Introduction to Perturbative QCD

Lecture 4

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Outline for Lecture 4

- ❑ Cross section with two or more identified hadrons – Drell-Yan process as an example
- ❑ Factorization for hadronic cross sections
- ❑ Cross sections with two observed hard scales
- ❑ Resummation of large logarithms in pQCD

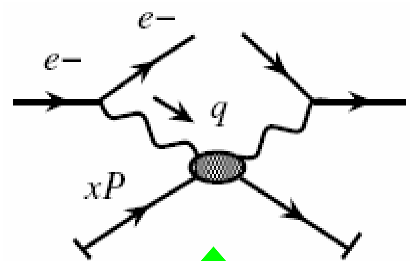
Excellent resource – CTEQ summer school website

http://www.phys.psu.edu/~cteq

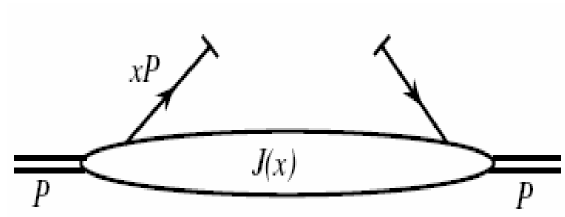
Cross section with identified hadrons

- **One hadron:**

$$\sigma^{\rm DIS}_{\rm tot} \sim$$



$$\otimes$$



$$+\,O\left(\frac{1}{QR}\right)$$

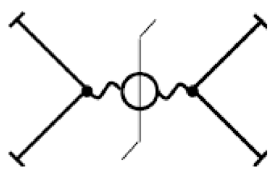
Now
Hard-part

Past
Parton-distribution

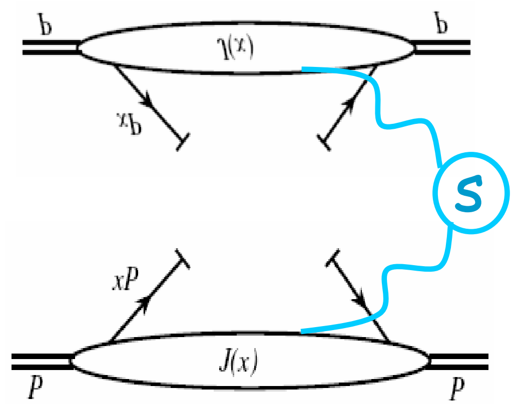
Connection
Power corrections

- **Two hadrons:**

$$\sigma^{\rm DY}_{\rm tot} \sim$$



$$\otimes$$



$$+\,O\left(\frac{1}{QR}\right)$$

Soft interactions between incoming hadrons break the universality of PDFs

Drell-Yan cross section

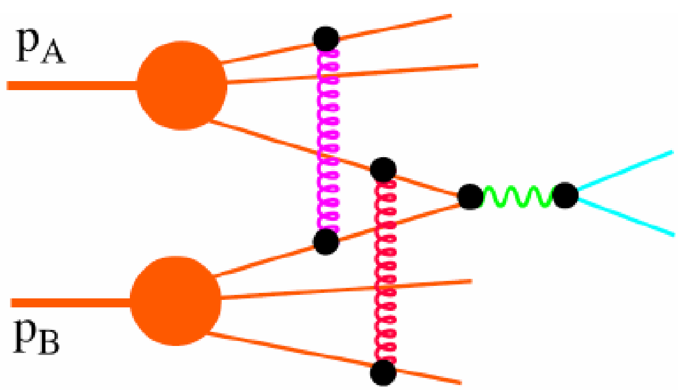
- Drell-Yan process:

$$h(p_A)+h'(p_B)\rightarrow \ell^+\ell^-(q)+X \qquad \text{with} \quad Q^2=q^2$$

- Parton model formula:

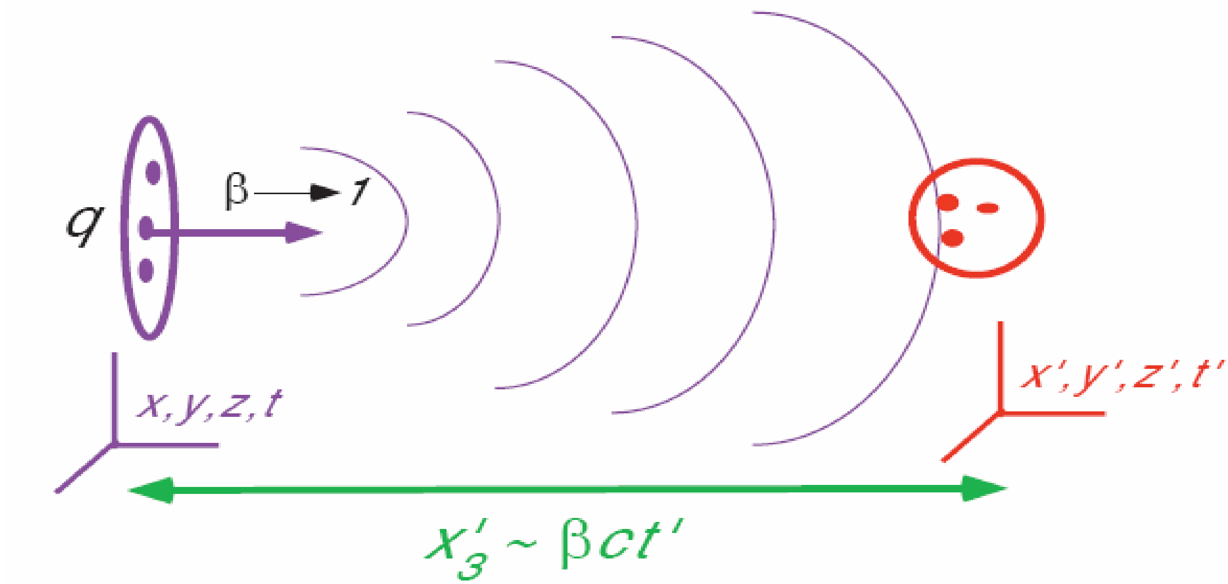
$$\frac{d\sigma_{hh'}^{\rm DY}(p_A,p_B,q)}{dQ^2}=\sum_{f,f'}\int_0^1 dx\int_0^1 dx'\,\phi_f(x)\,\frac{d\hat{\sigma}_{ff'}^{\rm el}(xp_A,x'p_B,q)}{dQ^2}\,\phi_{f'}(x')$$

- Long-range soft interactions before the hard collision could break the PDF's universality – loss of prediction



$$K_{\rm factor}=\frac{\sigma_{\rm Exp}^{\rm DY}}{\sigma_{\rm PM}^{\rm DY}}\sim 2$$

Heuristic argument for factorization



Field	x-Frame	x'-Frame
Scalar	$V(x) = \frac{e}{\lvert\vec{x}\rvert}$	$V'(x') = \frac{e}{(x_T^2 + \gamma^2\Delta^2)^{1/2}}$
		$\Longrightarrow \frac{1}{\gamma}$ "contracted like a ruler"
Gauge	$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2 + \gamma^2\Delta^2)^{1/2}}$
		$\Longrightarrow 1$ "not contracted!"

Field strength is strongly contracted

Field	x-Frame	x'-Frame
Field Strength	$E_3(x) = \frac{e}{\|\vec{x}\|^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2 + \gamma^2\Delta^2)^{3/2}}$
		$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

➡ *Lorentz contracted fields of incident particles do not overlap until the moment of the scattering!*

the $1/\gamma^2$ translates into a suppression factor of $1/Q^4$

➡ *Initial-state interaction disappear at high enough energies!*

$$\sigma(Q) = \sigma_0(Q) + \sigma_2(Q)\frac{1}{Q^2} + \sigma_4(Q)\frac{1}{Q^4} + \ldots$$

➡ the factorization should be valid at the order of $1/Q^2$

Leading power (twist): Collins, Soper, and Sterman; Bodwin

Next leading power: Qiu and Sterman

Factorization is violated at $1/Q^4$ via explicit calculation: Taylor et al.

Factorization for Drell-Yan cross section

❑ Factorization at leading power:

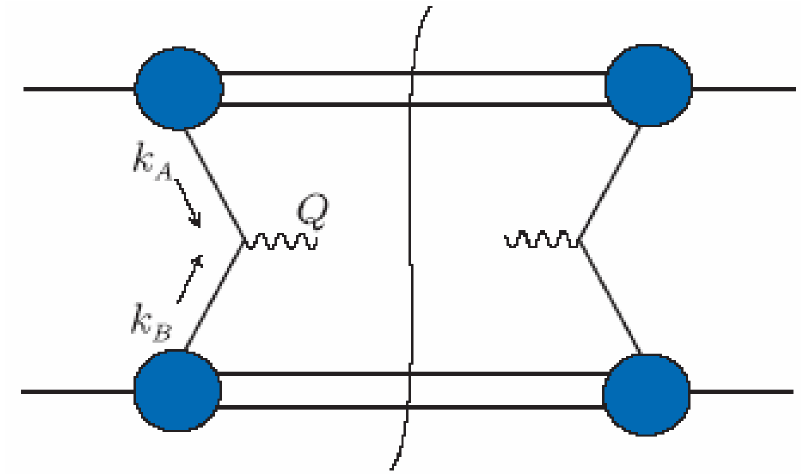
$$\frac{d\sigma_{hh'}^{\mathrm{DY}}\left(p_A, p_B, q\right)}{dQ^2} = \sum_{f,f'} \int_0^1 dx \int_0^1 dx' \; \phi_f\left(x, \mu_F^2\right) \; \frac{d\hat{\sigma}_{ff'}^{\mathrm{DY}}\left(xp_A, x' p_B, q, \mu_F^2\right)}{dQ^2} \; \phi_{f'}\left(x', \mu_F^2\right)$$

- ❖ **This is not "leading log approximation", corrections to this factorized formula are power suppressed $1/Q^2$**

- ❖ **Parton distributions are non-perturbative, but, defined in terms of matrix elements of universal and gauge invariant operators – same as those defined in DIS**

- ❖ $d\hat{\sigma}$ **has an expansion in powers of α_s**

Why Drell-Yan process makes sense?



