Effective Temperature of Non-equilibrium Steady States from AdS/CFT Correspondence

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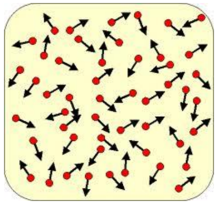
Ref. S. N. and H. Ooguri (Caltech/KIPMU), arXiv:1309.4089, to appear in PRD.

We employ the natural unit: $k_B=c=\hbar=1$.

Statistical Systems

If a system is at thermal equilibrium, the macroscopic nature of the system can be characterized by using only few macroscopic variables.
One such variable is temperature.



T, ...

How about non-equilibrium systems?

Non-equilibrium systems

Time-dependent systems　　Time-independent systems

Non-equilibrium steady states (NESS)

Do we still have a notion of (generalized) temperature that characterizes the physics of NESS?

This talk

We are going to show the following facts by using the AdS/CFT correspondence:

- A notion of generalized temperature (effective temperature) exists at least for some examples of NESS, even outside the linear-response regime.
- The effective temperature shows curious behaviors that may be counter-intuitive, in some cases.
- A useful/new picture of effective temperature is provided in the framework of AdS/CFT.

What is temperature?

Definitions of equilibrium temperature:

$$P \propto e^{-E/T},\ \ t_E \approx t_E + 1/T$$ Distributions

$$dE = TdS$$ Thermodynamics

$$D = T\mu$$ Fluctuation-dissipation relation

diffusion const. mobility

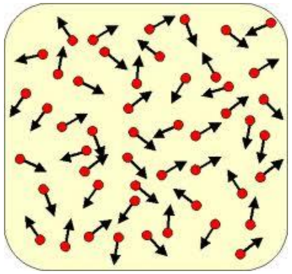
Another definition of temperature?

Yes, from AdS/CFT
in terms of black hole.

AdS/CFT correspondence

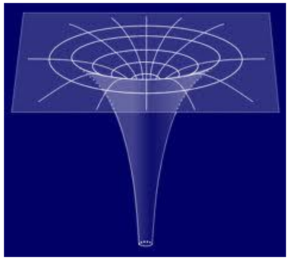
[Maldacena, 1997]

A strongly-interacting quantum gauge theory



equivalent

A classical gravity (general relativity) on a curved spacetime in higher dimensions.



Advantages in gravity picture

- Computations beyond the linear-response regime are possible.
- Different picture for physics is available.

General relativity

Einstein has formulated the theory of gravity in terms of geometry of space and time: general relativity.

“Energy-momentum deforms the spacetime.”

Einstein’s equation:

