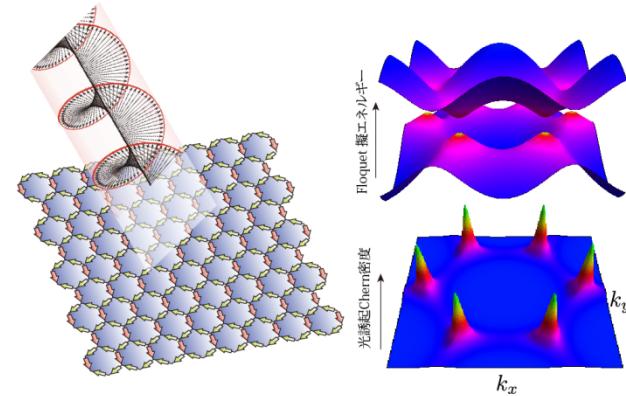
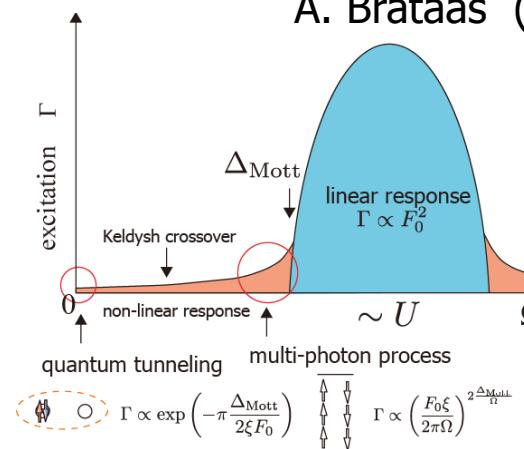


# 固体電子中の強電場物理 --- 多体Schwinger機構、光誘起ホール効果

Takashi Oka (The University of Tokyo)

H. Aoki (University of Tokyo)  
N. Tsuji (ETH)  
P. Werner (ETH)  
T. Kitagawa (Harvard)  
L. Fu (Harvard)  
E. Demler (Harvard)  
A. Brataas (Norwegian University)



解説:

岡、青木、日本物理学会2012出版予定「強相関系の非平衡現象」

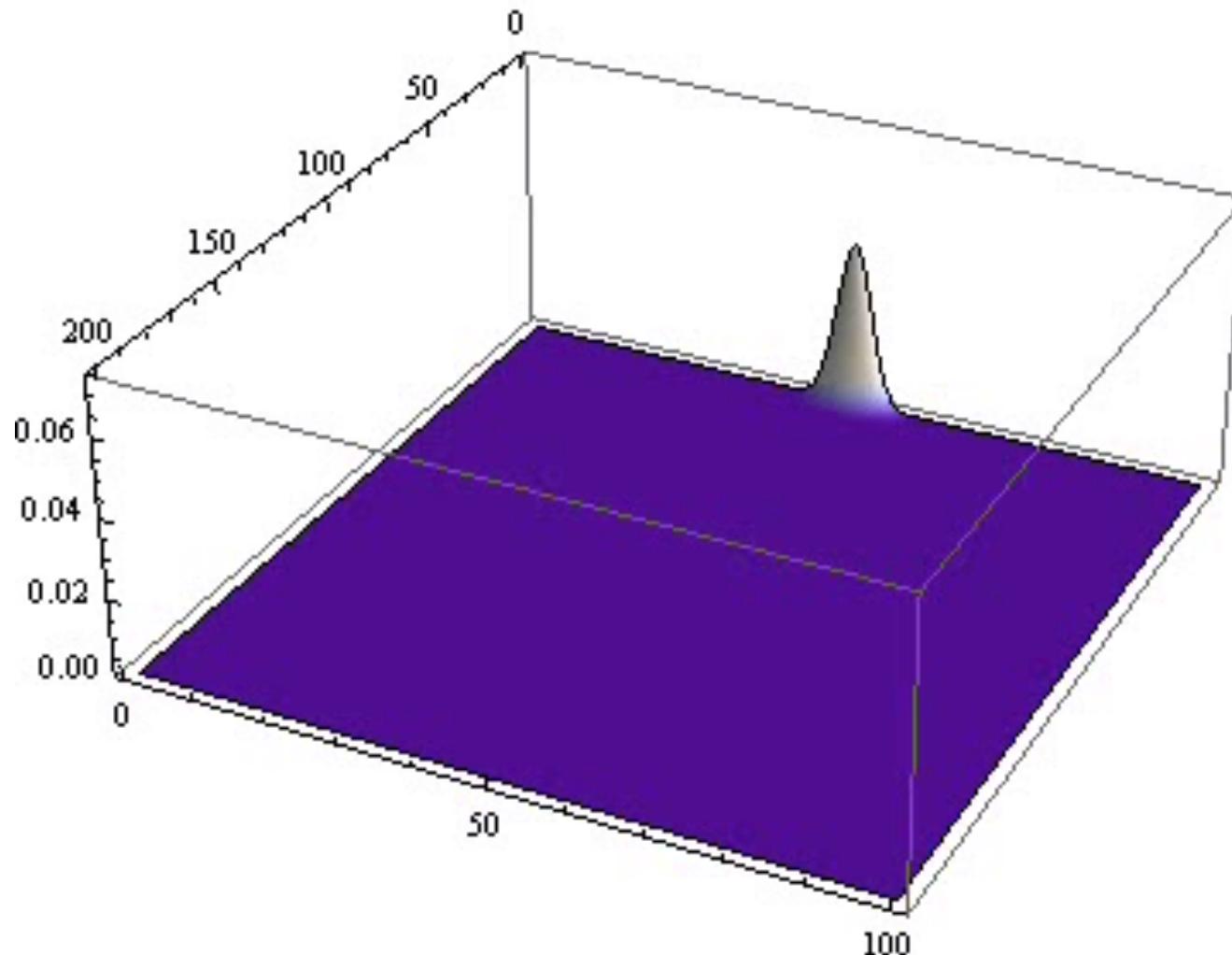
岡、北川、固体物理2011, 11月号「光誘起トポロジカル相転移の理論--Floquet描像の立場から--」

Aoki, TO, Tsuji, Eckstein, Werner RMP in prep.

## 2+1d Dirac equation

$$\mathcal{L} = \bar{\psi} i \gamma_\mu \partial^\mu \psi$$

wave packet dynamics, with a boundary



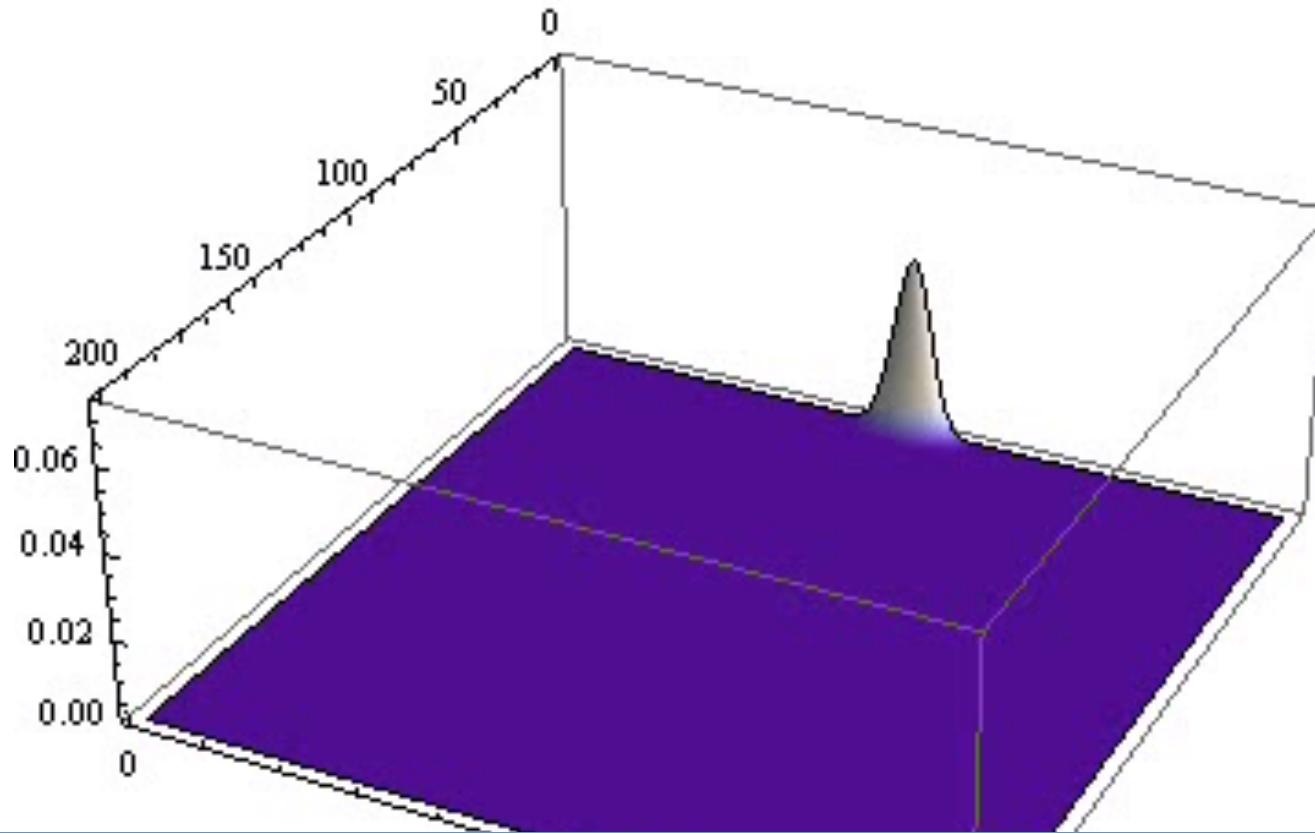
---

calculation is done in graphene (honeycomb lattice)

2+1d Dirac equation + circularly polarized light

$$\mathcal{L} = \bar{\psi} \gamma_\mu (i\partial^\mu - A^\mu(t)) \psi$$

$$A^\mu = (0, \frac{F}{\Omega} \cos \Omega t, \frac{F}{\Omega} \sin \Omega t)$$



Aim of Strong field physics  
= “**Control**” of Quantum states

# 1. Introduction

## 1.1 Condensed matter VS Hadron Physics

Mott transition       $\longleftrightarrow$       confinement  
topological phase transition       $\longleftrightarrow$       quantum anomaly

## 1.2 (Old, New) Strong field physics

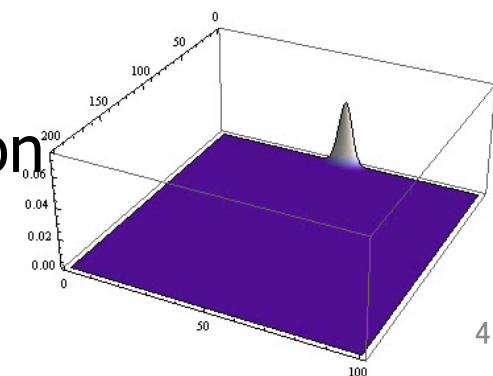
## 1.3 Numerical methods for non-equilibrium

# 2. Many-body Schwinger mechanism

Nonlinear transport/Photo-induced Mott transition

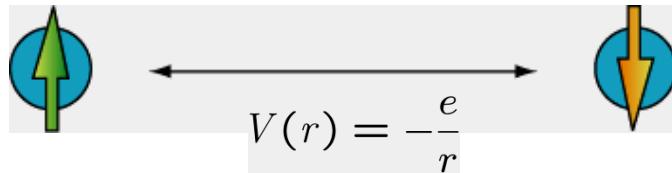
# 3. Photo-induced Hall effect

Floquet topological phase transition



# Introduction: Strongly Correlated Electron Systems

What are strongly correlated electrons?

