Thermalization of Gluonic Matter with the Kadanoff-Baym Approach

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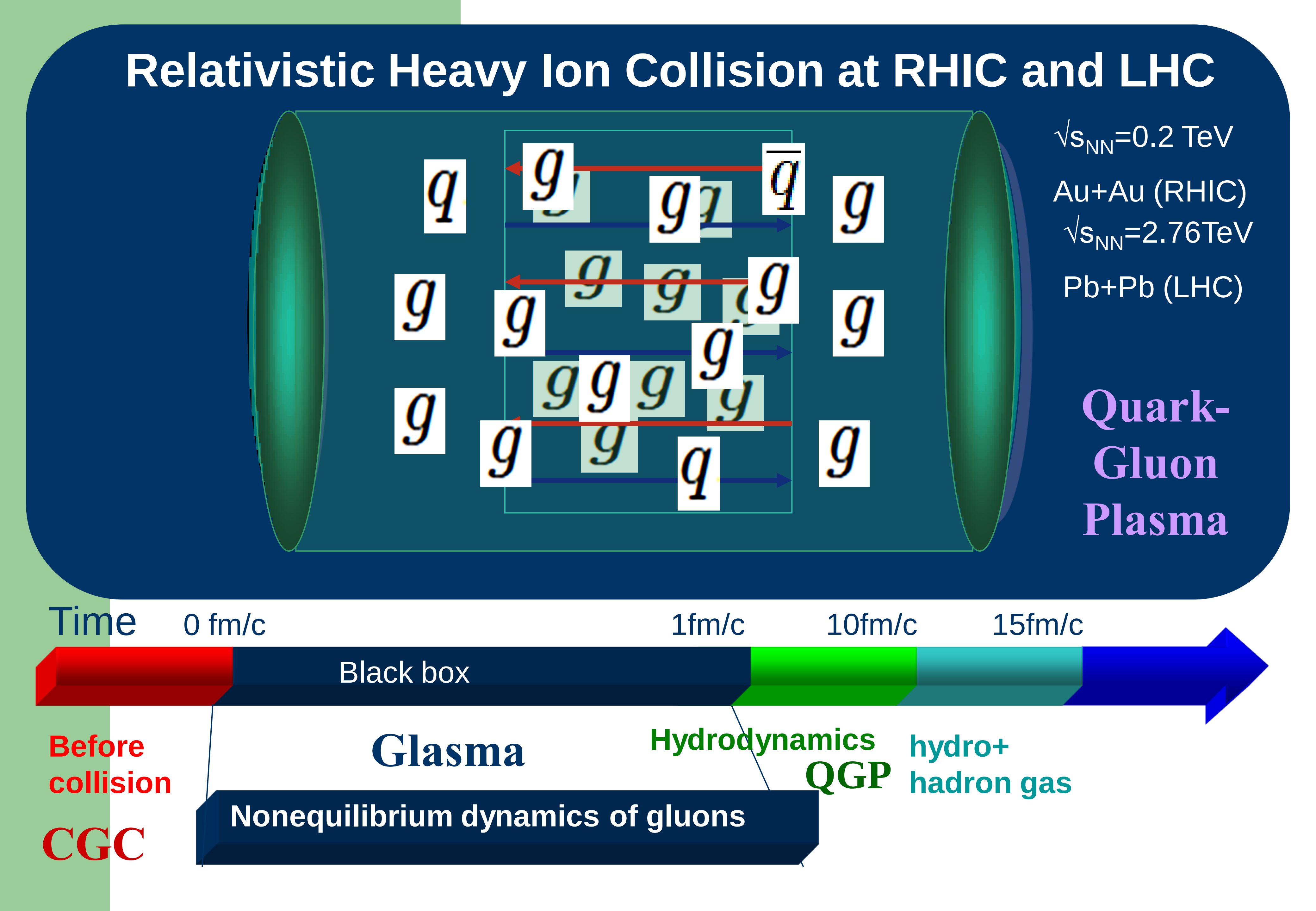
University of Tsukuba

Feb. 18th, 2012.


Relativistic Heavy Ion Collision at RHIC and LHC
$\sqrt{s_{NN}}$=0.2 TeV
Au+Au (RHIC)
$\sqrt{s_{NN}}$=2.76TeV
Pb+Pb (LHC)
Quark-Gluon Plasma
Time
0 fm/c
1fm/c
10fm/c
15fm/c
Black box
Before collision
CGC
Glasma
Hydrodynamics
QGP
hydro+ hadron gas
Nonequilibrium dynamics of gluons

Formation of Quark-Gluon Plasma (QGP)

Success of nearly ideal hydrodynamics after thermalization.
Early Thermalization of gluons (0.6-1fm/c)! (RHIC and LHC)
Kolb and Heinz (2002), Hirano et al. (2010)

Comparative to formation time of partons (1/Qs~0.2fm/c)
Semi-Classical Boltzmann eq. should not be applied,
since 2-3fm/c is predicted for **gg→gg, gg→ggg (Boltzmann).**

Decoherence: Muller, Schafer (2006)

Baier, Mueller, Schiff, Son (2001 and 2011)

New method is needed.

Quantum nonequilibrium processes based on field theory

Application of Kadanoff-Baym eq.
to early thermalization of gluons.

Purpose of this talk

To introduce the Kadanoff-Baym equation and to apply it to gluodynamics.

To show entropy production of gluons in Numerical Simulation and estimate order of time of instability.

To show nonequilibrium process in expanding system with classical field

Rest of this talk

- Kadanoff-Baym equation
- Application to non-Abelian gauge theory, H-theorem
- 3+1 dimension, Numerical Analyses
- Expanding system with classical fields
- Summary and Remaining Problems

Kadanoff-Baym equation

- Quantum evolution equation of 2-point Green's function (fluctuations).

statistical (distribution) and **spectral** functions

$$F(x,y) = \frac{1}{2}\left\langle\left\{\tilde{\phi}(x), \tilde{\phi}(y)\right\}\right\rangle$$

$$\rho(x,y) = \left\langle\left[\tilde{\phi}(x), \tilde{\phi}(y)\right]\right\rangle$$

$$F(p^0,p) = 2\pi\delta(p^2-m^2)\left(1+\underbrace{\frac{1}{e^{\beta|p^0|}-1}}_{\text{Boson}}\right)$$

$$\rho(p^0,p) = \frac{\gamma}{(p^0-\omega)^2+\gamma^2/4} \xrightarrow{\gamma\to 0} 2i\pi\epsilon(p^0)\delta(p^2-m^2)$$

Breit-Wigner type

$$\left(-G_0^{-1}+\Sigma_{\rm loc}\right)F(x,y) = \int_0^{y^0} dz\Sigma_F(x,z)\rho(z,y) - \int_0^{x^0} dz\Sigma_\rho(x,z)F(z,y)$$

$$\left(-G_0^{-1}+\Sigma_{\rm loc}\right)\rho(x,y) = \int_{x^0}^{y^0} dz\Sigma_\rho(x,z)\rho(z,y)$$

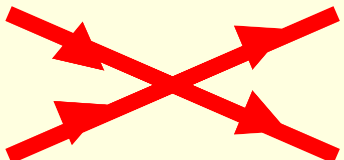
Memory integral

$G_0^{-1} = -\partial^2 - m^2$ **Σ=Self-energies**

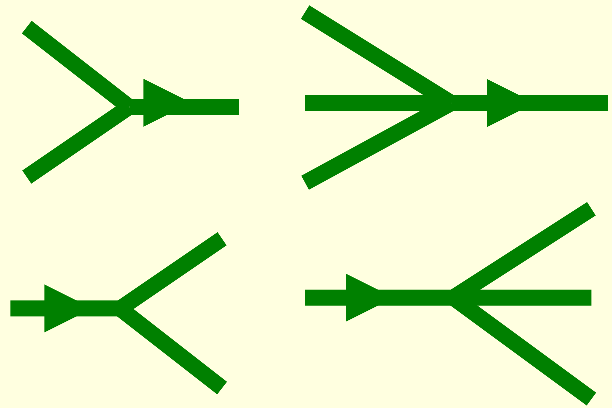
Self-energies: local $\Sigma_{\rm loc}$ mass shift, nonlocal real Σ_F and imaginary part Σ_ρ

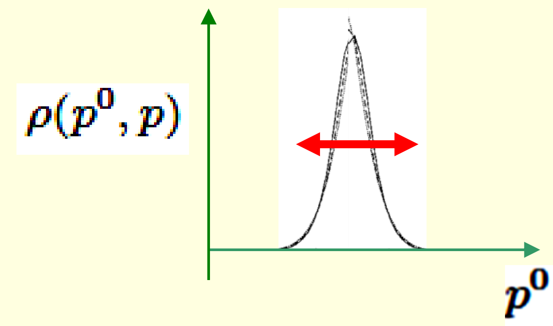
Merit

- **Quantum evolution with conservation law**
- **Evolution of spectral function with decay width + distribution function**



binary





Finite decay width

- **Off-shell effect**

Decay width ⇒ particle number changing process (gg⇔g (2-to-1) and ggg⇔g (3-to-1))+ binary collisions (gg⇔gg).

They are prohibited kinematically in Boltzmann simulation. This process might contribute to the early thermalization.

Demerit

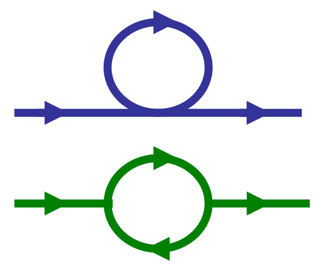
Numerical simulation needs much memory of computers.

Application to Non-Abelian Gauge Theory

- **Temporal Axial Gauge $A^0=0$**
- **No classical field $\langle A \rangle = 0$**
- **Leading Order Self Energy of coupling**

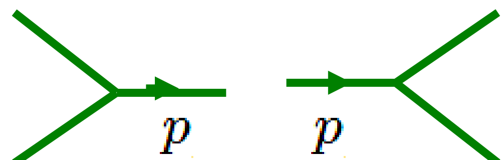
LO (local)

LO (nonlocal)



$O(g^2)$

$\Leftrightarrow$

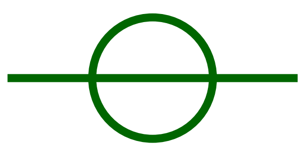


gg⇔g

$O(g^2)$

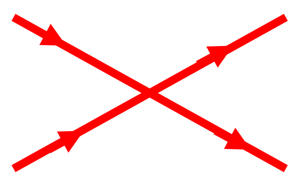
If necessary, we use NLO as shown here.

NLO

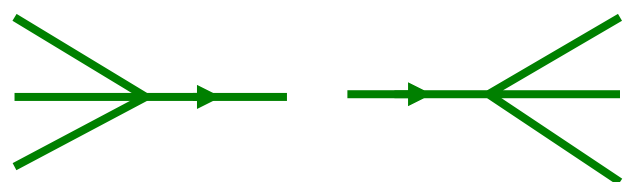


\+ …

$\Leftrightarrow$



gg⇔gg



ggg⇔g

Numerical analysis for KB eq. in 3+1 dim.

- Without classical fields
- Initial condition

  Nonthermal distribution (Gaussian configuration, anisotropic in momentum space)

Transverse

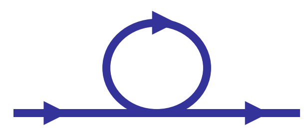
$$n^0_{\mathbf{k}} = \frac{C}{\Delta_\perp^2 \Delta_z} \exp\left(- \frac{k_z^2}{2\Delta_z^2} - \frac{k_x^2 + k_y^2}{2\Delta_\perp^2} \right)$$

Longitudinal

$$n^0_{\mathbf{k}} = 0$$

$$\boxed{x \equiv \frac{\Delta_\perp^2}{\Delta_z^2} = 100}$$

Uniform Space



$\fallingdotseq g^2NT^2/9 \rightarrow m^2$

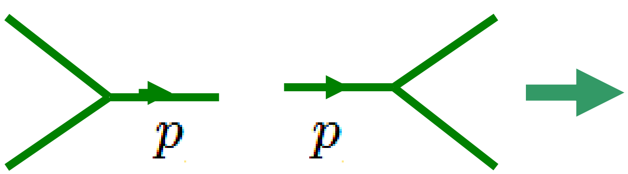
Set $T = 360\mathrm{MeV}, g^2 = 1.0$

$\rightarrow$ m=210MeV

thermal mass

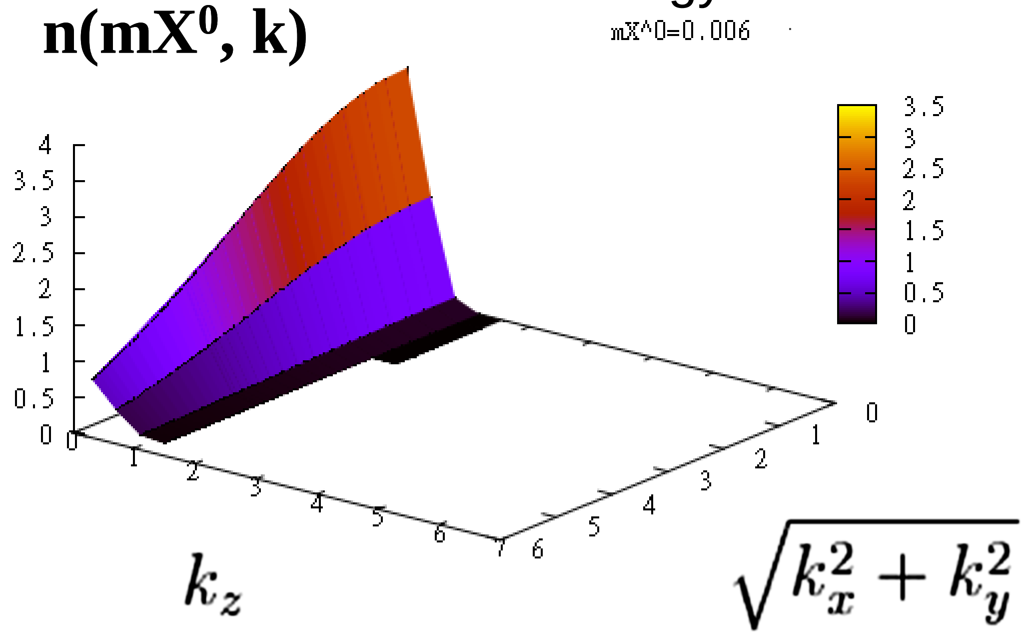
ε=12GeV/fm^3

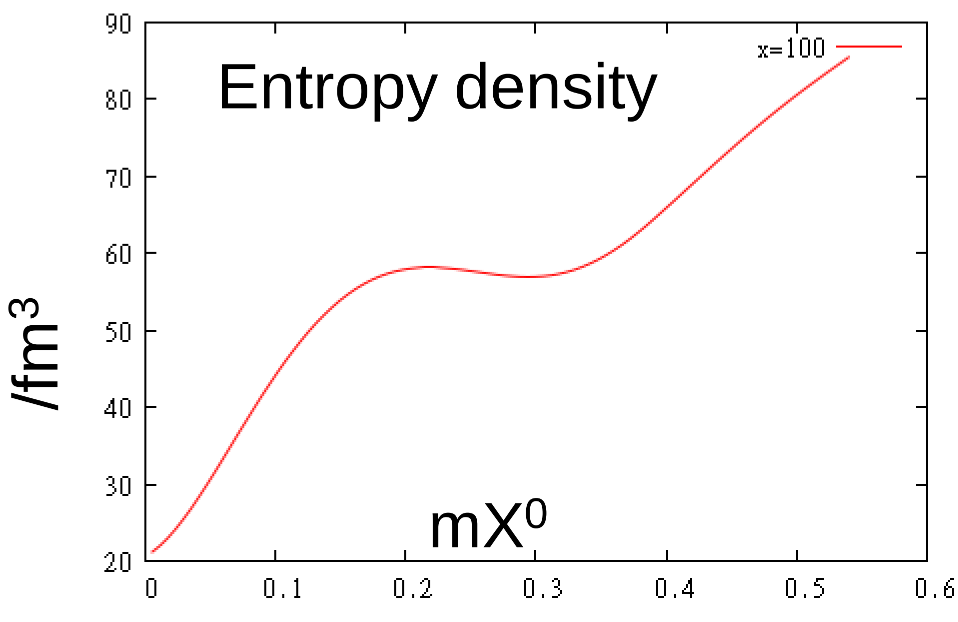
- Without expansion



3 transverse splitting and 2 transverse 1 longitudinal splitting

Energy error is 2 %.