Lowest order diagram in QCD perturbation theory, and kinematics determines the process

$$\frac{d\sigma}{dQ^2dy} = \int dk_{A,T}\, dk_{B,T} dk_A^- dk_B^+ \; H_{\mu,\nu}(Q^+, Q^-, k_{A,T} + k_{B,T})$$
$$\times \mathrm{Tr}\{\gamma^\mu \Phi_A(Q^+ - k_B^+, k_{A,T}, k_A^-)\gamma^\nu \Phi_B(k_B^+, k_{A,T}, Q^- - k_A^-)\}$$

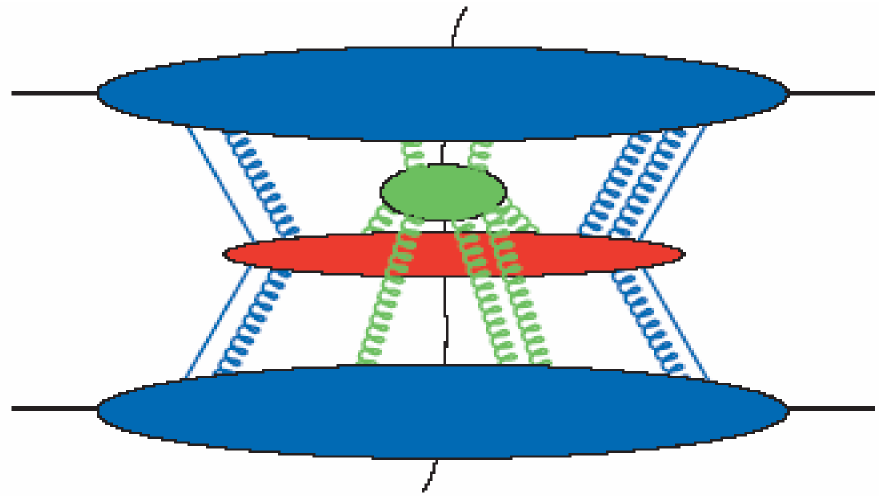
Approximation:

$$k_{A,T}^2 \;,\; k_{B,T}^2 \ll Q^2$$
$$k_A^- \ll Q^-$$
$$k_B^+ \ll Q^+$$

$\Rightarrow$ **Drell-Yan formula**

QCD dynamics is rich and complicate

Analysis of leading (pinch or singular) integration regions gives the following:



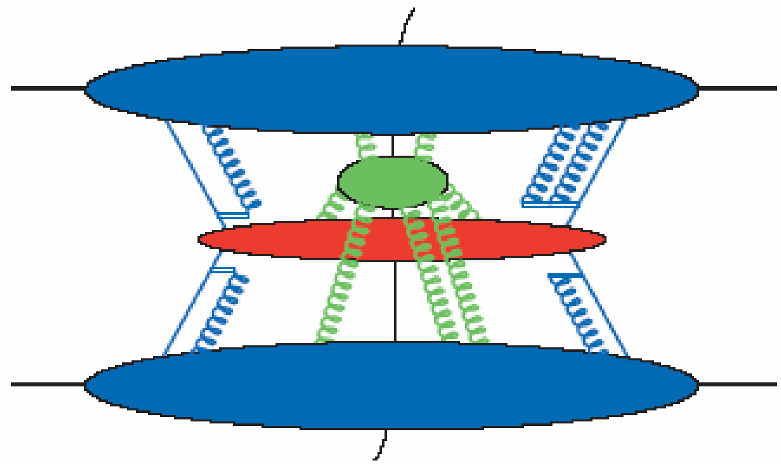
Hard (Large P_T or way off shell)

Collinear (to A or to B, small P_T)

Soft (All components small, includes "Glauber.")

Eikonalization of collinear gluons

The extra collinear gluons would be a big problem because the factorization formula contemplates collisions of only one parton from each hadron.



But the collinear gluons are OK

- The extra collinear gluons have $\epsilon^{\mu} \propto k^{\mu}$.

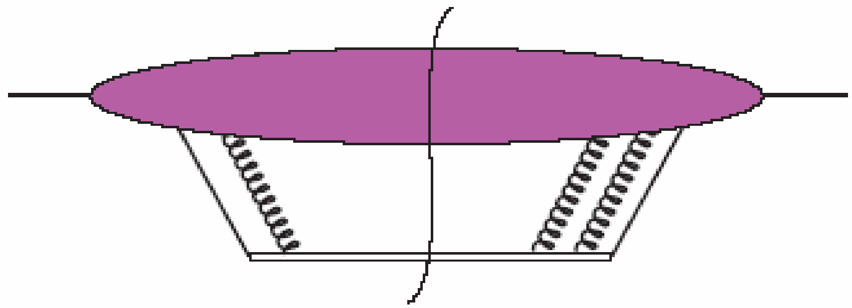
- There effect can be approximated as shown with eikonal lines, with u in the $-$ direction for hadron A, u in the $+$ direction for hadron B,