Metric: defines unit length in the geometry

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}^{(\text{matter})}$$

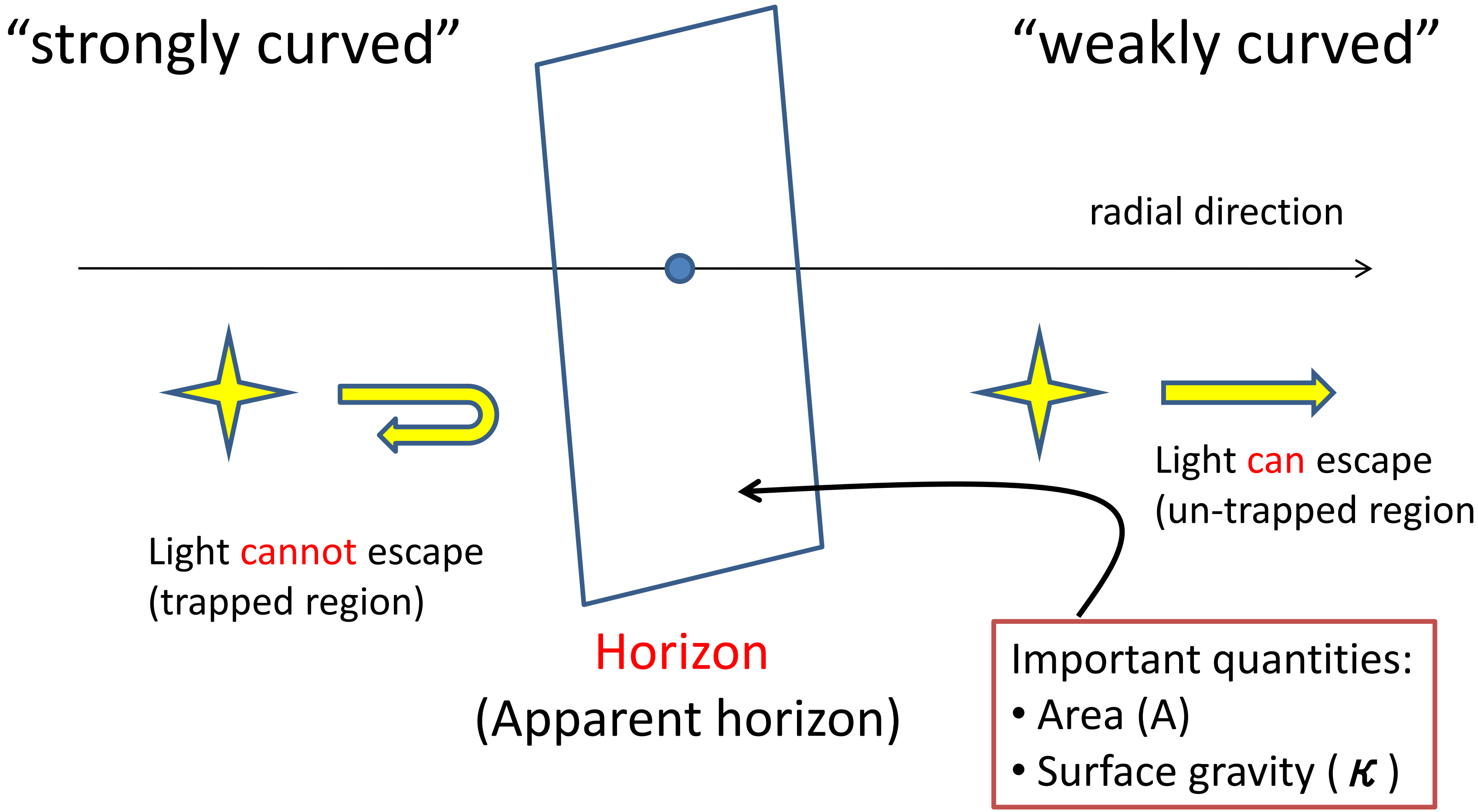
Curvature:~combination of second derivatives of the metric

Cosmological constant

Solutions to the vacuum Einsteins’ equation include black hole geometries.

Black hole

A solution to the Einstein's equation.



Temperature in gravity

Black holes obey:

Area of the horizon

$$dM = T_H dS_{BH} \qquad S_{BH} = \frac{k_B c^3}{\hbar G_N}\frac{A}{4}$$

Mass of the black hole

Hawking temperature

Newton's constant

This resembles of the first-law of thermodynamics

$$dE = TdS$$

This is not only an analogy. A black hole radiates a black-body radiation at Hawking temperature: we can assign a temperature to a black hole.

Hawking, S. W. (1974). "Black hole explosions?". *Nature* **248** (5443): 30.

We can introduce a notion of temperature into the theory of gravity.

Black hole thermodynamics

	Thermodynamics	Black hole
0-th law	T= const. at the equilibrium.	κ is constant in the static solution.
1st law	dE=T dS	dM=[κ /(8πG$_N$)]dA
2nd law	Entropy never decreases.	The area of horizon (A) never decreases.
3rd law	We cannot reach T=0 in any physical process.	κ cannot reach zero in any physical process.

κ : surface gravity (the gravitational acceleration at the horizon of the black hole)
G_N: Newton's constant, M: mass of the black hole
A: area of the horizon

κ and A mimic T and S, respectively.

$$\boxed{T = \frac{\kappa}{2\pi}, \quad S = \frac{A}{4G_N}}$$

Hawking found that the BH emits thermal radiation
with T= κ /2π, if we quantize the fields around the BH.