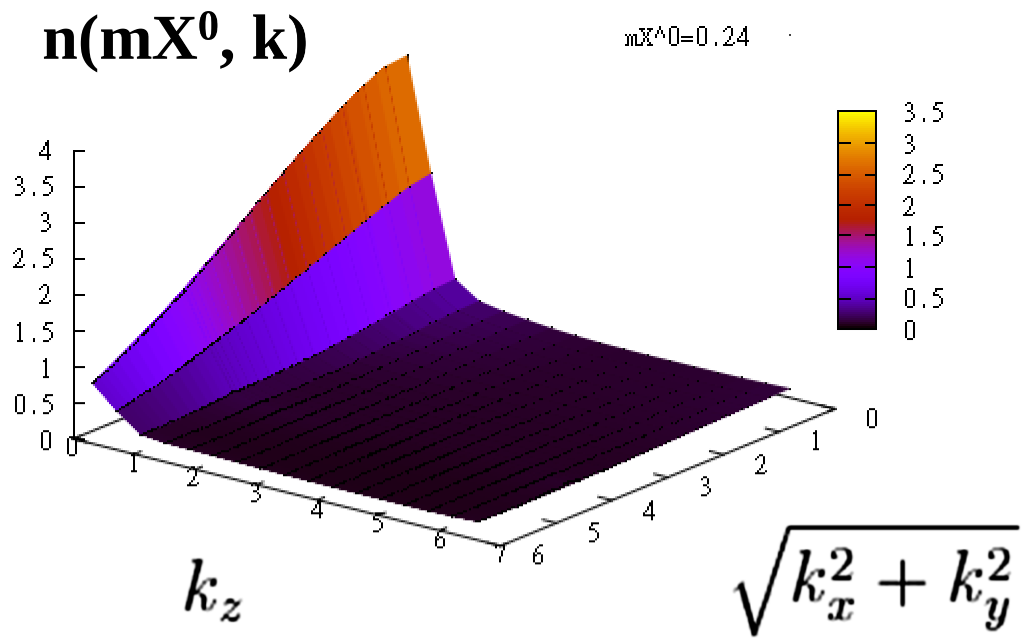


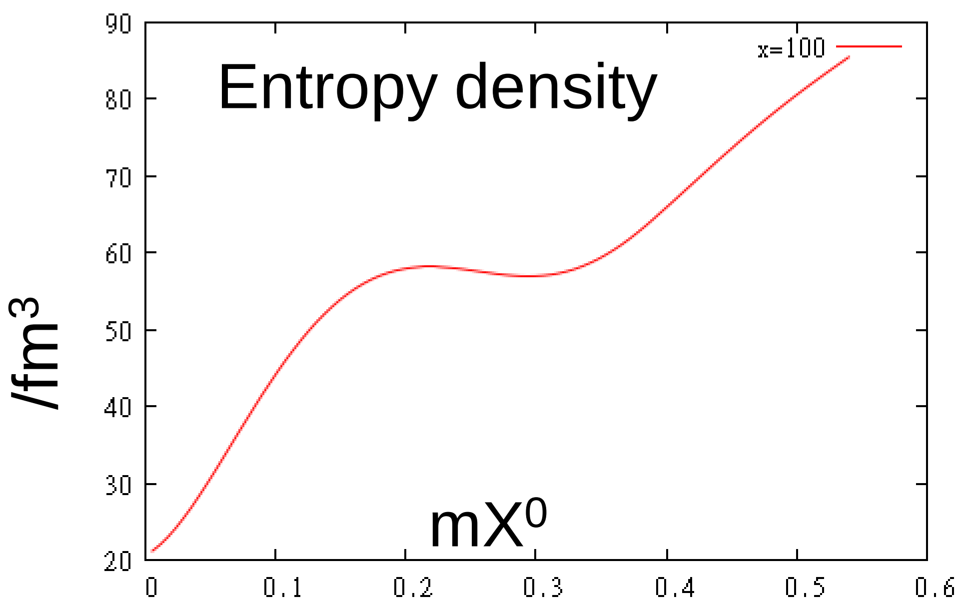


The X-Y mode → Z mode.

The n_k approaches Bose distribtuion due to off-shell g⇔gg (1⇔2) in 3+1 dim.

Entropy production occurs at early time $mX^0 \leqq 1$.





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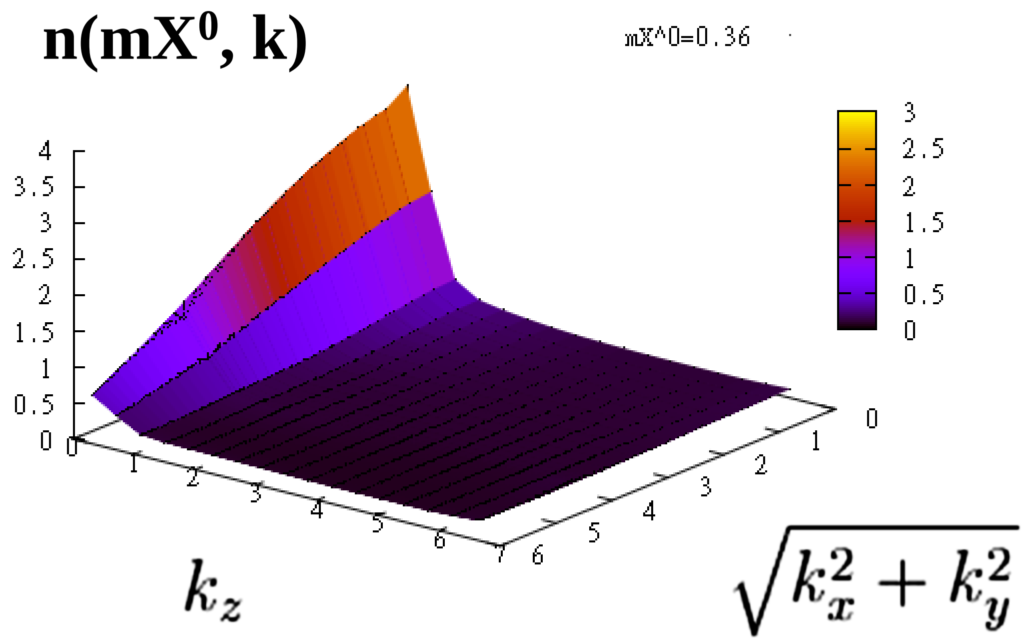

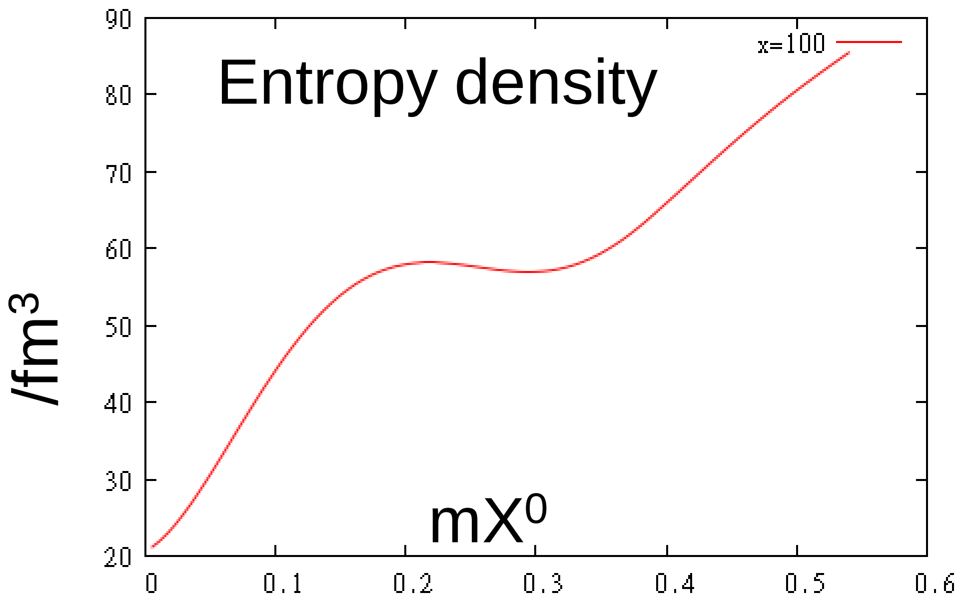

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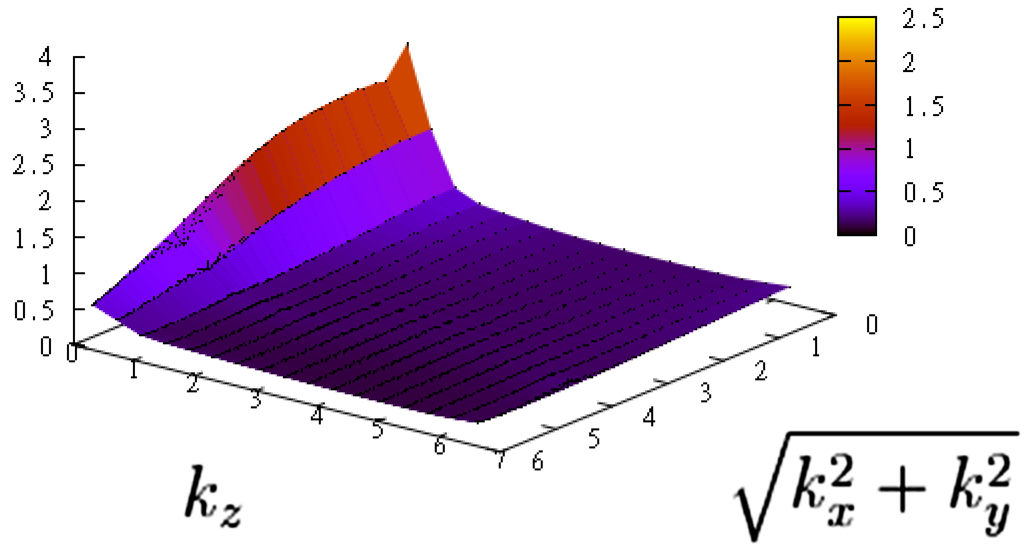
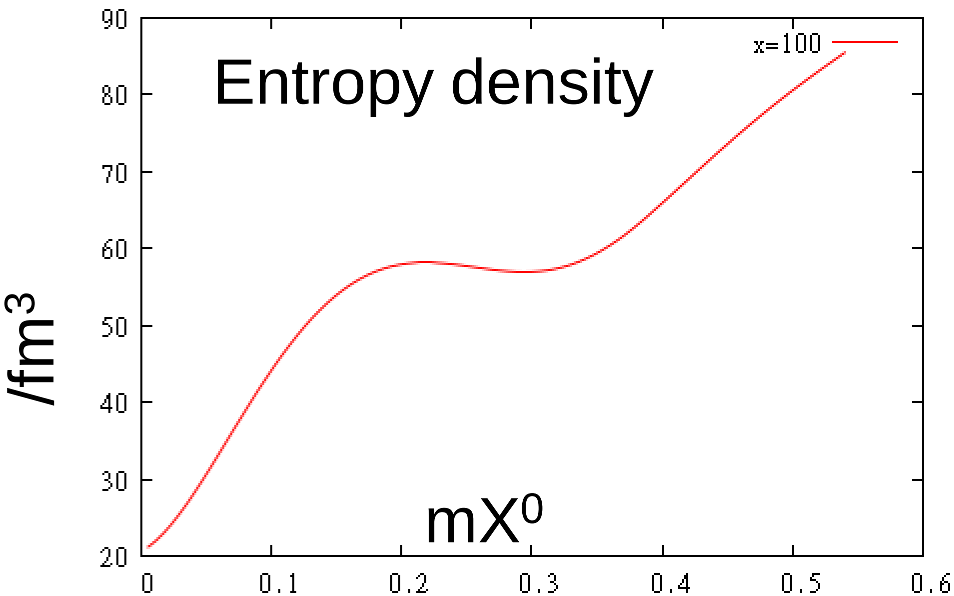
The n_k approaches Bose distribtuion due to off-shell g⇔gg (1⇔2) in 3+1 dim.

