RHICのエネルギー走査実験とバリオン数ゆらぎ

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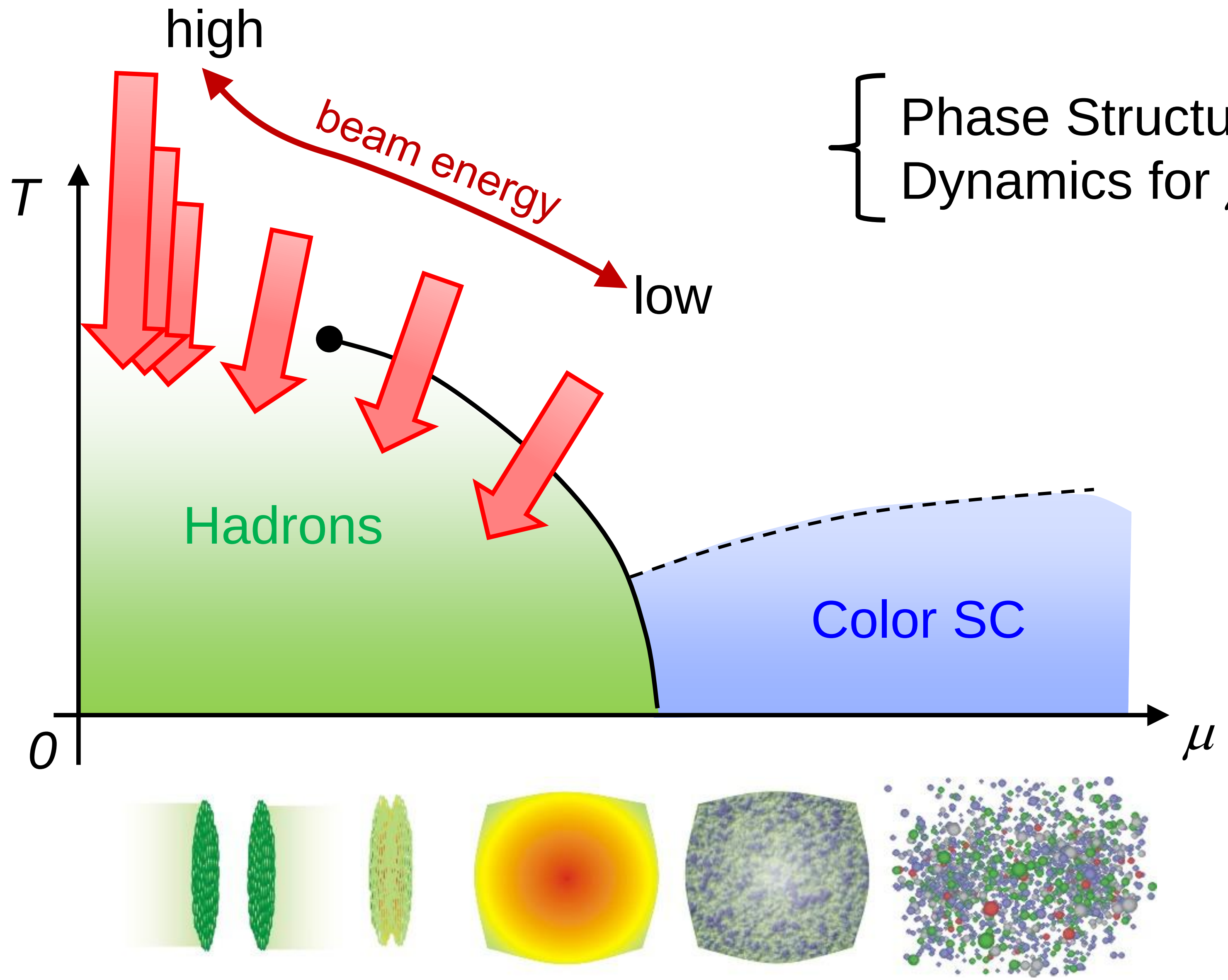
(阪大理)

MK, Asakawa, arXiv:1107.2755 (PRC, in press)

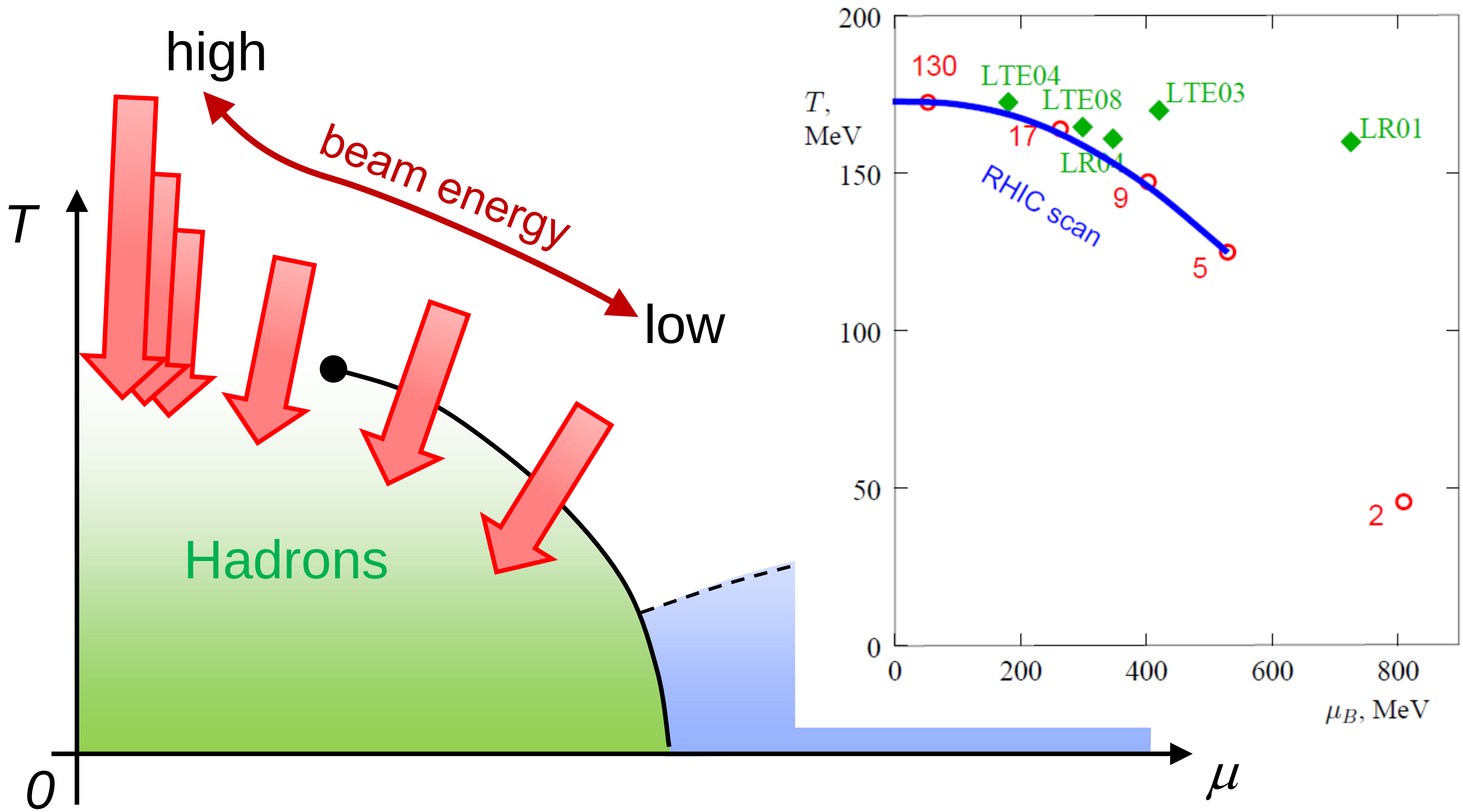
理研非平衡研究会, 18, Feb, 2012

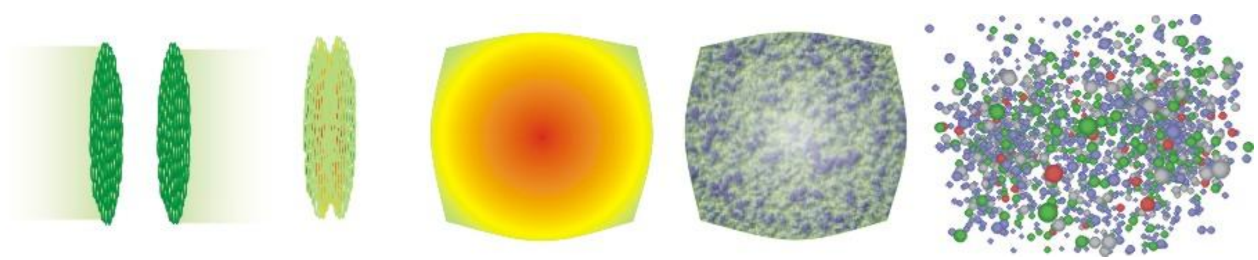
Energy Scan Program @ RHIC



Phase Structure
Dynamics for $\mu\sim 0$

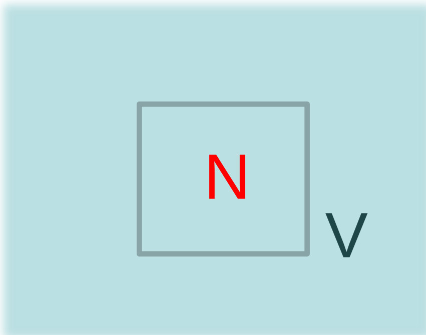
Energy Scan Program @ RHIC

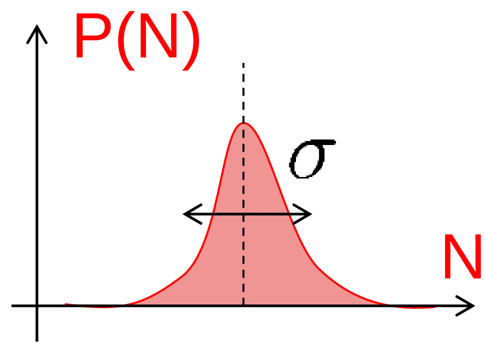




Fluctuations

Observables in equilibrium is fluctuating.





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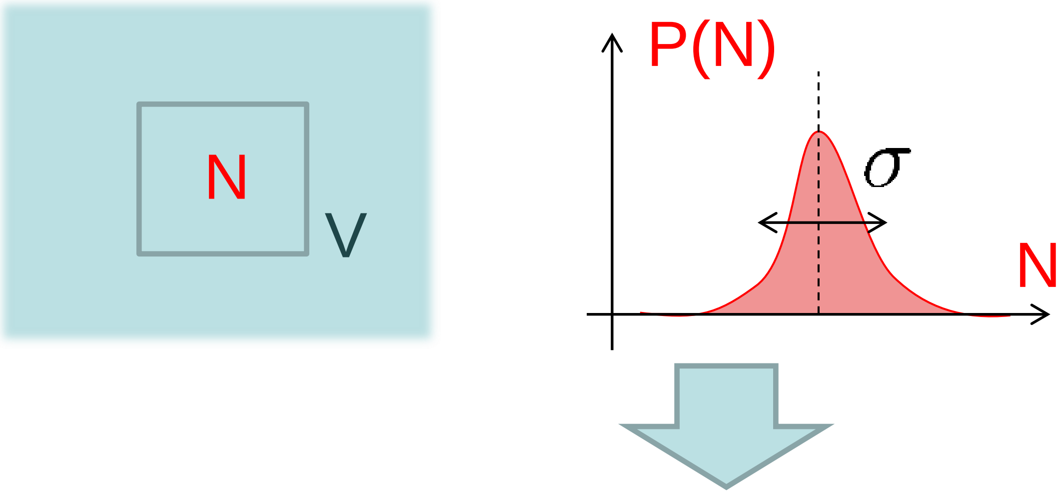


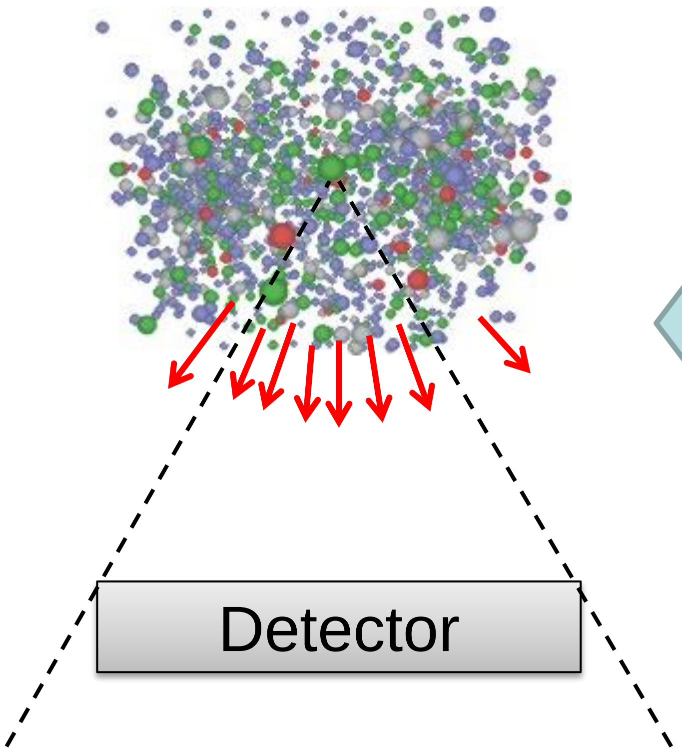
- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$

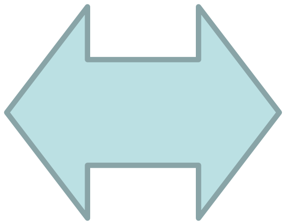
$$\delta N = N - \langle N \rangle$$

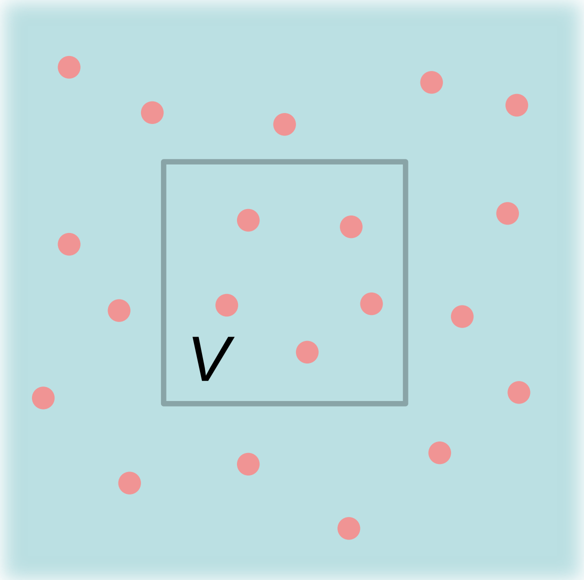
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in HIC.