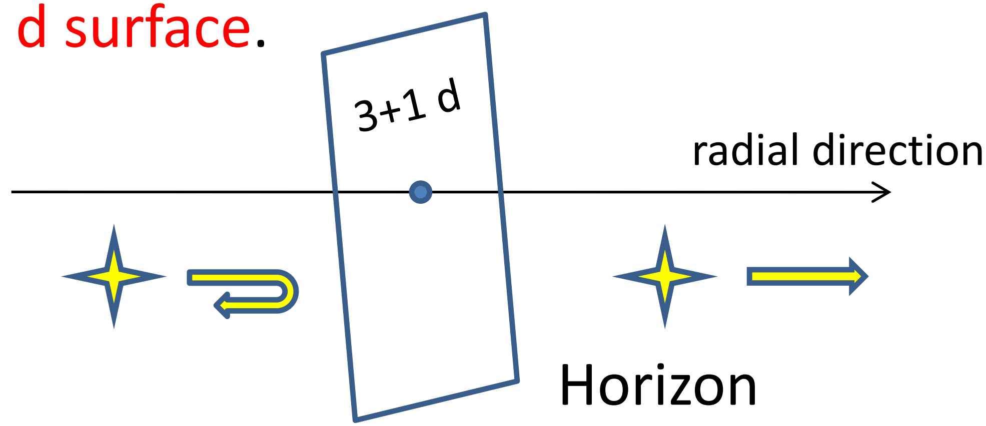
Black hole in AdS/CFT

AdS/CFT says some gauge theory is equivalent to a gravity theory. The entropy of the gauge-theory system will be given by the area of the corresponding black hole.

Let us consider a 3+1 d gauge theory.

$$dM = T_H \frac{1}{4G_N} d(\text{area})$$

The entropy need to be proportional to the 3d volume: the horizon has to be 3+1 d surface.



We need at least one more direction (the radial direction) to define the horizon : 4+1 d gravity

Black hole in AdS/CFT

However, black holes in a flat spacetime
(BH's in an asymptotically flat spacetime)
have negative specific heat.

This problem is cured if the BH is embedded in
a negatively curved spacetime.
(BH in an asymptotically anti de Sitter (AdS) spacetime)

Black hole in 4+1d AdS

is actually the simplest example of gravity dual of a
3+1d finite-temperature system.

AdS/CFT correspondence

[Maldacena, 1997]

A typical example

Type IIB supergravity on $AdS_5 \times S^5$
at the classical level

conjectured to be equivalent

4d SU(Nc) N=4 supersymmetric Yang-Mills (SYM) theory
at the large-Nc and the large λ (‘t Hooft coupling) limit
at the quantum level

At the finite temperature, AdS_5 becomes AdS-BH.

What is temperature?

Definitions of equilibrium temperature:

$$P \propto e^{-E/T}, \ t_E \approx t_E + 1/T$$

Distributions

$$dE = TdS$$

Thermodynamics

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diffusion const. mobility

Fluctuation-dissipation relation

We have another definition of temperature:

$$\xi^a \nabla_a \xi^b \Big|_{\text{Horizon}} = 2\pi T_{\text{eff}} \, \xi^b \Big|_{\text{Horizon}}$$

Killing vector

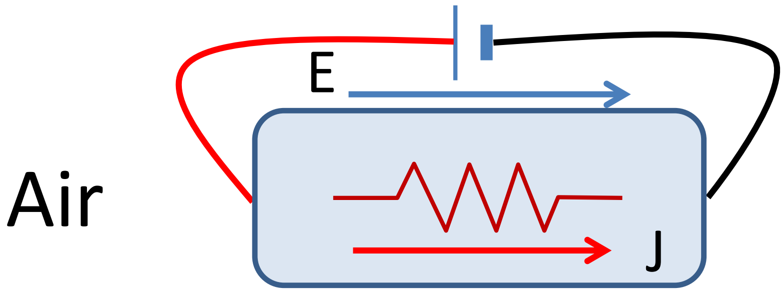
Hawking temperature

Non-equilibrium steady state (NESS)

Non-equilibrium, but time-independent.

A typical example:

A system with a constant current along the electric field.