$$\text{propagator} = \frac{i}{k \cdot u + i\epsilon}$$

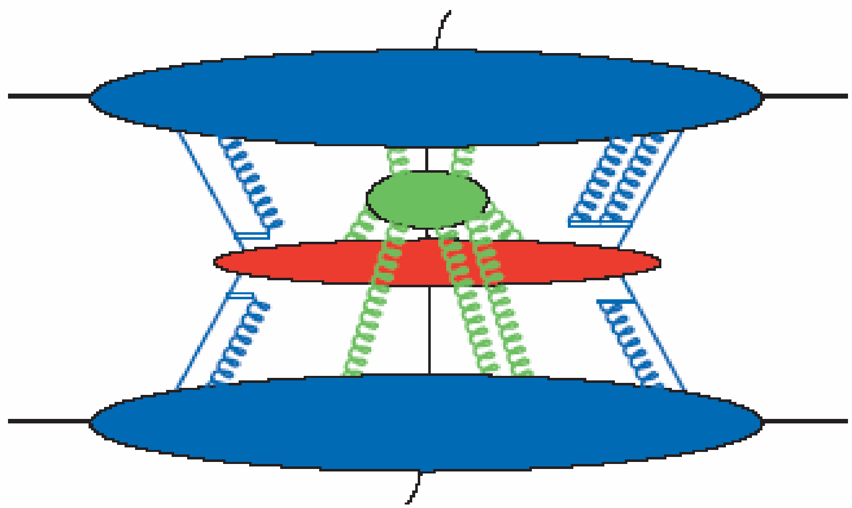
$$\text{vertex} = -igt_a u^{\mu}$$

Factorization of PDFs

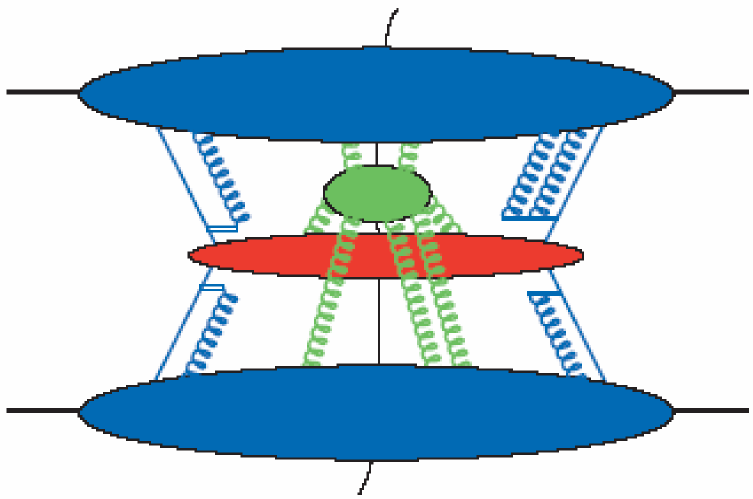
Parton distribution in diagrams



Compare



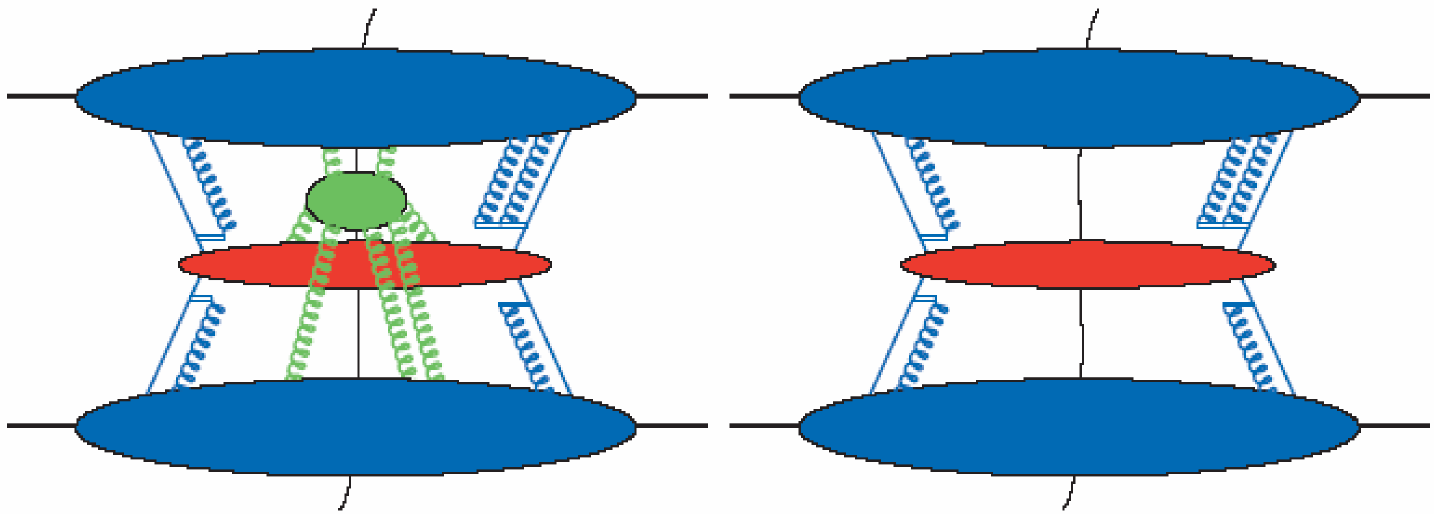
Trouble from soft gluons



- It seems that a soft gluon exchanged from a spectator quark in hadron A to the active quark in hadron B can rotate the quark's color and thus keep it from annihilating.

- Soft gluon approximations (with eikonal lines) needs $q^{\pm}$ not too small. But $q^{\pm}$ contours can be trapped in "too small" region.

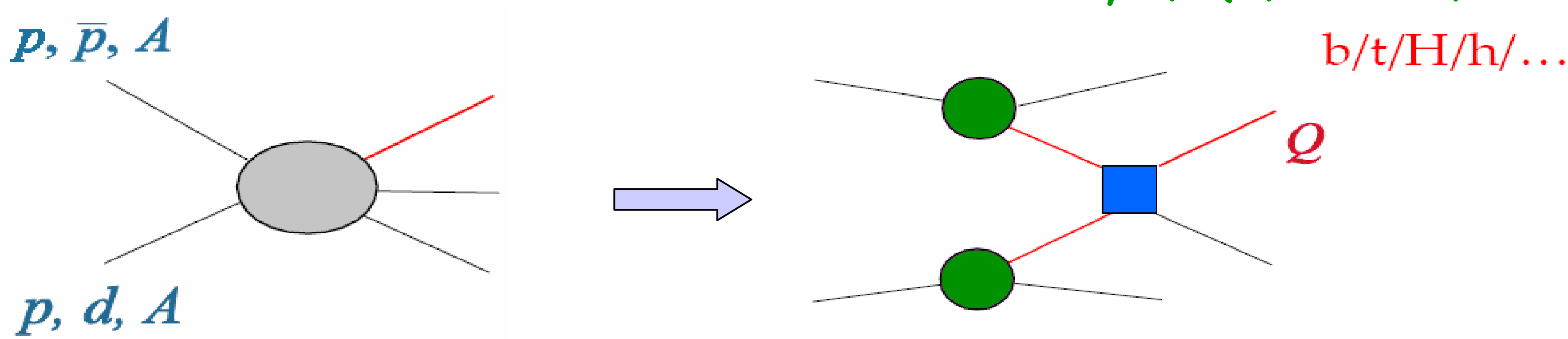
The soft gluons take care of themselves



- This part is quite technical.
- Ingredients: unitarity, causality, gauge invariance.
- We use the fact that the initial state is a color singlet bound state and that we can sum over all final states.

Factorization for high p_T single hadron/jet

Nayak, Qiu, Sterman, 2006



$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x, \mu_F^2) \otimes \phi_{B\to b}(x', \mu_F^2)$$

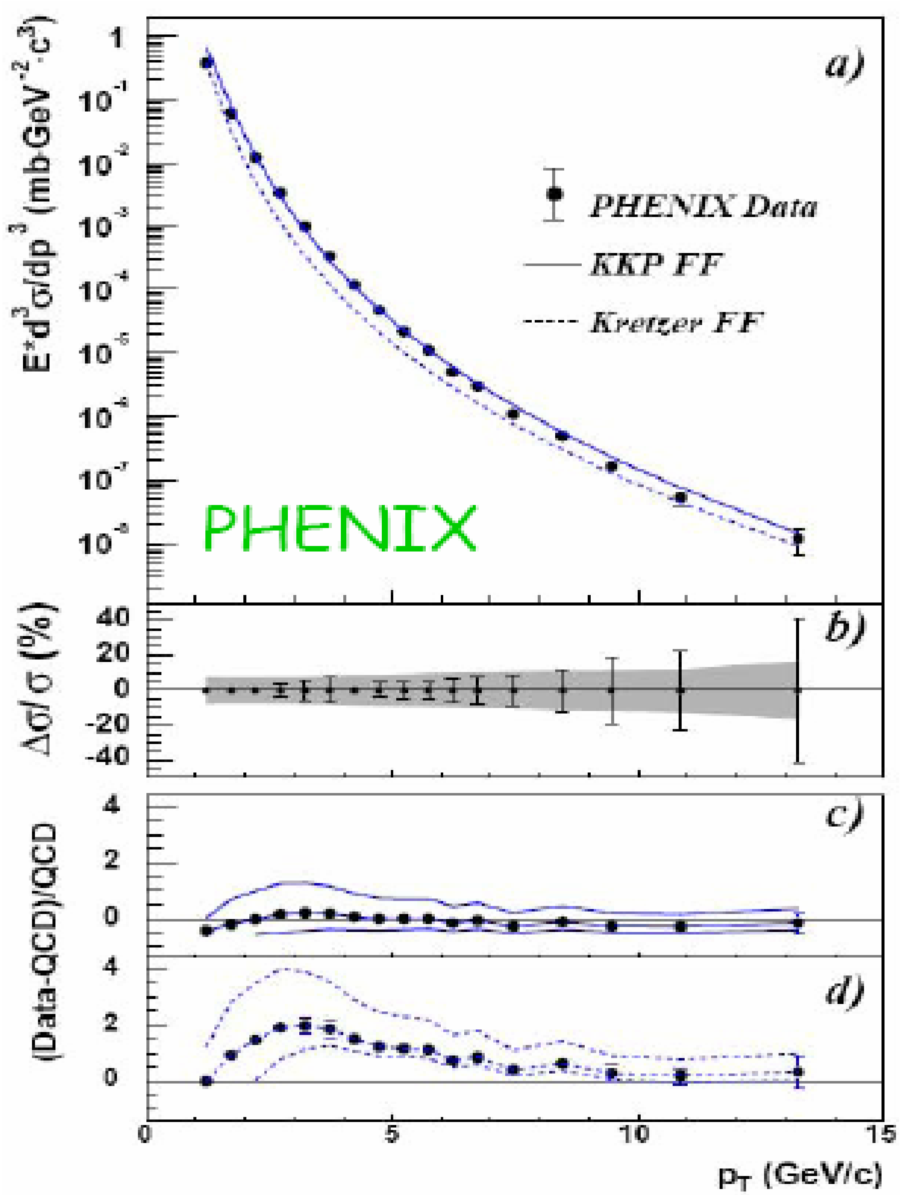
$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x, x', z, y, p_T^2 \mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z, \mu_F^2)$$

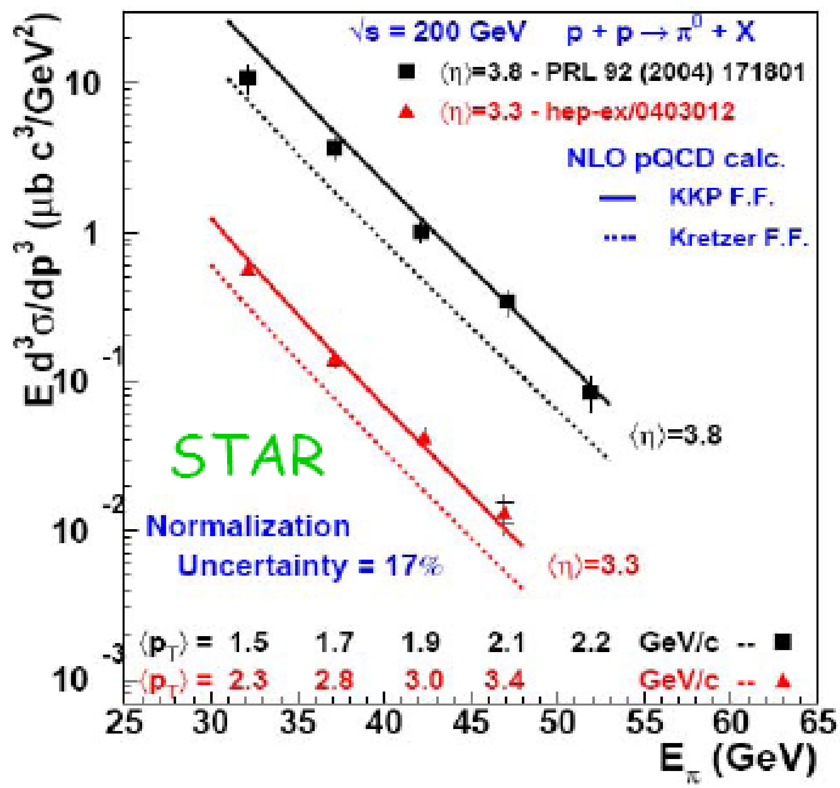
Fragmentation function: $D_{c\to C}(z, \mu_F^2)$