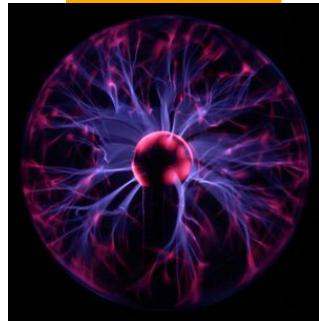


Strong effects of Coulomb interaction

space



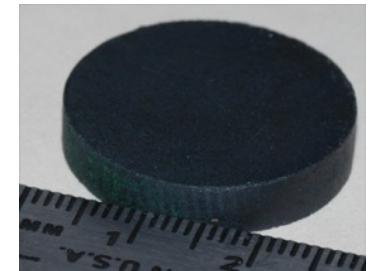
plasma



semiconductor



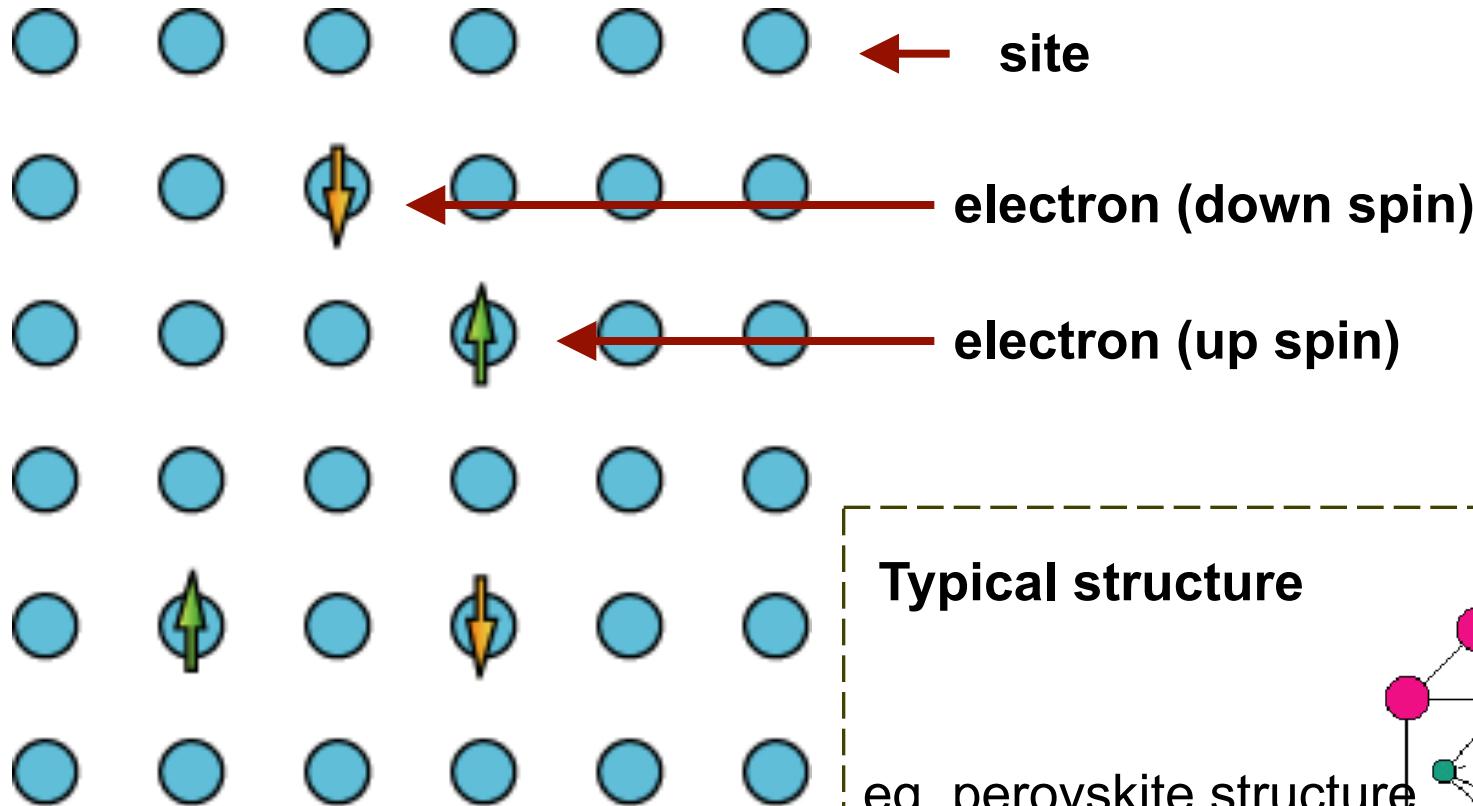
strongly  
correlated  
material



density of electrons

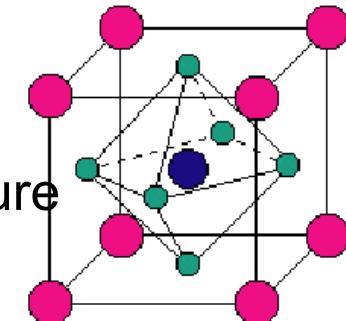
large

# Basic degrees of freedom



## Typical structure

eg. perovskite structure  
of manganite oxides



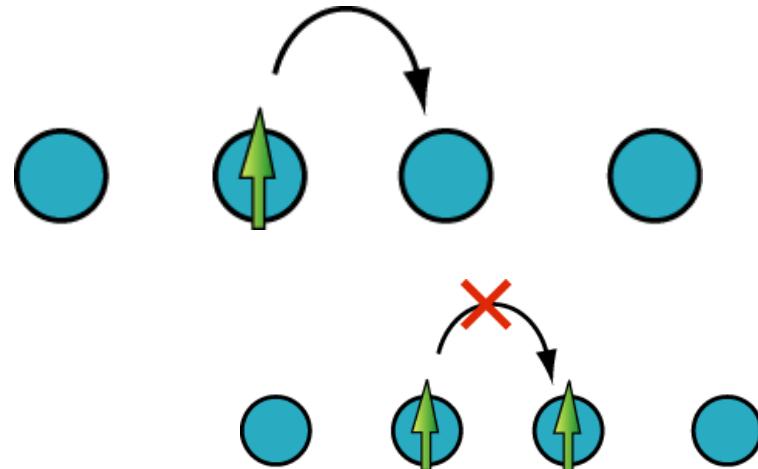
The electrons exist  
on the R-atom =site.

- : M
- : O
- : R

# Basic rules

## 1. Hopping between lattice sites

$$-t \sum_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.})$$



Fermi statistics: Pauli principle

## 2. On-site Coulomb interaction

$$U \sum_i n_{i\uparrow} n_{i\downarrow}$$



$$U$$



energy  
>



## Hubbard Hamiltonian:

minimum model of strongly correlated electron system.

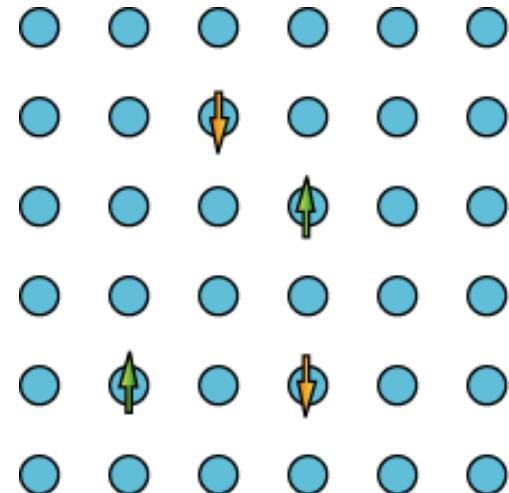
$$H = -t \sum_{\langle ij \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

# Other parameters

## 3. Band filling

**$n$  = フィリング(filling)**

= number of electrons per lattice site



## 4. Temperature

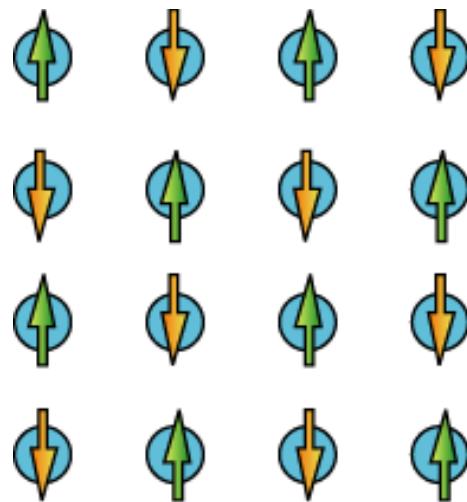
typical temp. range:  $T = 0.1\text{-}1 \text{ eV} = 1000\text{-}10000 \text{ K}$

# Equilibrium phase transitions

## Magic filling

When the filling takes certain values, the groundstate show non-trivial orders.

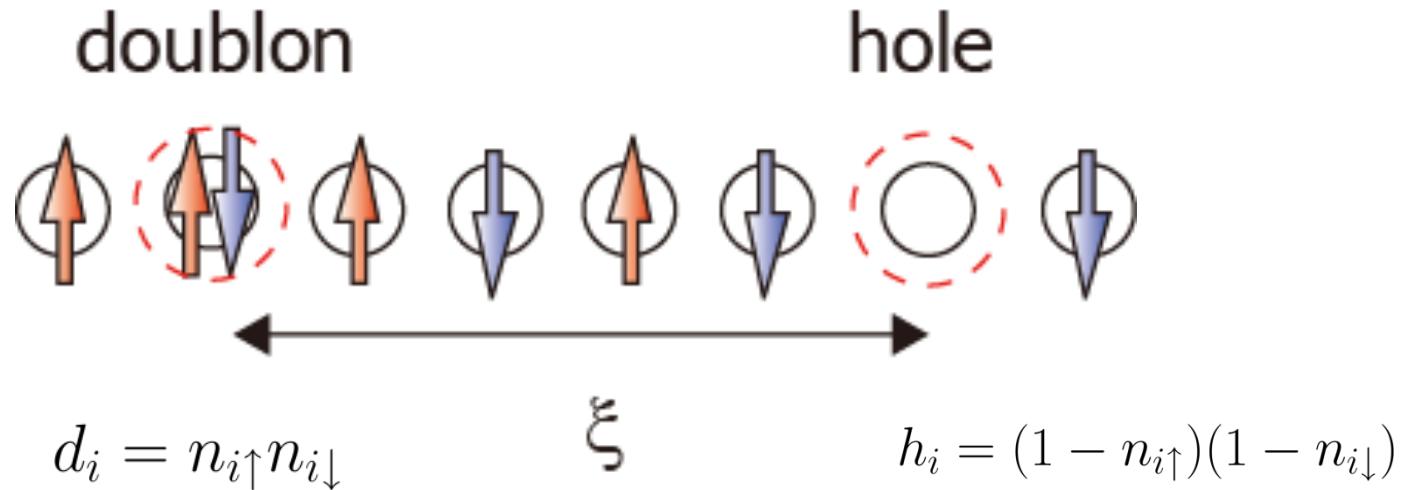
$n = 1$  (half-filling)