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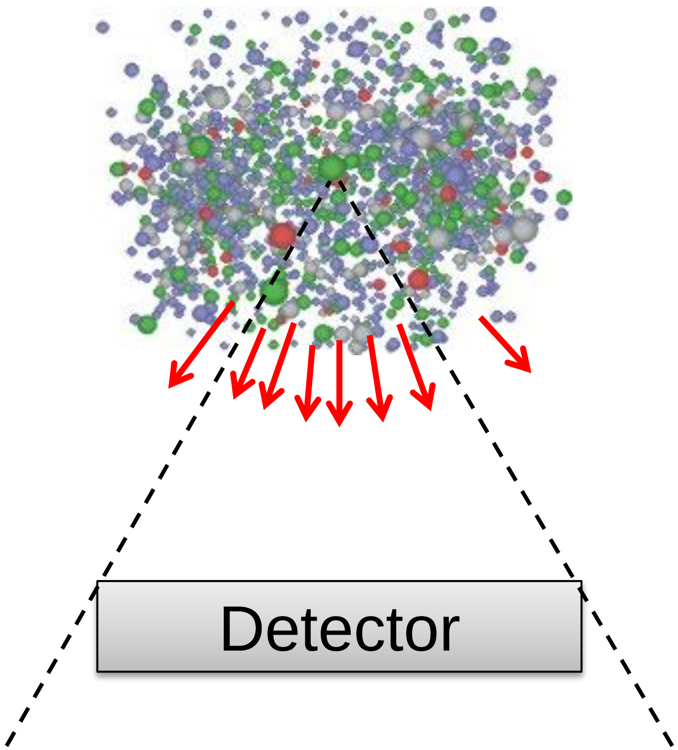
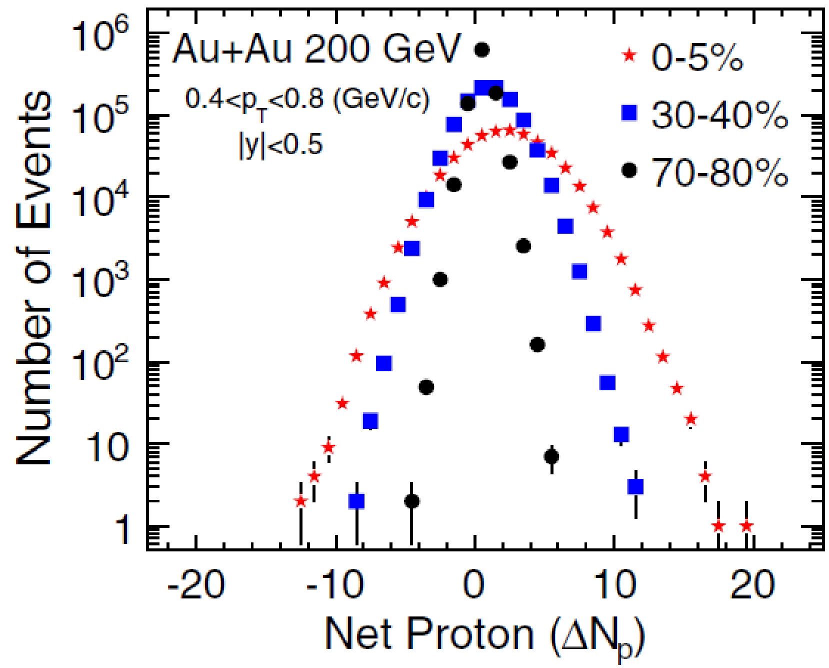
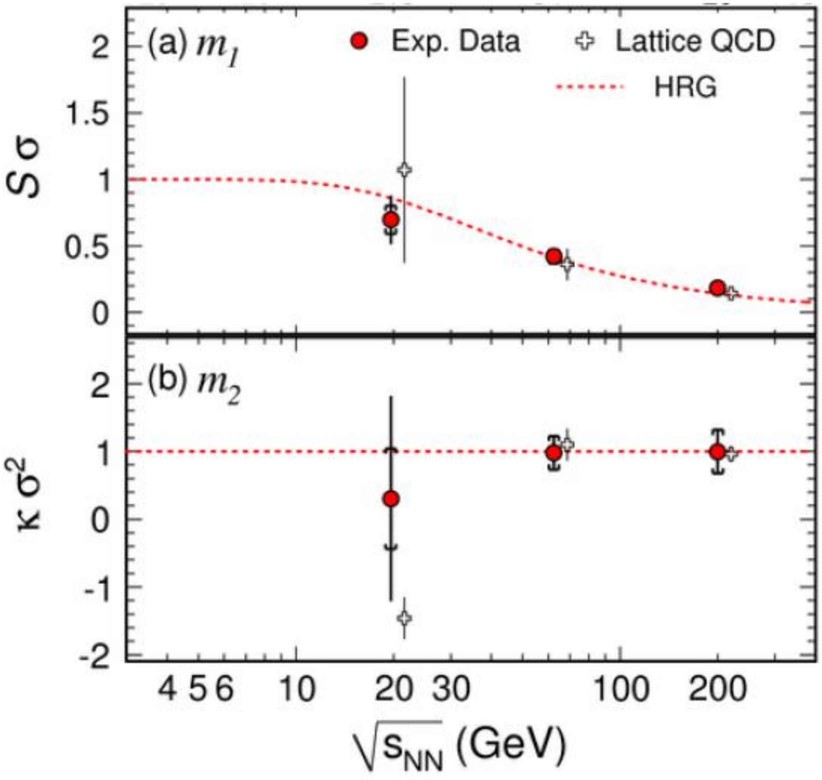


STAR, PRL**105** (2010)



Proton # Fluctuations @ STAR

STAR, 2010



$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}, \quad \kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

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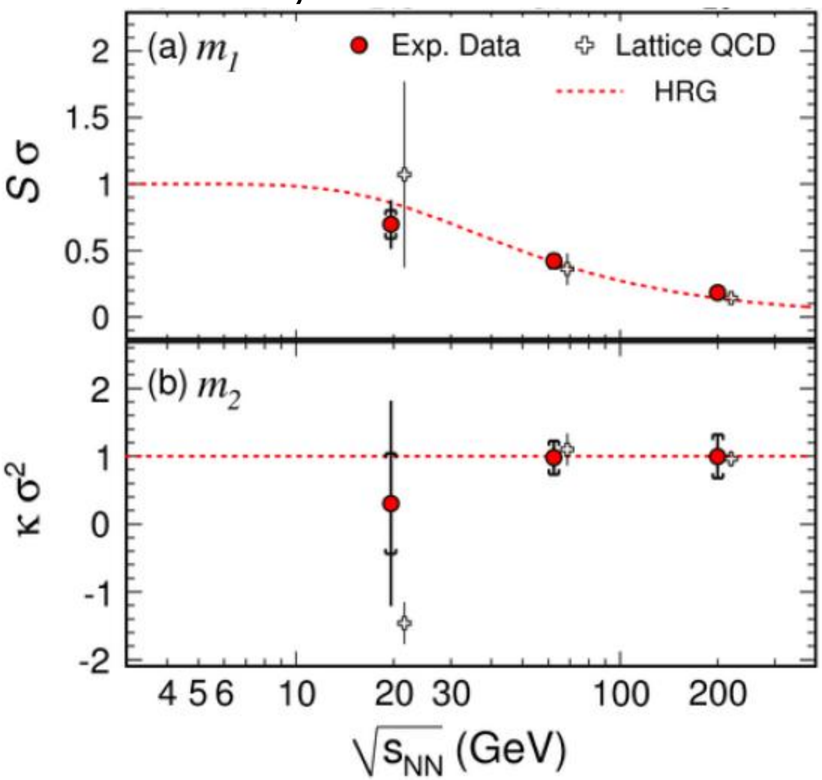
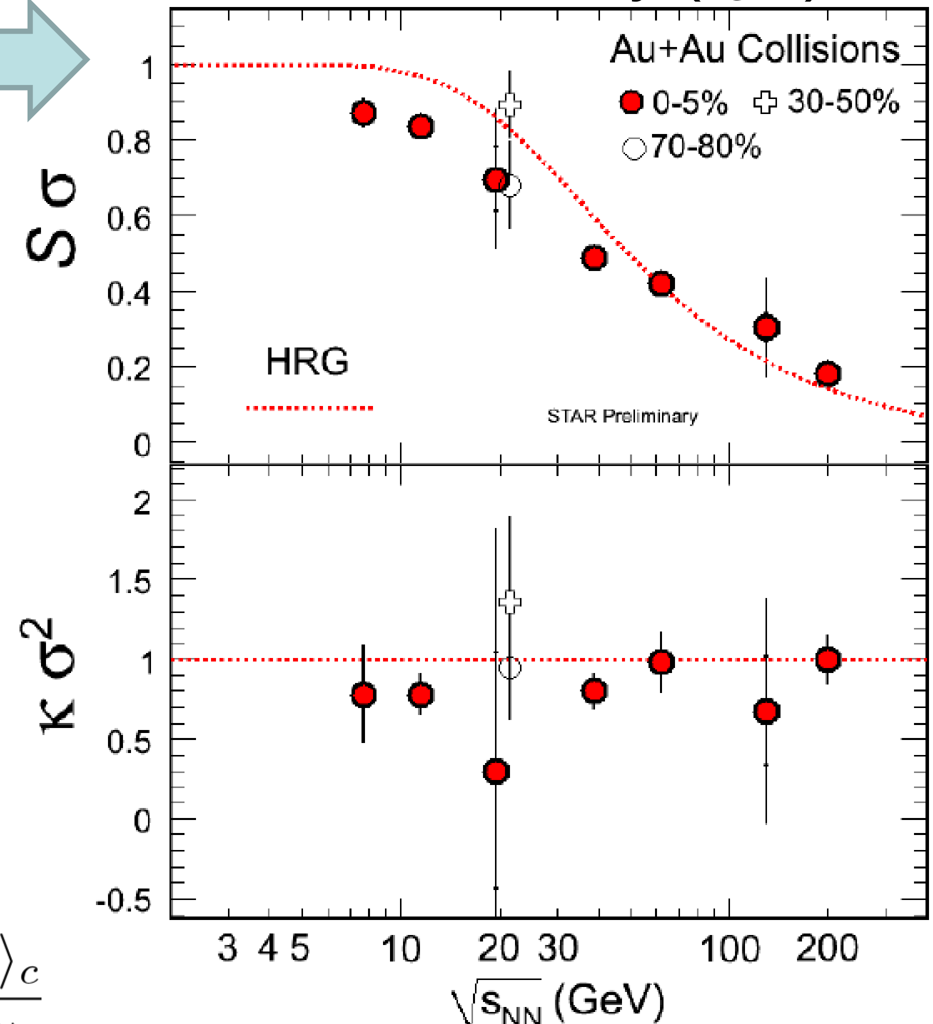


STAR, 2011, May (QM)



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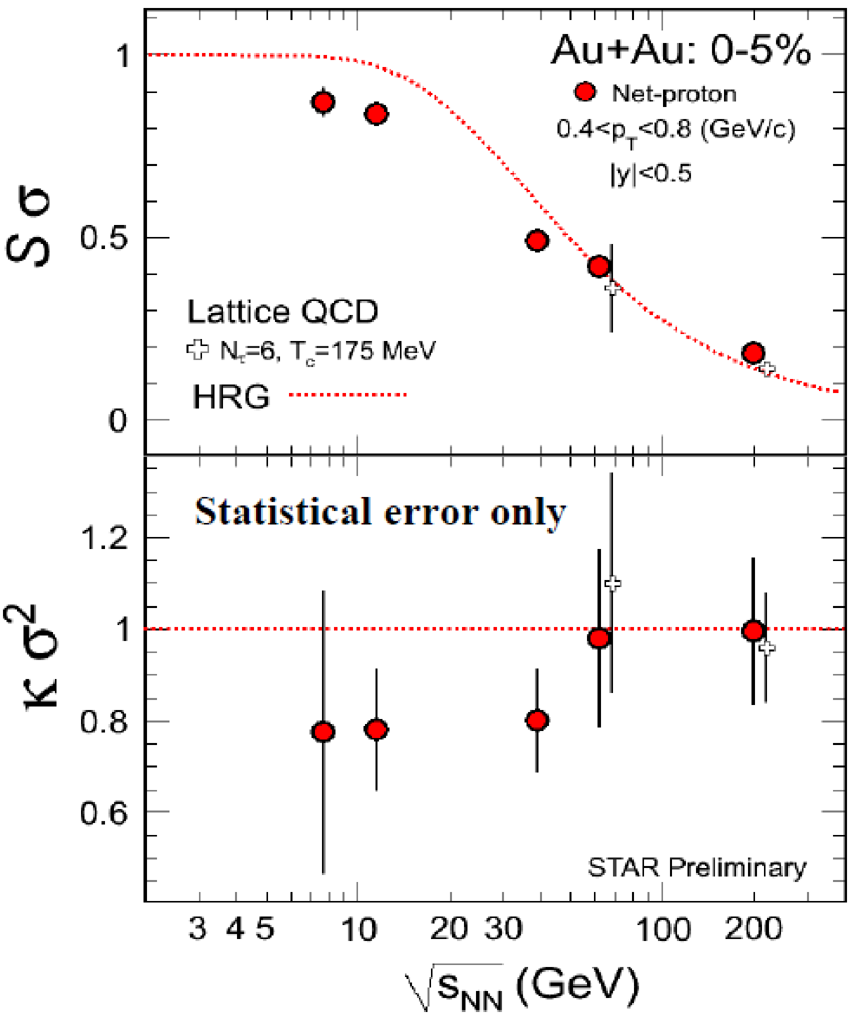
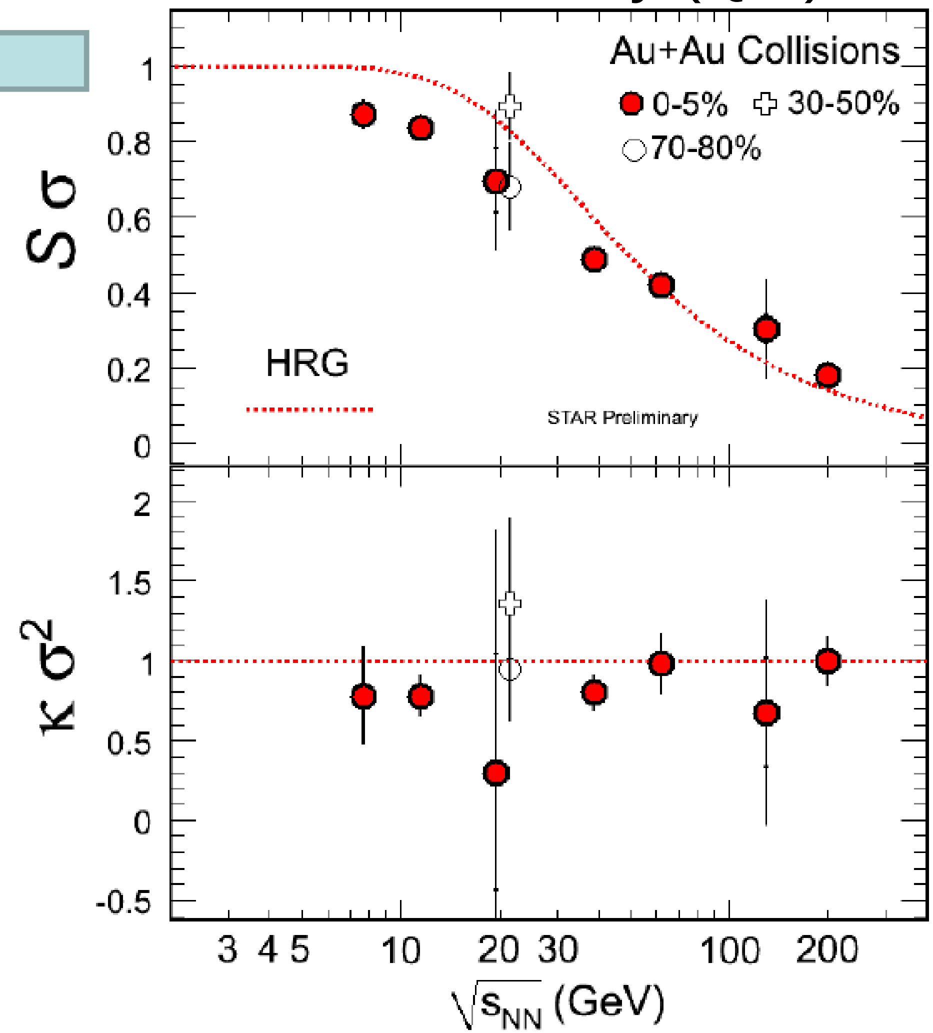
high μ ← → low μ