Entropy production occurs at early time $mX^0 \lesssim 1$.

n(mX⁰, k)





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$n(mX^0, k)$

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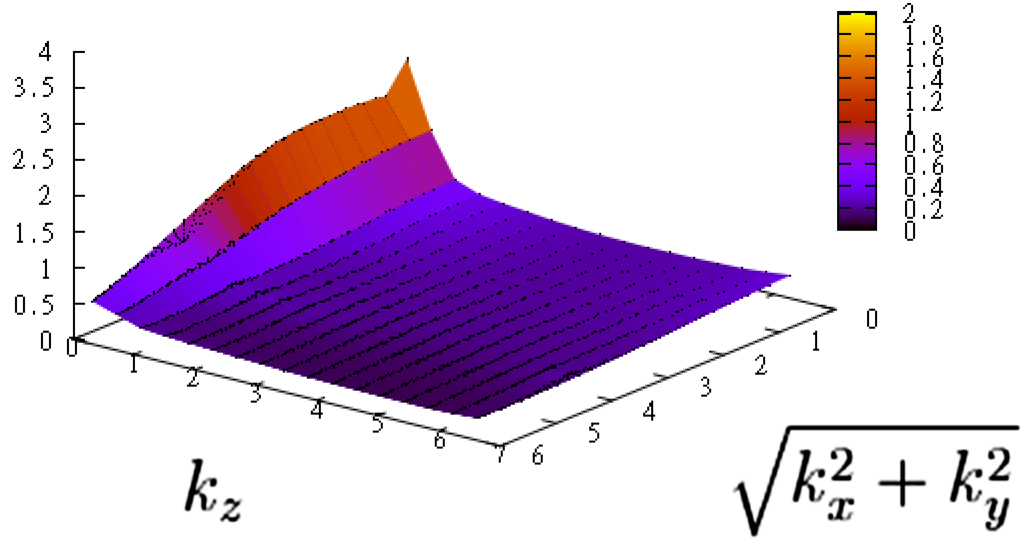
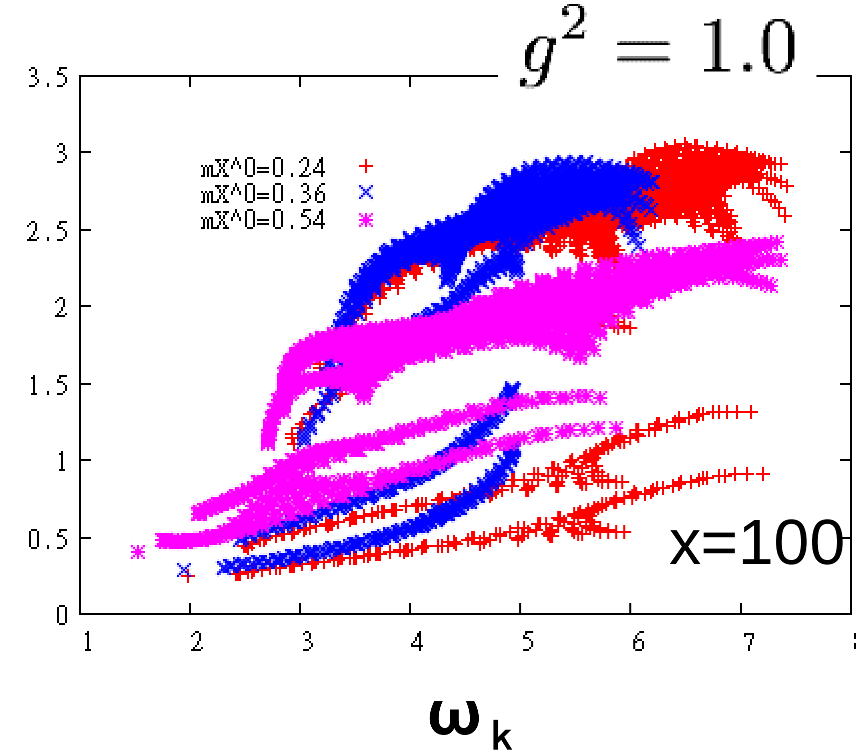
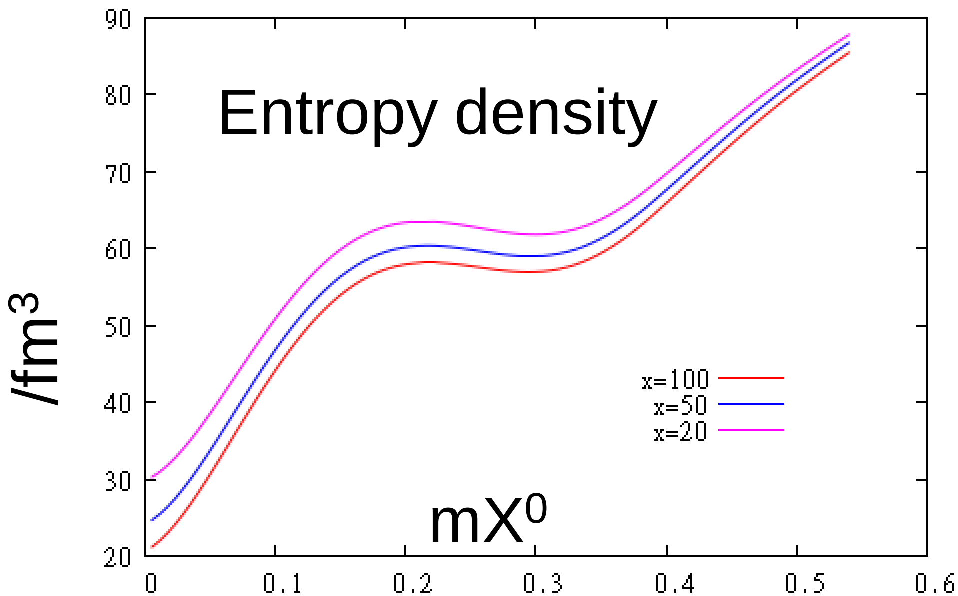


log[1+1/n(mX^0,k)], logplot





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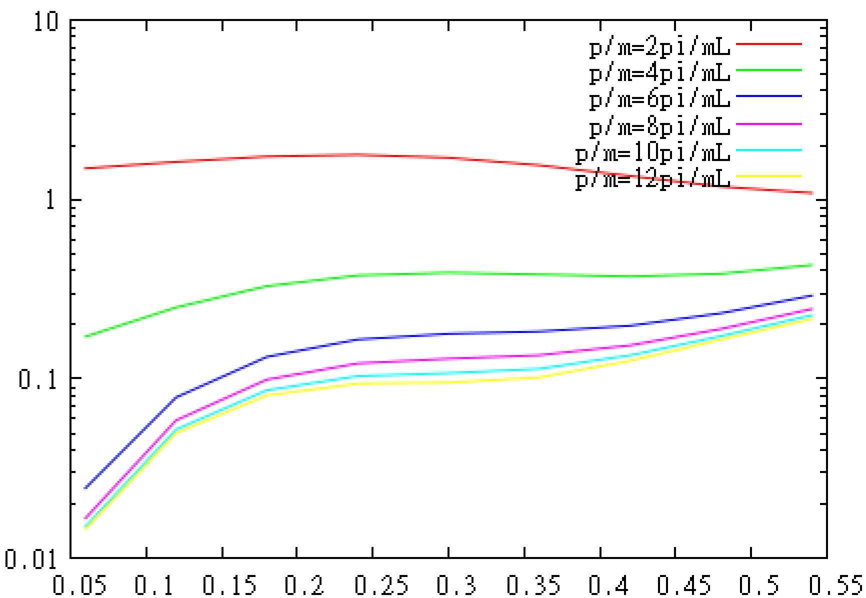
Evolution of each mode in distribution

Transverse mode

$n(p_x=p_y=p_z=p,X^0)$



mX^0

$n(p,X^0)=A\exp(\gamma(p)X^0)$

$1/\gamma_{max}=0.3/m$ fm/c

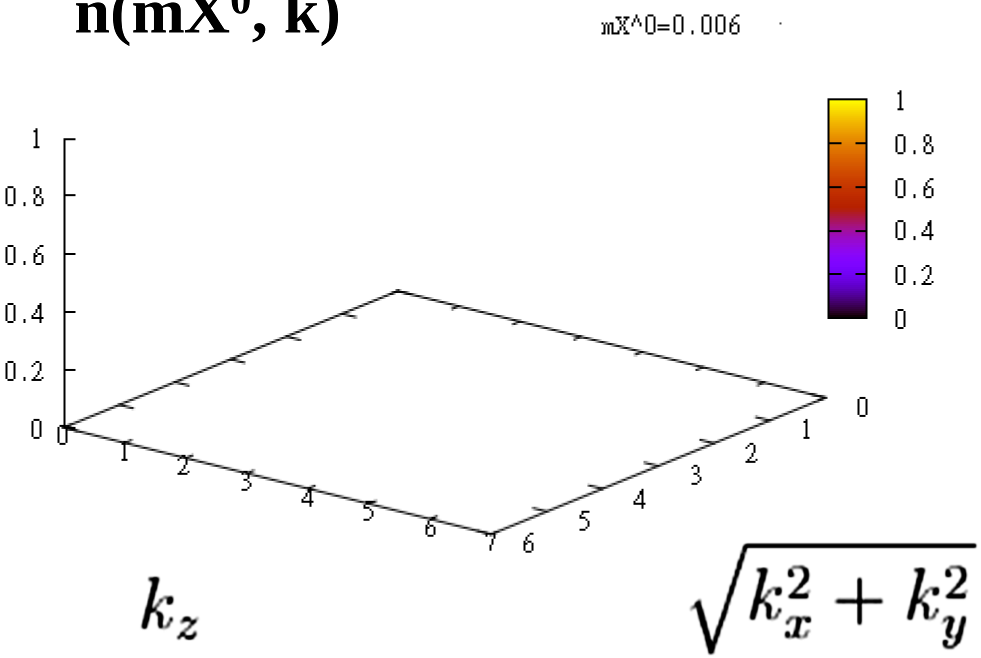
$(p=(8\sim22)\pi/L=1.5\sim4.3m)$

Smaller time scale is realized.

Higher momentum mode $32\pi/mL \geqq p/m > 24\pi/mL$ is still not monotonically increasing functions.

Time evolution of longitudinal distribution function

$n(mX^0, k)$



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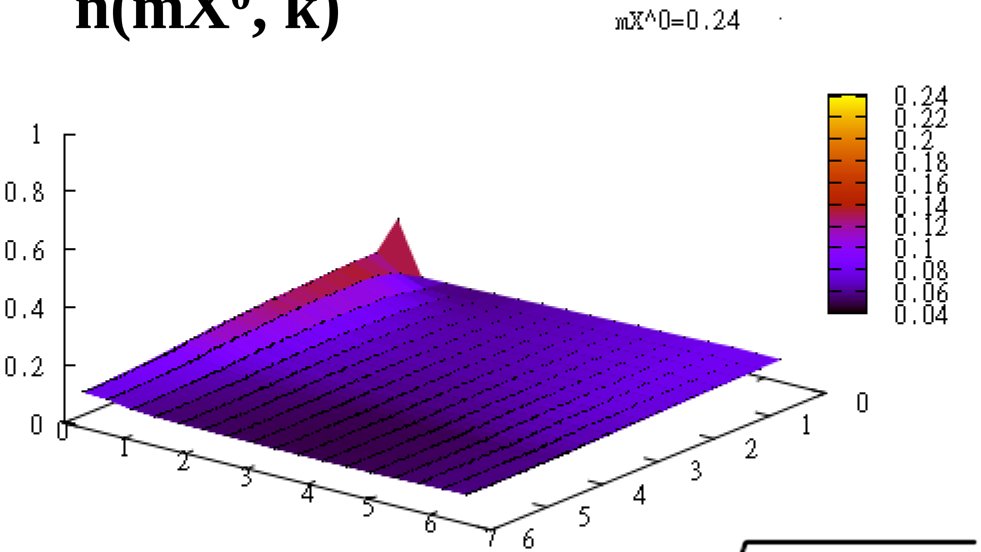
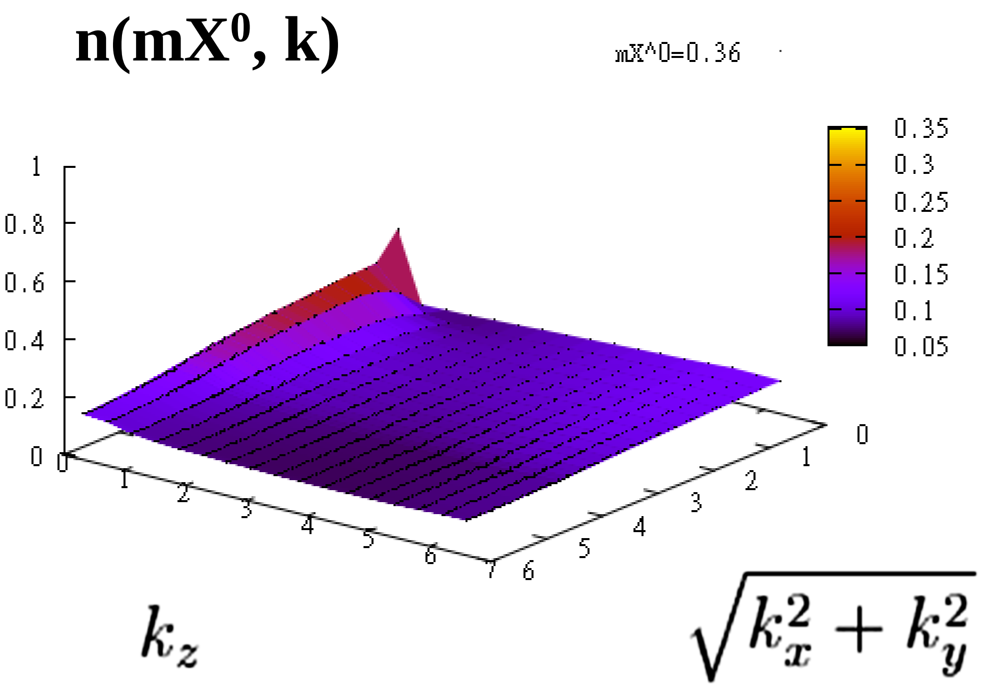


k_z $\sqrt{k_x^2 + k_y^2}$

Time evolution of longitudinal distribution function



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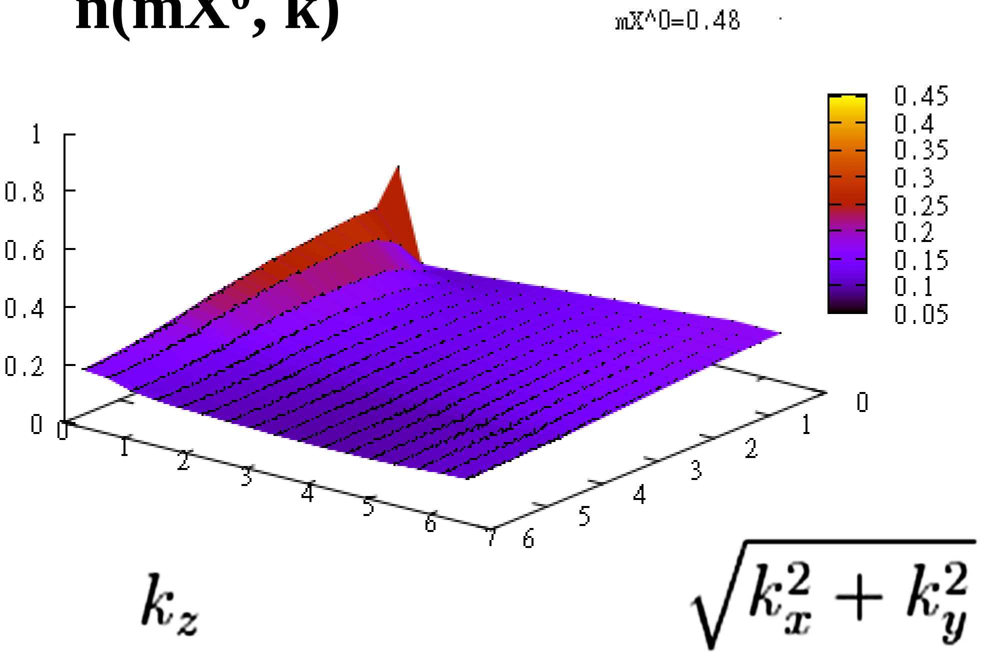