***Mott Insulator***

1. **Insulator:**  
no free carriers
2. **Anti-ferromagnetic order**

## Doublon-hole “confinement”

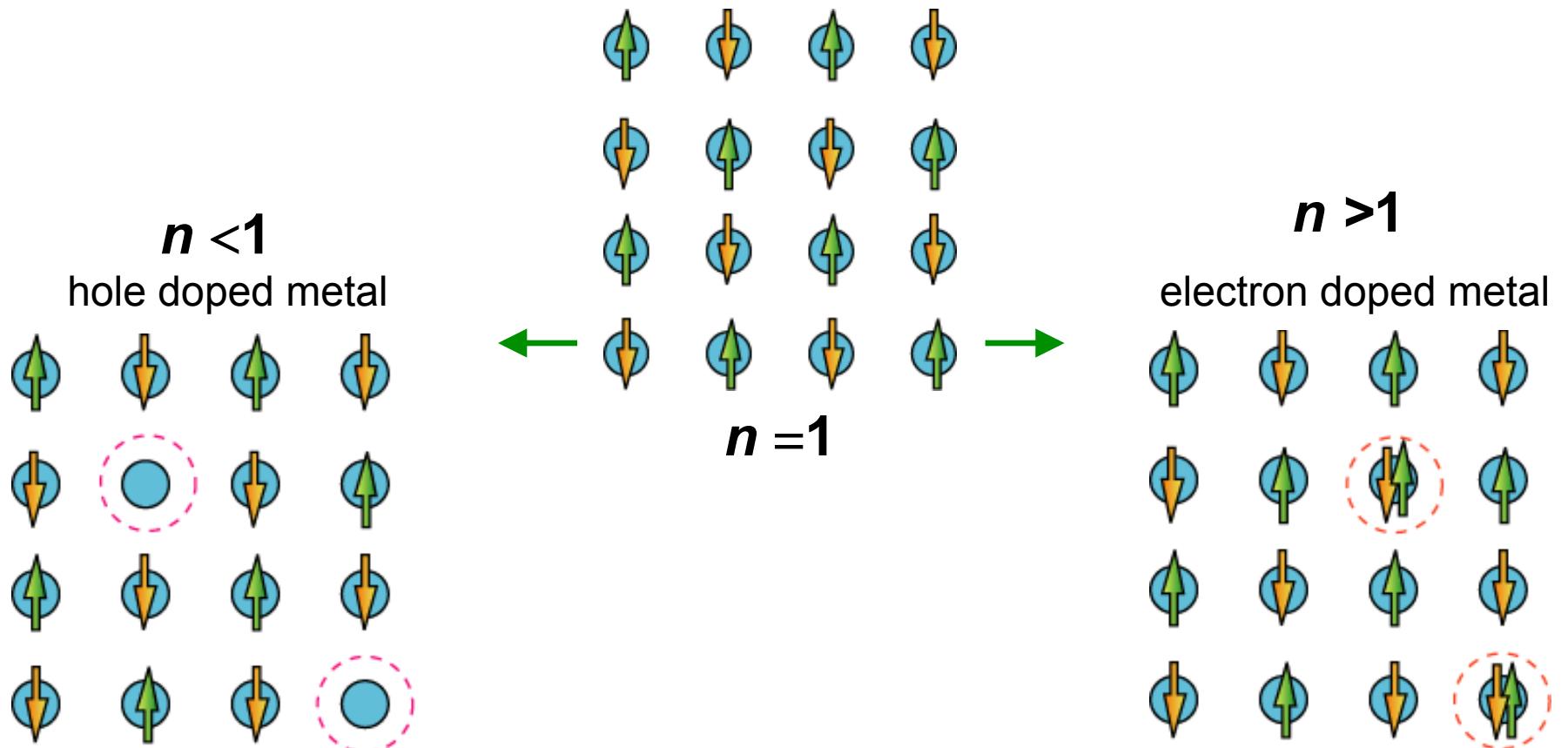


$$\langle d_x^\dagger h_0 \rangle = \exp(-x/\xi)$$

$\xi$ : correlation length

doublons and holes are bound to each other

# Metal-insulator transition due to doping



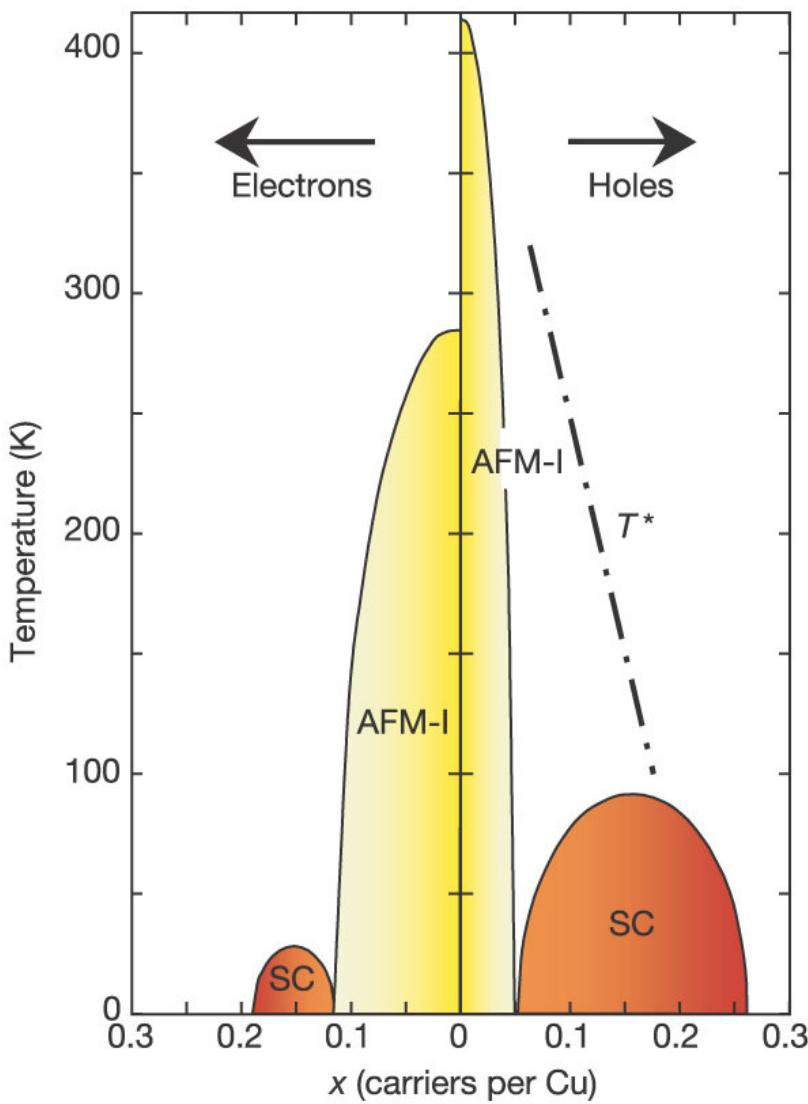
**carrier = hole**

$$h_i = \langle (1 - n_{i\uparrow})(1 - n_{i\downarrow}) \rangle \neq 0$$

**carrier = double occupied state (doublon)**

$$d_i = \langle n_{i\uparrow} n_{i\downarrow} \rangle \neq 0$$

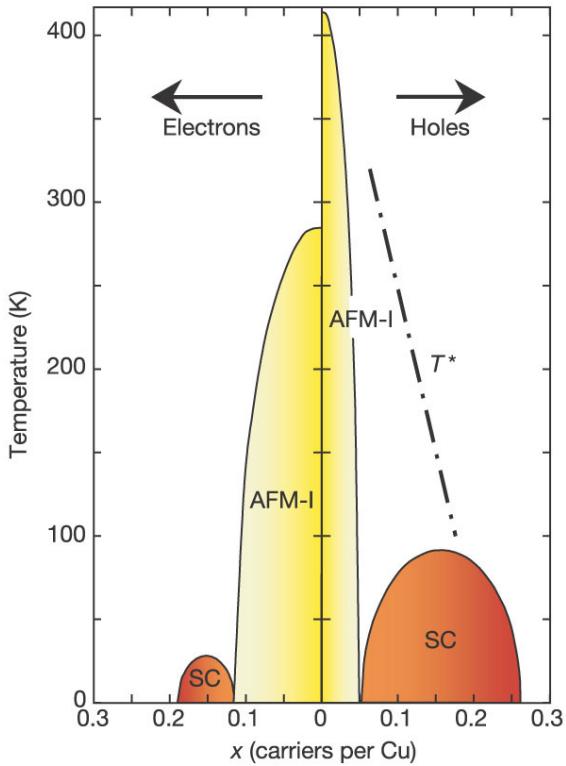
# Equilibrium phase diagram



**Phase diagram of copper oxides  
(2 dim material)**

1. **AFM = Mott insulator with anti-ferromagnetic order**
2. **Metal-insulator transition due to doping**
3. **Superconductivity (SC) near the Mott insulator phase**

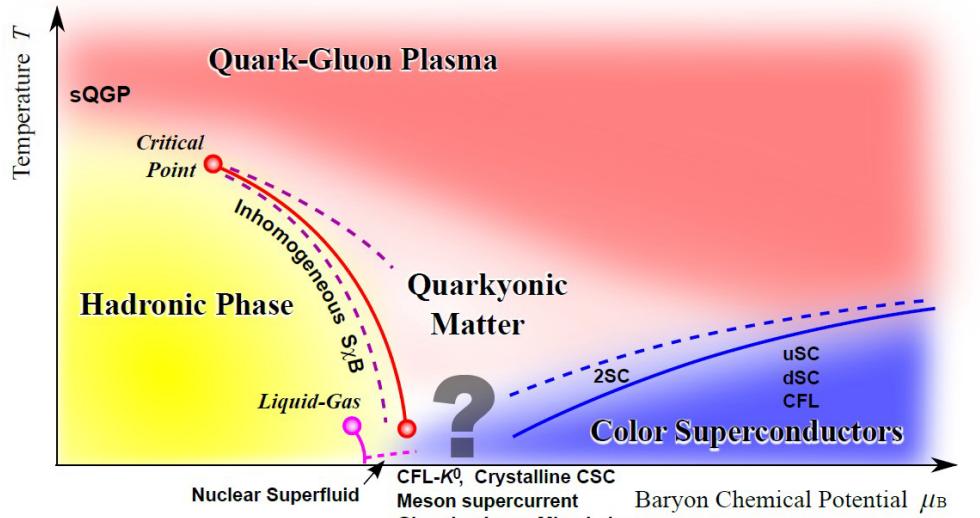
# Condensed matter VS Hadron Physics



electron+Coulomb interaction

$\downarrow$   
Hubbard model

$$H = -t \sum_{\langle ij \rangle} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



from Fukushima-Hatsuda (2010)

Hatsuda-Kunihiro ('94)

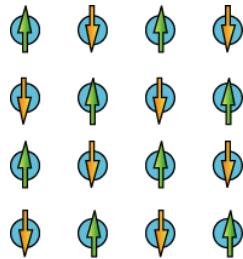
QCD (quark-gluon)

$\downarrow$   
Nambu-Jona Lasinio (NJL) model

$$\mathcal{L} = \bar{q} i \gamma \cdot \partial q + g/2 [(\bar{q} q)^2 + (\bar{q} i \gamma_5 \boldsymbol{\tau} q)^2]$$

# Condensed matter VS Hadron Physics 2

magnetic phase transition



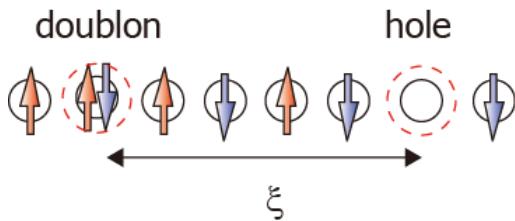
$$M = \sum_i \langle (-1)^i s_{iz} \rangle$$

chiral phase transition

$$\langle \bar{q}q \rangle \neq 0$$

SSB

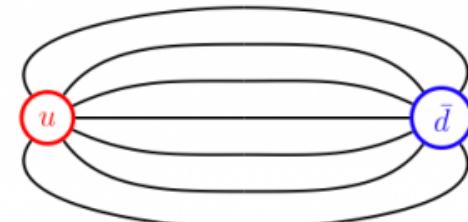
Mott transition



$$\langle d_x^\dagger h_0 \rangle = \exp(-x/\xi)$$

No Mean Field!

confinement



- Mott transition w/o magnetic phase transition is possible (1d Hubbard)
- confinement w/o chiral phase transition is possible (e.g., 1+1d QED)

# (old) Strong field physics

1936 Heisenberg Euler

Effective Lagrangian for static fields  
vacuum instability (electron-positron pair creation)

1935 Volkov

sol. of Dirac equation in electro-magnetic fields

1951 Schwinger

Heisenberg-Euler's result by path integral

Schwinger mechanism = vacuum instability (electron-positron pair creation)