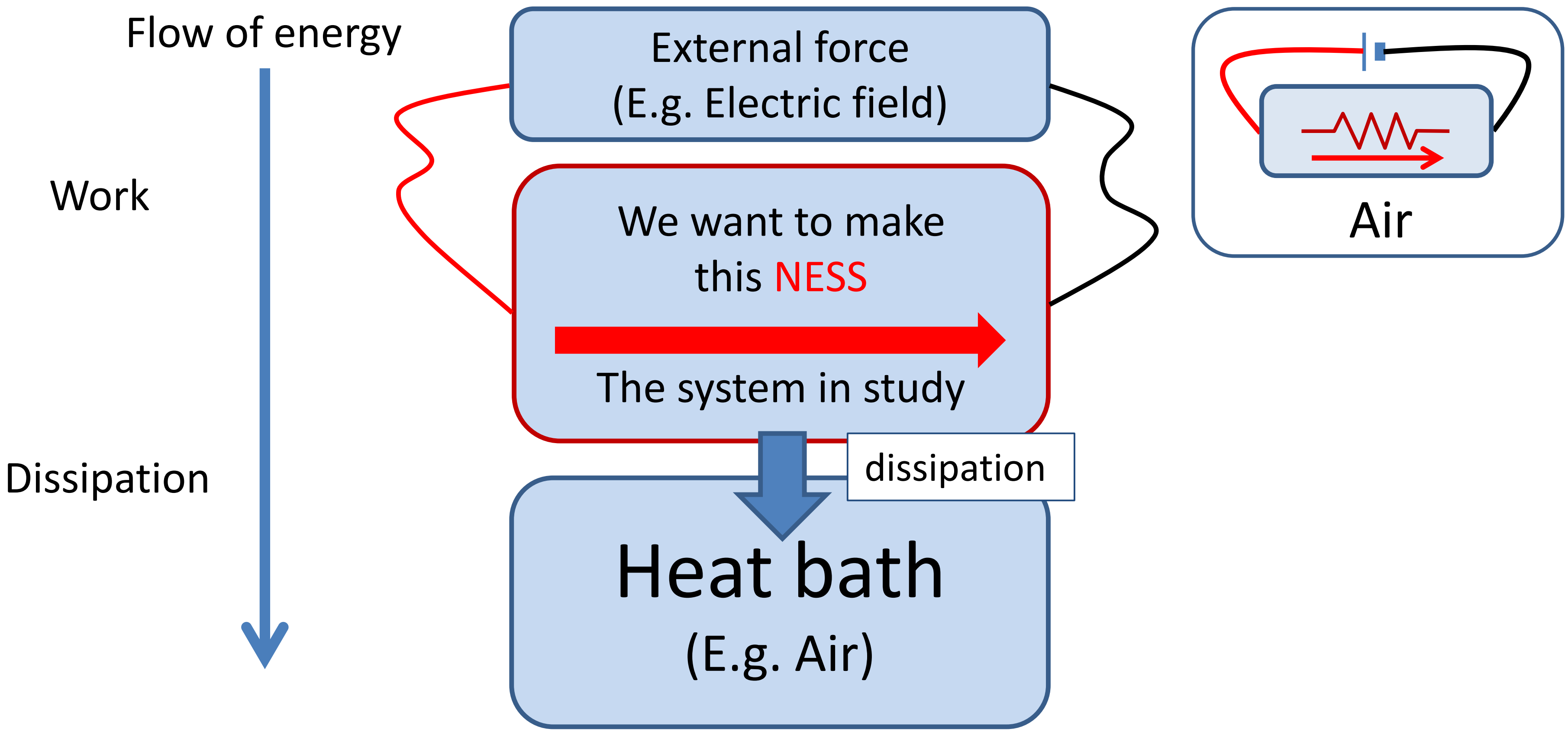
- It is non-equilibrium, because heat and entropy are produced.
- The macroscopic variables can be time independent.

In order to realize a NESS, we need
an external force and a heat bath.

Setup for NESS

External force and heat bath are necessary.

Power supply drives the system our of equilibrium.

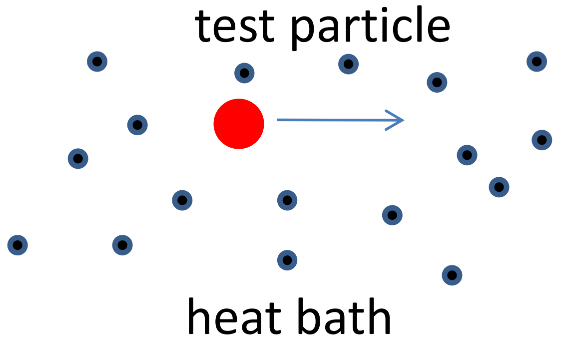


The subsystem can be NESS if the work of the source and the energy dissipated into the heat bath are in balance.