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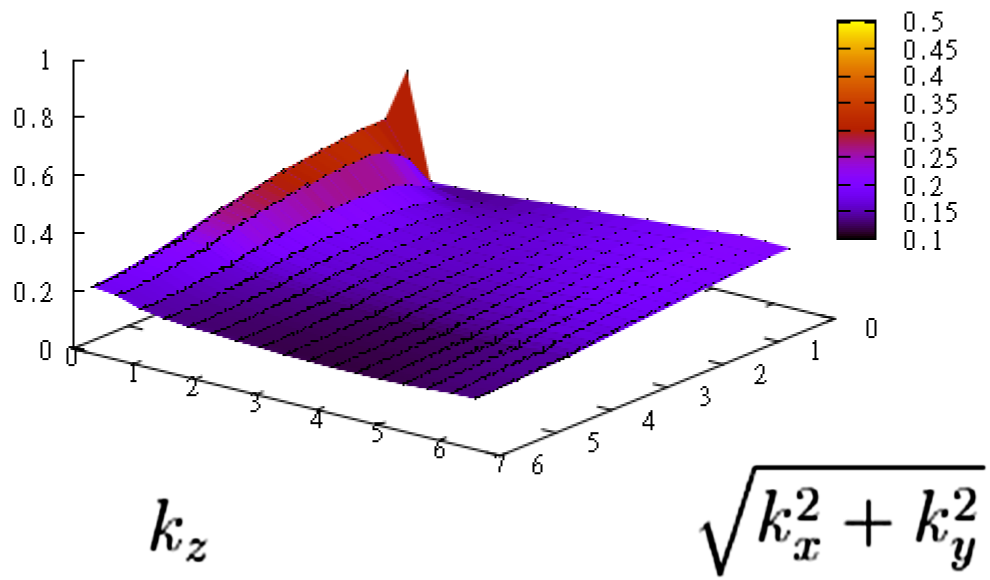
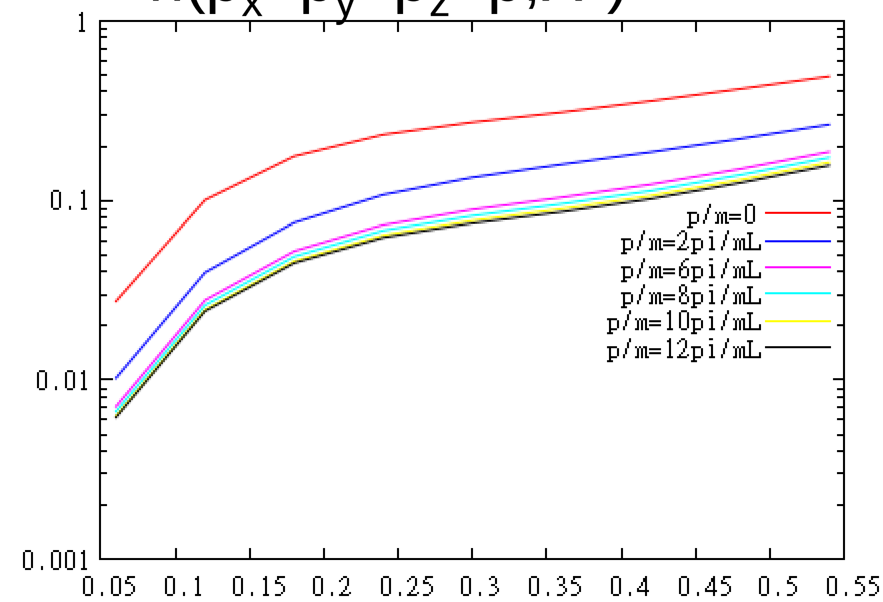


Longitudinal mode.

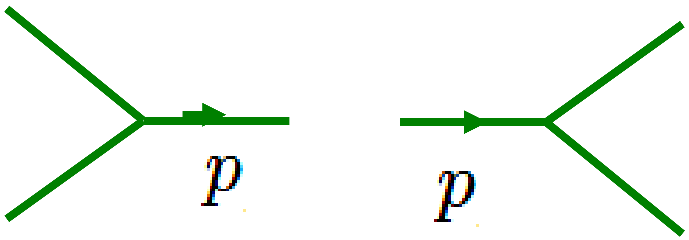
$n(p_x=p_y=p_z=p, X^0)$



$n(p,X^0)=A\exp(\gamma(p)X^0)$

$1/\gamma_{max}=0.3/m$ for each mode.

Numerical analysis for KB eq. in 3+1 dim.



$$n_0(\mathbf{k}) = \frac{C}{\Delta_z \Delta_\perp^2} \exp\left[-\frac{k_x^2 + k_y^2}{2\Delta_\perp^2} - \frac{k_z^2}{2\Delta_z^2} \right]$$

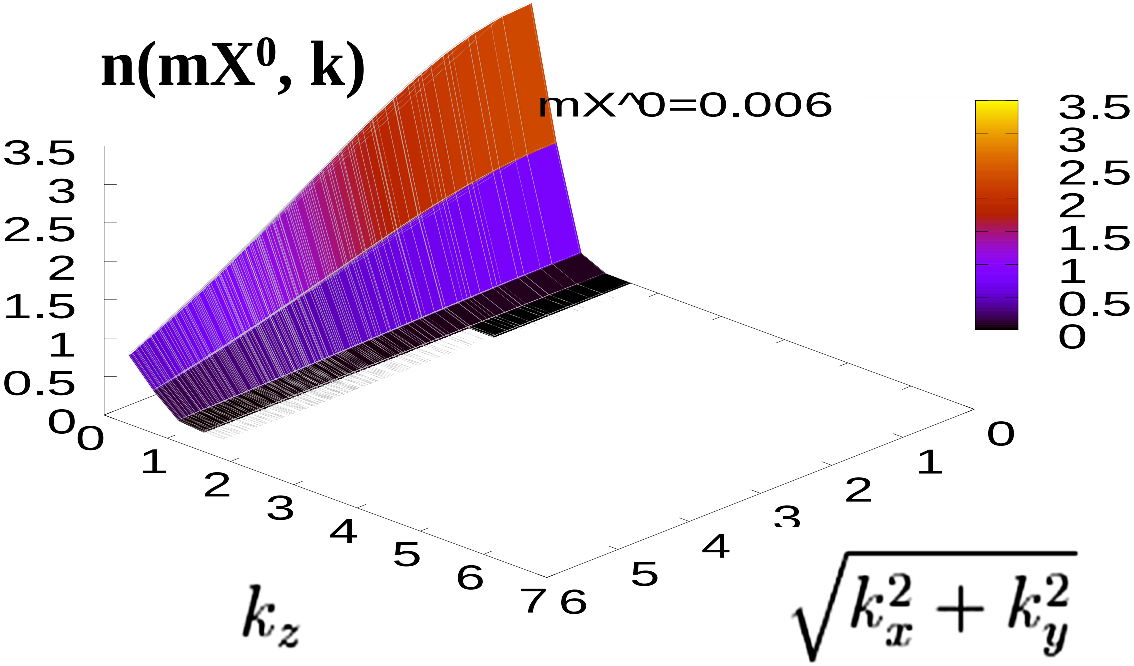
$$\boxed{x \equiv \frac{\Delta_\perp^2}{\Delta_z^2} = 100}$$

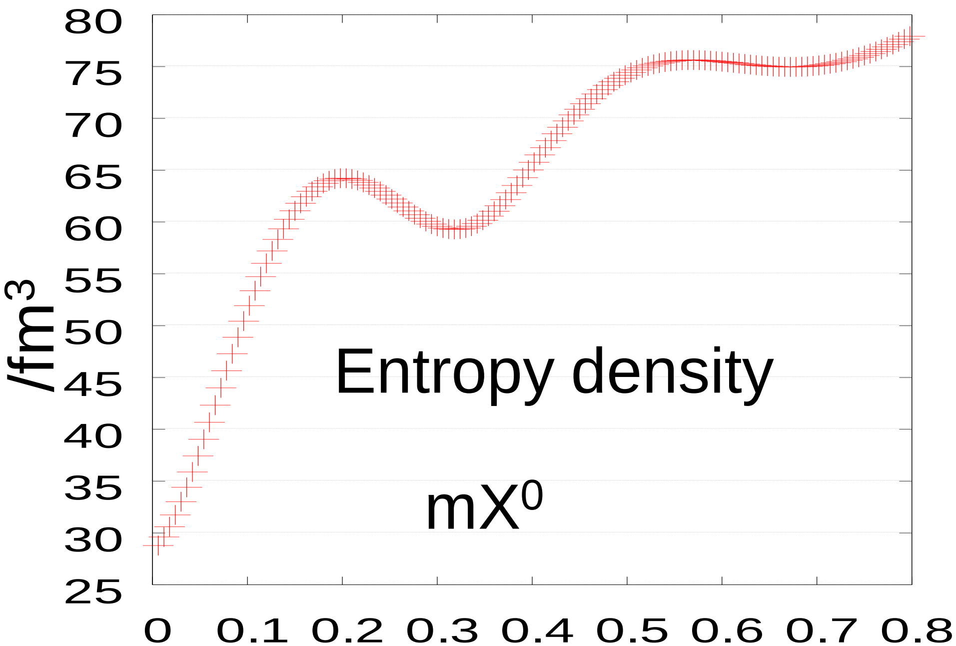
→ Only transverse Part

m=210MeV

thermal mass

ε=12GeV/fm^3





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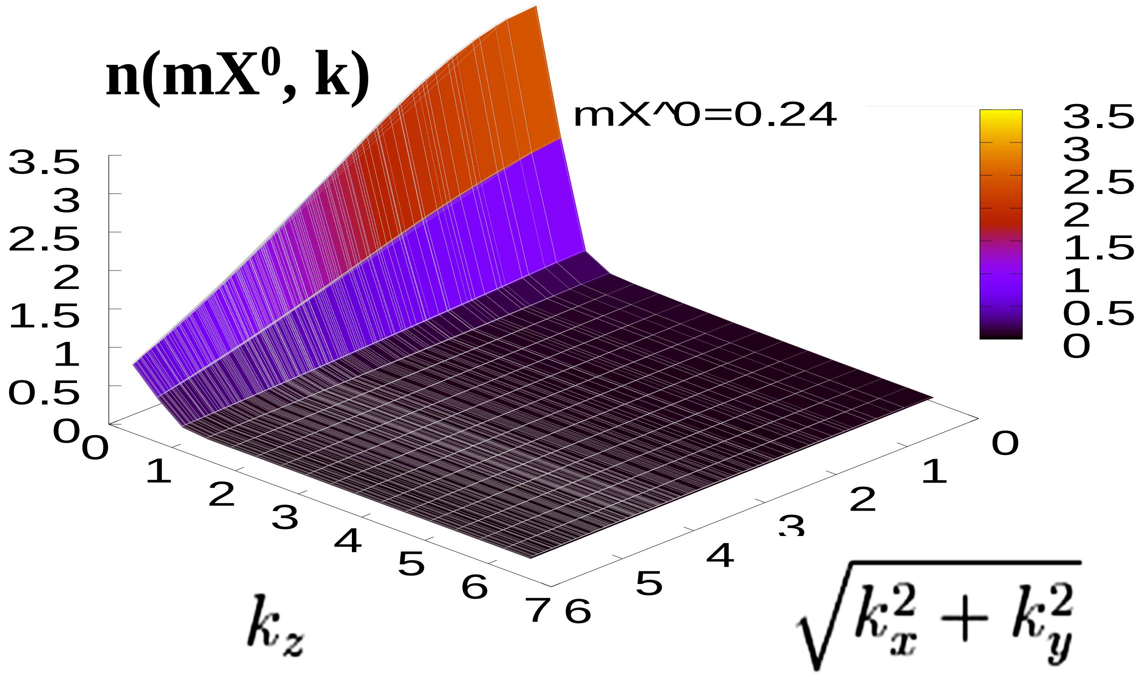