Proton # Fluctuations @ STAR

STAR, 2011, Nov. (CPOD)



STAR, 2011, May (QM)

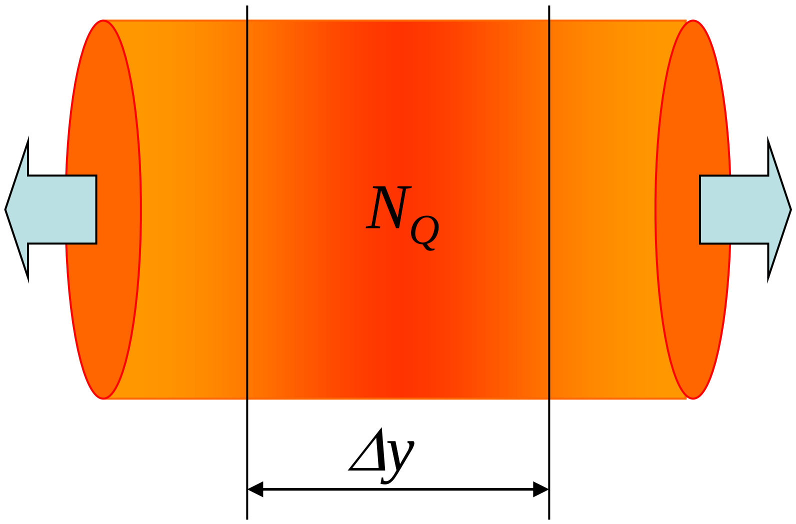


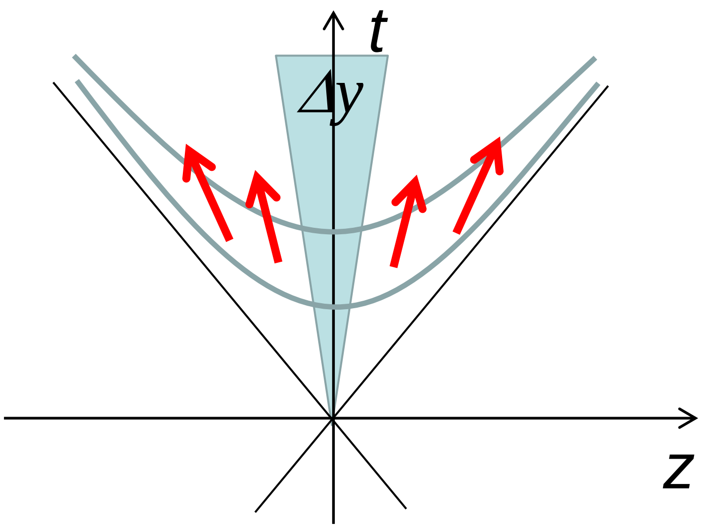
high μ

low μ

Fluctuations of Conserved Charges

- When is experimentally measured *D* formed?





- **Conserved charges** can remember fluctuations at early stage, if diffusions are sufficiently slow.

Asakawa,Heintz,Muller ('00); Jeon,Koch ('00); Shuryak, Stephanov ('02)

Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point

    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges

    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants

    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)

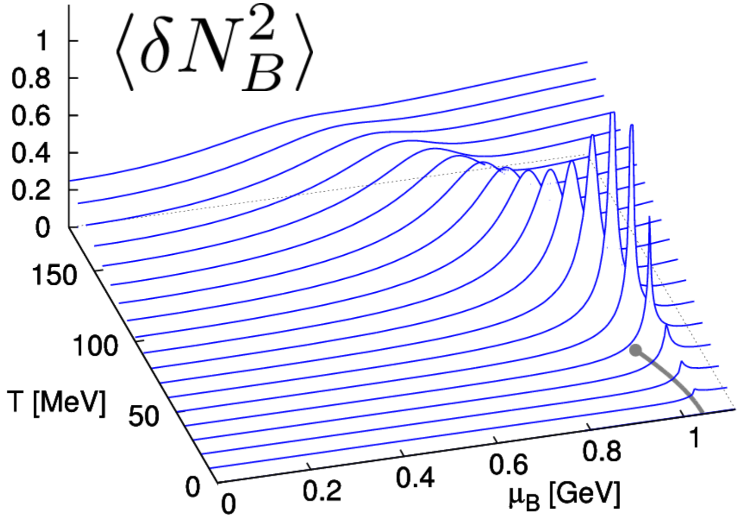
Fluctuations at QCD Critical Point

Stephanov, Rajagopal, Shuryak '98,'99

- 2^{nd} order phase transition at the CP.

⇒

- divergences in fluctuations of
  - p_T distribution
  - freezeout T
  - baryon number, chage, …



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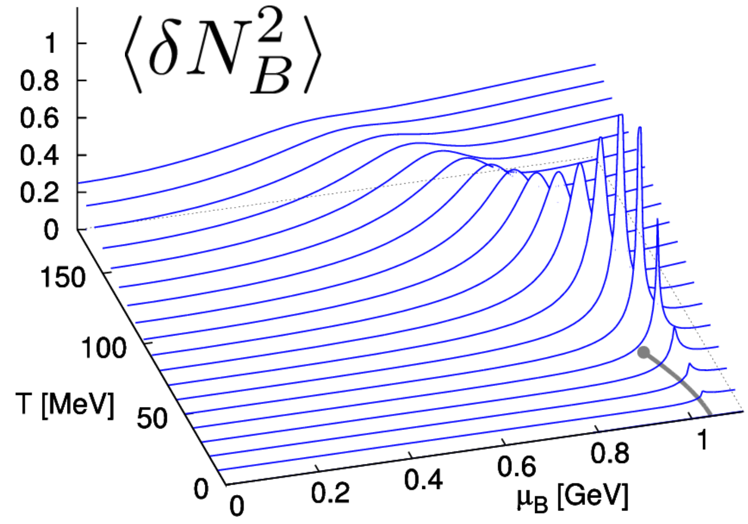


- Singular part in proton number fluctuations. Hatta, Stephanov, '02

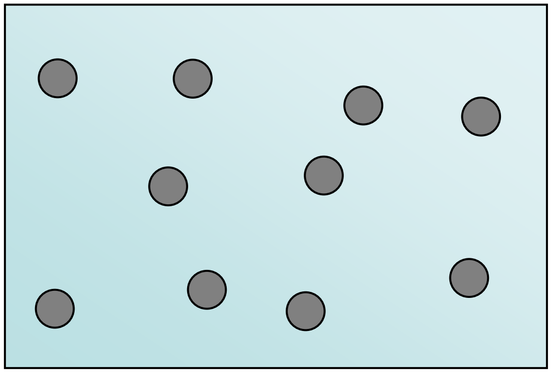
$$\langle \delta N_p^2 \rangle \sim A\xi^2 + \langle \delta N_p^2 \rangle_{\rm regular}$$

- Higher order moments has stronger ξ dep near the CP. Stephanov, '09

$$\langle \delta N^2 \rangle \sim \xi^2 \quad \langle \delta N^3 \rangle = \xi^{4.5} \quad \langle \delta N^4 \rangle_c = \xi^7$$

Ratios of Cumulants

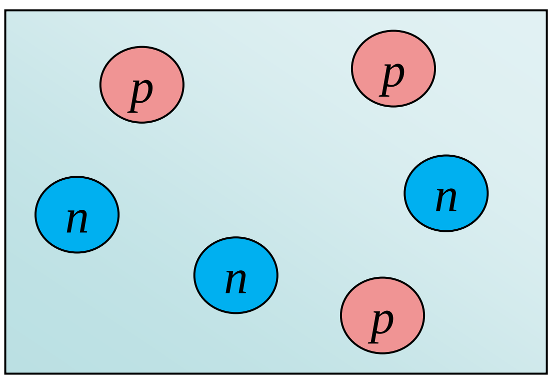
Asakawa, Heinz, Muller, ’00
Jeon, Koch, ’00
Ejiri, Karsch, Redlich, ’06



Boltzmann gas $(T, \mu \ll M)$
(=Poisson distribution)

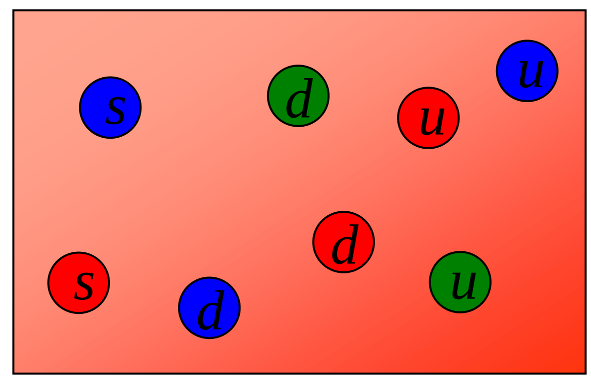
$$\langle N \rangle = \langle \delta N^2 \rangle = \langle \delta N^3 \rangle = \langle \delta N^4 \rangle_c = \cdots$$