NESS to consider

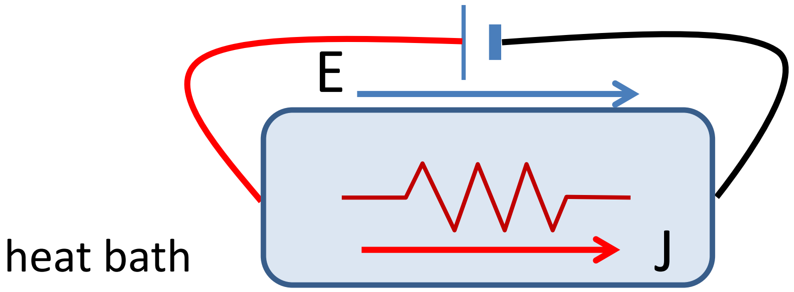
Langevin system

A test particle immersed in
a heat bath is driven by
a constant external force.



System with constant current

A system of charged particles
immersed in a heat bath is
driven by a constant external
electric field.



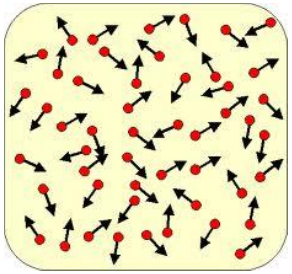
Strategy

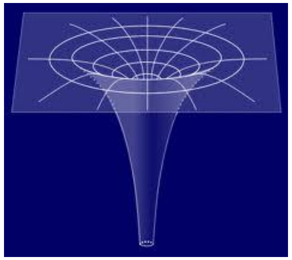
A strongly-interacting quantum gauge theory

AdS/CFT

equivalent

A classical gravity (general relativity) on a curved spacetime in higher dimensions.





Heat bath (gluons) → Black Hole

Charged particle(s) (quarks) in the heat bath → Object immersed in the black hole geometry

What kind of object in the gravity side?

Objects in gravity side

The idea of AdS/CFT is coming from superstring theory.

For the Langevin system

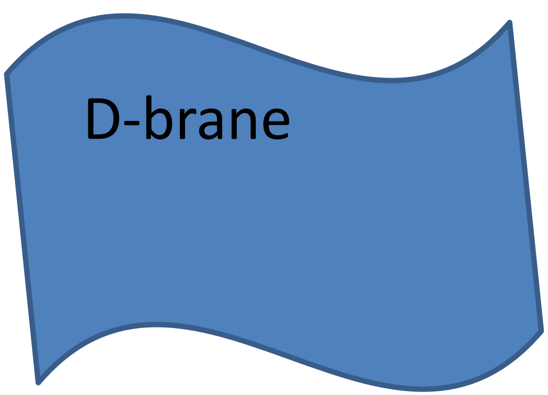
A single quark/anti-quark,
as a test particle

→ A single string

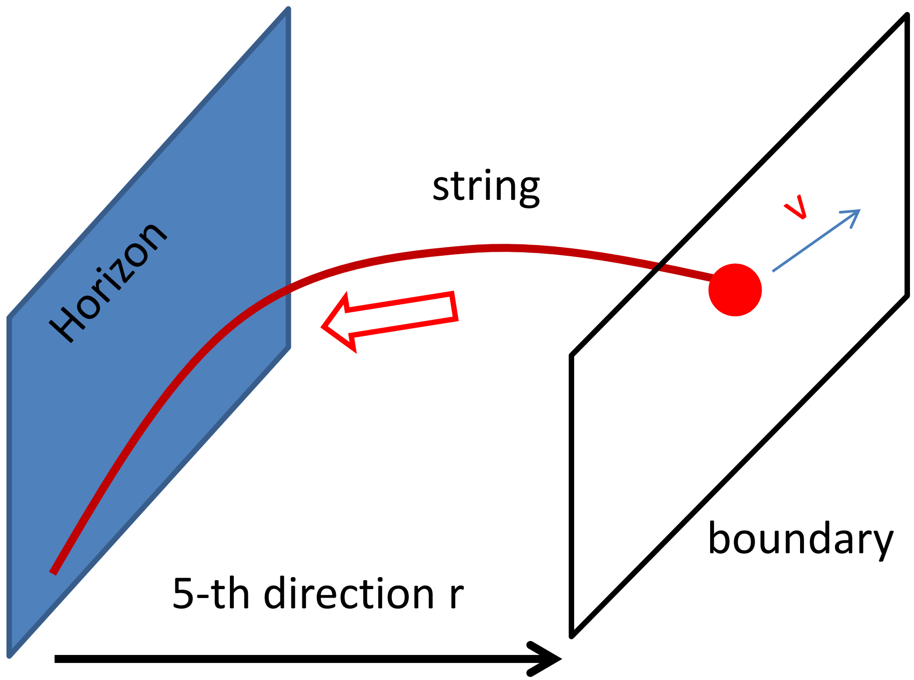
For the system of conductor

A single system of (many)
quarks and anti-quarks

→ A single D-brane



Langevin system



[Gubser, 2006]
[Herzog et al., 2006]