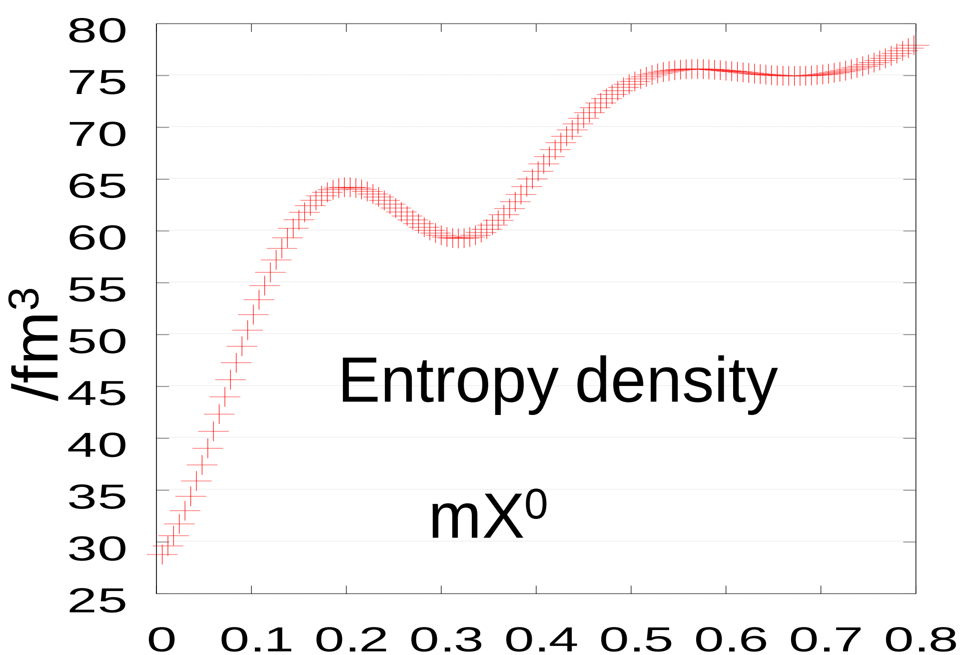

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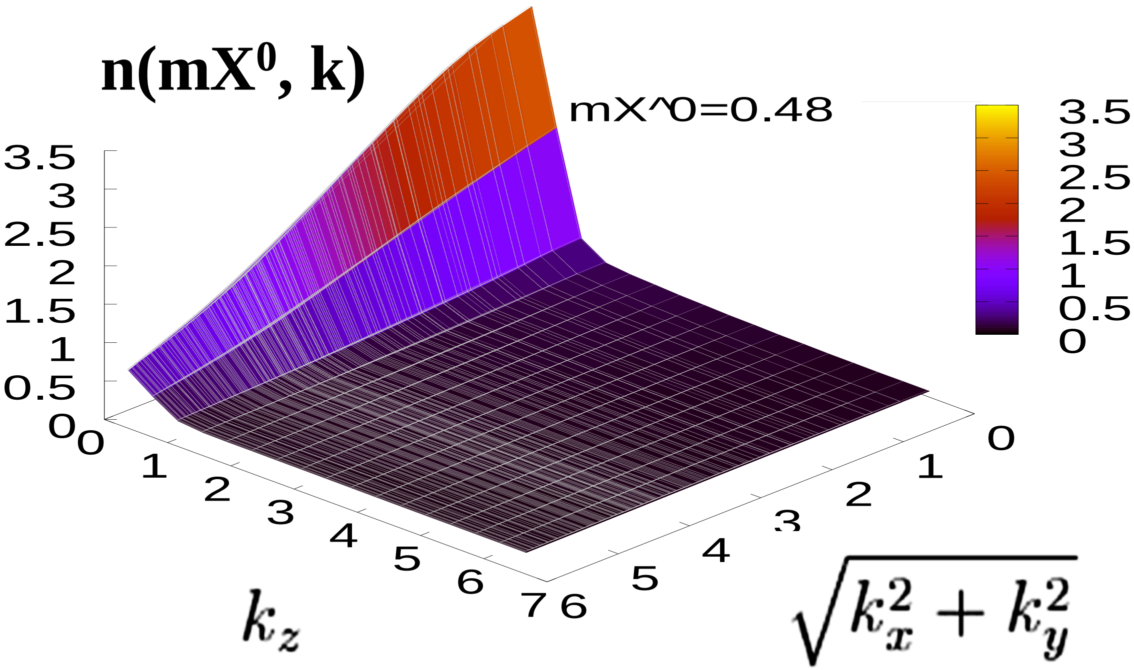

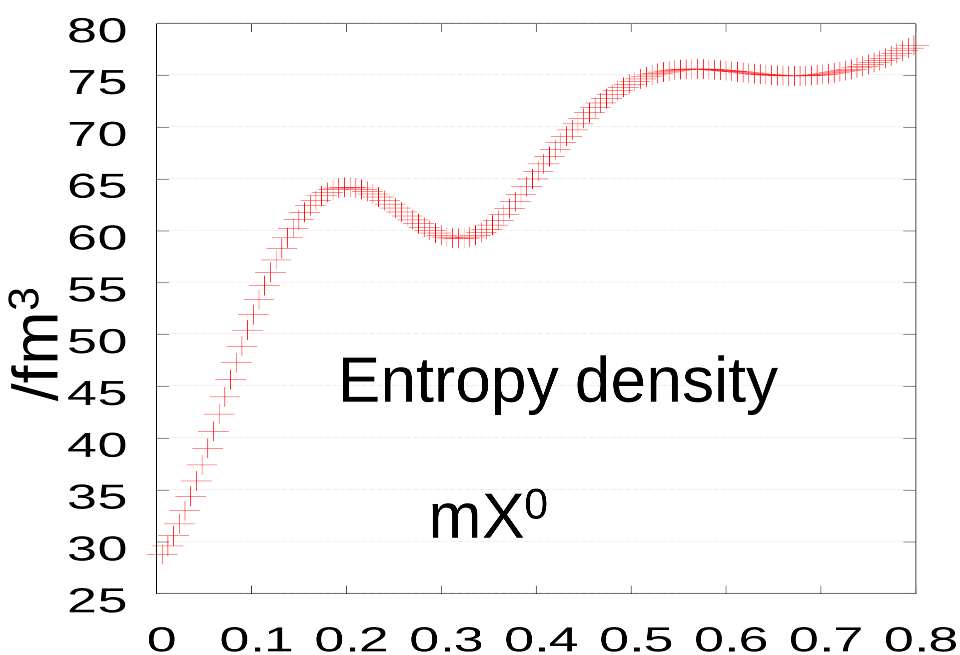

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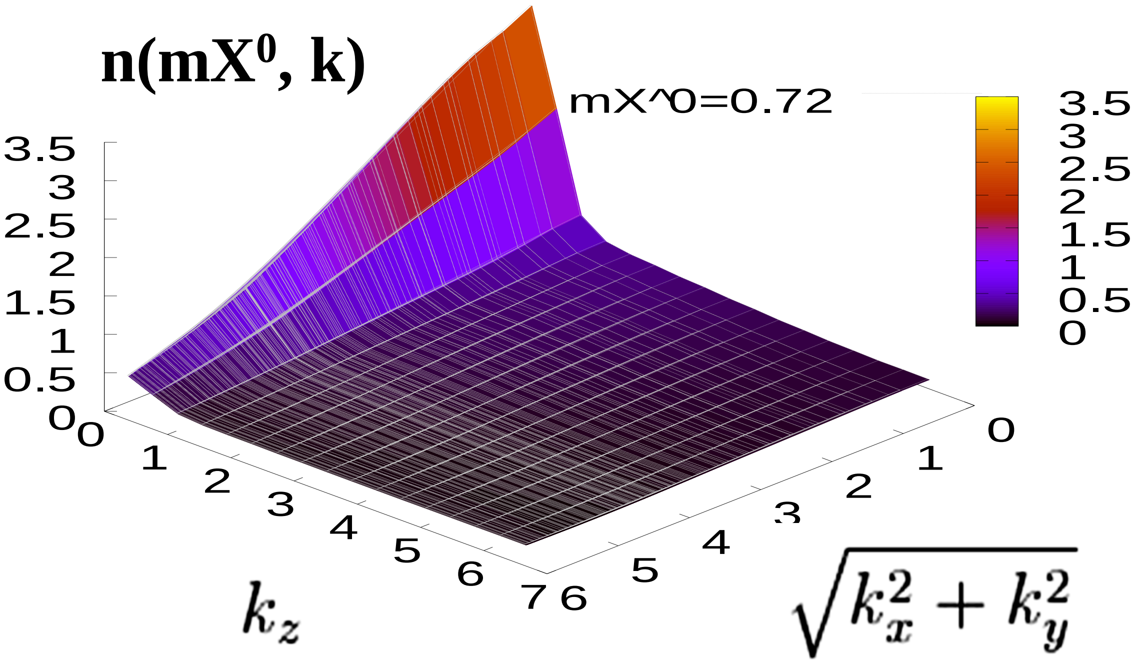

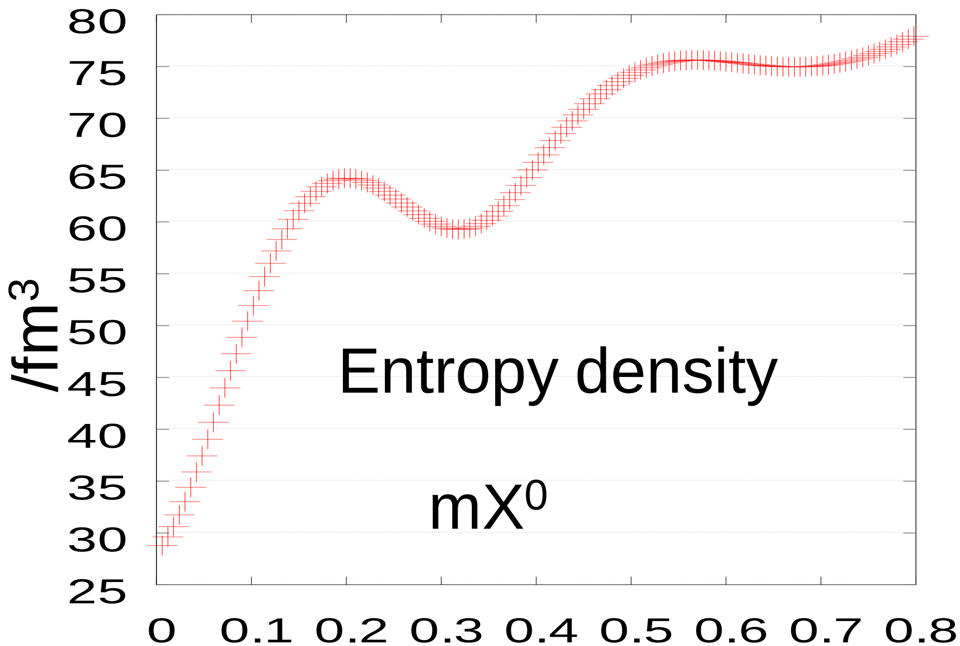

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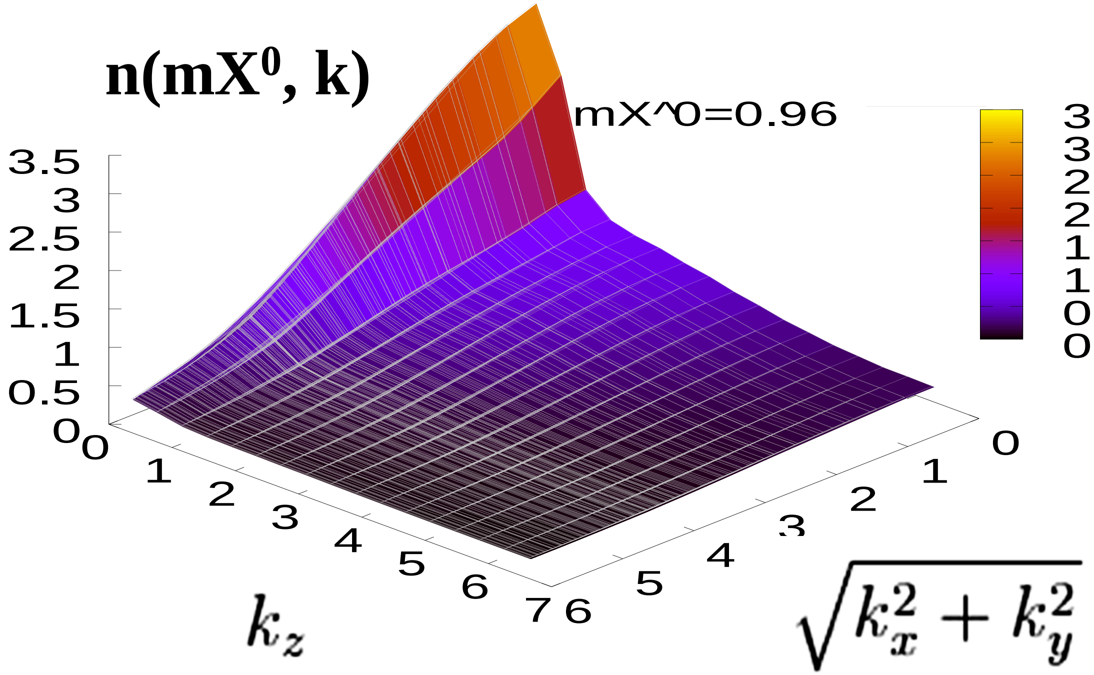
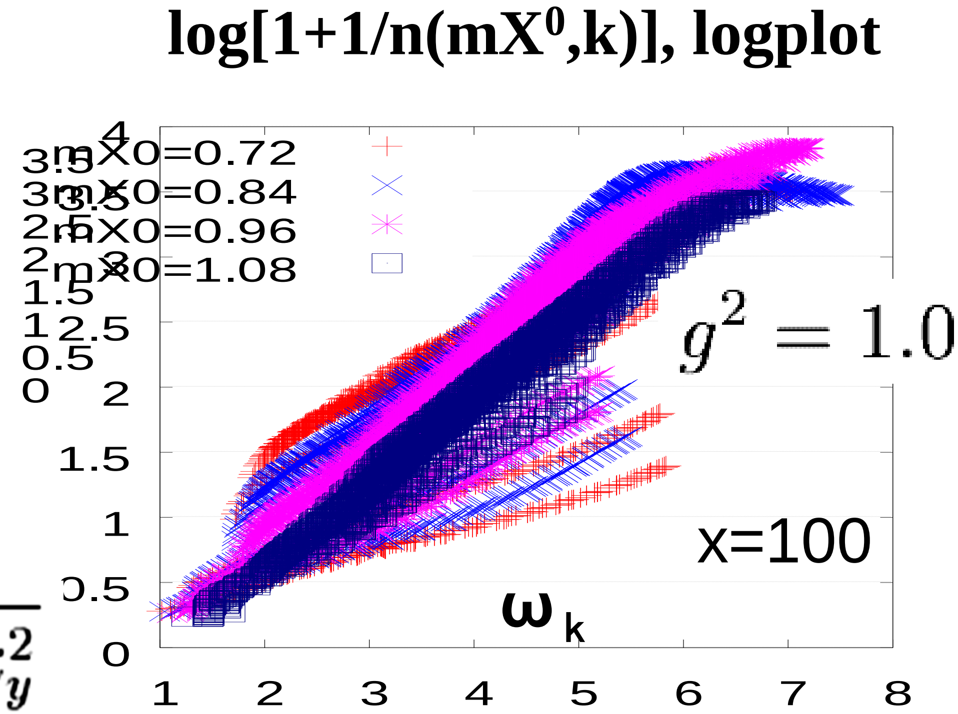
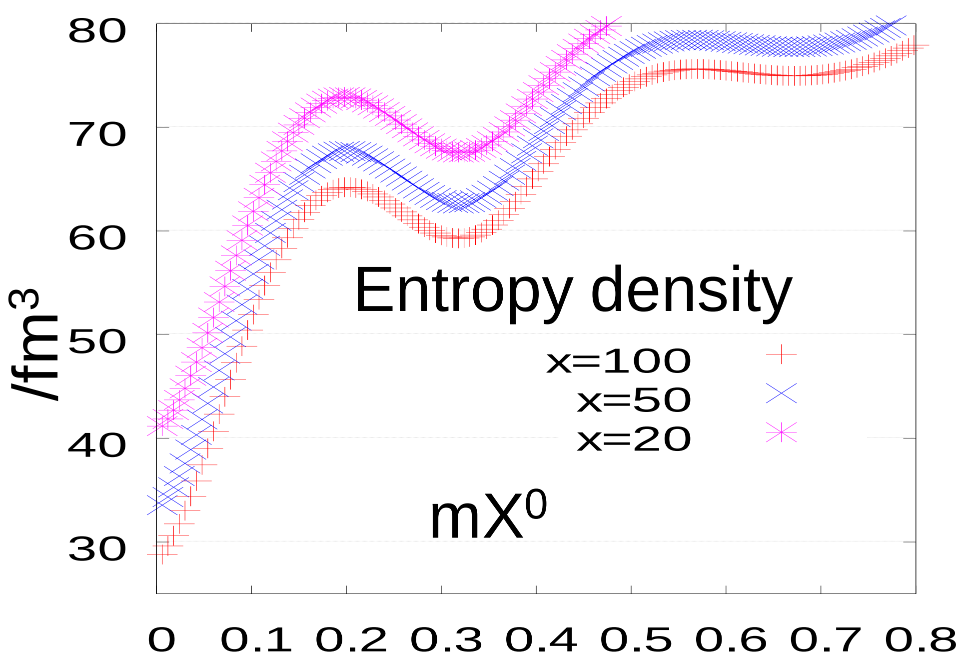