Hadrons: $N_B = N$



$$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} = 1$$

Quark–gluon: $N_B = N/3$

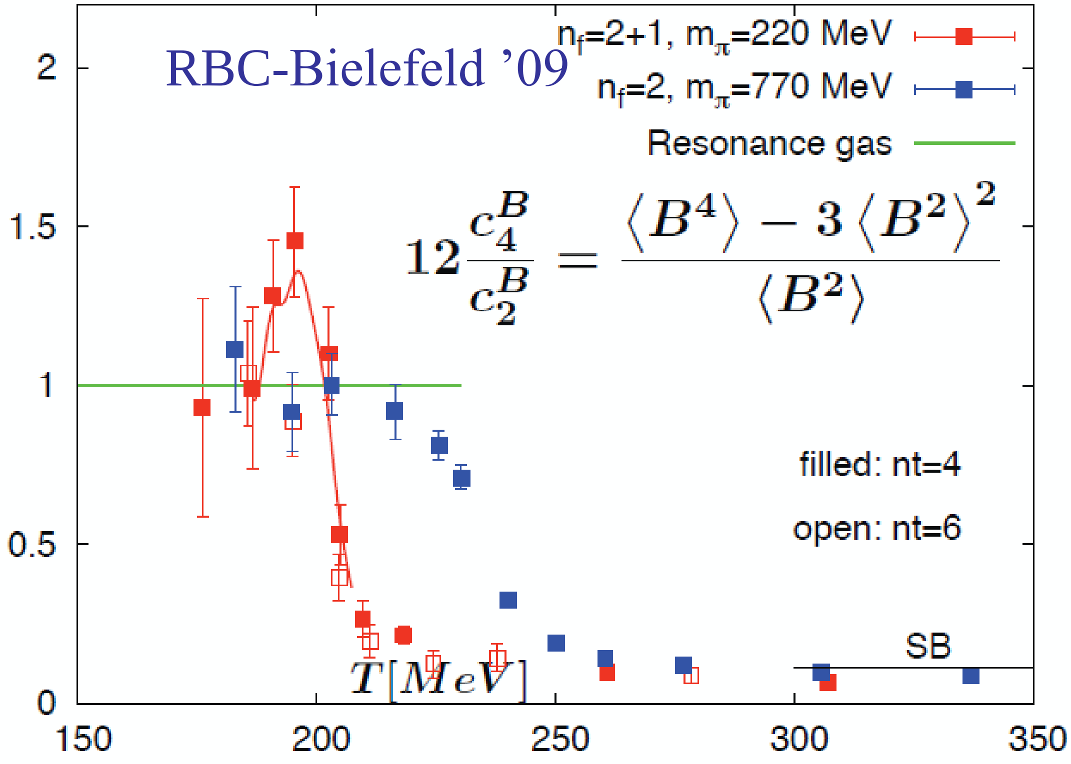


$$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} = \frac{1}{3^{n-m}}$$

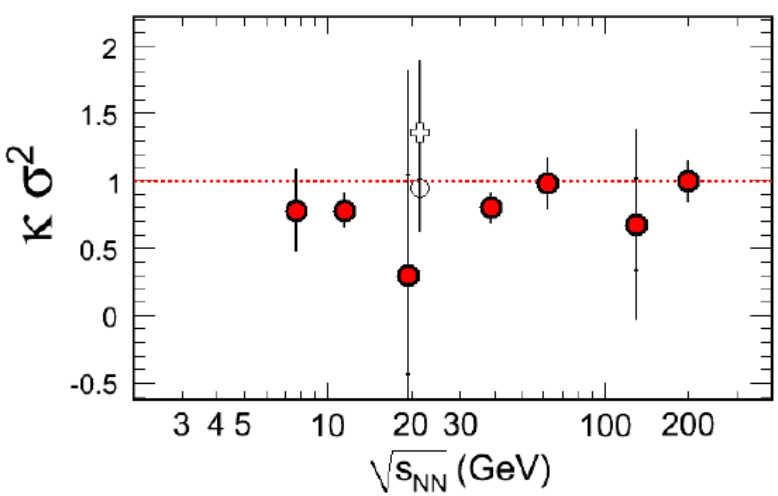
Baryon Number 4th/2nd

Ejiri, Karsch, Redlich, ’05

- Ratios between baryon number cumulants



Hadrons:1 ⟺ Quarks:$1/3^2$



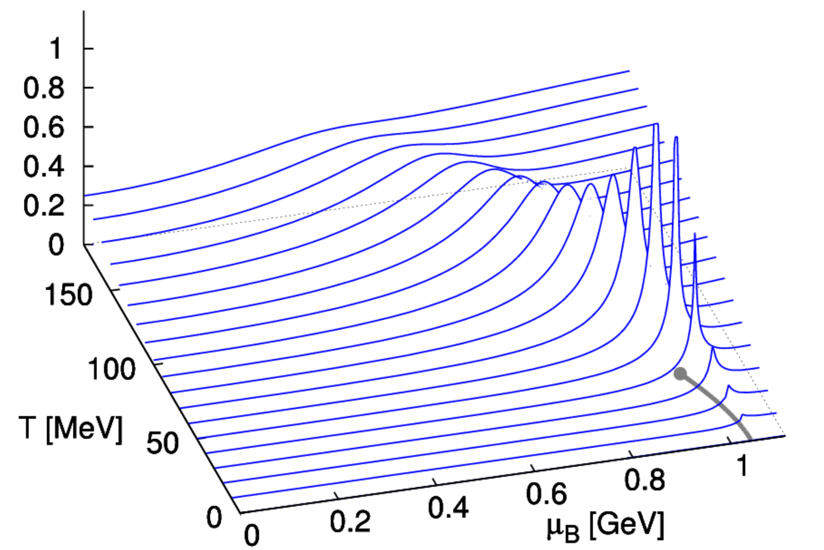
Take a Derivative of χ_B

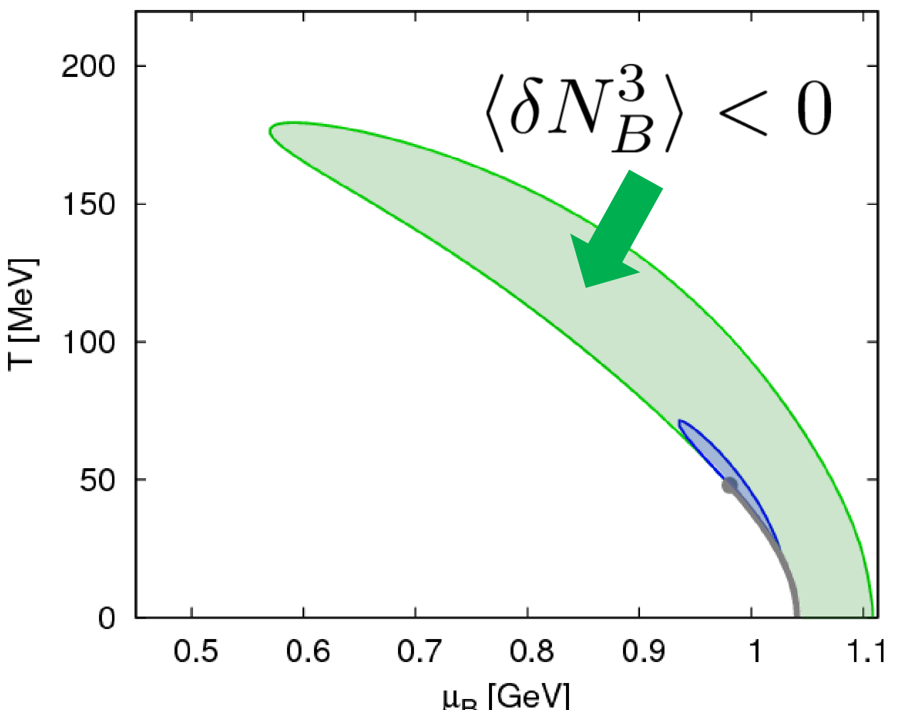
Asakawa, Ejiri, MK, 2009

- χ_B has an edge along the phase boundary

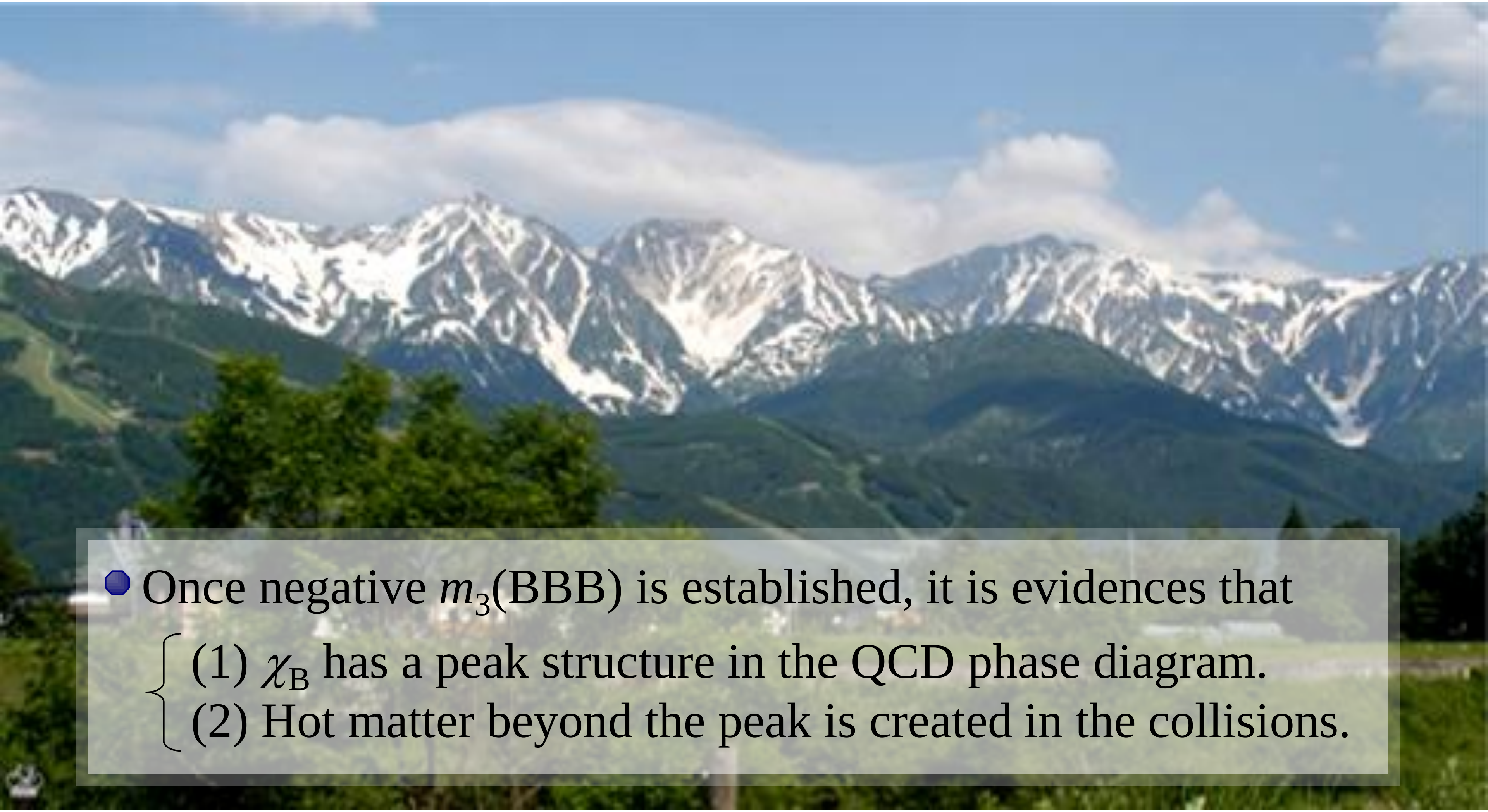
- $$\begin{cases} \chi_B = -\dfrac{1}{V}\dfrac{\partial^2\Omega}{\partial\mu_B^2} = \dfrac{\langle(\delta N_B)^2\rangle}{VT} \\ \dfrac{\partial\chi_B}{\partial\mu_B} = -\dfrac{1}{V}\dfrac{\partial^3\Omega}{\partial\mu_B^3} = \dfrac{\langle(\delta N_B)^3\rangle}{VT^2} \end{cases}$$

$\dfrac{\partial\chi_B}{\partial\mu_B}$ changes the sign at QCD phase boundary!





Impact of Negative Third Moments

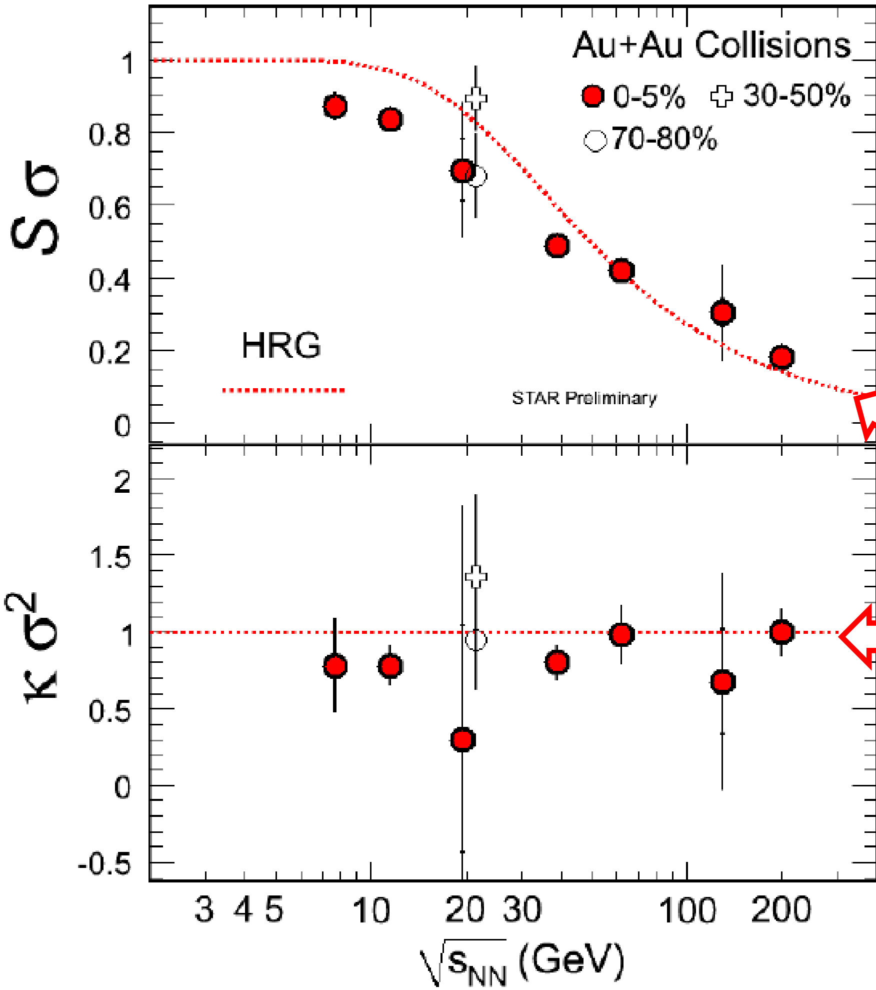


- Once negative m_3(BBB) is established, it is evidences that
  - (1) χ_B has a peak structure in the QCD phase diagram.
  - (2) Hot matter beyond the peak is created in the collisions.

- •**No** dependence on any specific models.
  •Just the sign! **No** normalization (such as by N_{ch}).