Schwinger limit = threshold for e-p pair creation

1965 Keldysh

tunneling problem in time-periodic potential

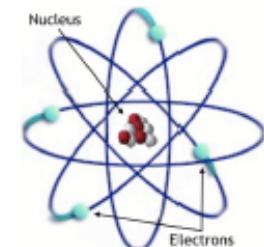
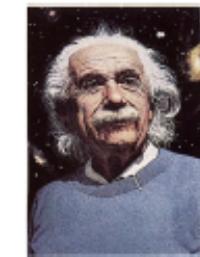
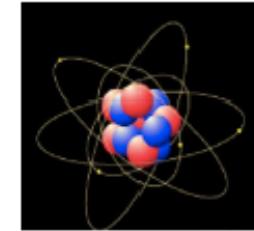
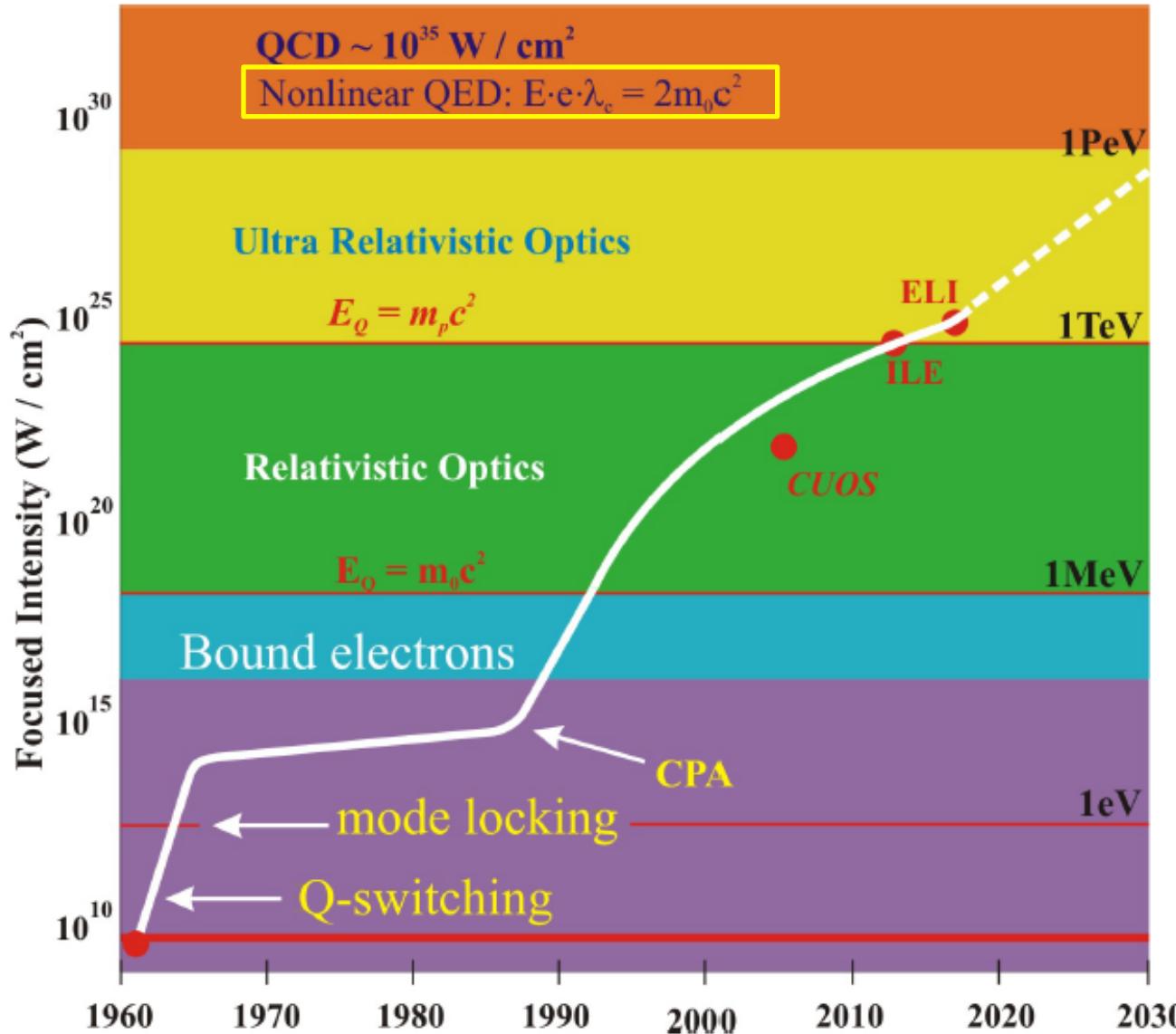
1970 Brezin Itzykson

1972, 1974 V.S. Popov

Schwinger mechanism in time-periodic potential

Keldysh, Brezin-Itzykson, Popovの仕事はPIF2010でDunneさんに教えてもらいました。

# Laser elementary particle physics



From a talk at PIF2010 by  
Gerard A. Mourou (Institut de Lumiere Extreme, ILE) Physics Today 51 (1998)

# NEW Strong field physics

## **Strong correlation/interaction**

Non-linear doublon-hole excitation in Mott insulators

RHIC

Gauge/gravity duality (S. Nakamura's talk)

## **Topology (quantum anomaly)**

Photo-induced topological phase transition

**Control of phase transitions by light**

# New Strong field physics

1936 Heisenberg Euler

1935 Volkov

1951 Schwinger

Oka Aoki PRB 2009

“photovoltaic Hall effect”

\*This is a non-relativistic effect

Oka, Aoki PRL 2003, 2005, PRB2011

1965 Keldysh

1970 Brezin Itzykson

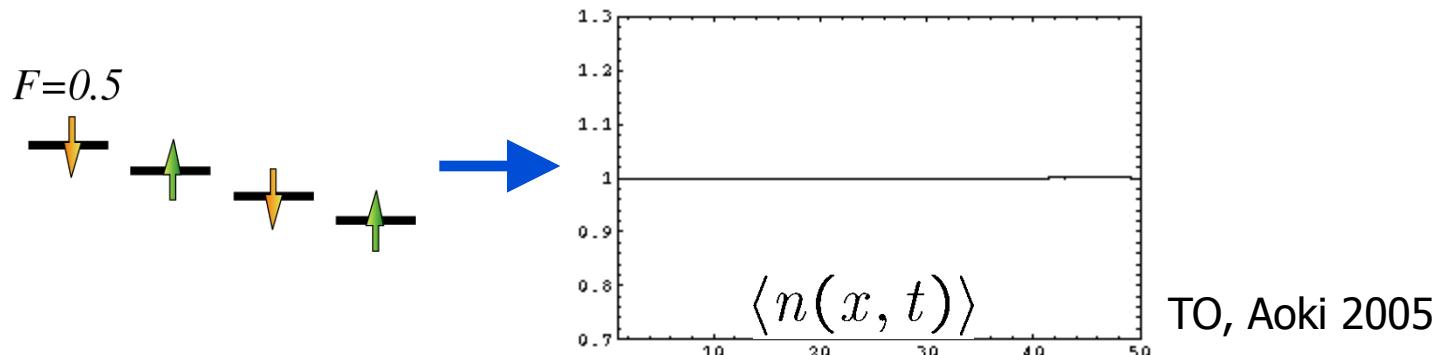
1972, 1974 V.S. Popov

Oka arXiv2011

# Numerically exact methods to study real time dynamics

## one dimension

Time-dependent density matrix renormalization group (Feiguin, White 2004)



## infinite dimension

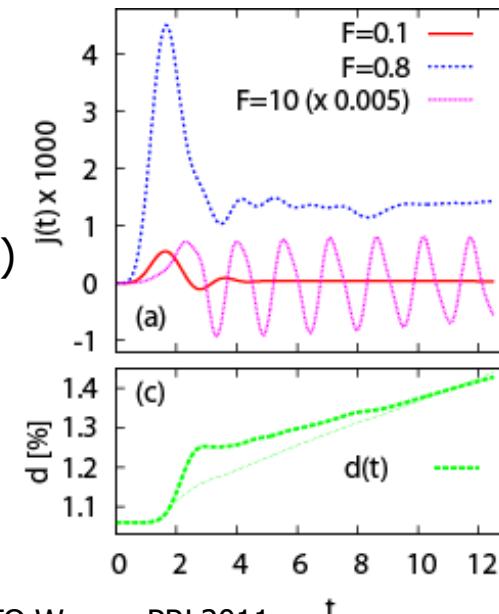
Non-equilibrium dynamical mean field theory

(Freericks 2008, Eckstein,TO,Werner 2011)

real time diagrammatic QMC (Werner, TO, Millis 2008)

$$\Sigma(k; t, t') \rightarrow \Sigma(0; t, t')$$

Simplification of Feynman diagrams



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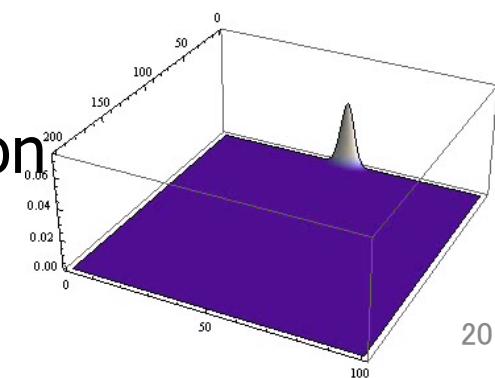
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## → 2. Many-body Schwinger mechanism

Nonlinear transport/Photo-induced Mott transition

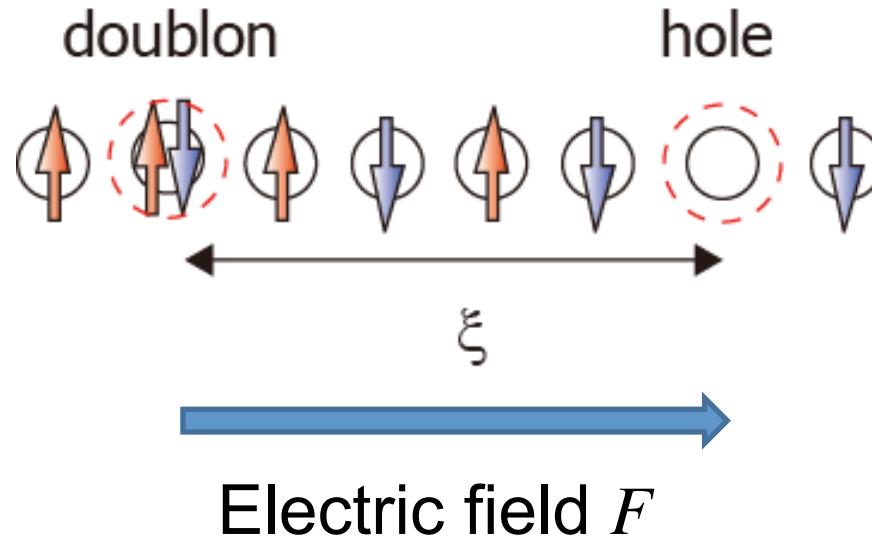
## 3. Photo-induced Hall effect

Floquet topological phase transition



# Doublon-hole “de-confinement”

In strong electric fields, doublon-hole pair production leads to instability of the Mott phase