Energy-momentum tensor of string

$T^0{}_r$ = energy flow into the black hole in unit time: dissipation

= Work in unit time by the force acting on the test particle

$$f = \left.\frac{\partial L}{\partial(\partial_r x)}\right|_{\text{boundary}} \neq 0 \quad \text{at} \quad v \neq 0.$$

[Gubser, 2006]
[Herzog et al., 2006]

Computation of drag force

[Gubser, 2006], [Herzog et al., 2006]

$$L_{\text{string}} = -(\text{tension})\sqrt{-\det\left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}\right)}$$

$$X(t,r) = vt + x(r)$$

$$\partial_r \frac{\partial L}{\partial(\partial_r x)} = 0 \Rightarrow \frac{\partial L}{\partial(\partial_r x)} = f$$

$$(\partial_r x)^2 = f^2 \frac{g_{rr}}{-g_{tt} g_{rr}} \frac{(-g_{tt}) - g_{xx} v^2}{(-g_{tt}) g_{xx} - f^2}$$

Right-hand-side can be negative.

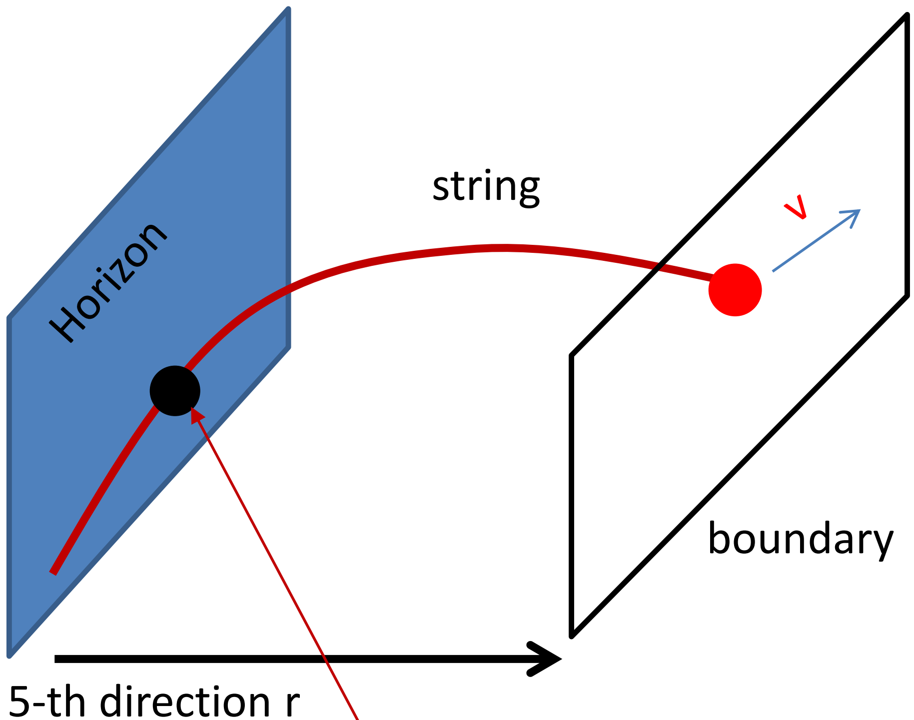
Let us define a point r=r_* by $(-g_{tt}) - g_{xx} v^2\Big|_{r_*} = 0$.

$$(-g_{tt}) g_{xx} - f^2\Big|_{r_*} = 0$$

If f satisfies this, $\partial_r x$ can be real.

f is given as a function of v.

Langevin system



There is a special point ($r=r_*$).

What is this point?

"Special point" $r=r_*$

The point r=r* plays a role of horizon for the fluctuation of the string.

[Gubser, 2008]
See also, [Kim-Shock-Tarrio 2011, Sonner-Green 2012].