Proton # Fluctuations @ STAR

2011 (Quark Matter)



$$
\begin{cases}
S\sigma = \dfrac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}, \\[2ex]
\kappa\sigma^2 = \dfrac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}
\end{cases}
$$

HRG:
Hadron Resonance Gas

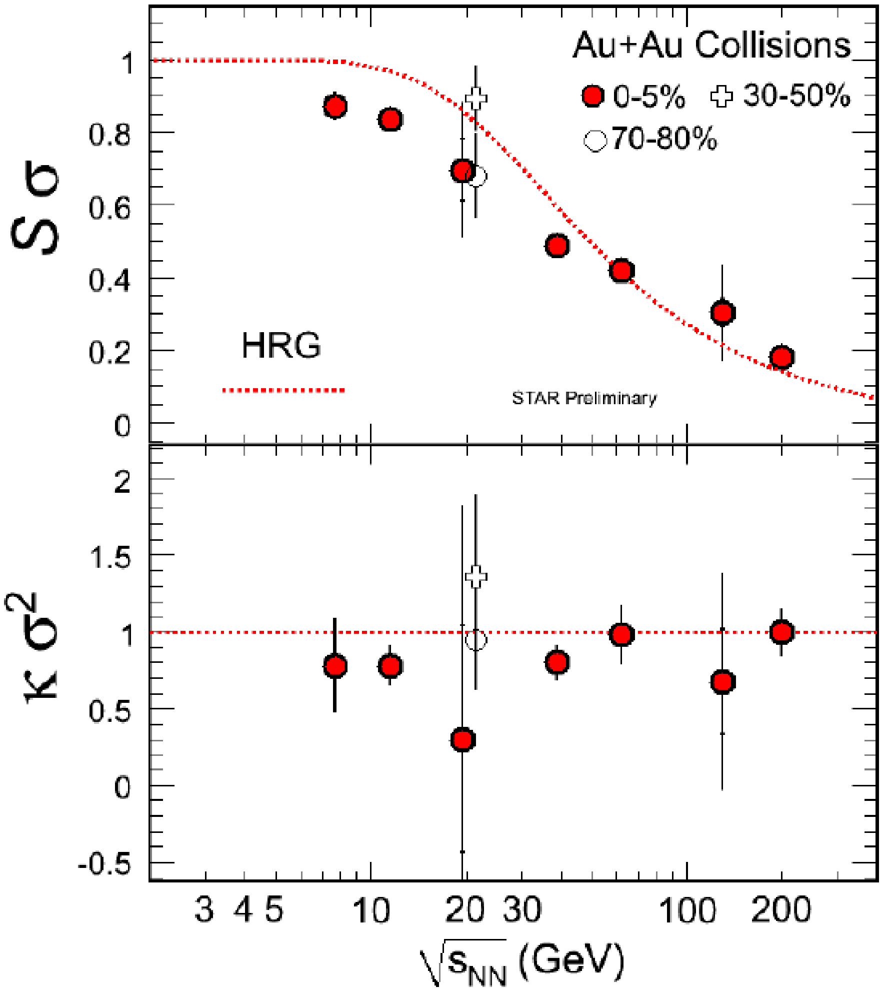
||

Free gas composed of all hadrons & resonances

high μ ← → low μ

Proton # Fluctuations @ STAR

2011 (Quark Matter)



high μ ← → low μ

$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle},$$

$$\kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

- No negative *S*
- No suppression of $\kappa\sigma^2$

Do fireballs forget all information in the early stage?

$$\langle \delta N_p^n \rangle$$

$$\langle \delta N_{\mathrm{B}}^n \rangle$$

- In equilibrated free nucleon gas,

$$\langle \delta N_{\mathrm{B}}^n \rangle_c = 2 \langle \delta N_p^n \rangle_c$$

- If the medium is not equilibrated,

$$\langle \delta N_{\mathrm{B}}^n \rangle_c \neq 2 \langle \delta N_p^n \rangle_c$$

Baryon Number Fluctuations are Better

- Since it is a conserved charge
  - Expectation values are well defined
  - possible slow diffusion in hadronic stage

Asakawa,Heintz,Muller ('00); Jeon, Koch ('00)

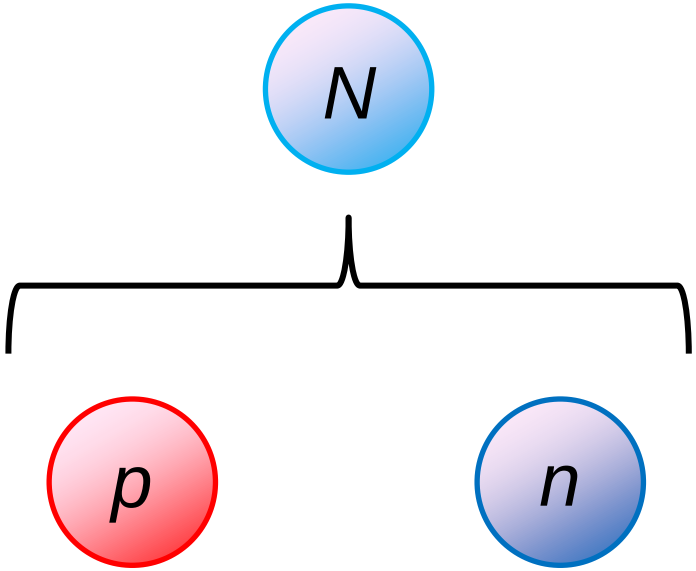
- Clearest observable for enhancement near CP
  - Same critical exponents for baryon, charge, proton #

Hatta, Stephanov ('02)

- But singular contribution is largest in baryon #

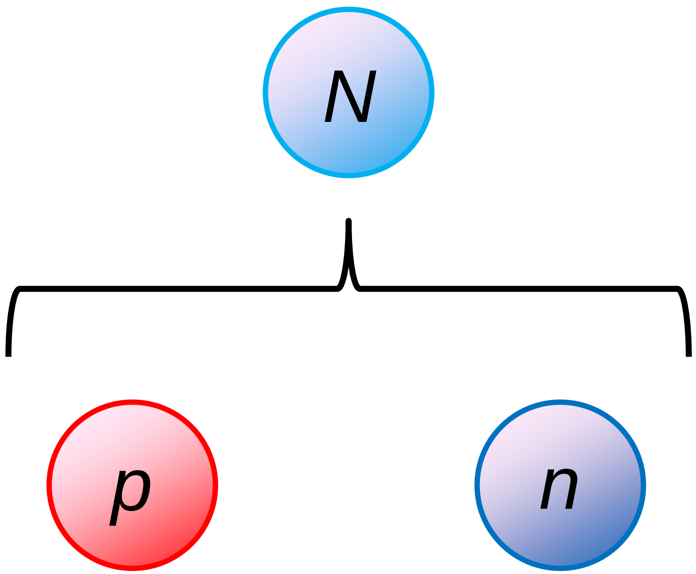
- Ratio of cumulants behaves most drastically

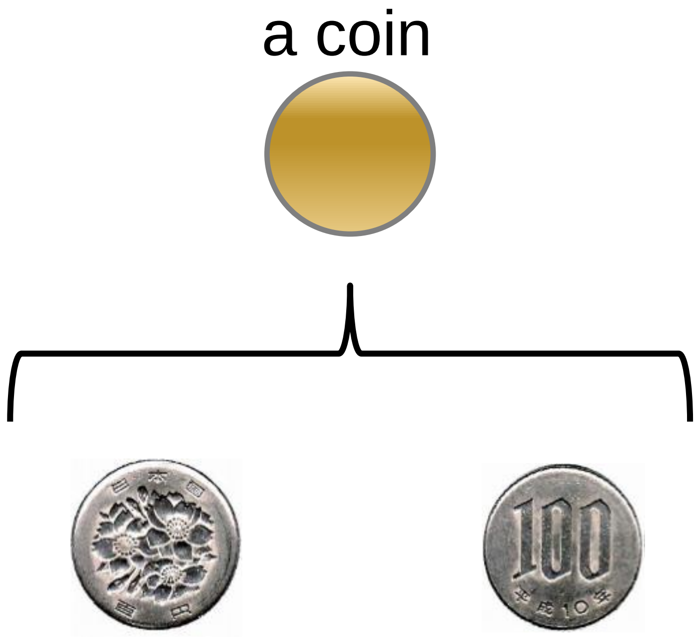
Nucleon Isospin as Two Sides of a Coin



Nucleons have
two isospin states.

Nucleon Isospin as Two Sides of a Coin

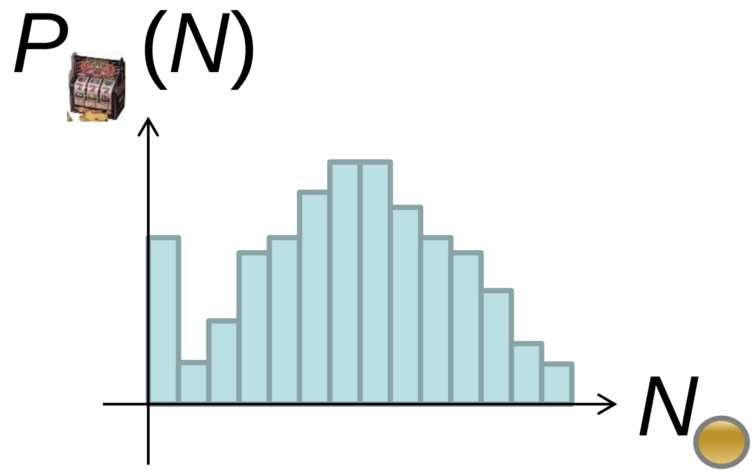


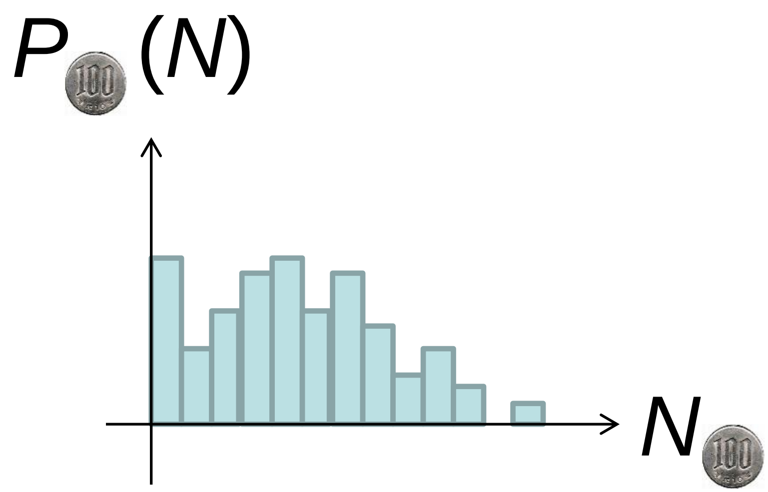


Nucleons have
two isospin states.

Coins have two sides.