Choice of the scales: $\mu_{\rm Fac}^2 \approx \mu_{\rm ren}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

High p_T pion production from RHIC





Collinear factorization
at NLO works well
at RHIC energies

Processes with two large scales

$$Q_1^2 \gg Q_2^2 \gg \Lambda_{\rm QCD}^2$$

- **We could choose: $\mu = Q_1$ or Q_2, or somewhere between**

→ $\alpha_s\left(Q_1^2\right)$ is small, $\alpha_s\left(Q_1^2\right)\ell n(Q_1^2/Q_2^2)$ is not necessary small

Cannot remove the logarithms by choosing a proper μ

→ **Resummation of the logarithms is needed**

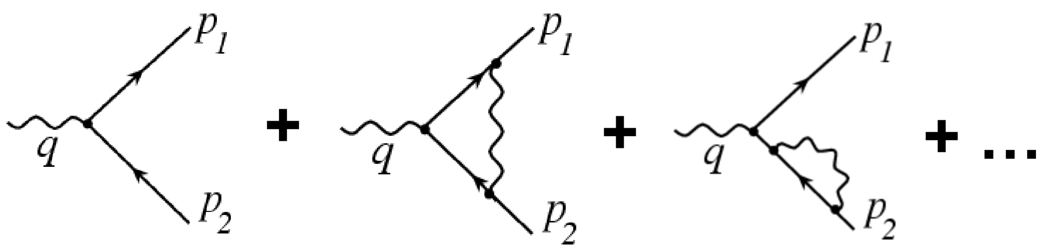
- **For a massless theory, we can get <u>two</u> powers of the logarithms at each order in perturbation theory:**

$$\alpha_s\left(Q_1^2\right)\ell n^2(Q_1^2/Q_2^2)$$

because of an overlap region of IR and CO divergences

Double logarithms

❑ **Consider electromagnetic form factor:**



$$\Gamma_\mu(q^2,\epsilon) = -ie\mu^\epsilon\ \bar{u}\ (p_1)\gamma_\mu v(p_2)\ \rho(q^2,\epsilon)$$

❑ **For massless quark at one loop:**

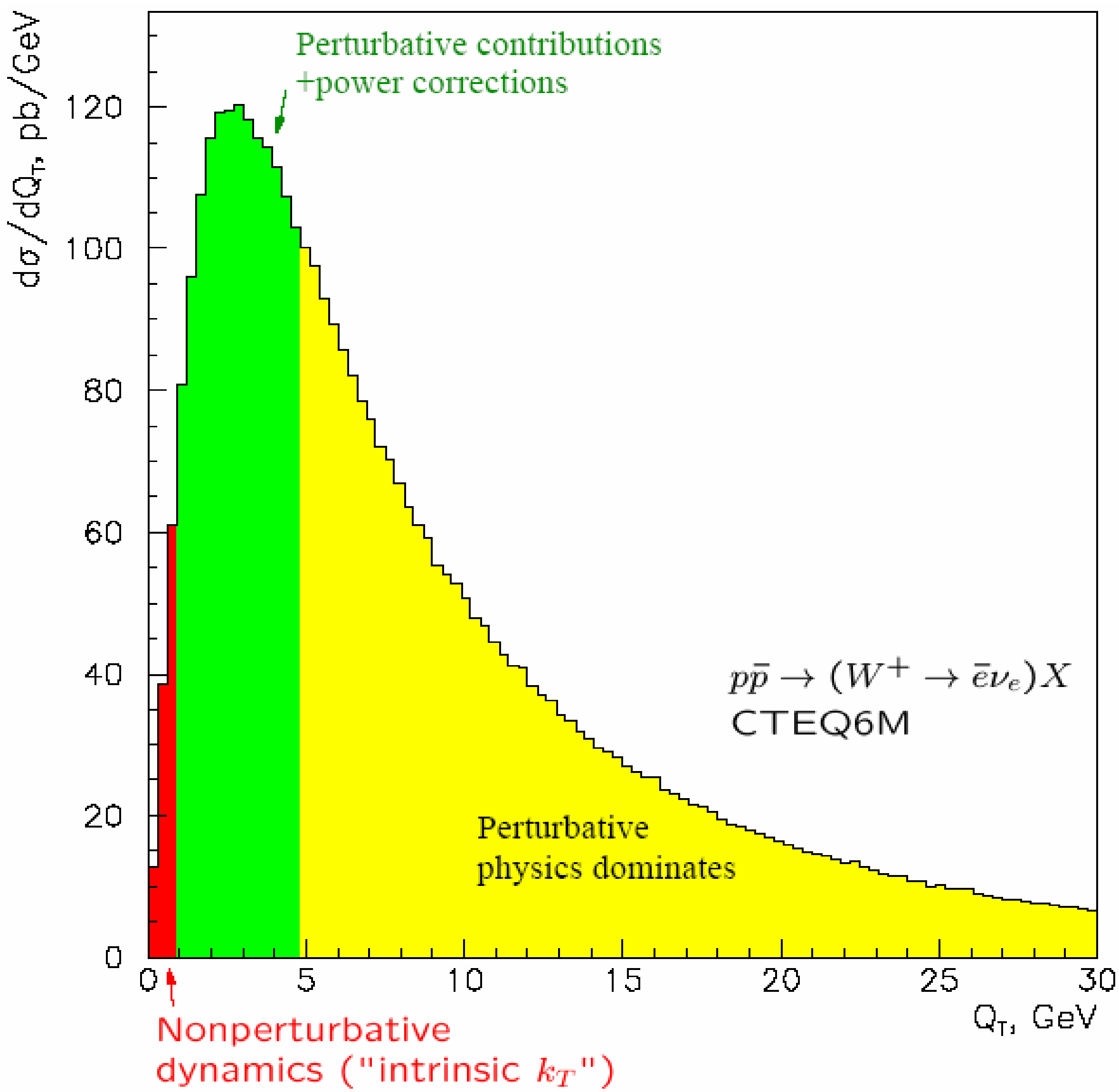
$$\rho(q^2,\epsilon) = -\frac{\alpha_s}{2\pi}C_F\left(\frac{4\pi\mu^2}{-q^2-i\epsilon}\right)^\epsilon \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\left\{\frac{1}{(-\epsilon)^2}-\frac{3}{2(-\epsilon)}+4\right\}$$

$$= 1 - \frac{\alpha_s}{4\pi}C_F\ \ln^2(q^2/\mu^2) + \ldots$$

Overlap of IR and CO singularities ➡ Double logarithms

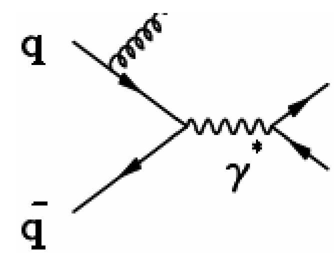
- ❖ **known as Sudakov double logarithms**
- ❖ **common in a massless theory**

Drell-Yan (W/Z …) Q_T distribution



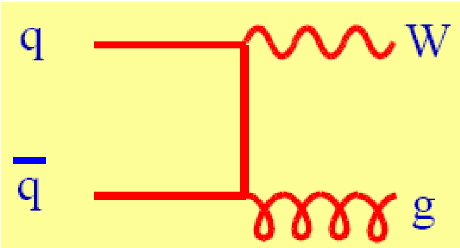
Showing the different theoretical regions in momentum space

Drell-Yan type subprocess



Photon can replaced by W, Z, Higgs, etc.