*Many-body* Schwinger mechanism

What let's calculate the dh-pair production rate

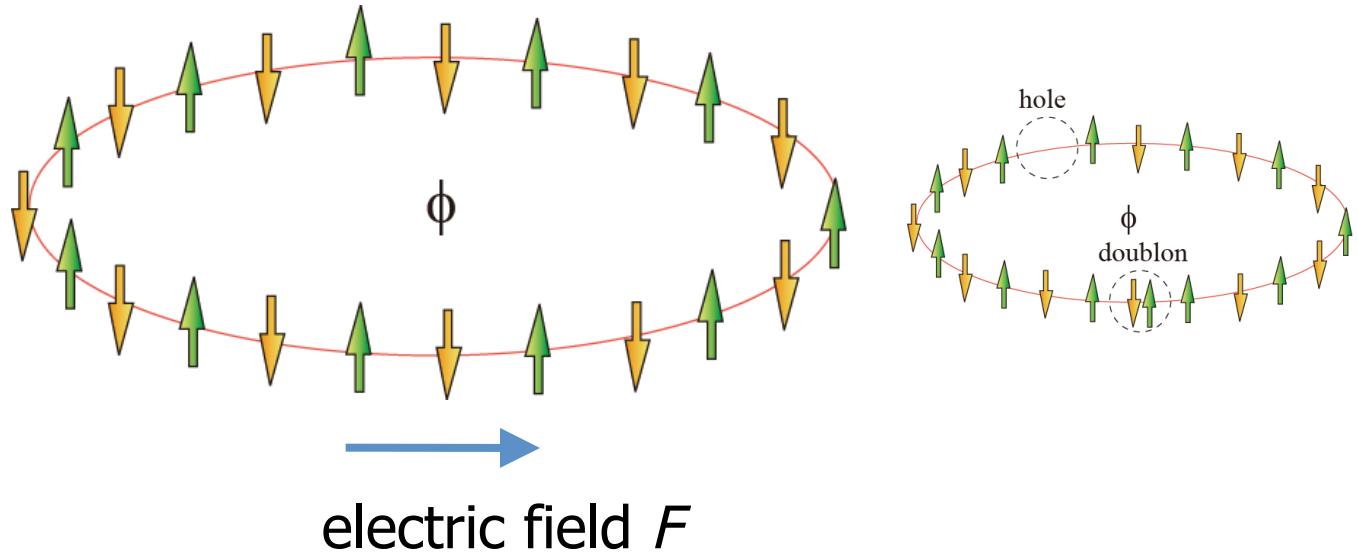
use the integrability of the 1dHubbard model

# Electric fields applied to a Mott insulator

(1dim) TO, Aoki: PRL (2003), PRL(2005)  
(DMFT) Eckstein, TO, Werner: PRL (2010)

half-filled Hubbard model (1-dim)

$$H(t) = - \sum_i \left[ e^{i\phi(t)} c_{i+1}^\dagger c_i + e^{-i\phi(t)} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

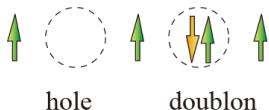


$$F(t) = -\frac{d\Phi(t)}{dt}$$

# Bethe ansatz solution in the 1d half-filled Hubbard model

doublon-hole (holon-antiholon) excitations

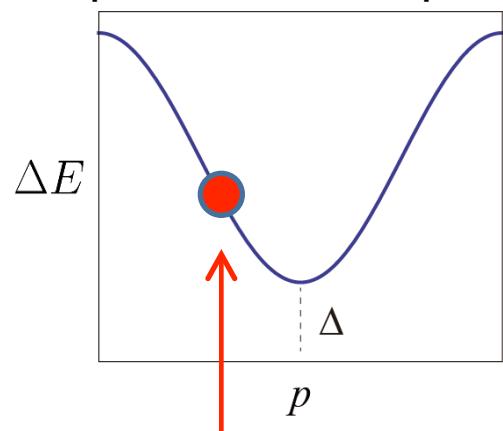
$|0\rangle$       “ $d^\dagger(p)h^\dagger(p)|0\rangle$ ”



“ $d^\dagger(p_1)d^\dagger(p_2)b^\dagger(p_3)b^\dagger(p_4)|0\rangle$ ”

we neglect these contributions

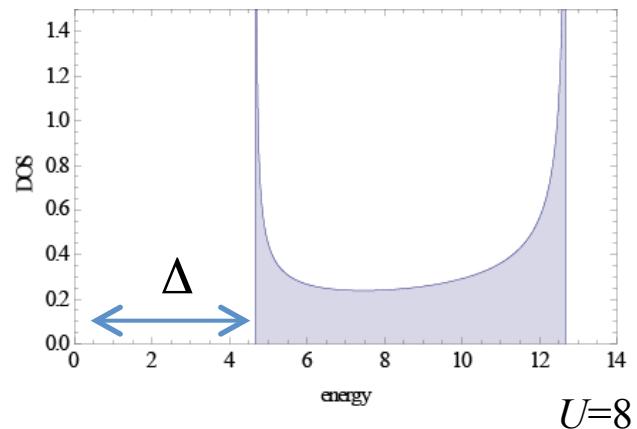
spectrum of d-h pairs



$$\Delta E = \varepsilon_h(p) + \varepsilon_{\bar{h}}(p)$$

We want to calculate the tunneling probability  $\mathcal{P}_p$

Density of states



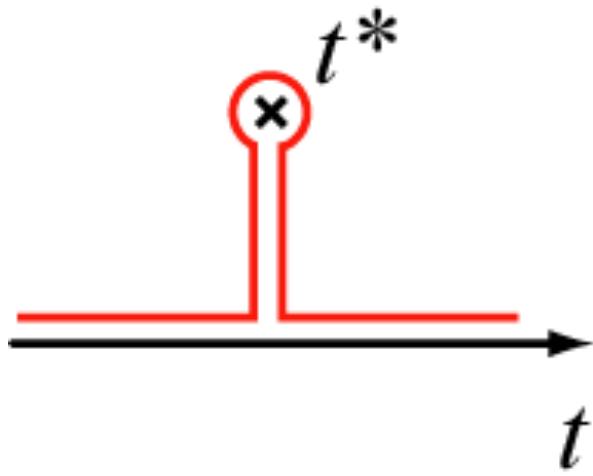
$$\begin{aligned} \varepsilon_h(k) &= \varepsilon_d(k) = U/2 + 2 \cos k \\ &+ 2 \int_0^\infty \frac{d\omega}{\omega} \frac{J_1(\omega) \cos(\omega \sin k) e^{-U\omega/4}}{\cosh(\omega U/4)} \end{aligned}$$

# Landau-Dykhne theory of tunneling

Dykhne JETP (1962), Daviis, Pechukas, J.Chem.Phys. (1976)  
Landau-Lifshitz *Quantum mechanics*

generalization of Landau-Zener formula

singularity



1. Use complex time
2. Find the singular point

$$E_2(t^*) = E_1(t^*)$$

3. Tunneling probability

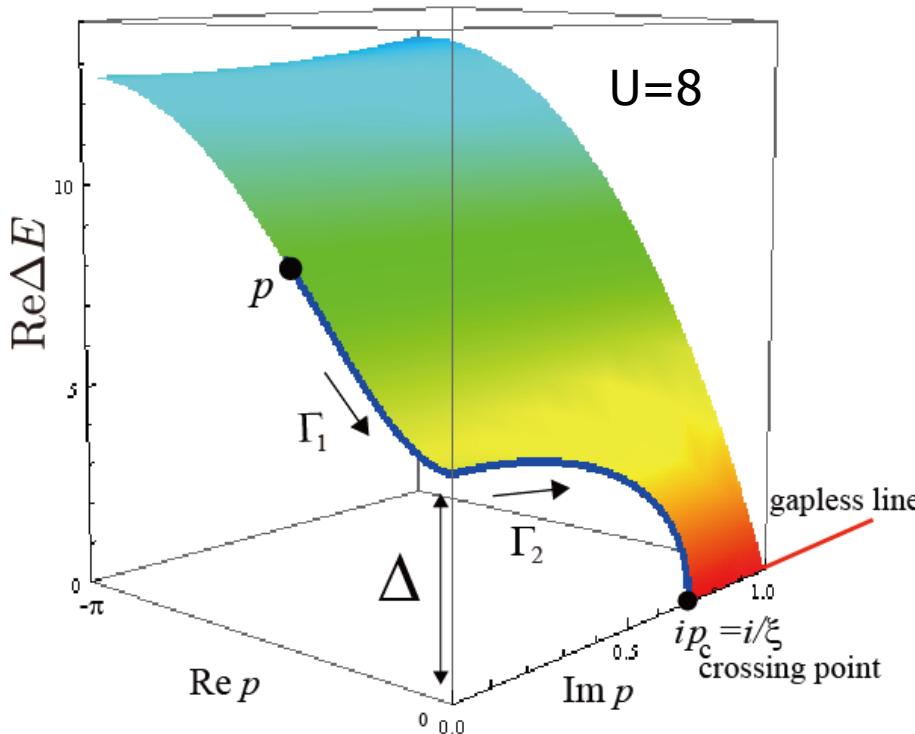
$$p = \exp(-2\text{Im}S_{1,2}/\hbar)$$

$$S_{1,2} = \int_{t_0}^{t^*} dt' [E_2(\Phi(t')) - E_1(\Phi(t'))]$$

imaginary part of the dynamical phase

# Singular point in the 1d Hubbard model

Fukui Kawakami 1998, TO Aoki 2010



$\Phi \rightarrow i\Psi$  Non-Hermitian Hubbard model

$$H(t) = - \sum_i \left[ e^{\Psi} c_{i+1}^\dagger c_i + e^{-\Psi} c_i^\dagger c_{i+1} \right] + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

# dh-pair production rate

tunneling probability

$$\mathcal{P}_p = \exp \left( -2\text{Im} \int_0^{\Phi_c} \Delta E(p - \Phi) \frac{-1}{F(\Phi)} d\Phi \right)$$

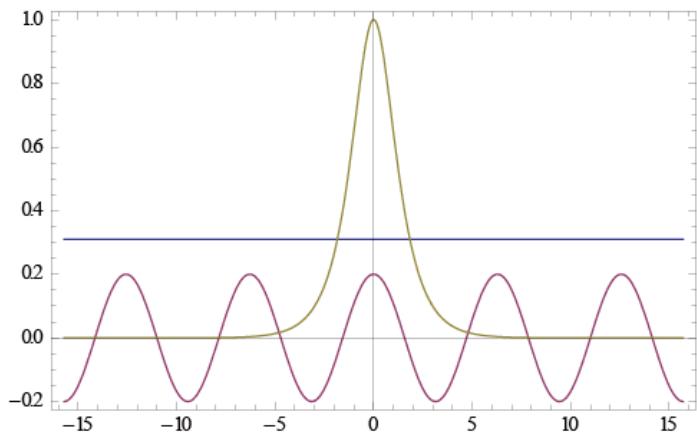
production rate

$$\Gamma_p = f \mathcal{P}_p$$

TO arXiv:1105.3145

Dirac model: Popov (1972)

electric field  $F(t)$



type	$F(t)$	$F(\Phi)$	attempt frequency $f$
DC-field	$F_0$	$F_0$	$F_0/2\pi$
AC-field	$F_0 \sin \Omega t$	$\pm \sqrt{F_0^2 - \Omega^2 \Phi^2}$	$\Omega/\pi$
single pulse	$F_0 \cosh^{-2}(t/\sigma)$	$F_0 \left(1 - \frac{\Phi^2}{\sigma^2 F_0^2}\right)$	1(single process)

# DC-result: Schwinger limit

$$\Gamma_p = \frac{F_0}{2\pi} \exp(-\pi \frac{F_{\text{Sch}}}{F_0}) \quad F_{\text{Sch}} = \frac{2}{\pi} \int_0^{p_c=1/\xi} \Delta E(il) dl$$

strong/weak coupling limits

$$\Gamma_p = \frac{F_0}{2\pi} \exp\left(-\frac{\pi}{2} \frac{(\Delta_{\text{Mott}})^2}{c_{\text{eff}} F_0}\right) \quad (\text{DC, Small } U)$$

$$\Gamma_p = \frac{F_0}{2\pi} \left(\frac{g\tau}{U}\right)^{\frac{\pi}{2} \frac{U}{F_0}} \quad (\text{DC, Large } U)$$

