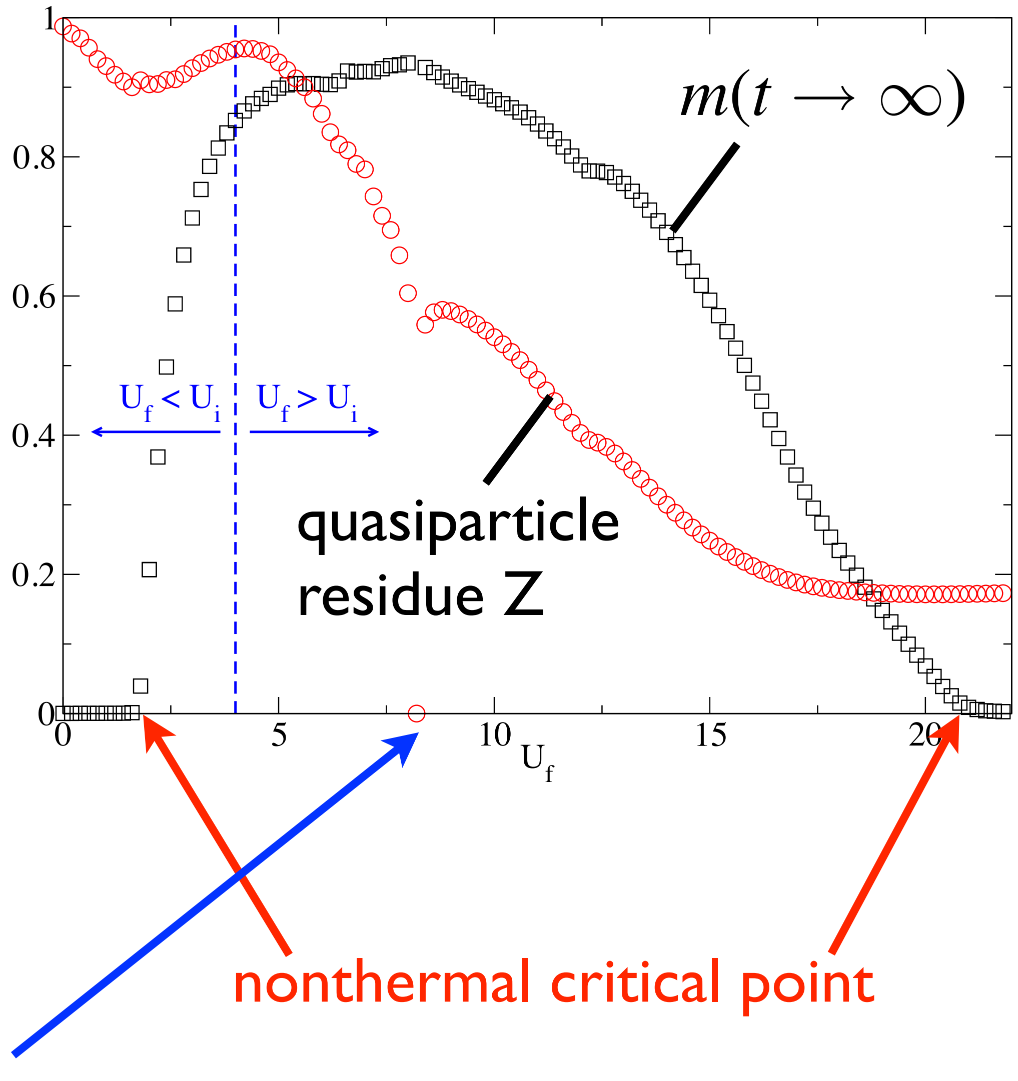
Time-dep. Gutzwiller approximation

Sandri, Fabrizio, PRB (2013).