Leading double log contribution



LO Differential Q_T-distribution as $Q_T \to 0$:

$$\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\text{LO}} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F \left(\frac{\alpha_s}{\pi}\right) \frac{\ell n\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

$$\int_0^{Q^2} \left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\text{real+virutal}} dQ_T^2 \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} + O(\alpha_s) \qquad \text{with } Q^2 \approx M_W^2$$

Integrated Q_T-distribution:

$$\int_0^{Q_T^2} \left.\frac{d\sigma}{dy\,dp_T^2}\right|_{\text{real+virutal}} dp_T^2 \equiv \left[\int_0^{Q^2} - \int_{Q_T^2}^{Q^2}\right] \left.\frac{d\sigma}{dy\,dp_T^2}\right|_{\text{real+virutal}} dp_T^2$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \left[1 - \int_{Q_T^2}^{Q^2} 2C_F \frac{\alpha_s}{\pi} \frac{\ell n\left(Q^2/p_T^2\right)}{p_T^2} dp_T^2\right] = \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \left[1 - C_F \frac{\alpha_s}{\pi} \ell n^2\left(Q^2/Q_T^2\right)\right]$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \exp\left[-C_F \frac{\alpha_s}{\pi} \ell n^2\left(Q^2/Q_T^2\right)\right]$$

Effect of gluon emission

Assume this exponentiates

Resummed Q_T-distribution

- **Differentiate the integrated Q_T-distribution:**

$$\frac{d\sigma}{dy\,dQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ell n\left(Q^2/Q_T^2\right)}{Q_T^2} \times \exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ell n^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- **compare to the explicit LO calculation:**

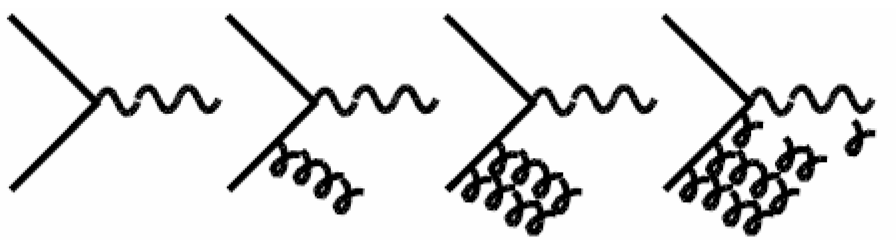
$$\frac{d\sigma}{dy\,dQ_T^2}_{\rm LO} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ell n\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

Q_T-spectrum (as $Q_T \to 0$) is completely changed!

- **We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms**

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2\,!} - \frac{(\alpha_s L^2)^3}{3\,!} + \ \ldots$$

$L \propto \ell n\left(Q^2/Q_T^2\right)$



Soft gluon emission treated as uncorrelated

Still a wrong Q_T-distribution

❑ **Experimental fact:** $\frac{d\sigma}{dy\,dQ_T^2} \Rightarrow$ finite [neither ∞ nor 0!] as $Q_T \to 0$

- Double Leading Logarithms Approximation (DLLA)

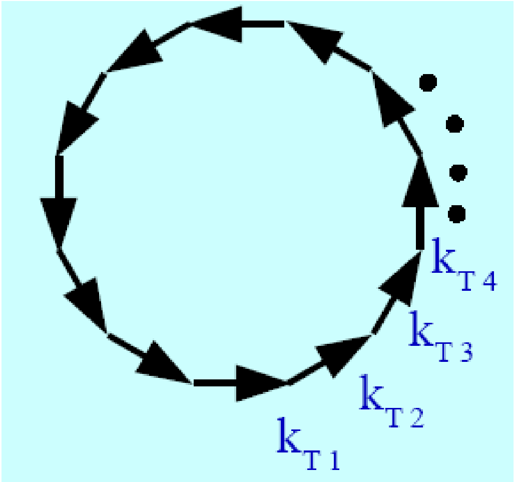
  radiated gluons are both soft and collinear with strong ordering in their transverse momenta

- Strong ordering in transverse momenta in DLLA
  - overly constrains the phase space ofthe emitted gluons
  - ignores the overall transverse momentum conservation

  $\Rightarrow$ DLLA over suppresses small Q_T region

Resummation of uncorrelated soft gluon emission leads to too strong suppression at Q_T=0

- **Why?**
  Particle can receive many finite k_T kicks via soft gluon radiation yet still have Q_T=0
  – Vector sum!

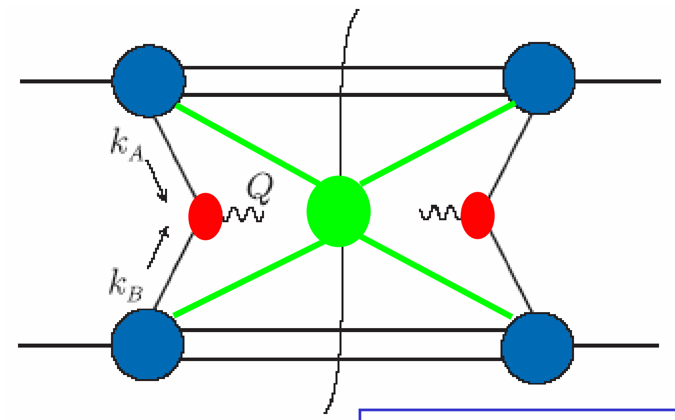


- **Subleading logarithms are equally important at Q_T=0**

- **Solution:**
  impose 4-momentum conservation at each step of soft gluon resummation

k_T-factorization and resummation

❑ Leading order K_T-factorized cross section:



$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2 k_{A_T} d^2 k_{B_T} d^2 k_{s,T}}{(2\pi)^6}$$
$$\times P_{f/A}(\xi_a, k_{A_T}) P_{\bar{f}/B}(\xi_b, k_{B_T}) H_{f\bar{f}}(Q^2) S(k_{s,T})$$
$$\times \delta^2(\vec{Q}_T - \vec{k}_{A_T} - \vec{k}_{B_T} - \vec{k}_{s,T})$$

$$\delta^2(\vec{Q}_T - \prod_i \vec{k}_{i,T}) = \frac{1}{(2\pi)^2} \int d^2 b \; \mathrm{e}^{i\vec{b}\cdot\vec{Q}_T} \prod_i \mathrm{e}^{-i\vec{b}\cdot\vec{k}_{i,T}}$$

❑ Factorized cross section in "impact parameter space":

$$\frac{d\sigma_{AB}(Q,b)}{dQ^2} = \sum_f \int d\xi_a d\xi_b \bar{P}_{f/A}(\xi_a, b, n) \bar{P}_{\bar{f}/B}(\xi_b, b, n) H_{f\bar{f}}(Q^2) U(b,n)$$

❑ Resummation:

Two equations, two resummation of log's

$$\mu_{\mathrm{ren}} \frac{d\sigma}{d\mu_{\mathrm{ren}}} = 0 \qquad\qquad n^\nu \frac{d\sigma}{dn^\nu} = 0$$

CSS b-space resummation formalism

❑ **Solve those two equations and transform back to Q_T:**

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T}\tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2,Q^2)$$

$$= \frac{1}{(2\pi)}\int_0^\infty db\, J_0(bQ_T)\, b\tilde{W}_{AB}(b,Q) + \left[\frac{d\sigma_{AB}^{(\text{Pert})}}{dQ^2 dQ_T^2} - \frac{d\sigma_{AB}^{(\text{Asym})}}{dQ^2 dQ_T^2}\right]$$

resummed

No large log's

❑ **b-space distribution:** $\tilde{W}_{AB}(b,Q) \equiv \sum_{i,j}\tilde{W}_{ij}(b,Q)\hat{\sigma}_{ij}(Q)$

The Q_T-distribution is determined by the b-space function:

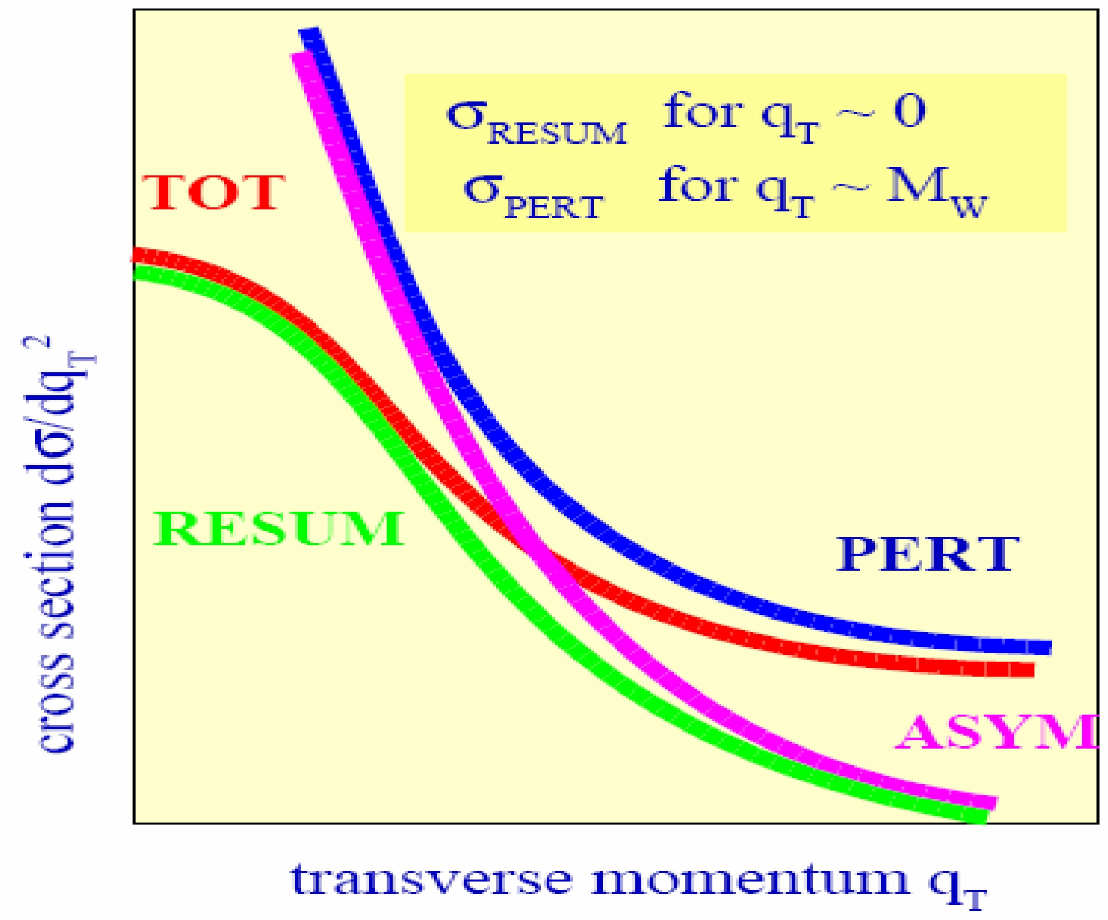
$$\frac{\partial}{\partial \ln Q^2}\tilde{W}_{ij}(b,Q) = [K(b\mu,\alpha_s) + G(Q/\mu,\alpha_s)]\,\tilde{W}_{ij}(b,Q) \quad (1)$$

$$\frac{\partial}{\partial \ln \mu^2}K(b\mu,\alpha_s) = -\frac{1}{2}\gamma_K(\alpha_s(\mu)) \quad (2)$$

$$\frac{\partial}{\partial \ln \mu^2}G(Q/\mu,\alpha_s) = \frac{1}{2}\gamma_K(\alpha_s(\mu)) \quad (3)$$