X^0=1.08/m



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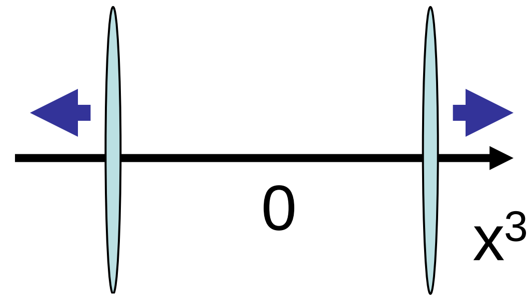
Expanding system with classical field

- Metrics (expansion in x^3 direction)

$$\tau = \sqrt{t^2 - (x^3)^2} \quad \eta = \tanh^{-1} \frac{x^3}{t}$$



- O(N) scalar model.

$$S = \int d^4x \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \varphi_a \partial_\nu \varphi_a - \frac{m^2}{2} \varphi_a \varphi_a - \frac{\lambda (\varphi_a \varphi_a)^2}{4! N} \right] \quad a = 1, \ldots, N$$

- Evolution equation of classical field and Kadanoff-Baym eq.

$$\left[\partial_\tau^2 + \frac{1}{\tau} \partial_\tau + m^2 + \frac{\lambda}{6N} \phi^2(\tau) \right] \phi(\tau) = 0$$

- Initial condition: Classical field with vacuum quantum fluctuations (Color Glass Condensate ?)

$$\phi_a(\tau) = \phi(\tau) \delta_{a1}$$

$$F_{ab} = \mathrm{diag}(F_{\parallel}, F_{\perp}, \ldots, F_{\perp})$$

$$\phi(\tau_0) = \sqrt{\frac{6N}{\lambda}} \sigma_0$$

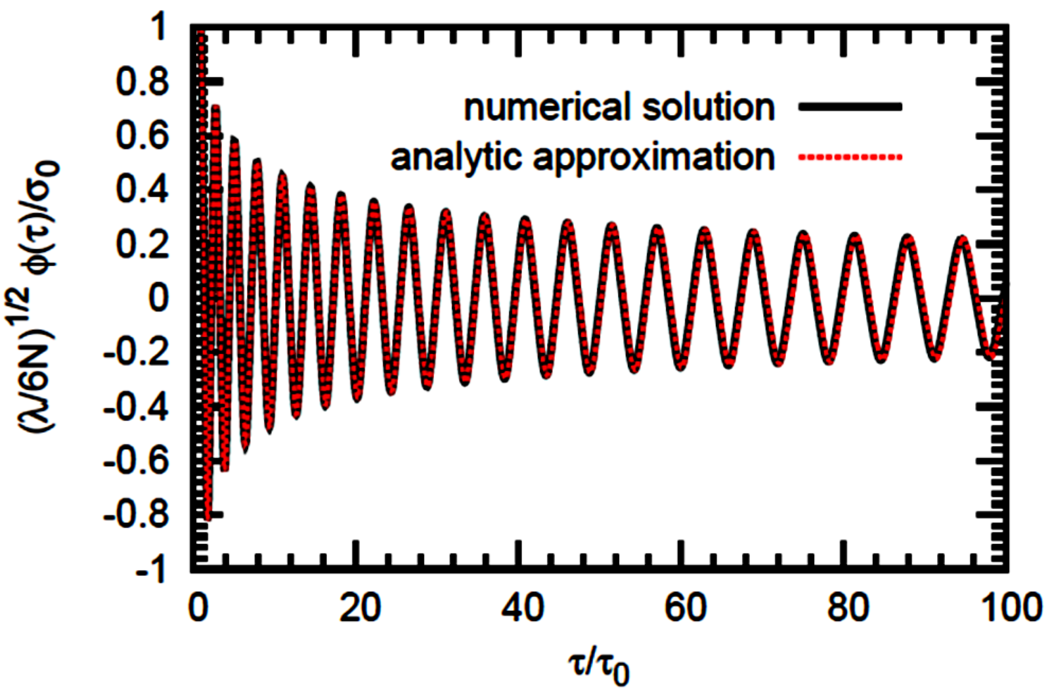
$$\lambda = 10^{-4}$$

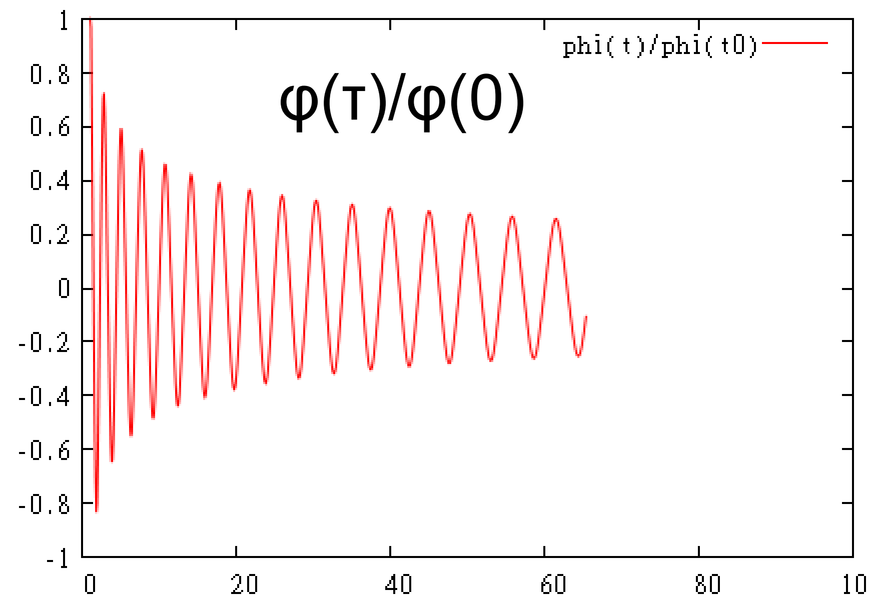
$$N = 4$$ massless

Evolution of classical field and fluctuation

Reproduction of J. Berges, K. Boguslavski, S. Schlichting, hep-ph 1201.3582.

Case I (without collision term)





τ/τ_0

$\varphi \sim \tau^{-1/3}$

Parametric Resonance instability

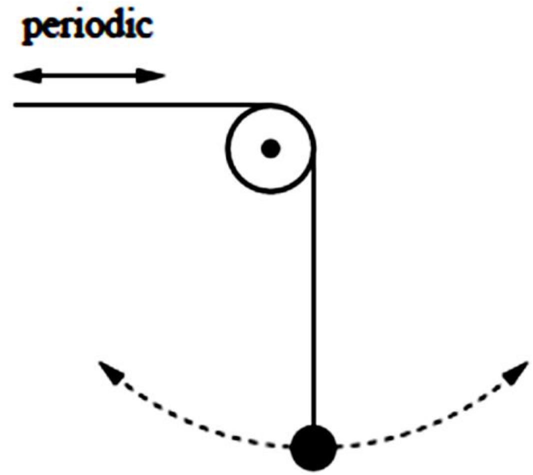
Fluctuation $\ddot{y} + \omega^2(t) y = 0$

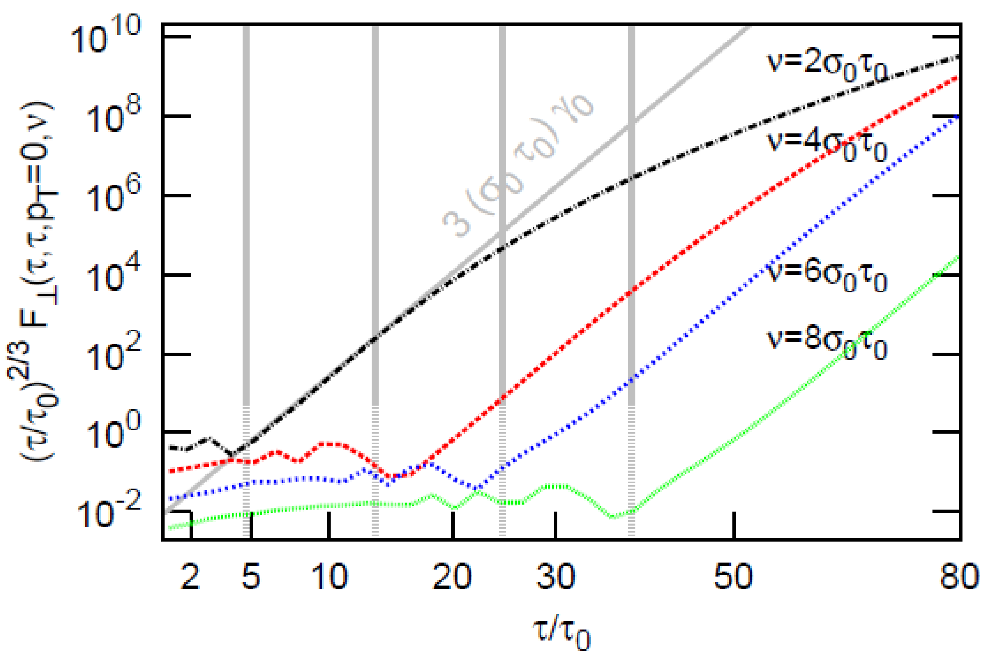
periodic $\omega(t+T) = \omega(t)$

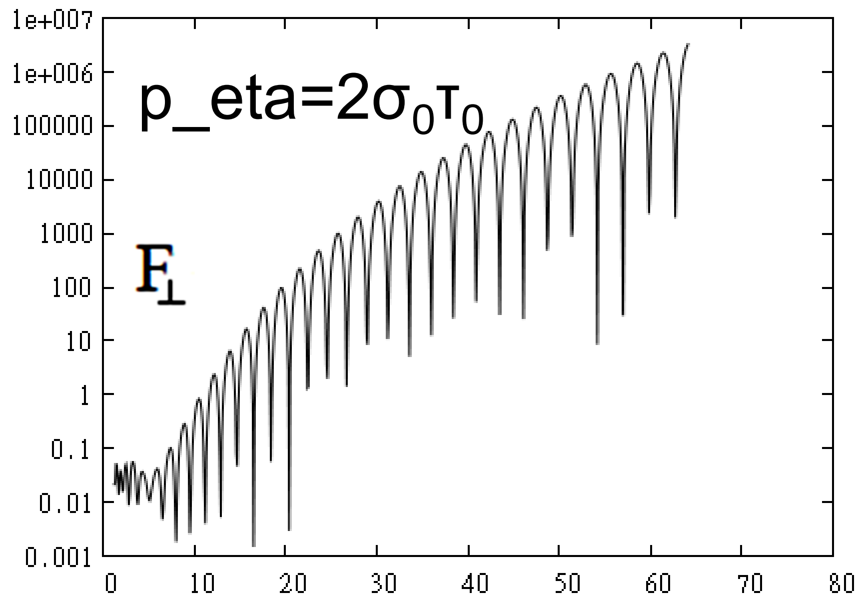
$y(t) = c^{t/T}\Pi(t) \qquad c > 1$

$\omega^2(t) \sim \varphi^2(t) + \ldots$

Flat







Curved $\exp(\gamma_0 \tau^{2/3})$

Case II (with collision term)

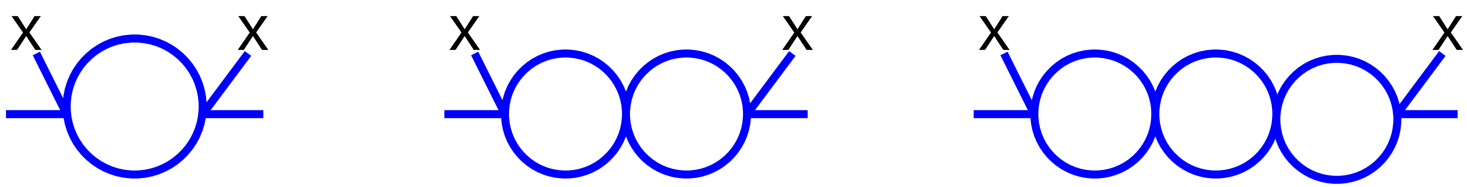
Berges et al. : Classical Statistical Lattice. Neq~T/k

Our results (collaboration with Y. Hatta): Quantum collision term.

We investigate the difference in boost invariant metric. (in progress.)