Linearized equation of motion for the small fluctuation δX

$$\partial_a\left(\sqrt{-\tilde{g}}\,\tilde{g}^{ab}\partial_b\,\delta X^\mu\right)=0,$$

Klein-Gordon equation
on a curved spacetime
given by the induced metric.

$$\tilde{g}_{ab}=\partial_a X^\mu \partial_a X^\nu g_{\mu\nu}$$

Induced metric on the string
depends on v.

Now we have two temperatures

Black hole horizon gives the temperature of the heat bath.

The effective horizon on the string gives a different "Hawking temperature" that governs the fluctuations of the test particle.

We call this effective temperature T_{eff} of NESS.

> If the system is driven to NESS,
> $r_H < r_*$ at the order of v^2.

Computation of T_{eff}

Can never been understood as a Lorentz factor.

Beyond the linear-response regime

$$T_{\mathrm{eff}} = \left(1-v^2\right)^{\frac{1}{7-p}}\left(1+Cv^2\right)^{\frac{1}{2}} = T + \frac{1}{2}\left(C - \frac{2}{7-p}\right)v^2 T + O(v^4)$$

$$c_0 = \frac{4\pi}{7-p}, \quad C = \frac{1}{2}\left(q+3-p+\frac{p-3}{7-p}n\right)$$

This factor can be negative!

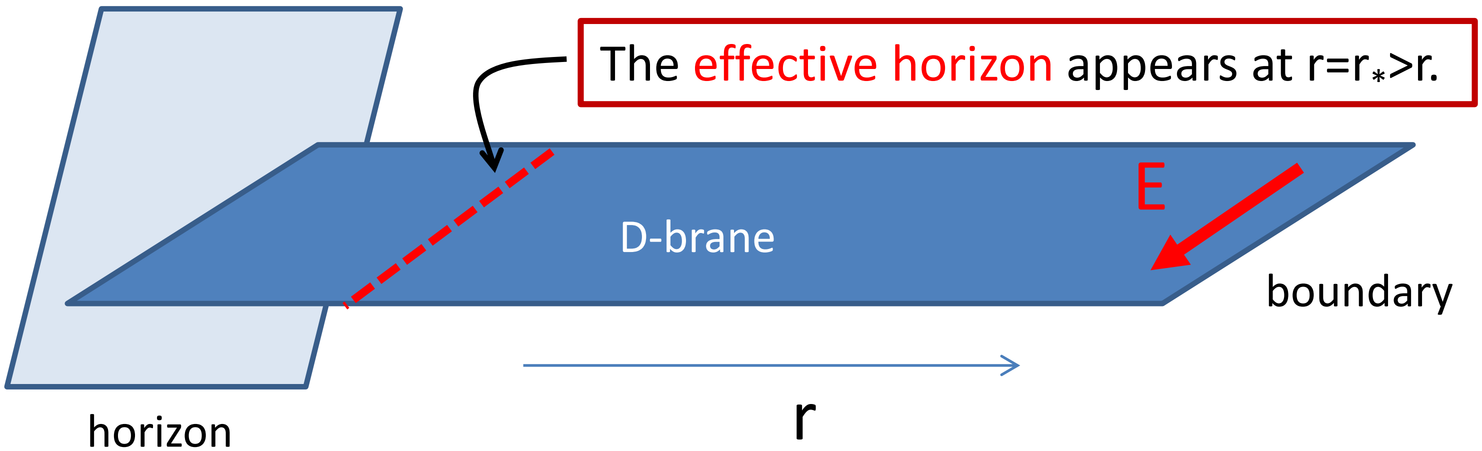
$T_{\mathrm{eff}} < T$ can be realized.

For example, for the test quark in N=4 SYM: [Gubser, 2008]

$$T_{\mathrm{eff}} = T\Big/\sqrt{\gamma} < T \qquad \gamma = 1\Big/\sqrt{1-v^2}$$

The temperature seen by the fluctuation can be made smaller by driving the system into out of equilibrium.

For conductors



Small fluctuation of electro-magnetic field on D-brane δA_b obeys to the linearized Maxwell equation on a curved geometry:

$$\partial_a\left(\sqrt{-\bar{g}}\,\bar{g}^{ab}\,\delta f_{bc}\,\bar{g}^{cd}\right)=0, \qquad \delta f_{bc}=\partial_b\delta A_a-\partial_a\delta A_b\,.$$

The geometry has a horizon at $r=r_*>r$.