Slot Machine Analogy

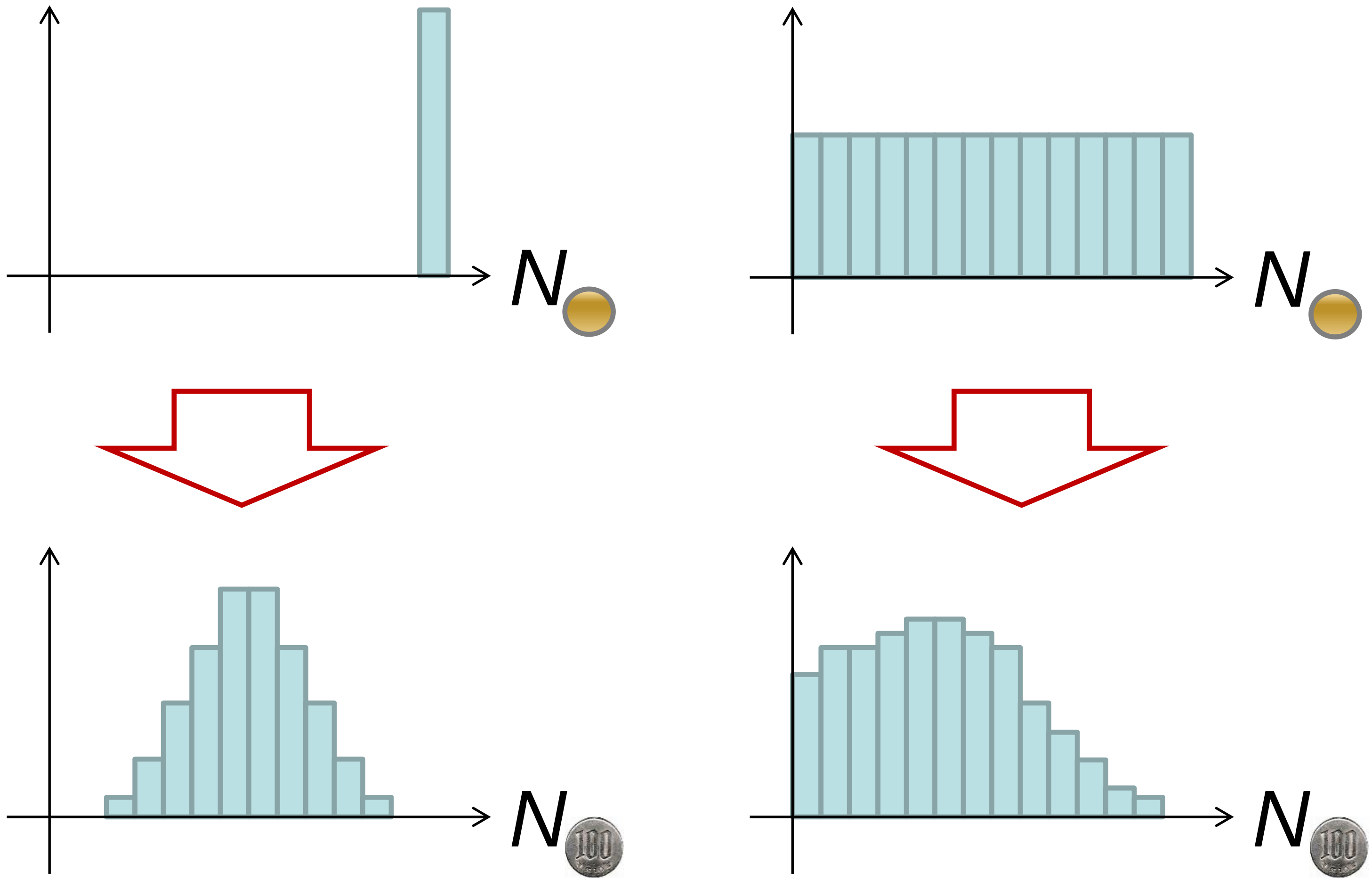




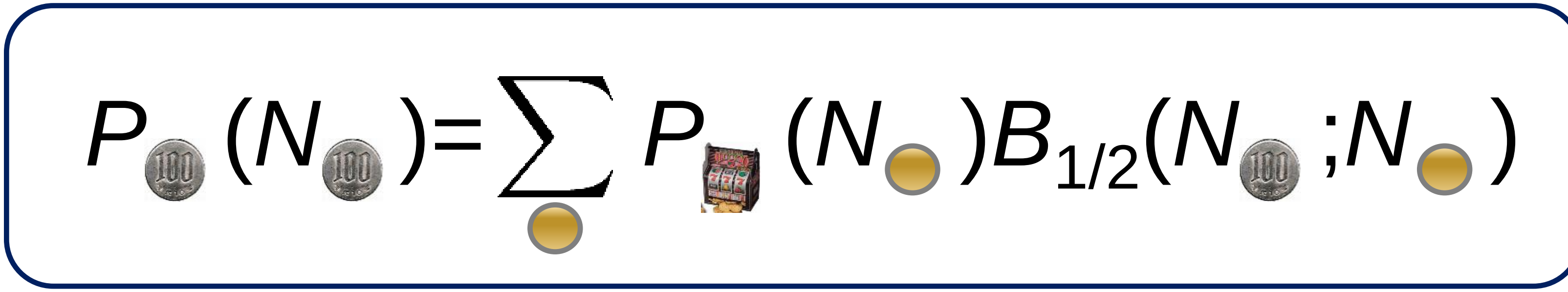
Extreme Examples

Fixed # of coins

Constant probabilities



Reconstructing Total Coin Number



$$B_p(k;N) = p^k(1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

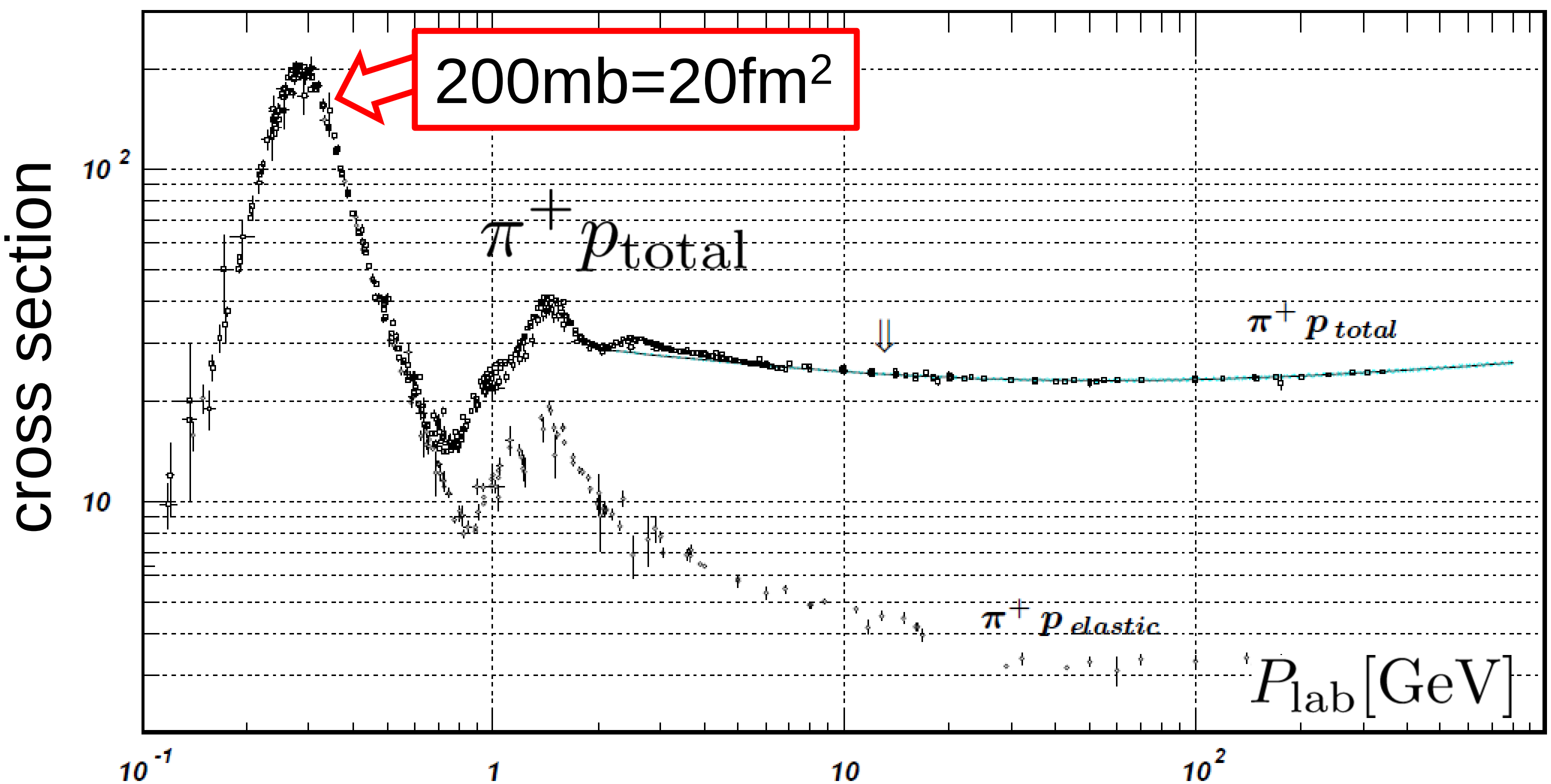
Nucleon Isospin in Hadronic Medium

- Isospin of baryons can vary <u>after chemical freezeout</u> via charge exchange reactions mediated by Δ(1232):

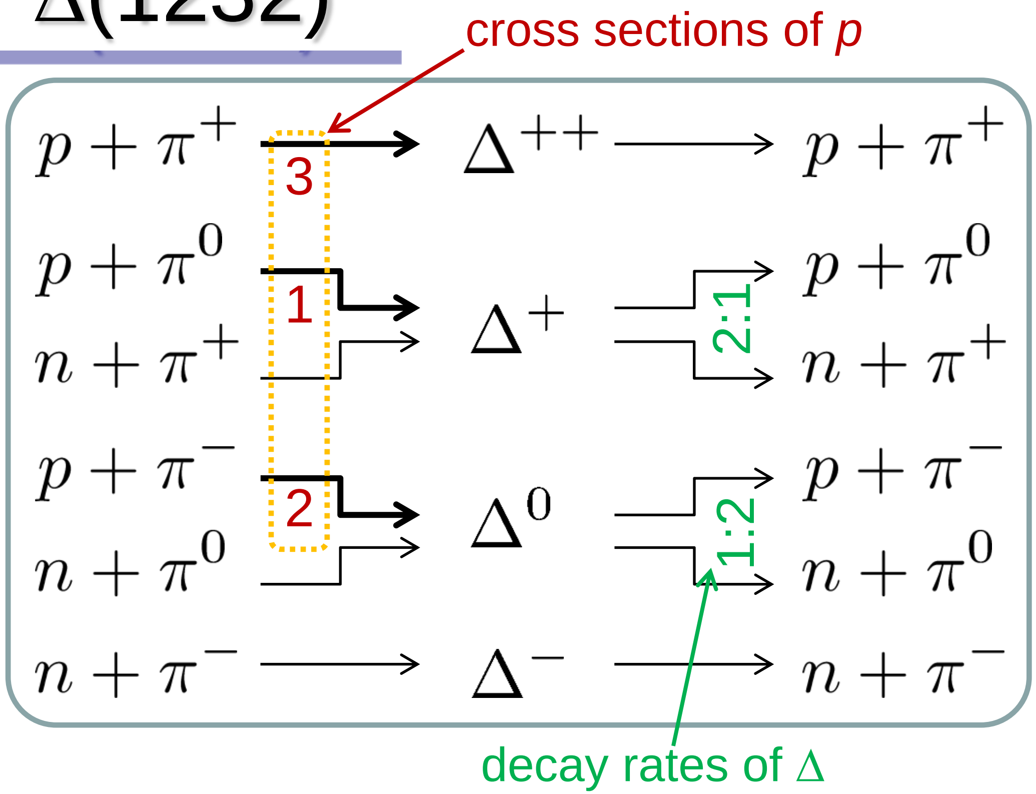
$$p, n + \pi \rightarrow \Delta(1232) \rightarrow p, n + \pi$$

$$I = 3/2$$

$$\Gamma \simeq 1.8 \text{ [fm]}$$

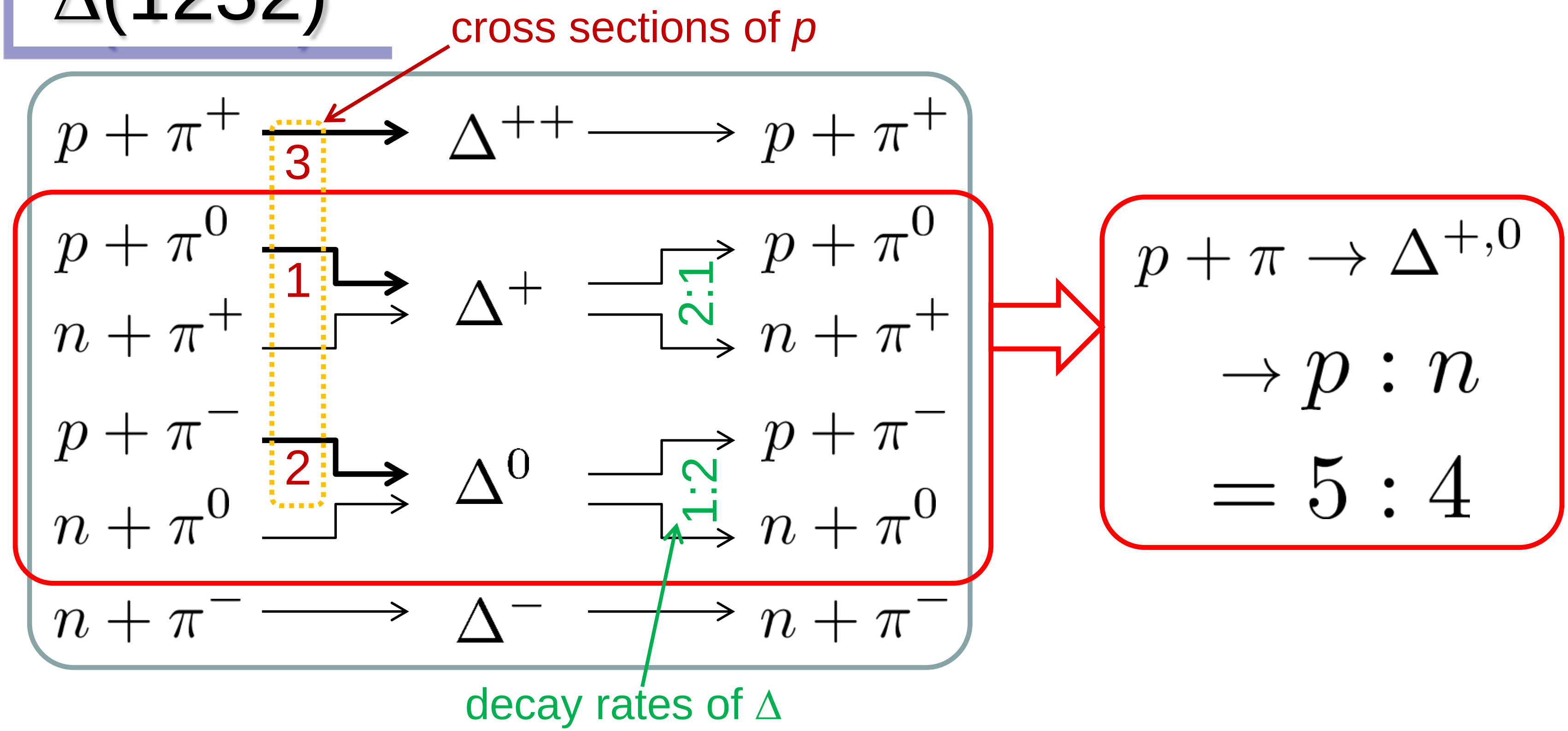


Δ(1232)

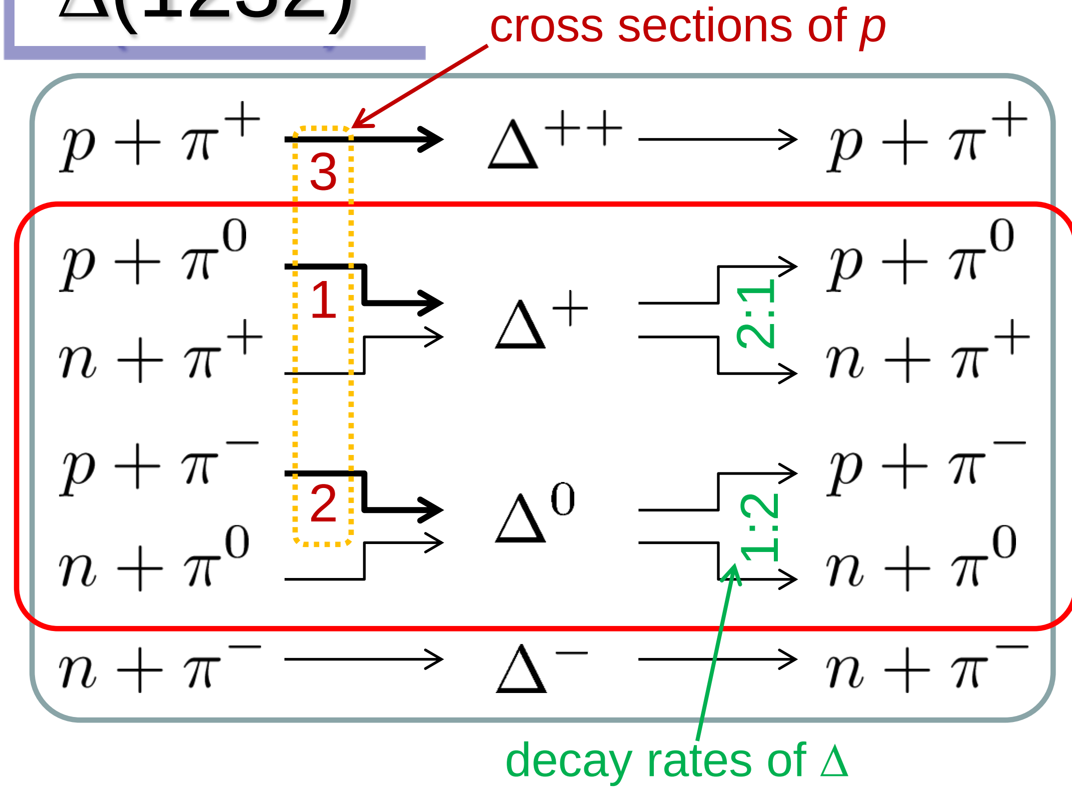


cross sections of p
$p + \pi^+$
Δ^{++}
$p + \pi^+$
3
$p + \pi^0$
1
Δ^+
2:1
$p + \pi^0$
$n + \pi^+$
$n + \pi^+$
$p + \pi^-$
2
Δ^0
1:2
$p + \pi^-$
$n + \pi^0$
$n + \pi^0$
$n + \pi^-$
Δ^-
$n + \pi^-$
decay rates of Δ