$\Sigma =$ 

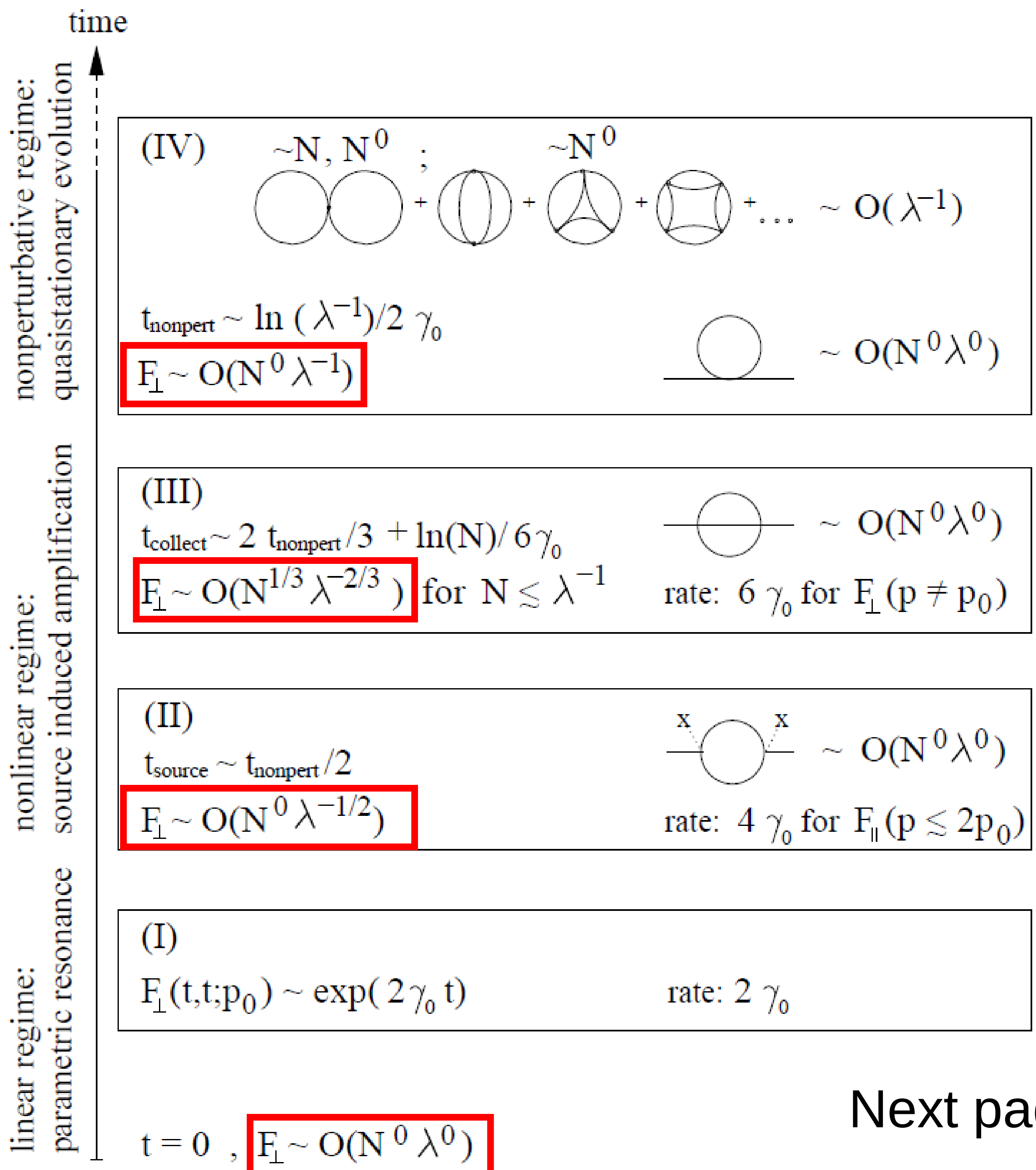
Normal collision term.

$+$ 

$+\cdots$

Source induced amplification.

Summation of Next-to-Leading Order of 1/N expansion. This approach covers all evolution of F from O(1) to $O(\lambda^{-1})$



Berges 2004.

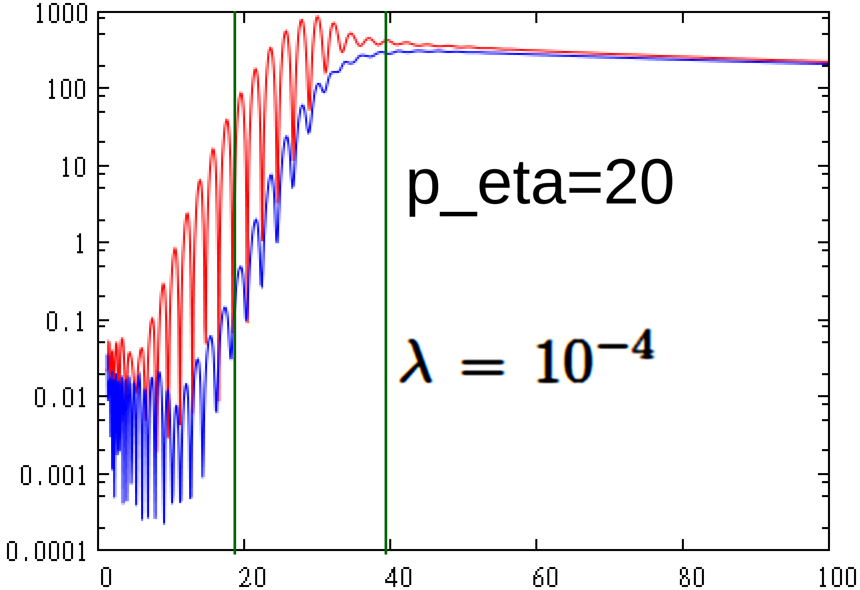
(Flat metric)

Next page: Our results

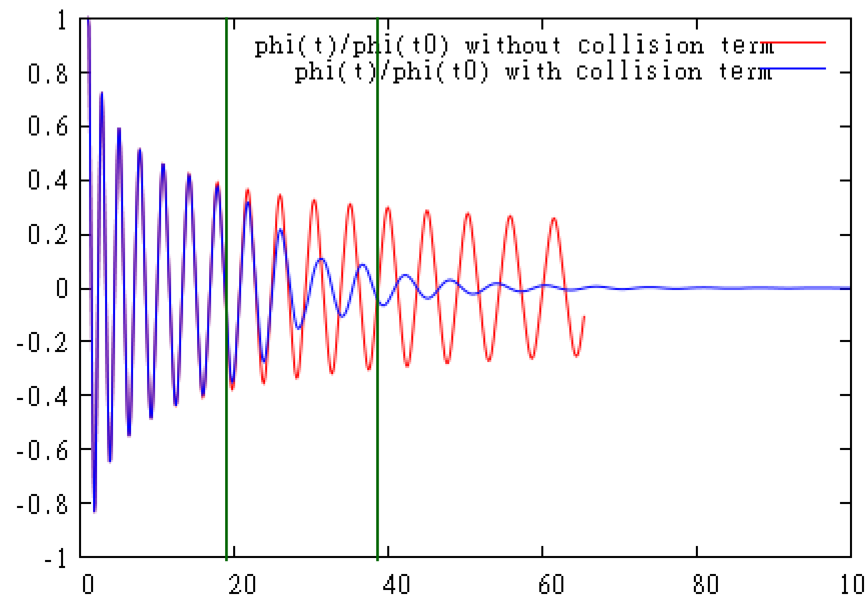
Numerical Results (1+1 dimensions)

p_eta=10 $\tau^{-2/3}$

$F_\perp$



τ/τ_0



τ/τ_0

$\tau/\tau^0<20$, parametric resonance.

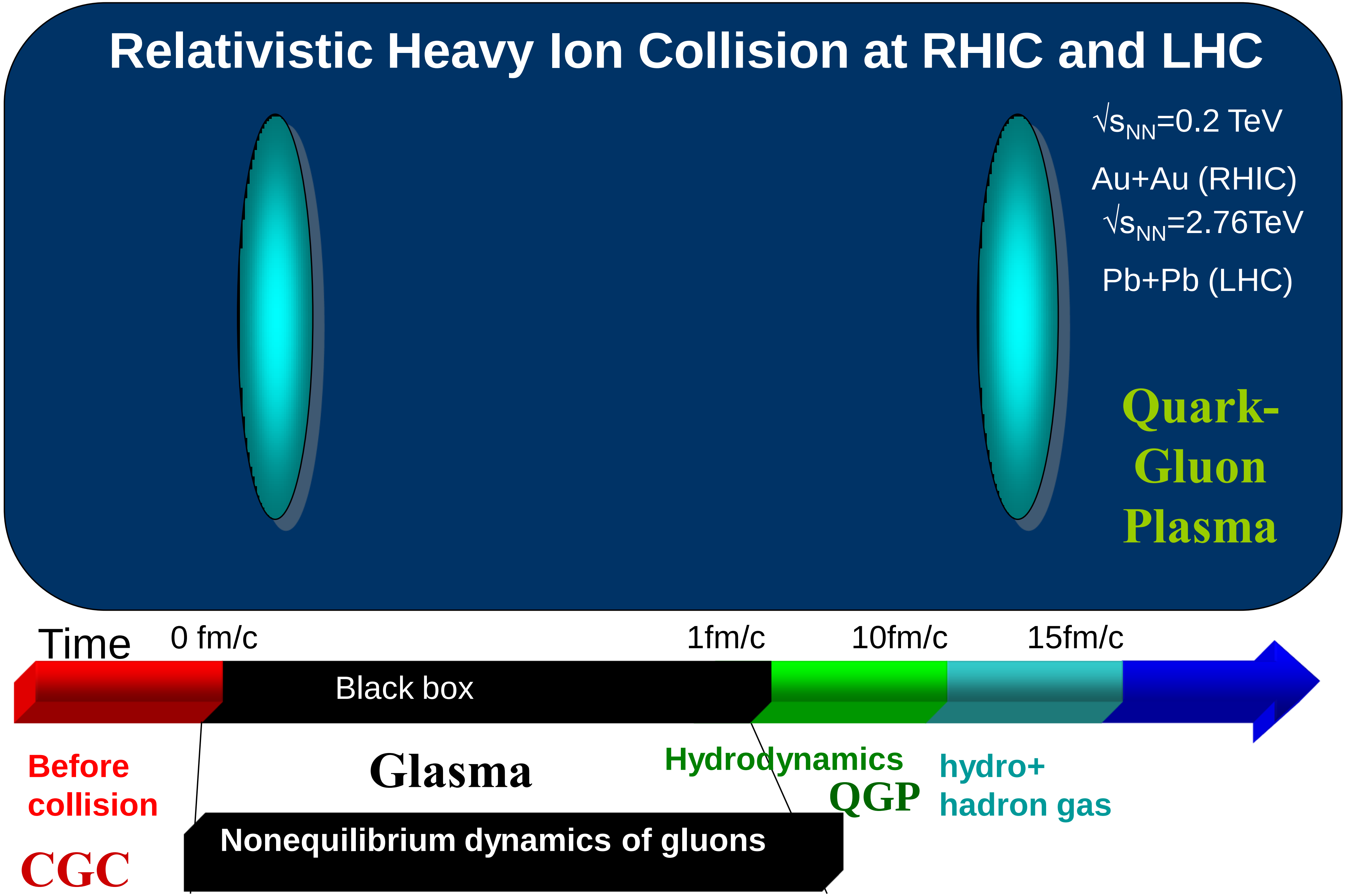
$20<\tau/\tau^0<40$, source induced amplification.

Summary

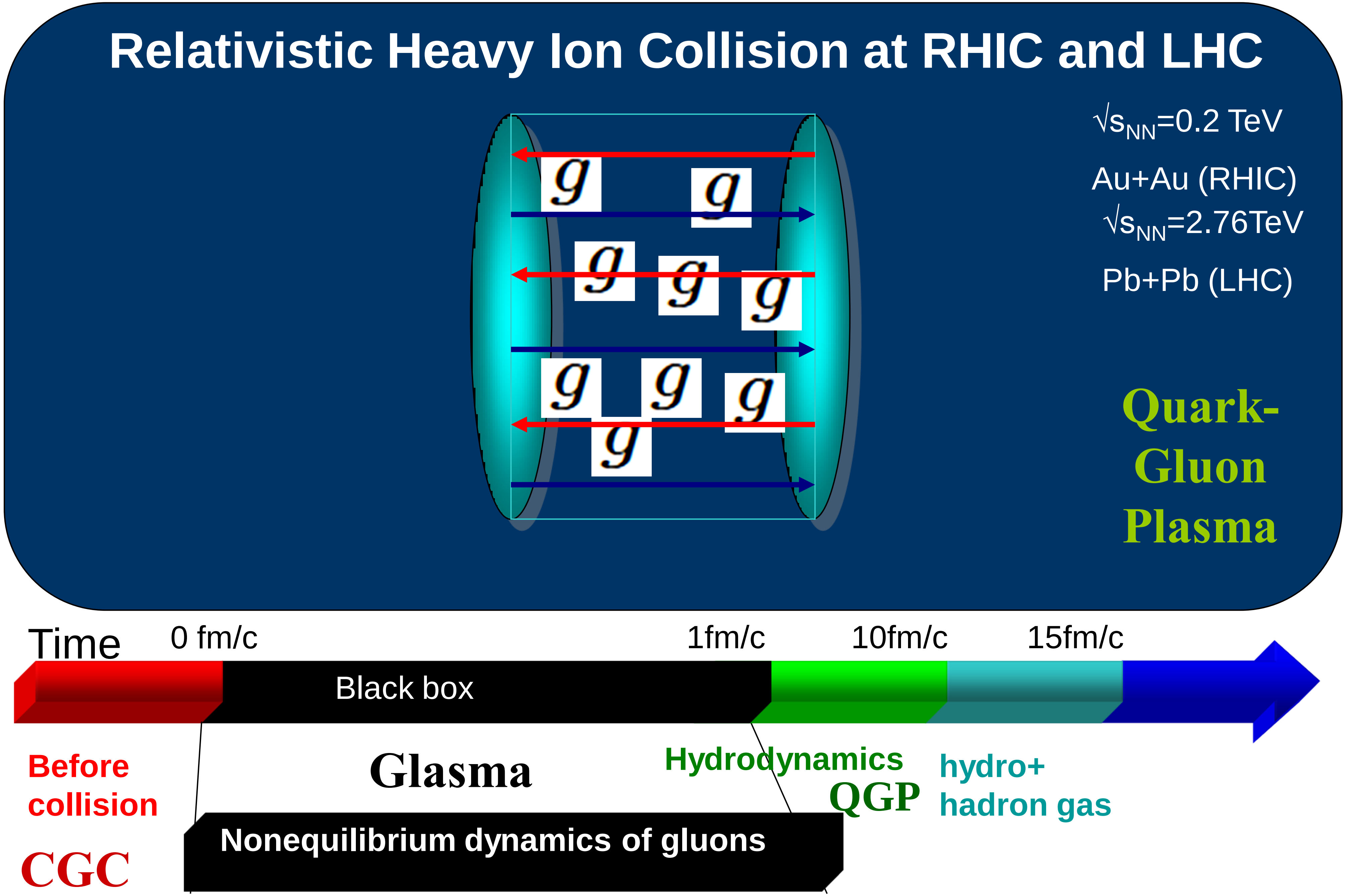
- **We have considered the Kadanoff-Baym approach to thermalization of dense nonequilibrium gluonic system.**
- **Entropy production occurs with the Kadanoff-Baym dynamics with off-shell 1-to-2 processes although it has been neglected in on-shell Boltzmann dynamics. This property may help the understanding of the early thermalization.**
- **It is possible to perform calculation in 3+1 dimension in gauge theory in temporal axial gauge. Then KB eq with the off-shell process shows instability mt^0~0.3 at early stage in the time evolution.**

Remaining Problems

- **Long time behavior of KB equation in gauge theory with longitudinal mode.**
- **Solution for the KB eq. in and out of equilibrium for the NLO of g^2 for the gauge theory (2+1 and 3+1dimensions).**
- **Renormalization procedure in expanding system**
- **Field-particle conversion in expanding system for gauge theory. Thermalization from Color Glass Condensate.**



Relativistic Heavy Ion Collision at RHIC and LHC
$\sqrt{s_{NN}}$=0.2 TeV
Au+Au (RHIC)
$\sqrt{s_{NN}}$=2.76TeV
Pb+Pb (LHC)
Quark-Gluon Plasma
Time
0 fm/c
1fm/c
10fm/c
15fm/c
Black box
Before collision
CGC
Glasma
Hydrodynamics
QGP
hydro+ hadron gas
Nonequilibrium dynamics of gluons