Role of each term in CSS formalism

$$\sigma_{TOT} = \sigma_{RESUM} + \sigma_{PERT} - \sigma_{ASYM}$$



The b-space resummation

- homogeneous evolution equation
  $\Rightarrow$ solution proportional to boundary condition

$$W_{ij}(b,Q) = W_{ij}(b,\frac{1}{b})\,\mathrm{e}^{-S_{ij}(b,Q)}$$

- if $b \ll 1/\Lambda_{\rm QCD}$, boundary condition $W_{ij}(b,1/b)$
  - depends only on one perturbative scale $\sim 1/b$
  - should be fully perturbative, and
  - have no large logarithms

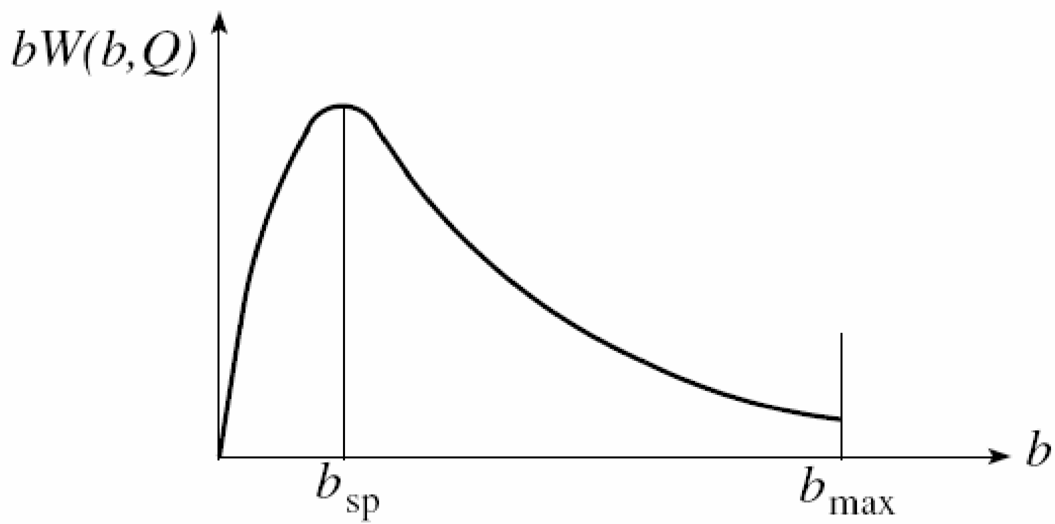
  $\Rightarrow$ perturbative b-distribution

$$W^{\rm pert}(b,Q) = \sum_{a,b,i,j} \sigma^{(LO)}_{ij\to C}\left[\phi_{a/A}\otimes C_{a\to i}\right] \otimes \left[\phi_{b/B}\otimes C_{b\to j}\right] \times \mathrm{e}^{-S(b,Q)}$$

❑ Sudakov form factor:

$$S(b,Q) = \int_{c^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A\left(\alpha_s\left(\mu^2\right)\right) \ell n \left(\frac{Q^2}{\mu^2}\right) + B\left(\alpha_s\left(\mu^2\right)\right) \right]$$

- all large logarithms are summed into $S(b,Q)$, and $S(b,Q)$ is perturbative for b not too large
- functions: $C_{a\to i}$ and $C_{b\to j}$ are perturbative



- Need non-perturbative input at large b:

Fourier transform back to Q_T-space

❑ **Nonperturbative input when b ~ 1/Λ_{QCD}:**

1) Work in Q_T-space directly to some approximation

   The originals: Dokshitzer, Diakanov & Troyan
   Revived by Ellis & Veseli Kulesza & Stirling
   who re-derived it from b-space.

2) Insert a "soft landing" on the k_T integral by replacing

$$1/b \to \sqrt{1/b^2 + 1/b_*^2}$$

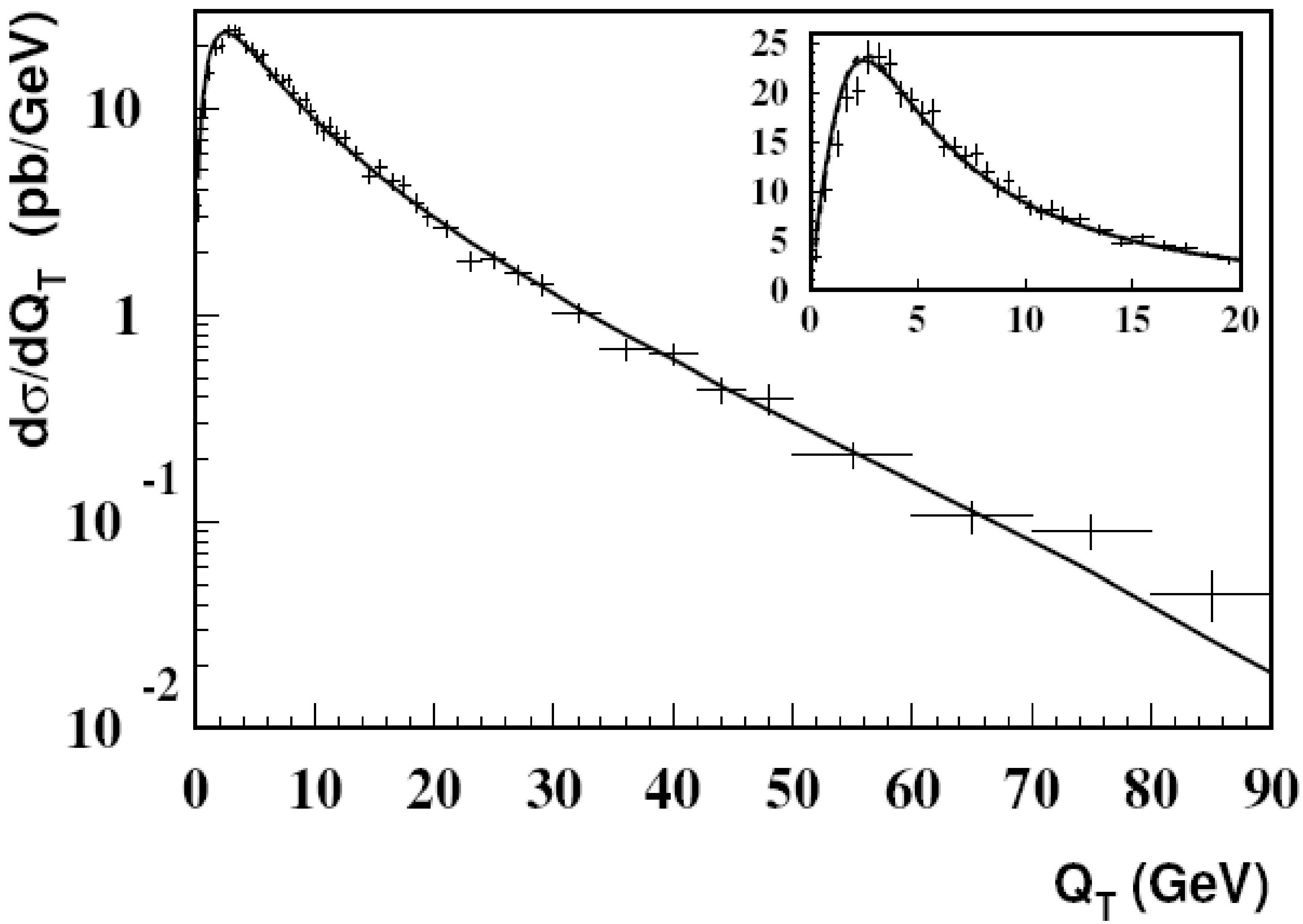
   for some fixed b_*. (CS, CSS "b_*" prescription, ResBos)

3) Extrapolation of E^{PT} into NP region (Qiu, Zhang)

4) Minimal: avoid the singularity at $1/b = \Lambda_{\mathrm{QCD}}$
   by monkeying with the b-space contour integral

   (This technique introduced in threshold resummation;
   then adapted by Laenen, GS and Vogelsang,
   and Bozzi, Catani, de Florian and Grazzini.)