Estimate for 1d Mott insulators

	$\tau$ (eV)	$U$ (eV)	$a$ (Å)	$\Delta$ (eV)	$\xi(a)$	$F_{\text{Sch}}$ (V/Å)
ET-F <sub>2</sub> TCNQ	0.1	1	10	0.7	1.1	0.06
[Ni(cnxn) <sub>2</sub> Br]Br <sub>2</sub>	0.22	2.4	5	1.6	1.0	0.3
Sr <sub>2</sub> CuO <sub>3</sub>	0.52	3.1	4	1.5	2.1	0.18

$$F_{\text{Sch}}^{\text{QED}} = m^2 c^3 / e\hbar \sim 10^6 \text{ V/Å}$$

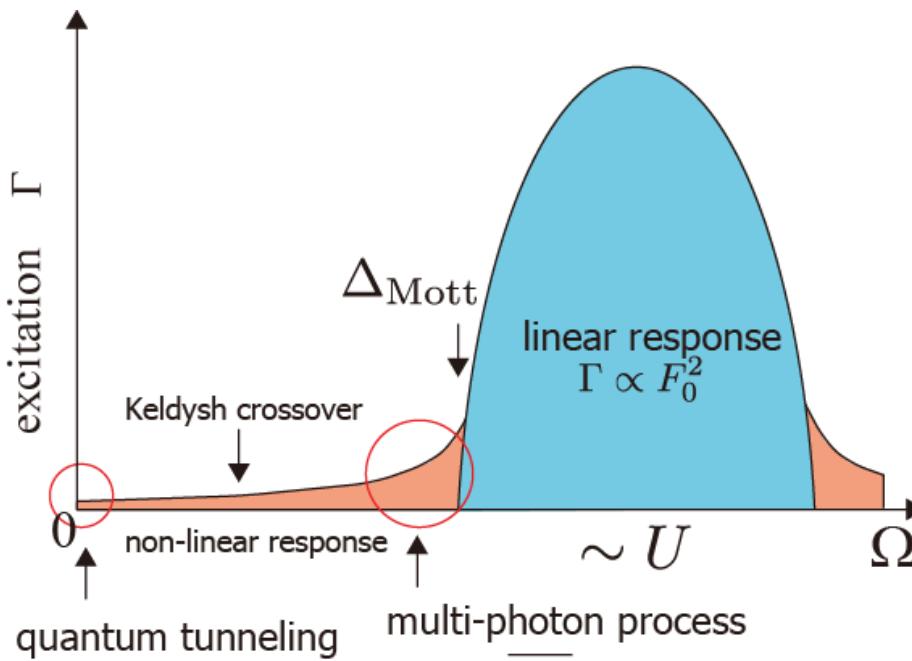
Easily accessible with ultra-short pulse lasers!

# AC-result: Keldysh Crossover

AC-field  $\phi(t) = F_0 \sin(\Omega t)$

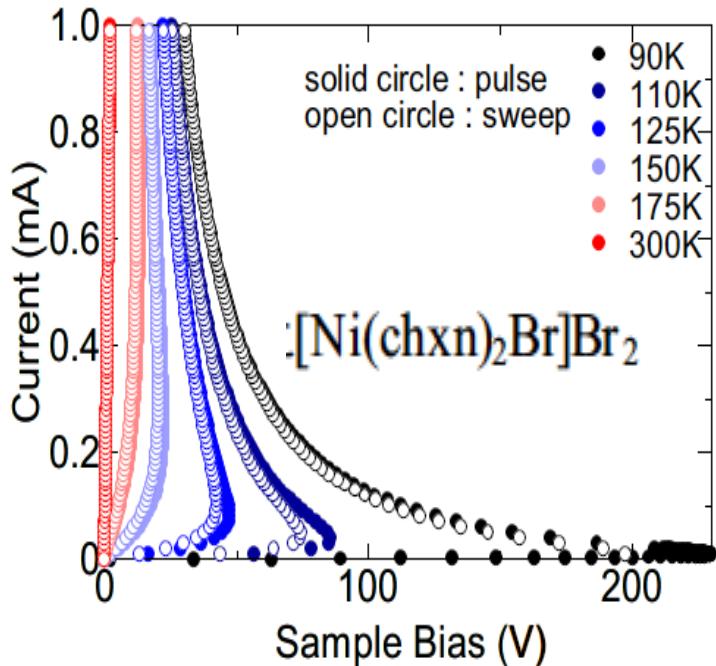
$$\mathcal{P}_{p=0} \rightarrow \begin{cases} \left( \frac{F_0 \xi}{b\Omega} \right)^2 \frac{\Delta_{\text{Mott}}}{\Omega} & \gamma \gg 1, \text{ mulit-photon} \\ \exp \left( -\frac{\pi}{2} \frac{\Delta_{\text{Mott}}}{\xi F_0} \left( 1 - \frac{\pi}{16} \gamma^2 + \dots \right) \right) & \gamma \ll 1, \text{ tunneling} \end{cases}$$

Keldysh parameter  $\gamma = \Omega / \xi F_0$

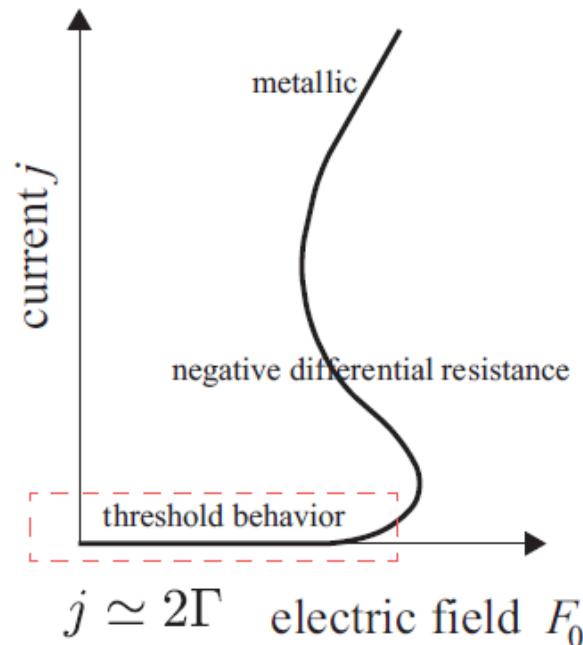


$\Gamma \propto \exp \left( -\pi \frac{\Delta_{\text{Mott}}}{2\xi F_0} \right)$        $\Gamma \propto \left( \frac{F_0 \xi}{2\pi\Omega} \right)^2 \frac{\Delta_{\text{Mott}}}{\Omega}$

# Comparison with Experiments (DC)



H. Kishida, *et al.* J. Appl. Phys. **106**, 016106 (2009);



possible origin of negative differential resistance

1. Heat transfer between fermion/bosons: phonon=heat reservoir.

c.f.) Altshuler et al. PRL 102 (2009)  
Mori et al. PRB (2009)

2. non-equilibrium phase transition c.f.) Ajisaka et al. (2009)

see S. Nakamura's talk

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## 1.2 (Old) Strong field physics

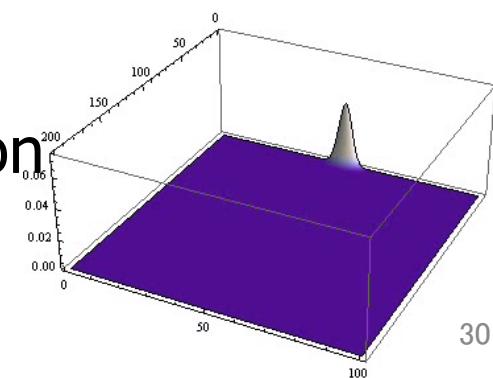
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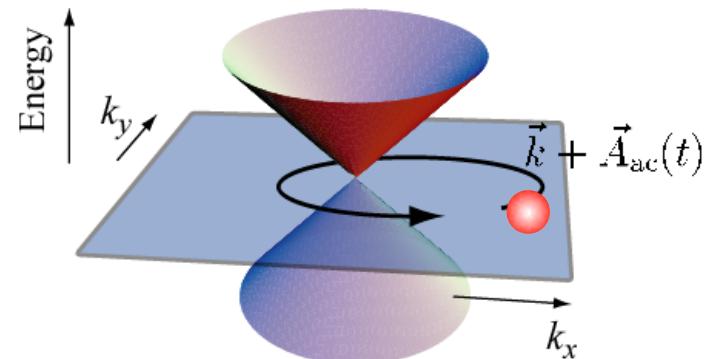
Nonlinear transport/Photo-induced Mott transition

# 3. Photo-induced Hall effect

Floquet topological phase transition



Question:  
What happens to Dirac particles  
in circularly polarized light?



High energy answer:

the Volkov state

D. M. Volkov, Z. Phys. **94**, 250 1935

$$\Psi_{p,r}(x) = \sqrt{\frac{m}{QV}} \sum_{s=-\infty}^{\infty} \left[ J_s(\bar{\alpha}) e^{is\varphi} + \frac{e \hat{\varkappa} \hat{a}_1}{2\varkappa \cdot p} J_s^+(\bar{\alpha}, \varphi) \right. \\ \left. + \frac{e \hat{\varkappa} \hat{a}_2}{2\varkappa \cdot p} J_s^-(\bar{\alpha}, \varphi) \right] u_r(p) e^{-i(q-s\varkappa) \cdot x}.$$

Condensed matter answer:

A dynamical gap opens at the Dirac point of graphene  
making it a photo-induced quantum Hall state.

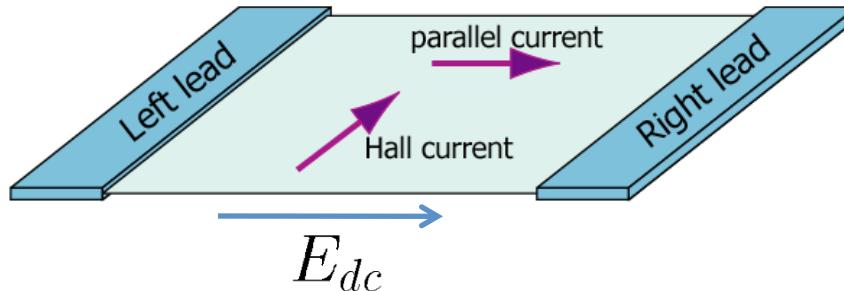
TO, H. Aoki, PRB 79, 081406 (R) (2009)

The two are different:

in graphene, speed of massless fermion  $\ll$  speed of light

momentum of light  $l^\mu = (1, 0, 0, \cancel{1})$

# Topology and Transport: Kubo formula and the TKNN formula



$$J_{\text{dc}}^i = \sigma_{ij} E_{\text{dc}}^j$$

## Kubo formula (linear response)

$$\sigma_{ab} = i \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha, \beta \neq \alpha} \frac{[f_\beta(\mathbf{k}) - f_\alpha(\mathbf{k})]}{E_\beta(\mathbf{k}) - E_\alpha(\mathbf{k})} \frac{\langle \Phi_\alpha(\mathbf{k}) | J_b | \Phi_\beta(\mathbf{k}) \rangle \langle \Phi_\beta(\mathbf{k}) | J_a | \Phi_\alpha(\mathbf{k}) \rangle}{E_\beta(\mathbf{k}) - E_\alpha(\mathbf{k}) + i\eta}$$



$\Phi_\alpha(\mathbf{k})$  Bloch wave function

$$f_\alpha(\mathbf{k}) = (\exp(\beta E_\alpha(\mathbf{k})) + 1)^{-1}$$

Particle anomaly diagram for 2+1 d Dirac model

# TKNN formula in a 2d Dirac system

Thouless, Kohmoto, Nightingale, Nijs 1982

$$\sigma_{xy} = e^2 \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha} f_{\alpha}(\mathbf{k}) [\nabla_{\mathbf{k}} \times \mathcal{A}_{\alpha}(\mathbf{k})]_z$$

Berry curvature

artificial gauge field

$$\mathcal{A}_{\alpha}(\mathbf{k}) \equiv -i \langle \Phi_{\alpha}(\mathbf{k}) | \nabla_{\mathbf{k}} | \Phi_{\alpha}(\mathbf{k}) \rangle$$