Δ(1232)


cross sections of p
$p + \pi^+ \longrightarrow \Delta^{++} \longrightarrow p + \pi^+$
3
$p + \pi^0$
1
$n + \pi^+$
Δ^+
2:1
$p + \pi^0$
$n + \pi^+$
$p + \pi^-$
2
$n + \pi^0$
Δ^0
1:2
$p + \pi^-$
$n + \pi^0$
$n + \pi^- \longrightarrow \Delta^- \longrightarrow n + \pi^-$
decay rates of Δ
$p + \pi \rightarrow \Delta^{+,0}$
$\rightarrow p : n$
$= 5 : 4$

Δ(1232)

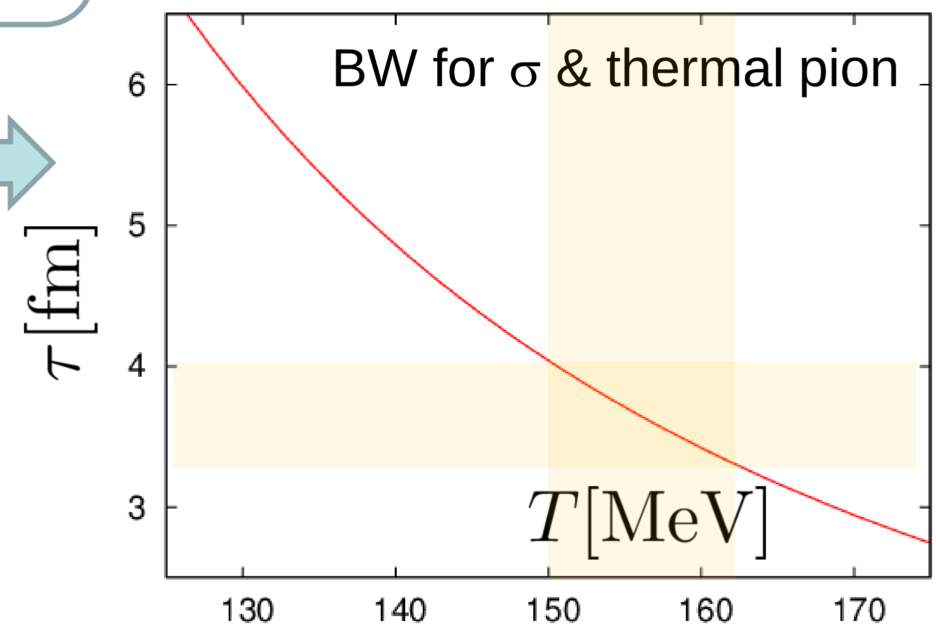


$$p+\pi \to \Delta^{+,0}$$
$$\to p : n$$
$$= 5 : 4$$

Lifetime to create Δ^+ or Δ^0

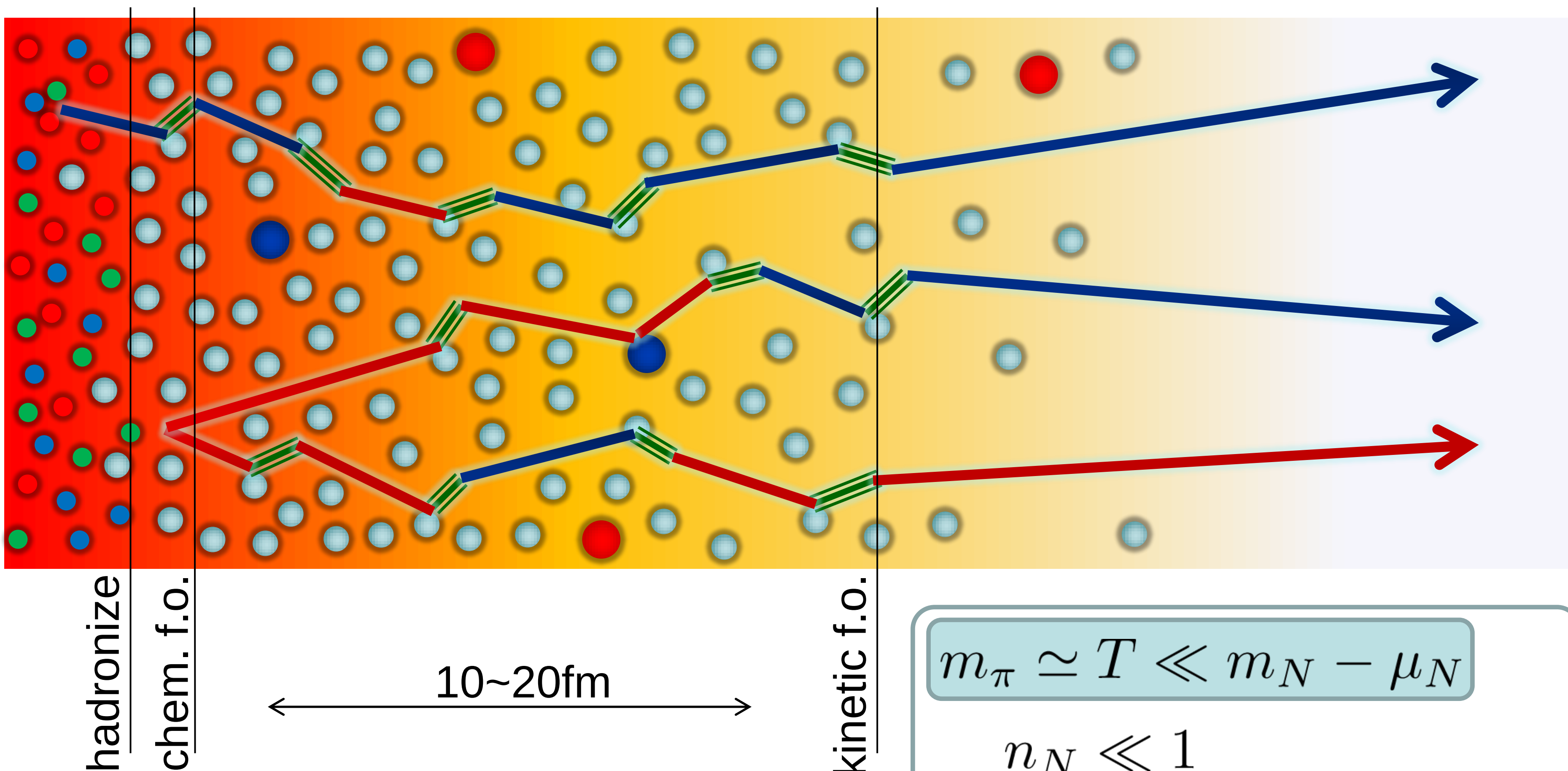
$$\tau^{-1} = \int \frac{d^3k_\pi}{(2\pi)^3} \sigma(E_{\rm cm}) v_\pi n(E_\pi)$$

$$\tau \ll (\text{freezeout time}) \simeq 20[\text{fm}]$$



c.f.) Nonaka, Bass, 2007

Nucleons in Hadronic Phase



- $p, \bar{p}$
- $n, \bar{n}$
- $\Delta(1232)$
- mesons
- baryons

$$m_\pi \simeq T \ll m_N - \mu_N$$

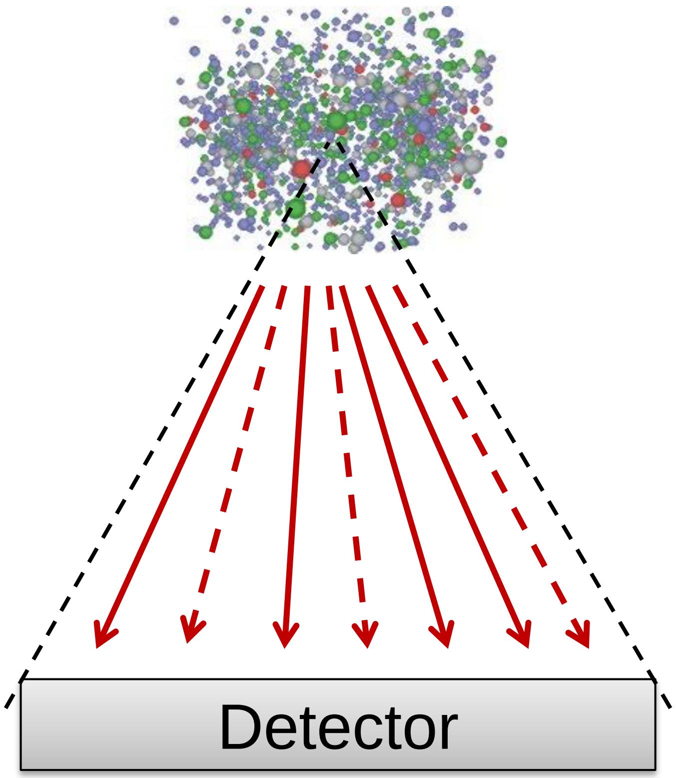
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\left\{ \begin{array}{l} \longrightarrow \ N_N \text{ nucleons} \\ \dashrightarrow \ N_{\bar{N}} \text{ anti-nucleons} \end{array} \right.$

  ⇨ $F(N_N, N_{\bar{N}})$

- $N_N \left\{ \begin{array}{l} N_p \text{ protons} \\ N_n \text{ neutrons} \end{array} \right.$

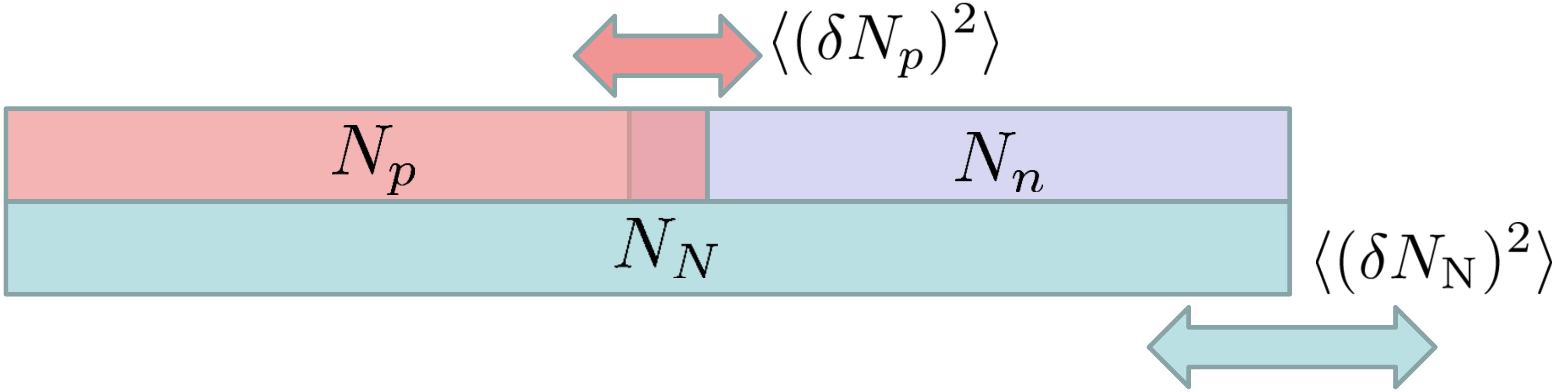
  ⇨ $B(N_p; N_N)$

  binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

- for any phase space in the final state.

Nucleon & Proton Number Fluctuations



$$\left\{\begin{aligned} \langle(\delta N_p^{(\mathrm{net})})^2\rangle &= \frac{1}{4}\langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^2\rangle + \frac{1}{4}\langle N_{\mathrm{N}}^{(\mathrm{tot})}\rangle \\ \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^2\rangle &= 4\langle(\delta N_p^{(\mathrm{net})})^2\rangle - 2\langle N_p^{(\mathrm{tot})}\rangle \end{aligned}\right.$$

- for isospin symmetric medium
- effect of isospin density <10%
- Similar formulas up to any order!

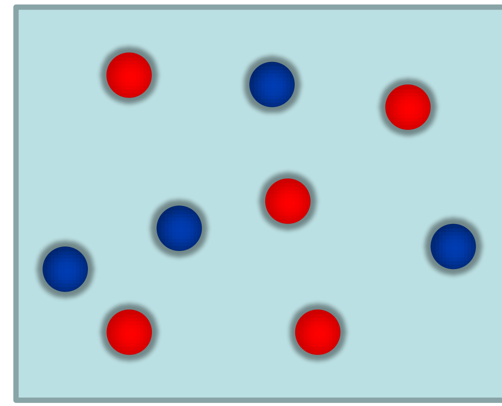
For free gas

$$\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^2\rangle$$

Free Nucleon Gas

$T, \mu_\mathrm{B} \ll m_\mathrm{N}$ ⇒ Poisson distribution $P_\lambda(N)$

$$\mathcal{P}(N_p, N_n) = P_\lambda(N_p) P_\lambda(N_n)$$
$$= P_{2\lambda}(N_p + N_n) B_{1/2}(N_p; N_p + N_n)$$



- The factrization is satisfied in free nucleon gas.