Relativistic Heavy Ion Collision at RHIC and LHC
g
g
g
g
g
g
g
g
g
√sNN=0.2 TeV
Au+Au (RHIC)
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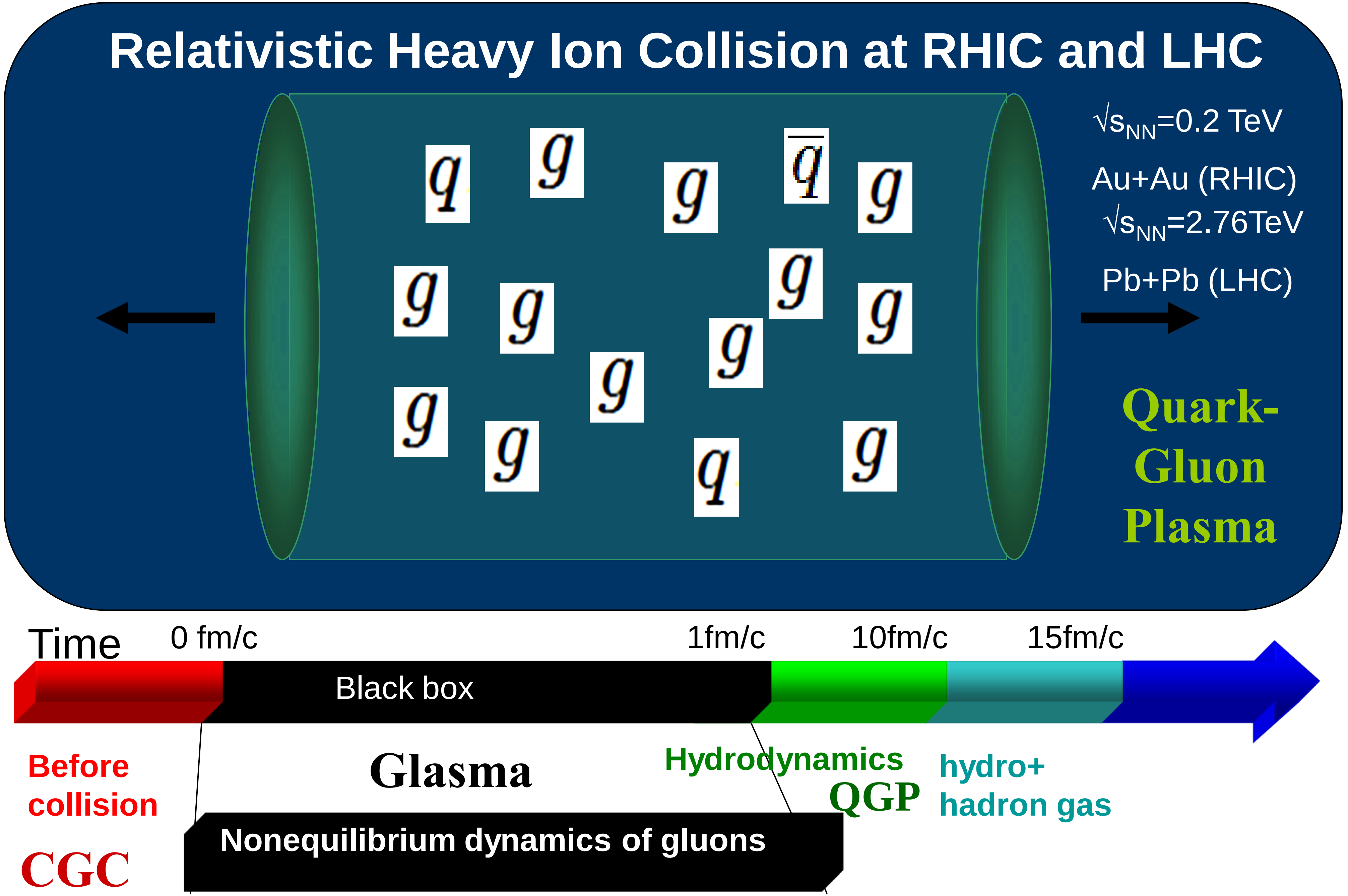

Relativistic Heavy Ion Collision at RHIC and LHC
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Hydrodynamics
QGP
hydro+
hadron gas

However does the dynamics contribute to thermalization? To confirm it,

H-theorem for Gauge Theory

- Introduction of kinetic entropy current based on relativistic Kadanoff-Baym eq for gauge theory. **A.N. Nucl. Phys. A 832:289-313, 2010.**
- 1st order gradient expansion of KB eq.
- Extension of nonrelativistic case (Ivanov, Knoll and Voskresenski (2000), Kita (2006)) and relativistic scalar φ4 (Nishiyama (2010)) and O(N) case (Nishiyama and Ohnishi (2010)). H-therorem has been shown in these cases

In temporal axial gauge, when we divide Green function D and self-energy Π to transverse (T) and longitudinal (L) part, we obtain

I. Offshell

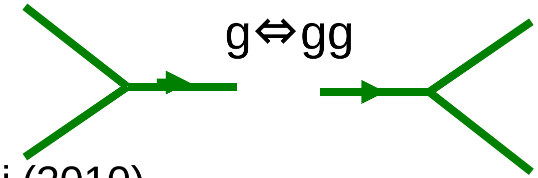
[ ] : Entropy flow spectral function

$$s^{\mu} \equiv \int \frac{d^{d+1}k}{(2\pi)^{d+1}}(d-1)\left[\left(k^{\mu}-\frac{1}{2}\frac{\partial \mathrm{Re}\,\Pi_{T,\mathrm{Re}}}{\partial k_{\mu}}\right)\frac{\rho_T}{i}+\frac{1}{2}\frac{\partial \mathrm{Re}\,D_{T,\mathrm{Re}}}{\partial k_{\mu}}\frac{\Pi_{\rho,T}}{i}\right]\sigma[f_T](X,k)$$
$$+\int \frac{d^{d+1}k}{(2\pi)^{d+1}}\left[\left(k^{0}\delta^{\mu 0}-\frac{1}{2}\frac{\partial \mathrm{Re}\,\Pi_{L,\mathrm{Re}}}{\partial k_{\mu}}\right)\frac{\rho_L}{i}+\frac{1}{2}\frac{\partial \mathrm{Re}\,D_{L,\mathrm{Re}}}{\partial k_{\mu}}\frac{\Pi_{\rho,L}}{i}\right]\sigma[f_L](X,k)$$

$$\sigma[f_{T,L}] \equiv (1+f_{T,L})\log(1+f_{T,L}) - f_{T,L}\log f_{T,L}$$

For LO self-energy

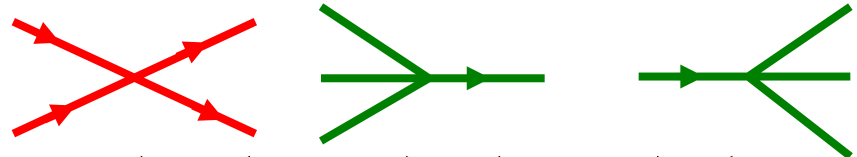
$$\partial_\mu s^\mu = g^2 N[\boxed{(TTT)} + (TTL) + (TLL)] \geq 0.$$



Each term is positive definite. Nishiyama and Ohnishi (2010)

H-theorem is derived at the level of Green's function with off-shellness.

(For NLO self-energy



$$\partial_\mu s^\mu = g^4 N^2[(TTTT) + (TTTL) + (TTLL) + (TLLL) + (LLLL)]$$)

Controlled gauge dependence of our entropy density with a certain constant term is assured at thermal equilibrium.

For gauge transformation $\delta s^0_{\mathbf{eq}} \sim g^2 s^0_{\mathbf{eq}}$ (Smit and Arrizabaraga (2002), Carrington et al (2005))

Gauge dependence is higher order of coupling.

Proof of controlled gauge dependence **out of equilibrium** is still remaining problem. (Blaiziot, Iancu and Rebhan (1999))

In the quasiparticle limit (small coupling)

We reproduce the entropy for the boson.

II. $$\boxed{s^\mu \to \int \frac{d^d p}{(2\pi)^d} v^\mu \left[-n_{\mathbf{p}} \ln n_{\mathbf{p}} + (1 + n_{\mathbf{p}}) \ln(1 + n_{\mathbf{p}})\right]}$$

$v^\mu = p^\mu/\varepsilon_p$ **:velocity**

Kita's Entropy

$$s \equiv \hbar k_{\mathrm{B}} \int \frac{d^3p\, d\varepsilon}{(2\pi\hbar)^4} \sigma \left[A \frac{\partial (G_0^{-1} - \mathrm{Re}\Sigma^{\mathrm{R}})}{\partial \varepsilon} + A_\Sigma \frac{\partial \mathrm{Re} G^{\mathrm{R}}}{\partial \varepsilon} \right],$$

$$\boldsymbol{j}_s \equiv \hbar k_{\mathrm{B}} \int \frac{d^3p\, d\varepsilon}{(2\pi\hbar)^4} \sigma \left[-A \frac{\partial (G_0^{-1} - \mathrm{Re}\Sigma^{\mathrm{R}})}{\partial \boldsymbol{p}} - A_\Sigma \frac{\partial \mathrm{Re} G^{\mathrm{R}}}{\partial \boldsymbol{p}} \right],$$

$$\frac{\partial s_{\mathrm{coll}}}{\partial t} \equiv \hbar k_{\mathrm{B}} \int \frac{d^3p\, d\varepsilon}{(2\pi\hbar)^4} \mathcal{C} \ln \frac{1 \pm \phi}{\phi}.$$

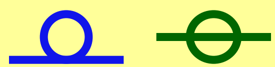
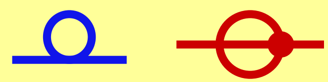
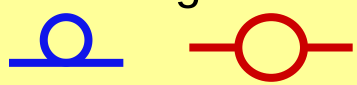
$$\sigma[\phi] \equiv -\phi \ln \phi \pm (1 \pm \phi) \ln(1 \pm \phi).$$

Equilibrium at

$$\ln \frac{1 \pm \phi_1}{\phi_1} = \alpha + \beta(\varepsilon_1 - \boldsymbol{v} \cdot \boldsymbol{p}_1),$$

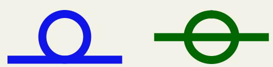
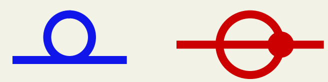
Time irreversibility

Symmetric phase ⟨Φ⟩=0

	$\lambda\Phi^4$	**O(N)**	**SU(N)**
Exact 2PI (no truncation)	×	×	×
Truncation	NLO of λ  △	NLO of 1/N  △	LO of g^2  △(TAG)
LO of Gradient expansion **H-theorem**	○	○	△ (TAG)

Numerical Simulation for KB eq.

Symmetric phase $\langle\Phi\rangle=0$

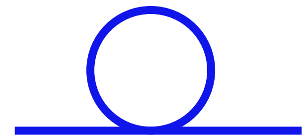
	$\lambda\Phi^4$	O(N)	SU(N)
Truncation	NLO of λ 	NLO of 1/N 	LO of g^2 
Others' Code	1+1 dim 2+1 dim 3+1 dim	1+1 dim 3+1 dim	?
Our Code	1+1 dim 2+1 dim 3+1 dim	1+1 dim	Part of 2+1, 3+1 dim

Renormalization (φ^4 model)

A. Arrizabalaga, J. Smit and A. Tranberg (2004,2005)

Mass

$$m_0^2 = m^2 - \boxed{\frac{\lambda}{2a^2}} I_1(am)$$



Quadratic divergent

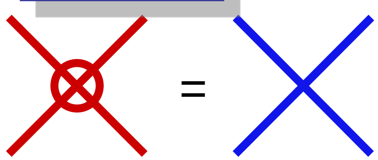
➡ Subtraction

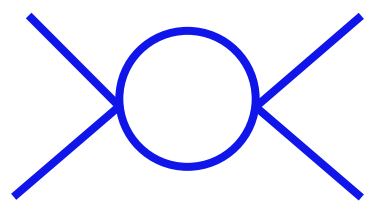
In symmetric phase

$a\,m$: lattice spacing

$$I_1(am) = \int_{-\pi}^{\pi} \frac{d^3k}{(2\pi)^3} \frac{1}{2\sqrt{a^2m^2 + k_{\mathrm{lat}}^2}},$$

Vertex





Logarithmic divergent

➡ Bethe-Salpeter eq.

$$I_2(am) = \frac{1}{16\pi^2} \ln (a^2m^2) + O(a^2m^2)$$

For $0.5 \leqq a\,m \leqq 1$ the difference between λ_0 and λ is less than 10% $(\lambda = 1, 6)$

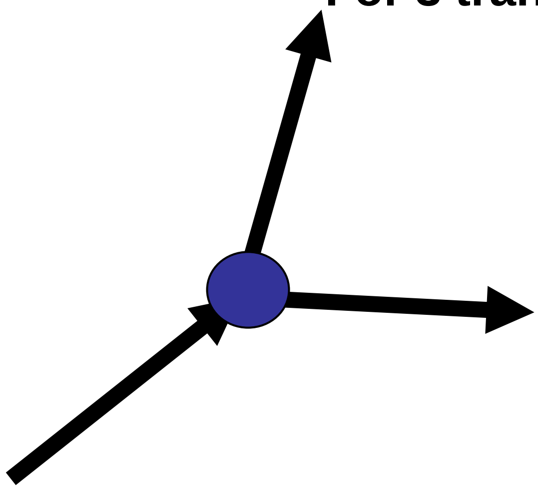
We can use bare coupling as if it were renormalized coupling when the relevant length scale is larger than a in numerical simulation. ($a < m^{-1}$)

<u>It is expected that the above analysis might hold at gauge theory with coupling expansion.</u>

Microscopic process (Non-Abelian)

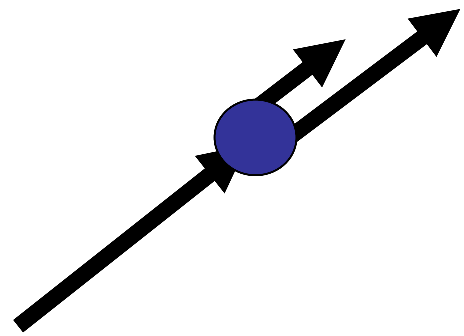
Each microscopic process is possible in 2+1 and 3+1 dimensions.

For 3 transverse fluctuations,



$C \neq 0$

Entropy production



$C = 0$

No entropy production

The 0-to-3 and 1-to-2 might contribute to isotropization with entropy production. These processes are prohibited in Boltzmann limit without spectral width and memory integral.