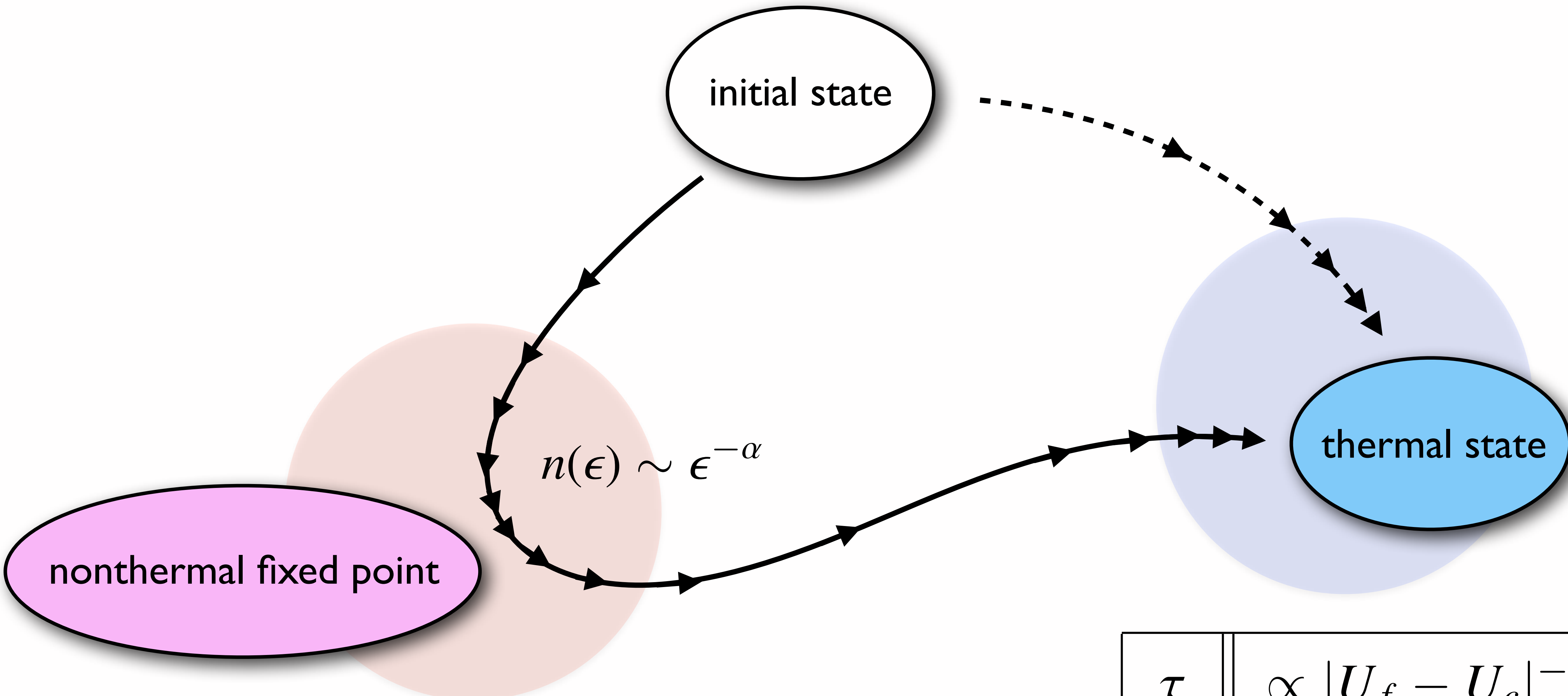
Qualitatively similar results have been obtained by other methods.



$$U_i = 4 \longrightarrow U_f = 3.8, 3.2, 2.6, 2.2, 2.0, 1.8, 1.6$$



Summary



τ	$\propto \lvert U_f - U_* \rvert^{-1}$
m	$\propto \lvert U_f - U_* \rvert^{1}$
ω	$\propto \lvert U_f - U_* \rvert^{1}$

τ	$\propto \lvert U_f - U_c \rvert^{-1}$
m	$\propto \lvert U_f - U_c \rvert^{1/2}$
ω	—

Acknowledgments

University of Fribourg
Philipp Werner

University of Hamburg-CFEL
Martin Eckstein

University of Geneva
Peter Barmettler

University of Tokyo
Hideo Aoki
Takashi Oka

University of Augsburg
Marcus Kollar

Tsuji, Eckstein, Werner, Phys. Rev. Lett. 110, 136404 (2013).
Tsuji, Werner, Phys. Rev. B 88, 125126 (2013).
Werner, Tsuji, Eckstein, Phys. Rev. B 86, 205101 (2012).
Aoki, Tsuji, Eckstein, Kollar, Oka, Werner, arXiv:1310.5329 (review).
Tsuji, Barmettler, Aoki, Werner, arXiv:1307.5946.