First Chern number if  $f=1$

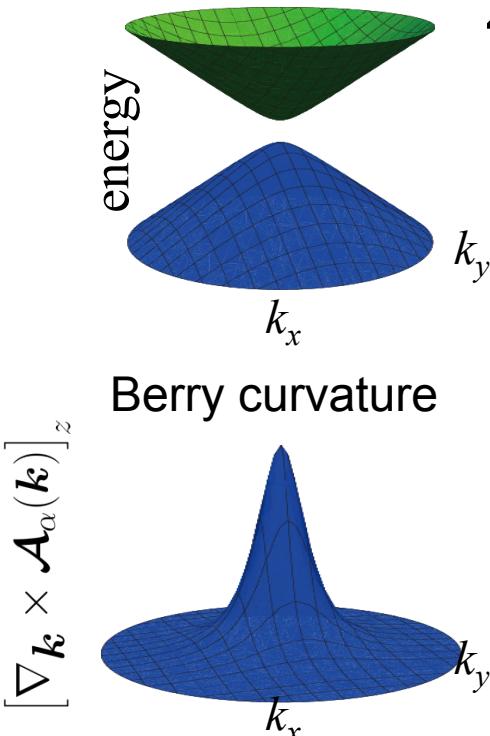
2d Dirac system has a non-trivial Chern number

Niemi Semenoff '83, Redlich '84, Ishikawa '84

$$H = \begin{pmatrix} m & \pm k_x - ik_y \\ \pm k_x + ik_y & -m \end{pmatrix}$$

$$\sigma_{xy} = e^2 \int \frac{d\mathbf{k}}{(2\pi)^d} [\nabla_{\mathbf{k}} \times \mathcal{A}_1(\mathbf{k})]_z$$

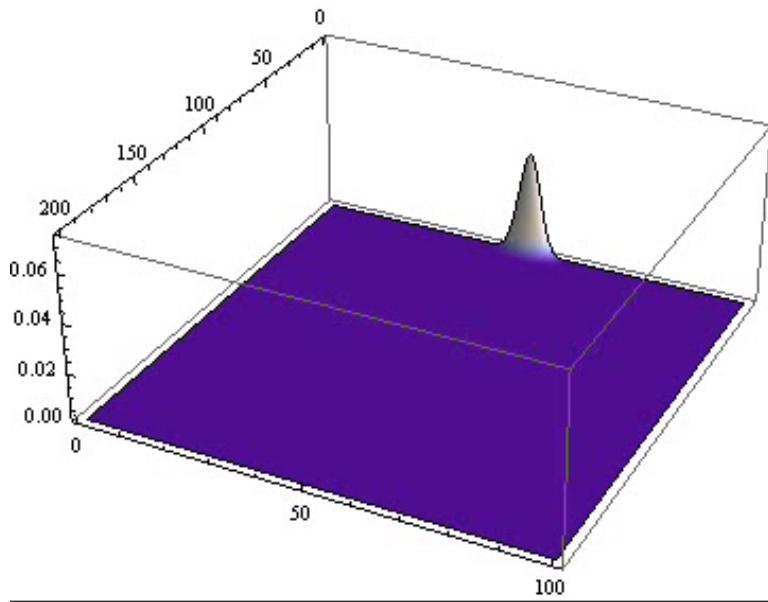
$$= \pm \frac{1}{2} \frac{e^2}{h} \frac{m}{|m|}$$



1. Sign of the mass = direction of the Hall current
2. Dirac cone = half quantum unit
3. parity anomaly in field theory

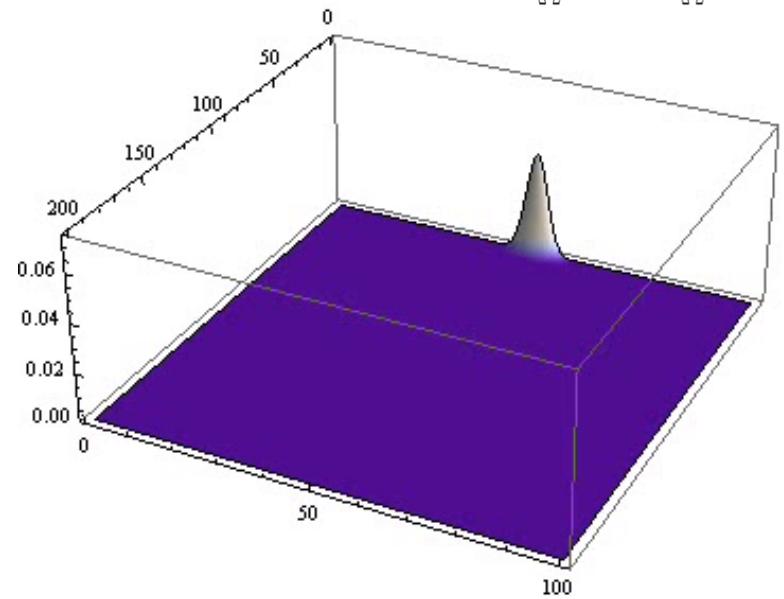
# photo-induced Chern number

$$\mathcal{L} = \bar{\psi} i \gamma_\mu \partial^\mu \psi$$



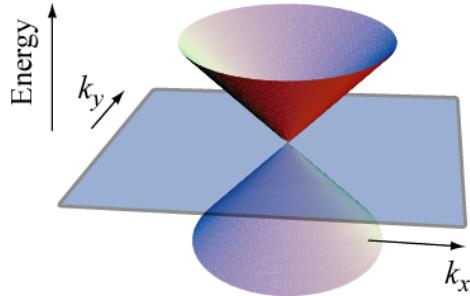
Chern number = 0

$$\mathcal{L} = \bar{\psi} \gamma_\mu (i \partial^\mu - A^\mu(t)) \psi$$
$$A^\mu = (0, \frac{F}{\Omega} \cos \Omega t, \frac{F}{\Omega} \sin \Omega t)$$



Chern number  $\neq 0$

# 2+1 Dirac + circular polarization



$$i\partial_t \psi_k = \begin{pmatrix} 0 & k + Ae^{i\Omega t} \\ \bar{k} + Ae^{-i\Omega t} & 0 \end{pmatrix} \psi_k$$

$$k = k_1 - ik_2 \quad A = F/\Omega$$

Floquet Hamiltonian (Fourier 变換)

$$H^{mn} = \frac{1}{T} \int_0^T dt H(t) e^{i(m-n)\Omega t} + m\delta_{mn}\Omega I$$

$$H_{CP}^{Floquet} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

# circular polarization

$$H_{CP}^{\text{level repulsion}} = \begin{pmatrix} \Omega & k & 0 & A & 0 & 0 \\ \bar{k} & \Omega & 0 & 0 & 0 & 0 \\ 0 & k & 0 & 0 & 0 & A \\ A & 0 & \bar{k} & 0 & 0 & \text{level repulsion with } \Omega \\ 0 & 0 & 0 & 0 & -\Omega & k \\ 0 & 0 & A & 0 & \bar{k} & -\Omega \end{pmatrix}$$

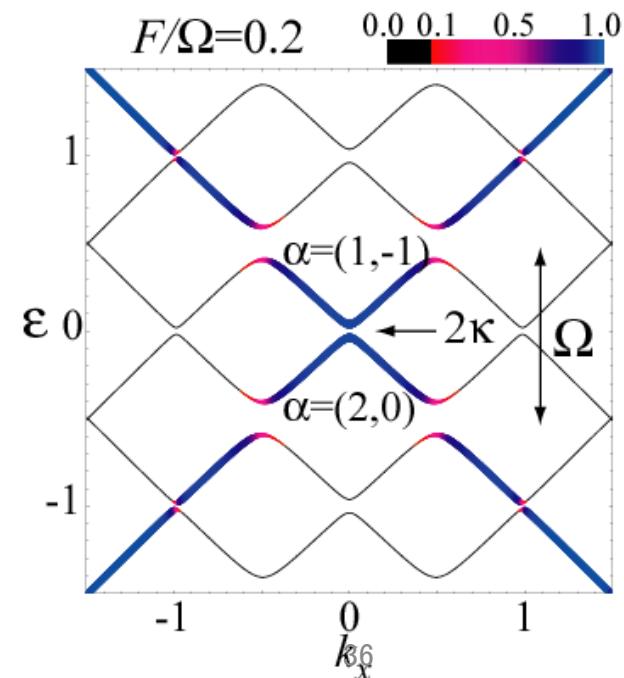
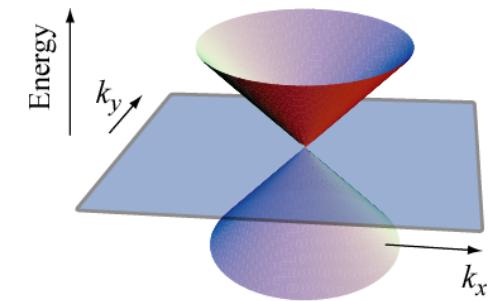
level repulsion with  $-\Omega$

level repulsion with  $\Omega$

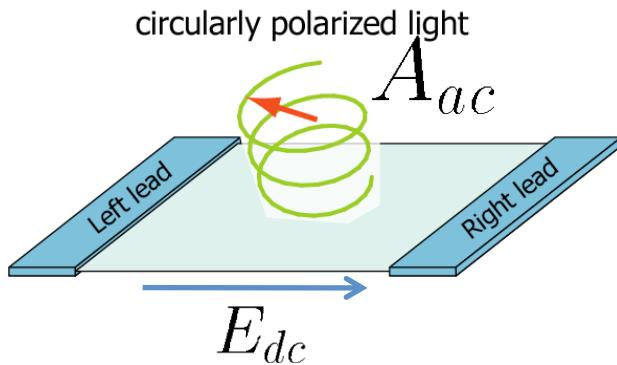
$$\sim \begin{pmatrix} \kappa/2 & k \\ \bar{k} & -\kappa/2 \end{pmatrix}$$

“Dynamical topological gap”

$$\kappa = \frac{\sqrt{4A^2 + \Omega^2} - \Omega}{2} \sim A^2/\Omega$$



# Kubo-formula for photo-induced transport



Large  $A_{ac}$  small  $E_{dc}$

$$J_{dc}^i = \sigma_{ij}(\mathbf{A}_{ac}) E_{dc}^j$$

$$\sigma_{ab}(\mathbf{A}_{ac}) = i \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha, \beta \neq \alpha} \frac{[f_\beta(\mathbf{k}) - f_\alpha(\mathbf{k})]}{\varepsilon_\beta(\mathbf{k}) - \varepsilon_\alpha(\mathbf{k})} \langle\langle \Phi_\alpha(\mathbf{k}) | J_b | \Phi_\beta(\mathbf{k}) \rangle\rangle \langle\langle \Phi_\beta(\mathbf{k}) | J_a | \Phi_\alpha(\mathbf{k}) \rangle\rangle$$

$\varepsilon_\alpha$  Floquet's quasi-energy

$f_\alpha$  occupation fraction

inner product = time average

$$\langle\langle \Phi_\alpha | \Phi_\beta \rangle\rangle = \frac{1}{T} \int_0^T \langle \Phi_\alpha(t) | \Phi_\beta(t) \rangle$$