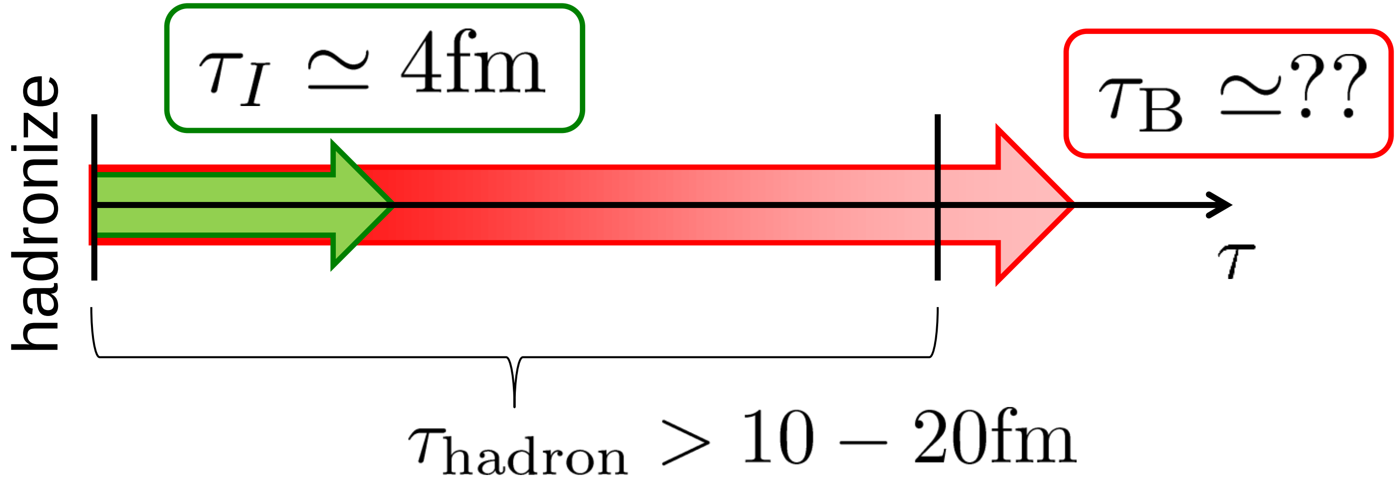
$$\mathcal{P}_\mathrm{free}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = \underbrace{P_{\bar{N}_\mathrm{N}}(N_\mathrm{N}) P_{\bar{N}_{\bar{\mathrm{N}}}}(N_{\bar{\mathrm{N}}})} B(N_p; N_\mathrm{N}) B(N_{\bar{p}}; N_{\bar{\mathrm{N}}})$$

$$F(N_\mathrm{N}, N_{\bar{\mathrm{N}}}) = P_{\bar{N}_\mathrm{N}}(N_\mathrm{N}) P_{\bar{N}_{\bar{\mathrm{N}}}}(N_{\bar{\mathrm{N}}})$$

Time Scales

- Time scales of fireballs:

$$\left\{ \begin{array}{ll} \tau_I & \text{: time scale to realize isospin binomiality} \\ \tau_{\rm B} & \text{: time scale of baryon number diffusion} \\ \tau_{\rm hadron} & \text{: life-time of hadronic medium in HIC} \end{array} \right.$$



Effect of Isospin Distribution

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

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(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

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$$\begin{cases} 2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^2\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^3\rangle = \frac{1}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^3\rangle_\mathrm{free} \\ 2\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c = \frac{1}{8}\langle(\delta N_\mathrm{B}^{(\mathrm{net})})^4\rangle_c + \cdots \end{cases}$$

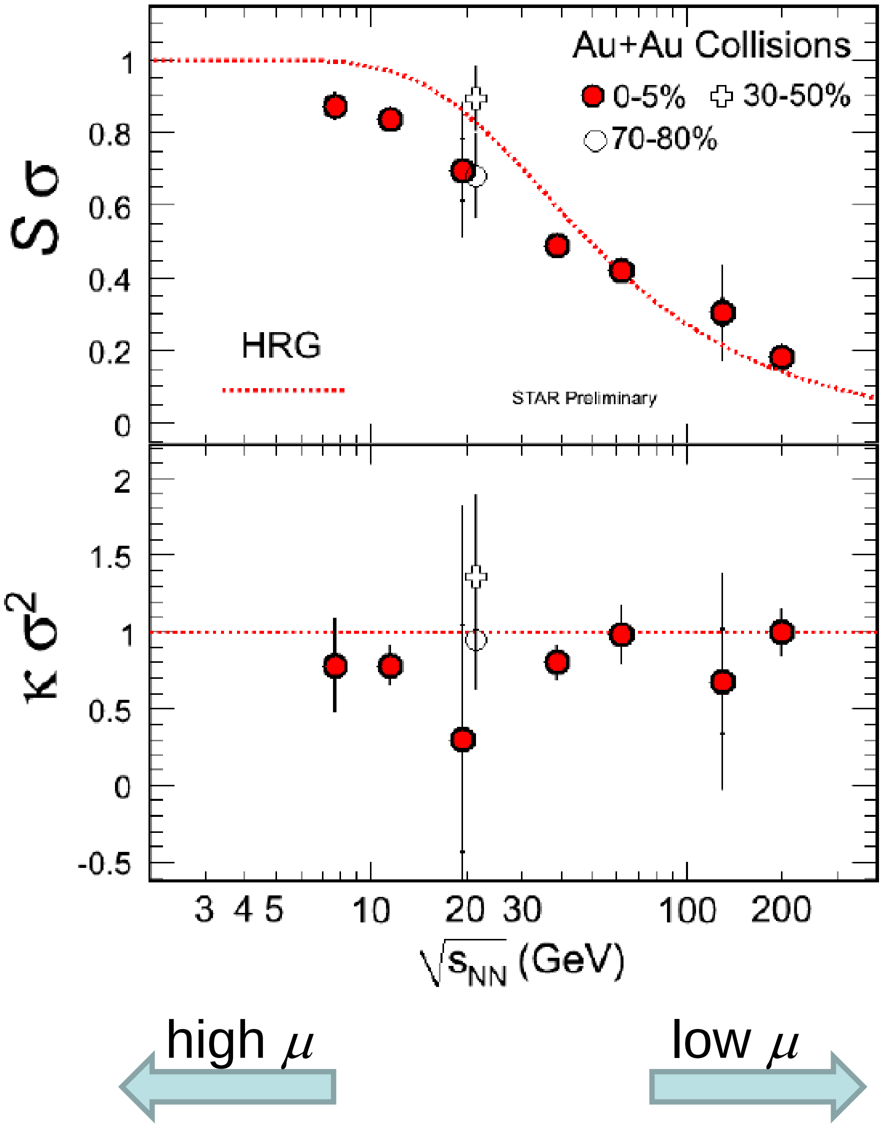
genuine info. noise

For free gas

$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_\mathrm{N}^{(\mathrm{net})})^n\rangle_c$$

Proton # Fluctuations @ STAR

2011 (Quark Matter)



$$S\sigma = \frac{\langle(\delta N_p^{(\mathrm{net})})^3\rangle}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle},$$

$$\kappa\sigma^2 = \frac{\langle(\delta N_p^{(\mathrm{net})})^4\rangle_c}{\langle(\delta N_p^{(\mathrm{net})})^2\rangle}$$

- No negative S
- No suppression of $\kappa\sigma^2$

Do fireballs forget all information in the early stage?

No. Not necessarily!

QCD臨界点近傍の陽子数臨界ゆらぎ

- バリオン数ゆらぎは高次キュムラントほど高い臨界指数で発散

Stephanov, '09

$$\langle \delta N^2 \rangle \sim \xi^2 \quad \langle \delta N^3 \rangle = \xi^{4.5} \quad \langle \delta N^4 \rangle_c = \xi^7$$

- 陽子数キュムラントに含まれるバリオン数キュムラントの割合は、高次になるほど抑制される。

$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \frac{1}{4} \langle (\delta N_{\mathrm{N}}^{(\mathrm{net})})^2 \rangle + \frac{1}{4} \langle N_{\mathrm{N}}^{(\mathrm{tot})} \rangle$$

$$\langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c = \frac{1}{16} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^4 \rangle_c + \frac{3}{8} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \delta N_{\mathrm{B}}^{(\mathrm{tot})} \rangle + \frac{3}{16} \langle (\delta N_{\mathrm{B}}^{(\mathrm{tot})})^2 \rangle - \frac{1}{8} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle$$

Strange Baryons

Decay Rates:

Λ $m_\Lambda \simeq 1116[\mathrm{MeV}]$

$$\Rightarrow \; p : n \simeq 1.6 : 1$$

Σ $m_\Sigma \simeq 1190[\mathrm{MeV}]$

$$\Rightarrow \; p : n \simeq 1 : 1.8$$

Decay modes:

$\Lambda \to p + \pi^-$ 64%

$\Lambda \to n + \pi^0$ 36%

$\Sigma^+ \to p + \pi^0$ 52%

$\Sigma^+ \to n + \pi^+$ 48%

$\Sigma^0 \to \Lambda \to p + \pi^-$ 64%

$\Sigma^0 \to \Lambda \to n + \pi^0$ 36%

$\Sigma^- \to n + \pi^-$

Regarding these ratios even, protons from these decays is incorporated into the binomial distribution. Then, $N_N \rightarrow N_B$

Summary

- **Baryon and proton number fluctuations are different**. To see non-thermal effects in heavy ion collisions, baryon number's is better.

- Formulas to reveal baryon # cumulants in experiments.

- Experimental analysis of baryon # fluctuations may verify
  - signals of QCD phase transition
  - speed of baryon number diffusion in the hadronic stage.

Future Work

- Distribution function itself
- Incorporating effects of efficiency and acceptance in exp. / correlation between isospin fluctuations of pions
- Discussion on dynamical evolution of fluctuations

3rd & 4th Order Fluctuations

$$N_{\rm B} \to N_p$$

$$\langle(\delta N_p^{(\rm net)})^3\rangle = \frac{1}{8}\langle(\delta N_{\rm B}^{(\rm net)})^3\rangle + \frac{3}{8}\langle\delta N_{\rm B}^{(\rm net)}\delta N_{\rm B}^{(\rm tot)}\rangle,$$

$$\langle(\delta N_p^{(\rm net)})^4\rangle_c = \frac{1}{16}\langle(\delta N_{\rm B}^{(\rm net)})^4\rangle_c + \frac{3}{8}\langle(\delta N_{\rm B}^{(\rm net)})^2\delta N_{\rm B}^{(\rm tot)}\rangle + \frac{3}{16}\langle(\delta N_{\rm B}^{(\rm tot)})^2\rangle - \frac{1}{8}\langle N_{\rm B}^{(\rm tot)}\rangle,$$

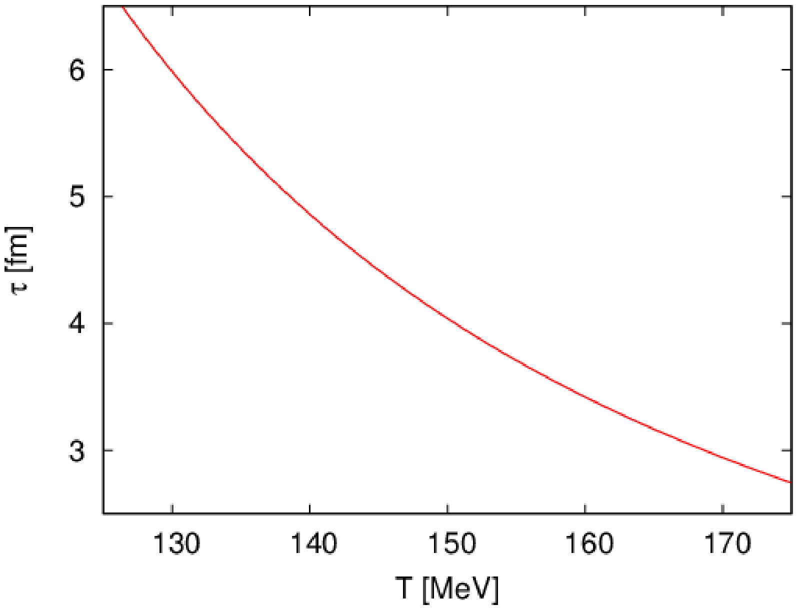
$$N_p \to N_{\rm B}$$

$$\langle(\delta N_{\rm B}^{(\rm net)})^3\rangle = 8\langle(\delta N_p^{(\rm net)})^3\rangle - 12\langle\delta N_p^{(\rm net)}\delta N_p^{(\rm tot)}\rangle + 6\langle N_p^{(\rm net)}\rangle,$$

$$\langle(\delta N_{\rm B}^{(\rm net)})^4\rangle_c = 16\langle(\delta N_p^{(\rm net)})^4\rangle_c - 48\langle(\delta N_p^{(\rm net)})^2\delta N_p^{(\rm tot)}\rangle + 48\langle(\delta N_p^{(\rm net)})^2\rangle + 12\langle(\delta N_p^{(\rm tot)})^2\rangle - 26\langle N_p^{(\rm tot)}\rangle,$$

Nucleon Time Scales in Fireballs

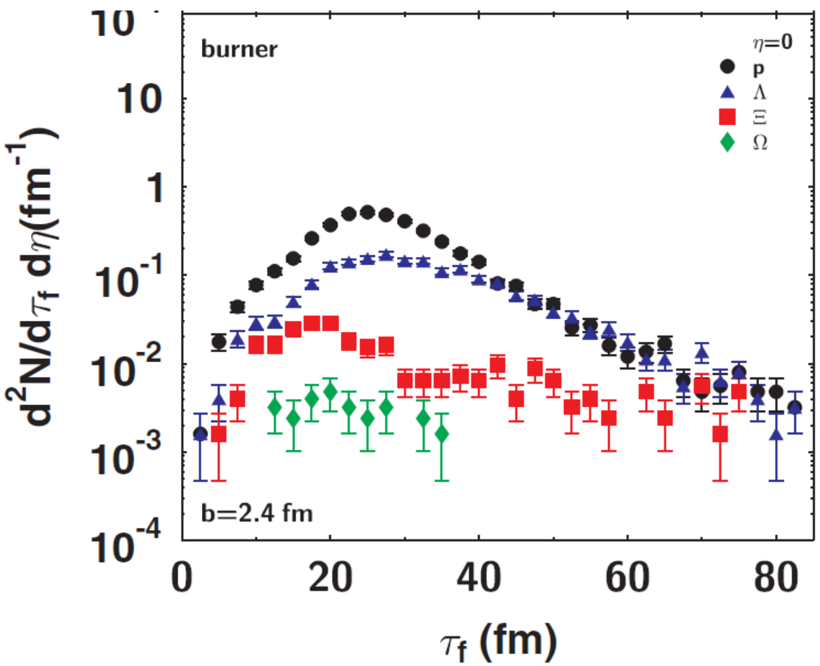
Mean time to create $\Delta^{+,0}$



$$\tau_{\Delta} = 3 \sim 4[\mathrm{fm}]$$

Freeze-out time

Nonaka, Bass, 2007



$$\tau_{\mathrm{f.o.}} > 20[\mathrm{fm}]$$

Isospin Distributions

- Large pion density
  - Small nucleon density because $M_N/T \ll 1$
  - For top RHIC energy, $N_\pi \sim 20 N_N$
- Nucleons exclusively interact with pions
  - Rare NN collisions
  - Huge $\pi\pi$ reactions

All formations and decays of Δ take place independently

Nonaka, Bass, '07