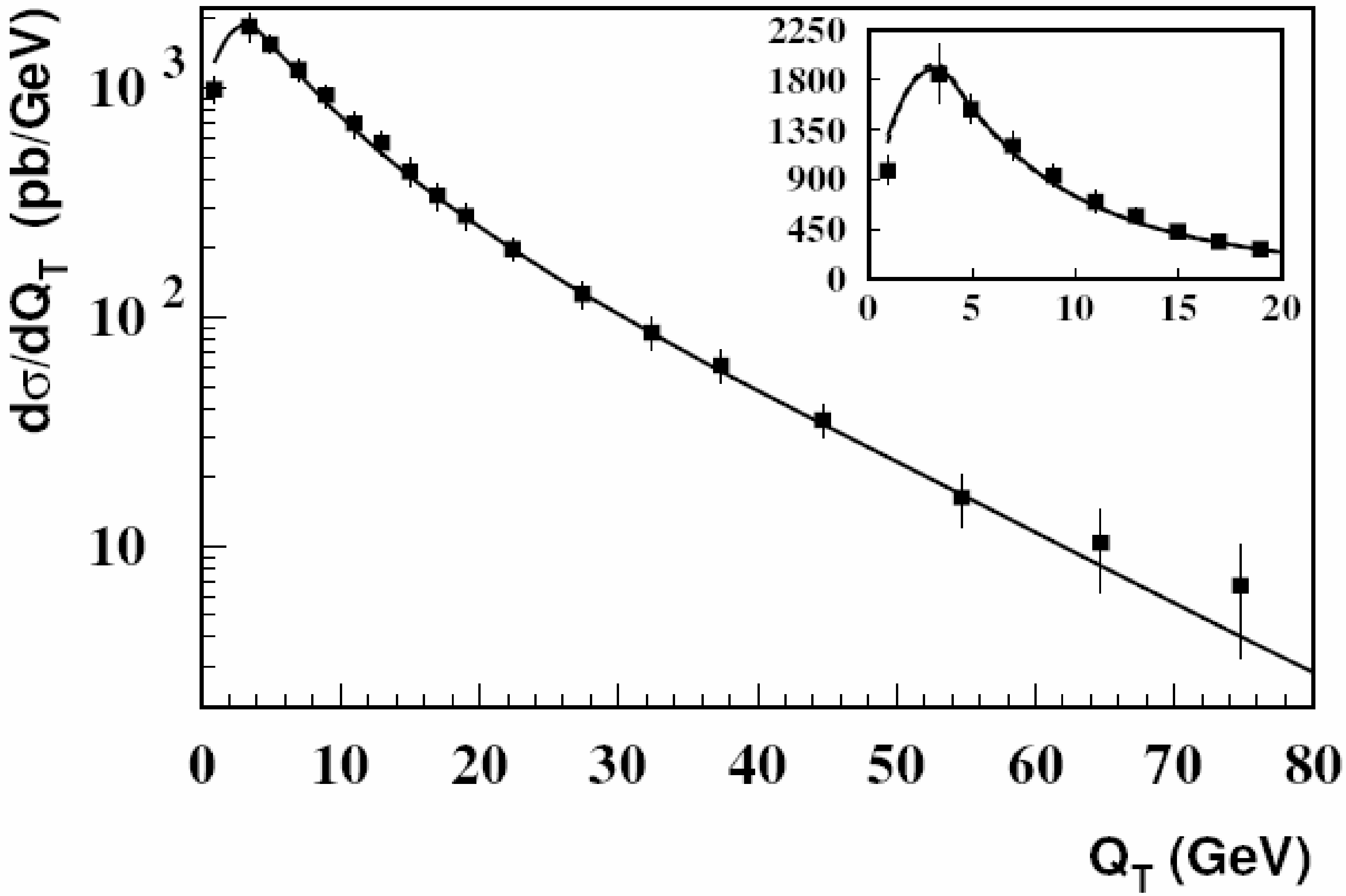
- Fermilab CDF data on Z at $\sqrt{S} = 1.8$ TeV



Power correction is very small, excellent prediction!

Qiu, Zhang, PRL 2001

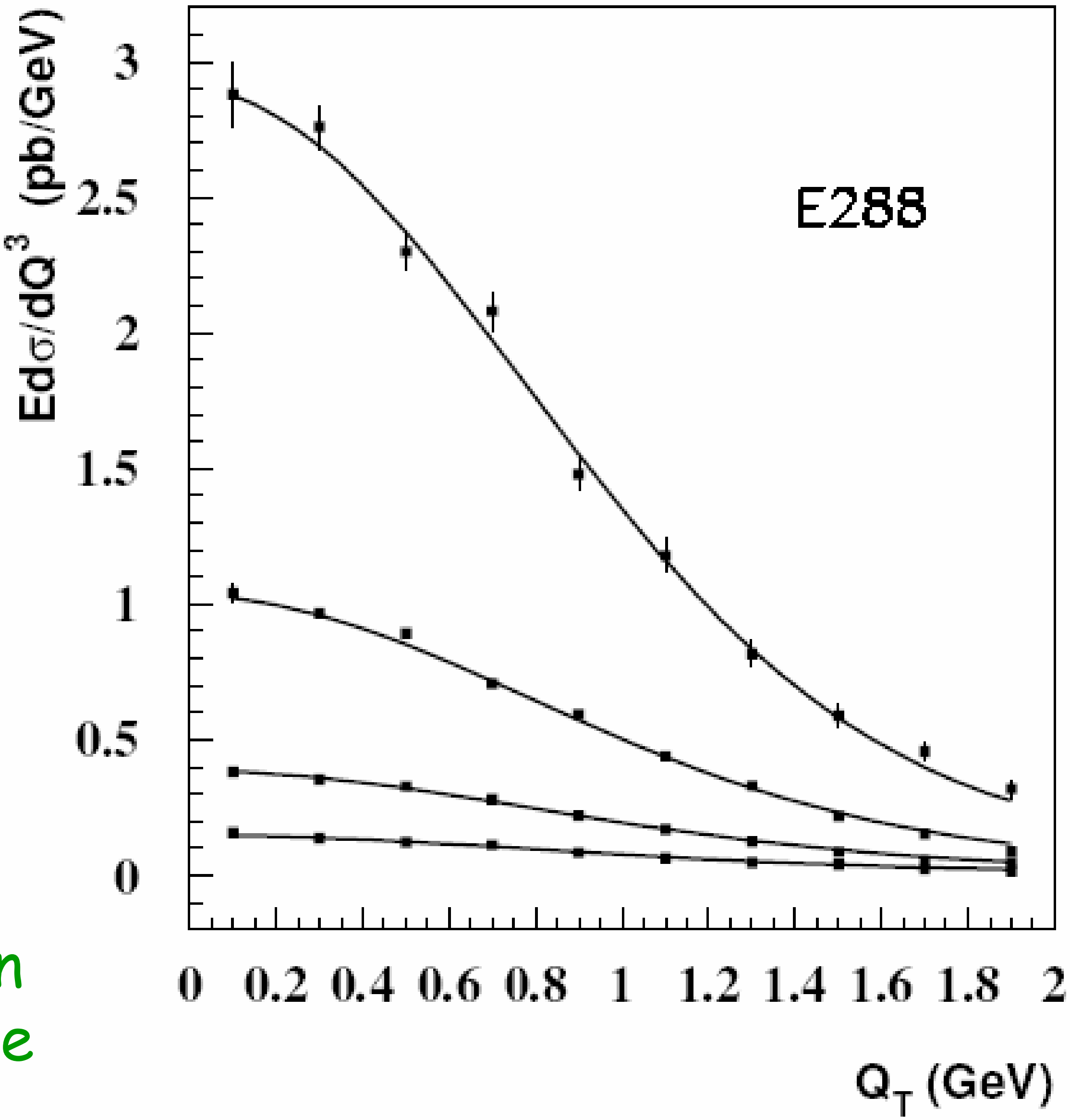
- Fermilab D0 data on W at $\sqrt{S} = 1.8$ TeV



No free fitting parameter!