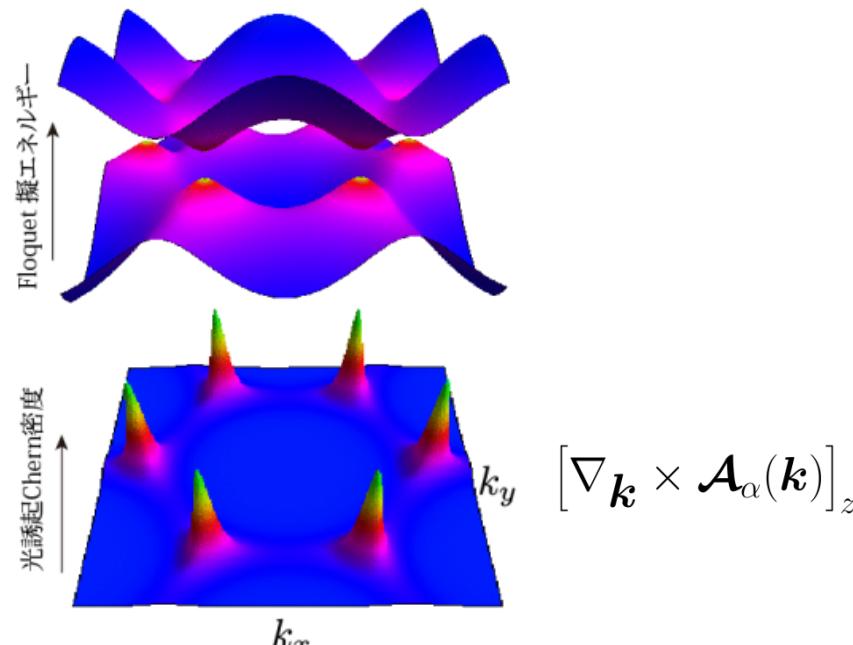
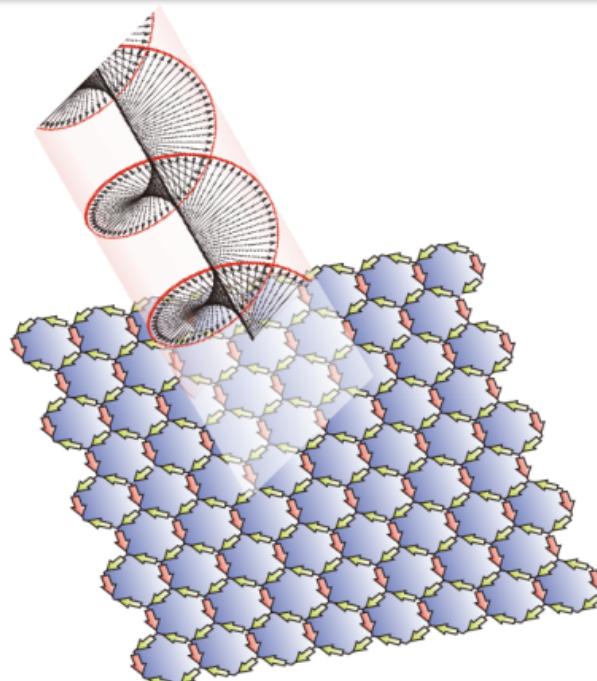
Floquet method = “Furry picture” in HEP

# Extended Thouless-Kohmoto-Nightingale-Nijis formula for photo-induced Hall conductivity (photo-induced Chern form)

$$\sigma_{xy}(A_{\text{ac}}) = e^2 \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha} f_{\alpha}(\mathbf{k}) [\nabla_{\mathbf{k}} \times \mathcal{A}_{\alpha}(\mathbf{k})]_z$$

photo-induced gauge field  $\mathcal{A}_{\alpha}(\mathbf{k}) \equiv -i \langle \langle \Phi_{\alpha}(\mathbf{k}) | \nabla_{\mathbf{k}} | \Phi_{\alpha}(\mathbf{k}) \rangle \rangle$

Floquet states (time-dependent solution)



# Static current in circularly polarized light

DC-component of the current

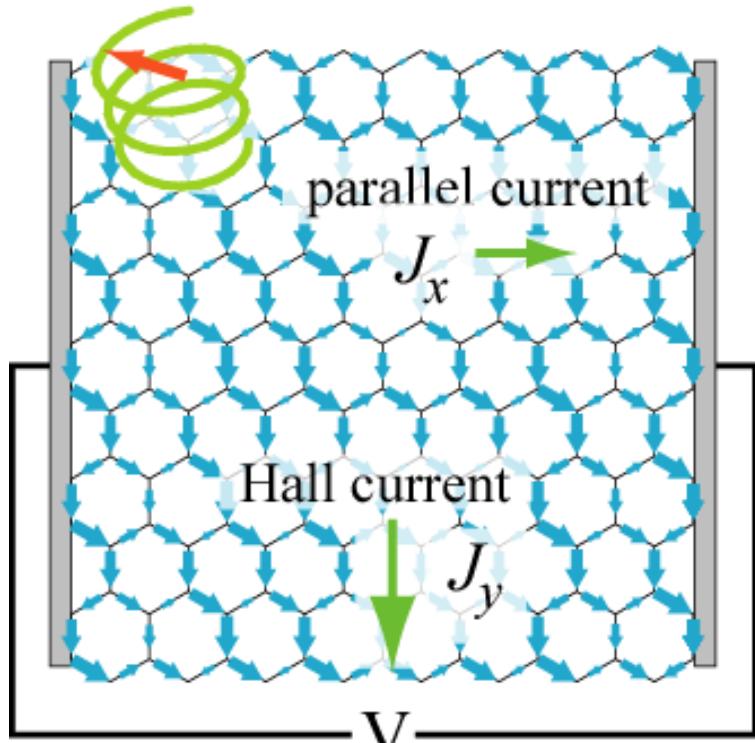


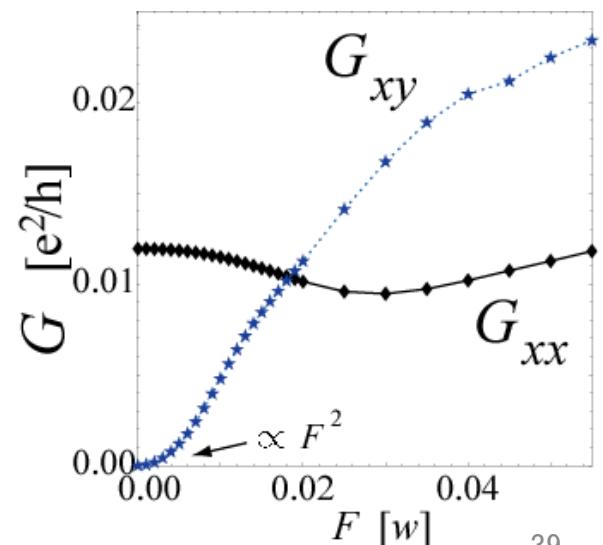
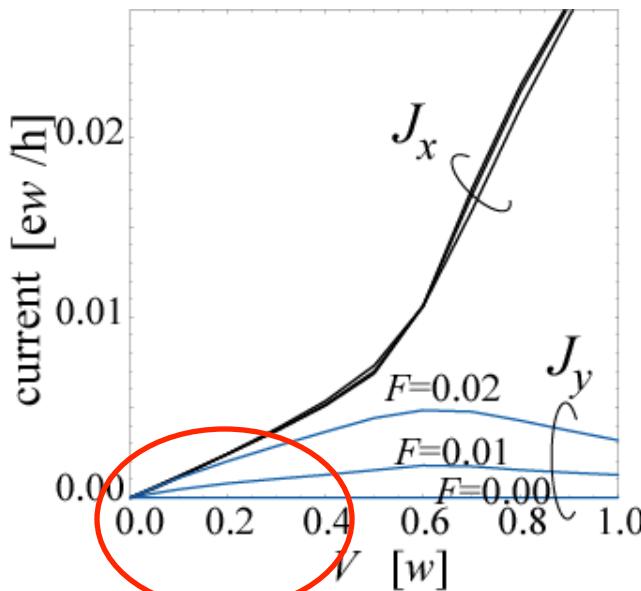
Photo-induced Hall conductivity

$$J_y = G_{xy} V$$

$$G_{xy} \propto F^2$$

Experimentally observable!

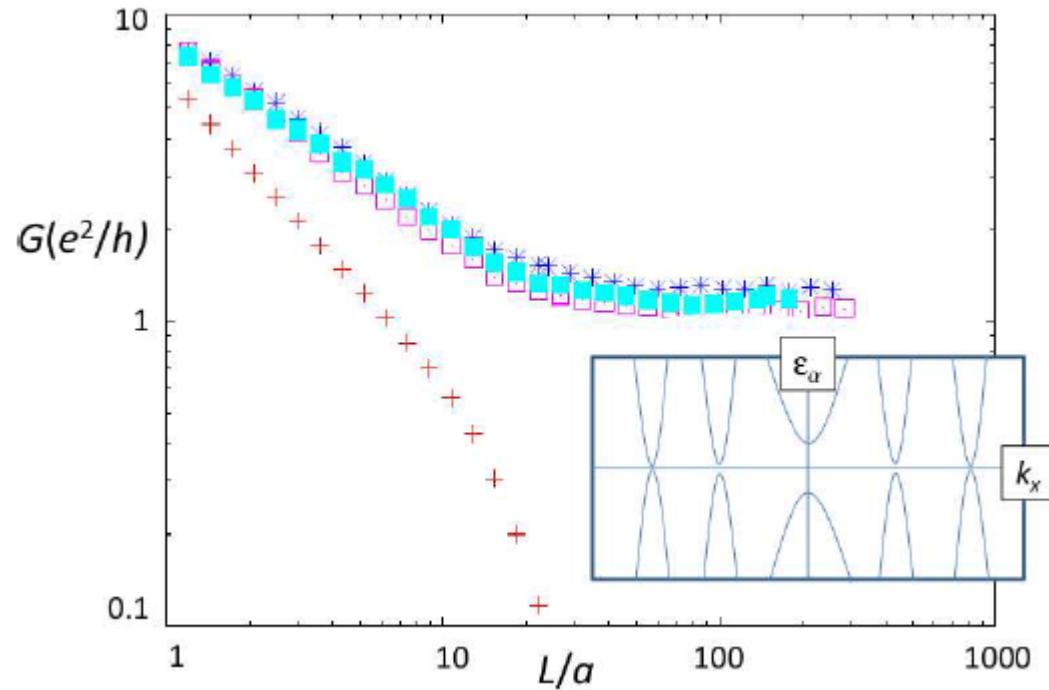
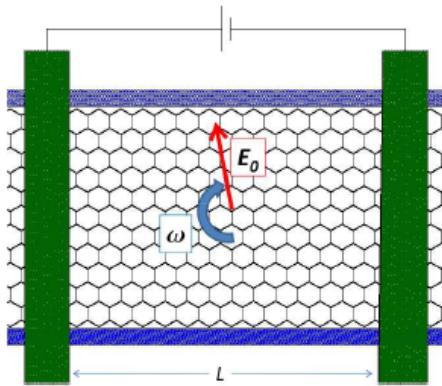
I/V-characteristics



# Quantization of the current

Z. Gu, H.A. Fertig, D. P. Arovas, A. Auerbach, PRL 2011

open boundary condition (ribbon geometry)  
Floquet Landauer formalism



Quantized transport when size is *large* (Edge state)

# Proposals for experimental detection

necessary field strength

$$\Omega \sim 1 - 3 eV \quad F \sim 10^7 V/m$$

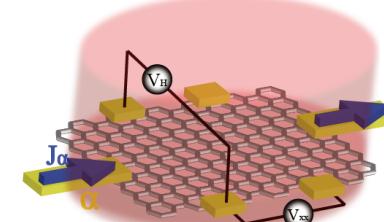
graphene, multilayer-graphene, graphite, surface of TI, etc.

TO, Aoki, arXiv:1007.5399

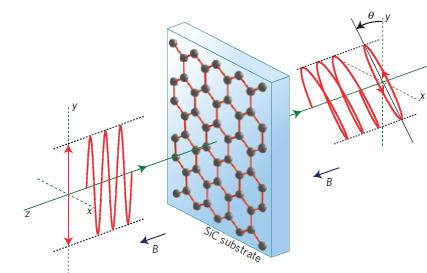
Gapless is best, but gapped system is OK

Possible detection

1. Photo-induced Transport



2. Pump-probe (Kerr effect, MOKE)



3. Pump-probe (photoemission)

# Summary

## Strong field physics in condensed matter

1. Many-body Schwinger mechanism
2. photo-induced topological phase transition

### Future problems:

1. Comparison with hadron physics (RHIC, gauge/gravity)
2. Fast thermalization, appearance of a correlated liquid
3. Non-equilibrium superconductor transition

TO Aoki PRB2008