The metric is proportional to the open-string metric, but is different from the induced metric.

We find many examples of $T_{eff} < T$ at the order of E^2.

[S. N. and H. Ooguri, arXiv:1309.4089]

Is T_{eff}<T allowed?

It is not forbidden.

Some examples of smaller effective temperature:

[K. Sasaki and S. Amari, J. Phys. Soc. Jpn. 74, 2226 (2005)]

[Also, private communication with S. Sasa]

Is it OK with the second law?

No contradiction.

- NESS is an open system.
- The second law of thermodynamics applies to a closed system.
- The definition of entropy in NESS (beyond the linear response regime) is not clear.

What is the physical meaning of T_{eff}?

Fluctuation of string $\Longleftrightarrow$ Fluctuation of external force acting on the test particle

Fluctuation of electro-magnetic fields on the D-brane $\Longleftrightarrow$ Fluctuation of current density

Computations of correlation functions of fluctuations in the gravity dual is governed by the ingoing-wave boundary condition at the effective horizon.

$$\boxed{\int dt \langle \delta f(t) \delta f(0) \rangle \Big|_{v \neq 0} = 2T_{\mathrm{eff}} \frac{\mathrm{Im}\, G^R(\omega)}{-\omega} \Bigg|_{\substack{\omega \to 0, \\ v \neq 0}}}$$

fluctuation dissipation

See also, [Gursoy et al.,2010]

The fluctuation-dissipation relation at NESS is characterized by the effective temperature (at least for our systems).

What is temperature?

Definitions of equilibrium temperature:

$$P \propto e^{-E/T}, \; t_E \approx t_E + 1/T$$

Distributions

$$dE = TdS$$

Thermodynamics

$$D = T\mu$$

diffusion const. mobility

Fluctuation-dissipation relation

We have another definition of temperature:

$$\xi^a \nabla_a \xi^b \Big|_{\text{Horizon}} = 2\pi T_{\text{eff}} \, \xi^b \Big|_{\text{Horizon}}$$

Killing vector

Hawking temperature

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diffusion const. differential mobility

They give the same temperature.

$$\xi^a \nabla_a \xi^b \Big|_{\text{Horizon}} = 2\pi T_{\text{eff}} \, \xi^b \Big|_{\text{Horizon}}$$ Hawking temperature

Killing vector

What is temperature?

Definitions of effective temperature:

$$P \propto e^{-E/T_{\mathrm{eff}}},\ \cancel{t_E \approx t_E + 1/T}$$

Distributions of small fluctuations

$$dE = TdS$$

Thermodynamics

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They give the same temperature.

$$\xi^a \nabla_a \xi^b \Big|_{\mathrm{Horizon}} = 2\pi T_{\mathrm{eff}}\, \xi^b \Big|_{\mathrm{Horizon}}$$

Killing vector

Hawking temperature

Then, some thermodynamics?

$$dE \overset{??}{=} T_{\mathrm{eff}}\, dS$$

It is highly nontrivial.

Hawking radiation (Hawking temperature) is more general than the thermodynamics of black hole.

Hawking radiation:
It occurs as far as the "Klein-Gordon equation" of fluctuation has the same form as that in the black hole.

Thermodynamics of black hole:
We need the Einstein's equation. It relies on the theory of gravity.

Example of "non-gravity" Hawking radiation

Sonic black hole in liquid helium.

Fast — Sonic horizon where the flow velocity exceeds the velocity of sound. — Slow

- The sound cannot escape from inside the "horizon".
- It is expected that the sonic horizon radiates a "Hawking radiation" of sound at the "Hawking temperature".

[W. G. Unrhu, PRL51(1981)1351]

However, any "thermodynamics" associated with the Hawking temperature of sound has not been established so far.

[See for example, M. Visser, gr-qc/9712016]

Summary

- In AdS/CFT, non-equilibrium dynamics of many-body systems can be reduced to a classical dynamics of strings/D-branes on a black hole geometry in some cases.

Usually, the microscopic theory in gauge-theory side is different from what we have in our world. However, we can still ask a basic statistical properties of many-body systems.

- The notion of temperature naturally/automatically appears in terms of Hawking temperature, even for NESS beyond the linear-response regime.

- The wisdom of general relativity may tell us important hints for non-equilibrium physics if we translate it by using AdS/CFT.