Qiu, Zhang

- Fermilab E288 data at $p_{\text{beam}} = 400$ GeV



Power correction
is important here

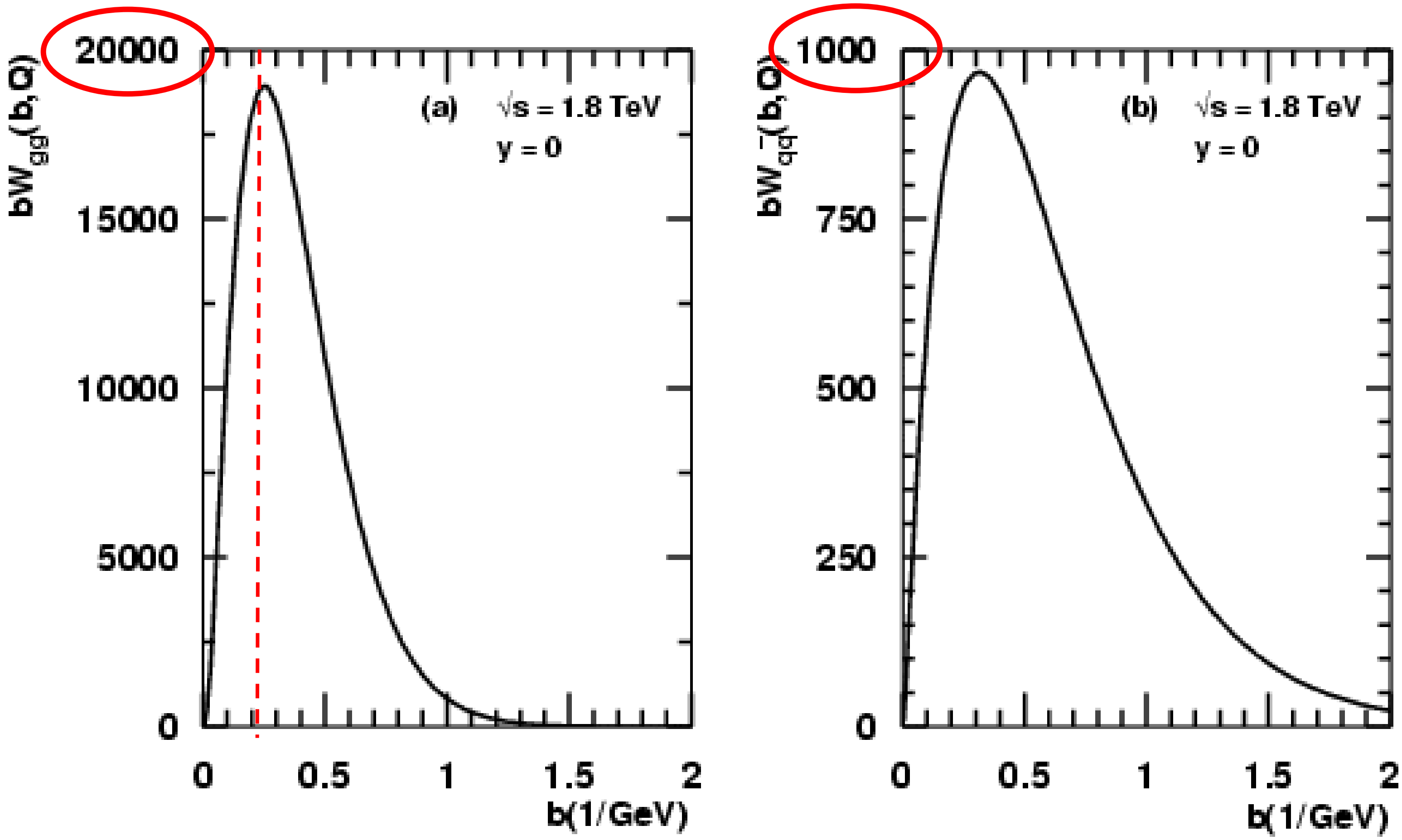
Hadronic Upsilon production

- **Process:** $A(p_A) + B(p_B) \to b\bar{b}(Q)[\to \Upsilon(p) + \bar{X}] + X'$
- **Similarities and differences from W/Z, or Higgs**
  - **Events are dominated by low Q_T region**
  - **Gluon shower should play an important role in determine the Q_T – distribution**

  - **M_Y << M_W , or Q is now small**
  - **Heavy b-quark pair is not necessary color singlet**
  - **Additional nonperturbative physics from b-quark to Upsilon**
- **Key approximation:**

  Neglect gluon radiation from heavy quarks

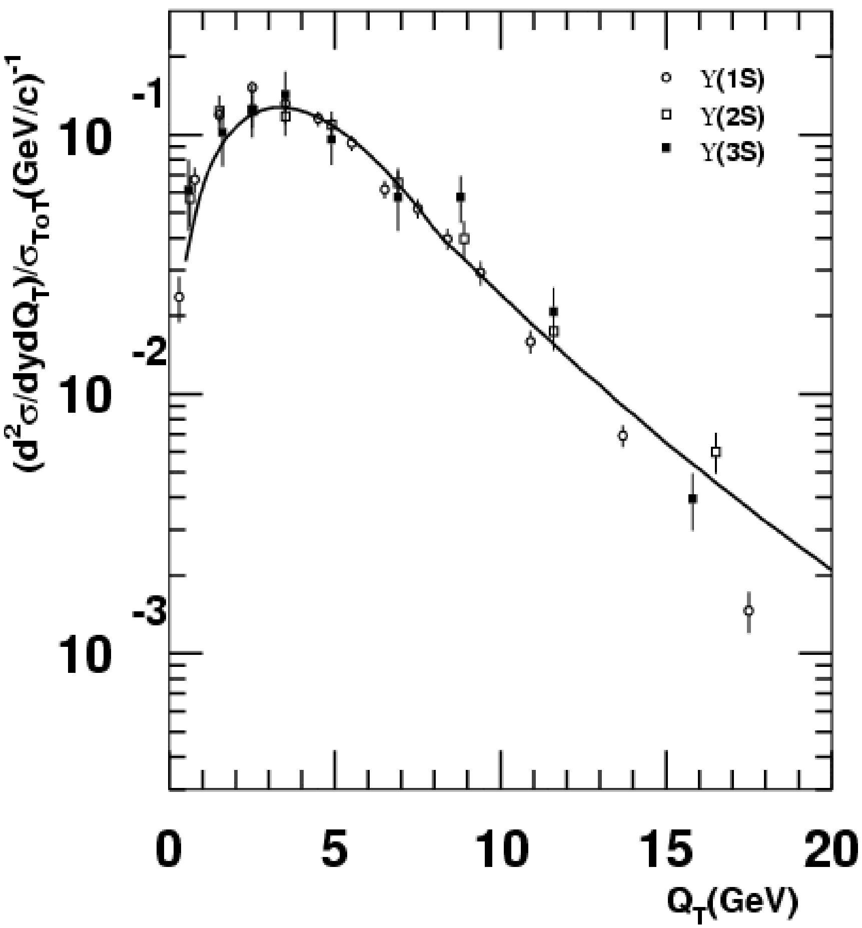
The b-space distribution



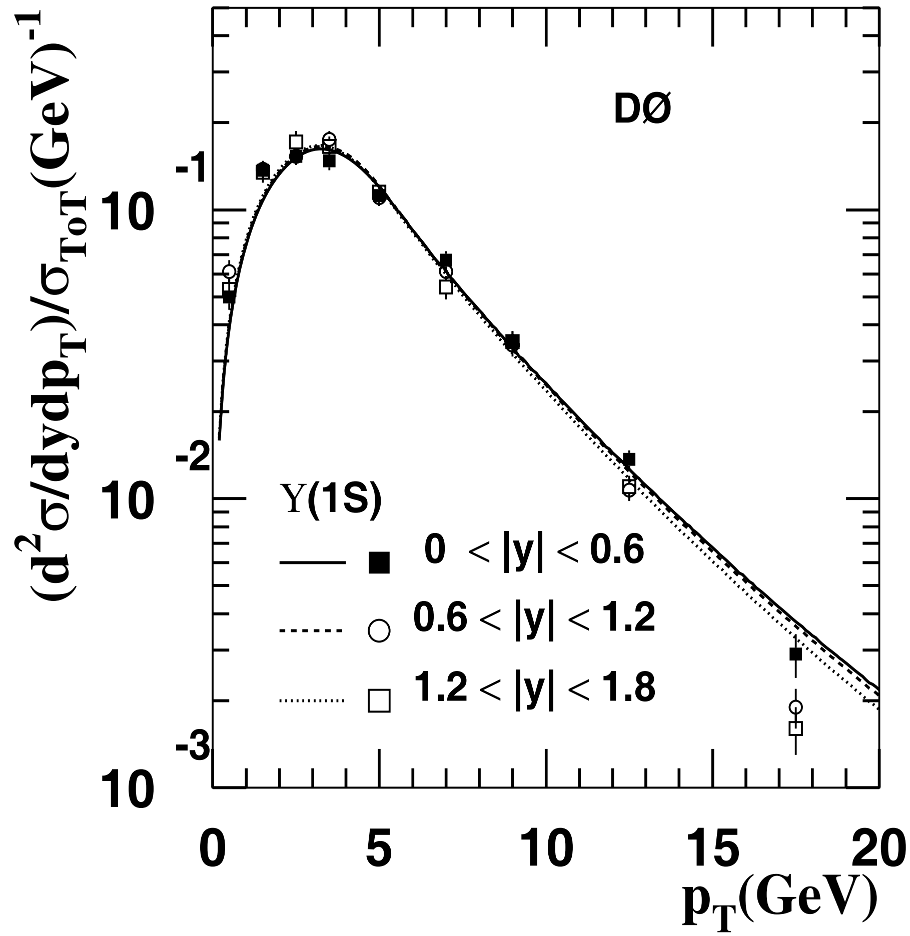
Gluon-gluon dominate the production

Dominated by perturbative contribution even M_Y~10 GeV

Upsilon production at Tevatron



CDF Run-I data



DO Run-II data

Summary

- Factorization for hadronic collision is more complicate
- Factorization works for Drell-Yan process
- Collinear factorization for hadronic single hadron production at high p_T is also valid
- k_T – factorization of DY allows to study processes with two different physical scales, and perform all order resummation
- k_T – factorization for jets in hadronic collisions is much more complicated

Collins and Qiu, 2007

Enjoy the rest of the school!